

Abstract

A major concern for the regulators and owners of financial institutions is the market risk of a portfolio consisting of risky assets, e.g. a stock market index or a mutual fund, and the adequacy of capital to meet such risk (see [27]). Market risk is the risk of losses in positions arising from the movements in market prices. A risk measure is used to determine the amount of capital to be kept in reserve. Some of the well known measures of market risk are: Value-at-Risk (VaR), Median Shortfall (MS) and Expected Shortfall (ES). VaR is an extreme quantile of the marginal loss distribution. Its use was recommended by the Basel Committee on Banking Supervision in 1996. MS is the median loss when the loss in the investment exceeds the VaR level. ES is the mean of the conditional loss distribution, given the event that the loss exceeds the VaR. In this thesis we study the problems of estimation of these risk measures based on asset or portfolio return data, and their applications in comparing the market risk of a wide variety of Indian mutual funds. In the Chapters 1-3, we study the background, motivation, properties and accuracy of a wide class of estimators of market risks. In the Chapters 4 and 5, appropriate estimators of VaR and ES are used to assess the market risk of different types of Indian mutual funds.

In Chapter 1, we introduce the concepts of the market risk measures such as VaR, MS and ES and their properties in detail. Cont in 2001 observed certain stylized properties exhibited by the asset return data, such as heavy tails, nonlinear dependence, skewness, volatility clustering etc (see [26]). We discuss these properties in Chapter 1. No one particular parametric model is known to capture all or most of these stylized properties of asset return data. Hence, we do not specify any particular model. Instead we assume that the asset return time series is a stationary α -mixing process. The estimators obtained under such general assumptions are nonparametric in nature, i.e. they are not dependent on exact specification of a model for the stochastic process.

The VaR and MS of an asset turn out to be extreme quantiles of the marginal distribution of the asset return time series. We assume that the asset return time series is a stationary

α -mixing process. A wide variety of common econometric models satisfy these assumptions. In Chapter 2, we study the accuracy of a wide class of estimators of extreme quantiles of the marginal distribution of a stationary α -mixing process. The estimators obtained under such assumptions are nonparametric, without requiring exact specification of the data generating process. There are a number of such simple, easy to implement nonparametric quantile estimators. The sample quantile and the quantile estimators discussed in Hyndman and Fan [58], defined as weighted averages of two consecutive order statistics, are examples of such estimators. Other nonparametric estimators have also been suggested based on the idea of L -statistics, kernel smoothing, extreme value theory (EVT) and transformation. We also propose a new quantile estimator $S-G_p$, based on Swanepoel and Grann's distribution function estimator (see [35]).

Sheather and Marron [95] observed that one can expect only modest improvement (upto 15 percent) over the sample quantile, even with the best possible kernel based L -estimator. In Chapter 2, our aim is to find out whether a significantly improved quantile estimate can be obtained by using any one of the above mentioned quantile estimators. We study the effect of both sample size (n) and the location of the quantile (choice of p) in the tail region on the precision of the nonparametric estimators. To compare the finite sample accuracy of the quantile estimators we compare their mean squared error (MSE) and the ratio bias/standard deviation. The exact bias, standard deviation and MSE of most of these estimators are difficult to compute, so it is approximated in a simulation study by Monte Carlo method. These comparisons provide some interesting insights into the finite sample performance of the competing estimators. Our proposed quantile estimator exhibits encouraging finite sample performance while estimating extreme quantile in the right tail region. We observe that for sample size less than 500 and p close to zero, the proposed estimator $S-G_p$ performs very well under all the different time series models considered in our simulation study (see [35]). Therefore this estimator appears to be useful for estimation of VaR and MS based on short term (less than one financial year) asset return data.

In Chapter 3, we address the problem of $100(1 - p)$ percent ES estimation based on asset return data, where p is positive and close to zero. In this chapter we review a number of nonparametric ES estimators and compare their finite sample performance using Monte Carlo simulations. The asymptotic properties of $100(1 - p)$ percent ES estimators are obtained under the assumption that p is fixed as $n \rightarrow \infty$. Not much seems to be known about the behaviour of the ES estimators under the condition that $p \rightarrow 0$ as $n \rightarrow \infty$. This condition, viz. $p \rightarrow 0$ as $n \rightarrow \infty$, implies that even for large sample size only a small proportion of the sample values is likely to be above the VaR level. Hence, under this condition the estimation of ES seems to be a challenging problem even for large sample size, without any extra model assumptions. Our simulations and real data analysis provide insight into the effect of varying p with n on the performance of nonparametric ES estimators. Our observations suggest that Brazauskas et al.'s [17], Yamai and Yoshioka's [107] and Filtered Historical [71] estimators are preferable choices for estimation of the ES for large sample size, $n > 1000$ and small $p = 0.001$, such that $np > 1$ (see [36]). However for $np < 1$, the gain in accuracy using these estimators *compared to the empirical estimator* varies widely with the process of generating the data. If the data is generated by a $GARCH(1, 1)$ model, the filtered historical estimator seems to perform well (see [36]). Though Yamai and Yoshioka's estimator and Filtered Historical estimator are preferable choices for estimation of ES (see [36]). But we observe that Yamai and Yoshioka's estimator depend on a positive constant β and in filtered historical estimator we need to fit a suitable time series model, such as an ARMA or a GARCH model, to the asset return data (see [36]). Hence we prefer estimator of Brazauskas et al. [17] for estimating ES for large sample and small p , which do not require the specification of any extra parameter or fitting a time series model. We also observe that the kernel based estimator performs poorly compared to the empirical ES estimator, and that there is no reason to use kernel smoothing for ES estimation (see [36]). We even observed that kernel smoothing does not yield an asymptotically efficient estimator (see [36]). So kernel smoothing is not recommended for ES estimation.

In Chapter 4, we assess and compare the market risk and risk adjusted returns of twenty one Indian index funds. An index fund is a mutual fund based on the concept of ‘indexing’ or ‘passive investment’, where the aim is to create a portfolio by replicating the composition of some benchmark index (see [13]). In India, two important stock market indices are the S & P BSE SENSEX in the Bombay Stock Exchange (BSE) and the CNX NIFTY in the National Stock Exchange (NSE). In India, there has been significant growth in the number of such funds since 2002. Since an index fund aims to replicate a market index, it is natural to expect the fund to have similar market risk and returns as the index that it imitates. In Chapter 4, we estimate two market risk measures, viz. the VaR and ES, and two measures of risk adjusted returns, viz. the Sharpe and the Treynor ratio, for twenty one Indian index funds based on daily and monthly return data. For each fund, the data on the daily and monthly net asset value (or closing price for an exchange traded fund) from 1 April 2007 to 31 March 2015, during which the Indian equity market experienced extreme volatility due to global recession and subsequent recovery. The comparison of the Sharpe ratio, the Treynor ratio provide insight into the performance of the index funds in terms of reaping risk adjusted returns. The estimates of the VaR and the ES of the index funds are compared with the benchmark indices. The comparisons enable us to identify the index funds exhibiting similar market risk and risk adjusted return as that of the SENSEX or NIFTY index, which the funds mimic (see [13]). We observe that among all the twenty one index funds only four such index funds satisfy this criteria. Bryant and Liu [19] observed that different fund management structures, such as unitary (single-fund) management and multiple-fund management, have significant impact on the risk exposure of the respective funds. The authors conclude that if a single-fund manager operates multiple funds, at least one of these funds can significantly deviate from its stated objective. However, most of the Indian index funds seems to be unaffected by multiple-fund management (see [13]).

In Chapter 5, we estimate and compare the market risk and risk adjusted returns of 20 balanced, 36 small & mid cap and 45 large cap funds in India over a period of 8 financial

years. A balanced fund is a mutual fund that invests in equity or stock, bond and, sometimes also in a money market instruments in a single portfolio. The idea is to reduce market risk through diversification in risky and fixed return instruments. In contrast, the small & mid cap funds are mutual funds which invest purely in equity and the investments in these funds are spread across the shares of small or mid cap companies or a mix of both. Small & mid cap refer to stocks with a small or medium market capitalization. The large cap funds are those funds which invest a larger proportion of their corpus in companies with large market capitalization. For each fund, we have collected the data on the daily and monthly net asset value and the closing price of the underlying index from 1 April 2007 to 31 March 2015. We use the proposed $S-G_p$ estimator to estimate the 99% VaR of the monthly returns of balanced, small & mid cap and large cap funds and the empirical estimator to estimate the 99% VaR of the daily returns of balanced, small & mid cap and large cap funds. The estimator of Brazauskas et al. is used to estimate the 99% ES of the daily returns of balanced, small & mid cap and large cap funds and the empirical estimator is used to estimate the 99% ES of the monthly returns of balanced, small & mid cap and large cap funds. We compare the market risk and risk adjusted returns of balanced, small & mid cap and large cap funds during the above mentioned period. We find that some of the balanced funds have exhibited much higher market risk, than a number of small & mid cap and large cap funds. This is an unusual finding, as balanced funds are expected to be less risky than pure equity based funds. No mutual fund uniformly out performs its peers in terms of market risk or risk adjusted returns. We are able to identify some small & mid cap funds that have exhibited much lower market risk than the balanced funds, and also generated healthy risk adjusted monthly returns during the period under study. We also observe that only three large cap funds exhibit much lower market risk and generate healthy risk adjusted monthly returns compared to the balanced and small & mid cap funds during the period under study. In Appendix A, we reported the returns of 122 mutual funds and two benchmark indices for monthly net asset values and the closing prices from 1 April 2007 to 31 March 2015.