Chapter 4

Neutrino Mixing Model in the context of $\Delta(27) \times Z_3$ Flavor Symmetry using Type-I Seesaw Mechanism

4.1 Introduction

The accuracy of the Standard Model is astounding, and it has been incredibly successful in predicting a wide range of events that have been confirmed experimentally up to the current accelerator energy. Nevertheless, the theory fell short in explaining several observed phenomena as well as theoretical questions such as dark matter, matter-antimatter asymmetry, and non-zero neutrino masses, etc. In Standard Model (SM), neutrinos are found to be massless, however experimental data on neutrino oscillations clearly shows the massiveness of neutrinos. Till now one of the biggest unresolved problems in particle physics is the explanation of the quark and lepton families' origins and the observed pattern of their masses and mixing. These masses and mixing in the Standard Model are obtained via Yukawa couplings in the SM, which are not defined by the gauge symmetry. As such, they are arbitrary parameters, and to account for the observed fermion flavor structures, another kind of symmetry known as flavor symmetry is needed.

Two large mixing angles, the solar angle θ_{12} and the atmospheric angle θ_{23} along with the relatively small reactor mixing angle θ_{13} characterize the neutrino oscillation phenomenon. Important information on three mixing angles θ_{12}, θ_{23} and θ_{13}, one Dirac type CP phase δ_{CP}, and two mass squared differences $\Delta^2_{21} = m^2_2 - m^2_1$ and $\Delta^2_{31} = m^2_3 - m^2_1$ are provided by the three flavor neutrino oscillation framework. However, there are still a few unidentified parameters, such as the octant of θ_{23}, precision of CP-phase δ_{CP}, neutrino mass hierarchy, and Majorana phases if neutrinos are Majorana fermions. Long baseline neutrino oscillation experiments [1, 2] can be used to determine the CP-violating Dirac phase and neutrino mass hierarchy. Two more Majorana phases must be added to the neutrino mixing matrix if neutrinos turn out to be Majorana fermions. This encourages us to extend the SM paradigm to account for unsolved problems in particle physics, including non-zero neutrino masses.

Although the tribimaximal mixing (TBM) [3, 4] scheme has proven to be a useful approximation of the data and an important stimulus for model building, in TBM θ_{13} is zero and the CP phase δ_{CP} is not determined. However, in 2012, the RENO Experiment [5] and the Daya Bay Reactor Neutrino Experiment [6] showed that $\theta_{13} \simeq 9^\circ$. In addition, several other neutrino oscillation experiments such as Double Chooz [7], T2K [8], MINOS [9], have consistently measured non-zero values for θ_{13}. Moreover, neutrino oscillations caused by nonvanishing neutrino masses are clearly seen in experiments involving solar, atmospheric, and reactor neutrinos [5–10]. And furthermore different models in light of non-abelian discrete flavor symmetries [11] like A_4 [12–14], S_3 [15], S_4 [16, 17], $\Delta(27)$ [18–21], $\Delta(54)$ [22], $\Delta(96)$ [23–25], D_4 [26–28], Q_6 [29–31], T_7 [32–34] and so forth. have been proposed to get tribimaximal mixing (TBM) and deviation from TBM.

One well-known mechanism for explaining the small neutrino masses is the seesaw mechanism [36, 37]. Type-I, Type-II, and Type-III seesaw variations can be found in the SM's extension with certain heavy fields [38–44]. The elegant theory of leptogenesis [45] explains the matter-antimatter asymmetry seen in the universe.

Parameters	NH (3σ)	IH (3σ)
$\Delta m^2_{21}[10^{-5}eV^2]$	$6.82 \rightarrow 8.03$	$6.82 \rightarrow 8.03$
$\Delta m^2_{31}[10^{-3}eV^2]$	$2.428 \rightarrow 2.597$	$-2.581 \rightarrow -2.408$
$\sin^2\theta_{12}$	$0.270 \rightarrow 0.341$	$0.270 \rightarrow 0.341$
$\sin^2\theta_{13}$	$0.02029 \rightarrow 0.02391$	$0.02047 \rightarrow 0.02396$
$\sin^2\theta_{23}$	$0.405 \rightarrow 0.620$	$0.410 \rightarrow 0.623$
δ_{CP}	$105 \rightarrow 405$	$192 \rightarrow 361$

Table 4.1: The 3σ ranges of neutrino oscillation parameters from NuFIT 6.0 (2024) [35]

Using seesaw mechanisms to build a relationship between neutrino physics at low and high energy is one of its attractive qualities. Lepton asymmetry results from the CP-violating out-of-equilibrium decay of the heavy messenger particle mediating the seesaw mechanism. This can subsequently be transformed into the necessary baryon asymmetry via the non-perturbative Sphaleron [46] process.

A few experiments like KamLand-ZEN, GERDA, EXO, COURE, and SNO+ [47–51] are trying to observe the rare event known as neutrinoless double beta $0\nu\beta\beta$ decay,[52, 53] which can establish the Majorana nature of large neutrinos. The values of mixing angles and mass-squared differences and δ_{CP} from the global analysis of data are summarized in given in Table 4.1.

The content material of our work is organized as follows: In section 2, we give the overview of the framework of our model by specifying the fields involved and their transformation properties under the symmetries imposed. In section 3, we do a numerical analysis and study the results for the neutrino phenomenology. We finally conclude our work in section 4.

4.2 Framework of the Model

Here we provide a concise overview of how the non-Abelian discrete symmetry $\Delta(27)$ group can be represented [19]. $\Delta(27)$ is the simplest non-trivial discrete symmetry group of $\Delta(3N^2)$. The $\Delta(27)$ group has nine singlets $1_{r,s}(r, s = 0, 1, 2)$ and two triplets, $3_{[0][1]}$ and $3_{[0][2]}$. Tensor products between triplets are obtained as-

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{3[0][1]} \otimes \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{3[0][1]} = \begin{pmatrix} x_1y_1 \\ x_2y_2 \\ x_3y_3 \end{pmatrix}_{3[0][2]} \oplus \begin{pmatrix} x_3y_1 \\ x_1y_2 \\ x_2y_3 \end{pmatrix}_{3[0][2]} \oplus \begin{pmatrix} x_1y_3 \\ x_2y_1 \\ x_3y_2 \end{pmatrix}_{3[0][2]} \tag{4.1}$$

$$\begin{pmatrix} \overline{x_1} \\ \overline{x_2} \\ \overline{x_3} \end{pmatrix}_{3[0][2]} \otimes \begin{pmatrix} \overline{y_1} \\ \overline{y_2} \\ \overline{y_3} \end{pmatrix}_{3[0][2]} = \begin{pmatrix} \overline{x_1}.\overline{y_1} \\ \overline{x_2}.\overline{y_2} \\ \overline{x_3}.\overline{y_3} \end{pmatrix}_{3[0][1]} \oplus \begin{pmatrix} \overline{x_3}.\overline{y_1} \\ \overline{x_1}.\overline{y_2} \\ \overline{x_2}.\overline{y_3} \end{pmatrix}_{3[0][1]} \oplus \begin{pmatrix} \overline{x_1}.\overline{y_3} \\ \overline{x_2}.\overline{y_1} \\ \overline{x_3}.\overline{y_2} \end{pmatrix}_{3[0][1]} \tag{4.2}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{3[0][1]} \otimes \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{3[0][2]} = \sum_r \Big(x_1.y_1 + \omega^{2r}x_2y_2 + \omega^r x_3y_3\Big)_{1_{r,0}} \oplus$$
$$\sum_r \Big(x_1.y_2 + \omega^{2r}x_2y_3 + \omega^r x_3y_1\Big)_{1_{r,1}} \oplus \sum_r \Big(x_1.y_3 + \omega^{2r}x_2y_1 + \omega^r x_3y_2\Big)_{1_{r,2}} \oplus \tag{4.3}$$

The tensor products between singlets and triplets are obtained as-

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{3[0][1]} \otimes (Z_{r,s})_{1_{r,s}} = \begin{pmatrix} x_1 Z_{r,s} \\ \omega^r x_2 Z_{r,s} \\ \omega^{2r} x_3 Z_{r,s} \end{pmatrix}_{3[s][s+1]} \tag{4.4}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{3[0][2]} \otimes (Z_{r,s})_{1_{r,s}} = \begin{pmatrix} y_1 Z_{r,s} \\ \omega^r y_2 Z_{r,s} \\ \omega^{2r} y_3 Z_{r,s} \end{pmatrix}_{3[s][s+2]} \tag{4.5}$$

The tensor products of singlets $1_{k,l}$ and $1_{k',l'}$ are obtained as-

$$1_{k,l} \otimes 1_{k',l'} = 1_{k+k',l+l'} \tag{4.6}$$

Here ω is the cubic root of unity, $\omega = exp(i2\pi)$, so that $1 + \omega + \omega^2 = 0$.

Field	L	e^c	μ^c	τ^c	h_u	h_d	N^c	Φ_S	Φ_T	ξ	ξ'
SU(2)	2	1	1	1	2	2	1	1	1	1	1
$\Delta(27)$	$3_{0,2}$	$1_{0,0}$	$1_{0,2}$	$1_{0,1}$	$1_{0,2}$	$1_{0,0}$	$3_{0,1}$	$3_{0,1}$	$3_{0,1}$	$3_{0,1}$	$3_{0,1}$
Z_3	ω	ω	ω	ω	1	ω	ω^2	ω^2	1	ω^2	ω^2

Table 4.2: Full particle content of our model

The particle content and their charge assignment under the symmetry group is given in Table 4.2. The left-handed lepton doublets L and right-handed charged leptons (e^c, μ^c, τ^c) are assigned to triplet $3_{[0,2]}$ and singlet $(1_{0,0}, 1_{0,2}, 1_{1_{0,1}})$ representation under $\Delta(27)$ respectively and other particles transform as shown in Table 4.2. Here, h_u and h_d are the standard Higgs doublets which is $1_{0,2}$ and $1_{0,0}$ under $\Delta(27)$. The right-handed neutrino field N^c is assigned to the triplet $3_{[0,1]}$ representation under $\Delta(27)$ flavor symmetry. There are four $SU(2) \otimes U_Y(1)$ Higgs singlets, ξ, ξ', Φ_S and Φ_T which transform as triplet $3_{0,1}$ under $\Delta(27)$.

Consequently, the invariant Yukawa Lagrangian is as follows:

$$-\mathcal{L} = [y_e e^c (\Phi_T L)_{1_{0,0}} + y_\mu \mu^c (\Phi_T L)_{1_{0,1}} + y_\tau \tau^c (\Phi_T L)_{1_{1_{0,2}}}](\frac{h_d}{\Lambda}) + y(N^c L) h_u + X_A \xi (N^c N^c) + X_N \xi' (N^c N^c) + X_B \Phi_S (N^c N^c) + h.c. \quad (4.7)$$

The terms y_e, y_μ, y_τ, y, X_A, X_N and X_B are coupling constant and Λ is the cut-off scale of the theorys. We assume Φ_T does not couple to the Majorana mass matrix and Φ_S does not couple to the charged leptons. After spontaneous symmetry breaking of flavor and electroweak symmetry, we obtain the mass matrices for the charged leptons and neutrinos. We assume the vacuum alignment of $\langle \Phi_T \rangle = (v_T, 0, 0)$ and $\langle \Phi_S \rangle = (v_s, v_s, v_s)$. Also, v_u, v_d are the VEVs of $\langle h_u \rangle$, $\langle h_d \rangle$ and u and u' are the VEVs of $\langle \xi \rangle$ and $\langle \xi' \rangle$, respectively.

The charged lepton mass matrix is given as

$$M_l = \frac{v_d v_T}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \quad (4.8)$$

where, v_d and v_T are the VEVs of h_d and Φ_T respectively.

The structure of the Majorana neutrino mass matrix:

$$M_R = \begin{pmatrix} c & a & b \\ b & c & a \\ a & b & c \end{pmatrix} \tag{4.9}$$

The form of the Dirac mass matrix:

$$M_D = \begin{pmatrix} 0 & 0 & d \\ d & 0 & 0 \\ 0 & d & 0 \end{pmatrix} \tag{4.10}$$

Where, $a = X_A u$, $b = X_B v_s$, $c = X_N u'$ and $d = y v_u$ respectively.

In section 3, we give a detailed phenomenological analysis of various neutrino oscillation parameters. Further, we present a numerical study of neutrinoless double-beta decay considering the allowed parameter space of the model.

4.3 Numerical Analysis and results

The neutrino mass matrix m_ν can be diagonalized by the PMNS matrix U as

$$U^\dagger m_\nu U^* = \text{diag}(m_1, m_2, m_3) \tag{4.11}$$

.

We can numerically calculate U using the relation $U^\dagger h U = \text{diag}(m_1^2, m_2^2, m_3^2)$, where, $h = m_\nu m_\nu^\dagger$. The neutrino oscillation parameters solar mixing angle θ_{12}, reactor mixing angle θ_{13}, atmospheric mixing angle θ_{23} and CP-phase δ_{CP} can be obtained from U as

$$s_{12}^2 = \frac{|U_{12}|^2}{1-|U_{13}|^2}, \qquad s_{13}^2 = |U_{13}|^2, \qquad s_{23}^2 = \frac{|U_{23}|^2}{1-|U_{13}|^2}, \tag{4.12}$$

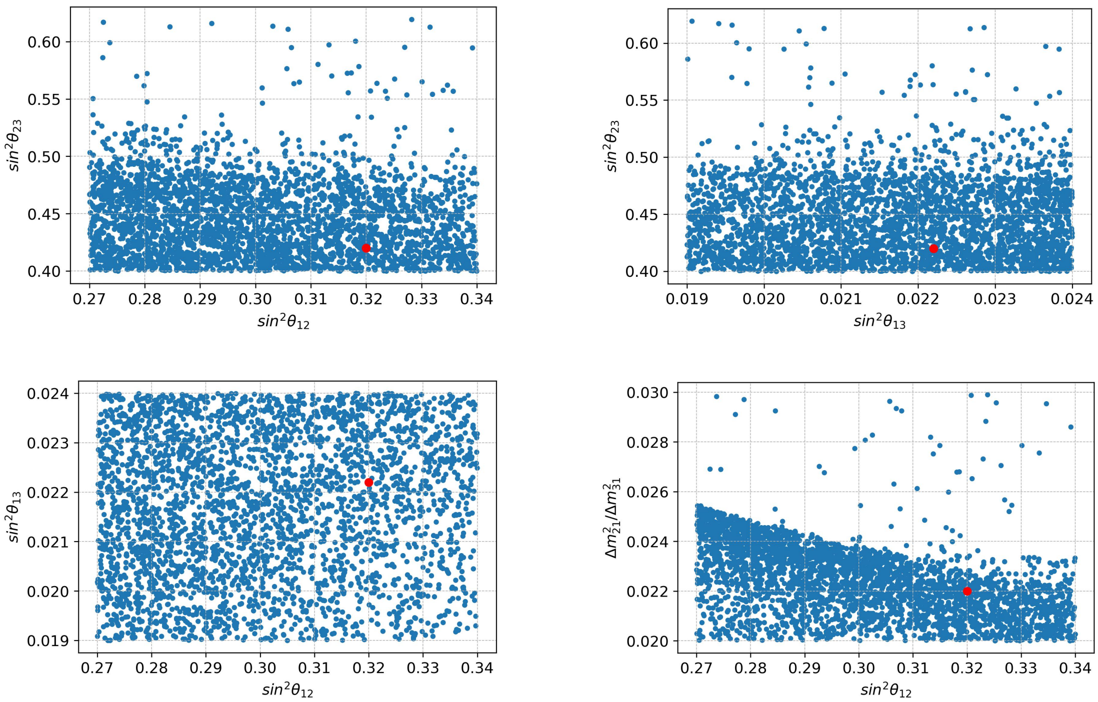

Figure 4.1: Correlation among the neutrino oscillation parameters $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$ and $\frac{\Delta m^2_{21}}{\Delta m^2 31}$

and δ may be given by

$$\delta = \sin^{-1}\left(\frac{8\,\mathrm{Im}(h_{12}h_{23}h_{31})}{P}\right) \tag{4.13}$$

with

$$P = (m_2^2 - m_1^2)(m_3^2 - m_2^2)(m_3^2 - m_1^2)\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13} \tag{4.14}$$

For the comparison of theoretical neutrino mixing parameters with the latest experimental data [35], the $\Delta(27)$ model is fitted to the experimental data by minimizing the following χ^2 function:

$$\chi^2 = \sum_i \left(\frac{\lambda_i^{model} - \lambda_i^{expt}}{\Delta\lambda_i}\right)^2. \tag{4.15}$$

where λ_i^{model} is the i^{th} observable predicted by the model, λ_i^{expt} stands for the i^{th} experimental best-fit value and $\Delta\lambda_i$ is the 1σ range of the i^{th} observable.

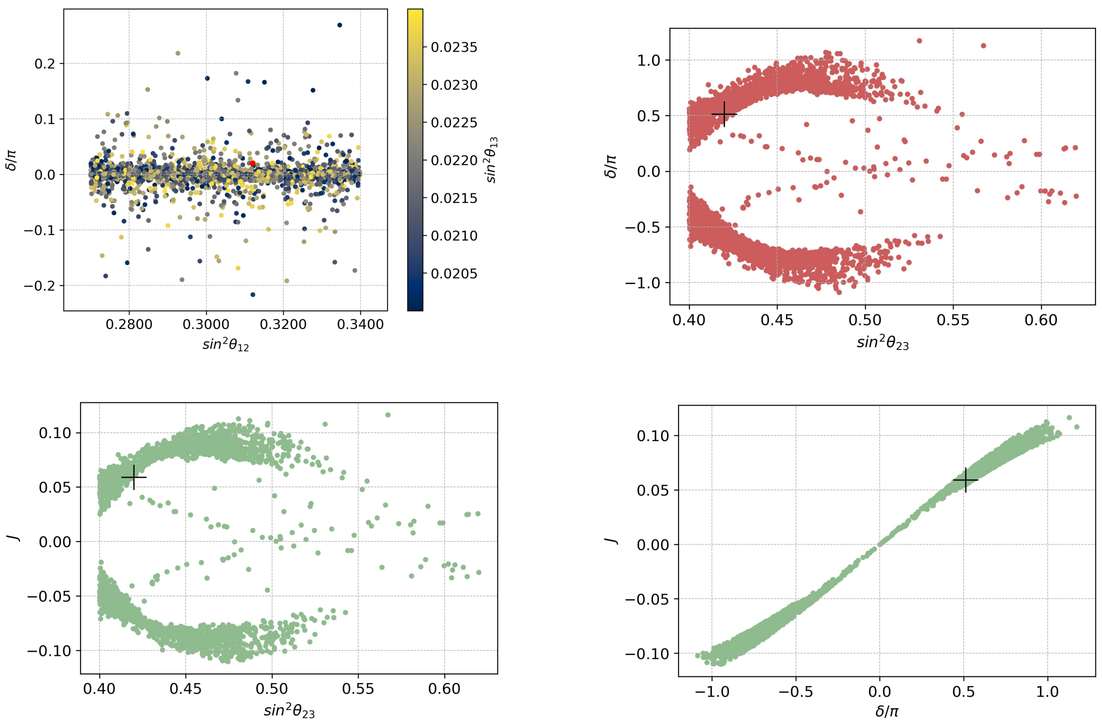

Figure 4.2: Correlation among the neutrino oscillation parameters $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$, Dirac CP phase and Jarlskog parameter.

In Fig. 4.1, correlation among the neutrino oscillation parameters $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\frac{\Delta m^2_{21}}{\Delta m^2_{31}}$ and $\sin^2\theta_{13}$ for NH has shown, which is constrained using the 3σ bound on neutrino oscillation data. Fig. 4.2 is the correlation among the neutrino oscillation parameters $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$, Dirac CP phase and Jarlskog parameter J for NH. We can see that there is a high correlation among different parameters of the model.

The calculated best-fit values of $\sin^2\theta_{12}$, $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ are (0.345, 0.0239, 0.551) which are within the 3 σ range of experimental values. Other parameters such as Δm^2_{21}, Δm^2_{31} and δ_{CP} have their best-fit values, corresponding to χ^2-minimum, at $(7.395 \times 10^{-5} eV^2,\ 2.59 \times 10^{-3} eV^2,\ -0.349\pi)$ respectively, which perfectly agreed with the latest observed neutrino oscillation experimental data. Thus, the model defined here clearly shows the deviation from exact tri-bimaximal mixing.

Neutrinoless double beta decay (NDBD):

Up until now, the question of whether neutrinos belong to the Dirac or Majo-

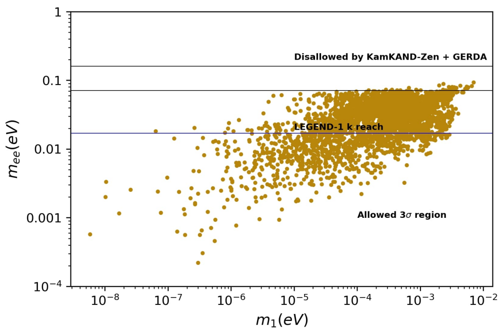

Figure 4.3: Variation of effective Majorana neutrino mass with lightest neutrino mass in NH with the KamLAND-Zen-Gerda bound on the effective mass.

rana category remains unanswered. If they are of the Majorana type, the investigation of Neutrinoless Double Beta Decay (NDBD) becomes highly significant. Several ongoing experiments are being conducted to ascertain the Majorana nature of neutrinos. The effective mass that controls this process is furnished by

$$m_{\beta\beta} = U_{Li}^2 m_i \tag{4.16}$$

where U_{Li} are the elements of the first row of the neutrino mixing matrix U_{PMNS} which is dependent on known parameters θ_{12}, θ_{13} and the unknown Majorana phases α and β. U_{PMNS} is the diagonalizing matrix of the light neutrino mass matrix m_ν so that,

$$m_\nu = U_{PMNS} M_\nu^{(diag)} U_{PMNS}^T \tag{4.17}$$

where, $m_\nu^{(diag)}$ =diag(m_1, m_2, m_3). The effective Majorana mass can be parameterized using the diagonalizing matrix elements and the mass eigenvalues as follows:

$$m_{\beta\beta} = m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta} \tag{4.18}$$

Using the constrained parameter space we have evaluated the value of $m_{\beta\beta}$ for our model. The variation of $m_{\beta\beta}$ with the lightest neutrino mass is shown in figure 4.3. The sensitivity reach of NDBD experiments like KamLAND-Zen [54], GERDA [55], LEGEND-1k [56] is also shown in figure 4.3. $m_{\beta\beta}$ is found to be well within the sensitivity reach of these NDBD experiments for our model.

4.4 Conclusions

We have developed a flavon-symmetric $\Delta(27) \times Z_3$ model using the seesaw Type-I mechanism. This model aims to explain the latest experimental data on neutrino oscillation, which shows a deviation from the Tribimaximal neutrino mixing pattern. To eliminate unwanted terms during the computations, the cyclic Z_3 symmetric component has been included. The calculated values clearly showed that the neutrino mixing parameters differ from the exact Tri-bimaximal neutrino mixing pattern. The generated mass matrices provide predictions for the neutrino oscillation parameters, and the χ^2 analysis is used to determine their best-fit values, which agree with the most recent experimental evidence on global neutrino oscillation data. In our model, we've delved into the idea of Neutrinoless Double Beta Decay (NDBD). The calculated value of the effective Majorana neutrino mass $|m_{\beta\beta}|$ aligns well with the capabilities of recent NDBD experiments such as KamLAND-Zen, GERDA, and LEGEND-1k, falling within their sensitivity range.

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