Chapter 3

Neutrino mass model in the context of $\Delta(54)$ flavor symmetries with Inverse Seesaw mechanism

Our analysis involves enhancing the $\Delta(54)$ flavor symmetry model with Inverse Seesaw mechanism along with two SM Higgs through the incorporation of distinct flavons. Additionally, we introduce supplementary $Z_2 \otimes Z_3 \otimes Z_4$ symmetries to eliminate any undesirable components within our investigation. The exact tribimaximal neutrino mixing pattern undergoes a deviation as a result of the incorporation of extra flavons, leading to the emergence of a non-zero reactor angle θ_{13} that aligns with the latest experimental findings. It was found that for our model the atmospheric oscillation parameter occupies the lower octant for normal hierarchy case. We also examine the parameter space of the model for normal hierarchy to explore the Dirac CP (δ_{CP}), Jarlskog invariant parameter (J) and the Neutrinoless double-beta decay parameter ($m_{\beta\beta}$) and found it in agreement with the neutrino latest data. Hence our model may be testable in the future neutrino experiments.

3.1 Introduction

Despite the fact that the standard model (SM) has been verified by various particle physics experiments and observations, there are still several unresolved questions. These unanswered questions include, but are not limited to, the source of flavor structure, the matter-antimatter imbalance in the universe, the existence of additional neutrino flavors, and the enigmatic properties of dark matter and dark energy. The detection of neutrino oscillations in 1998 marked the first sign of physics beyond the standard model. However, many uncertainties still surround neutrino physics, such as whether the symmetry related to leptonic CP has been broken, what the lightest absolute neutrino mass value is, and whether the atmospheric mixing angle is maximal or not. Additionally, it remains unclear whether the neutrino masses follow the normal hierarchy ($m_1 < m_2 < m_3$) or the inverted hierarchy ($m_3 < m_1 < m_2$). It is currently unknown whether neutrinos are Dirac particles or Majorana particles that violate lepton number. Neutrinoless double beta decay experiments, if observed, could confirm that neutrinos are indeed Majorana particles. The combined results of the KamLAND-Zen and GERDA experiments have established an upper limit for m_{ee} within the range of 0.071–0.161 eV. Several recent reviews on neutrino physics can be found in references[1–15].

The study of neutrino oscillation involves three mixing angles, two of which are large (the solar angle and the atmospheric angle) and one of which is relatively small (the reactor angle). In the context of tribimaximal mixing (TBM), the reactor angle is assumed to be zero and the CP phase cannot be determined or is not known. However, experiments such as the Daya Bay Reactor Neutrino Experiment ($sin^2 2\theta_{13} = 0.089 \pm 0.010 \pm 0.005$)[16] and RENO Experiment($sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$) [17]have shown that the reactor mixing angle is not zero. Other experiments such as MINOS [18], Double Chooz[19], and T2K[20] have also measured consistent nonzero values for the reactor mixing angle, rendering the TBM model unrealistic. As a result, alternative models or modifications must be considered to achieve realistic mixing[17, 19]. The PMNS matrix, which is generally acknowledged, represents the blending between the various neutrino flavor states and their corresponding mass eigenstates. Within the context of a three-flavored model, the PMNS matrix is characterized by three mixing angles and three CP phases.

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\iota\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\iota\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\iota\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\iota\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\iota\delta} & c_{23}c_{13} \end{pmatrix} \cdot U_{Maj} \quad (3.1)$$

where, $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$.The diagonal matrix $U_{Maj} = diag(1, e^{\iota\alpha}, e^{\iota(\beta+\gamma)})$ holds the Majorana CP phases, α and β, which become detectable if neutrinos act as Majorana particles. Discovering neutrinoless double beta decay is likely the key to proving that neutrinos are Majorana particles, but these decays are yet to be observed. Symmetry is an essential factor in explaining this issue, as Wendell Furry proposed the Majorana nature of particles and studied a kinetic process similar to neutrinoless double beta decay[21, 22]. By producing a pair of electrons, this process breaks the lepton number by two units, generating Majorana neutrino masses as the electroweak symmetry breaks $(A, Z) \rightarrow (A, Z + 2) + 2e^-$. The large value of the lepton number violation scale($\Lambda \sim 10^{14} - 10^{15}$GeV) is associated with the smallness of observed neutrino masses since they are zero in the standard model[23]. To generate non-zero neutrino mass, it is necessary to develop a new model beyond the standard model, such as effective theories that use the Weinberg operator[24].

Several alternative frameworks to the standard model exist that can explain the origin of neutrino masses. These include the Seesaw Mechanism [25–28], the Minimal Supersymmetric Standard Model (MSSM)[29],Supersymmetry[30], the Next-to-Minimal Supersymmetric Standard Model (NMSSM)[31], string theory[32], Radiative Seesaw Mechanism[33], models based on extra dimensions[34], and various other models. Numerous neutrino experiments have demonstrated the presence of tiny but non-zero neutrino masses and provided evidence of flavor mixing[35, 36].Several researchers have proposed different patterns of lepton mixing. Phenomenological patterns of neutrino mixing include Trimaximal (TM1/TM2)[37–39], Tri-bimaximal (TBM)[40, 41], Bi-large mixing patterns[42–44] and Quasidegenerate neutrino mass models. Additionally, several models based on non-abelian discrete symmetries[45] such as A_4[33, 46–48], S_3[33], S_4, $\Delta(27)$[49–52] and $\Delta(54)$[53–55] have been suggested to achieve tribimaximal mixing (TBM), and deviations from TBM are obtained by adding extra flavons.

This study introduces a new approach, the inverse seesaw, using the $\Delta(54)$ flavour symmetry framework. The $\Delta(54)$ symmetry can manifest itself in heterotic string models on factorizable orbifolds, such as the T^2/Z_3 orbifold. In these string models, only singlets and triplets are observed as fundamental modes, while doublets are absent as fundamental modes. However, doublets have the potential to become fundamental modes in magnetized/intersecting D-brane models. We can also suggest an extension to the Standard Model, utilizing $\Delta(54)$ symmetry. We have the option to engage with both the singlets $(1_1, 1_2)$ and doublets $(2_1, 2_2, 2_3, 2_4)$ representations of $\Delta(54)$, which allow us to represent quarks in different ways. This extension effectively incorporates the most recent experimental data for various properties within the quark sector, encompassing six quark masses, three quark mixing angles, and the CP-violating phase in this sector [56]. A previous proposal by the authors of [57] suggested Type I and inverse seesaw for Dirac neutrinos with A4 flavour symmetry. Lately, endeavors have been made in this particular area, particularly concerning the type I seesaw, type II seesaw and type I+II for Dirac neutrinos within the $\Delta(54)$ symmetry [58].The current work presents a more concise version of the inverse seesaw, utilizing two SM Higgs. To deviate from the specific TBM neutrino mixing pattern, we incorporated extra flavons χ, χ', η, ζ, ξ,Φ_S and ϕ under $\Delta(54)$ and expanded the flavon sector of the model. In addition, we incorporated a $Z_2 \otimes Z_3 \otimes Z_4$ symmetry into our model to prevent unwanted terms and facilitate the construction of specific coupling matrices. We modified the structure of the neutrino mass matrix M_ν that characterizes

Field	Q_{1L}	$Q_{\alpha L}$	$u_{\alpha R}$	u_{1R}	d_{1R}	$d_{\alpha R}$	H	ϕ
$\Delta(54)$	1_+	2_2	1_+	2_2	1_+	2_2	1_+	2_2
$U(1)$	1/6	1/6	2/3	2/3	$-1/3$	$-1/3$	1/2	1/2

Parameters	NH (3σ)	IH (3σ)
$\Delta m^2_{21}[10^{-5}eV^2]$	$6.82 \rightarrow 8.03$	$6.82 \rightarrow 8.03$
$\Delta m^2_{31}[10^{-3}eV^2]$	$2.428 \rightarrow 2.597$	$-2.581 \rightarrow -2.408$
$\sin^2\theta_{12}$	$0.270 \rightarrow 0.341$	$0.270 \rightarrow 0.341$
$\sin^2\theta_{13}$	$0.02029 \rightarrow 0.02391$	$0.02047 \rightarrow 0.02396$
$\sin^2\theta_{23}$	$0.406 \rightarrow 0.620$	$0.410 \rightarrow 0.623$
δ_{CP}	$108° \rightarrow 404°$	$192° \rightarrow 360°$

Table 3.1: The 3σ range of neutrino oscillation parameters from NuFIT 5.2 (2022) [59]

the masses of neutrinos and conducted symmetry-based studies. This approach allows us to thoroughly examine M_ν, and investigate the Jarlskog invariant parameter (J) and the NDBD parameter (m_{ee}). The approach we take sets our work apart from that of others.

Our chapter structure is arranged as follows: Section 3.2 presents an outline of the model's framework, including the fields and their symmetrical transformation properties. Section 3.3 contains a numerical analysis and examination of the neutrino phenomenology results. Lastly, in section 3.4, we offer our concluding remarks.

3.2 Model Framework

To achieve the realization of the Inverse seesaw mechanism, it is necessary to expand the fermion sector of the Standard Model. We introduced Vector like (VL) fermions N and S which are all Standard Model gauge singlets. The left and right-handed fields are distinguished with a subscript 1 and 2 respectively. We introduced ν_R which is a right handed neutrino. It gives us a Dirac mass matrix in the Lagrangian. The $\Delta(54)$ group includes irreducible representations 1_1, 1_2, 2_1, 2_2, 2_3, 2_4, $3_{1(1)}$, $3_{1(2)}$, $3_{2(1)}$ and $3_{2(2)}$. The product rules and Clebsch–Gordan coefficients are presented in Appendix A. The multiplication rules are as follows:

$$3_{1(1)} \otimes 3_{1(1)} = 3_{1(2)} \oplus 3_{1(2)} \oplus 3_{2(2)}$$

$$3_{1(2)} \otimes 3_{1(2)} = 3_{1(1)} \oplus 3_{1(1)} \oplus 3_{2(1)}$$

$$3_{2(1)} \otimes 3_{2(1)} = 3_{1(2)} \oplus 3_{1(2)} \oplus 3_{2(2)}$$

$$3_{2(2)} \otimes 3_{2(2)} = 3_{1(1)} \oplus 3_{1(1)} \oplus 3_{2(1)}$$

$$3_{1(1)} \otimes 3_{1(2)} = 1_1 \oplus 2_1 \oplus 2_2 \oplus 2_3 \oplus 2_4$$

$$3_{1(2)} \otimes 3_{2(1)} = 1_2 \oplus 2_1 \oplus 2_2 \oplus 2_3 \oplus 2_4$$

$$3_{2(1)} \otimes 3_{2(2)} = 1_1 \oplus 2_1 \oplus 2_2 \oplus 2_3 \oplus 2_4$$

$$3_{1(1)} \otimes 3_{2(2)} = 1_2 \oplus 2_1 \oplus 2_2 \oplus 2_3 \oplus 2_4$$

For two triplets

$$a = (a_1, a_2, a_3)$$

$$b = (b_1, b_2, b_3)$$

We can write

$$1_1 = (ab)_+ = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$1_2 = (ab)_- = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Our model is based on the $\Delta(54)$ model where we introduced additional flavons

Field	L	l	H_1	H_2	ν_R	N_1	N_2	S_1	S_2	χ	χ'	η	ζ	ξ	Φ_S	ϕ
$\Delta(54)$	$3_{1(1)}$	$3_{2(2)}$	1_1	1_2	$3_{2(2)}$	$3_{1(1)}$	$3_{2(2)}$	$3_{1(1)}$	$3_{2(2)}$	1_2	2_1	$3_{2(2)}$	1_2	$3_{2(1)}$	$3_{1(1)}$	1_2
Z_2	1	-1	1	1	-1	-1	1	-1	1	-1	-1	1	-1	-1	-1	-1
Z_3	ω	ω	1	1	1	1	1	1	1	1	1	1	1	ω	ω	1
Z_4	1	-1	1	1	-i	1	-i	-i	1	-1	-1	1	1	1	1	-i
U(1)	1	1	0	0	1	1	1	1	1	0	0	0	0	0	0	0

Table 3.2: Full particle content of our model

χ, χ', η , ζ, ξ, Φ_S and ϕ to get the deviation from the precise TBM pattern of neutrino mixing[60]. We put extra symmetry $Z_2 \otimes Z_3 \otimes Z_4$ to avoid unwanted terms. Table 3.2 provides details regarding the composition of the particles and corresponding charge assignment in accordance with the symmetry group. The triplet representation of $\Delta(54)$ is used to assign the right-handed charged leptons and the left-handed lepton doublets.

The invariant Yukawa Lagrangian is as follows [1]:

$$\mathcal{L} = \frac{y_1}{\Lambda}(l\bar{L})\chi H_1 + \frac{y_2}{\Lambda}(l\bar{L})\chi' H_1 + \frac{\bar{L}\tilde{H}_2 N_1}{\Lambda} y_\xi \xi + \frac{\bar{L}\tilde{H}_1 N_1}{\Lambda} y_s \Phi_s + \frac{\bar{L}\tilde{H}_2 N_1}{\Lambda} y_a \Phi_s$$
$$+ \frac{y_{_{RN}}}{\Lambda}\bar{\nu_R} N_2 \eta\zeta + y_{_{NS}}\bar{S}_1 N_2 \zeta + y'_{_{NS}}\bar{S}_2 N_1 \zeta + \frac{y_s}{\Lambda^2}\bar{S}_1 S_2 \phi$$

We analyze the scalar potential to find out the vacuum alignment in Appendix B. We consider the vacuum expectation values as,

$$\langle\chi\rangle = (v_\chi) \qquad \langle\chi'\rangle = (v_\chi, v_\chi) \qquad \langle\xi\rangle = (v_\xi, v_\xi, v_\xi) \qquad \langle\eta\rangle = (v_\eta, v_\eta, v_\eta)$$
$$\langle\Phi_S\rangle = (v_s, v_s, v_s) \qquad \langle\zeta\rangle = (v_\zeta) \qquad \langle\phi\rangle = (v_\phi)$$

The charged lepton mass matrix is given as [61]

$$M_l = \frac{y_1 v}{\Lambda}\begin{pmatrix} v_\chi & 0 & 0 \\ 0 & v_\chi & 0 \\ 0 & 0 & v_\chi \end{pmatrix} + \frac{y_2 v}{\Lambda}\begin{pmatrix} -\omega v_{\chi'} + v_{\chi'} & 0 & 0 \\ 0 & -\omega^2 v_{\chi'} + \omega^2 v_{\chi'} & 0 \\ 0 & 0 & -v_{\chi'} + \omega v_{\chi'} \end{pmatrix}$$

where, y_1 and y_2 are coupling constants.

3.2.1 Effective neutrino mass matrix

By utilizing the Lagrangian presented, it is possible to derive the mass matrices that pertain to the neutrino sector once both $\Delta(54)$ and electroweak symmetry breaking have occurred. The essence of the ISS theory lies in the assurance that the neutrino masses remain small by postulating a small M_S scale. To reduce the right-handed neutrino masses to the TeV scale, it is necessary for the M_S scale to be at the KeV level. The inverse seesaw model is a TeV-scale seesaw model that allows heavy neutrinos to stay as light as a TeV while permitting Dirac masses to

[1] Considering terms upto dimension-5.

be as substantial as those of charged leptons, all while maintaining compatibility with light neutrino masses in the sub-eV range.

$$M_{RN} = \frac{y_{_{RN}}}{\Lambda} \begin{pmatrix} v_\eta v_\zeta & 0 & 0 \\ 0 & v_\eta v_\zeta & 0 \\ 0 & 0 & v_\eta v_\zeta \end{pmatrix} \tag{3.2}$$

$$M_{NS} = y_{_{NS}} \begin{pmatrix} v_\zeta & 0 & 0 \\ 0 & v_\zeta & 0 \\ 0 & 0 & v_\zeta \end{pmatrix} \tag{3.3}$$

$$M'_{NS} = y'_{_{NS}} \begin{pmatrix} v_\zeta & 0 & 0 \\ 0 & v_\zeta & 0 \\ 0 & 0 & v_\zeta \end{pmatrix} \tag{3.4}$$

$$M_S = \frac{y_s}{\Lambda^2} \begin{pmatrix} v_\phi & 0 & 0 \\ 0 & v_\phi & 0 \\ 0 & 0 & v_\phi \end{pmatrix} \tag{3.5}$$

$$M_{\nu N} = \frac{v}{\Lambda} \begin{pmatrix} y_\xi v_\xi & y_s v_s + y_a v_a & y_s v_s - y_a v_a \\ y_s v_s - y_a v_a & y_\xi v_\xi & y_s v_s + y_a v_a \\ y_s v_s + y_a v_a & y_s v_s - y_a v_a & y_\xi v_\xi \end{pmatrix} \tag{3.6}$$

Effective neutrino mass matrix is given by

$$m_\nu = M_{RN} (M'_{NS})^{-1} M_S M_{NS}^{-1} M_{\nu N} \tag{3.7}$$

$$= \lambda \begin{pmatrix} x & s+a & s-a \\ s-a & x & s+a \\ s+a & s-a & x \end{pmatrix} \tag{3.8}$$

$$= \begin{pmatrix} x' & s'+a' & s'-a' \\ s'-a' & x' & s'+a' \\ s'+a' & s'-a' & x' \end{pmatrix} \tag{3.9}$$

where, $x = y_\xi v_\xi$, $s = y_s v_s$ and $a = y_a v_a$

We now define the Hermitian matrix as

$$M_\nu = m_\nu m_\nu^\dagger \tag{3.10}$$

$$= \begin{pmatrix} x'^2 + 2(s'^2 + a'^2) & s'^2 - a'^2 + 2s'x' & s'^2 - a'^2 + 2s'x' \\ s'^2 - a'^2 + 2s'x' & x'^2 + 2(s'^2 + a'^2) & s'^2 - a'^2 + 2s'x' \\ s'^2 - a'^2 + 2s'x' & s'^2 - a'^2 + 2s'x' & x'^2 + 2(s'^2 + a'^2) \end{pmatrix} \tag{3.11}$$

where, $\lambda = \frac{v v_\eta v_\phi y_{RN} y_s}{\Lambda^4 v_\zeta y_{NS} y'_{NS}}$, $x' = x\lambda$, $s' = s\lambda$ and $a' = a\lambda$

3.3 Numerical Analysis

In the earlier section, we illustrated the method of enhancing the $\Delta(54)$ model by incorporating supplementary flavons. The subsequent section comprises a quantitative investigation aimed at examining the effectiveness of the parameters in generating a departure from TBM in neutrino mixing. The outcomes of this analysis, restricted to the normal hierarchy scenario, will be deliberated.

The neutrino mass matrix m_ν can be diagonalized by the PMNS matrix U as

$$U^\dagger m_\nu^{(i)} U^* = \text{diag}(m_1, m_2, m_3) \tag{3.12}$$

We can numerically calculate U using the relation $U^\dagger M_\nu U = \text{diag}(m_1^2, m_2^2, m_3^2)$, where $M_\nu = m_\nu m_\nu^\dagger$. The neutrino oscillation parameters θ_{12}, θ_{13}, θ_{23} and δ can be obtained from U as

$$s_{12}^2 = \frac{|U_{12}|^2}{1 - |U_{13}|^2}, \qquad s_{13}^2 = |U_{13}|^2, \qquad s_{23}^2 = \frac{|U_{23}|^2}{1 - |U_{13}|^2} \tag{3.13}$$

and δ may be given by

$$\delta = \sin^{-1}\left(\frac{8\,\text{Im}(h_{12}h_{23}h_{31})}{P}\right) \tag{3.14}$$

with

$$P = (m_2^2 - m_1^2)(m_3^2 - m_2^2)(m_3^2 - m_1^2) \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \tag{3.15}$$

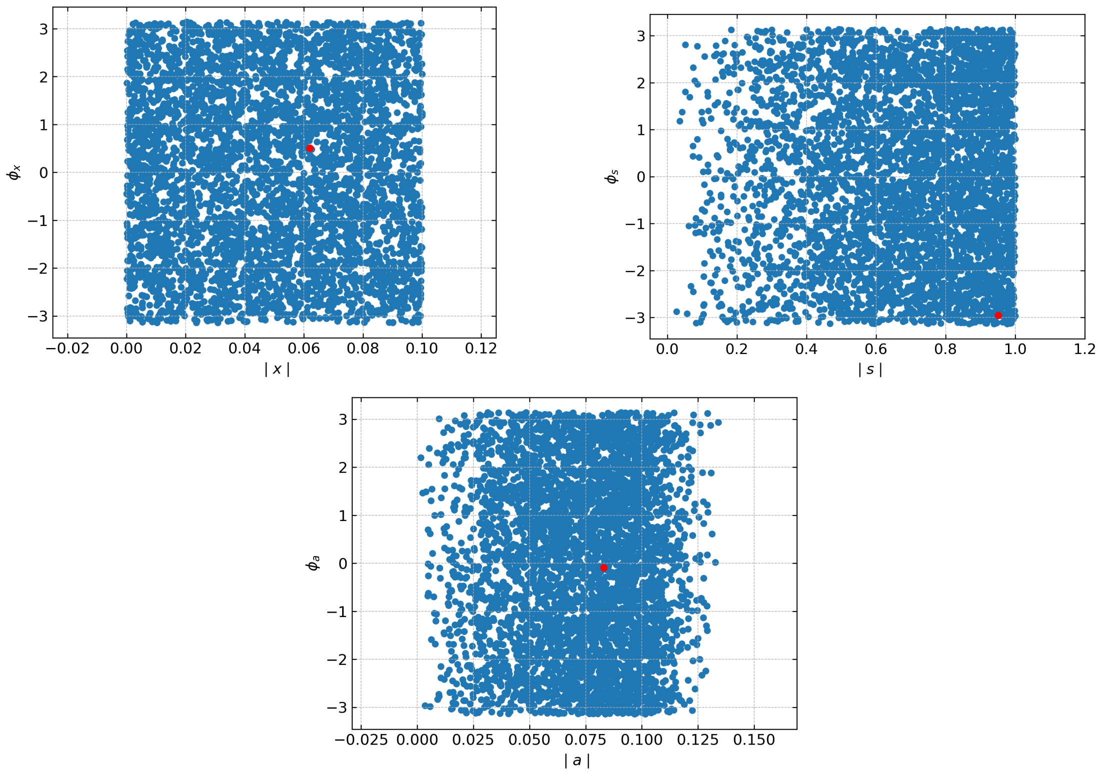

Figure 3.1: Allowed regions of the model parameters $|x|$, ϕ_x, $|s|$, ϕ_s and $|a|$, ϕ_a in NH. The best fit values are indicated by red dots.

To assess how the neutrino mixing parameters compare to the most recent experimental data [59], we adjusted the modified $\Delta(54)$ model to fit the experimental data by minimizing the subsequent χ^2 function:

$$\chi^2 = \sum_i \left(\frac{\lambda_i^{model} - \lambda_i^{expt}}{\Delta \lambda_i} \right)^2 , \tag{3.16}$$

where λ_i^{model} is the i^{th} observable predicted by the model, λ_i^{expt} stands for i^{th} experimental best-fit value and $\Delta\lambda_i$ is the 1σ range of the observable. The parameter of the model space is illustrated in Fig.3.1, with restrictions based on the 3σ limit of neutrino oscillation data presented in Table3.1. The illustration indicates a strong interdependence among various parameters of the model. The best-fit values for $|x|$, $|s|$, $|a|$, ϕ_x, ϕ_s and ϕ_a obtained are (0.062, 0.951, 0.083, 0.505π, -2.953π, -0.092π).

Fig. 3.2 predicts the expected values of the neutrino oscillation parameters within the model for NH. The best fit values of $\sin^2\theta_{12}$, $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ are

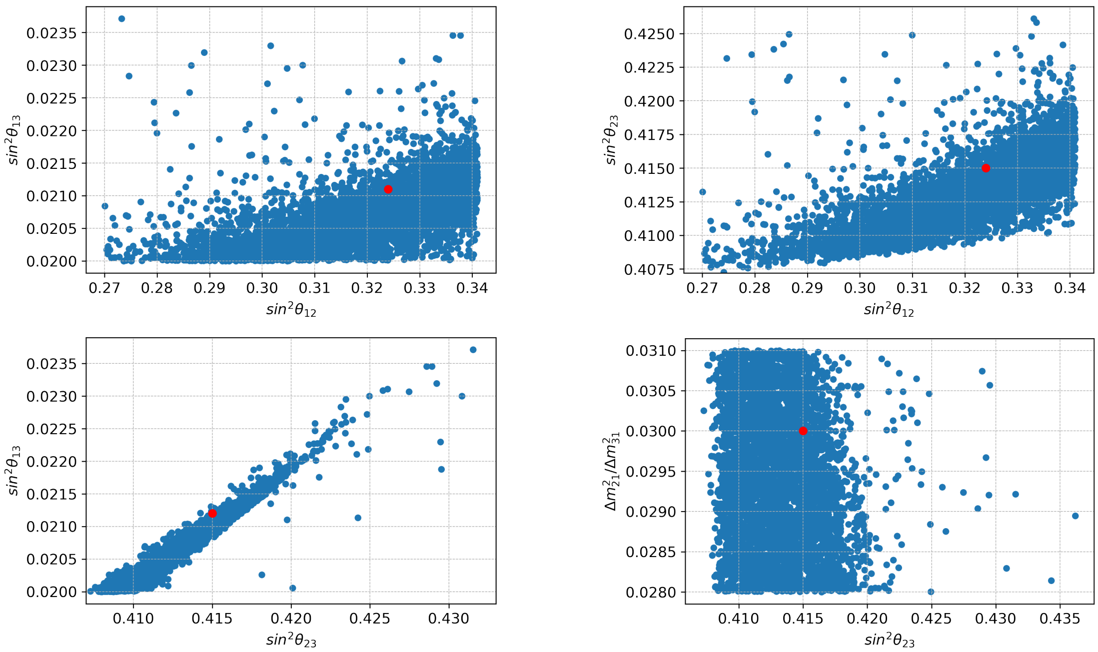

Figure 3.2: Correlation among the mixing angles and among mixing angle and ratio of mass-squared difference. The best fit value is indicated by the red dot.

(0.320, 0.0214, 0.417) which are within the 3σ range of experimental measurements. Additional parameters such as Δm^2_{21}, Δm^2_{31} and δ_{CP} have their best-fit values, corresponding to χ^2-minimum, at (6.88×10^{-5} eV^2, 2.50×10^{-3} eV^2, 0.115π).

Fig. 3.3 gives the correlation between the two mass square difference predicted by the model for NH. The Δm^2_{21} and Δm^2_{31} have their best-fit values, corresponding to χ^2-minimum, at (6.88×10^{-5} eV^2, 2.507×10^{-3} eV^2) respectively.

Fig. 3.4 gives the correlation between the Dirac CP phase and atmospheric angle and also Dirac CP with the ratio of the two mass squared difference. The best fit value of δ_{CP} is predicted to be around 0.115π.

Thus, the model defined in this work indicates clear deviation from tri-bimaximal mixing.

It is clear from the correlation plot among the neutrino oscillation parameters that the mixing of neutrinos deviates from the TBM mixing in NH. The prediction of mixing angle θ_{23} in NH scenario indicates a preference towards the lower octant. With the modification of $\Delta(54)$ model with additional term, it is possible to

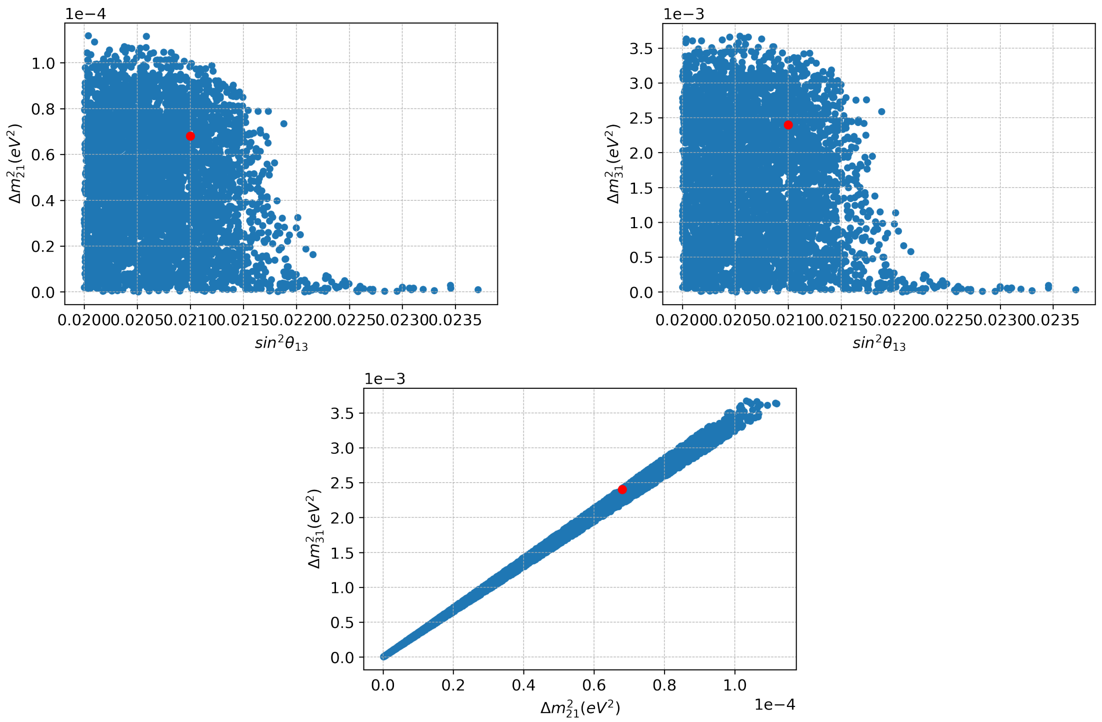

Figure 3.3: Variation of the mass squared differences with mixing angles and correlation between two mass squared differences.

deviate from TBM mixing. The Table 3.3 shows the best-fit values of the various oscillation parameters.

Parameter	NH
$\sin^2\theta_{12}$	0.320
$\sin^2\theta_{13}$	0.0214
$\sin^2\theta_{23}$	0.417
δ_{CP}/π	0.115
$\Delta m^2_{21}[10^{-5}eV^2]$	6.880
$\Delta m^2_{31}[10^{-3}eV^2]$	2.507

Table 3.3: Best-fit values for different parameters predicted by the model

Prediction for Jarlskog invariant Parameter: In Fig.3.5 We are sharing further forecasts generated by the revised model concerning CP-violation's Jarlskog

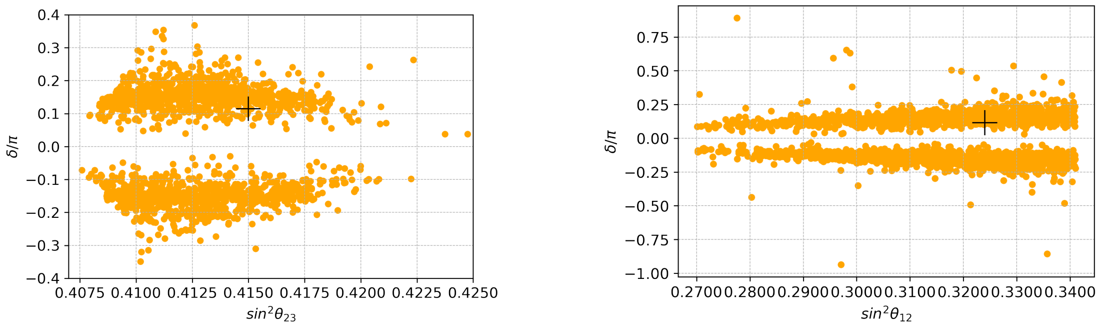

Figure 3.4: Correlation between Dirac CP (δ_{CP}) and atmospheric mixing angle($sin^2\theta_{23}$) and solar mixing angle($sin^2\theta_{12}$) respectively. The best fit value for 3σ range is given by $+$ symbol.

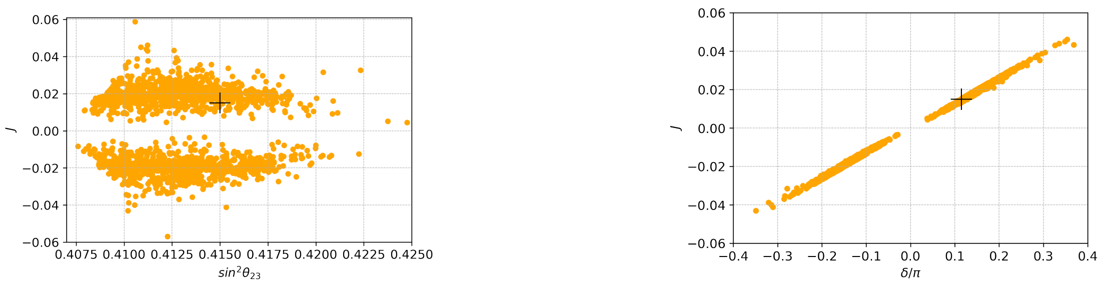

Figure 3.5: Correlation between the Jarlskog invariant parameter(J) with Dirac CP(δ_{CP}) phase and atmospheric mixing angle($sin^2\theta_{23}$) respectively. The best fit value for 3σ range is given by black $+$.

parameter(J) and the effective Majorana mass(m_{ee}) that defines the neutrinoless double beta decay. The Jarlskog constant is a quantity that remains unchanged even after a phase redefinition.

$$J = Im\{U_{11}U_{22}U_{12}^{*}U_{21}^{*}\} = s_{13}c_{13}^{2}s_{12}c_{12}s_{23}c_{23}sin\delta \tag{3.17}$$

Neutrinoless double beta decay (NDBD): The NDBD phenomenon is crucial in the field of neutrino physics, as it involves the light Majorana neutrinos. The process is governed by an effective mass, denoted as m_{ee}, which can be calculated using the equation:

$$|m_{ee}| = U_{Li}^{2}m_{i} \tag{3.18}$$

where U_{Li} are the elements of the first row of the neutrino mixing matrix U_{PMNS}.

This equation depends on certain known parameters such as θ_{12} and θ_{13}, as well as unknown Majorana phases denoted by α and β. The diagonalizing matrix of the light neutrino mass matrix, denoted by m_ν, is represented by U_{PMNS}, such that

$$m_\nu = U_{PMNS} M_\nu^{(diag)} U_{PMNS}^T \tag{3.19}$$

where, $M_\nu^{(diag)}$ =diag(m_1, m_2, m_3). The effective Majorana mass can be expressed applying the diagonalizing matrix elements and the mass eigenvalues as follows:

$$m_{ee} = m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2\iota\alpha} + m_3 s_{13}^2 e^{2\iota\beta} \tag{3.20}$$

where c_{12} and s_{12} are the cosine and sine of the mixing angle θ_{12}, respectively.

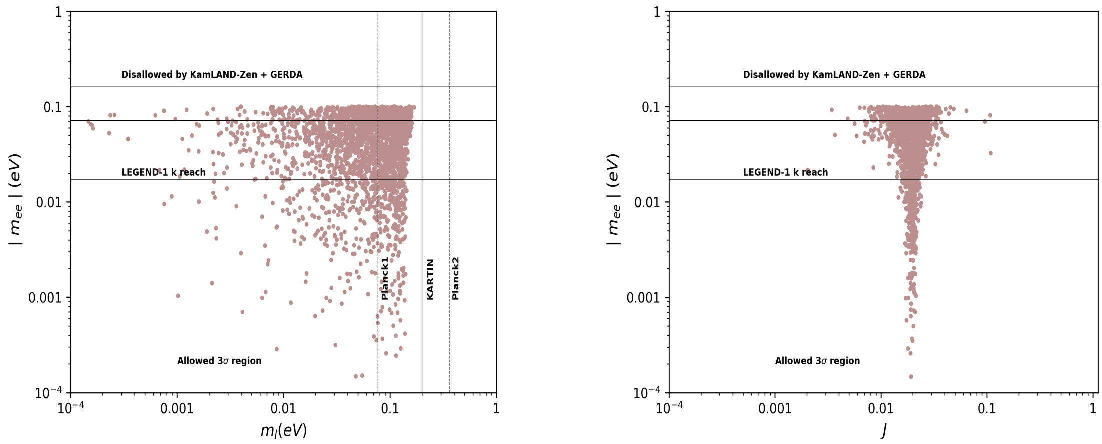

Figure 3.6: Correlation between Effective Majorana neutrino mass(m_{ee}) with lightest neutrino mass(m_1) and Jarlskog invariant(J).

After analyzing the restricted parameter space, we have computed the value of m_{ee} in the NH scenario. The figure labeled as 3.6 depicts the changes in m_{ee} as per the lightest neutrino mass. Additionally, the sensitivity range of experiments like GERDA, KamLAND-Zen and LEGEND-1k for neutrinoless double beta decay is also illustrated in the same figure. The combined constraints from KamLAND-Zen and GERDA experiments put an upper limit on m_{ee} in the range 0.071–0.161 eV. LEGEND-1k experiment value on m_{ee} is 0.017 eV. The vertical black solid line represents the future sentivity of KATRIN $m_{lightest} < 0.2eV$[62] and the other two vertical lines represent the bound $m_{lightest} < 0.077$eV and $m_{lightest} < 0.36$eV, following the two extreme bounds from Planck data set. As per the findings, m_{ee} falls well within the reach of these NDBD experiments for normal hierarchy.

3.4 Conclusion

We have introduced $\Delta(54)$ flavor model with two SM Higgs along with $Z_2 \otimes Z_3 \otimes Z_4$ symmetry that generate a neutrino mass matrix. We have constructed a flavor-symmetric approach in Inverse Seesaw mechanism to realize the neutrino masses and mixing to fit present neutrino oscillation data, including the non-zero reactor angle (θ_{13}) and Dirac CP (δ_{CP}). We use additional flavons to obtain the desired mixing pattern. The model under consideration clearly shows deviation from TBM mixing of the neutrino mixing matrix. The predictions for the neutrino oscillation parameters derived from the resulting mass matrix are in agreement with the best-fit values obtained through χ^2 analysis. The prediction of the mixing angles, mass-squared differences, and the Dirac CP phase in the IH case do not agree with the experimental data. The prediction of mixing angles in NH case indicates that for the parameter space under consideration, the model prefers lower octant of θ_{23}.

Furthermore, we investigated the Jarlskog invariant parameter and NDBD within the context of our $\Delta(54)$ models. The effective Majorana neutrino mass $|m_{ee}|$ is well within the sensitivity reach of the recent $0\nu\beta\beta$ experiments. In extension of this work, the model can be used to study leptogenesis and dark matter. The consistency of the model predictions with the latest neutrino data implies that model may be tested in future neutrino oscillation experiments.

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