Dedicated to my parents

Miraz Ali Ahmed

&

Phiroma Choudhury

DECLARATION BY THE CANDIDATE

I, Mr. Firdous Ee Jannat, hereby declare that the subject matter in this thesis entitled "Common neighborhood and distance spectral aspects of certain graphs defined on finite groups", is the record of work done by me, that the contents of this thesis did not form basis of the award of any previous degree to me or to the best of my knowledge to anybody else, and that the thesis has not been submitted by me for any research degree in any other university/institute.

This thesis is being submitted to the Tezpur University for the degree of Doctor of Philosophy in Mathematical Sciences.

Date: 12.06.2025

Place: Tezpur

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CERTIFICATE OF THE SUPERVISOR

This is to certify that the thesis entitled "*Common neighborhood and distance spectral aspects of certain graphs defined on finite groups*" submitted to the School of Sciences of Tezpur University in partial fulfillment for the award of the degree of Doctor of Philosophy in Mathematical Sciences is a record of research work carried out by Mr. Firdous Ee Jannat under my supervision and guidance.

All help received by him from various sources have been duly acknowledged.

No part of this thesis have been submitted elsewhere for award of any other degree.

Date: 12.06.2025
Place: Tezpur

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List of symbols

$\cup$	union of sets/graphs
$\cap$	intersection of sets
$\subseteq$	subset/subgraph
$\lceil x \rceil$	ceiling function of x
$\lfloor x \rfloor$	floor function of x
$\mathbb{N}$	set of natural numbers
$\mathbb{Z}$	set of integers
$\mathbb{R}$	set of real numbers
Γ	finite, simple and undirected graph
$\overline{\Gamma}$	complement of Γ
$\Gamma^{\dagger}$	derived graph of Γ
$\Gamma[S]$	subgraph of Γ induced by vertex set S
$v(\Gamma)$	vertex set of Γ
$e(\Gamma)$	edge set of Γ
$\deg(v)$	degree of vertex v
Δ	maximum degree of a graph
$m_{\Gamma}(v)$	average degree of the adjacent vertices of vertex v in Γ
$d(u, v)$	distance between the vertices u and v
K_n	complete graph of order n
$K_{n_1,n_2,\ldots,n_k}$	complete k-partite graph
P_n	path graph of order n
C_n	cycle graph of order n
$\mathrm{tr}(M)$	trace of a square matrix M
$\mathrm{diag}[d_1, d_2, \ldots, d_n]$	diagonal matrix of size n having diagonal entries $d_1, d_2, \ldots, d_n$
$\mathrm{A}(\Gamma)$	adjacency matrix of Γ
$\mathrm{D}(\Gamma)$	degree matrix of Γ

$\mathrm{L}(\Gamma)$	Laplacian matrix of Γ
$\mathrm{Q}(\Gamma)$	signless Laplacian matrix of Γ
$\mathcal{D}(\Gamma)$	distance matrix of Γ
$\mathrm{DL}(\Gamma)$	distance Laplacian energy of Γ
$\mathrm{DQ}(\Gamma)$	distance signless Laplacian matrix of Γ
$\mathrm{CN}(\Gamma)$	common neighborhood matrix of Γ
$\mathrm{CNRS}(\Gamma)$	common neighborhood row sum matrix of Γ
$\mathrm{CNL}(\Gamma)$	common neighborhood Laplacian matrix of Γ
$\mathrm{CNSL}(\Gamma)$	common neighborhood signless Laplacian matrix of Γ
$\mathrm{Ch}_{\mathrm{D}}(\Gamma, x)$	characteristic polynomial of $\mathcal{D}(\Gamma)$
$\mathrm{Ch}_{\mathrm{DL}}(\Gamma, x)$	characteristic polynomial of $\mathrm{DL}(\Gamma)$
$\mathrm{Ch}_{\mathrm{DQ}}(\Gamma, x)$	characteristic polynomial of $\mathrm{DQ}(\Gamma)$
$\mathrm{Spec}(\Gamma)$	spectrum of Γ
$\mathrm{L\text{-}spec}(\Gamma)$	Laplacian spectrum of Γ
$\mathrm{Q\text{-}spec}(\Gamma)$	signless Laplacian spectrum of Γ
$\mathrm{D\text{-}spec}(\Gamma)$	distance spectra of Γ
$\mathrm{DL\text{-}spec}(\Gamma)$	distance Laplacian spectra of Γ
$\mathrm{DQ\text{-}spec}(\Gamma)$	distance signless Laplacian spectra of Γ
$\mathrm{CN\text{-}spec}(\Gamma)$	common neighborhood spectrum of Γ
$\mathrm{CNL\text{-}spec}(\Gamma)$	common neighborhood Laplacian spectrum of Γ
$\mathrm{CNSL\text{-}spec}(\Gamma)$	common neighborhood signless Laplacian spectrum of Γ
$\mathrm{E}(\Gamma)$	energy of Γ
$\mathrm{LE}(\Gamma)$	Laplacian energy of Γ
$\mathrm{LE}^{+}(\Gamma)$	signless Laplacian energy of Γ
$\mathrm{E}_{\mathrm{D}}(\Gamma)$	distance energy of Γ
$\mathrm{E}_{\mathrm{DL}}(\Gamma)$	distance Laplacian energy of Γ
$\mathrm{E}_{\mathrm{DQ}}(\Gamma)$	distance signless Laplacian energy of Γ

$\mathrm{E_{CN}}(\Gamma)$	common neighborhood energy of Γ
$\mathrm{LE_{CN}}(\Gamma)$	common neighborhood Laplacian energy of Γ
$\mathrm{LE^{+}_{CN}}(\Gamma)$	common neighborhood signless Laplacian energy of Γ
$\Delta_{\mathrm{CN}}(\Gamma)$	$\frac{\mathrm{tr(CNRS(\Gamma))}}{\lvert v(\Gamma)\rvert}$
$W(\Gamma)$	Wiener index of Γ
$\Delta_{\mathrm{D}}(\Gamma)$	$\frac{\mathrm{tr(DL(\Gamma))}}{\lvert v(\Gamma)\rvert} = \frac{2W(\Gamma)}{\lvert v(\Gamma)\rvert}$
$M_1(\Gamma)$	first Zagreb index of Γ
G	finite non-abelian group
x^g	gxg^{-1}
x^G	conjugacy classes of $x \in G$
$\mathrm{Cl}(S)$	$\{x^G : x \in S\}$ for any subset S of G
$\Gamma_{\mathrm{c}}(G)$	commuting graph of G
$\Gamma_{\mathrm{ccc}}(G)$	commuting conjugacy class graph of G
$\Gamma_{\mathrm{nccc}}(G)$	non-commuting conjugacy class graph of G
$\Gamma^*_{\mathrm{ccc}}(G)$	the subgraph $\Gamma_{\mathrm{ccc}}(G)[\mathrm{Cl}(G \setminus Z(G))]$ of $\Gamma_{\mathrm{ccc}}(G)$
$\Gamma^*_{\mathrm{nccc}}(G)$	the subgraph $\Gamma_{\mathrm{nccc}}(G)[\mathrm{Cl}(G \setminus Z(G))]$ of $\Gamma_{\mathrm{nccc}}(G)$
$\mathbb{Z}_n$	group of integers modulo n
D_{2m}	$\langle x, y : x^m = y^2 = 1, y^{-1}xy = x^{-1}\rangle$ (Dihedral group)
Q_{4n}	$\langle x, y : x^{2n} = 1, x^n = y^2, y^{-1}xy = x^{-1}\rangle$ (Dicyclic group)
SD_{8n}	$\langle x, y : x^{4n} = y^2 = 1, y^{-1}xy = x^{2n-1}\rangle$ (Semidihedral group)
$U_{(n,m)}$	$\langle x, y : x^{2n} = y^m = 1, x^{-1}yx = y^{-1}\rangle$
U_{6n}	$\langle x, y : x^{2n} = y^3 = 1,\ x^{-1}yx = y^{-1}\rangle$
V_{8n}	$\langle x, y : x^{2n} = y^4 = 1, yx = x^{-1}y^{-1}, y^{-1}x = x^{-1}y\rangle$
QD_{2^n}	$\langle x, y : x^{2^{n-1}} = y^2 = 1, y^{-1}xy = x^{2^{n-2}}\rangle$ (Quasihedral group)
$Sz(2)$	$\langle x, y : x^5 = y^4 = 1, y^{-1}xy = x^2\rangle$ (Suzuki group of order 20)
$PSL(2, 2^k)$	projective special linear group
$GL(2, q)$	general linear group

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