Appendix A

$\Delta(54)$ group

The irreducible representations of $\Delta(54)$ follow the following Kronecker products. The tensor products of the non-trivial singlets with other representations are obtained as

$$1_1 \otimes S_i = S_i, \qquad 1_2 \otimes 1_2 = 1_1, \qquad 1_2 \otimes 3_{1(1)} = 3_{2(1)}$$

$$1_2 \otimes 3_{1(2)} = 3_{2(2)}, \qquad 1_2 \otimes 3_{2(1)} = 3_{1(1)}, \qquad 1_2 \otimes 3_{2(2)} = 3_{1(2)}$$

The tensor products between doublets are obtained as

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{2_s} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}_{2_s} = \begin{pmatrix} a_1b_2 + a_2b_1 \end{pmatrix}_{1_1} \oplus \begin{pmatrix} a_1b_2 - a_2b_1 \end{pmatrix}_{1_2} \oplus \begin{pmatrix} a_2b_2 \\ a_1b_1 \end{pmatrix}_{2_s}$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{2_1} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}_{2_2} = \begin{pmatrix} a_2b_2 \\ a_1b_1 \end{pmatrix}_{2_3} \oplus \begin{pmatrix} a_2b_1 \\ a_1b_2 \end{pmatrix}_{2_4}$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{2_1} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}_{2_3} = \begin{pmatrix} a_2b_2 \\ a_1b_1 \end{pmatrix}_{2_2} \oplus \begin{pmatrix} a_2b_1 \\ a_1b_2 \end{pmatrix}_{2_4}$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{2_1} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}_{2_4} = \begin{pmatrix} a_1b_2 \\ a_2b_1 \end{pmatrix}_{2_2} \oplus \begin{pmatrix} a_1b_1 \\ a_2b_2 \end{pmatrix}_{2_3}$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{2_2} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}_{2_3} = \begin{pmatrix} a_2 b_2 \\ a_1 b_1 \end{pmatrix}_{2_1} \oplus \begin{pmatrix} a_1 b_2 \\ a_2 b_1 \end{pmatrix}_{2_4}$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{2_2} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}_{2_4} = \begin{pmatrix} a_1 b_1 \\ a_2 b_2 \end{pmatrix}_{2_1} \oplus \begin{pmatrix} a_1 b_2 \\ a_2 b_1 \end{pmatrix}_{2_3}$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{2_3} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}_{2_4} = \begin{pmatrix} a_1 b_2 \\ a_2 b_1 \end{pmatrix}_{2_1} \oplus \begin{pmatrix} a_1 b_1 \\ a_2 b_2 \end{pmatrix}_{2_2}$$

The tensor products between triplets are obtained as

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_{1(1)}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_{1(1)}} = \begin{pmatrix} a_1 b_1 \\ a_2 b_2 \\ a_3 b_3 \end{pmatrix}_{3_{1(2)}} \oplus \begin{pmatrix} a_2 b_3 + a_3 b_2 \\ a_3 b_1 + a_1 b_3 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}_{3_{1(2)}} \oplus \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}_{3_{2(2)}}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_{1(2)}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_{1(2)}} = \begin{pmatrix} a_1 b_1 \\ a_2 b_2 \\ a_3 b_3 \end{pmatrix}_{3_{1(1)}} \oplus \begin{pmatrix} a_2 b_3 + a_3 b_2 \\ a_3 b_1 + a_1 b_3 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}_{3_{1(1)}} \oplus \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}_{3_{2(1)}}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_{2(1)}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_{2(1)}} = \begin{pmatrix} a_1 b_1 \\ a_2 b_2 \\ a_3 b_3 \end{pmatrix}_{3_{1(2)}} \oplus \begin{pmatrix} a_2 b_3 + a_3 b_2 \\ a_3 b_1 + a_1 b_3 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}_{3_{1(2)}} \oplus \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}_{3_{2(2)}}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_{2(2)}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_{2(2)}} = \begin{pmatrix} a_1 b_1 \\ a_2 b_2 \\ a_3 b_3 \end{pmatrix}_{3_{1(1)}} \oplus \begin{pmatrix} a_2 b_3 + a_3 b_2 \\ a_3 b_1 + a_1 b_3 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}_{3_{1(1)}} \oplus \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}_{3_{2(1)}}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_{1(1)}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_{1(2)}} = \begin{pmatrix} a_1b_1 + a_2b_2 + a_3b_3 \end{pmatrix}_{1_1} \oplus \begin{pmatrix} a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3 \\ \omega a_1b_1 + \omega^2 a_2b_2 + a_3b_3 \end{pmatrix}_{2_1}$$
$$\oplus \begin{pmatrix} a_1b_2 + \omega^2 a_2b_3 + \omega a_3b_1 \\ \omega a_1b_3 + \omega^2 a_2b_1 + a_3b_2 \end{pmatrix}_{2_2} \oplus \begin{pmatrix} a_1b_3 + \omega^2 a_2b_1 + \omega a_3b_2 \\ \omega a_1b_2 + \omega^2 a_2b_3 + \omega a_3b_1 \end{pmatrix}_{2_3}$$
$$\oplus \begin{pmatrix} a_1b_3 + a_2b_1 + a_3b_2 \\ a_1b_2 + a_2b_3 + a_3b_1 \end{pmatrix}_{2_4}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_{1(1)}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_{2(1)}} = \begin{pmatrix} a_1b_1 \\ a_2b_2 \\ a_3b_3 \end{pmatrix}_{3_{2(2)}} \oplus \begin{pmatrix} a_3b_2 - a_2b_3 \\ a_1b_3 - a_3b_1 \\ a_2b_1 - a_1b_2 \end{pmatrix}_{3_{1(2)}} \oplus \begin{pmatrix} a_3b_2 + a_2b_3 \\ a_1b_3 + a_3b_1 \\ a_2b_1 + a_1b_2 \end{pmatrix}_{3_{2(2)}}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_{1(1)}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_{2(2)}} = \begin{pmatrix} a_1b_1 + a_2b_2 + a_3b_3 \end{pmatrix}_{1_2} \oplus \begin{pmatrix} a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3 \\ -\omega a_1b_1 - \omega^2 a_2b_2 - a_3b_3 \end{pmatrix}_{2_1}$$
$$\oplus \begin{pmatrix} a_1b_2 + \omega^2 a_2b_3 + \omega a_3b_1 \\ -\omega a_1b_3 - \omega^2 a_2b_1 - a_3b_2 \end{pmatrix}_{2_2} \oplus \begin{pmatrix} a_1b_3 + \omega^2 a_2b_1 + \omega a_3b_2 \\ -\omega a_1b_2 - \omega^2 a_2b_3 - a_3b_1 \end{pmatrix}_{2_3}$$
$$\oplus \begin{pmatrix} a_1b_3 + a_2b_1 + a_3b_2 \\ -a_1b_2 - a_2b_3 - a_3b_1 \end{pmatrix}_{2_4}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_{1(2)}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_{2(1)}} = \begin{pmatrix} a_1b_1 + a_2b_2 + a_3b_3 \end{pmatrix}_{1_2} \oplus \begin{pmatrix} a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3 \\ -\omega a_1b_1 - \omega^2 a_2b_2 - a_3b_3 \end{pmatrix}_{2_1}$$

$$\oplus \begin{pmatrix} a_1b_2 + \omega^2 a_2b_3 + \omega a_3b_1 \\ -\omega a_1b_3 - \omega^2 a_2b_1 - a_3b_2 \end{pmatrix}_{2_2} \oplus \begin{pmatrix} a_1b_3 + \omega^2 a_2b_1 + \omega a_3b_2 \\ -\omega a_1b_2 - \omega^2 a_2b_3 - a_3b_1 \end{pmatrix}_{2_3}$$

$$\oplus \begin{pmatrix} a_1b_3 + a_2b_1 + a_3b_2 \\ -a_1b_2 - a_2b_3 - a_3b_1 \end{pmatrix}_{2_4}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_{1(2)}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_{2(2)}} = \begin{pmatrix} a_1b_1 \\ a_2b_2 \\ a_3b_3 \end{pmatrix}_{3_{2(1)}} \oplus \begin{pmatrix} a_3b_2 - a_2b_3 \\ a_1b_3 - a_3b_1 \\ a_2b_1 - a_1b_2 \end{pmatrix}_{3_{1(1)}} \oplus \begin{pmatrix} a_3b_2 + a_2b_3 \\ a_1b_3 + a_3b_1 \\ a_2b_1 + a_1b_2 \end{pmatrix}_{3_{2(1)}}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_{2(1)}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_{2(2)}} = \begin{pmatrix} a_1b_1 + a_2b_2 + a_3b_3 \end{pmatrix}_{1_1} \oplus \begin{pmatrix} a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3 \\ \omega a_1b_1 + \omega^2 a_2b_2 + a_3b_3 \end{pmatrix}_{2_1}$$

$$\oplus \begin{pmatrix} a_1b_2 + \omega^2 a_2b_3 + \omega a_3b_1 \\ \omega a_1b_3 + \omega^2 a_2b_1 + a_3b_2 \end{pmatrix}_{2_2} \oplus \begin{pmatrix} a_1b_3 + \omega^2 a_2b_1 + \omega a_3b_2 \\ \omega a_1b_2 + \omega^2 a_2b_3 + a_3b_1 \end{pmatrix}_{2_3}$$

$$\oplus \begin{pmatrix} a_1b_3 + a_2b_1 + a_3b_2 \\ a_1b_2 + a_2b_3 + a_3b_1 \end{pmatrix}_{2_4}$$

Appendix B

The Scalar Sector of the Model

The scalar potential of the model in Chapter-3 is written such that it is invariant under the symmetry $\Delta(54) \otimes Z_2 \otimes Z_3 \otimes Z_4$ and has the following form,

The invariant superpotential is given by

$$\begin{aligned} w = {} & \mu_1\zeta^2 + \mu_2\phi^4 + \mu_3\chi^2 \\ & + \beta_1\chi_1'\chi_2' + \beta_1'(\chi_1'^2 + \chi_2'^2) \\ & + \alpha_2(\eta_1^6 + \eta_2^6 + \eta_3^6) + \alpha_2'(\eta_1^2\eta_2^2\eta_3^2) \\ & + \alpha_3(\xi_1^6 + \xi_2^6 + \xi_3^6) + \alpha_3'(\xi_1^2\xi_2^2\xi_3^2) \\ & + \alpha_4(\Phi_{s1}^6 + \Phi_{s2}^6 + \Phi_{s3}^6) + \alpha_4'(\Phi_{s1}^2\Phi_{s2}^2\Phi_{s3}^2) \end{aligned}$$

which leads to the scalar potential

$$\begin{aligned} V = {} & |2\mu_1\zeta|^2 + |4\mu_2\phi^3|^2 + |2\mu_3\chi|^2 \\ & + |\beta_1\chi_2' + 2\beta_1'\chi_1'|^2 + |\beta_1\chi_1' + 2\beta_1'\chi_2'|^2 \\ & + |6\alpha_2\eta_1^5 + 2\alpha_2'\eta_1\eta_2^2\eta_3^2|^2 + |6\alpha_2\eta_2^5 + 2\alpha_2'\eta_2\eta_1^2\eta_3^2|^2 + |6\alpha_2\eta_3^5 + 2\alpha_2'\eta_3\eta_1^2\eta_2^2|^2 \\ & + |6\alpha_3\xi_1^5 + 2\alpha_3'\xi_1\xi_2^2\xi_3^2|^2 + |6\alpha_3\xi_2^5 + 2\alpha_3'\xi_2\xi_1^2\xi_3^2|^2 + |6\alpha_3\xi_3^5 + 2\alpha_3'\xi_3\xi_1^2\xi_2^2|^2 \\ & + |6\alpha_4\Phi_{s1}^5 + 2\alpha_4'\Phi_{s1}\Phi_{s2}^2\Phi_{s3}^2|^2 + |6\alpha_4\Phi_{s2}^5 + 2\alpha_4'\Phi_{s2}\Phi_{s1}^2\Phi_{s3}^2|^2 + |6\alpha_4\Phi_{s3}^5 + 2\alpha_4'\Phi_{s3}\Phi_{s1}^2\Phi_{s2}^2|^2 \end{aligned}$$

The conditions of the potential minimum are written as

$$2\mu_1\zeta = 0$$
$$4\mu_2\phi^3 = 0$$
$$2\mu_3\chi = 0$$
$$\beta_1\chi'_2 + 2\beta'_1\chi'_1 = 0; \qquad \beta_1\chi'_1 + 2\beta'_1\chi'_2 = 0$$
$$6\alpha_2\eta_1^5 + 2\alpha'_2\eta_1\eta_2^2\eta_3^2 = 0; \qquad 6\alpha_2\eta_2^5 + 2\alpha'_2\eta_2\eta_1^2\eta_3^2 = 0; \qquad 6\alpha_2\eta_3^5 + 2\alpha'_2\eta_3\eta_1^2\eta_2^2 = 0$$
$$6\alpha_3\xi_1^5 + 2\alpha'_3\xi_1\xi_2^2\xi_3^2 = 0; \qquad 6\alpha_3\xi_2^5 + 2\alpha'_3\xi_2\xi_1^2\xi_3^2 = 0; \qquad 6\alpha_3\xi_3^5 + 2\alpha'_3\xi_3\xi_1^2\xi_2^2 = 0$$
$$6\alpha_4\Phi_{s1}^5 + 2\alpha'_4\Phi_{s1}\Phi_{s2}^2\Phi_{s3}^2 = 0; \qquad 6\alpha_4\Phi_{s2}^5 + 2\alpha'_4\Phi_{s2}\Phi_{s1}^2\Phi_{s3}^2 = 0; \qquad 6\alpha_4\Phi_{s3}^5 + 2\alpha'_4\Phi_{s3}\Phi_{s1}^2\Phi_{s2}^2 = 0$$

A solution of these equations is

$$\chi'_1 = \chi'_2 \quad \text{with} \quad \beta_1 + 2\beta'_1 = 0$$
$$\eta_1 = \eta_2 = \eta_3 \quad \text{with} \quad 3\alpha_2 + \alpha'_2 = 0$$
$$\xi_1 = \xi_2 = \xi_3 \quad \text{with} \quad 3\alpha_3 + \alpha'_3 = 0$$
$$\Phi_{s1} = \Phi_{s2} = \Phi_{s3} \quad \text{with} \quad 3\alpha_4 + \alpha'_4 = 0$$

Therefore we can take Vacuum alignment as

$$\langle\chi'\rangle = (v_{\chi'}, v_{\chi'})$$
$$\langle\eta\rangle = (v_\eta, v_\eta, v_\eta)$$
$$\langle\xi\rangle = (v_\xi, v_\xi, v_\xi)$$
$$\langle\Phi_s\rangle = (v_s, v_s, v_s)$$

List of Publications

Journals

1. **Bora, H.**, Francis, Ng. K., Barman, A., and Thapa, B. Neutrino mass model in the context of $\Delta(54) \times Z_2 \times Z_3 \times Z_4$ flavour symmetries with inverse seesaw mechanism. *Physics Letters B*, **848**, 138329 (2023).

2. **Bora, H.**, Francis, Ng. K., Barman, A., and Thapa, B. Majorana neutrinos in Double Inverse Seesaw and $\Delta(54)$ flavour models. *International Journal of Modern Physics A*, **39**, 2450066 (2024).

3. **Bora, H.**, Francis, Ng. K., Barman, A., and Thapa, B. Neutrino Mixing and Resonant Leptogenesis in Inverse Seesaw and $\Delta(54)$ Flavor Symmetry. *arXiv: 2402.18906* (2024) (Under Review)

4. **Bora, H.**, Francis, Ng. K., Barman, A., and Thapa, B. Relic Abundance of Dark Matter with $\Delta(54)$ Flavor Symmetry. *Journal of Subatomic Particles and Cosmology*, **1**, 100011 (2024)

5. Barman, A., Francis, Ng. K., **Bora, H**. Neutrino mixing phenomenology: discrete flavor symmetry with type-I seesaw mechanism. *Modern Physics Letters A*, **39(07)**, 2350200 (2024).

Book Chapters

1. **Bora, H**., Francis, Ng. K. $\Delta(54)$ model for Dirac neutrinos: Inverse Seesaw. *Recent Trends in Physics Research*, ISBN: 978-93-90951-66-6.

2. Barman, A., Francis, Ng. K. and **Bora, H**. Study on Minimal modification of Tri-bimaximal Neutrino Mixing Matrix and its Phenomenological Implication. *Recent Trends in Physics Research*, ISBN: 978-93-90951-66-6.

Posters/ Oral Presentations/ Papers Presented in Conferences

1. H. Bora, Ng. K. Francis, **XIII Biennial National Conference of Physics Academy of North East (PANE-2022)**, Department of Physics, Manipur University, Manipur, November 8-10, 2022.
2. H. Bora, Ng. K. Francis, **National Conference on Symposium on Physics: Advances in Research and Knowledge (SPARK) 2023**, Department of Physics, North Lakhimpur College, Octuber 14, 2023.
3. H. Bora, Ng. K. Francis, **Physics Frontiers–2024: National Conference on Bridging Theories And Experiments**, Department of Physics, Bhawanipur Anchalik College, March 2-4, 2024.
4. H. Bora, Ng. K. Francis, **International Conference on Future Prospects In Neutrino And Astroparticle Physics (ICFPNAP-2024)**, Department of Physics, Assam Don Bosco University, November 23-24, 2024.

Workshops/ Schools Attended

1. *Workshop on Neutron Scattering (Elastic and Inelastic) and Muon Spectroscopy*, Indian Institute of Technology Guwahati, Febuary 16-17, 2024.