

Contents

Abstract	i
Declaration by the candidate	iii
Certificate of the supervisor	iv
Acknowledgements	v
1 Introduction	1
1.1 Ramanujan's theta functions	2
1.2 The Rogers-Ramanujan continued fraction	2
1.3 n -Dissection of power series	3
1.4 Partitions of positive integers	3
1.5 Arithmetic density	4
1.6 Some infinite products related to the Rogers-Ramanujan Identities . .	5
1.7 Some partition functions	6
1.8 Parity biases in integer partitions	9
2 Sign patterns and congruences of certain infinite products involving the Rogers-Ramanujan continued fraction	10
2.1 Introduction	10
2.2 Preliminary lemmas	14
2.3 Proof of Theorem 2.2	15
2.4 Proof of Theorem 2.3	18
2.5 Proof of Theorem 2.4	19
2.6 Proof of Theorem 2.6	21

2.7	Proof of Theorem 2.7	23
2.8	Concluding remarks	29
3	Differences of even and odd numbers of parts of the cubic and some analogous partition functions	30
3.1	Introduction	30
3.2	Dissection formulas	34
3.3	Proofs of Theorems 3.2–3.4	35
3.4	Proof of Theorem 3.6	39
3.5	Proof of Theorem 3.7	40
3.6	Proof of Theorem 3.8	42
3.7	Proof of Theorem 3.9	46
4	Arithmetic properties of 5-regular partitions into distinct parts	51
4.1	Introduction	51
4.2	Proof of Theorem 4.1	54
4.3	Proof of Theorem 4.2	54
4.4	Proof of Theorem 4.3	57
4.5	Proof of Theorem 4.4	59
5	Arithmetic properties of Andrews' integer partitions with even parts below odd parts	64
5.1	Introduction	64
5.2	Proof of Theorem 5.6	68
5.3	Proof of Theorem 5.7	71
5.4	Proofs of Theorems 5.9 and 5.11	73
5.5	Proof of Theorem 5.13	74
5.6	Proof of Theorems 5.15 and 5.16	76
6	Arithmetic properties of overcubic partition triples	78
6.1	Introduction	78
6.2	Proofs of Theorems 6.1 and 6.3	83
6.3	Proof of Theorem 6.4	86

6.4	Proof of Theorem 6.8	88
6.5	Sketch Proof of Theorem 6.7	89
6.6	Concluding Remarks	90
7	Parity biases in integer partitions	92
7.1	Introduction	92
7.2	Preliminaries	94
7.3	Biases in Ordinary Non-Unitary Partitions	95
7.4	Inequalities between Partitions with Parts Separated by Parity	98
7.5	Concluding Remarks	102
List of Publications		104
Bibliography		105