ABSTRACT

Small-x physics and its implications for high-density quantum chromodynamics (QCD) are among the most fascinating areas in high-energy particle physics. Understanding small-x physics is crucial for understanding the entirety of hadronic wavefunctions at high energy. Significant theoretical and experimental endeavors have been undertaken to comprehend the implicit consequences of small-x physics, ranging from deep inelastic scattering (DIS) experiments at the Hadron Electron Ring Accelerator (HERA) to heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). The most significant characteristic in the high-energy small-x region ($x \leq 10^{-2}$) is the enhancement of parton density, as evidenced by HERA's data on proton structure functions. The behavior of parton densities at small-x is an intriguing aspect of high-energy QCD. Small-x physics elucidates the parton evolution framework, where parton density significantly increases as x approaches zero, leading to nonlinear phenomena like parton density saturation. However, gluons are predominant among partons within hadrons at small-x. Consequently, since the gluon distribution dictates the physics at small-x, determining gluon distribution is essential. A thorough understanding of gluon distribution is crucial in evaluating backgrounds and uncovering new physics at the LHC and other experimental facilities.

QCD evolution equations determine the parton distribution functions (PDFs) and the DIS structure functions in QCD phenomenology. The two well-known QCD evolution equations, DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) and BFKL (Balitsky-Fadin-Kuraev-Lipatov), provide the framework for studying PDFs on their respective kinematic domains in perturbative QCD (pQCD). For the evolution of Q^2 (virtuality of photons), the DGLAP equation is used. Since it sums up higher-order α_s contributions enhanced by the leading logarithmic powers of $\ln Q^2$, it is valid for large Q^2. The BFKL approach, in contrast to DGLAP evolution,

is more effective in the small-x and semi-hard Q^2 kinematic region, where it becomes crucial to sum the leading logarithmic contributions of $(\alpha_s \ln 1/x)^n$.

Since the DGLAP and BFKL evolution equations are linear, they are not appropriate for small-x nonlinear physics. These equations do not take into consideration the underlying physics in high-density QCD at small-x since they use solely the Feynman diagrams for gluon emission, ignoring gluon fusion scenarios completely. Furthermore, it is found that neither of the linear evolution equations can satisfy the unitarity bound nor the conventional Froissart bound. A formulation of QCD needs to slow down the gluons' rapid growth in order to preserve unitarity. This eventually leads to the nonlinear phenomena of gluon recombination and saturation at small-x. In this context, a number of nonlinear evolution equations based on the BFKL and DGLAP evolutions have been developed recently to address the nonlinear phenomena in high-density QCD at small-x.

Gribov, Levin, and Ryskin (GLR) and Muller and Qiu (MQ) performed the first pQCD calculations that took into account gluon recombination. They proposed a new nonlinear evolution equation with an additional nonlinear quadratic in the gluon density. This new nonlinear evolution equation, also known as the GLR-MQ equation, is an updated version of the DGLAP equation with some corrections for gluon recombination. There are several other nonlinear evolution equations that use nonlinear corrections to the DGLAP and BFKL equations. For example, Modified DGLAP (MD-DGLAP) is a DGLAP-based nonlinear equation, whereas Balitsky-Kovchegov (BK), Modified-BFKL (MD-BFKL), and Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov (JIMWLK) are BFKL-based nonlinear evolution equations. The BK equation is among the most thoroughly studied evolution equations in the context of gluon saturation at small-x, which is a mean-field approximation of the more complex JIMWLK equation.

Despite numerous numerical explorations of the BK equation, its complexity still eludes an accurate analytical solution. The necessity for a reliable analytical solution to an issue that is otherwise CPU (Central Processing Unit)-intensive and technically difficult to solve is essential. Thus, analytical studies of the BK equation are essential to comprehending nonlinear physics at small-x. As a result, in this PhD thesis, we investigated and presented an analytical solution to the BK equation, as well as some phenomenological implications for various phenomena inside hadrons at small-x. The thesis is organized in the following chapters:

Chapter 1 gives an overview of basic particle physics. It talks briefly about the building blocks of matter, quantum chromodynamics (QCD), deep inelastic scattering (DIS), parton distribution functions (PDFs), nucleon structure functions, and parton evolution equations. Furthermore, this chapter provides a brief overview of small-x physics and discusses nonlinear processes like gluon recombination and saturation. The importance of nonlinear parton evolution equations is explained in this chapter.

In chapter 2, we investigate the BK evolution equation, which is a critical nonlinear parton evolution equation for small-x. We presented an approximate analytical solution to the BK problem using the homotopy perturbation method (HPM). Based on the resulting solution, we extracted the gluon saturation momentum and compared it to the numerical analysis of the BK equation.

In chapter 3, we investigate the proton structure function $F_2^p(x, Q^2)$ at small-x using the analytical solution of the BK equation. In the color dipole description of DIS, the structure function $F_2^p(x, Q^2)$ is determined using the analytical expression for the scattering amplitude $N(k, Y)$ obtained from the solution of the BK equation. Furthermore, we have extracted the integrated gluon density $xg(x, Q^2)$ using the BK solution and compared it to the LHAPDF global data fits, NNPDF3.1sx and CT18. This chapter provides the behavior of $F_2^p(x, Q^2)$ in the kinematic area of $10^{-5} \leq x \leq 10^{-2}$ and $2.5 \leq Q^2 \leq 60$ GeV^2. The presented results for $F_2^p(x, Q^2)$ within the specific kinematic region are compared with the recent high-precision data for $F_2^p(x, Q^2)$ from HERA (H1 collaboration) and the LHAPDF global parametrization group NNPDF3.1sx.

In chapter 4, we investigate the theoretical prediction of exclusive vector meson production. Employing the color dipole description of DIS for vector meson production, we have calculated both the differential cross-section and total cross-section of J/Ψ and ρ^0 vector mesons, employing the analytical solution of the BK equation. Furthermore, we have presented the ratio of the longitudinal to the transverse cross-section for J/Ψ and ρ^0 vector mesons as a function of Q^2. We have integrated two well-known vector meson wave function models, Boosted Gaussian and Gaus-LC (Light-Cone), into our analysis, demonstrating slight sensitivity to the selected vector meson wave functions. We compared our theoretical predictions with the existing experimental data for vector meson production.

In chapter 5, we summarize and outline our conclusions as well as possible future prospects.