STUDIES ON BALITSKY-KOVCHEGOV EQUATION AND ITS APPLICATIONS TO VARIOUS PHENOMENA INSIDE HADRONS AT SMALL-X

A thesis submitted in partial fulfillment of the requirements for award of the degree of

Doctor of Philosophy

Ranjan Saikia

Registration No: TZ23001226 of 2023

Department of Physics

School of Sciences

Tezpur University

Napam, Tezpur-784028

Assam, India

November 2024

"All of physics is either impossible or trivial. It is impossible until you understand it, and then it becomes trivial."

— Ernest Rutherford

CHAPTER 5

CONCLUSION AND OUTLOOK

In this thesis, we conducted phenomenological studies of the nonlinear QCD evolution equation, or more precisely, the BK evolution equation, at high-density QCD within the framework of nonlinear processes inside hadrons at small x. To address the implicit physics of nonlinear phenomena at small-x, it is crucial to study nonlinear QCD evolution equations. The BK equation is one of the most suitable QCD evolution equations for small-x physics. This thesis examines the properties and applications of the BK equation at small-x, along with its importance. Although the BK equation has been studied numerically in great detail, thorough analytical research has not yet been carried out. Thus, in this thesis, we investigated the BK equation and attempted to solve it analytically. The achieved analytical solution is then applied to the gluon saturation region, where nonlinear effects are likely to play a significant role. In addition, we employed its solution to some of the phenomenological aspects at small-x.

In Chapter 2, we derived an approximate analytical solution to the BK evolution equation. The original BK problem is an integro-differential equation in coordinate space that may be converted into momentum space, yielding a partial differential equation. To obtain an approximation analytical solution to the BK equation, it is advisable to transform the equation into momentum space. The momentum space BK equation can be converted into the FKPP equation with certain variable transformations. The transition of the scattering amplitude into the saturation area resembles the creation of the wave front in the FKPP equation. We provided an analytical solution to the momentum space BK equation via the homotopy perturbation method in relation to the FKPP equation. We graphically represented the solution of the BK equation, specifically the dipole-proton scattering amplitude at different rapidities, and contrasted its characteristics with those forecasted by the numerical analysis of the BK equation. A similar nature of the scattering amplitude (travelling wave) with the scattering amplitude estimated by the numerical solution to the BK equation is found. This is a vital physical result of the travelling wave approach. In addition, saturation momentum is extracted from the obtained solution to check its nature as rapidity changes. The saturation momentum extracted in this work is compared with the saturation momentum predicted by the numerical analysis of the BK equation, showing a good agreement between results. It is found that the saturation momentum is an increasing function of the rapidity. It would be interesting to observe whether this nature of travelling wave solution and geometric scaling exists at very high energies when Electron-Ion Collider (EIC) and other future projects run operations. Nonetheless, the BK equation with truncation of the BFKL kernel successfully explains the observed geometric scaling and the travelling wave nature of its solution at current accelerator facilities. We must rely on future accelerator facilities for precise measurements of observed phenomena and their confirmation.

As a phenomenological application, the analytical solution provided in this work is used to measure DIS structure functions. Understanding the substructure of the nucleon has relied heavily on the structure functions of the nucleon. The DIS cross-section is associated with the structure functions of the nucleon in relation to parton distributions. The measurements of the proton's structure functions at the HERA

Collider facility have begun a new era of parton density measurements within the nucleon. The structure functions can be correlated with the momentum distributions of partons within the nucleon, and thus the parton distribution functions (PDFs). At small-x, gluons contribute the most significant contribution to proton structure functions, as gluon density dominates among partons. Therefore, the measurement of structure functions at small-x x is essential for calculating gluon distribution functions and visualizing the overall hadronic wave function in high-density QCD.

Chapter 3 employs the analytical solution to the BK equation to ascertain the gluon density at small-x and the proton structure function F_2^p within the color dipole framework of deep inelastic scattering in quantum chromodynamics. Under the constraints of the kinematic region: $10^{-5} \leq x \leq 10^{-2}$ and $2.5 \leq Q^2 \leq 60$ GeV^2, we computed the proton structure function F_2^p and compared the findings with the measurements of F_2^p from the H1 Collaboration at HERA and the LHAPDF global parameterization group NNPDF3.1sx. The anticipated outcomes align well within the limited kinematic domain. Additionally, the x evolution of the integrated gluon density $xg(x, Q^2)$ for various Q^2 values is obtained, and the findings are compared with the LHAPDF global data fits CT18 and NNPDF3.1sx. NNPDF3.1sx and CT18 incorporate HERA and LHC data in their analyses. This study demonstrates that our analytical solution of the BK equation effectively characterizes physics at small-x. Our results can be evaluated in forthcoming experimental facilities, such as the FCC-ch, EIC, and LHeC. The proton structure function will be measured with enhanced precision at far lower values of x in these facilities. We conclude that the analytical solution of the BK equation can serve as a valuable instrument for future investigations in small-x and high-density QCD. We expect that the BK equation, in conjunction with forthcoming experimental facilities, will enhance our understanding and exploration of events within hadrons at small-x in the near future.

Chapter 4 examines an additional phenomenological application of the analytical solution to the BK equation. We investigated vector meson production utilizing the color dipole framework of deep inelastic scattering. Exclusive vector meson production serves as a means to examine the nonlinear dynamics of quantum chromodynamics in deep inelastic scattering investigations. The H1 Collaboration at HERA and the LHCb Collaboration at CERN's LHC delivered high-precision data on the production of exclusive vector mesons at the highest energy recorded to date. In the color dipole framework of deep inelastic scattering, we examined the theoretical assessment of the differential and total cross sections for the production of J/Ψ and ρ^0 vector mesons, taking into account the different scales involved. Utilizing two established models, the Gaus-LC (Light Cone) and the BG (Boosted Gaussian), for the vector meson wave functions and deriving the dipole-proton scattering amplitude from the resolution of the BK equation, we calculated the cross-sections for the production of J/Ψ and ρ^0 vector mesons, as well as the ratios of longitudinal to transverse cross-sections. We evaluated the theoretical predictions for the cross-sections of J/Ψ and ρ^0 vector meson production against current experimental data, observing a satisfactory concordance. Our analytical solution to the BK equation accurately calculates vector meson production in a low Q^2 range.

The BK equation performs well in low Q^2 and small-x domains. However, for high Q^2, corrections from the DGLAP evolution equation are necessary and should be incorporated into the calculations. Nevertheless, our analytical solution remains valid for measuring the production of vector meson cross-sections within the specified range. Moreover, our results highlight the sensitivity of outcomes to the Gaus-LC and BG models. While differences between these models are less noticeable for the ρ^0 meson as Q^2 increases, distinctions are evident for the J/Ψ meson even at high Q^2. Additionally, it is observed that the longitudinal to transverse cross-section ratios for the production of J/Ψ and ρ^0 vector mesons reveal a significant dependency on the two wave function models as Q^2 increases. Our findings indicate that the results from the BG with BK solution offer a superior representation of the data compared to those from the Gaus-LC with BK solution. In conclusion, exclusive vector meson production serves as an exceptional technique to investigate the characteristics of nuclear materials. The cross-section is proportional to the proton's parton distribution functions. Our analytical solution to the BK equation can facilitate more phenomenological investigations at current and forthcoming experimental facilities. We expect that diffractive processes, such as exclusive vector meson production at upcoming experimental facilities, will illuminate various facets of QCD.

Thus, in this thesis, we looked at nonlinear physics at small-x using BK evolution theory. We demonstrate the utilization of the BK evolution framework for phenomenological applications. As a future perspective, our findings motivate more extensive research in the high gluon density region using more current experimental data in which nonlinear effects play a crucial part. The planned LHeC and EIC programs will undoubtedly provide a great chance for particle physicists to investigate the smallest x regimes. In this thesis, we dealt with the BK equation containing the BFKL kernel with leading logarithmic x (LLx) accuracy. However, higher-order corrections to the BK equation have already been evaluated up to NLLx accuracy. The BK equation in LLx is inherently a complex nonlinear equation, and it becomes even more complicated when we consider the NLLx BK equation. Furthermore, it is unfortunate that the physical ramifications of NLLx BK equation are not widely understood right now. Nonetheless, LLx approximations perform reasonably well with HERA energies. However, for ultra-high-energy future colliders, it would be intriguing to include higher twist effects and investigate the phenomenological implications.