Chapter 4

Majorana neutrinos in Double Inverse Seesaw and $\Delta(54)$ flavor models

The current work involves augmenting the $\Delta(54)$ flavor symmetry model by incorporating two Standard Model Higgs particles into the Inverse Seesaw mechanism. The mass matrices are discussed numerically in the framework of $\Delta(54)$ flavor for Majorana neutrinos. We introduced Vector like fermions and Weyl fermions, which are gauge singlets in the Standard Model and produces Majorana mass terms. We restrict the undesirable terms in our Lagrangian by using additional symmetry. Due to these additional terms, the mass matrices deviate from the tribimaximal neutrino mixing pattern, resulting in the production of a non-zero reactor angle θ_{13}. We found that the atmospheric oscillation parameter (θ_{23}) occupies the upper octant under the normal hierarchy situation. We also study the CP violation (δ_{CP}) , Jarlskog invariant parameter (J), and Neutrinoless double-beta decay parameter (m_{ee}) in the parameter space of the normal hierarchy model to see whether they concur with the most recent neutrino observations.

4.1 Introduction

The detection of neutrino oscillations was an early and compelling experimental indication of deficiencies in the Standard Model (SM). This is because the SM initially proposed neutrinos to have no mass at all, but the undeniable evidence of neutrino oscillations unequivocally demonstrated that at least two out of the three recognized neutrinos must have distinct, non-zero masses. However, theorists had already predicted the existence of neutrino masses long before experimental verification occurred. Three mixing angles are used in the research of neutrino oscillation, two of which are large and one of which is very comparatively small. The reactor mixing angle is not zero, as evidenced by investigations like the Daya Bay Reactor Neutrino Experiment [1] and RENO Experiment[2]. The TBM model is unreliable since several other tests, including MINOS[3], Double Chooz[4], and T2K[5], consistently observed nonzero values for the reactor mixing angle. To accomplish realistic blending, other models or adjustments must be taken into account.

Neutrinoless double beta decay is the key to establishing that neutrinos are Majorana particles, but it has yet to be observed. Wendell Furry proposed the Majorana constitution of particles and investigated a kinetic process analogous to neutrinoless double beta decay, both of which require symmetry to explain[6, 7]. This process, expressed as $(A, Z) \rightarrow (A, Z + 2) + 2e^-$ disrupts the lepton number by two units by generating a pair of electrons, resulting in Majorana neutrino masses. Since neutrino masses are zero in the standard model, the large value of the lepton number violation scale is associated with the smallness of observed neutrino masses($\Lambda \sim 10^{14} - 10^{15}$GeV). To generate neutrino masses greater than zero, a model beyond the standard model, such as effective theories employing the Weinberg operator is required.

Various theoretical models have been proposed to accomplish tribimaximal mixing (TBM) by using non-abelian discrete symmetries, including A_4[8–11], S_3[10], S_4[12], $\Delta(27)$[13–16], and $\Delta(54)$[17–19]. In order to introduce departures from TBM, additional flavons are included into these models. This paper presents a methodology by using the $\Delta(54)$ flavor symmetry framework. The $\Delta(54)$ symmetry can manifest itself in heterotic string models on factorizable orbifolds, such as the T^2/Z_3 orbifold [20]. In these string models, only singlets and triplets are observed as fundamental modes, while doublets are absent as fundamental modes. However, doublets have the potential to become fundamental modes in magnetized/intersecting D-brane models. We can also suggest an extension to the Standard Model, utilizing $\Delta(54)$ symmetry. We have the option to engage with both the singlets $(1_1, 1_2)$ and doublets $(2_1, 2_2, 2_3, 2_4)$ representations of $\Delta(54)$, which allow us to represent quarks in different ways. This extension effectively incorporates the most recent experimental data for various properties within the quark sector, encompassing six quark masses, three quark mixing angles, and the CP-violating phase [19].

Field	Q_{1L}	$Q_{\alpha L}$	$u_{\alpha R}$	u_{1R}	d_{1R}	$d_{\alpha R}$	H	ϕ
$\Delta(54)$	1_+	2_2	1_+	2_2	1_+	2_2	1_+	2_2
$U(1)$	1/6	1/6	2/3	2/3	−1/3	−1/3	1/2	1/2

The authors of a prior study proposed the use of the Inverse Seesaw mechanism in conjunction with the $\Delta(54)$ flavor model for Dirac neutrinos[21]. This study provides a demonstration of the Inverse Seesaw mechanism for Majorana neutrinos, using two Standard Model Higgs bosons. We introduced a Weyl fermion and this additional component produces a Majorana mass term after the symmetry breaking. We introduced Vector like fermions which also produces second Majorana mass term. In order to depart from the prescribed TBM neutrino mixing pattern, we included additional flavons, namely χ, χ' ζ, ζ', ξ, ϕ, ϕ' and Φ_S, within the framework of the $\Delta(54)$ symmetry. Furthermore, we have integrated a symmetry of $Z_2 \otimes Z_3 \otimes Z_4$ into our model in order to minimize the presence of undesired components and simplify the process of constructing coupling matrices with certain properties. The structure of the neutrino mass matrix, denoted as m_ν, was altered in our study, which pertains to the characterization of neutrino masses. Our investigation primarily focused on symmetry-based analyses. This methodology enables a comprehensive analysis of the neutrino mass (m_ν), as well as an exploration of the Jarlskog invariant parameter (J) and the neutrinoless double beta decay parameter (m_{ee}). The methodology used distinguishes our study from that of other researchers.

The organization of our chapter is as follows: The framework of the model, including the fields and their symmetrical transformation characteristics, is outlined in Section 4.2. The neutrino phenomenology findings are numerically analyzed and examined in Section 4.3. Section 4.4 concludes with our final remarks.

4.2 Framework of the Model

In order to attain the implementation of the Inverse seesaw mechanism, it is essential to enlarge the fermion sector within the framework of the Standard Model. Our analysis involves enhancing the $\Delta(54)$ flavor symmetry model with Inverse Seesaw mechanism along with two SM Higgs H and H' through the incorporation of distinct flavons. In this study, we have presented Vector-like (VL) fermions denoted as N_1 and N_2, which possess the characteristic of being gauge singlets inside the framework of the Standard Model. We also introduced a Weyl fermion denoted as S_1. In fact, the ϕ VEV induces a Majorana mass term for the S_1 fermion. The fields associated with right-handedness and left-handedness are denoted by subscripts 1 and 2, respectively. The $\Delta(54)$ group includes irreducible representations 1_1, 1_2, 2_1, 2_2, 2_3, 2_4, $3_{1(1)}$, $3_{1(2)}$, $3_{2(1)}$ and $3_{2(2)}$. The products of $3_{2(1)} \otimes 3_{2(2)}$, $3_{1(1)} \otimes 3_{1(2)}$, $3_{1(2)} \otimes 3_{2(1)}$ and $3_{1(1)} \otimes 3_{2(2)}$ lead to the trivial singlets 1_1, 1_1, 1_2 and 1_2 respectively. The trivial singlet is the invariance of $\Delta(54)$ symmetry that all terms in Lagrangian must respect.

The following are the rules for multiplication:

$$3_{1(1)} \otimes 3_{1(1)} = 3_{1(2)} \oplus 3_{1(2)} \oplus 3_{2(2)}$$

$$3_{1(2)} \otimes 3_{1(2)} = 3_{1(1)} \oplus 3_{1(1)} \oplus 3_{2(1)}$$

$$3_{2(1)} \otimes 3_{2(1)} = 3_{1(2)} \oplus 3_{1(2)} \oplus 3_{2(2)}$$

$$3_{2(2)} \otimes 3_{2(2)} = 3_{1(1)} \oplus 3_{1(1)} \oplus 3_{2(1)}$$

$$3_{1(1)} \otimes 3_{1(2)} = 1_1 \oplus 2_1 \oplus 2_2 \oplus 2_3 \oplus 2_4$$

$$3_{1(2)} \otimes 3_{2(1)} = 1_2 \oplus 2_1 \oplus 2_2 \oplus 2_3 \oplus 2_4$$

$$3_{2(1)} \otimes 3_{2(2)} = 1_1 \oplus 2_1 \oplus 2_2 \oplus 2_3 \oplus 2_4$$

$$3_{1(1)} \otimes 3_{2(2)} = 1_2 \oplus 2_1 \oplus 2_2 \oplus 2_3 \oplus 2_4$$

Field	L	l	H	H'	N_1	N_2	S_1	χ	χ'	ζ	ζ'	ξ	Φ_S	ϕ	ϕ'
$\Delta(54)$	$3_{1(1)}$	$3_{2(2)}$	1_1	1_2	$3_{1(1)}$	$3_{2(1)}$	$3_{2(2)}$	1_2	2_1	1_2	1_1	$3_{2(1)}$	$3_{1(1)}$	$3_{1(2)}$	$3_{2(1)}$
Z_2	1	-1	1	1	-1	1	1	-1	-1	-1	1	-1	-1	1	-1
Z_3	ω	ω	1	1	1	ω	1	1	1	1	ω	ω	ω	1	ω^2
Z_4	1	-1	1	1	1	-1	1	-1	-1	1	-1	1	1	1	-1
U(1)	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0

Table 4.1: Full particle content of our model

The model we have developed is derived on the $\Delta(54)$ model, which incorporates the inclusion of additional flavons in order to account for deviations from the exact Tri-Bimaximal (TBM) pattern of neutrino mixing [22]. We put extra symmetry $Z_2 \otimes Z_3 \otimes Z_4$ to avoid undesirable terms. Table 4.1 provides details regarding the composition of the particles and corresponding charge assignment in accordance with the symmetry group. The triplet representation of $\Delta(54)$ is used to assign the left-handed leptons doublets and the right-handed charged lepton. The representations of $\Delta(54)$ symmetry are real that guarantees the construction of the effective Lagrangian.

The Lagrangian is as follows [1]:

$$\mathcal{L} = \frac{y_1}{\Lambda}(l\bar{L})\chi H + \frac{y_2}{\Lambda}(l\bar{L})\chi' H + \frac{\bar{L}\tilde{H}' N_1}{\Lambda} y_\xi \xi + \frac{\bar{L}\tilde{H} N_1}{\Lambda} y_s \Phi_s$$
$$+ y_{_{NS}} \bar{N}_1 S_1^c \zeta + y'_{_{NS}} \bar{N}_2 S_1 \zeta' + y_{_N} \bar{N}_1 N_2 \phi' + \frac{y_s}{2\Lambda^2} \bar{S}_1 S_1^c \phi + h.c$$

In this context, the vacuum expectation values are considered naturally as,

$$\langle\chi\rangle = (v_\chi) \quad \langle\chi'\rangle = (v_{\chi'}, v_{\chi'}) \quad \langle\xi\rangle = (v_\xi, v_\xi, v_\xi) \quad \langle\phi\rangle = (v_\phi, v_\phi, v_\phi)$$
$$\langle\Phi_S\rangle = (v_s, v_s, v_s) \quad \langle\zeta\rangle = (v_\zeta) \quad \langle\zeta'\rangle = (v'_\zeta) \quad \langle\phi'\rangle = (v'_\phi, v'_\phi, v'_\phi)$$

The charged lepton mass matrix is given as [18]

$$M_l = \frac{y_1 v}{\Lambda}\begin{pmatrix} v_\chi & 0 & 0 \\ 0 & v_\chi & 0 \\ 0 & 0 & v_\chi \end{pmatrix} + \frac{y_2 v}{\Lambda}\begin{pmatrix} -\omega v_{\chi'} + v_{\chi'} & 0 & 0 \\ 0 & -\omega^2 v_{\chi'} + \omega^2 v_{\chi'} & 0 \\ 0 & 0 & -v_{\chi'} + \omega v_{\chi'} \end{pmatrix}$$

where, y_1 and y_2 are coupling constants and $v \simeq 55$ GeV.

4.2.1 Effective neutrino mass matrix

The mass matrices relevant to the neutrino sector may be obtained by using the above Lagrangian, after the implementation of both $\Delta(54)$ and electroweak symmetry breaking. The essence of the ISS theory lies in the assurance that the neutrino masses remain small by postulating a small M_S scale. To reduce the right-handed neutrino masses to the TeV scale, it is necessary for the M_S scale to be at the KeV level. The inverse seesaw model is a TeV-scale seesaw model that allows heavy neutrinos to stay as light as a TeV while permitting Dirac masses to be as substantial as those of charged leptons, all while maintaining compatibility with light neutrino masses in the sub-eV range.

$$M_{NS} = y_{_{NS}} \begin{pmatrix} v_\zeta & 0 & 0 \\ 0 & v_\zeta & 0 \\ 0 & 0 & v_\zeta \end{pmatrix} \tag{4.1}$$

[1] Considering terms upto dimension-5.

$$M_N = y_{_N} \begin{pmatrix} v'_\phi & 0 & 0 \\ 0 & v'_\phi & 0 \\ 0 & 0 & v'_\phi \end{pmatrix} \tag{4.2}$$

$$M_S = \frac{y'_s}{\Lambda^2} \begin{pmatrix} v_\phi & 0 & 0 \\ 0 & v_\phi & 0 \\ 0 & 0 & v_\phi \end{pmatrix} \tag{4.3}$$

$$M'_{NS} = y'_{_{NS}} \begin{pmatrix} v'_\zeta & 0 & 0 \\ 0 & v'_\zeta & 0 \\ 0 & 0 & v'_\zeta \end{pmatrix} \tag{4.4}$$

$$M_{\nu N} = \frac{v}{\Lambda} \begin{pmatrix} y_\xi v_\xi & y_s v_s & y_s v_s \\ y_s v_s & y_\xi v_\xi & y_s v_s \\ y_s v_s & y_s v_s & y_\xi v_\xi \end{pmatrix} \tag{4.5}$$

Effective neutrino mass matrix is given by

$$m_\nu = M_{\nu N}^2 \frac{M_{NS}^2}{2M_N M_{NS} M'_{NS} - M_N^2 M_S} \approx M_{\nu N}^2 \frac{M_{NS}^2}{M_N^2 M_S} \tag{4.6}$$

$$m_\nu = \lambda \begin{pmatrix} 2c^2 + x^2 & c^2 + 2cx & c^2 + 2cx \\ c^2 + 2cx & 2c^2 + x^2 & c^2 + 2cx \\ c^2 + 2cx & c^2 + 2cx & 2c^2 + x^2 \end{pmatrix} \tag{4.7}$$

where $\lambda = \frac{v^2 v_\zeta^2 y_{_{NS}}^2}{v_\phi v_\phi'^2 y_{_N}^2 y'_s}$, $x = y_\xi v_\xi$, $c = y_s v_s$. The neutrino masses get suppressed by M_{NS}^2. Therefore this model is known as double inverse seesaw [23].

4.3 Numerical Analysis and results

In the previous section, we demonstrated how to improve the $\Delta(54)$ model by integrating extra flavons. The next section is a quantitative examination of the

efficiency of the factors in producing a deviation from TBM in neutrino mixing. The results of this research, which is limited to the normal hierarchical situation, will be discussed.
The neutrino mass matrix m_ν can be diagonalized by the PMNS matrix U as

$$U^\dagger m_\nu U^* = \text{diag}(m_1, m_2, m_3) \tag{4.8}$$

We can numerically calculate U using the relation $U^\dagger M_\nu U = \text{diag}(m_1^2, m_2^2, m_3^2)$, where $M_\nu = m_\nu m_\nu^\dagger$. The neutrino oscillation parameters θ_{12}, θ_{13}, θ_{23} and δ can be obtained from U as

$$s_{12}^2 = \frac{|U_{12}|^2}{1-|U_{13}|^2}, \qquad s_{13}^2 = |U_{13}|^2, \qquad s_{23}^2 = \frac{|U_{23}|^2}{1-|U_{13}|^2} \tag{4.9}$$

and δ may be given by

$$\delta = \sin^{-1}\left(\frac{8\,\text{Im}(h_{12}h_{23}h_{31})}{P}\right) \tag{4.10}$$

with

$$P = (m_2^2 - m_1^2)(m_3^2 - m_2^2)(m_3^2 - m_1^2)\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13} \tag{4.11}$$

Parameters	NH (3σ)	IH (3σ)
$\Delta m_{21}^2[10^{-5}eV^2]$	$6.82 \to 8.03$	$6.82 \to 8.03$
$\Delta m_{31}^2[10^{-3}eV^2]$	$2.428 \to 2.597$	$-2.581 \to -2.408$
$\sin^2\theta_{12}$	$0.270 \to 0.341$	$0.270 \to 0.341$
$\sin^2\theta_{13}$	$0.02029 \to 0.02391$	$0.02047 \to 0.02396$
$\sin^2\theta_{23}$	$0.406 \to 0.620$	$0.410 \to 0.623$
δ_{CP}	$108^\circ \to 404^\circ$	$192^\circ \to 360^\circ$

Table 4.2: The neutrino oscillation parameters from NuFIT 5.2 (2022) [24]

The 3σ ranges of neutrino oscillation parameters from NuFIT 5.2 [24]. We adjusted the modified $\Delta(54)$ model to suit the experimental data by minimizing

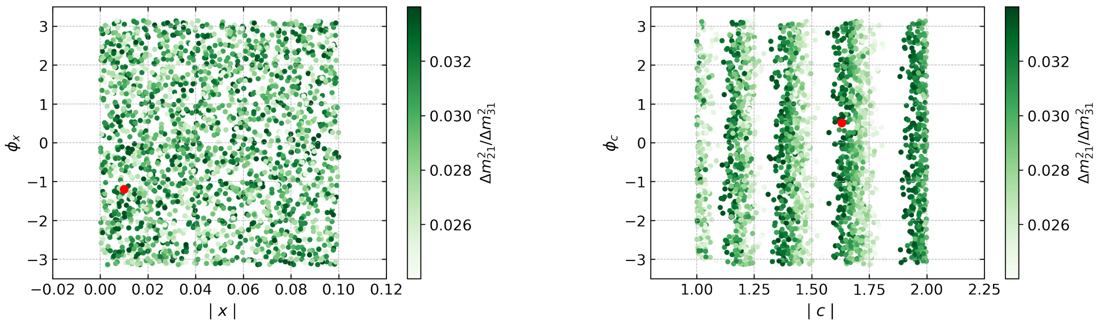

Figure 4.1: Allowed regions of the model parameters $|x|$, $|c|$, ϕ_x and ϕ_c in NH. The best fit values are indicated by red dot.

the ensuing χ^2 function in order to evaluate how the neutrino mixing parameters contrast with the most current experimental data:

$$\chi^2 = \sum_i \left(\frac{\lambda_i^{model} - \lambda_i^{expt}}{\Delta \lambda_i} \right)^2, \tag{4.12}$$

where λ_i^{model} is the i^{th} observable predicted by the model, λ_i^{expt} stands for i^{th} experimental best-fit value and $\Delta\lambda_i$ is the 1σ range of the observable.

The parameter of the model space is illustrated in Fig.4.1, with restrictions based on the 3σ limit of neutrino oscillation data. The illustration indicates a strong interdependence among various parameters of the model. The best-fit values for $|x|$, $|c|$, ϕ_x and ϕ_c obtained are (0.0124, 1.6354, -1.2841π, 0.5123π).

The anticipated values of the neutrino oscillation parameters for NH are shown in Fig.4.2. The values of $sin^2\theta_{12}$, $\sin^2\theta_{13}$, and $\sin^2\theta_{23}$ that best suit the experimental measurements 3σ range are 0.3201, 0.0238, and 0.5128 respectively. The best-fit values for other parameters, such as $\Delta m^2_{21}/\Delta m^2_{31}$, are correspond to the χ^2-minimum. Fig. 4.3 gives the correlation between the CP phase with reactor mixing angle and atmospheric mixing angle respectively. Thus, the model defined in this work indicates clear deviation from tri-bimaximal mixing. The best fit value of δ_{CP} is predicted to be around 0.0961π.

The correlation between the neutrino oscillation parameters makes it obvious that the neutrino mixing differs from the TBM mixing in NH. The upper octant is preferred in the NH scenario, according to the prediction of mixing angle θ_{23}.

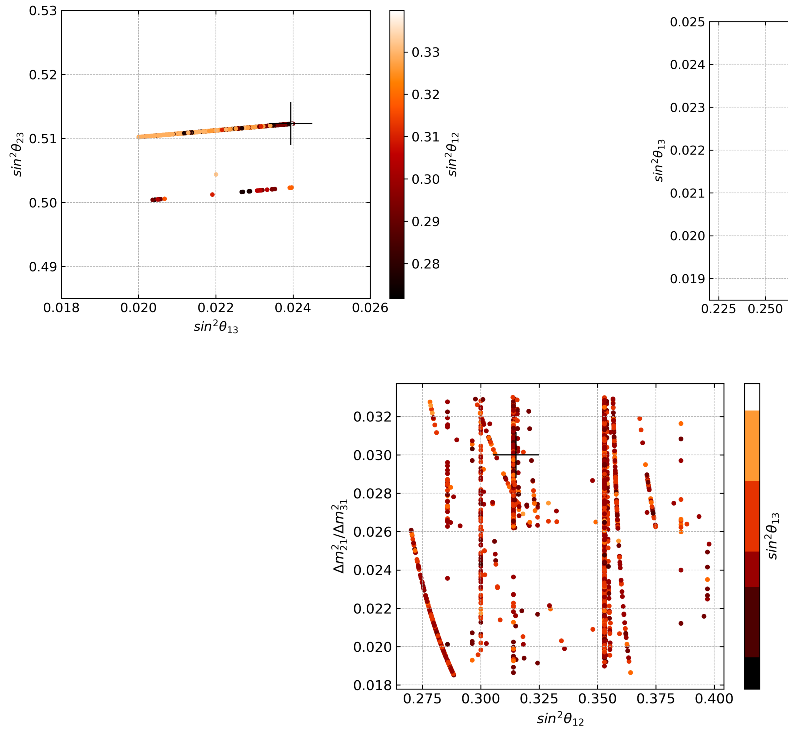

Figure 4.2: Correlation among the oscillation parameters predicted by the model at 3σ in normal hierarchy. The best fit value is indicated by the + marker.

It is feasible to depart from TBM mixing by modifying the $\Delta(54)$ model.

Jarlskog invariant Parameter: In Fig. 4.4 we further estimates CP-violation's Jarlskog parameter (J). This parameter is completely defined by the mixing angles and the Dirac phase. The Jarlskog constant is a quantity that remains unchanged even after a phase redefinition[25].

$$J = Im\{U_{11}U_{22}U_{12}^*U_{21}^*\} = s_{13}c_{13}^2 s_{12}c_{12}s_{23}c_{23}sin\delta \tag{4.13}$$

Neutrinoless double beta decay (NDBD): The NDBD phenomena has significant importance within the context of neutrino physics due to its association with the light Majorana neutrinos. The process is regulated by an efficient mass $|m_{ee}|$, which may be determined by the use of the following equation:

$$|m_{ee}| = U_{Li}^2 m_i \tag{4.14}$$

where U_{Li} are the elements of the first row of the neutrino mixing matrix U_{PMNS}.

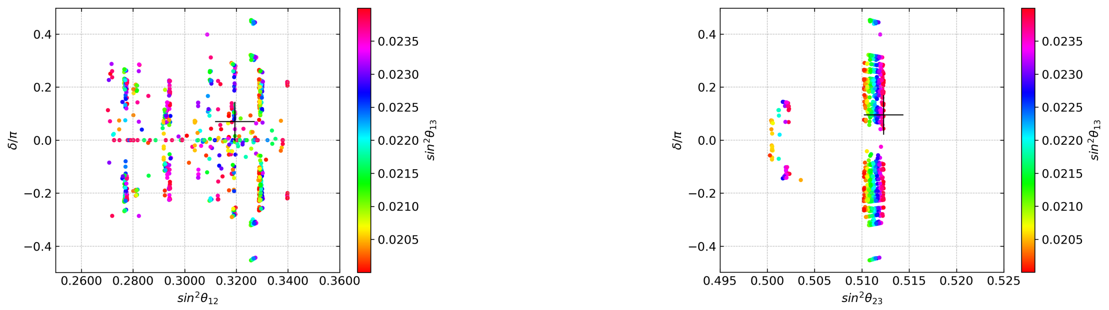

Figure 4.3: Correlation between Dirac CP (δ_{CP}) with solar mixing angle($sin^2\theta_{12}$) and atmospheric mixing angle($sin^2\theta_{23}$) respectively. The best fit value for 3σ range is given by the + marker.

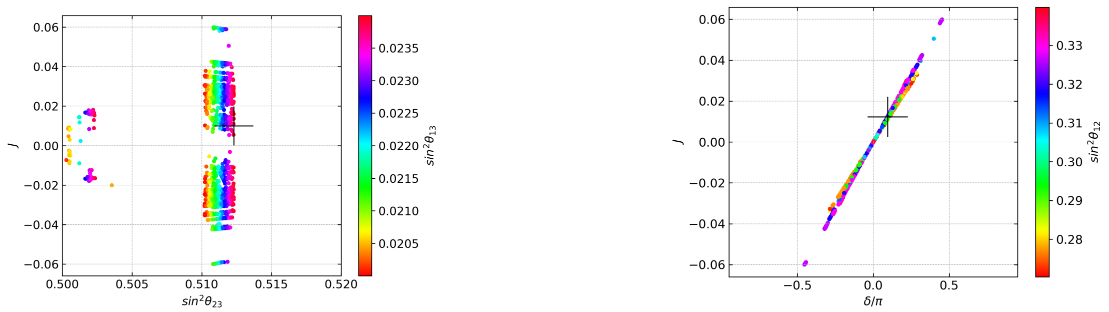

Figure 4.4: Correlation between the Jarlskog invariant parameter(J) with atmospheric mixing angle($sin^2\theta_{23}$) and CP phase (δ_{CP}) and respectively. The best fit value for 3σ range is given by the + marker.

This equation depends on certain known parameters such as θ_{12} and θ_{13}, as well as unknown Majorana phases denoted by α and β. The diagonalizing matrix of the light neutrino mass matrix, denoted by m_ν, is represented by U_{PMNS}, such that

$$m_\nu = U_{PMNS} M_\nu^{(diag)} U_{PMNS}^T \tag{4.15}$$

where, $M_\nu^{(diag)}$ =diag(m_1, m_2, m_3). The effective Majorana mass can be expressed applying the diagonalizing matrix elements and the mass eigenvalues as follows:

$$|m_{ee}| = m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2\iota\alpha} + m_3 s_{13}^2 e^{2\iota\beta} \tag{4.16}$$

where c_{12} and s_{12} are the cosine and sine of the mixing angle θ_{12}, respectively.

Upon investigating the constrained parameter domain, we have computed the value of $|m_{ee}|$ in the NH scenario. Figure 4.5 illustrates variations in $|m_{ee}|$ corresponding to the lightest neutrino mass (m_l). Furthermore, it demonstrates the sensitivity range of experiments such as GERDA, KamLAND-Zen, nEXO and LEGEND-1k for neutrinoless double beta decay within the same diagram. The combined constraints from KamLAND-Zen and GERDA experiments puts an upper limit on $|m_{ee}|$ in the range 0.071–0.161 eV. The LEGEND-1k experiment puts a band in $|m_{ee}|$ with lowest value as 0.017 eV. The future sentivity of KATRIN $m_{lightest}$ is around $0.2eV$ [26]. According to the results, $|m_{ee}|$ falls well within the detection capabilities of these NDBD experiments in the case of the normal hierarchy.

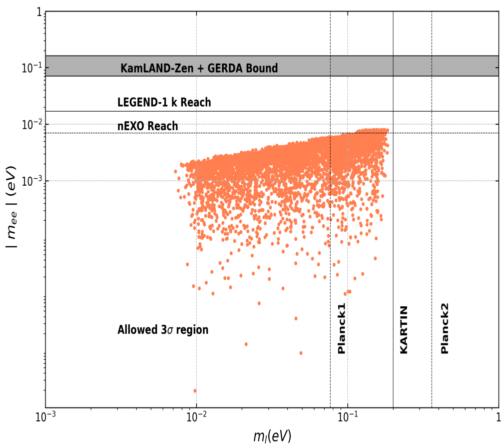

Figure 4.5: Correlation between Effective Majorana neutrino mass(m_{ee}) and the lightest neutrino mass(m_l).

4.4 Conclusion

In order to produce a neutrino mass matrix, we have proposed the $\Delta(54)$ flavor with SM Higgs boson and $Z_2 \otimes Z_3 \otimes Z_4$ symmetry. Using the ISS mechanism, we developed a flavor-symmetric approach to achieve neutrino masses and mixing that align with current neutrino oscillation data. This includes accounting for the non-zero reactor angle (θ_{13}) and CP violation (δ_{CP}). We have incorporated additional flavons to achieve the intended mixing pattern in our study. The examined model clearly departs from the Tribimaximal (TBM) mixing pattern in the neutrino mixing matrix. The anticipated values for neutrino oscillation parameters derived from the resultant mass matrix align with the best-fit values obtained through χ^2 analysis. However, the anticipated mixing angles, mass-squared differences, and the CP violation phase in the Inverted Hierarchy (IH) scenario do not concur with experimental data. In the Normal Hierarchy (NH) scenario, the model projected mixing angles indicate a preference for the upper octant of atmospheric angle (θ_{23}) within the specified parameter space.

Furthermore, we investigated the Jarlskog invariant parameter (J) and the Neutrinoless Double Beta Decay (NDBD) phenomenon within the framework of our $\Delta(54)$ flavor model. It is worthy to note that the effective Majorana neutrino mass, denoted as $|m_{ee}|$, falls within the sensitivity range of contemporary Neutrinoless Double Beta Decay ($0\nu\beta\beta$) experiments. Our estimations are consistent with the existing neutrino oscillation parameters. Future work is reserved for examining the model to explore phenomena such as Leptogenesis and Asymmetric Dark Matter.

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