CHAPTER 6
PHENOLOGICAL MODELLING

Objective IV: To examine the impact of weather parameters on different phenophases of plants using statistical models

6.1 Introduction

The occurrences and durations of phenophases in plants in association with changes in one or more climatic variables such as temperature, precipitation, relative humidity, insolation, etc. signify the sensitivity of phenophases to climatic conditions. Consequently, this sensitivity highlights the adaptation of plants to the climate of a specific area [1, 2]. Therefore, due to the presence of local adaptations in plants along with the impacts of the current scenario of climate change, modeling the phenological responses of plants to climatic variables has become an important aspect of phenological studies [2, 3, 4, 5, 6]. Statistical analysis of phenophases with climatic variables and the subsequent development of phenological models tend to provide significant insights into the roles of climatic variables as phenological cues [7]. These phenological models effectively establish the correlation between the timing of phenological events and the prevailing climatic conditions [9]. According to Roberts [7], statistical models can be classified into two broad categories: (a) regression-based models, where the underlying biological processes are not prioritized; (b) mechanistic models, which are developed to relate different biological processes inferred from experimental studies.

Phenological studies frequently use regression-based models because of their flexibility and robustness, which makes them applicable for small datasets [8]. Traditionally, phenological models are developed based on the assumption of linear relationships between climatic variables and phenophases [9]. But besides linear regression, other regression models such stepwise regression, multiple regression, penalized regression, and others are also used to obtain more accurate projections of phenological events [8, 9]. A commonly observed limitation in developing regression-based models is the presence of interdependent variables or regressors [10]. In the process of developing regression models, where several variables are required and their removal reduces the models's utility, the presence of collinearity between the required variables can decrease the model's effectiveness [10, 11]. Therefore, to address the issues associated with the multicollinearity of variables and subsequently obtain stable regression coefficients, it is necessary to implement regularization methods such as lasso regression, ridge regression, etc. as alternatives to the ordinary least squares method [10, 11].

6.2 Methodology

Two phenophases of trees, i.e., leaf initiation and flowering, displaying distinct seasonality in the Sonai Rupai Wildlife Sanctuary were selected to develop regression models using climatic variables: mean air temperature, precipitation amount, mean relative humidity (RH) and mean photosynthetically active radiation (PAR) for each month of the study period (2021-2023) as regressors. Three types of regression models: simple linear regression, binomial generalized linear models and ridge regression models were developed in R using stats, lmtest and lmridge packages [10, 12, 13].

The linear regression models between the phenophases and climatic variables were developed after applying square root normalization to the variables. The ridge regression models were created using standardized variables and optimal biasing parameters (k). The addition of a penalty term to the loss function results in constraining the ridge regression models' coefficients, ensuring that the coefficients are not skewed by outliers, thereby reducing their variances [10, 14]. The standardizations of the variables were done as described by Belsley et al. [15] and Draper and Smith [16],

$$X_j = \frac{x_{ij} - \bar{x}_j}{\sqrt{\Sigma(x_{ij} - \bar{x}_j)^2}} \qquad \text{(Eq. 1)}$$

where, j = 1, 2, ..., p such that Xj=0 and $\bar{X}j'Xj = 1$, given X_j is the jth column of the matrix X (Eq. 1) [10]. The coefficients of the independent variables were determined using the formula

$$\hat{\beta}_{R_k} = (X'X + kI_p)^{-1}X'y \qquad \text{(Eq. 2)}$$

where the vector $\hat{\beta}_{R_k}$ is the ridge regression's standardized coefficients of order $p \times 1$ and kI_p is a positive semi-definite matrix added to the $X'X$ matrix (Eq. 2) [10]. The ridge regression model for the phenophases along with the meteorological parameters is

$$y = \beta_0 + \beta_1 Precipitation + \beta_2 RH + \beta_3 Temperature + \beta_4 PAR \qquad \text{(Eq. 3)}$$

where, y is the phenophase, β_0 is the constant, β_1, β_2, β_3, and β_4 are variable coefficients for precipitation, RH, temperature, and PAR, respectively (Eq. 3).

For the evaluation of models that best fit the observed phenological data, the root mean square error (RMSE), AIC, and corrected AIC (AICc) of the regression models were calculated using the AICcmodavg and Metrics packages in R [17, 18]. The RMSE determines the deviation of the predicted values of the model from the observed values. The calculation formula of RMSE uses observed value (Oi), predicted value (Pi) and number of samples (n) (Eq. 4).

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(O_i - P_i)^2}{n}} \quad \text{(Eq. 4)}$$

The AIC and AICc are also used to evaluate the goodness of the models based on the concept of information entropy [19, 20]. The formulae of AIC and AICc contain K i.e. the number of parameters equipped in the models (Eqs. 5 and 6).

$$AIC = n * log(RMSE^2) + 2K + \frac{2K(K+1)}{n-K-1} \quad \text{(Eq. 5)}$$

$$AICc = AIC + \frac{2K^2+2K}{n-K-1} \quad \text{(Eq. 6)}$$

The binomial generalized linear models were constructed using the logit link function, and the phenological observations were represented as the proportion of species displaying a particular phenophase relative to the total number of species considered for the study.

6.3 Results

In the phenological study, four climatic variables were selected that displayed weak negative to strong positive correlation among themselves. A weak negative correlation (-0.06) was observed between RH and PAR (Table 6.1). Moderate positive correlation was observed between precipitation and RH (0.52), and temperature and RH (0.43) (Table 6.1). However, a strong positive correlation of 0.72 was observed between temperature and PAR, indicating a close relationship between these two variables. The presence of moderate and strong correlations between the variables highlights the interplays between the climatic variables which act as cues for the occurrences of phenophases.

Table 6.1: Correlation matrix of the selected climatic variables

Climatic variables	Precipitation	Temperature	RH	PAR
Precipitation	1.00	0.63	0.52	0.05
Temperature	0.63	1.00	0.43	0.72
RH	0.52	0.43	1.00	-0.06
PAR	0.05	0.72	-0.06	1.00

Simple regression models of leaf initiation with precipitation, relative humidity, temperature, and photosynthetically active radiation were generated. The goodness of the generated models was evaluated based on the values of R^2, adjusted R^2, AIC, AICc, and RMSE (Table 6.2). Among the four models of leaf initiation, the R^2 (0.54) and adjusted R^2 (0.51) values were highest for the RH vs Leaf initiation model. Additionally, this model showed the lowest AICc value of 16.79 and RMSE value of 0.30. Therefore, among the models of leaf initiation, the RH vs Leaf initiation model with a p-value of 0.00 displayed the best-fitting relationship between a phenophase and a climatic parameter (Fig. 6.1 (d), Table 6.2). On the contrary, the R^2 and adjusted R^2 values of 0.18 and 0.14 respectively were the lowest, and the values of AICc and RMSE of 30.53 and 0.40 respectively were highest for the Precipitation vs Leaf initiation model (Table 6.2). Thus, it is implied that the Precipitation vs Leaf initiation model is a lesser fit to the observed variables in comparison to the different models of leaf initiation (Fig. 6.2 (a)). The AIC of the null model for leaf initiation is 111.39, yielding ΔAIC of 95.17 with RH vs Leaf initiation model. The Shapiro-Wilk test of the residuals for this model gives a value of 0.93 with a p-value of 0.10, thereby upholding the fitness of the model.

Similarly, simple regression models were generated for the flowering phenophase using the selected climatic parameters. Among the four models, three models, i.e., Precipitation vs Flowering, Temperature vs Flowering and PAR vs Flowering hold p-values<0.05. The R2 and adjusted R2 values of Temperature vs Flowering and PAR vs Flowering models are higher than those of Precipitation vs Flowering model (Table 6.2). However, the Shapiro-Wilk test of the residuals of the considered models for flowering shows that only Precipitation vs Flowering model has a value of 0.94 with a p-value of 0.14. As the null model for flowering has an AIC of 94.89, the ΔAIC between the Precipitation vs Flowering model and the null model is 43.79, thereby implying that the aforementioned model is a better fit for the observed data (Table 6.2). Ridge regression models were also generated for the two phenophases: leaf initiation and flowering using all the selected climatic variables. The fittings of the selected models were determined through the comparison and assessment of the values of R^2, adjusted R^2, AIC, and RMSE. The p-values of both the ridge regression models were estimated to be 0.00 and 0.05, respectively. The R^2 and adjusted R^2 values of both leaf initiation and flowering models were higher compared to the simple regression models of RH vs Leaf initiation and PAR vs Flowering (Table 6.2). The ridge regression models had marginally higher RMSE values compared to the simple regression models. However, in ridge regression models, the AIC values for the leaf initiation and flowering models were comparatively lower, having 3.17 and 17.72, respectively (Table 6.2). The equations for ridge regression models are:

Leaf Initiation = -14.8526 - 0.0015 * Precipitation + 0.1995 * RH + 0.1187 * Temperature + 0.0579 * PAR, where k = 1.87 (5)

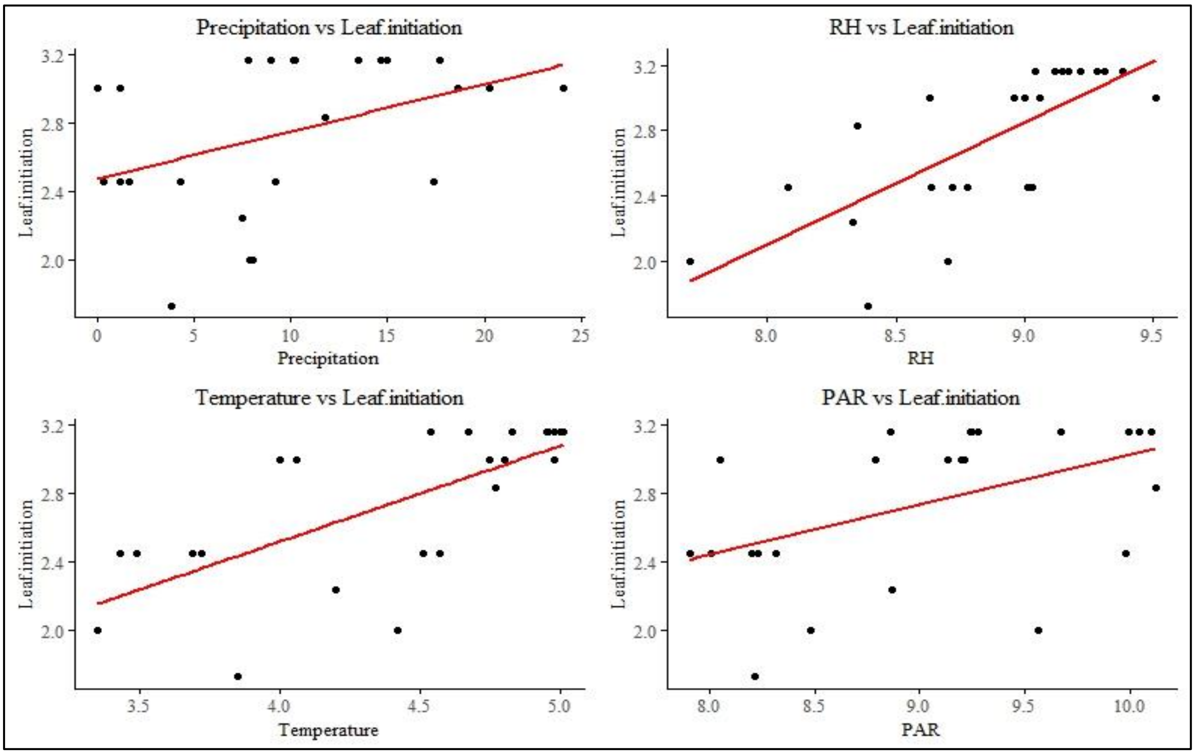
Flowering = 5.8913 + 0.0027 * Precipitation – 0.0999 * RH + 0.1051 * Temperature + 0.0125 * PAR, where k = 2.50 (6)

The binomial generalized linear model for leaf initiation with all the climatic variables yielded an AIC of 59.435 in comparison to the AIC of 122.94 for the null model of leaf initiation. The null deviance and residual deviance of the leaf initiation model were 80.99 on 23 degrees of freedom (df) and 9.49 on 19 df, respectively. Similarly, for flowering, the AIC of the model was 76.87, while the AIC of the null model was 87.97. The null deviance and the residual deviance of the flowering model were 46.66 on 23 df and 27.56 on 19 df. The GLM plots of the predicted proportion of leaf initiation and flowering show that responses of the phenophases differ for different climatic variables (Fig.. 6.2, 6.3). The predicted leaf initiation showed positive responses to temperature, RH, and PAR, while a negative response was observed with precipitation (Fig.. 6.2). Similarly, flowering showed positive responses to temperature and precipitation and negative responses to PAR and RH (Fig.. 6.3). However, the broadening of the confidence intervals for both the phenophases at extreme values of climatic variables indicates the uncertainty of responses through the model predictions.

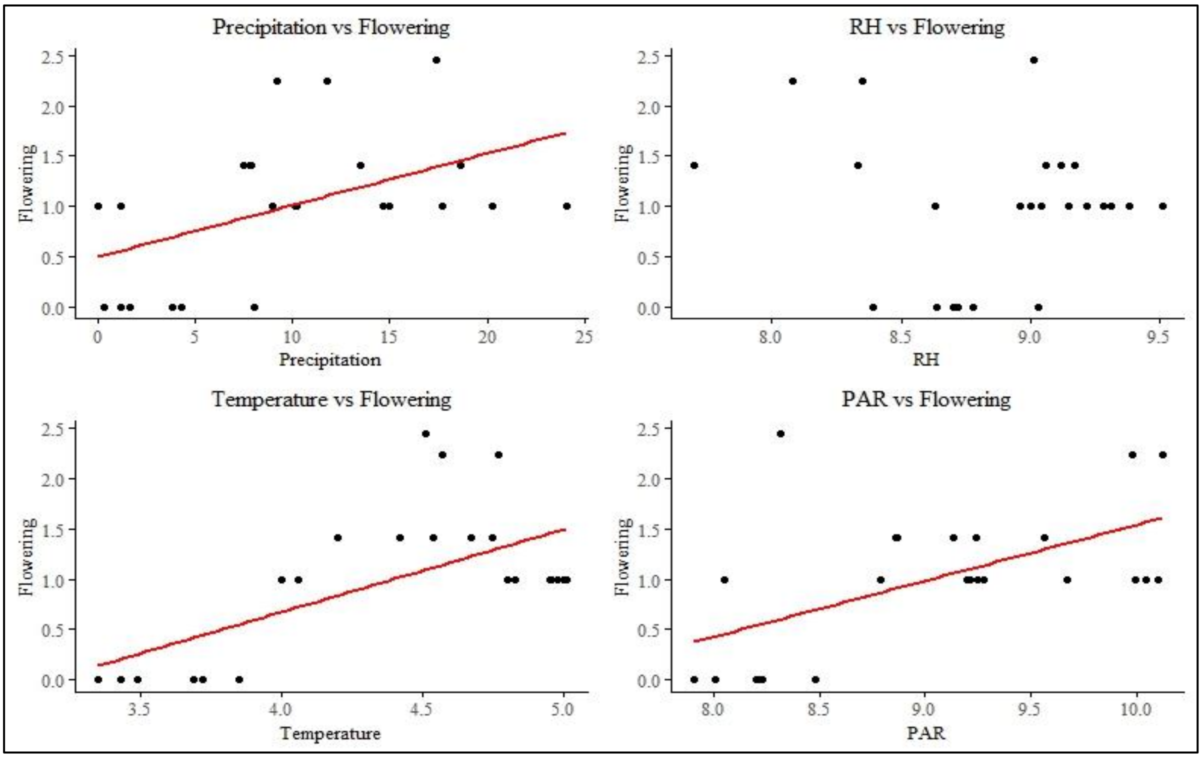
Table 6.2: Regression models of leaf initiation and flowering with different climatic variables

Model	Coefficients				R^2	Adjusted R^2	AIC	AICc	ΔAIC	RMSE	p-value
	Precipitation	Temperature	RH	PAR							
Simple Regression models											
Precipitation vs Leaf initiation	0.03				0.18	0.14	29.96	30.53	81.43	0.40	0.04
Temperature vs Leaf initiation		0.56			0.49	0.47	18.47	19.04	92.92	0.31	0.00
RH vs Leaf initiation			0.74		0.54	0.51	16.22	16.79	95.17	0.30	0.00
PAR vs Leaf Initiation				0.29	0.23	0.19	28.40	28.97	82.42	0.39	0.02
Precipitation vs Flowering	0.05				0.23	0.20	51.10	51.67	43.79	0.62	0.02
Temperature vs Flowering		0.82			0.41	0.38	45.01	45.58	49.88	0.55	0.00
RH vs Flowering			-0.19		0.01	-0.03	57.18	57.75	37.71	0.70	0.59
PAR vs Flowering				0.56	0.32	0.29	48.32	48.89	46.57	0.58	0.00
Ridge regression models											
Leaf initiation ~ Precipitation +	-0.0015	0.1187	0.1995**	0.0579*	0.73	0.69	3.17	5.28	106.11	0.94	0.00

Temperature + RH + PAR											
Flowering ~ Precipitation + Temperature + RH + PAR	0.0027	0.105	-0.0999*	0.0125	0.29	0.18	17.72	19.83	75.06	1.28	0.05
Binomial generalized linear models											
Leaf initiation ~ Precipitation + Temperature + RH + PAR	-0.004	0.15	0.19	0.06	—	—	59.44	62.77	63.50	—	—
Flowering ~ Precipitation + Temperature + RH + PAR	0.001	0.26	-0.10	-0.03	—	—	76.87	80.20	11.10	—	—

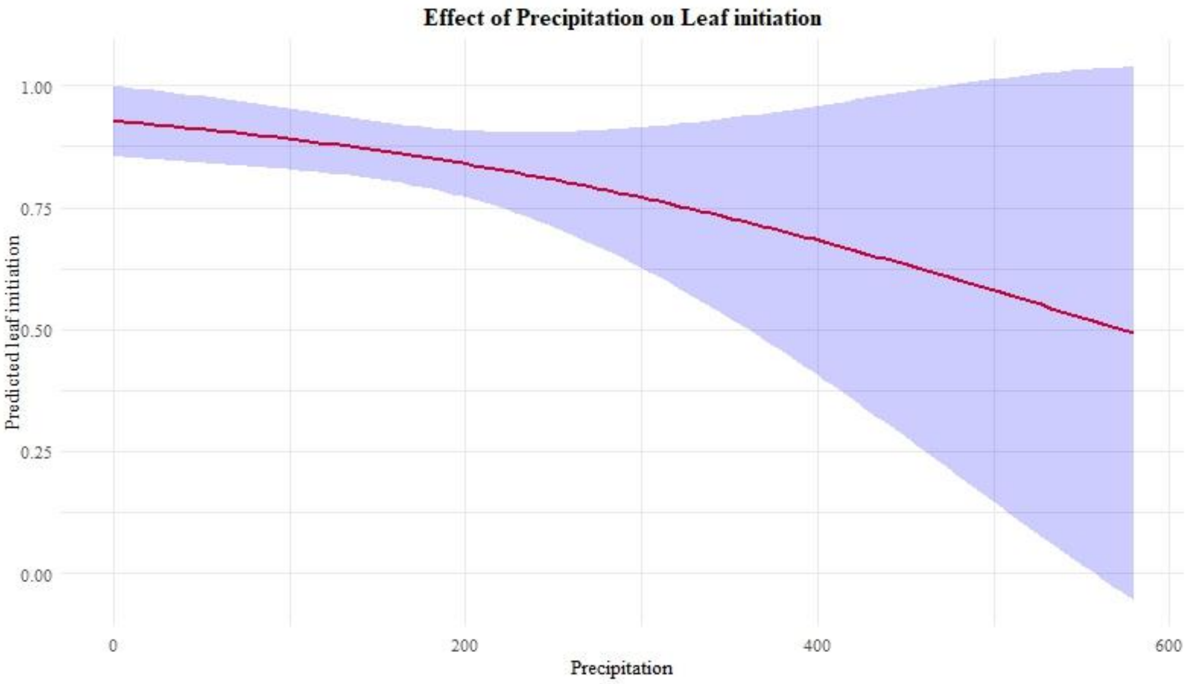


6.1 (a)

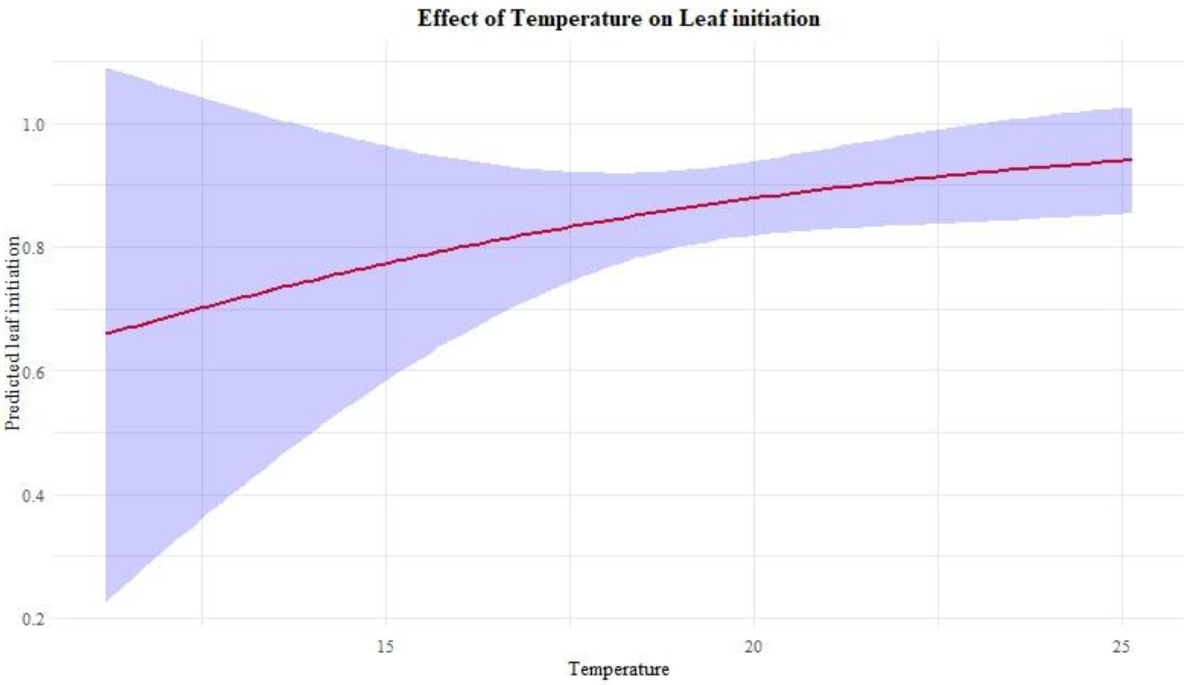


6.1 (b)

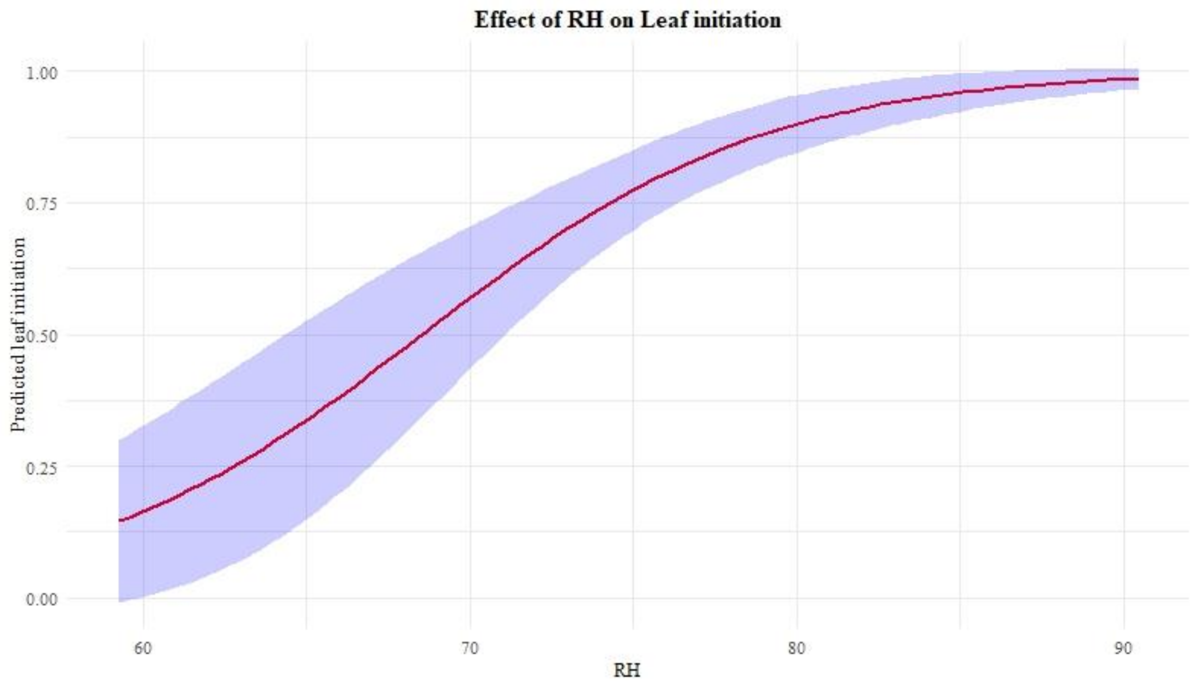
Fig. 6.1: Linear regression plot displaying significant relationships ($p < 0.05$) between phenophases: (a) leaf initiation, and (b) flowering with climatic variables


Effect of Precipitation on Leaf initiation
1.00
0.75
0.50
0.25
0.00
Predicted leaf initiation
0
200
400
600
Precipitation

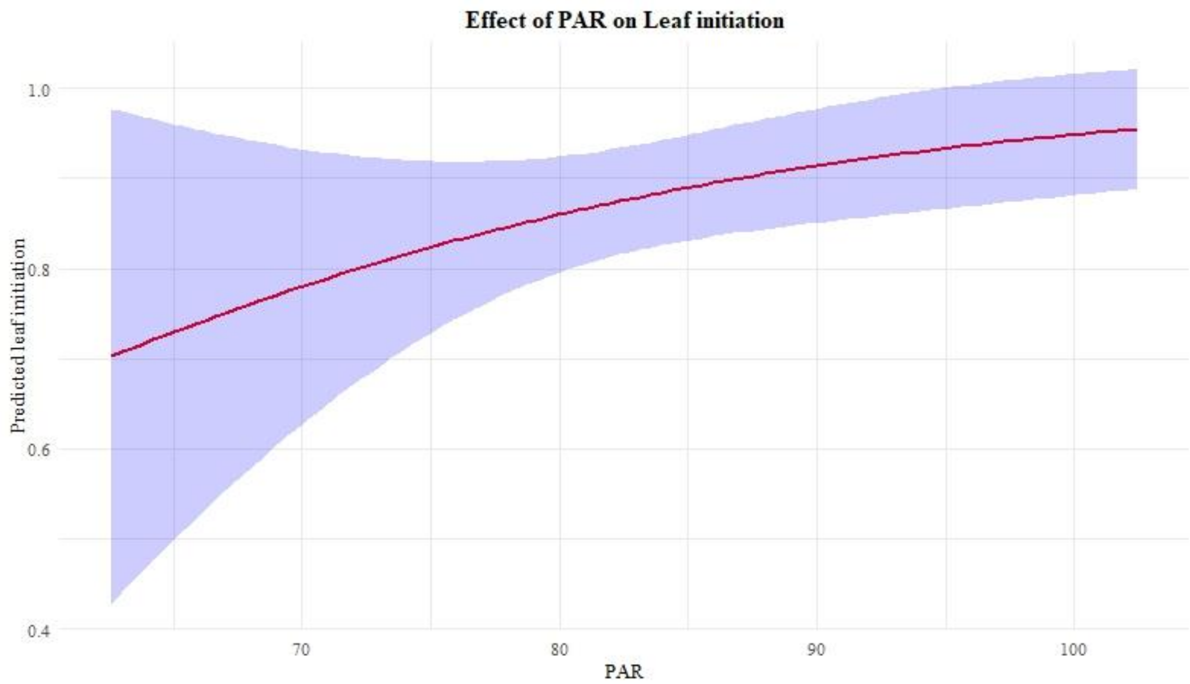
6.2 (a)


Effect of Temperature on Leaf initiation
1.0
0.8
0.6
0.4
0.2
Predicted leaf initiation
15
20
25
Temperature

6.2 (b)

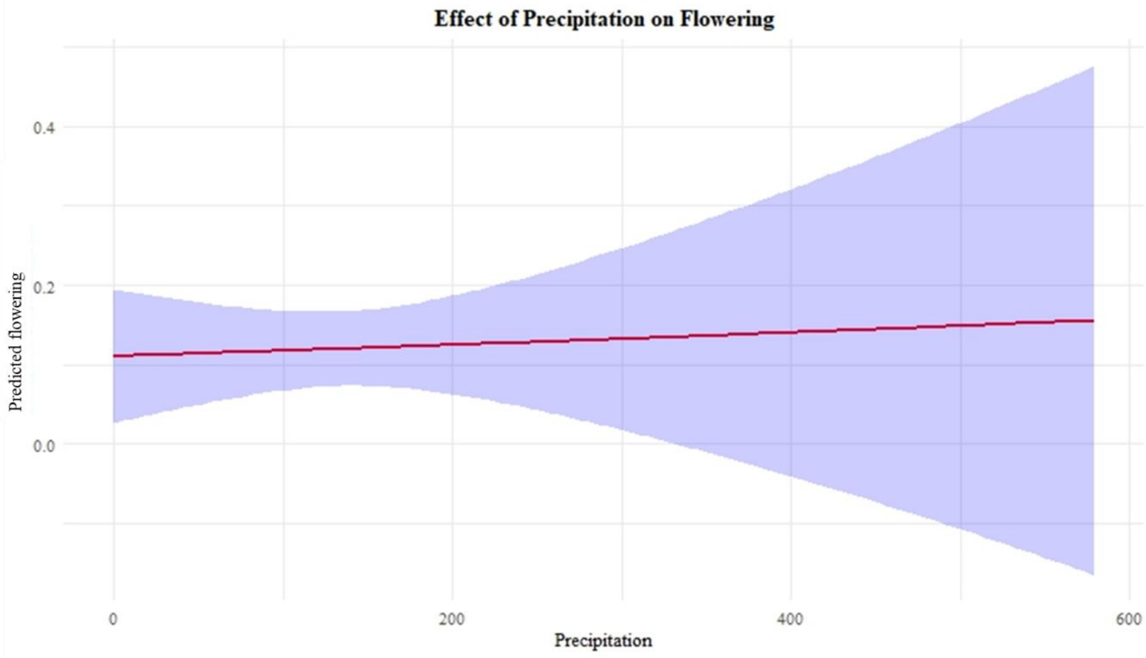


6.2 (c)

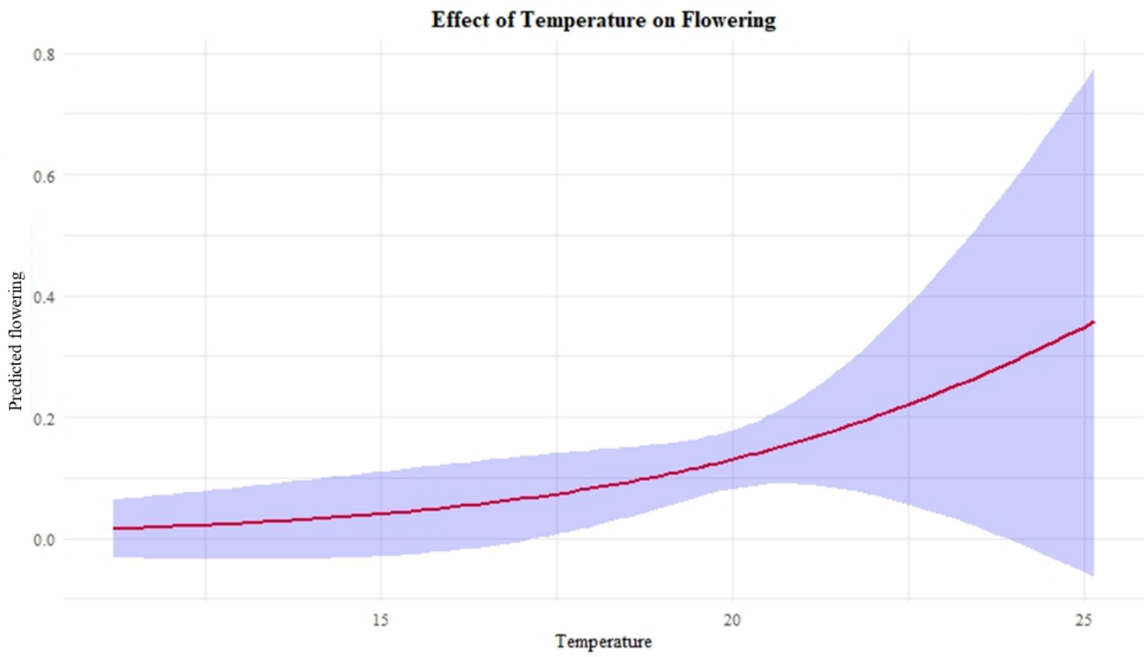


6.2 (d)

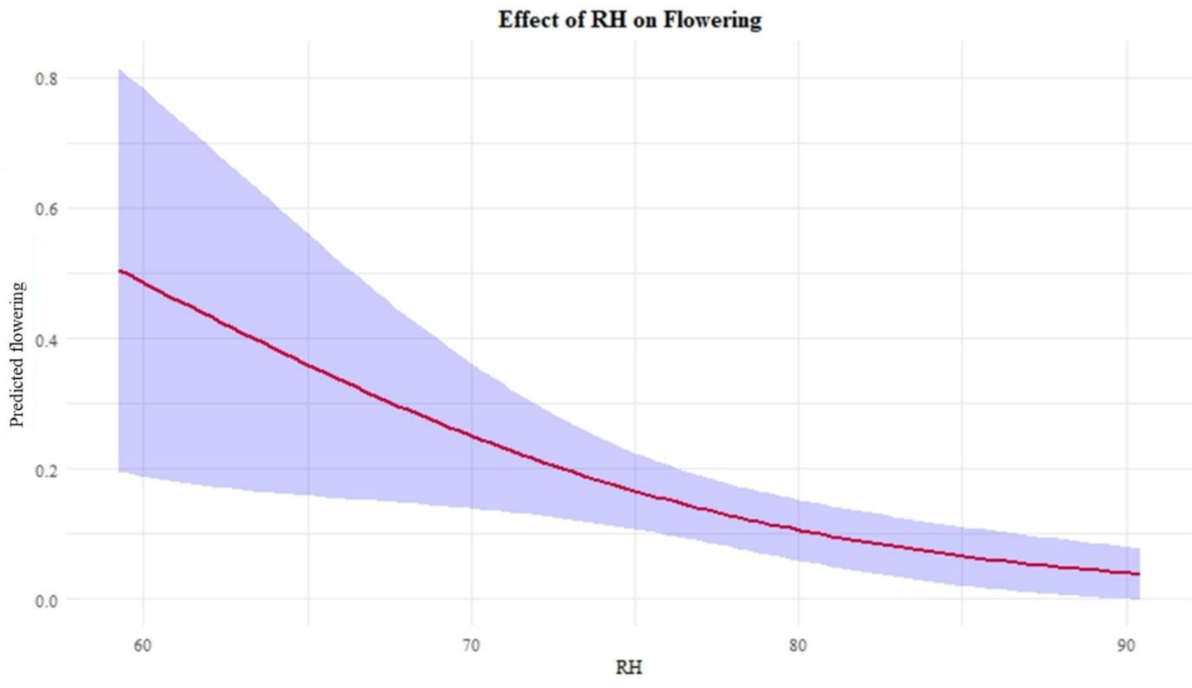
Fig. 6.2: GLM plots of predicted leaf initiation in response to changes in (a) precipitation, (b) temperature, (c) PAR and (d) RH


Effect of Precipitation on Flowering
Predicted flowering
0.4
0.2
0.0
0
200
400
600
Precipitation

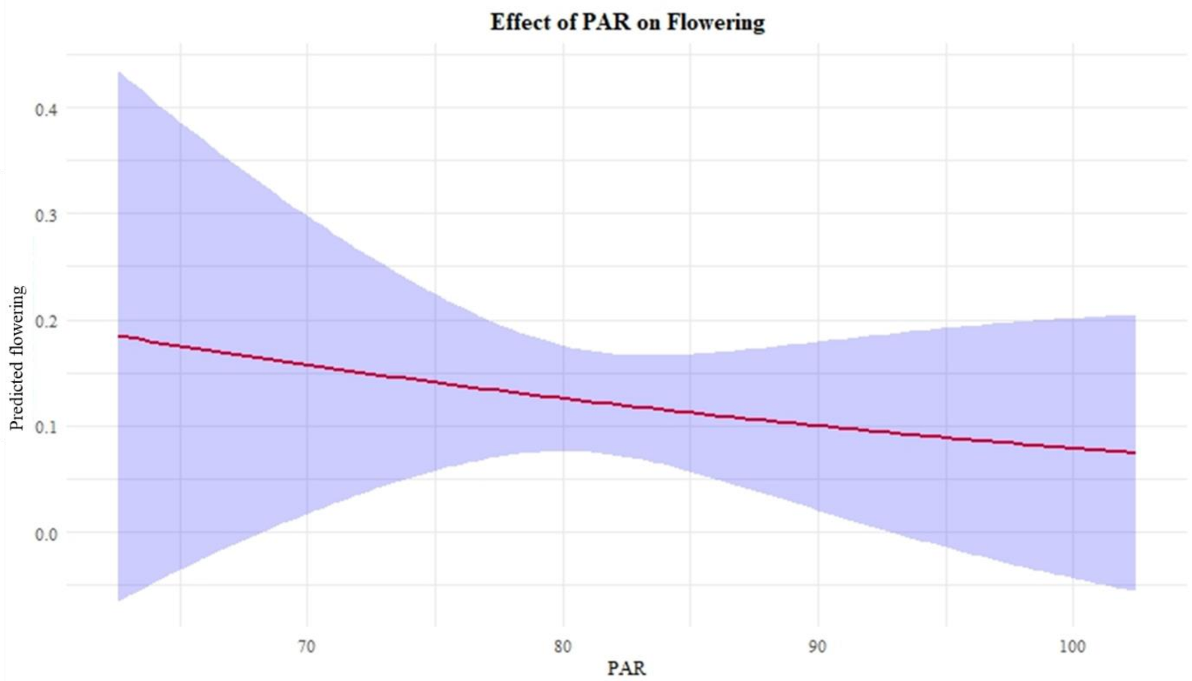
6.3 (a)


Effect of Temperature on Flowering
Predicted flowering
0.8
0.6
0.4
0.2
0.0
15
20
25
Temperature

6.3 (b)



6.3 (c)



6.3 (d)

Fig. 6.3: GLM plots of predicted flowering in response to changes in (a) precipitation, (b) temperature, (c) PAR and (d) RH

6.4 Discussion

The application of Spearman's rank correlation highlights the existence of positive and negative associations, as it does not prove a cause-and-effect or any type of potential relationship between the phenophases and changes in the climatic variables [21]. The occurrences and durations of the phenophases are responses to complex interactions with both the abiotic and biotic environmental variables [22]. The implementation of regression techniques on phenological observations allows the establishment of potential relationships amongst different variables and the generation of predictive models. The selection of the predictive models is done on the basis of several criteria, such as adjusted R^2, AIC, AICc, etc., which can indicate a better goodness of fit [23]. According to Gregorich et al. [11], information criteria such as AIC can be used to determine the most predictive model. Although the higher value of adjusted R^2, and lower values of AIC as well as AICc signify better models, certain cons, such as the influence of outliers as well as sizes of the datasets, are associated with the model selection criteria [23].

Although the development of regression models by multiple regression is a sensible approach to understand the effects of several independent variables on a dependent variable, the presence of collinearity between the variables causes significant variances in the model's predictions [10, 11]. In this study, it is observed that interdependence is present among the selected climatic variables. To address the underlying issue of multicollinearity among the variables, multiple regression can be performed, either considering the ambiguity associated with the coefficients of regression or with stepwise selection of variables [11]. Therefore, simple linear regression models for phenophases have the benefits of being devoid of the issue of multicollinearity, as they have only one independent variable.

Several studies have reported the occurrences and durations of vegetative phenophases in response to seasonal variations in temperature, precipitation, photoperiod, etc. [24, 25, 26, 27]. In this study, simple regression models for leaf initiation show that all 4 climatic variables have a significant relationship with the occurrence of leaf initiation. Compared to the temperate regions, the presence of weak seasonality in the tropical region along with a mosaic distribution of evergreen and deciduous trees results in a less prominent onset of green waves [9, 28]. However, in the Indian subcontinent, specifically Northeast India, the occurrence of the monsoon causes significant changes in the climatic variables thereby influencing the vegetative phenological patterns [28, 29]. This is clearly reflected in this study, and other studies carried out in parts of the Eastern Himalayan region of Northeast India agree that seasonal variations in rainfall and temperature during different times of the year causes phenophases to occur [30, 31]. However, the reproductive phenophases of plants do not follow the same pattern. The flowering models had lower R^2 and adjusted R^2 values than the leaf initiation models. This indicates that the reproductive phenophases are affected by climatic variables, but their timings and durations vary with species. The asynchronous flowering of plants in communities often results in inconspicuous peaks or two peaks, one major and one minor [32, 33]. According to Kikim and Yadava [32], this is because in the case of the evergreen trees the onset of flowering succeeds the occurrences of leaf flushing but for the deciduous species different patterns are observed regarding flowering making it a highly species-specific response to the biotic factors as well as climatic variables. Although the simple linear regression technique is effective to generate predictive models, it falls short when the non-normal data cannot be transformed to meet the required normality assumption. When the required assumption of normality of data is not fulfilled, GLMs are effective in determining the relationships and influences of the variables on the dependent variable. However, the effects of collinearity on estimation of parameters are also observed in GLMs which reduces the reliability of the models [34].

Ridge regression is an efficient technique to develop phenological models because of its ability to suppress the collinearity among the relevant variables and its robustness towards overfitting [14, 35]. This study shows that this technique can be used because it includes all the climatic variables and gets better values for the model performance metrics [36]. Therefore, with the integration of the ridge penalty, the ridge regression models effectively convey the dynamic relationships between the phenophases i.e. leaf initiation and flowering with precipitation, temperature, RH and PAR.

6.5 Conclusion

This study demonstrates that regression-based models are efficient in depicting the influence of variables on the dependent variables. However, the type and distribution of the data plays a significant role in the applicability of the models. The ridge regression models effectively demonstrate the influence of climatic variables on the phenophases of the tropical semi-evergreen forest in the Eastern Himalayan region of Northeast India while addressing underlying issue of collinearity among the variables. Furthermore, the species wise studies will enable to comprehend the responses along with the physiological mechanisms of the tree species to the seasonal variations in the climatic conditions in this region where phenological studies are scare and in dire necessity in the current scenario of anthropogenic induced climate change.

6.6 References

[1] Geßler, A., Keitel, C., Kreuzwieser, J., Matyssek, R., Seiler, W., and Rennenberg, H. Potential risks for European beech (*Fagus sylvatica* L.) in a changing climate. *Trees* 21(1):1–11, 2007.

[2] García-Mozo, H., Galán, C., Belmonte, J., Bermejo, D., Candau, P., Díaz de la Guardia, C., Elvira, B., Gutiérrez, M., Jato, V., Silva, I., Trigo, M. M., Valencia, R., and Chuine, I. Predicting the start and peak dates of the Poaceae pollen season in Spain using process-based models. *Agricultural and Forest Meteorology*, 149(2):256-262, 2009.

[3] Hänninen H. Modelling bud dormancy release in trees from cool and temperate regions. *Acta Forestalia Fennica*, 213:1-47,1990.

[4] Chuine, I., Cambon, G., and Comtois, P. Scaling phenology from the local to the regional level: Advances from species-specific phenological models. *Global Change Biology*, 6:943–952, 2000.

[5] Galán, C., García-Mozo, H., Vázquez, L., Ruiz, L., De La Guardia, C. D., and Trigo, M. M. Heat requirement for the onset of the Olea europaea L. pollen season in several sites in Andalusia and the effect of the expected future climate change. *International Journal of Biometeorology*, 49:184-188, 2005.

[6] García-Mozo, H., Galán, C., and Vázquez, L. The reliability of geostatistic interpolation in olive field floral phenology. *Aerobiologia*, 22:95-106, 2006.

[7] Roberts, A. M. I., Tansey, C., Smithers, R. J., and Phillimore, A. B. Predicting a change in the order of spring phenology in temperate forests. *Global Change Biology*, 21(7):2603-2611, 2015.

[8] Roberts, A. M. I. Comparison of regression methods for phenology. *International Journal of Biometeorology*, 56(4):707-717, 2012.

[9] Fitchett, J. M., Grab, S. W., and Thompson, D. I. Plant phenology and climate change: Progress in methodological approaches and application. *Progress in Physical Geography*, 39(4):460-482, 2015.

[10] Ullah, M. I., Aslam, M., and Altaf, S. lmridge: A comprehensive R package for ridge regression. *R Journal*, 10(2):326-346, 2019.

[11] Gregorich, M., Strohmaier, S., Dunkler, D., and Heinze, G. Regression with highly correlated predictors: Variable omission is not the solution. *International Journal of Environmental Research and Public Health*, 18(8), 2021.

[12] Zeileis, A., and Hothorn, T. Diagnostic checking in regression relationships. *R News*, 2(3):7–10, 2002.

[13] R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2024

[14] Van De Vijver, E., Landschoot, S., Smagghe, G., De Baets, B., Temmerman, F., Dillen, J., and Haesaert, G. Potentials and Limitations of a Growing Degree Day Approach to Predict the Phenology of Cereal Leaf Beetles. *Environmental Entomology*, 47(4):1039-1046, 2018.

[15] Belsley, D.A., Kuh, E. and Welsch, R.E. *Diagnostics: Identifying Influential Data and Sources of Collinearity*. John Wiley & Sons, New York, 1980.

[16] Draper, N.R. and Smith, H. *Applied Regression Analysis*. 2nd edition, John Wiley & Sons, New York, 1998.

[17] Hamner, B., and Frasco, M. *Metrics: Evaluation Metrics for Machine Learning*. R package version 0.1.4, 2018

[18] Mazerolle, M.J. *AICcmodavg: Model selection and multimodel inference based on (Q)AIC(c)*. R package version 2.3.3, 2023.

[19] Akaike, H. A new look at the statistical model identification. *IEEE Transactions on Automatic Control*, 19, 716–723, 1974.

[20] Mo, Y., Zhang, J., Jiang, H., and Fu, Y. H. A comparative study of 17 phenological models to predict the start of the growing season. *Frontiers in Forests and Global Change*, 5(January)2023.

[21] Tanni, S.E., Patino, C.M. and Ferreira, J.C. Correlation vs. regression in association studies. Jornal Brasileiro de Pneumologia, 46(1):e20200030, 2020.

[22] Suresh, H. S., and Sukumar, R. Vegetative Phenology of Tropical Montane Forests in the Nilgiris, South India. *Journal of the National Science Foundation of Sri Lanka*, 39(4):333-343, 2011.

[23] Sanquetta, C.R., Dalla Corte, A.P., Behling, A., de Oliveira Piva, L.R., Péllico Netto, S., Rodrigues, A.L., and Sanquetta, M.N.I. Selection criteria for linear regression models to estimate individual tree biomasses in the Atlantic Rain Forest, Brazil. *Carbon balance and management*, 13:1-15, 2018.

[24] van Schaik, C.P., Terborgh, J.W., and Wright, S.J. The phenology of tropical forests: Adaptive significance and consequences for primary consumers. *Annual Review of Ecology and Systematics*, 24:353–377, 1993.

[25] Borchert, R., and Rivera, G. Photoperiodic control of seasonal development and dormancy in tropical stem-succulent trees. *Tree Physiology*, 21(4):213-221, 2001.

[26] Adole, T., Dash, J., Rodriguez-Galiano, V., and Atkinson, P. M. Photoperiod controls vegetation phenology across Africa. *Communications Biology*, 2(1)2019.

[27] Singh, K.P., and Kushwaha, C.P. Paradox of leaf phenology: *Shorea robusta* is a semi-evergreen species in tropical dry deciduous forests in India. *Current Science*, 88(11):1820-1824, 2005.

[28] Devi, N.L., Brearley, F.Q., and Tripathi, S.K. Phenological diversity among sub-tropical moist forest trees of northeastern India. Journal of Tropical Ecology, 39(e29):1–10, 2023

[29] Aravind, N. A., Ganeshaiah, K. N., and Shaanker, R. U. Indian monsoons shape dispersal phenology of plants. Biology Letters, 9(6), 2013.

[30] Deka, J., Kalita, S., and Khan, M. L. Vegetation Phenological Characterization of Alluvial Plain Shorea robusta-dominated Tropical Moist Deciduous Forest of Northeast India Using MODIS NDVI Time Series Data. *Journal of the Indian Society of Remote Sensing*, 47(8):1287-1293, 2019

[31] Bhuyan, M., Singh, B., Vid, S., and Jeganathan, C. Analysing the spatio-temporal patterns of vegetation dynamics and their responses to climatic parameters in Meghalaya from 2001 to 2020. *Environmental Monitoring and Assessment*, 195(1), 2023.

[32] Kikim, A., and Yadava, P. S. Phenology of tree species in subtropical forests of manipur in north eastern India. *Tropical Ecology*, 42(2):269-276, 2001.

[33] Pereira, C. C., Boaventura, M.G., Cornelissen, T., Nunes, Y.R.F., and de Castro, G.C. What triggers phenological events in plants under seasonal environments? A study with phylogenetically related plant species in sympatry. *Brazilian Journal of Biology*, 84:e257969, 2022.

[34] Kurtoğlu, F., and Özkale, M.R. Liu estimation in generalized linear models: application on gamma distributed response variable. *Statistical Papers*, 57:911-928, 2016.

[35] Pei, T., Ji, Z., Chen, Y., Wu, H., Hou, Q., Qin, G., and Xie, B. The sensitivity of vegetation phenology to extreme climate indices in the loess plateau, China. *Sustainability (Switzerland)*, 13(14):1-18, 2021.

[36] Portet, S. A primer on model selection using the Akaike Information Criterion. *Infectious Disease Modelling*, 5:111-128, 2020.