Chapter 2

Frequency-magnitude distribution factor (b-value)

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2.1. Introduction

Earthquakes cause severe damage and disruptions, making it crucial to study their patterns for effective risk assessment and mitigation. A fundamental tool in this endeavor is the GR power law **[1]**. This empirical relationship quantifies the FMD within a given region over a specified period. The GR law is expressed mathematically as:

$$Log_{10}N = a - bM \ldots (1)$$

The a-value represents the logarithm of the total number of earthquakes exceeding a certain magnitude threshold within a specific region and time period. It signifies the level of seismic occurrences in that region, serving as a measure of the baseline seismicity. The a-value varies between 2.0 to 8.0 for any seismically active region **[2]**. This parameter is essential for characterizing the seismicity of a region **[3]**. Meanwhile, 'b' indicates the relative frequency of large to small earthquakes. The significance of the GR law lies in its ability to reveal the power-law nature of earthquake occurrences. This relationship is pivotal for seismologists, as it provides a framework for estimating the probability of future seismic events based on historical data, thereby facilitating earthquake preparedness strategies, and contributing to the development of building codes and land-use planning. Despite its widespread use and acceptance, the GR law also invites ongoing research and discussion. Even though GR law has been widely used by researchers, there are certain areas such as variations in the b-value across different regions and time scale, the physical process underlying these variations, and the law's applicability to timescales. Variations in the b-value across different regions and timescales, the physical processes underlying these variations, and the law's applicability to induced seismicity and non-traditional seismic environments are areas of active investigation. The b-value, derived from the GR relationship typically hovers around 1.0 for most tectonic settings. However, variations in the b-value can offer critical clues about the state of stress in the Earth's crust, the heterogeneity of the seismogenic layer, and the prevailing tectonic processes. For instance, lower b-values often indicate higher stress levels and a greater likelihood of larger earthquakes, while higher b-values suggest a predominance of smaller seismic events, potentially linked to more heterogeneous fault conditions. Understanding the b-value is essential for several reasons. Firstly, it enhances our ability to assess seismic hazards by providing a statistical measure of earthquake size distribution, which is crucial for risk management and mitigation strategies. Secondly, temporal and spatial variations in the b-value can signal changes in the stress regime or the onset of significant seismic activity, serving as a potential tool for earthquake forecasting. Thirdly, the b-value is relevant for examining induced seismicity associated with human activities such as mining, reservoir-induced seismicity, and hydraulic fracturing, where deviations from typical b-values can indicate altered stress conditions. Recent advances in seismological data collection and analysis techniques have allowed more precise and localized estimations of the b-value, enhancing our understanding of its variability and underlying causes. Additionally, the integration of b-value analysis with other geophysical and geological data has opened new avenues for interdisciplinary research, providing a more comprehensive picture of seismic hazard and risk. Considering the paramount significance of b value as reflected the past research, an attempt has been made in this chapter to revisit the theoretical foundations, methodological approaches of this parameter with simultaneous emphasis in practical applications. We review the factors influencing its variation, discuss its significance in different tectonic and induced seismic environments, and explore recent case studies that highlight its role in advancing our understanding of earthquake processes. Through this exploration, we aim to underscore the importance of the b-value as a vital tool in the ongoing effort to mitigate the impacts of seismic hazards.

2.2. Data Analysis

Reliable b-value estimation necessitates the meticulous compilation of an earthquake catalog. This process involves multiple detailed steps, each crucial for ensuring the accuracy and completeness of the data. The subsequent sections outline these steps in detail.

2.2.1. Homogenizing an earthquake catalog

Homogenizing an earthquake catalog compiled from diverse sources such as the International Seismological Centre (ISC), the United States Geological Survey (USGS) etc. is a critical preparatory step for seismic analysis. This process involves reconciling discrepancies in event parameters like origin time, location, magnitude, and depth, which may arise due to differences in data collection methods, reporting standards, and computational techniques across various agencies. Harmonization typically requires converting all magnitudes to a common scale, such as moment magnitude (Mw), and applying consistent criteria for event selection and classification. Additionally, it may involve cross-referencing and merging overlapping records, removing duplicates, and ensuring uniformity in the metadata format. The aim is to create a cohesive, high-quality dataset that enhances the accuracy and reliability of seismic hazard assessments and facilitates comprehensive scientific research into earthquake behavior and trends. In the thesis we have used the region specific and globally accepted homogenization equations for the northeast India, Nepal, and the EAFZ. The different equations of homogenization used in the thesis are listed in the Table 2.1.

Table 2.1. The different equations proposed for homogenization of the earthquake catalog.

Author	Equation	Region
[4]	$M_W = 0.625\ Ms + 2.350$ $M_W = 0.862\ mb + 1.034$ $Mw = 1.926\ M_L - 0.943$	Northeast India
[5]	$M_W = 1.080\ mb - 0.325,\ 4.0 \leq mb \leq 7.2$ $M_W = 0.815\ M_L + 0.767,\ 3.3 \leq M_L \leq 7.0$ $M_W = 0.693\ M_S + 1.922,\ 3.7 \leq M_S \leq 8.8$	Nepal
[6] [7]	$M_W = 0.6495\ M_S + 2.163,\ 3.5 \leq M_S \leq 6.6$ $M_W = 1.157\ M_S - 1.176,\ 6.7 \leq M_S \leq 8.5$ $M_W = 0.499 M_L + 2.880,\ 3.5 \leq M_L \leq 7.3$ $M_W = (0.930 \pm 0.040)\ M_D + (0.35 \pm 0.14)$	Northeast India

	$M_W = 1.160\ mb - 0.663,\ 3.8 \leq mb \leq 7.0$	
[8]	$M_W = 1.043\ (\pm 0.02)\ mb - 0.080 \pm (0.08),\ 4.0 \leq mb \leq 7.0$ $Mw = 0.827\ (\pm 0.05)\ M_S + 1.181\ (\pm 0.21),\ 4.0 \leq M_S \leq 7.7$ $Mw = 1.111\ (\pm 0.03)\ M_D - 0.459\ (\pm 0.14),\ 2.8 \leq M_D \leq 7.3$ $M_W = 1.099\ (\pm 0.13)\ M - 0.354\ (\pm 0.68),\ 3.4 \leq M \leq 6.9$ $Mw = 1.017\ (\pm 0.02)\ M_L - 0.012\ (\pm 0.07),\ 2.8 \leq M_L \leq 7.2$	Türkiye

2.2.2. Completeness analysis of an earthquake catalog

Subsequent to homogenization, completeness analysis of an earthquake catalog is an essential prelude to seismic analysis, as it assesses the temporal and spatial consistency of recorded seismic events to determine the reliability of the dataset. This involves identifying the magnitude of completeness (M_C) above which all earthquakes are reliably detected and recorded over the study period and area. Techniques such as the GR frequency-magnitude distribution (FMD), visual inspection of cumulative event plots, and statistical methods like the Maximum Curvature (MAXC) method [9] or the Entire Magnitude Range (EMR) method are commonly employed to estimate Mc. The MAXC method is a commonly used technique for estimating the M_C in seismology, which represents the lowest magnitude at which all earthquakes in a catalog are reliably recorded. This method involves plotting the FMD of earthquakes and identifying the point of MAXC on the cumulative frequency curve. The MAXC point, often seen as a noticeable kink or bend in the curve, signifies the shift from incomplete to complete data. Earthquakes below this magnitude are frequently underreported due to detection limitations, while those above this magnitude are deemed complete. The MAXC method is straightforward and effective, providing a clear estimate of Mc, which is crucial for accurate seismic hazard assessments and b-value calculations. However, it is important to validate the results with other methods to ensure robustness and reliability. The EMR method is a sophisticated technique for estimating the M_C in seismology. Unlike simpler MAXC methods, the EMR method uses the entire range of recorded magnitudes to provide a statistically robust estimate of Mc. It involves fitting the observed magnitude distribution to a theoretical model that accounts for both the complete and incomplete parts of the dataset. By doing so, it can more accurately identify the magnitude threshold above which the catalog is complete. The EMR method is advantageous because it leverages all available data, reducing biases associated with underreporting of smaller earthquakes and providing a more precise and reliable estimate of Mc. This makes it particularly useful for detailed seismic hazard assessments and for ensuring the accuracy of other seismic parameters derived from earthquake catalogs. A robust completeness analysis ensures that subsequent seismic hazard assessments and modeling efforts are based on a dataset that accurately reflects the seismicity of the region, allowing for more precise predictions and risk mitigation strategies. Likewise, after the estimation of M_C value, another step ensuring earthquake catalog completeness is the analysis of the temporal completeness of the earthquake catalog. Various methods, as proposed by [10] and [11] have been developed to evaluate the temporal completeness of earthquake catalogs. These methods aid in understanding the presence of any gaps or inconsistencies in the recording of seismic events over time, which is essential for ensuring the accuracy and reliability of the catalog. The [10] methodology involve a series of essential steps for the evaluation of catalog completeness and reliability. Initially, earthquake events within the catalog are organized in ascending order based on their magnitudes. Subsequently, a graphical representation is created, with time depicted on the x-axis and the cumulative event count on the y-axis. The graph is then visually scrutinized for any alterations in the slope of the cumulative event count curve, indicating transitions from periods characterized by high magnitude incompleteness to phases of relative completeness. The specific point at which this change in slope occurs is recognized as the completeness point for the catalog. This approach offers a swift and visual mechanism for estimating the completeness point, thereby facilitating the assessment of data quality and catalog completeness, which proves valuable for researchers engaged in seismic analysis and interpretation. Furthermore, the methodology for [11] completeness test involves plotting the cumulative number of earthquakes against time for various magnitude ranges. By observing these plots, one identifies the point at which the cumulative frequency curve flattens, indicating a stable rate of earthquake occurrences and thus catalog completeness. Specifically, [11] approach requires dividing the earthquake catalog into time intervals and calculating the frequency of earthquakes within each interval. The completeness magnitude is then determined as the lowest magnitude for which the cumulative frequency stabilizes over the longest time period. This allows for the assessment of the time period during which the earthquake catalog is complete and reliable for seismic hazard analysis.

2.2.3. Declustering an earthquake catalog

In similitude to completeness of catalog, declustering an earthquake catalog is a crucial step before seismic analysis, aimed at isolating mainshock events by removing dependent events such as aftershocks and foreshocks. This process enhances the accuracy of seismic hazard assessments by focusing on the primary seismic activity, thus avoiding the inflation of seismicity rates due to closely spaced, temporally linked events. Common declustering methods include the windowing technique, where events within a specific time and spatial window around a mainshock are classified as aftershocks. Another one is the stochastic method, which uses probabilistic models to distinguish between independent and dependent events. By applying these techniques, the catalog is refined to better reflect the true rate of mainshock occurrences, which is critical for generating reliable seismic hazard models and understanding the underlying tectonic processes. The declustered catalog thus provides a clearer picture of the seismicity patterns, enabling more precise risk assessments and aiding in the development of effective mitigation strategies. Several declustering methods can be used to separate time-dependent from non-time-dependent events namely [12], [13], [14], and [15] to detect and dissociate dependent events. The above-mentioned declustering methods are used with the help of Zmap tool [16]. Each method takes different distance and time value for the declustering of the earthquake catalog. Likewise, the default values for the [12] declustering method is listed in the Table 2.2.

Table 2.2. The default parameters and its corresponding values for [12] Reasenberg (1985) declustering methods in the Zmap tool.

Parameter	**Standard**	**Simulation range**	
		Min	**Max**
T_{min} (days)	1.0	0.5	2.5
T_{max} (days)	10	3.0	15
P	0.95	0.9	0.99
Xmeff	4.0	0	1.0
X_K	0.5	1.6	1.8
R_{factor}	10	5	20

Where 'T_{min}' represents the minimum look-ahead time value for forming clusters when the initial event is not part of a cluster, and 'T_{max}' denotes the maximum look-ahead time value for cluster formation. 'P' is the probability of identifying the next clustered event, which is used to determine the look-ahead time 'T'. 'X_K' is the increment of the lower cut-off magnitude during clusters, and the effective lower magnitude cutoff for the catalog is adjusted as 'Xmeff = Xmeff + X_{KM}', where 'M' is the magnitude of the largest event in the cluster. Additionally, 'R_{factor}' specifies the number of crack radii around each earthquake within which new events are considered part of the cluster. Furthermore, the equations proposed by namely [13], [14], and [15] for the declustering are as follows (Table 2.3):

Table 2.3. The standard equations proposed for the different declustering methods are listed in the table.

S.no	**Method**	**Equation**
1	[13]	$t = 10^{0.032M+2.7389}$, M ≥ 6.5 $t = 10^{0.5409M-0.547}$, M< 6.5 $d = 10^{0.1238M+0.983}$

2	[14]	$t = e^{-2.87+1.235M}$ $d = e^{-1.024+0.804M}$
3	[15]	$t = e^{-3.95+(0.62+17.32M)^2}$, $M \geq 6.5$ $t = e^{2.8+0.024M}$, $M < 6.5$ $d = e^{1.77+(0.037+1.02M)^2}$

The practical examples for these different declustering methods are elaborated in the following chapters.

2.3. Methodology

There are several methods to estimate b-value. Here are some commonly used techniques:

2.3.1. Least Square Fit (LSF) Method:

This method involves fitting a straight line to the cumulative FMD in a log-linear plot using linear regression. The least squares method is a widely used technique for estimating the b-value in seismology, which represents the slope of the FMD of earthquakes. This method involves fitting the GR relationship, $Log_{10}N = a - bM$. This approach provides a straightforward and computationally efficient way to estimate the b-value, making it a popular choice among seismologists. Despite its advantages, the least squares method has several disadvantages when applied to b-value estimation in seismology. One significant drawback is its sensitivity to data quality and distribution. Earthquake catalogs often contain incomplete or biased data, particularly at lower magnitudes where detection is less reliable. This can lead to inaccuracies in the b-value estimation as the least squares method does not inherently account for such biases. Moreover, the method assumes that the errors in the magnitude data are normally distributed and that the relationship between $\log_{10}N(M)$ and M is linear, which may not always be the case in real-world seismic data. Another disadvantage of the least square method is its vulnerability to the influence of outliers. Large, rare earthquakes can have a disproportionate impact on the fitted line, potentially causing an inaccurate estimation of the b-value. This issue is particularly problematic in regions with complex tectonic settings where the FMD may deviate from the idealized GR model. Additionally, the least squares method does not provide a measure of uncertainty for the estimated b-value, making it difficult to assess the reliability of the results. To address these limitations, alternative methods such as MLE are often used. MLE is less sensitive to data incompleteness and outliers, providing a more robust estimation of the b-value. While being simple, it is less accurate than MLE due to its sensitivity to data binning and outliers.

2.3.2. Maximum Likelihood Estimation (MLE):

This is the most statistically robust method for b-value estimation. It accounts for the entire dataset and provides a more accurate and less biased b-value. The MLE approach proposed by [16] and [17] is commonly used in the investigations. [16] and [17] proposed the Aki-Utsu MLE relation, which describes a particular mathematical relationship employed in seismology to analyze seismic activity:

$$b = \frac{\log_{10} e}{\bar{M} - (M_c - \frac{\Delta M_{bin}}{2})} \dots (2)$$

Here, $\bar{M}$ is the median magnitude of earthquakes larger than the M_C, and M_{bin} is the binning width. The Aki-Utsu law suggests that the earthquake occurrence rate is influenced by both magnitude and the time elapsed since the previous earthquake. Larger magnitudes and shorter inter-event times lead to higher occurrence rates, while smaller magnitudes and longer inter-event times result in lower occurrence rates. The Aki-Utsu law has been widely used in seismic hazard assessment, earthquake forecasting, and the study of earthquake occurrence patterns. It provides a statistical framework for understanding the relationship between earthquake magnitudes, time intervals, and their impact on earthquake occurrence rates. The SD in b-value, which can be determined by applying the formula that [18] came up with and is given as:

$$\delta b = 2.3b^2 \sqrt{\Sigma \frac{M_i - \bar{M}}{n(n-1)}} \dots (3)$$

where 'n' represents the total number of occurrences in the data set.

2.4. Application of b-value analysis

The b-value, derived from the GR FMD, is a fundamental parameter in seismology that provides insights into the characteristics and underlying mechanics of earthquake occurrences. Its applications in seismology are diverse and impactful, extending from hazard assessment to understanding the physics of earthquake generation. Here are some key applications:

2.4.1. Earthquake Forecasting and Early Warning:

By monitoring temporal changes in the b-value, seismologists can detect anomalies that may precede significant seismic events. The b-value, which quantifies the relative frequency of small to large earthquakes, often exhibits notable changes prior to significant seismic events. Typically, a decrease in the b-value is observed before a major earthquake, suggesting an increased proportion of larger earthquakes relative to smaller ones. This change can be interpreted as a sign of increasing stress and strain accumulation in the Earth's crust, making it more likely to release a substantial amount of energy in the form of a large earthquake. Various studies ([19], [20], [21]) have been carried out over the globe as well as in northeast India for the spatio-temporal analysis of b-value before the occurrence of the major event. The recent studies carried over the major events in the northeast India includes the study carried by [19] in which they observed a temporal decline in the b-value before the occurrence of the Manipur 2016 earthquake (M_W 6.7). Furthermore, the b-value Thus, from the past studies it can be inferred that the temporal variations in the b-value before major earthquakes can serve as a potential precursor to major events.

2.4.2. Gumbel extreme value approach

In seismic hazard assessment, the application of the b-value in the GEV approach is pivotal for estimating the probability of rare, large-magnitude earthquakes. The GEV approach, rooted in the GR relationship, utilizes the b-value to model the FMD of seismic events and extrapolate this distribution to predict extreme events lying beyond the range of historical observations. With a lower b-value indicating a higher proportion of large earthquakes relative to small ones, regions characterized by such values are anticipated to have a steeper slope in their GR relationship, suggesting a greater likelihood of experiencing extreme seismic events. By integrating statistical methods with seismicity data, this approach offers valuable insights into seismic hazard assessment, aiding in disaster preparedness and risk mitigation strategies for communities residing in seismically active regions. Earlier, Emil Julius Gumbel's extreme value theory ([22], [23]) was used for hydrological studies but nowadays its use has been extended to various fields of science. The extreme value theory (EVT) proposed by Gumbel is used in various probabilistic seismic hazard studies for the estimation of seismic parameters including T(m), H(t), P(t), etc. Before using the Gumbel method some conditions need to be fulfilled initially the earthquake magnitude should not be limited secondly each event in the dataset should be independent. In addition, as the magnitude of the earthquakes increases, there should be a fall in the number of earthquake events observed per year. The conditions required for the Gumbel method are satisfied, allowing the application of the first asymptotic type distribution of extreme values. The probability of non-exceedance of an earthquake having a magnitude greater than 'm' observed within a time limit of 1.0 year is given by the relation:

$$G(m) = e^{-\alpha^{e^{-\beta m}}} \ldots (4)$$

Where α and β denote the Gumbel coefficients that are used for the seismic hazard studies. The relationship between Gumbel coefficients ('α';'β') and GR constants ('a';'b') is established by [24] is given as:

$$\alpha = 10^{a}, \beta = \frac{b}{\log e} \ldots (5)$$

$$N = \alpha e^{-\beta M} \ldots (6)$$

On further simplification equation (4) reduce to

$$\ln G(m) = -\alpha e^{-\beta m} \ldots (7)$$

Equation (7) can be written as $-\ln G(m) = \alpha e^{-\beta m} \ldots (8)$

In addition, modification equation (8) can be written as

$$\ln(-\ln G(m)) = \ln \alpha - \beta m \ldots (9)$$

Again equation (6) can be written as

$$\ln N(m) = \ln \alpha - \beta m \ldots (10)$$

Comparing equation (10) and equation (1) leads to

$$a = \frac{\ln \alpha}{\ln 10}, b = \frac{\beta}{\ln 10} \ldots (11)$$

The value of the most probable annual maximum magnitude (H) in terms of α and β is given by

$$H = \frac{\ln \alpha}{\beta} \ldots (12)$$

Similarly, the most probable magnitude H(t) of an earthquake during the period (t) is given as:

$$H(t) = \frac{\ln(\alpha t)}{\beta} = H + \frac{\ln t}{\beta} \ldots (13)$$

The mean return period (T(m)) between earthquake events is given as:

$$T(m) = \frac{1}{N(m)} = \frac{e^{\beta m}}{\alpha} \ldots (14)$$

Finally, the likelihood of an earthquake of magnitude (m) during the period (t) is given as (P(t)):

$$P_t = 1 - e^{-\alpha t^{e^{-\beta m}}} = 1 - e^{-tN} \ldots (15)$$

The above-mentioned equations are used for the seismic parameterization of different seismic zones and included in the fifth chapter of the thesis.

2.4.3. Seismic Hazard Assessment:

The b-value is vital for seismic hazard analysis as it helps estimate the probability of various earthquake magnitudes in a region ([25], [26]). Representing the slope of the earthquake FMD, the b-value is a key parameter in Probabilistic Seismic Hazard Analysis (PSHA). In PSHA, the b-value is used to measure the relative occurrence rates of smaller versus larger earthquakes within a region, which is essential for assessing seismic hazard. The b-value is incorporated into the characterization of seismic sources, defining the FMD for each source. This distribution, combined with ground motion prediction equations (GMPEs), helps evaluate the probability of exceeding different ground shaking levels at a site over a specified period. Accurately estimating the b-value enhances the modeling of earthquake occurrence, resulting in more reliable seismic hazard curves. These curves represent the likelihood of exceeding various levels of ground motion and are crucial for designing earthquake-resistant structures and informing public safety and emergency preparedness strategies.

2.4.4. Stress Analysis and Tectonic Studies:

Variations in the b-value can indicate changes in stress conditions within the Earth's crust. A lower b-value often signifies high-stress regions where larger earthquakes are more likely, while higher b-values suggest lower stress levels and a predominance of smaller quakes. This information aids in comprehending the stress regime and tectonic characteristics of a region. Variations in the b-value can provide insights into the changing stress conditions in a region. Specifically, the b-value tends to decrease in response to increasing tectonic stress. This implies that the earthquakes with large magnitudes are more as compare to earthquakes with small magnitudes. In regions experiencing rising stress levels, the b-value typically shows a downward trend. This decrease suggests that the Earth's crust is undergoing greater strain accumulation, making it more susceptible to releasing energy through larger seismic events. Conversely, a higher b-value is often associated with lower stress levels, where small earthquakes are more prevalent. Monitoring b-value variations involves calculating the b-value over successive time windows and observing trends. A consistent decrease in the b-value over time can signal increasing stress, potentially leading to a major earthquake. Several studies ([27], [28], [3]) have documented the association between b-value changes and the stress conditions. However, it is important to note that the b-value is influenced by various factors, including the heterogeneity of the Earth's crust, the presence of fluids, and local tectonic settings. Therefore, while b-value variations are a valuable stress indicator, they should be considered alongside other seismic and geophysical data for a comprehensive understanding of earthquake potential.

2.4.5. Volcanic Seismology:

In volcanic regions, the b-value can be significantly higher due to the high frequency of small earthquakes associated with magmatic activities. Observing variations in the b-value can offer insights into volcanic activities and assist in predicting eruptions. Various studies ([29], [30], [31], [32], [33]) have inferred that for volcanic regions the b-value varies up to 3.0.

Likewise, the detailed application of b-value analysis is discussed in the next chapters of the thesis.

2.5. References

[1] Gutenberg, B. and Richter, C. F. Frequency of earthquakes in California. *Bulletin of the Seismological Society of America,* 34 (4): 185–188, 1944.

[2] Allen, J. R. L. Earthquake magnitude-frequency, epicentral distance, and soft-sediment deformation in sedimentation basins. *Sedimentology,* 46: 67–77, 1986.

[3] Bora, D. K., Borah, K., Mahanta, R. and Borgohain, J. M. Seismic b-values and its correlation with seismic moment and Bouguer gravity anomaly over Indo-Burma ranges of Northeast India: tectonic implications. *Tectonophysics,* 728–729: 130–141, 2018.

[4] Sitharam, T. G. and Sil, A. Comprehensive seismic hazard assessment of Tripura and Mizoram states. *Journal of Earth System Science,* 123(4): 837–857, 2014.

[5] Kolathayar, S., Sitharam, T. G. and Vipin, K. S. Spatial variation of seismicity parameters across India and adjoining areas. *Natural Hazards,* 60(3): 1365–1379, 2012.

[6] Nath, S. K., Mandal, S., Adhikari, M. D. and Maiti, S. K. A unified earthquake catalogue for South Asia covering the period 1900–2014. *Natural Hazards,* 85(3): 1787–1810, 2017.

[7] Bora, D. K. Scaling relations of moment magnitude, local magnitude, and duration magnitude for earthquakes originated in Northeast India. *Earthquake Science,* 29(3): 153–164, 2016.

[8] Tan, O. A homogeneous earthquake catalogue for Turkey. *Natural Hazards and Earth System Sciences,* 21(7): 2059–2073, 2021.

[9] Wiemer, S. and Wyss, M. Minimum magnitude of completeness in earthquake catalogs: Examples from Alaska, the Western United States, and Japan. *Bulletin of the Seismological Society of America,* 90: 859–869, 2000.

[10] Tinti, S. and Mulargia, F. Effects of magnitude uncertainties in the Gutenberg-Richter frequency-magnitude law. *Bulletin of the Seismological Society of America,* 75: 1681–1697, 1985.

[11] Stepp, J.C. Analysis of completeness of the earthquake sample in the Puget Sound area and its effect on statistical estimates of earthquake hazard. *Int. Conf. Microzonat* II, 897–909, 1972.

[12] Reasenberg, P. Second-order moment of central California seismicity 1969–1982. *Journal of Geophysical Research: Solid Earth,* 90(B7): 5479–5495, 1985.

[13] Gardner, J.K. and Knopof, L. Is the sequence of earthquakes in southern California, with aftershocks removed, Poissonian? *Bulletin of the Seismological Society of America,* 64(5): 1363–1367, 1974.

[14] Uhrhammer, R. A. Characteristics of Northern and Central California Seismicity. *Earthquake Notes,* 57(1):21, 1986.

[15] Grünthal, G. The history of historical earthquake research in Germany. *Annals of Geophysics,* 47(2–3):631– 643, 2004.

[16] Aki, K. Maximum likelihood estimate of b in the formula log N = a - bm and its confidence limits. *Bulletin of Earthquake Research,* 43: 237–239, 1965.

[17] Utsu, T. A method for determining the value of the formula logN = a - bM showing the magnitude-frequency relation for earthquakes. *Geophys. Geophysical bulletin of Hokkaido University,* 13: 99–10, 1965.

[18] Shi, Y. and Bolt, B. A. The standard error of the magnitude-frequency b value. *Bulletin of the Seismological Society of America,* 72(5):1677–1687, 1982.

[19] Borgohain, J. M., Borah, K., Biswas, R. & Bora, D. K. Seismic b-value anomalies prior to the 3rd January 2016, Mw = 6.7 Manipur Earthquake of Northeast India. *Journal of Asian Earth Sciences,* 154:42–48, 2018.

[20] Chen, J. and Zhu, S. Spatial and temporal b-value precursors preceding the 2008 Wenchuan, China, earthquake (Mw = 7.9): implications for earthquake prediction. *Geomatics, Natural Hazards and Risk, 11*(1): 1196–1211, 2020.

[21] Tsukakoshi, Y. and Shimazaki, K. Decreased b-value prior to the M 6.2 Northern Miyagi, Japan, earthquake of 26 July 2003. *Earth Planet Space,* 60: 915–924, 2008.

[22] Gumbel, E. J. Les valeurs extrêmes des distributions statistiques. *Annales de l'Institut Henri Poincaré,* 5: 115–158, 1935.

[23] Gumbel, E. J. 1958. *Statistics of Extremes*. 375. Columbia University Press.

[24] Tezcan, S. S. 1996. *Probability Analysis of Earthquake Magnitudes*. 26. Turkish Earthquake Foundation, Istanbul.

[25] Nicknam, A., Khanzadi, M., Motaghed, S. and Yazdani, A. Applying b-value variation to seismic hazard analysis using closed-form joint probability distribution. *Journal of Vibroengineering*, *16*(3): 1376–1386, 2014.

[26] Taroni, M. and Akinci, A. Good practices in PSHA: declustering, b-value estimation, foreshocks and aftershock inclusion; a case study in Italy. *Geophysical Journal International, 224*(2): 1174–1187, 2021.

[27] Khan, P. K., Ghosh, M., Chakraborty, P. P. and Mukherjee, D. Seismic b-value and the assessment of ambient stress in Northeast India. *Pure and Applied Geophysics,* 168: 1693–1706, 2011.

[28] Scholz, C. H. On the stress dependence of the earthquake b value. *Geophysical Research Letters, 42*(5): 1399–1402, 2015.

[29] Wyss, M., Shimazaki, K. and Wiemer, S. Mapping active magma chambers by b-value beneath the off-Ito volcano. *Journal of Geophysical Research,* 102(20): 20413–20422, 1997.

[30] Frohlich, C. and Davis, S. Teleseismic b-values: or much ado about 1.0. *Journal of Geophysical Research,* 98: 631–644, 1993.

[31] McNutt, S. R. 2005. Volcanic seismology. *Annual Review of Earth and Planetary Sciences,* 33: 461–491.

[32] Bridges, D. L. and Gao, S. S. Spatial variation of seismic b-values beneath Makushin volcano Unalaska Island, Alaska. *Earth and Planetary Science Letters,* 245: 408–415, 2006.

[33] Farrell, J., Husen, S. and Smith, R. B. Earthquake swarm and b-value characterization of the Yellowstone volcano-tectonic system. *Journal of Volcanology and Geothermal Research,* 188: 260–276, 2009.