Abstract

This thesis is dedicated to the study of hybrid functional fractional differential equations and inclusions, with a focus on the existence, uniqueness, stability, and controllability of solutions. The work is divided into eight chapters, each addressing a different type of fractional differential system or inclusion, extending existing results in fractional calculus to hybrid systems.

Chapter 1: In this chapter, we present essential foundational concepts and preliminary results, including key definitions from fractional calculus and techniques that are required to analyse qualitative properties of solutions in the remaining chapters. We introduce various fixed point theorems and provide an overview of semigroup theory, which serves as a foundation for the analysis of fractional differential equations. Additionally, in this chapter we review relevant literature that has inspired and motivated the exploration of fractional differential equations, highlighting the significance of this growing field of study.

Chapter 2: The second chapter begins with the analysis of a Caputo type hybrid functional fractional differential equations, where the fractional derivative is of order $1 < q \leq 2$. The equation is accompanied by a nonlocal boundary condition, which is used for modelling some real-world phenomena. The primary objective of this chapter is to establish the existence of solutions using Dhage's fixed point theorem for two operators in the framework of Banach algebra. This powerful fixed point result is well-suited for the hybrid systems.

Chapter 3: In this chapter, we explore another type of hybrid fractional differential equations, specifically a Volterra-Fredholm type involving Caputo fractional derivative with order $1 < q < 2$ accompanied by nonlocal boundary conditions. The existence of solutions is established using Dhage's fixed point theorem for three operators in Banach algebras. An example is provided to illustrate the theoretical results, demonstrating how the established method can be applied, involving fractional order systems.

Chapter 4: Moving to the fourth chapter, where we investigate a class of nonlinear p-Laplacian implicit hybrid fractional differential equations. The fractional derivatives

considered here are of mixed types: Caputo and Riemann-Liouville. The main objective of this chapter is to establish the existence and uniqueness of solutions, as well as their Hyers-Ulam stability, which ensures that solutions remain reliable even when there are small perturbations in the initial conditions. Krasnoselskii's fixed point method is employed, with specific adaptations to handle the p-Laplacian operator and the hybrid nature of the system. To establish the uniqueness result, Banach contraction principle is used. The theoretical results are supported by an example that highlights the implications of the findings.

Chapter 5: In the fifth chapter, we address a more specialized class of problems, namely hybrid non-instantaneous impulsive fractional differential equations involving the p-Laplacian operator and Caputo-Katugampola fractional derivatives. The impulsive effects considered here are non-instantaneous, meaning that they occur over intervals rather than at discrete points. In this chapter, the existence of solutions is established using Schauder's fixed point theorem, while the uniqueness of these solutions is ensured through the Banach contraction principle. As with previous chapters, an example is provided to demonstrate how the theoretical results apply.

Chapter 6: We devote this chapter to explore a semilinear hybrid evolution inclusion involving a Hilfer fractional derivative. The main focus of this chapter is on proving the existence and exact controllability of solutions, which are used for understanding how such systems can be manipulated to achieve desired outcomes. The nonlocal condition included in the problem adds further complexity. The results are obtained using Dhage's fixed point theorem for multivalued operators. An illustrative example is included to highlight the practical significance of the results.

Chapter 7: In this chapter, we focus on analysing a hybrid fractional differential equation with multi-point boundary conditions, formulated using the ψ-Caputo derivative, which is a generalized fractional derivative. To emphasize the originality and significance of this work, we employ a purely mathematical approach based on topological degree theory to establish the existence of solutions. Additionally, the uniqueness of the solution is examined through the Banach contraction principle. Finally we conclude this chapter with two illustrative examples. The first example provides an analytical validation of the main results and the second example complements this analysis by presenting numerical simulations.

Chapter 8: In the final chapter, we summarize the key points from all the chapters and discuss potential directions for future research in this field.