SIGN PATTERNS AND CONGRUENCES OF CERTAIN INFINITE PRODUCTS INVOLVING THE ROGERS-RAMANUJAN CONTINUED FRACTION, ARITHMETIC PROPERTIES OF SOME PARTITION FUNCTIONS, AND PARITY BIASES IN INTEGER PARTITIONS

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BY

ABHISHEK SARMA

Roll No. MSP21107

(Reg. No. TZ23001235)

DEPARTMENT OF MATHEMATICAL SCIENCES

TEZPUR UNIVERSITY, NAPAAM 784028, ASSAM, INDIA

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Summary

The thesis mainly consists of three topics. The first topic that we studied is sign patterns and congruences of certain infinite products related to the Rogers-Ramanujan continued fraction. In the next four chapters, we study arithmetic properties of some partition functions which we shortly explain in this chapter. And finally, Finally, in the last chapter, we study parity biases in non-unitary partitions (namely, the partitions where 1 is not allowed to be a part).

In Chapter 2, we studied the behavior of the signs of the coefficients of certain infinite products involving the Rogers-Ramanujan continued fraction. For example, if

$$\sum_{n=0}^{\infty} A(n)q^n := \frac{(q^2;q^5)_\infty^5 (q^3;q^5)_\infty^5}{(q;q^5)_\infty^5 (q^4;q^5)_\infty^5},$$

then $A(5n+1) > 0$, $A(5n+2) > 0$, $A(5n+3) > 0$, and $A(5n+4) < 0$. We also proved a few congruences satisfied by some coefficients. For example, we proved that, for all nonnegative integers n, $A(9n+4) \equiv 0 \pmod 3$, $A(16n+13) \equiv 0 \pmod 4$, and $A(15n+r) \equiv 0 \pmod{15}$, where $r \in \{4, 8, 13, 14\}$.

In Chapters 3–6, we proved various congruence properties and density results for various partition functions. Specifically, we found such results for the cubic partition function, 5-regular partitions into distinct parts, Andrews' integer partitions with even parts below odd parts and overcubic partitions triples. In Chapter 6, we also extended the definition overcubic partition k-tuples.

Finally in Chapter 7, we proved parity biases for non-unitary partitions. We

also looked at inequalities between two classes of partitions introduced by Andrews (2019) where the parts are seperated by parity (either all odd parts are less than all even parts or vice versa).