CHAPTER 8

Conclusion and Future Study

In this thesis, we have conducted a comprehensive study on hybrid functional fractional differential equations and inclusions, focusing on the existence, uniqueness, stability, and controllability of solutions. By employing advanced mathematical tools such as Dhage's fixed point theorem, Krasnoselskii's fixed point theorem, Schauder's fixed point theorem, Banach contraction principle, and topological degree theory, we have extended existing results in fractional calculus to hybrid systems. The analysis was further supported by techniques in Banach algebras and semigroup theory, demonstrating the mathematical depth of our approach.

One of the key contributions of this thesis is the use of Dhage's fixed point theorem for both single-valued and multivalued operators in different hybrid systems. In each chapter we have addressed a distinct type of fractional differential equations or inclusions, incorporating nonlocal boundary conditions, impulsive effects, p-Laplacian operators, and multi-point boundary conditions.

The study also incorporates various fractional derivatives, such as Caputo, Riemann-Liouville, and more generalized forms like Hilfer, ψ-Caputo, and Caputo-Katugampola. This broadens the scope of hybrid fractional calculus and its real-world applications. Additionally, the analysis of existence and controllability in hybrid fractional evolution inclusions offers valuable insights into guiding and managing these systems, addressing a gap in existing research.

The use of multivalued operators and topological degree theory further strengthens the study, enhancing our understanding of the solvability of these systems. Beyond the theoretical results, illustrative examples are included throughout to demonstrate the implications of the developed results.

While this research has established fundamental results, several open directions remain for further investigation. Future studies may extend these models to stochastic systems, develop efficient numerical schemes for solving complex hybrid fractional equations, and explore applications in engineering, physics, and control theory. The growing interest in fractional calculus shows that hybrid systems will continue to be important in mathematical modelling and applied sciences. This also encourages further study of more generalized fractional derivatives and finding suitable fixed point theorems, also the combined effects of impulses and delays in fractional hybrid models, as well as new problems in hybrid evolution systems.

In conclusion, this thesis contributes to a solid foundation for future advancements in the theory and applications of hybrid fractional differential equations and inclusions.