

*In loving memory of my father, and with gratitude to  
my mother for her unwavering support.*



## Declaration by Candidate

I, **Hirakjyoti Sarma**, declare that the thesis entitled “**Study on Yukawa clusters of charged dust grains immersed in a plasma environment**” is a record of my own work carried out under the supervision of **Dr. Nilakshi Das**. I, hereby, confirm that:

- This work was done wholly or mainly while in candidature for a research degree in Doctor of Philosophy at Tezpur University. Where no part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work. I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

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## Certificate by the Supervisor

This is to certify that the thesis entitled “**Study on Yukawa clusters of charged dust grains immersed in a plasma environment**”, submitted to the School of Sciences, Tezpur University in partial fulfillment of the requirements for the award of the degree of **Doctor of Philosophy** in **Physics** is a record of original research work carried out by **Mr. Hirakjyoti Sarma** under my supervision and guidance.

All help received by him from various sources has been duly acknowledged.

No part of the thesis has been submitted elsewhere for the award of any other degree.

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