This thesis is dedicated to my parents

For their endless love, support and encouragement

DECLARATION

I hereby declare that the thesis entitled ***"Studies on Balitsky-Kovchegov equation and its applications to various phenomena inside hadrons at small-x"*** being submitted to Tezpur University, Tezpur, Assam in partial fullfillment of the requirements for the award of the degree of Doctor of Philosophy, has previously not formed the basis for the award of any degree, diploma, associateship, fellowship or any other similar title or recongnition.

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Date: 09/04/2025

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CERTIFICATE

This is to certify that the thesis entitled ***"Studies on Balitsky-Kovchegov equation and its applications to various phenomena inside hadrons at small-x"*** submitted to the School of Sciences, Tezpur University in partial fulfillment for the award of the degree of Doctor of Philosophy in Physics is a record of research work carried out by Mr. Ranjan Saikia under my supervision and guidance.

All help received by him from various sources have been duly acknowledged. No part of this thesis has been submitted elsewhere for award of any other degree/diploma.

Signature of the Supervisor: Jayanta Kumar Sarma

Name of the Supervisor: Dr. Jayanta Kumar Sarma

Designation: Professor

School: Sciences

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Date: 9.4.2025

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