Chapter 1

Introduction

Beta decay is one of the important nuclear reactions in particle physics. Through this reaction, ${}^A_Z X \rightarrow^A_{Z+1} X + e^-$, an unstable nucleus converts into a stable daughter nucleus by emitting a beta particle (e^-). Energy and momentum conservation required that all the energy of the reaction, determined by Q-value of the decay, should be associated to the emitted electron. On the contrary, the energy spectrum of the electrons was found to be continuous. This indicated that some part of total energy of the reaction was missing. In the year 1927 Charles Drummond and William Alfred Rooster performed an experiment using radioactive element radium E (bismuth-210). The observations drawn from this experiment confirmed this unstability in electron energy distribution and hinted at the prospect for development of new physics that could provide a better understanding of the same. This unsettled problem of missing energy got a possible breakthrough due to the genius of Wolfgang Pauli. According to a letter that he sent to his friends in the month of December 1930, Pauli mentioned that he believed there might be another particle being emitted along with electron in β-decay [21]. He went a little further, with some hesitation, that this particle must be electrically neutral and exhibit weak interactions. Also it became very difficult for theorist and experimentalists to detect these unknown mysterious particles because of its very small interactions via weak force. Enrico Fermi, another brilliant minds in physics, named this particle 'neutrino' at the Solvay conference in the year 1933. Based on the hypothesis of Pauli, Fermi developed a new theory of beta decay. This new theory elegently incorporated the problem of missing energy and provided a suitable explanation by associating this energy to neutrinos. As neutrino is an electrically neutral particle, it is immune to any kind of electromagnetic interactions. Also due to its very feeble weak interaction it becomes immensely difficult to detect these ghost particles. But situation changed during the second world war when a new dawn for neutrinos happened. At this time due to great advent in science, particularly in nuclear power, a large source of radioactive nucleus was available to the scientific community. This immense supply of radioactive nucleus, apart from its use in nuclear weapons, became a tremendous source for neutrinos and increased its number manifold. Despite the tremendous theoretical efforts of Pauli, Fermi and Pontecarvo there were no signs of neutrinos in any of the experiments. However a remarkable event took place in 1956 when Frederick Reines and Clyde Cowan detected for the first time electron anti-neutrinos in their experimental setup which was situated near the Savannah river, South Carolina [22]. Unfortunately Cowan died in 1974 when he was 54 years of age. In recognition to their profound contribution in neutrino physics, Reines was honored with the Nobel Prize in 1995. After this groundbreaking discovery, the particle physics community began studying the nature, origin, and various sources of neutrinos. Sun is one of the major stellar sources of neutrinos that is close to earth. The reactions taking place deep in the interior of the sun produces electron neutrinos. In 1964 Raymond David Jr and John Bahcall decided to examine the number of neutrinos that were emitted from interactions taking place in the sun. Surprisingly there was a big mismatch between the number they got after examination to that predicted from solar reactions. They could only observe one third of the actual predicted value of neutrinos, which implies that majority of them were not detected. This is called the "solar neutrino problem" [23]. Later on two other types of neutrinos, namely muon-neutrino and taon-neutrino were discovered. The existence of muon-neutrino came into light after the discovery of muon decay, whereas the third genration of neutrinos (atmospheric neutrinos) was discovered much later in 2000 at the Fermi Lab [24]. Thus three different types of neutrino corresponding to three generations of leptons came into being. It was Pontecorvo who first proposed a solution to the solar neutrino problem. According to him when a flavor of neutrino travells a distance through space there is a possibility that it gets converted into other types of neutrino [25, 2]. Mathematically for each particle there is a wave associated to it which depends on the mass and speed at which it is moving. This hypothesis suggested that neutrinos should have masses and also there should be some mixing between the different flavors. This mechanism was termed as neutrino oscillations. But the Standard Model which was constructed around 1970 required neutrinos to be massless [26, 27, 28]. Therefore the prospect of new physics Beyond the Standard Model became inevitable. Moreover experiments at the Super-Kamiokande detectors verified and confirmed the concept of neutrino oscillation. Finally in the year 2002 Arthur McDonald, the then professor at Princeton University and his team at the Sadbury Neutrino Observatory confirmed the change of solar (electron) neutrinos into muon or tau neutrinos as they travelled through space from the sun. The flux of neutrinos detected experimentally matched with the theoretical predictions. For this outstanding contribution, Arthur McDonald and Takaaki Kajita, one of the pioneers whose work led to the discovery of neutrino oscillations at the Super-Kamiokande experiments, were honoured with the Nobel Prize in Physics in 2015. [29, 30]

1.1 Current Status of Neutrino

1.1.1 Theoretical Advancements:

The idea of neutrino oscillation put forwarded by one of the greatest minds, Pontecorvo around 1957 marked a historic theoretical breakthrough in neutrino physics. This concept of neutrino changing its flavor while travelling grew very rapidly among particle physicists and gained tremendous significance among both theorists and experimentalists. In contrast to the Standard Model where neutrinos are massless, this theory required them to be massive, thereby advocating the need to go for physics beyond the Standard Model. The experiments such as Super-Kamiokande and Sudbury Neutrino Observatory which are associated with detection of neutrino oscillations gave a new set of results. The interpretation of these results along with the formulation of a systematic approach can be attributed to a framework consisting of three active neutrinos. In this scenario the mass eigenstates of the neutrinos can be related to the three flavor eigenstates through a unitary mixing matrix of order 3×3. This matrix named after Pontecarvo, Maki, Nakagawa and Sakata is commonly referred to as PMNS matrix or leptonic mixing matrix [31]. This important matrix which defines the mixing between flavor eigenstates is parameterised by the oscillating parameters: three mixing angles and a CP-violating phase (δ_{CP}). These mixing angles are solar mixing angle (θ_{12}), atmospheric mixing angle (θ_{23}) and the reactor mixing angle (θ_{13}). If the neutrinos are Majorana type then there are two more additional phases, α and β. At the beginning the reactor mixing angle was considered to be zero. But later on with the gradual developments in experiments, such as RENO and DayaBay, θ_{13} was found to have a non-zero value. Moreover the current neutrino oscillation experiments are sensitive only to mass-squared differences. As a result, determining absolute mass of neutrinos is still a challenging issue and remains undetected till date. There are two mass-squared differences which are referred to as solar mass splitting (Δm^2_{21}) and atmospheric mass splitting (Δm^2_{31}). From the results of these experiments it is confirmed that the value of $\Delta m^2_{21} > 0$. Surprisingly this is not confirmed for the atmospheric mass splitting which can take on both + or - sign. Because of this peculiar behavior of atmospheric mass-squared differences, there are two spectrums of neutrino masses. One of these is termed as normal hierarchy which refers to the condition $m_1 << m_2 < m_3$; whereas the other mass spectrum is called inverted hierarchy and it indicates the condition $m_3 << m_1 < m_2$ [8]. However, from the cosmological standpoint, there is an upper limit on the sum of three neutrino masses. This value given by the Planck data at the confidence level 95% is found to be $\sum m_\nu \leq 0.12$ eV [20]. The following Table (1.1) represents the latest 3σ nu-fit values of oscillation parameters [32].

There has been remarkable developments in particle physics in the last few decades. The formulation of numerous frameworks Beyond the Standard Model (BSM) which aims to address neutrino masses and leptonic mixing are a direct manifestation of these developments, as well as understanding of the subject. These are extensions of Standard Model(SM) which not only explain neutrino phenomenology, but also serves as a feasible mechanism for possible explanations of some of the relevant mysteries in cosmology. In this process, these mechanisms act as bridges for a possible connection between these two different fields. Some of the

Parameters	bfp$\pm 1\sigma$	3σ (NH)	bfp$\pm 1\sigma$	3σ (IH)
$\Delta m^2_{21}[10^{-5}eV^2]$	$7.42^{+0.21}_{-0.20}$	$6.82 - 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 - 8.04$
$\Delta m^2_{31}[10^{-3}eV^2]$	$+2.514^{+0.028}_{-0.027}$	$+2.431 - +2.589$	$-2.497^{+0.028}_{-0.028}$	$-2.583 - 2.412$
$\sin^2\theta_{12}/10^{-1}$	$3.04^{+0.013}_{-0.012}$	$2.69 - 3.43$	$3.04^{+0.013}_{-0.012}$	$2.69 - 3.43$
$\sin^2\theta_{23}/10^{-1}$	$5.70^{+0.018}_{-0.024}$	$4.07 - 6.18$	$5.75^{+0.017}_{-0.021}$	$4.11 - 6.21$
$\sin^2\theta_{13}/10^{-2}$	$2.221^{+0.00068}_{-0.00052}$	$2.034 - 2.430$	$2.240^{+0.00062}_{-0.00062}$	$2.053 - 2.436$
$\delta_{CP}/^0$	195^{+51}_{-25}	$107 - 403$	286^{+27}_{-32}	$192 - 360$

Table 1.1: Latest nu-fit values of the oscillation parameters.

popular mechanisms include seesaw mechanism [33, 34], radiative seesaw mechanism [35], left-right symmetric model (LRSM) [36], inverse seesaw [37, 38]. The results obtained from experiments like LSND [39] and MiniBooNE [40, 41] suggest that a new flavor, called sterile neutrino, exists in nature. Although the mass and number of generations are yet to be determined, sterile neutrinos play a very significant role in physics beyond the Standard Model. Some of these unsolved BSM phenomena includes neutrinoless double beta decay (NDBD), dark matter (DM), lepton number violation (LNV), lepton flavor violation (LFV), dark energy etc.

1.1.2 Experimental Advancements:

For a holistic growth of science both theory and experiments should go hand-in-hand. Neither theory nor experiments alone can justify the happenings in nature. In a similar manner, with the theoretical developments in neutrino physics, there were simultaneous progresses made in the experimental sector also. In particular the solar neutrino experiments (Homestake [42], SAGE [43], GALLEX [44], Kamiokande [45], Super-Kamiokande [46]), the atmospheric neutrino experiments (Kamiokande, Super-Kamiokande, MARCO [47], Soudan [48]) and the accelerator experiments like LSND [39] played a very important role in discovering neutrino oscillation. This breakthrough discovery of neutrino oscillation confirmed the massiveness of neutrinos and also the important aspect of mixing between the different flavors. It has to be mentioned that the reactor mixing angle (θ_{13}) was considered to be zero for a long time. The first signals in support of a non-zero θ_{13} was provided by reactor experiments such as Daya Bay [49], RENO [5] and Double-Chooz [7]. Also Tokai-to-Kamioka (T2K) [6], which is a long-baseline accelerator located in Japan, plays a crucial role in this aspect.

In addition to the above discussion, there are ample literatures which suggest the possibility for existence of a fourth generation/flavor of neutrinos. This extra flavor is called sterile neutrino [50]. Though the number of sterile fermions is not known, its mass lies in a range extending from eV to keV which are expected to be detected in future experiments KATRIN [51]. Moreover experimental evidences for the existence of sterile neutrinos can be found in signals provided by MiniBoone [52], LSND experiments. This hypothetical particle when considered in a suitable framework has the potential to shed light on some of the BSM phenomena. For example, a sterile neutrino in the keV range can be a possible DM candidate [53, 54]. But of course, in order to do so, it has to satisfy different constraints that are given by Lyman-α, X-ray and from the perspective of structure formation.

In the following sections, we will discuss the most successful model in particle physics i.e. Standard Model. Subsequently we will talk about its limitations and shortcomings which demands the need for physics beyond the Standard Model. Along with these, we will also discuss the popular BSM frameworks, related neutrino and cosmological phenomena and symmetry that is used in the thesis.

1.2 Standard Model

It was Glashow, Weinberg and Salam who first proposed the standard model of particle physics [26, 27, 28]. This relativistic quantum field theory gives a collective picture of all the known fundamental particles and explains the interactions (except gravitaional force) that govern them. The gauge groups $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ play a very significant role in development of this model [55]. The group $SU(3)_C$ corresponds to the strong interaction that exists between the quarks and gluons. Similarly, the groups $SU(2)_L \otimes U(1)_Y$ together denote the electroweak interactions of the particles. The subscripts C, L and Y represent the color charge of quarks and gluons, left-handed chirality and weak hypercharge associated with the particles, respectively. After the discovery of Higgs boson at the LHC experiments in 2012 [1], the SM became very popular among physicists and was established as one of the most successful theories in particle physics.

The SM contains all the elementary particles, their antiparticles and also the mediators of different interactions. This pool of fundamental particles can be broadly classified into three groups: fermions, gauge bosons and scalars. The strong interaction is mediated by the gluons. They are eight in numbers. The massive $W^{\pm}$ and Z bosons are responsible for weak interactions, whereas the electromagnetic interactions between charged particles are mediated by a massless vector boson, the photon. These mediators are together called the gauge bosons of the SM. The fermionic sector includes the spin half quarks, leptons and their corresponding antiparticle counterparts. Under $SU(3)_C$ the quarks transform as triplets and the leptons which are color neutral transform as singlets. In a similar manner, the left-handed fermions are considered as doublets under $SU(2)_L$ and right-handed fermions are taken as singlets. In the SM masses of these fermions and gauge bosons are generated through Higgs mechanism. The scalar Higgs boson which is doublet under $SU(2)_L$ plays a very crucial role in mass generation of the fundamental particles. The table (1.2) below shows the transformation of particles under different groups considered in the model.

1.2.1 The Electroweak sector

The electromagnetic and weak interactions collectively form the electroweak sector of the SM. All the interactions that take place between different fields in this sector can be explained by the gauge group $SU(2)_L \otimes U(1)_Y$. It is interesting to note that there is no mixing between strong and electroweak interactions. As a result they can be discussed separately without any loss of physics. We know that the Dirac fermions can be expressed by a four component field, ψ. It is possible to break this field into two different components for left-handed and right-handed fermions,

$$\psi_L = \frac{1-\gamma_5}{2}\psi \qquad \text{and} \qquad \psi_R = \frac{1+\gamma_5}{2}\psi$$

Fields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Spin	Charge
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	3	2	$\frac{1}{3}$	$\frac{1}{2}$	$\begin{pmatrix} \frac{2}{3} \\ \frac{-1}{3} \end{pmatrix}$
$q_R = u_R$ $q_R = d_R$	3 3	1 1	$\frac{4}{3}$ -$\frac{2}{3}$	$\frac{1}{2}$ $\frac{1}{2}$	$\frac{2}{3}$ -$\frac{1}{3}$
$L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	1	2	-1	$\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
$l_R = e_R$	1	1	-2	$\frac{1}{2}$	-1
$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	1	2	1	0	0

Table 1.2: This table shows the charge assignments of quarks, leptons and Higgs boson under different goups of SM. The first two rows are for the left-handed and right-handed quark family. The third and fourth rows represent the leptons (left-handed doublets and right-handed singlets). The field in the final row is the Higgs boson.

In the SM a quark doublet is formed under $SU(2)_L$ by taking together a left-handed *up-quark* and left-handed *down-quark*. For the leptonic part, a doublet is formed by a left-handed charged fermion and its corresponding neutrino. All the right-handed fermions are considered to be singlets under the group $SU(2)_L$. Now the complete Lagrangian of the electroweak sector of Standard Model consists of four different parts, each part corresponds to specific interactions of the model. As a result, the complete Lagrangian can be written in the following way:

$$\mathcal{L}_{EW} = \mathcal{L}_{fermions} + \mathcal{L}_{gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa} \tag{1.1}$$

Some of the principles that govern the SM are local gauge symmetry, spontaneous symmetry breaking and Higgs mechanism. It is important to note that under local gauge transformation, the Lagrangian of SM remains invariant. The fermionic part in eq. (1.1) denotes the kinetic energy and interactions of the fermions with different gauge bosons. This relevant component of SM Lagrangian can be written as:

$$\mathcal{L}_{fermions} = i\bar{Q_L}\gamma^\mu D^L_\mu Q_L + i\bar{L}\gamma^\mu D^L_\mu L + i\bar{q_R}\gamma^\mu D^R_\mu q_R + i\bar{l_R}\gamma^\mu D^R_\mu l_R \tag{1.2}$$

In the eq. (1.2) D_μ^L and D_μ^R are the covariant derivatives for left-handed and right-handed chiral fields. Mathematically, these covariant derivatives take the following forms:

$$D_\mu^L = (\partial_\mu + ig_2 T^a W_\mu^a + ig_1 \frac{Y}{2} B_\mu) \quad \text{and} \qquad D_\mu^R = (\partial_\mu + ig_1 \frac{Y}{2} B_\mu) \tag{1.3}$$

In the above equation W_μ^a and B_μ are the gauge fields associated with the groups $SU(2)_L$ and $U(1)_Y$. These new fields have a significant role in the theory. W_μ^a ($a = 1, 2, 3$) represents three gauge bosons that correspond to the three generators of $SU(2)_L$ group. Similarly, B_μ denotes the gauge boson related to $U(1)_Y$. Moreover T^a and Y are the generators of these two groups and coupling strength of electromagnetic and weak interactions are represented by the gauge constants g_1 and g_2, respectively. For $SU(2)_L$ the generators are 2×2 matrices, such that $T^a = \frac{1}{2}\tau^a$, where τ^a are the Pauli spin matrices,

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1.4}$$

Again, the weak hypercharge operator Y of $U(1)_Y$ group is a linear combination of the third operator T^3 of $SU(2)_L$ and electromagnetic charge operator Q, such that,

$$\frac{Y}{2} = Q - T^3 \tag{1.5}$$

Similarly the gauge part of the Lagrangian for the gauge bosons can be written as:

$$\mathcal{L}_{gauge} = -\frac{1}{4} W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \tag{1.6}$$

where the field tensors $W_{\mu\nu}^a$ and $B_{\mu\nu}$ have the form:

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g_2 \epsilon^{abc} W_\mu^b W_\nu^c \tag{1.7}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{1.8}$$

ϵ^{abc} in eq. (1.7) is the strucutre constant for the group $SU(2)_L$, with the relation $[T^a, T^b] = i\epsilon^{abc} T^c$. Another important component of SM formulation is the invariant Higgs Lagrangian. This sector includes the complex scalar field ϕ, which is a doublet under $SU(2)_L$ transformation. Accordingly, the expression for Higgs Lagrangian can be written in the following way:

$$\mathcal{L}_{Higgs} = (D^{L,\mu}\phi)^\dagger (D_\mu^L \phi) - V(\phi) \tag{1.9}$$

$V(\phi)$ in the above equation is the Higgs potential of Standard Model. This potential can be written as:

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \tag{1.10}$$

There are some peculiar interactions between the fermions and scalar field ϕ in the Standard Model. These kind of interactions, which are popularly known as Yukawa interactions, are described by the Yukawa Lagrangian and can be written as:

$$\mathcal{L}_{Yukawa} = -Y_d(\bar{Q_L}\phi d_R) - Y_u(\bar{Q_L}\tilde{\phi} u_R) - Y_L(\bar{L}\phi l_R) + h.c. \tag{1.11}$$

The Higgs potential in eq. (1.10) plays a very important role in generating mass of the fermions and gauge boson of SM. The process of generating masses of these particles are done through spontaneous symmetry breaking (SSB) and Higgs mechanism. As there are no right-handed neutrinos in the SM, therefore, a Yukawa term for neutrinos is not possible for this model. Consequently these neutral particles remain massless in the SM. We will discuss the mass generation mechanism in detail in the following section.

1.2.2 Masses of Fermions and Gauge Bosons

We know that the electroweak theory is a non-abelian theory which is governed by the gauge group $SU(2)_L \otimes U(1)_Y$. This theory is responsible to produce the masses of the gauge bosons and fermions of the SM. The weak interaction has three massive gauge particles viz. $W^\pm$ and Z bosons. The mediator of electromagnetic force is the massless photon (γ). In order to explain the massiveness of these particles, Peter Higgs and his colleagues proposed a special mechanism called the Higgs mechanism [56, 57]. According to this mechanism, the real part of the neutral component of Higgs doublet obtains a non-vanishing VEV. This leads to symmetry breaking of the theory and subsequently gives masses to the particles. Also this VEV breaks the gauge group $SU(2)_L \otimes U(1)_Y$ into the electromagnetic symmetry group $U(1)_{EM}$, thereby keeping the theory intact.

The $SU(2)_L$ doublet complex scalar field ϕ can be expressed as:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \frac{\phi_1 + i\phi_2}{\sqrt{2}} \\ \frac{\phi_3 + i\phi_4}{\sqrt{2}} \end{pmatrix} \tag{1.12}$$

where ϕ^+ and ϕ^0 are the charged and neutral components of the Higgs field. Under the SM gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ the Higgs field transforms as (1,2,1). The expression in eq. (1.9) represents the gauge invariant Higgs component of the SM Lagrangian. This expression contains the important Higgs potential which is given by:

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2, \quad \phi^\dagger\phi = \frac{1}{2}[\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2] \tag{1.13}$$

The mass of the SM particles are dependent on this potential. In order to generate the masses, one has to minimise this potential with the condition $\mu^2 < 0$ and $\lambda > 0$, such that ϕ obtains a non-zero VEV. For this condition, the minimum of the potential is found to be $|\phi| = \sqrt{< 0|\phi^\dagger\phi|0 >} = \sqrt{\frac{-\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}$. This minimum value is obtained by the real part ϕ_3 and the other three fields do not acquire any value. The unphysical fields ϕ_1, ϕ_2, ϕ_4 correspond to three Goldstone bosons which are eventually converted into the massive gauge bosons of weak interactions. After minimising the Higgs field can be written as:

$$< \phi >= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}; \quad v = \sqrt{\frac{-\mu^2}{\lambda}}; \quad [\phi_1 = \phi_2 = \phi_4 = 0, \phi_3 = v] \tag{1.14}$$

Now the field ϕ is excited about its VEV by a physical field, h. This new field is called the physical Higgs field. Accordingly the VEV of ϕ transforms as:

$$< \phi >= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix} \tag{1.15}$$

Using the expressions in eq. (1.14) and (1.15), the mass terms from Higgs Lagrangian of the SM can be derived as:

$$\mathcal{L}_{Higgs} = M_W^2 W_\mu^+ W_\mu^- + \frac{1}{2}M_Z^2 Z_\mu Z^\mu + \frac{1}{2}M_h^2 h \tag{1.16}$$

where,

$$\begin{aligned} W^+ &= \frac{W_\mu^1 - iW_\mu^2}{\sqrt{2}} \\ W^- &= \frac{W_\mu^1 + iW_\mu^2}{\sqrt{2}} \\ Z_\mu &= cos\theta_W W_\mu^3 - sin\theta_W B_W \end{aligned} \tag{1.17}$$

and

$$M_W = \frac{g_1 v}{\sqrt{2}}, \qquad M_Z = \frac{g_1 v}{cos\theta_W}, \qquad M_H = 2v\sqrt{\lambda}. \tag{1.18}$$

The Weinberg angle θ_W is given by $tan\theta_W = \frac{g_1}{g_2}$. Moreover the field ($A_\mu$) of the massless photon is an orthogonal combination of the fields W_μ^3 and B_μ. As a result the expression for A_μ is [58]:

$$A_\mu = cos\theta_W W_\mu^3 + sin\theta_W B_\mu \tag{1.19}$$

In a similar manner we can obtain the masses of fermions of the SM. For this purpose we need the Yukawa interactions that take place between the scalar and fermion fields. The gauge invariant Yukawa interaction of the SM is given by the eq. (1.11). Thus the masses of the electrons and quarks which are generated after the scalar ϕ acquires VEV can be written as:

$$M_e = Y_L v, \qquad M_u = Y_u v, \qquad M_d = Y_d v \tag{1.20}$$

$Y_{L,u,d}$ in the above equation are the Yukawa couplings of lepton, up-quark and down-quark, respectively. In this way one can explain how the particles obtain masses through Higgs mechanism in the SM. However, neutrinos are an exception in this theory. As right-handed neutrinos are absent in SM, there cannot be Yukawa interaction for them. Therefore, their mass cannot be generated and remain massless in the SM.

1.2.3 Shortcomings of Standard Model

The SM is a very successful model in the field of particle physics. In the current scenerio it has the potential to adequately address the elementary particles and three of the fundamental interactions that govern the behavior of nature. Though this seems to be a self-consistent theory, there are many inefficiencies which are associated with this model. The sources of these relevant inefficiencies can be theoretical, experimental, observational etc. For instance, gravitational interaction is one of the important fundamental forces present in nature. The SM not only fails to incorporate this interaction, but also cannot provide a suitable justification as to why the gravitational force is very weak when compared to electromagnetic or strong forces. Quantum chromodynamics implies the conservation of CP-symmetry in strong interaction. Due to lack of experimental evidences, the reason for possible conservation of such symmetry is not yet known. Another limitation can be found in terms of mass of the neutrinos [59]. Contrary to what the SM dictates, neutrino oscillations confirmed that these neutral particles should possess mass i.e. they are massive in nature. Some other phenomena whose origin cannot be obtained from the SM, but are observed in the universe includes baryon asymmetry [60], dark matter [61, 62], dark energy [63] etc. All these observations and drawbacks suggest the possibility of new physics that extend far beyond the Standard Model. In this thesis, we have studied some of these mysteries which are unsolved in the SM. Moreover, we have briefly discussed some of these phenomena in the following sections.

1.3 Neutrino Oscillation

The quantum mechanical phenomena of oscillations among different flavors of neutrinos is a path breaking discovery in the field of neutrino physics. This idea of neutrino oscillation was first proposed by Bruno Pontecarvo in the year 1967 as a possible solution to explain the solar neutrino problem. This phenomena which was originally inspired by kaon-antikaon oscillations, apart from successfully explaining the solar neutrino problem, threw light on new properties of neutrinos that were earlier thought to be absent in them. In SM neutrinos are considered to be massless and there is no mixing between them. But according to neutrino oscillation, neutrinos have a non-zero mass and there is mixing among the three generations as they propagate in space. Though they are produced at source and detected in experiments as flavor eigenstates, they travel in between this distance as a combination of mass eigenstates. As the SM fails to accomodate these relevant properties of neutrinos, therefore, one has to go for BSM frameworks by extending the Standard Model. By now it is clear that there are two eigenstates associated with neutrinos: flavor eigenstates and mass eigenstates. These two forms of neutrino eigenstates are connected by a unitary matrix known as PMNS matrix or leptonic mixing matrix.

$$\begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \\ \nu_\tau(x) \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1(x) \\ \nu_2(x) \\ \nu_3(x) \end{pmatrix} \tag{1.21}$$

In the above eq. (1.21) the column matrices on the left and right sides represent the flavor and mass eigenstates, respectively. This unitary mixing matrix can be parameterised in terms of three mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$ and Dirac CP-phase δ. This can be written as:

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} U_{Maj} \quad (1.22)$$

In the above matrix $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. $U_{Maj} = \mathrm{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)})$ is the diagonal matrix for Majorana neutrinos with phases α and β.

1.3.1 Neutrino Oscillation in Vacuum:

When a particular flavor of neutrino (say α) travels through a distance in vacuum it gets converted into some other flavors (say β) for the time it stays on the path of its journey. Mathematically this propagation can be expressed by the equation:

$$|\nu_\alpha(x) >= U_{\alpha i} e^{-ip_i x} |\nu_i(x) > \quad (1.23)$$

In eq. (1.23) x represents the space-time co-ordinate and p_i is the momentum associated with the mass eigenstate of neutrinos. The amplitude of probability of this transition for a neutrino travelling a distance L is calculated by the following equation:

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}[U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}]\sin^2(\frac{\Delta m^2_{ij}L}{4E}) \\ &+2\sum_{i>j} \mathrm{Im}[U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}]\sin(\frac{\Delta m^2_{ij}L}{2E}) \end{aligned} \quad (1.24)$$

However for two neutrinos with a single mixing angle the above expression takes the following form:

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta_{ij})\sin^2(\frac{\Delta m^2_{ij}L}{4E}) \quad (1.25)$$

$\Delta m^2_{ij} = m^2_i - m^2_j$ denotes the mass squared differences of the neutrinos involved in the process. The above equation shows that for neutrino oscillation to occur in nature the mixing angle and mass of neutrinos should have non-zero values. Thus this particluar expression confirms that neutrinos should have some mass.

1.3.2 Neutrino Oscillation in Matter:

The propagation of neutrinos is dependent on the medium in which it is travelling. In contrast to propagation in vacuum, there are many factors which tend to alter some of the properties of this process when neutrino travels through matter. The properties of matter, such as its nature, the number density of different components, density etc. are described by the effective potential of the medium. Owing to the difference in charge-current interactions, these effects vary with respect to different flavors of particles. The effective potential essentially explains the effect that any matter medium can have when neutrinos propagate through them. For electron neutrino (ν_e) this effective potential can be expressed in terms of the Fermi constant G_F and electron number density (n_e) in the following way:

$$V_C = \pm\sqrt{2}G_F n_e \tag{1.26}$$

Similarly the effective potential induced by neutral-current interaction for the active neutrinos can be expressed as:

$$V_N = \mp\sqrt{2}G_F n_n \tag{1.27}$$

n_n in eq. (1.27) represents neutron number density. Unlike neutrino propagation in vacuum, these effective potentials in matter tend to modify the neutrino eigenstates and affects the flavor evolution in matter. Thus the form of effective mass becomes:

$$M^2_{\nu_e} = M^2_{\nu_e} \pm 4EV_M \tag{1.28}$$

where,

$$V_M = \begin{pmatrix} V_e = V_C + V_N & 0 & 0 \\ 0 & V_\mu = V_N & 0 \\ 0 & 0 & V_\tau = V_N \end{pmatrix} \tag{1.29}$$

Accordingly the expressions for mixing angles and mass eigenvalues can be written down for desired number of flavors. For the simple case of two flavors of neurinos, the expression for light neutrino mass is:

$$M^2_\nu \simeq O^T M^{diag}_\nu O + \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix} \tag{1.30}$$

In eq. (1.30) O is an orthogonal mixing matrix which can be taken as:

$$O = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \tag{1.31}$$

Accordingly, the mass-squared matrix (M_ν^2) can be written as:

$$M_\nu^2 = \frac{m_0}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} A - \Delta m_{12} cos2\theta & \Delta m_{12} sin2\theta \\ \Delta m_{12} sin2\theta & -A + \Delta m_{12} cos2\theta \end{pmatrix} \tag{1.32}$$

Here $m_0 = m_1^2 m_2^2 + A$ and $\Delta m_{12} = |m_1^2 - m_2^2|$. Thus the eigenvalues are found to be:

$$m_{1,2} = \frac{m0}{2} \mp \frac{1}{2}\sqrt{(\Delta m_{12}\mathrm{cos}2\theta - A)^2 + \Delta m_{12}^2 \mathrm{sin}^2 2\theta} \tag{1.33}$$

Also the effective mixing angle can be expressed as:

$$\mathrm{tan}2\theta = \frac{\Delta m_{12}\mathrm{sin}2\theta}{\Delta m_{12}\mathrm{cos}2\theta - A} \tag{1.34}$$

These modifications that are observed in neutrino oscillations when they propagate through matter are well explained by the MSW (Mikheyev, Smirnov, Wolfenstein) effect [64].

1.3.3 Neutrino mass: Dirac and Majorana

Dirac Mass

All the fermions in SM have two different chiral fields, left-handed and right-handed components. The mass of these Dirac particles are generated through the popular Higgs mechanism where both of the chiral fields through Yukawa interactions with the Higgs boson play a major role. But because of the absence of right-handed chiral fields the Dirac mass term is not possible for neutrinos within the Standard Model. Therefore in order to incorporate Dirac mass term SM has to be augmented with three right-handed neutrinos (ν_R) such that its charge assignment under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is (1,1,0). Accordingly the Yukawa term can be written as:

$$-\mathcal{L}_{Yuk} = Y_\nu \bar{\nu_R}\tilde{\phi}L + h.c. \tag{1.35}$$

where $\tilde{\phi} = i\sigma_2\phi$. L and ϕ are the lepton doublets and Higgs boson, respectively. After electroweak symmetry breaking ϕ acquires VEV and then Dirac mass term for neutrinos can be written as:

$$-\mathcal{L}_{Dirac} = \bar{\nu_R} m_\nu \nu_L + h.c. \tag{1.36}$$

Though the above setup seems to be fine in generating neutrino Dirac mass, but it has a serious drawback. To produce sub-eV neutrino mass, the Yukawa couplings need to be vey small i.e. of the order of $Y_\nu \sim 10^{-12}$. This very small order of Yukawa coupling does not have any natural explanation and thus led to the fine tuning problem in SM. Nevertheless, in order to have Dirac mass any particle should have both the chiral fields.

Majorana Mass

In simple language, if a particle by nature is its own anti-particle then such particles are called Majorana particles. Neutrinos are the only fermions in SM which are chargeless and they serve as the best candidate to be a Majorana particle. The mass associated to these particles are termed as Majorana mass [65]. One of the interesting features of this type of mass is that they can be expressed by a single type of chiral field i.e. either by a left-handed or right-handed field alone. For the case of neutrino, right-handed chiral field can be expressed as $\nu_R = \nu_L^C = C\nu_L^T$. Therefore, Lagrangian for the Majorana mass term can be written as:

$$\mathcal{L}_{Majorana} = -\frac{1}{2} m \nu_L \nu_L^C \tag{1.37}$$

On careful observation one can find that this term violates the Lepton number by two units i.e. $\Delta L = \pm 2$. Because of this reason it is forbidden in the Standard Model. Moreover the double counting of the identical Hermitian conjugate is taken into consideration by the factor half present in the expression.

1.3.4 Neutrino Mass Models

Soon after the discovery of neutrino oscillation in various experiments, the idea that neutrinos should have a tiny non-zero mass gained much popularity among the neutrino physics community. In absence of right-handed counter parts the SM fails to provide a suitable mechanism that could give rise to mass of neutrinos. As a consequence of this failure of Standard Model, people began to explore the possible mechanisms outside of the SM regime that could incorporate massive nature of this mysterious particle. Apart from generating the tiny neutrino mass, these mechanisms must also have the potential to tackle the issue of large mixing angle in leptonic sector as well as the hierarchical discrepency that exists between neutrinos and charged lepton masses. These BSM frameworks are essentially extensions of the SM with new fields being added to its particle content. Many of these frameworks have been successfully used to include the mass of neutrinos in different models. Additionally many of these models have provided a way to study some other Beyond Standard Model phenomena which are observed in the universe. Among all the frameworks, the most popular theoretical developments was achieved in formulation of the well-known Seesaw mechanism. This breakthrough mechanism is further divided into three categories: Type I [66, 67], Type II [34] and Type III [33, 68] seesaw. Some other interesting frameworks found in literatures are inverse seesaw [10], left-right symmetric model (LRSM) [69, 70], neutrino two Higgs doublet model (ν2HDM) [71] etc. Below we have discussed briefly some of these mechanisms that are frequently used in constructing models to describe neutrino phenomenology.

Type I Seesaw: It is also called the conventional seesaw mechanism. The Type I seesaw framework is formed by adding three gauge singlet right-handed neutrinos (ν_R) to the SM. These additional particles interact with the lepton doublets (L) and the Higgs boson (ϕ) to give Dirac mass for neutrinos. They also have a Majorana mass (M_R) associated with them. The Yukawa interaction term for this setup is written as:

$$-\mathcal{L}_{Yukawa} = Y_\nu \bar{\nu_R}\tilde{\phi}L + \frac{1}{2}M_R\bar{\nu_R}\nu_R^C + h.c. \tag{1.38}$$

After symmetry breaking the scalar ϕ acquires VEV as $<\phi>= v$. As a result the Dirac mass term takes the form $M_D = Y_\nu v/\sqrt{2}$, where Y_ν is a matrix of order 3×3. The complete mass matrix for neutrinos can be obatined from eq. (1.38) as:

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \tag{1.39}$$

Block diagonalisation of the above mass matrix gives two mass eigenvalues. One is the Majorana mass term M_R and the other eigenvalue denotes the mass of active neutrinos $m_\nu = -M_D.M_R^{-1}.M_D^T$. Moreover to generate sub-eV SM neutrino mass, the right-handed neutrino mass matrix (M_R) must be very heavy i.e. of the order of GUT scale.

Type II Seesaw: Extension of the SM by adding a $SU(2)_L$ Higgs triplet field forms the Type II seesaw mechanism. This triplet scalar field $\Delta = (\Delta^{++}, \Delta^{+}, \Delta^{0})$ with unit hypercharge couples with the lepton doublet (L) and plays a very significant role in generating mass of the neutrinos. Under the SM gauge group the three components of Δ transform as (1,3,1). In a matrix form this triplet is expressed as:

$$\Delta = \begin{pmatrix} \frac{\Delta^{+}}{\sqrt{2}} & \Delta^{++} \\ \Delta^{0} & -\frac{\Delta^{+}}{\sqrt{2}} \end{pmatrix} \tag{1.40}$$

The Lagrangian for Type II seesaw mechanism is:

$$\mathcal{L}_{TypeII} = (Y_\Delta L \Delta L + \mu \phi \Delta \phi + h.c.) + M_\Delta^2 \Delta^{+} \Delta \tag{1.41}$$

Thus the mass of neutrino can be expressed in terms of VEV of the neutral component of the Higgs triplet, $<\Delta^0>= \frac{v_\Delta}{\sqrt{2}}$ as

$$m_\nu = \frac{Y_\Delta v_\Delta}{\sqrt{2}} \tag{1.42}$$

Type III Seesaw: This formulation contains a hyperchargeless fermion triplet, $\Sigma = (\Sigma^{+}, \Sigma^{0}, \Sigma^{-})$. This is expressed as:

$$\sum = \begin{pmatrix} \frac{\Sigma^{0}}{\sqrt{2}} & \Sigma^{+} \\ \Sigma^{-} & -\frac{\Sigma^{0}}{\sqrt{2}} \end{pmatrix} \tag{1.43}$$

Accordingly the Lagrangian for this mechanism is:

$$\mathcal{L}_{typeIII} = \frac{1}{2} \begin{pmatrix} \bar{\nu}_L^C & \bar{\Sigma}_R^0 \end{pmatrix} M_\nu \begin{pmatrix} \nu_L \\ \Sigma^{0C} \end{pmatrix} + h.c. \tag{1.44}$$

This expression has some resemblance to the Type I seesaw mechanism. Here the right-handed neutrinos are replaced by the fermion triplet. Thus, the mass matrix for the active neutrinos can be written as:

$$m_\nu = M_D.M_\Sigma^{-1}.M_D^T \tag{1.45}$$

where $M_{\sum}$ is the mass of fermion triplet and the Dirac mass matrix is $M_D = Y_\Sigma v/\sqrt{2}$.

Inverse Seesaw: Among the many BSM frameworks, inverse seesaw is one of the most popular mechanism that is used to generate the tiny neutrino mass. Along with the right-handed neutrinos, this framework includes sterile fermions (s) which are singlet under the SM gauge group [72, 73, 74, 38, 75]. Though the number of these particles vary, each of them are considered to be present in three generations in the standard notation. One of the salient feature of this mechanism is the presence of the lepton number violating mass (M_S) of the sterile fermions. Because of this term mass of the SM neutrinos can be produced by lowering the energy scale of right-handed neutrinos to TeV. The Lagrangian in the basis (ν_L, ν_R, s) can be written as:

$$\mathcal{L}_{ISS} = Y L\tilde{\phi}\nu_R + M_R\bar{\nu_R}s + M_S\bar{s}s + h.c. \tag{1.46}$$

M_S in the above equation is the lepton number violating mass of sterile fermions. From this equation the complete neutrino mass matrix can be expressed as:

$$M_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M_R \\ 0 & M_R^T & M_S \end{pmatrix} \tag{1.47}$$

In eq. (1.47) M_D is the Dirac mass and M_R is the mass matrix for right-handed neutrinos. One important condition among the different mass matrices of this setup is $M_S << M_D < M_R$. Following this condition M_ν can be block diagonalised to arrive at the expression of the three active neutrinos,

$$m_\nu = M_D M_R^{-1} M_S (M_R^T)^{-1} M_D^T. \tag{1.48}$$

Minimal Inverse Seesaw, ISS(2,3): ISS(2,3) is a minimal version of the standard inverse seesaw mechanism. The difference between the two mechanisms lie in the particle content of ISS(2,3). Unlike inverse seesaw, there are two right-handed neutrinos and three sterile fermions in ISS(2,3) [76, 77]. As a result, although the Lagrangian and structure of the complete neutrino mass matrix remains unchanged, order of the Dirac (M_D) and right-handed neutrino mass matrices (M_R) associated with this system changes. The new order for these matrices are: 3×2 for M_D and 2×3 for M_R.

As M_R is a rectangular matrix, its inverse is not properly defined. Because of this limitation the expression for effective mass of the light neutrinos change as compared to the usual form in eq. (1.48). The modified expression takes the form:

$$m_\nu = M_D.d.M_D^T \tag{1.49}$$

where d is a 2×2 matrix obtained from the heavy mass matrix M_H in the following way:

$$M_H^{-1} = \begin{pmatrix} 0 & M_R \\ M_R^T & M_S \end{pmatrix}^{-1} = \begin{pmatrix} d_{2\times 2} & ..... \\ .... & ..... \end{pmatrix} \tag{1.50}$$

1.4 Baryon Asymmetry of the Universe

Observational evidences suggest that the present universe around us is mainly dominated by matter. This matter domination can be found from the microscopic regime (atoms, molecules, amoeba, viruses etc.) to the macroscopic level (rivers, mountains,buildings etc.) and extends to outer space entities, such as planets, stars, galaxies, cluster of galaxies etc. But the scenario prevalent during early stages of the universe was very different from the present one. It is believed that matter and anti-matter both were present in equal numbers in the thermal bath. The standard Big-Bang theory reveals that the expansion of the universe started from extremely hot plasma where all the particles were in thermal equilibrium with each other. In contrast to this matter dominance, the presence of anti-matter is very very small. Several cosmological observations hint at an imbalance that occurs in the number of baryons and anti-baryons in the universe. This inequality may be caused by factors that disturbed the equilibrium state of the plasma. In cosmology this difference in number between matter and anti-matter is called Baryon Asymmetry of the Universe (BAU) [78, 79, 80]. From the Planck satellite observations the latest value of this asymmetry comes out to be:

$$\eta_B = (6.04 \pm 0.08) \times 10^{-10} \tag{1.51}$$

Andrew Sakharov was one of the pioneers who attempted to explain this observed asymmetry in the universe. Around 1967 in one of his works he pointed out three most essential ingredients that a mechanism has to fulfill to successfully explain this asymmetry [15, 81]. These important conditions are: Baryon (B) number violation, C and CP violation and departure from thermal equilibrium.

- Baryon number (B) violation is a very trivial requirement which ensures that in any reaction more number of baryons are produced than anti-baryons to have a non-zero asymmetry.

- If C and CP are conserved than an equal number of particles and anti-particles will be produced in any reactions. This is not favored for generation of the observed asymmetry. A remedy for this problem lies in the violation of C and CP which ensures that there are no reactions possible that can produce these particles in equal amounts.

- The out-of-equilibrium decay of heavy particles is a necessary condition to generate this asymmetry. This departure from the equilibrium state makes sure that the forward reactions are not counter-balanced by the inverse reactions.

The above three conditions are collectively called as Sakharov conditions. As discussed in literatures [82, 83], one of the mechanisms that respects all the three conditions and provides a possible explanation of generating this asymmetry of the universe is baryogenesis. There are several ways of realizing baryogenesis. Some of the popular mechanisms are GUT baryogenesis, electroweak baryogenesis, Affleck-Dine mechanism, leptogenesis. Among them baryogenesis via leptogenesis is the most interesting and popular mechanism. It was first proposed by Fukugita and Yanagida in their work [60, 84, 85]. This refers to an asymmetry created in the leptonic sector by decay of heavy fermions which is then transformed into baryon asymmetry via $B + L$ violating sphaleron process. In this thesis we have calculated BAU in the framework of resonant leptogenesis, which we have briefly discussed in the next section.

1.4.1 Resonant Leptogenesis

First proposed by Fukugita and Yanagida in 1986, leptogenesis has been one of the most favoured processes which is applied to study baryon asymmetry of the universe. Among its different types, a lot of works have been done in various formalisms using resonant leptogenesis. This type of leptogenesis is possible in those cases where mass of the decaying right-handed neutrinos are almost degenerate, such that it is of the order of their decay widths. Also this process can occur at low energy scales ($\sim$ TeV) and is suitable for mechanisms like inverse seesaw, linear seesaw etc [86, 87, 88].

In resonant leptogenesis the CP-asymmetry of the decaying right-handed neutrino N_1 is enhanced due to its mass degeneracy with N_2. The expression for CP-asymmetry for the decay of N_1 into any lepton flavor is given by [89]:

$$\epsilon_i = \frac{1}{8\pi} \sum_{i \neq j} \frac{Im[(hh^\dagger)^2_{ij}]}{(hh^\dagger)_{ii}} f_{ij} \tag{1.52}$$

where $f_{ij} = \frac{(M_i^2 - M_j^2) M_i M_j}{(M_i^2 - M_j^2)^2 + (M_i \Gamma_i + M_j \Gamma_j)^2}$ is the self-energy correction term h_{ij} are the Yukawa couplings in diagonal mass basis. Γ_i represents the decay width of the particles. The final expression for calculating BAU is [90]:

$$Y_B = 10^{-2} \sum k_i \epsilon_i \tag{1.53}$$

where k_i is the washout parameter associated with the heavy fermions.

1.5 Dark Matter

In modern cosmology one of the major challenges is to deal with the enigmatic phenomena of dark matter and also to provide a suitable particle that can serve as probable candidate. Observations from different sources through satellite, along with the signals obtained from various experiments in cosmology, demonstrate that a large portion of the universe is not formed by ordinary matter. These areas do not interact with radiation and so appear as dark, non-radiative and non-luminous large patches in space. These regions which constitute around 26% of the universe is termed as dark matter. Early indications of dark matter was first made by astronomer Fritz Zwicky in 1933 which was based on the observations he made in the galaxies of Coma clusters [91]. The discrepencies seen in velocity distribution of the galactic rotation curves at the centre and far-outer regions of many spiral galaxies is another indication of the presence of dark matter in the universe. These spiral galaxies rotate around their vertical axes. According to Newtonian mechanics, the expression for this velocity distribution at a distance

"r" from the centre is:

$$v(r) = \sqrt{\frac{GM(r)}{r}} \tag{1.54}$$

The above equation clearly demonstrates the dependence of $v(r)$ on the distance r i.e. $v(r) \propto r^{\frac{-1}{2}}$. On the same grounds, two astronomers Rubin and Ford in 1970 tried to examine the rotation curves of Andromeda galaxy [92]. To their surprise they found that at the regions far from the center of the galaxy the curves did not follow the above relation. Rather they turned out to be flat at the outer regions of Andromeda galaxy. This crucial observation confirmed that there must be additional invisible masses which prevents the stars at the periphery from flying away and also stopping the galaxy from breaking apart.

Gravitational lensing is another prime source of evidence that validates the existence of DM in the universe. This refers to the effect of observing distorted images of far away galaxies because of the presence of massive galaxies at the foreground in outer space. According to Einstein, light coming from a distant source gets bent when there are massive objects in its path. These large massive bodies behave as gravitational lens. Different galaxies, cluster of galaxies at the cosmological scale show these patterns of lensing. Analysing these patterns confirms the existence of DM in the universe. The most strong evidence is drawn from weak lensing effect that is observed in the bullet clusters [93, 94]. This cluster was formed about 150 million years ago due to the collision of two different clusters that contained a mixture of visible and dark matter, respectively. But during the formation of bullet clusters this visible and DM got spatially separated. Also these observations help in determining the density profiles of DM halos.

One of the early pictures of the universe after the Big-Bang is Cosmic Microwave Background (CMB) [95]. This presents a broad display of the composition of universe. The fluctuations (photon-baryon fluid) observed in CMB provide information about structure formation in the universe. Later on this fluctuations were found to be negligibly small and failed to account for large scale structure formations. This discrepency in CMB had to be compensated with a massive neutral form of matter, thus signaling the existence of DM.

The data that we receive from various cosmological and astrophysical observations and experiments tell us the amount of DM present in the universe. It could only tell how much of dark matter is present and what should be its characteristics. Though the presence of dark matter is now accepted by all across the community, but there is no consensus about its composition and what it is made of. The possible DM candidates include both dense baryonic matter and non-baryonic matter, the later being more favourable. A section of studies show that DM particles were in thermal equilibrium with the SM particles during early phases of the universe. In due course of time when this equilibrium state was disturbed, DM got annihilated and its number reduced significantly. This number became saturated once these particles were far apart from each other and reduced the possibility of interaction among them. This phase is called freeze-out and the number of particles left at this time are called thermal relics of DM. Another class of studies believe that there were negligible amount of DM in the early universe. The interactions(decay) that took place among the Standard Model particles gradually produced DM particles. This process of DM production is called the freeze-in mechanism. As per nomenclature DM particles associated with the freeze-out mechanism are called Weakly interacting massive particles (WIMPs) and those which are produced through freeze-in are called Feebly interacting massive particles (FIMPs). As per the latest data obtained from the Planck satellite, current DM relic abundace in the universe is found to be [96, 78]:

$$\Omega h^2 = 0.1199 \pm 0.0027 \tag{1.55}$$

The baryonic components of dark matter include Massive Astrophysical Compact Halo Objects (MACHOs), neutron stars, white dwarfs, black holes etc. Baryonic matter forms a very negligible amount of dark matter and this makes them less interesting. Most of the dark matter present in the universe is of non-baryonic nature. These include WIMPs, FIMPs, scalars, vectors etc. Based on large structure formation and speed of the particles, non-baryonic DM is classified into three different categories:

Hot Dark Matter(HDM): This type of DM is formed by relativistic particles whose masses (m $<$ T) is less than their kinetic energy. Neutrinos are a good example of this category of DM.

Cold Dark Matter(CDM): This type of DM is formed by slow moving non-relativistic paricles during the time of freeze-out. They are heavier (m > T) than their kinetic energies.

Warm Dark Matter(WDM): These are the kind of DM which lie in an intermediate region. They are neither too fast moving nor very slow like CDM.

It is to be noted that among the three categories of DM, Cold Dark Matter (CDM) is the most popular form of DM among the community. The reason being its capability to justify and form large structures as seen in different cosmological experiments and observations. Moreover in recent decades there have been rapid progess in experimental aspects of various fields. Therefore in near future, there are high chances of getting more insight about the particle constituent of this mysterious form of matter which occupies around 26% of the universe.

1.6 Neutrinoless double beta decay (NDBD/$0\nu\beta\beta$): Lepton Number Violation

Neutrinoless double beta decay is one of the few reactions that play a very prominent role in particle physics. It is a radioactive reaction that converts a parent nucleus $N(A, Z)$ into a daughter nucleus $N(A, Z+2)$ without emitting any neutrinos. This reaction can be expressed as:

$$N(A, Z) \rightarrow N(A, Z+2) + 2e^- \tag{1.56}$$

Wendell H Furry was the first person who considered this type of decay processes in the year 1939. It is evident from the reaction in eq. (1.56) that it is a lepton number violating decay process. The entire process involves violation of lepton number by 2 units. From the neutrino perspective, this decay process has the potential to determine the nature of neutrinos i.e. whether they are Dirac or Majorana type. This reaction is in support of Majorana nautre of neutrinos. Therefore, if this decay process is detected in experiments than it will confirm that neutrinos are Majorana type of fermions. One of the important observable for a nuclear reaction is its time period. The expression for time period for NDBD can be expressed in terms of electron mass (m_e) and nuclear matrix element (M_ν) as [97]:

$$\frac{\Gamma_{0\nu\beta\beta}}{ln2} = G|\frac{M_\nu}{m_e}|^2|m_{\beta\beta}|^2 \tag{1.57}$$

In eq. (1.57) G denotes phase-space factors. $m_{\beta\beta}$ represents effective mass for Majorana neutrinos. The value of $m_{\beta\beta}$ have contribution from the active neutrinos and first row elements of the mixing matrix. Mathematically this relation is expressed as:

$$m_{\beta\beta} = |\sum_i^3 m_i U_{ei}^2| \tag{1.58}$$

The above expression is valid for those cases where there are only three generations of neutrinos in the model. If additional neutrinos are added then the expression of effective neutrino mass gets modified. For example, in a framework which contains sterile neutrinos the expression gets modified in the following way [98]:

$$m_{\beta\beta} = |\sum_i^{3+s} m_i U_{ei}^2 \ \frac{p^2}{p^2 - m_i^2}| \tag{1.59}$$

p^2 in the above equation represents virtual momentum of neutrinos. The value of this quantity is -125 MeV^2. Currently there are many ongoing experiments which are actively looking for neutrinoless double beta decay using different nucleus. Some of these popular experiments are KamLAND-ZEN [17], GERDA [99], CUROE [19] etc. These experiments also put some stringent bounds on the effective neutrino mass, $m_{\beta\beta} < 0.061 - 0.165$ eV. Inspite of repeated continuous efforts there has been no success in detecting this type of beta decays till date. But by increasing the sensitivities of these experiments there might be some possibilities of detecting this important class of lepton number violating decay reactions in the future.

1.7 Lepton Flavor Violation (LFV)

In SM there are three generations of leptons viz. electron, muon and taon. Together with their corresponding neutrino partners they are considered as discrete doublets in the model. Within this regime, the possibility of a reaction where a charged lepton changes its flavor from one to another generation is very rare.

This type of interactions are strongly suppressed in the Standard Model. But with the discovery of neutrino oscillation there has been a shift in viewing these flavor violating processes. Moreover experiments, such as solar [100, 101], reactor [102, 103, 104] and atmospheric neutrino experiments [3] gave new hints about the existence of such flavor violating processes. Subsequently these findings opened up windows for possibilities of charged lepton flavor violation interactions, thereby creating a demand for physics beyond the Standard Model. The ongoing experiments search for two body ($l_j \to l_k \gamma$) and three body ($l_j \to l_i l_k l_k$) lepton flavor violating decays. $\mu \to e\gamma$ is one of the popular muonic two body decay processes. Similarly the three body decay reaction for muons is written as $\mu \to eee$. Another possible class of decays includes muon atom ($\mu^+ e^-$) converting into electron ($\mu \to e$) and the flavor violating decay, $\mu^- e^- \to e^- e^-$. The two body muonic decays are investigated by MEG collaboration experiments [105, 106]. These set of experiments also provide the bounds on the branching ratios of the decay. For the case of three body muonic decay, SINDRUM II [107] experiment plays a vital role in determining values of the branching ratios. But more precise bounds come from the highly sensitive Mu3e [108] collaboration experiments. Other experiments like Mu2e [109], DeeMe [110], COMET [111] focuses on detecting muon-electron conversion. Though there are no signals from experiments for $\mu^- e^- \to e^- e^-$ decays, but with improved sensitivity there might be some postitive response from COMET, Mu2e in near future.

cLFV processes involving the third flavor of leptons (τ) give rise to many decaying channels that favor flavor violation. Some of the common channels are $\tau \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to 3e$, $\tau \to 3\mu$. Apart from these, there are few other rare channels associated with taon decay that produce hadrons in the final states, $\tau \to l\pi^0$ and $\tau \to l\pi^+\pi^-$. Signals of flavor violation involving taons can also be found in theoretical model which predict cLFV in muons. Moreover the amplitudes of branching ratios of these decay channels are enhanced compared to muonic decays. These challenging decays are searched for in the experiments like BaBar [112, 113], Belle [114]. The table in (1.3) shows the latest bounds of the different decyas associated with cLFV.

cLFV process	Present Bound	Future Sensitivity
$\mu \to e\gamma$	5.7×10^{-13}	6.0×10^{-14}
$\mu \to eee$	1.0×10^{-12}	$\sim 10^{-16}$
$\tau \to e\gamma$	3.3×10^{-8}	$\sim 3 \times 10^{-9}$
$\tau \to \mu\gamma$	4.4×10^{-8}	$\sim 10^{-9}$
$\tau \to eee$	2.7×10^{-8}	$\sim 10^{-9}$

Table 1.3: This table shows current bounds of different cLFV processes. It also highlights the future sensitivities of these processes.

1.8 Flavor Symmetry in Particle Physics

Symmetry is an inseparable segment of high energy physics. It plays a very vital role in representing the particles and their interactions in a systematic mathematical order. One of the branches of mathematics i.e. Group theory which helps to realize symmetry is the backbone of model building in particle physics. The interactions that exist between different class of particles, such as weak, electromagnetic and strong interactions can be explained elegantly with the help of continuous symmetry groups. Common examples of continuous symmetries are Poincare, Lorentz and gauge groups. In addition to these groups, discrete symmetry such as Charge Conjugation (C), Parity (P) and Time Reversal (T) have significant contribution in simplifying particle physics to its present form.

The gauge groups $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ govern the dynamics of the Standard Model. Here $SU(N)$ and $U(N)$ are non-abelian continuous symmetry groups. However, there are certain exceptions when it comes to neutrino masses and mixings. These massive particles, otherwise considered massless in SM, require extension of SM for a better understanding of the underlying physics which can provide an appropriate explanation of the massive nature of neutrinos. In these BSM frameworks, discrete abelian symmetry groups Z_N are very useful in constructing different constrains of the model. Along with the Z_N groups, there is another class of discrete groups which are widely used in paticle physics and play a very important role in BSM frameworks. These groups are non-abelian in nature and serve as important tools in determining the flavor structure of any mechansim. Some of the most popular non-abelian discrete symmetry groups are A_N [115, 116, 117], S_N [118, 119, 120], D_N [121, 122], T' [123, 124] etc. Many models have been developed using these groups which focus on the studies of neutirno (quark) masses and mixings. The extra particles (flavons) considered in these models acquire vacuum expectation value (VEV) at different energy scales. Moreover these models are successful in producing the masses and mixings of neutrinos and quarks which are compatible with the results that are obtained from different experiments.

In the following sections we briefly discuss some of these groups that we have used in our work.

1.8.1 Abelian Z_N group

Z_N is a finite sub-group of the larger SO(2) group [12]. This abelian discrete symmetry group which is cyclic in nature represents the symmetry of a plane figure. Any figure which bears this symmetry remains invariant when rotated by an angle of $\frac{2\pi}{n}$. The simplest group of this symmetry is Z_2. It is characterised by two elements +1 and -1. The Z_2 group serves as an important tool in stabilizing the dark matter candidate in a model. Another important group in this category is Z_3. It has three elements $(1, \omega, \omega^2)$. Here 1 is denotes the identity element and ω is the cube root of unity i.e. $\omega = \exp(\frac{2i\pi}{3})$.

1.8.2 A_4 Discrete Symmetry Group

The group A_4 is an even permutation of four objects. It is a symmetry group of the tetrahedron and has (4!/2)=12 elements [125]. These twelve elements can be generated by repeatedly myltiplying its generators, $S = (14)(23)$ and $T = (123)$. These two generators of the group satisfy the relations:

$$S^2 = (ST)^3 = T^3 = 1$$

This family of discrete symmetry group has four irreducible representations. Among these representations, three are singlets $(1, 1', 1'')$ and one is triplet (3) which is further divided into symmetric (3_S) and anti-symmetric (3_A) parts. These irreducible representations play a very important role in building models in particle physics. Now in order to build a model, one must be able to determine singlets from the products of these representations, which will make the Lagrangian of the model invariant. This can be achieved from the Clebsch-Gordan decomposition scheme, which leads to the tensor product rules for the irreducible representations in the following way:

$$1' \otimes 1' = 1'', \quad 1' \otimes 1'' = 1, \quad 1'' \otimes 1'' = 1'$$

$$3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_S \oplus 3_A$$

The explicit representations of the product of two triplets can be done in two ways. These two different ways are known as *Ma-Rajasekaran* and *Altarelli-Feruglio* basis.

Ma-Rajasekaran Basis

In this particular basis, both of the generators of A_4 have real entries and S is considered to be diagonal. They are expressed through the following matrices,

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \tag{1.60}$$

For two triplets, $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$, the expressions for singlets and triplets in Ma-Rajasekaran basis can be written as:

$$\begin{aligned} 1 &\equiv (ab) = a_1 b_1 + a_2 b_2 + a_3 b_3 \\ 1' &\equiv (ab)' = a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3 \\ 1'' &\equiv (ab)'' = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3 \\ 3_S &\equiv (ab)_S = (a_2 b_3, a_3 b_1, a_1 b_2) \\ 3_A &\equiv (ab)_A = (a_3 b_2, a_1 b_3, a_2 b_1) \end{aligned} \tag{1.61}$$

Altarelli-Feruglio Basis

In Altarelli-Feruglio basis, the generator T' is diagonal. The generators (S', T') of this basis are related to the Ma-Rajasekaran basis through a unitary transformation. This relation among the generators of the two basis can be expressed as:

$$T' = V^\dagger T V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \qquad S' = V^\dagger S V = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$
$$\text{where} \qquad V = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \tag{1.62}$$

For two triplets, $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$, the expressions for singlets and triplets in this particular basis can be written as:

$$\begin{aligned}
1 &\equiv (ab) = a_1b_1 + a_2b_3 + a_3b_2 \\
1' &\equiv (ab)' = a_3b_3 + a_1b_2 + a_2b_1 \\
1'' &\equiv (ab)'' = a_2b_2 + a_1b_3 + a_3b_1 \\
3_S &\equiv (ab)_S = (2a_1b_1 - a_2b_3 - a_3b_2, 2a_3b_3 - a_1b_2 - a_2b_1, 2a_2b_2 - a_1b_3 - a_3b_1) \\
3_A &\equiv (ab)_A = (a_2b_3 - a_3b_2, a_1b_2 - a_2b_1, a_1b_3 - a_3b_1)
\end{aligned} \tag{1.63}$$

Both of these basis, Ma-Rajasekaran and Altarelli-Feruglio, are quite popular in constructing different models in the field of particle physics. In this thesis, we have focused more on the Altarelli-Feruglio basis and have built the models of our work using this representation.

1.8.3 Modular Symmetry

In recent times modular symmetry has become quite popular in the study of neutrino phenomenology. One of the salient features of this symmetry is its potential to reduce the number of flavons that are used in a model. There are also instances of such models where no extra flavon has been used. In this group of symmetry, neutrino masses and Yukawa couplings are not free. Rather they are modular forms of the complex modulus τ [126, 14, 127]. In general the modular group $\Gamma(N)$ $(N = 1, 2, 3...)$ can be defined in the following way:

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, Z), \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} (\text{mod N}) \right\} \tag{1.64}$$

such that $ad - bc = 1$. These groups act on the upper half of the complex plane, (Im(τ)>0) and transforms the complex variable τ linearly as:

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

The matrix form of the two generators of modular symmetry are:

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}. \tag{1.65}$$

These operators act on τ and transform them in the following ways:

$$S \xrightarrow{\tau} -\frac{1}{\tau}, \qquad T \xrightarrow{\tau} 1 + \tau. \tag{1.66}$$

It is interesting to note that the finite modular groups ($N \leq 5$) and non-abelian discrete groups are isomorphic to each other [128, 129]. As a result $\Gamma_2 \approx S_3$, $\Gamma_3 \approx A_4$, $\Gamma_4 \approx S_4$, $\Gamma_5 \approx A_5'$. For a group of level N, the number of modular forms varies with respect to their weights. The table (1.4) shows how to find out the number of modular forms that a particular group with a particular level can have. The modular forms $f(\tau)$ of modular level N, weight k transform under the

N	No. of modular forms	$\Gamma(N)$
2	$k+1$	S_3
3	$2k+1$	A_4
4	$4k+1$	S_4
5	$10k+1$	A_5
6	$12k$	
7	$28k-2$	

Table 1.4: No. of modular forms of weight $2k$.

action of $\Gamma(N)$ in the following way:

$$f(\gamma\tau) = (c\tau + d)^k f(\tau) \tag{1.67}$$

These modular forms form a linear space of finite dimension i.e. $f_i(\tau)$. For a certain finite modular group, $f_i(\tau)$ transform under a certain unitary representation of that group in the following way:

$$f_i(\gamma\tau) = (c\tau + d)^k \rho_{ij}(\gamma) f_j(\tau) \tag{1.68}$$

where $\rho_{ij}(\gamma)$ is the irreducible representation of that particular group in concern. This particular relation is the foundation of model building of lepton masses and mixing in modular symmetry.

1.8.4 $\Gamma(3)$ Modular Group

It is a level three modular group which is isomorphic to discrete symmetry group A_4. As evident from table (1.4), $\Gamma(3)$ has three Yukawa modular forms of weight 2 and they form a triplet under A_4 symmetry group. These modular forms present in this group are expressed in terms of Dedekind eta-function ($\eta(\tau)$) as [130, 131]:

$$\begin{aligned} Y_1(\tau) &= \frac{i}{2\pi}[\frac{\eta'(\frac{\tau}{3})}{\eta(\frac{\tau}{3})} + \frac{\eta'(\frac{\tau+1}{3})}{\eta(\frac{\tau+1}{3})} + \frac{\eta'(\frac{\tau+2}{3})}{\eta(\frac{\tau+2}{3})} - 27\frac{\eta'(3\tau)}{\eta(3\tau)}] \\ Y_2(\tau) &= \frac{-i}{\pi}[\frac{\eta'(\frac{\tau}{3})}{\eta(\frac{\tau}{3})} + \omega^2\frac{\eta'(\frac{\tau+1}{3})}{\eta(\frac{\tau+1}{3})} + \omega\frac{\eta'(\frac{\tau+2}{3})}{\eta(\frac{\tau+2}{3})}] \\ Y_3(\tau) &= \frac{-i}{\pi}[\frac{\eta'(\frac{\tau}{3})}{\eta(\frac{\tau}{3})} + \omega\frac{\eta'(\frac{\tau+1}{3})}{\eta(\frac{\tau+1}{3})} + \omega^2\frac{\eta'(\frac{\tau+2}{3})}{\eta(\frac{\tau+2}{3})}] \end{aligned} \tag{1.69}$$

where $\eta(\tau)$ is the Dedekind eta-function and is defined in the following way:

$$\eta(\tau) = q^{\frac{1}{24}}\prod_{n=1}^{\infty}(1-q^n), \qquad q = e^{2\pi i\tau}. \tag{1.70}$$

The eta functions satisfy the equations

$$\eta(\tau+1) = \exp^{i\pi/12}\eta(\tau), \qquad \eta(-1/\tau) = \sqrt{-i\tau}\eta(\tau) \tag{1.71}$$

Another way of expanding these modular forms is the q-expansions, where q is expressed in terms of τ in the following way $q = \exp(2i\pi\tau)$. These expansions take the form:

$$\begin{aligned} Y_1(\tau) &= 1 + 12q + 36q^2 + 12q^3 + ... \\ Y_2(\tau) &= -6q^{1/3}(1 + 7q + 8q^2 + ...) \\ Y_3(\tau) &= -18q^{2/3}(1 + 2q + 5q^2 + ...) \end{aligned} \tag{1.72}$$

The above q-expansions play a crucial role in calculating the values of Yukawa modular forms. One can also have modular forms which have weights greater than 2. These forms can be constructed easily with the help of lower weight modular forms [132, 133]. For example, weight 4 modular forms can be constructed as:

$$Y_1^4 = ((Y_1^2)^3 + 2Y_2^2Y_3^2), \quad Y_{1'}^4 = ((Y_3^2)^2 + 2Y_1^2Y_2^2)$$

$$Y_3^4 = \begin{pmatrix} (Y_1^2)^2 - Y_2^2 Y_3^2 \\ (Y_3^2)^2 - Y_1^2 Y_2^2 \\ (Y_2^2)^2 - Y_1^2 Y_3^2 \end{pmatrix}$$

Similarly with the help of these lower weight modular forms we can construct modular forms of higher weights.

1.9 Thesis Outline

This thesis is organised in the following manner:

In **Chapter 1** we outline the different theoretical and experimental aspects of particle physics which served as motivation for our work. We begin by giving a brief introduction about the mysterious neutral particle called neutrino, followed by its latest theoretical and experimental developments. We then discuss the Standard Model which is considered to be one of the most successful theories of particle physics in present time. We then highlight some of the drawbacks of SM that presses the need for theories beyond the Standard Model. Thereafter we discuss the important concept of neutrino oscillation and how it gets modified in matter. In this section we have discussed some of the important mechanisms that are able to generate tiny neutrino masses. We proceed ahead with some of the BSM phenomena, such as baryon asymmetry of the universe, dark matter, lepton number violation, lepton flavor violation. Study of these events are one of the main objectives of this thesis. As we know, symmetry plays a very vital role in the field of model building, so we have described the groups that are used in this thesis. This mainly includes the abelian Z_3 discrete symmetry group, non-abelian A_4 group, modular symmetry and $\Gamma(3)$ modular group.

In **Chapter 2** we have discussed a model that has been built using modular symmetry in the framework of minimal inverse seesaw, ISS(2,3). We have used $\Gamma(3)$ modular group which is isomorphic to non-Abelian discrete symmetry group A_4. In this group, there are three Yukawa modular forms of weight 2. Due to the use of modular symmetry, the dependence of our model on flavons reduced significantly. Consequently, we have used only a single flavon in our work. The role of this flavon is to obtain a desired diagonal charged lepton mass matrix without affecting the neutrino sector. Along with A_4 symmetry group, we have also used Z_3 to restrict certain interaction terms in the Lagrangian. In this model we have studied neutrino masses and mixings for both normal and inverted hierarchies. Accordingly, we have evaluated the three neutrino mixing angles i.e. solar, reactor and atmospheric mixing angles, the Jarlskog invariant. The correlations among these parameters are shown through various plots present in the chapter. Apart from neutrino phenomenology, we have studied two important BSM phenomena in this model i.e. neutrinoless double beta decay (NDBD) and lepton flavor violation (LFV). We have evaluated effective Majorana mass of electron neutrino for NDBD and calculated the branching ratio of cLFV process $\mu \to e\gamma$ for this model. Further, we have also checked for possible deviations from unitarity conditions for the mixing matrix.

In **Chapter 3** contains an important part of the thesis. The works discussed in this chapter are done in a model that has been constructed by extending the minimal inverse seesaw with Higgs-like scalar triplet field $\eta = (\eta_1, \eta_2, \eta_3)$. In order to realize this model, we have used $N = 3$ modular group, $\Gamma(3)$. As it is isomorphic to non-abelian symmetry group A_4, therefore, the properties of A_4 play a very crucial role in defining the Lagrangian of the model. Henceforth, we determined the complex variable τ and subsequently those of the Yukawa modular forms. We then calculated the mixing angles and tried to find a common region of real and imaginary parts of τ that could accommodate all the three angles. Also we have shown the variations between these mixing angles and Yukawa modular forms through different graphs for both normal and inverted orderings. Analysing these graphs we found a common space of the modular forms that can produce all the angles. In this model we have also studied dark matter and baryon asymmetry of the universe (BAU). After symmetry breaking, only one of the $\eta's$ acquire VEV and the other two components remain neutral. These neutral components of η serve as the probable dark matter candidate for our work. So we calculated their relic density and checked its consistency with the observed values. In order to study BAU, We have used the popular resonant leptogenesis mechanism. In our work, the asymmetry created by decay of the light quasi-Dirac pair is converted into the observed asymmetry of the universe. For this purpose, we calculated the CP-asymmetry associated with the decaying pair and then evaluated the numerical value of BAU from the model. We found that the outcomes of our model are consistent with the results obtained from different cosmological observations and ongoing experiments.

In **Chapter 4** we have presented our study on texture zeros of neutrino mass matrix. For this study we have constructed a model by using the non-abelian discrete flavor symmetry group A_4 in the framework of ISS(2,3). Along with this group, Z_3 plays a crucial role in restricting certain unwanted interactions among the fields of the model. In this work, along with the Higgs-like triplet scalar field $\eta = \eta_1, \eta_2, \eta_3$, we have used five flavons i.e. $\phi, \chi, \chi', \zeta, \zeta'$. The VEV alignment of these flavon fields determines the mass matrices of the model. We fixed the structures of M_{NS} and M_S by taking specific VEV alignments of ζ, ζ' and then try to study the origin of neutrino textures by implementing 2-0 conditions on the Dirac mass matrix (M_D) of neutrinos. Out of the fifteen possible 2-0 structures of M_D, only six of them are able to generate 1-0 textures of neutrino mass matrix. Accordingly we calculated the model parameters and mixing angles for these six cases. Interestingly, we found that only two among these six cases produced all the three mixing angles within the 3σ allowed range for normal hierarchy. So we focused our further studies on these two cases in normal hierarchy. Additionally, we have also studied dark matter in these two cases. As discussed in the previous chapter, the neutral components of η also serve as the probable dark matter candidates for this work. We found that the results obtained from the model, which have been represented through various graphs in the chapter, are consistent with those of the experiments.

Finally in **chapter 5** we conclude by giving an overview of the entire work of this thesis. Along with this, we also provide a glimpse of future prospects and scope of this work.