Chapter 4

Texture Zeros using Discrete Flavor Symmetry in ISS(2,3)

In this work we have realized texture zero structures of neutrino mass matrix through our study of neutrino phenomenology and dark matter. For analysing these processes, we have constructed a model in minimal inverse seesaw, ISS(2,3) by using non-abelian A_4 discrete symmetry group. The particles of ISS(2.3) has been augmented by a scalar triplet $\eta = (\eta_1, \eta_2, \eta_3)$, along with the flavons ϕ, χ, χ', ζ and ζ'. The probable dark matter candidates in this model are the neutral components of η. The three mass matices of ISS(2,3), M_D, M_{NS} and M_S contribute to the structure of light neutrino mass matrix m_ν. Here we try to examine the impact on texture structures of m_ν due to different possible 2-0 structures of M_D. To examine further possible contraints, we have evaluated the neutrino parameters and calculated relic density of dark matter for the favourable cases. From our analysis we find that out of the fifteen possible 2-0 structures, only two of them (M_{D3} and M_{D6}) successfully generates all the mixing angles in the allowed ranges.

4.1 Introduction

Over the past few decades, particle physics has achieved significant advancements, especially in the field of neutrino physics. The landmark discovery of neutrino oscillation revealed critical insights about the nature of neutrinos and their properties. It showed that neutrinos should be massive in nature which challenged the idea of it being massless in the Standard Model. It also highlighted the fact that a possible change of one flavor of neutrinos into another during its propagation through space demands the existence of mixing angles. These new developments emphasized the role of neutrinos in different cosmological and astrophysical processes. Moreover, these findings are one of the many reasons which compelled physicists to opt for beyond Standard Model frameworks.

Although many of the neutrino parameters, including the mixing angles and mass-squared differences, have been precisely measured in experiments, there are some aspects that remain unanswered till date [59]. Prominent among them are about the dynamics and source of neutrino mass and flavor structures of the family of fermions. In this regard, symmetry plays a very crucial role in defining the dynamics of the leptons. It becomes very necessary to understand the underlying symmetry that can explain the generation of tiny neutrino mass and their mixing angles. In the absence of flavor symmetry, the BSM frameworks produces a general structure of light neutrino mass matrix with many free parameters. As a result, it becomes important to have a specific process that can connect two or more of them and reduce the number of free parameters, thereby increasing the predictive power of the models. One of the possible remedy is to impose texture zeros in the mass matrix by using flavor symmetry [186, 187, 188, 189]. For a symmetric mass matrix m_ν, there are six independent entries. If we consider n of them to be zero then there are ${}^6C_n$ different texture structures for m_ν. This approach of applying texture zeros has been a feasible mechanism in studying fermion masses and mixings. Moreove, texture zeros for n $>$ 2 are found to be inconsistent with the experimental data. As a result, one-zero and two-zero texture conditions are more popular in literatures. A detailed study on texture zero can be found in the literatures [190, 191, 192, 193, 194, 195, 196].

In this chapter we present the effect of texture zeros in our model that we have constructed by using A_4 flavor symmetry in minimal inverse seesaw. We have extended ISS(2,3) with five flavons and a Higgs-type scalar field η. Here we have tried to produce the texture zero conditions of m_ν by imposing 2-0 conditions on Dirac mass matrix (M_D) of the neutral sector. Also we have fixed the structures of M_{NS} and M_S by taking specific VEV alignments of the flavons ζ and ζ'. Finally we have evaluated the neutrino mixing angles and studied the effect of dark matter in this model. For analysing the relic density of dark matter we have used the same methodology as discussed in the previous chapter.

This chapter is organised in the following manner: in section (4.2) we have discussed the model of this chapter in detail. Section (4.3) contains the fifteen possible 2-0 conditions of M_D. Here we have also shown the six cases of M_D which successfully produces 1-0 textures of neutrino mass matrix. In section (4.4) we have discussed the results of this work. Finally we conclude this chapter by giving an overview in section (4.5).

4.2 The Model

In this work we have extended the mechanism of minimal inverse seesaw, ISS(2.3), with a Higgs-type scalar triplet $\eta = (\eta_1, \eta_2, \eta_3)$. This new addition plays a vital role in our study of neutrino phenomenology and aids in testing possible constraints from dark matter. Apart from this, we have used a few flavons which form important interactions with other fields of the model. These flavon fields are ϕ, χ, χ', ζ and ζ'. In order to describe the relevant interactions among different particles we have considered the non-abelian discrete symmetry group A_4. For this purpose the particles of the model have specific assignments of the irreducible representation of the group. Accordingly, the lepton doublets (L), sterile fermions (S_i), scalar field η and the flavons (χ, χ', ζ, ζ') are considered to be triplets under the group A_4. The right-handed neutrinos (N_1, N_2) transform as $1'$ and $1''$; whereas the Higgs field H transforms as trivial singlet (1) under this group. Along with A_4 we have also used the abelian symmetry group Z_3 in this work. The assignment of charges to particles have been highlighted in Table (4.1).

One of the aim of this work is to study the effect of dark matter in our setup. For this case the neutral components of η are the probable dark matter candidates. We know that the discrete symmetry group Z_2 plays a crucial role in stabilizing the dark matter candidate. Under Z_2 group, all the Standard Model particles are

Fields	L	N_1	N_2	S_i	H	ϕ	η	χ	χ'	ζ	ζ'
A_4	3	$1'$	$1''$	3	1	1	3	3	3	3	3
Z_3	ω^2	ω	ω	1	ω	ω	ω^2	ω	ω	ω^2	ω^2

Table 4.1: Charge assignments of the particles under the various groups considered in the model.

considered even and rest of the extra additional particles are taken to be odd. After electroweak symmetry breaking, only one component of η acquires VEV and the other two components do not acquire any such value [172, 171]. At this state the η fields can be expressed as:

$$\eta_1 = \begin{pmatrix} \eta_1^+ \\ \frac{v_\eta + h_1 + iA_1}{\sqrt{2}} \end{pmatrix}, \quad \eta_2 = \begin{pmatrix} \eta_2^+ \\ \frac{h_2 + iA_2}{\sqrt{2}} \end{pmatrix}, \quad \eta_3 = \begin{pmatrix} \eta_3^+ \\ \frac{h_3 + iA_3}{\sqrt{2}} \end{pmatrix} \tag{4.1}$$

From eq. (4.1) we see that only one of the η's acquire VEV i.e. $<\eta> = v_\eta(1,0,0)$. Now the Lagrangian for charged lepton sector can be written as:

$$L_L = a_1 E_1^c H_d (L\phi)_1 + a_2 E_2^c H_d (L\phi)_{1'} + a_3 E_3^c H_d (L\phi)_{1''} \tag{4.2}$$

For obtaining a diagonal charged lepton mass matrix, VEV of ϕ is considered as $\phi = (u, 0, 0)$. So the mass matrix takes: $M_L = diag(a_1, a_2, a_3)uv$. Moreover the parameters a_1, a_2, a_3 can be adjusted as per the need of the model. Finally the Yukawa Lagrangian for the neutrinos can be written as:

$$\mathcal{L} = \frac{y_1}{\Lambda} N_1 (L\chi)_3 \eta + \frac{y_2}{\Lambda} N_2 (L\chi')_3 \eta + \gamma_1 N_1 (S\zeta)_{1''} + \gamma_2 N_2 (S\zeta')_{1'} + p(SS)_1 \tag{4.3}$$

In eq.(4.3) $y_1, y_2, \gamma_1, \gamma_2$ are the coupling constants. Λ which is present in the above equation is the cut-off scale. The first two terms in the equation represents the Dirac mass term M_D. Third and the fourth terms are for mixing between right-handed neutrinos and sterile fermions M_{NS}. The final part denotes the mass term for sterile fermions (M_S). Following the A_4 multiplication rules, along with the VEV of $<\chi>= (\chi_1, \chi_2, \chi_3)$ and $<\chi'>= (\chi'_1, \chi'_2, \chi'_3)$, matrix for the Dirac mass can be written as:

$$M_D = \frac{v_\eta}{\Lambda} \begin{pmatrix} -\chi_3 y_1 & -\chi'_2 y_2 \\ 2\chi_2 y_1 & -\chi'_1 y_2 \\ -\chi_1 y_1 & 2\chi'_3 y_2 \end{pmatrix} = \begin{pmatrix} a & b \\ 2c & d \\ e & 2f \end{pmatrix} \tag{4.4}$$

where $a = -\frac{v_\eta}{\Lambda}\chi_3 y_1$, $b = -\frac{v_\eta}{\Lambda}\chi_2' y_2$, $c = \frac{v_\eta}{\Lambda}\chi_2 y_1$, $d = -\frac{v_\eta}{\Lambda}\chi_1' y_2$, $e = -\frac{v_\eta}{\Lambda}\chi_1 y_1$ and $f = \frac{v_\eta}{\Lambda}\chi_3' y_2$.

In a similar way, VEV alignment of the flavons ζ and ζ' are considered to be $<\zeta>= v_\zeta(1,0,1)$ and $<\zeta'>= v_{\zeta'}(1,0,0)$. With the help of these alignments, we can express the matrices M_{NS} and M_S in the following way:

$$M_{NS} = \begin{pmatrix} \gamma_1 v_\zeta & 0 & \gamma_1 v_\zeta \\ 0 & \gamma_2 v_{\zeta'} & 0 \end{pmatrix} = \begin{pmatrix} g & 0 & g \\ 0 & h & 0 \end{pmatrix}, \qquad M_S = p\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{4.5}$$

In the above eq. (4.5) , $g = \gamma_1 v_\zeta$ and $h = \gamma_2 v_{\zeta'}$. Now using these three matrices, we can construct the 8×8 neutrino mass matrix. For ISS(2,3) this matrix takes the following form:

$$\mathcal{M} = \begin{pmatrix} 0 & M_D^T & 0 \\ M_D & 0 & M_{NS} \\ 0 & M_{NS}^T & M_S \end{pmatrix}_{8\times 8} \tag{4.6}$$

This 8×8 matrix can be diagonalised with the help of a unitary matrix. The eight eigenvalues obtained after diagonalisation will correspond to the mass of the eight particles that are involved in the matrix. From eq. (4.5) it can be seen that M_{NS} is a rectangular matrix. As a result, inverse of this matrix is not possible. Due to this issue, the expression for light neutrino mass matrix slightly changes to that in eq. (4.3). So in minimal inverse seesaw this expression is written as:

$$m_\nu = M_D.d.M_D^T \tag{4.7}$$

In the above equation, d is a 2×2 matrix which can be derived from the 5×5 heavy neutrino mass matrix M_H. The form of d can be obtained in the following way:

$$M_H^{-1} = \begin{pmatrix} 0 & M_{NS} \\ M_{NS}^T & M_S \end{pmatrix}^{-1} = \begin{pmatrix} d_{2\times 2} & ..... \\ ...... & ..... \end{pmatrix} \tag{4.8}$$

The matrix m_ν is symmetric in nature. Thus the elements of m_ν in terms of model parameters can be written as:

$$
\begin{aligned}
m11 &= -\frac{2abp}{gh} + \frac{b^2p}{h^2} \\
m12 &= -\frac{bcp}{gh} + \frac{bdp}{h^2} - \frac{adp}{gh} \\
m13 &= -\frac{bep}{gh} + \frac{bfp}{h^2} - \frac{afp}{gh} \\
m22 &= -\frac{2cdp}{gh} + \frac{d^2p}{h^2} \\
m23 &= -\frac{dep}{gh} + \frac{dfp}{h^2} - \frac{cfp}{gh} \\
m33 &= -\frac{2efp}{gh} + \frac{f^2p}{h^2}
\end{aligned} \tag{4.9}
$$

In this way, by using A_4 symmetry group, we have constructed a model in the mechanism of ISS(2,3). The eigenvalues of eq. (4.7) are the masses of three light neutrinos. Once this is done, it will be helpful to study other related phenomenologies.

4.3 Texture Zero Structures of the Matrices

Texture zeros is an effective method that helps to reduce the number of free parameters associated with the light neutrino mass matrix. These texture structures are classified as one-zero, two-zero, three-zero etc [197, 198]. This classification is based on the number of elements of the mass matrix that are considered to be zero. However, as stated earlier, only one-zero and two-zero textures are able to produce the neutrino parameters; the other texture structures are discarded as they are not compatible with the current experimental data [187, 199]. In this work we try to find out the impact on these texture structures by applying two-zero textures to Dirac mass matrix M_D i.e. two elements of M_D are simultaneously taken to be zero. We find that there are fifteen such possibilites of M_D. These are presented in the table (4.2).

It is interesting to note that out of all the fifteen possibilities, only six of them are able to produce one-zero textures of neutrino mass matrix. These structures are M_{D3}, M_{D5}, M_{D6}, M_{D8}, M_{D12} and M_{D13}. They successfully produce diagonal one-zero textures of the mass matrix. Below we divide them into six different cases and try to explain the texture origin of neutrino mass matrix for our work.

Possible two zero textures of Dirac mass matrix M_D		
$M_{D1}=\begin{pmatrix}0&0\\c&d\\e&f\end{pmatrix}$	$M_{D2}=\begin{pmatrix}0&b\\0&d\\e&f\end{pmatrix}$	$M_{D3}=\begin{pmatrix}0&b\\c&0\\e&f\end{pmatrix}$
$M_{D4}=\begin{pmatrix}0&b\\c&d\\0&f\end{pmatrix}$	$M_{D5}=\begin{pmatrix}0&b\\c&d\\e&0\end{pmatrix}$	$M_{D6}=\begin{pmatrix}a&0\\0&d\\e&f\end{pmatrix}$
$M_{D7}=\begin{pmatrix}a&0\\c&0\\e&f\end{pmatrix}$	$M_{D8}=\begin{pmatrix}a&0\\c&d\\0&f\end{pmatrix}$	$M_{D9}=\begin{pmatrix}a&0\\c&d\\e&0\end{pmatrix}$
$M_{D10}=\begin{pmatrix}a&b\\0&0\\e&f\end{pmatrix}$	$M_{D11}=\begin{pmatrix}a&b\\0&d\\0&f\end{pmatrix}$	$M_{D12}=\begin{pmatrix}a&b\\0&d\\e&0\end{pmatrix}$
$M_{D13}=\begin{pmatrix}a&b\\c&0\\0&f\end{pmatrix}$	$M_{D14}=\begin{pmatrix}a&b\\c&0\\e&0\end{pmatrix}$	$M_{D15}=\begin{pmatrix}a&b\\c&d\\0&0\end{pmatrix}$

Table 4.2: The possible two zero textures of Dirac mass matrix M_D.

Case 1:

For the Dirac mass matrix of the form M_{D3}, the VEV of the flavons after symmetry breaking must be in the order:

$$<\chi>=v_\chi(1,1,0) \qquad <\chi'>=v_{\chi'}(0,1,1)$$

This particular VEV alignment gives 1-0 texture at (2,2) position,

$$\begin{pmatrix}X&X&X\\X&0&X\\X&X&X\end{pmatrix}$$

Case 2:

For the matrix of the form M_{D5}, the required VEV alignments of the flavons are:

$$<\chi>= v_\chi(1,1,0) \qquad <\chi'>= v_{\chi'}(1,1,0)$$

This VEV arrangement of the flavons produces 1-0 textures at (3,3) position,

$$\begin{pmatrix} X & X & X \\ X & X & X \\ X & X & 0 \end{pmatrix}$$

Case 3:

For the Dirac mass matrix of the form M_{D6}, the VEV of the flavons after symmetry breaking must be in the order:

$$<\chi>= v_\chi(1,0,1) \qquad <\chi'>= v_{\chi'}(1,0,1)$$

This particular VEV alignment gives 1-0 texture at (1,1) position,

$$\begin{pmatrix} 0 & X & X \\ X & X & X \\ X & X & X \end{pmatrix}$$

Case 4:

For the matrix of the form M_{D8}, the required VEV alignments of the flavons are:

$$<\chi>= v_\chi(0,1,1) \qquad <\chi'>= v_{\chi'}(1,0,1)$$

This VEV arrangement of the flavons produces 1-0 textures at (1,1) position,

$$\begin{pmatrix} 0 & X & X \\ X & X & X \\ X & X & X \end{pmatrix}$$

Case 5:

For the Dirac mass matrix of the form M_{D12}, the VEV of the flavons after symmetry breaking must be in the order:

$$<\chi>= v_\chi(1,0,1) \qquad <\chi'>= v_{\chi'}(1,1,0)$$

This particular VEV alignment gives 1-0 texture at (3,3) position,

$$\begin{pmatrix} X & X & X \\ X & X & X \\ X & X & 0 \end{pmatrix}$$

Case 6:

For the Dirac mass matrix of the form M_{D13}, the VEV of the flavons after symmetry breaking must be in the order:

$$< \chi >= v_\chi(0,1,1) \qquad < \chi' >= v_{\chi'}(0,1,1)$$

This particular VEV alignment gives 1-0 texture at (2,2) position,

$$\begin{pmatrix} X & X & X \\ X & 0 & X \\ X & X & X \end{pmatrix}$$

We perform our study on these six different cases. We try to find further constraints on these matrices based on our study of neutrino parameters, mixing angles and masses. This work is also extended to probe the effect of dark matter in this system.

4.4 Results and Discussions

In this section we present the results of our work. For numerical evaluation of different quantities of the model, we have used the latest 3σ nu-fit values of neutrino oscillation parameters [158]. These values are highlighted in table (4.3).

In our model the charged lepton mass matrix is diagonal. Therefore the mixing matrix in this case is the unitary PMNS matrix, U_{PMNS}. Using this unitary mixing matrix, we diagonalise the light neutrino mass matrix of the model via the standard relation $m_\nu = U^T \text{diag}(m_1, m_2, m_3) U$, where m_1, m_2, m_3 are the masses of the active neutrinos. These masses have different forms for normal and inverted hierarchies. For normal hierarchy these are expressed as: $\text{diag}(0, \sqrt{m_1^2 + \Delta m_{solar}^2}, \sqrt{m_1^2 + \Delta m_{atm}^2})$ and it becomes $\text{diag}(\sqrt{m_3^2 + \Delta m_{atm}^2}, \sqrt{\Delta m_{atm}^2 + \Delta m_{solar}^2}, 0)$ for inverted hierarchy [184]. Through this process we can calculate the masses of

Oscillation parameters	Normal Ordering	Inverted Ordering
$\sin^2\theta_{12}$	[0.269,0.343]	[0.269,0.343]
$\sin^2\theta_{23}$	[0.407,0.618]	[0.411,0.621]
$\sin^2\theta_{13}$	[0.02034,0.02430]	[0.020530,0.02436]
$\Delta m^2_{21}/10^{-5}\mathrm{eV}^2$	[6.28,8.04]	[6.82,8.04]
$\Delta m^2_{31}/10^{-3}\mathrm{eV}^2$	[2.431,2.598]	[2.412,2.583]

Table 4.3: The latest 3σ nu-fit values of oscillation parameters.

neutrinos, their mixing angles and other parameters of the model. Additional constraints for these quantities come from solar and atmospheric mass squared differences. Once these values are obtained, they are then used to study the cosmological event of dark matter from the model. Furthermore, we have considered the values of p in the range (10-20) KeV and VEV of η is taken to be (1-10) GeV. The mixing angles can be calculated from the elements of mixing matrix. Thus, they can be expressed as:

$$\sin^2\theta_{13} = |U_{e3}|^2, \qquad \sin^2\theta_{23} = \frac{|U_{\mu 3}|^2}{1-|U_{e3}|^2}, \qquad \sin^2\theta_{12} = \frac{|U_{e2}|^2}{1-|U_{e3}|^2}, \tag{4.10}$$

As mentioned earlier, only six of the two-zero structures of Dirac mass matrix are able to produce 1-0 texture conditions of light neutrino matrix. We have performed our study in these six cases and have used the expressions in eq. (4.10) to find out the values of mixing angles for all these possible cases in both normal and inverted hierarcy. Also the parameters g and h are considered in the ranges $[10^5, 10^6]$ GeV and $[10^3, 10^4]$ GeV. Interestingly we found that only two of the 1-0 textures corresponding to M_{D3} and M_{D6} structures of Dirac matrix successfully produce all the three mixing angles in the allowed range for normal hierarchy. The other cases fail to produce one or two of these angles. Of the three mixing angles, two of them are possible in these cases, while the third is not possible and vice versa. This is true for both normal and inverted hierarchy. Because of this reason, we focus our study on normal hierarchy of the two cases that can generate all the mixing angles. In Table (4.4) we have highlighted the possibilities of obtaining the mixing angles in different 2-0 textures of Dirac mass matrix. In the following paragraphs we have presented the plots that describe the

Dirac Mass	1-0 Texture	Normal Hierarchy			Inverted Hierarchy		
		θ_{12}	θ_{13}	θ_{23}	θ_{12}	θ_{13}	θ_{23}
$M_{D3} = \begin{pmatrix} 0 & b \\ c & 0 \\ e & f \end{pmatrix}$	$\begin{pmatrix} X & X & X \\ X & 0 & X \\ X & X & X \end{pmatrix}$	✔	✔	✔	✔	✘	✔
$M_{D5} = \begin{pmatrix} 0 & b \\ c & d \\ e & 0 \end{pmatrix}$	$\begin{pmatrix} X & X & X \\ X & X & X \\ X & X & 0 \end{pmatrix}$	✘	✘	✔	✘	✘	✔
$M_{D6} = \begin{pmatrix} a & 0 \\ 0 & d \\ e & f \end{pmatrix}$	$\begin{pmatrix} 0 & X & X \\ X & X & X \\ X & X & X \end{pmatrix}$	✔	✔	✔	✘	✘	✘
$M_{D8} = \begin{pmatrix} a & 0 \\ c & d \\ 0 & f \end{pmatrix}$	$\begin{pmatrix} 0 & X & X \\ X & X & X \\ X & X & X \end{pmatrix}$	✘	✘	✔	✘	✘	✔
$M_{D12} = \begin{pmatrix} a & b \\ 0 & d \\ e & 0 \end{pmatrix}$	$\begin{pmatrix} X & X & X \\ X & X & X \\ X & X & 0 \end{pmatrix}$	✘	✔	✘	✘	✘	✘
$M_{D13} = \begin{pmatrix} a & b \\ c & 0 \\ 0 & f \end{pmatrix}$	$\begin{pmatrix} X & X & X \\ X & 0 & X \\ X & X & X \end{pmatrix}$	✔	✘	✔	✔	✘	✔

Table 4.4: The table above shows the possibility of occurence of mixing angles for the six 2-0 textures of M_D.

variations among different parameters of the model.

4.4.1 Possible Values of Model Parameters

In Fig. (4.1) we have shown the variation of model parameters with respect to sum of neutrino mass ($\sum m_\nu$). From cosmological observations it is found that the upper limit for $\sum m_\nu$ is ≤ 0.12 eV. From the figures we see that there are sufficient amounts of these parameters within the allowed range. For b this range is found to be aroung (0.02-1) eV. The lower limit of the parameters c and e are almost same, around 5 eV. But the upper limit for c is about 500 eV, whereas for e it is around 900 eV. Also the range for f is found to be (0.1-10) eV. Similarly the figures in (4.2) show the relation between model parameters and $\sum m_\nu$ for *Case 3* (M_{D6}) of Dirac mass matrix. We find that b lies in the range (0.05-1) eV. For this case, the parameters c and e lie in the same range (5-500) eV. Finally the values of d are restricted in the region (0.2-10) eV. The common region of space for these parameters have been highlighted in Table (4.5).

Model Parameters	Common Region (NH)
b	(0.05 - 1) eV
c	(5 - 500) eV
d	(0.2 - 10) eV
e	(5 - 500) eV
f	(0.1 - 10) eV

Table 4.5: This table shows the favourable space of the model parameters.

4.4.2 Analysis of DM for M_{D3} and M_{D6}

The plots in Fig. (4.3) and (4.4) shows the relation between relic density of dark matter (Ωh^2) with respect to sum of neutrino mass and lightest right-handed neutrino M_1 for both the 2-0 textures of Dirac mass matrix. From the figures it is clear that there are reasonable amounts of these quantities in the allowed range. In other words, the model is able to generate the relic density of dark matter for both the cases of M_{D3} and M_{D6}. Also for the right-handed neutrino, the relic density is producible for a range of about (1000-5000) GeV of its mass.

4.5 Conclusions

Here we have realized the texture zero structures of neutrino mass matrix by implementing 2-0 conditions in Dirac mass matrix. For this purpose we have constructed a model in the framework of minimal inverse seesaw using the non-

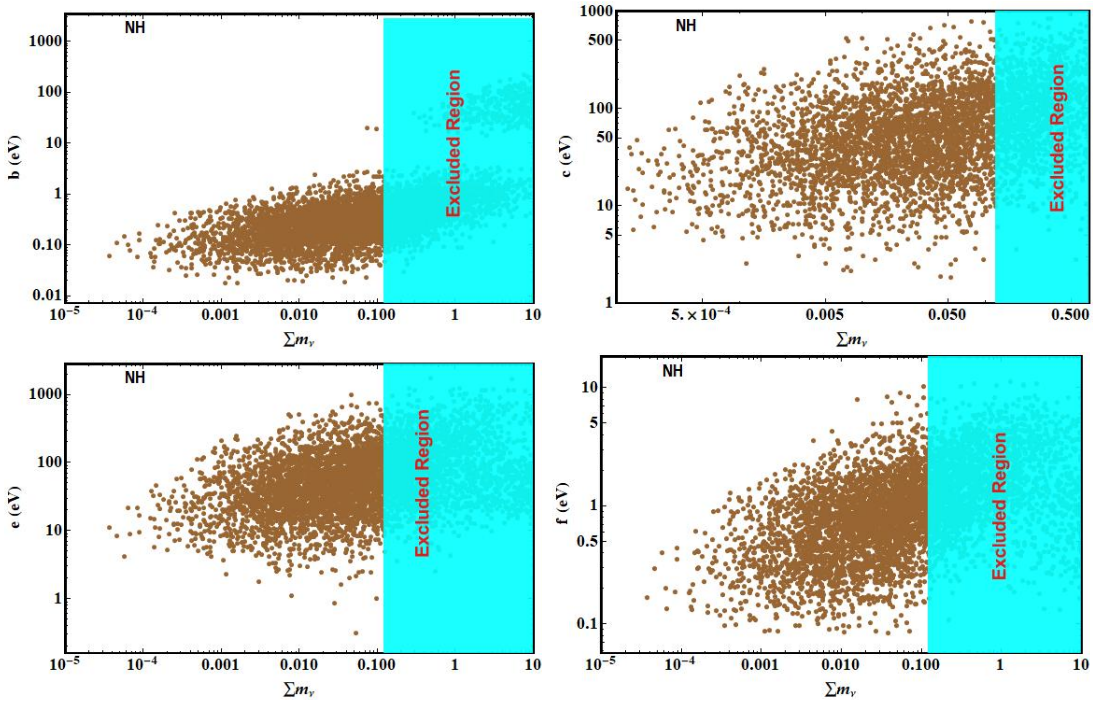

Figure 4.1: Corelation between the model parameters and $\sum m_\nu$ for Case 1 i.e. M_{D3}.

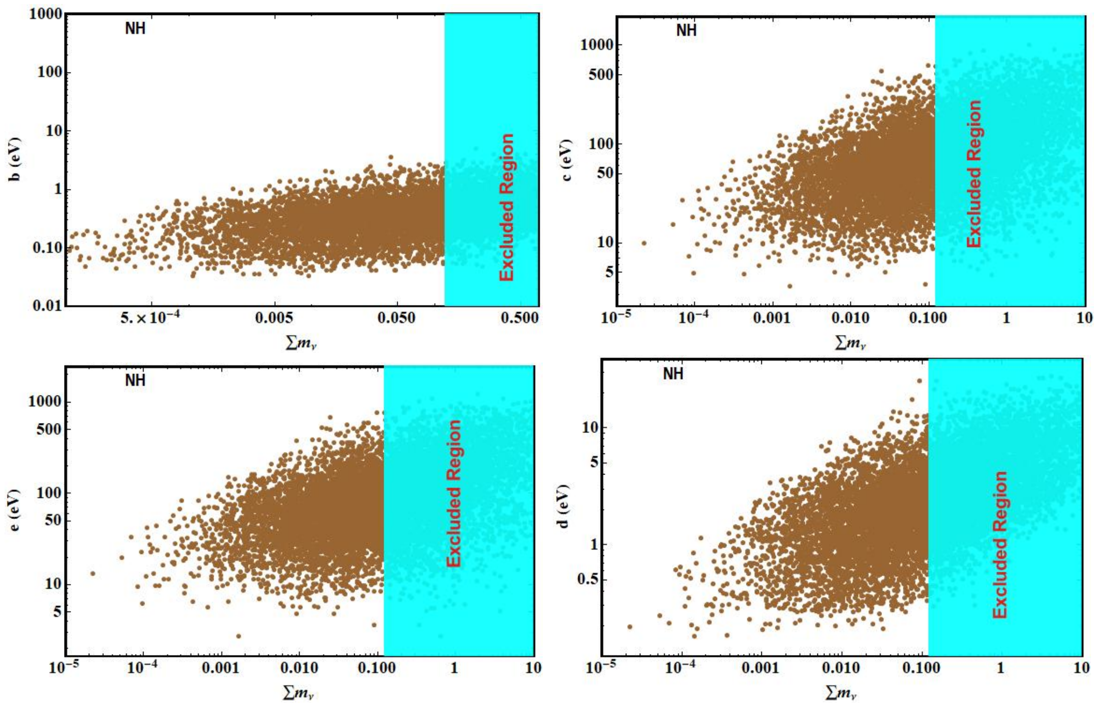

Figure 4.2: Corelation between the model parameters and $\sum m_\nu$ for Case 3 i.e. M_{D6}.

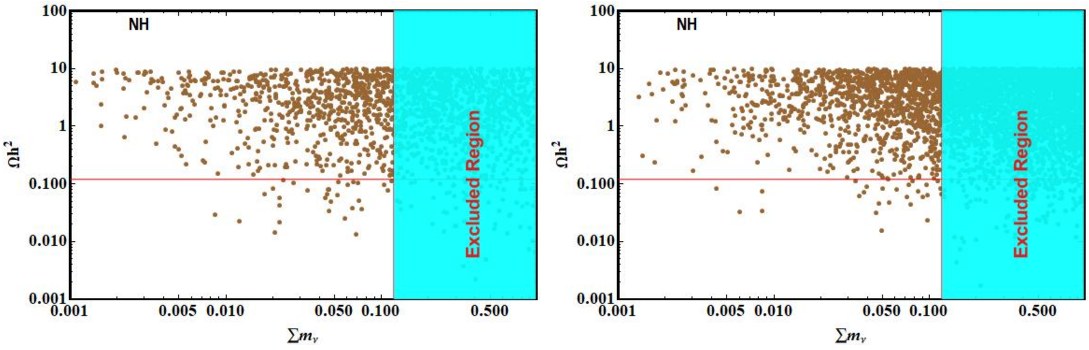

Figure 4.3: Corelation between relic density (Ωh^2) and sum of neutrino mass ($\sum m_\nu$). The left (right) figure is for M_{D3} (M_{D6}).

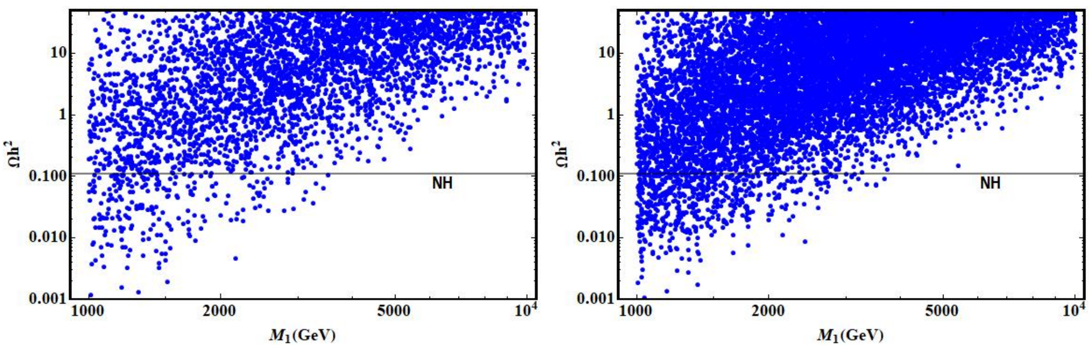

Figure 4.4: Corelation between relic density (Ωh^2) and right-handed neutrino M_1. The left (right) figure is for M_{D3} (M_{D6}).

abelian discrete symmetry group A_4. There are five flavons present in this model $\chi, \chi', \zeta, \zeta', \phi$. We find that out of the fifteen possible 2-0 structures of M_D, only six of them are able to generate diagonal 1-0 texture structures of light neutrino mass matrix. Accordingly we evaluated the model parameters for these cases. Furthermore we calculated the three mixing angles and masses to obtain any possible constraints on them. Interestingly we find that only two of the structures of M_D i.e. M_{D3} and M_{D6} favourably produce all the mixing angles in the allowed ranges for normal hierarchy. The other cases could generate two of the angles, but failed to produce the third angle and vice versa. As a result, we concentrated our study of dark matter in these two cases and found that they generated relic density in the desired range. So we can say that these results validate the compatibility of this model and it can be used to explore other phenomenological studies.