

Chapter 1

Introduction

1.1 Motivation

A major concern for the regulators and the owners of financial institutions is the market risk of a portfolio of assets and the adequacy of capital to meet such risk (see [27]). Market risk is the risk of losses (negative returns) in positions arising from the movements in market prices of assets in a portfolio. Orange County is a county in the U. S. state of California. In December 1994, Orange County stunned the markets by announcing that its investment pool had suffered a loss of \$1.7 billion of Orange County's \$7.4 billion investment portfolio (see [64], [61]). This was the largest loss ever recorded by a local government investment pool, and led to the bankruptcy of the county shortly thereafter. The bankruptcy was due to Robert L. Citron's investment strategies, which seemed to be an effort to earn high incomes for the county, without raising taxes, through risky, leveraged positions in bonds (see [64], [61]). The investment strategy worked excellently until 1994. However a series of interest rate hikes by the Federal Reserve Bank caused severe losses to the pool. In 1994, Procter & Gamble reported that it had lost \$157 million before taxes (\$102 million after taxes) as a result of derivatives transactions. Procter & Gamble filed suit against Bankers Trust in October 1994 after suffering losses from a Bankers Trust interest-rate swap with the Bank, amended its complaint to include a Deutschmark swap with Bankers Trust (See [24]). NatWest lost an estimated 50 million pound in options trading in 1996 (See [76]). Sumitomo Corporation reported in 1996 that it had lost \$1.8 billion over a ten year period as a result of unauthorised copper trading by its senior trader, Yasuo Hamanaka (See [84]). Baring Bank collapsed in 1995 after suffering losses of \$1.3 billion resulting from poor speculative investments, primarily in futures contracts, conducted by an employee named Nick Leeson working at the Singapore office of the company. In 1994, John Meriwether, the famed Salomon Brothers bond trader, founded a hedge fund called Long-Term Capital Management (LTCM) (see [70]). Meriwether assembled an all-star team of traders and academics in an attempt to

create a fund that would profit from the combination of the academics' quantitative models and the traders' market judgement and execution capabilities. Other members were Nobel-prize winning economists Myron Scholes and Robert Merton, as well as David Mullins, a former vice-chairman of the Federal Reserve Board who had quit his job to become a partner at LTCM. Sophisticated investors, including many large investment banks, flocked to the fund, investing \$1.3 billion at inception. But four years later, at the end of September 1998, the fund had lost substantial amounts of the investors' equity capital and was teetering on the brink of default. The main cause of LTCM's debacle was "flight to liquidity" across the global fixed income markets and subsequent poor risk management by the fund managers. Well publicized losses incurred by several institutions such as Orange County, Procter and Gamble and NatWest, through inappropriate derivatives pricing and management, as well as fraudulent cases such as Barings Bank and Sumitomo, have brought risk management and regulation of financial institutions to the forefront of policy making and public discussions (see [27]). In a financial market, a risk measure is used to determine the amount of capital to be kept in reserve. The purpose of this reserve is to make the risks taken by financial institutions, such as banks and insurance companies, acceptable to the regulator. Some of the well known risk measures of market risk are: Value-at-Risk(VaR), Median Shortfall(MS) and Expected Shortfall(ES).

1.2 Risk measures

A risk measure is a mapping that assigns real numbers to the possible outcomes of a random financial quantity, such as an insurance claim or loss of a portfolio (see [17]). Loss due to price fluctuations or insurance claim size are usually represented by random variables. Let ψ denote the set of real valued random variables on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

Definition 1.2.1. (Delbaen [30]) *A risk measure ρ is a mapping from ψ to $\mathbb{R}$ satisfying certain properties, viz.*

1. $X \geq 0 \Rightarrow \rho(X) \leq 0$.
2. $X \geq Y \Rightarrow \rho(X) \leq \rho(Y)$, $X, Y \in \psi$.
3. $\rho(\lambda X) = \lambda \rho(X)$, $\forall \lambda \geq 0$, $X \in \psi$.
4. $\rho(X + k) = \rho(X) - k$, $\forall k \in \mathbb{R}$, $X \in \psi$.

The term "coherent" risk measure is reserved for risk measures that satisfies one more additional property, viz. subadditivity. Artzner et al. introduced the concept of coherent risk measure (see [4], [5]).

Definition 1.2.2. (Delbaen [30]) A risk measure ρ on ψ is said to be coherent if in addition to the properties 1 – 4, ρ also satisfies the following “subadditivity” property, viz.

$$\rho(X + Y) \leq \rho(X) + \rho(Y), \forall X, Y \in \psi.$$

1.3 Some well known risk measures

VaR, ES and MS are three well known measures of market risk, and they are defined as follows.

- **Value-at-risk (VaR):** VaR is a popular measure of market risk associated with an asset or portfolio of assets (see [57], [65]). It is defined as an extreme quantile of the marginal loss distribution. It is a cut-off value that separates future loss events into risky and non-risky scenarios (see [98]). VaR’s use was recommended by the Basel Committee on Banking Supervision in 1996 as a benchmark risk measure and has been widely used by financial institutions for asset management and minimization of risk.

Let $\{Y_t\}_{t=0}^n$ be the price or market value of a portfolio over n consecutive periods of a time unit (say daily closing values of an index or stock). In the sequel we assume that $X_t = -\log(Y_t/Y_{t-1})$, $t = 1, \dots, n$, are the negative proxy returns or returns over these n consecutive time units (say days). We assume that $\{X_t\}_{t=1,2,\dots}$ is a stationary process with a continuous marginal distribution function F . For $0 < p < 1$, the $(1 - p)$ th quantile of the distribution with distribution function F is defined as

$$Q_p = \inf\{x : F(x) \geq 1 - p\},$$

the $100(1 - p)$ percent VaR, denoted by VaR_p , is the negative $(1 - p)$ th quantile of the marginal distribution of X_t (see [33]), i.e.

$$VaR_p = -Q_p. \tag{1.1}$$

Therefore

$$P(\log(Y_t/Y_{t-1}) < VaR_p) = P(X_t > Q_p) = p,$$

i.e. the chance of getting return less than VaR_p is equal to p . Hence estimation of VaR_p based on $X_1, \dots, X_n$ essentially reduce to the problem of estimation of the quantile Q_p .

VaR_p satisfies the properties 1 – 4 in Definition (1.2.1) (see [5]). But it fails to satisfy the “subadditivity” property. Hence, VaR is not a coherent risk measure.

- **Expected shortfall (ES):** Artzner et al. ([4], [5]) have shown that VaR does not provide any information about the size of the potential loss when it exceeds the VaR level. More importantly, VaR is not able to distinguish portfolios which bear different levels of risk (see [1], [90]). To address these issues, another risk measure called the expected shortfall was introduced by Artzner et al. (see [5]). *ES is defined as the mean of the conditional return distribution, given the event that the return is less than the VaR.* Under the assumption that $E|X_1| < \infty$, the $100(1 - p)$ expected shortfall, denoted by ES_p , is given by

$$\begin{aligned}
ES_p &= E(\log(Y_t/Y_{t-1}) | \log(Y_t/Y_{t-1}) < VaR_p)^1 \\
&= -E\{X_t | X_t > -VaR_p\} = -E\{X_t | X_t > Q_p\} \\
&= -\frac{1}{p} \int_{x > Q_p} x dF(x) = -\frac{1}{p} \int_{1-p}^1 Q_u du.
\end{aligned} \tag{1.2}$$

It is closely linked to VaR, and is regarded as a good supplement to the VaR (See [1], [2]). ES is a coherent risk measure (see [5]). Yamai and Yoshida showed that ES is easily decomposed and optimized, while VaR is not (see [107]). The decomposition of risk is a useful tool for managing portfolio risk (see [107]). For example, risk decomposition enables risk managers to select assets that provide the best risk-return trade-off, or to allocate economic capital to individual risk factors (see [107]). The concept of VaR decomposition was proposed by Garman in 1997 (see [46]). Yamai and Yoshida described the method of decomposing VaR and ES which was developed by Hallerbach in 1999 and Tasche in 2000 (see [51], [101]).

- **Median shortfall (MS):** So and Wong introduced this risk measure and named it Median Shortfall(MS) (see [98]). By definition, MS is the median of the conditional return distribution, given that the return is less than the VaR level (see [98]). Let Θ_p denote the distribution function of this conditional return distribution. It is defined as follows

$$\begin{aligned}
\Theta_p(x) &= P\{\log(Y_t/Y_{t-1}) \leq x | \log(Y_t/Y_{t-1}) < VaR_p\}^2 \\
&= P\{X_t \geq -x | X_t > Q_p\} \\
&= \frac{1}{p} P\{X_t \geq -x, X_t > Q_p\}.
\end{aligned}$$

¹ X is an integrable random variable on $(\Omega, \mathcal{F}, P)$. $\mathcal{F}$ is a σ -field and $\mathcal{G} \subset \mathcal{F}$ and $\mathcal{G}$ is a σ -field then $E(X|\mathcal{G})$ is a random variable on $(\Omega, \mathcal{G}, P)$ such that it is integrable and satisfies the equation $\int_G E[X|\mathcal{G}]dP = \int_G XdP$, $G \in \mathcal{G}$.

The median of this distribution is called the Median Shortfall, denoted by MS_p (see [98]). Under the assumptions that F is continuous and strictly increasing on $\mathbb{R}$, it is easy to verify that for $y \geq -Q_{0.5p}$, $\Theta_p(y) \geq \frac{1}{2}$ and for $y < -Q_{0.5p}$, $\Theta_p(y) < \frac{1}{2}$. Hence

$$MS_p = -\inf\{x : \Theta_p(x) \geq 0.5\} = -Q_{0.5p}. \quad (1.3)$$

The marginal loss distribution F and the quantile function Q are unknown. Therefore Var_p , ES_p and MS_p are unknown in practice. From (1.1) and (1.3), we see that estimation of Var_p and MS_p based on $X_1, \dots, X_n$ essentially reduce to the problem of estimation of the quantiles of the marginal loss distribution.

1.3.1 Literature review

With an appropriate model for the unknown F , the estimation of the above mentioned risk measures become straight forward (viz. with a model for F , one can approximate the above risk measures by Monte Carlo methods). Over the years, different authors have suggested different distributions to model F . The normal distribution is not an appropriate model for skewed or heavy tailed data. Several skewed extensions of the normal distribution have been proposed in the literature. For instance, the skew-normal distribution due to Azzalini (see [8]). Broda and Paoletta, Wilhelmsson, Gurny and Zhu and Galbraith discussed several parametric estimators of ES (see [12], [18], [105], [50], [109]). The skew-normal distribution was used to estimate the ES by Bernardi (see [12]). The skew t -distribution was used to estimate the ES by Azzalini and Capitanio and Jones and Faddy (see [9], [63]). Asset return data are known to exhibit heavy tails (see [26]). To model this feature, Mandelbrot assumed the marginal return distribution to be stable distributions (see Mandelbrot [74]). Other popular choices for modeling the marginal loss distribution include the Student's t , Weibull, Pareto distributions, mixtures of lognormal and Pareto distributions and hyperbolic distributions. See Blattberg and Gonnede [15], Eberlein et al. [38] and Prause [83] for a discussion on these distributions. Normal Inverse Gaussian distribution and exponentially truncated stable distributions were suggested by Barndorff-Nielsen [11] and Cont et al. [25], respectively, as models for the marginal loss distribution. Simonato in 2011 suggested an ES estimator based on the Johnson family of distributions developed by Johnson (see [62], [96]).

However a drawback of the model based approach is that the resulting quantile estimates, and hence the estimated risk measures, are sensitive to model mis-specification. A mis-specification of the model can induce substantial errors (see [21]). Modarres et al. (see [80])

²Given two events A and B from the σ -field $\mathcal{F}$ of a probability space $(\Omega, \mathcal{F}, P)$ with $P(B) > 0$, the conditional probability of A given B is defined as $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

considered the accuracy of estimators of upper quantiles of a right skewed distribution under model uncertainty. The authors compared the bias and MSE of the maximum likelihood, the sample quantile and tail-exponential method based quantile estimates. The authors observed that fitting log-normal distribution to the asset return data, when the return distribution is actually log-logistic or log double exponential, is fairly *robust* for estimating quantiles of sample sizes $100 \leq n < 1000$. The authors use the criterion of bias and root mean squared error (RMSE) to assess the accuracy of the quantile estimators. However for small samples $n \leq 30$, the model based extreme quantile estimators are not reliable in the presence of model misspecification (see [80]).

In actuarial science and in financial risk management, practitioners are interested in measuring downside risks (see [21]). Therefore their interest is focused on modeling the tails of the asset return distributions (see [21]). Gençay and Selçuk, Matthys and Beirlant, McNeil, McNeil et al. and Embrechts et al. use the Extreme Value Theory (EVT) for modeling tails of loss distributions and for estimating extreme quantiles (see [45], [75], [77], [78], [39]). From Pickands-Balkema-de Haan theorem (see [10]) it is known that tails of loss distribution should be either Pareto type or exponential type. Therefore the EVT based approach also depends on correct specification of the tails of the loss distribution.

In order to have a distribution-free approach, several authors have considered nonparametric methods for estimating the extreme quantiles of the marginal loss distribution. Hyndman and Fan ([58]) in 1996, defined quantile estimators as weighted averages of two consecutive order statistics. Harrell-Davis in 1982 introduced a quantile estimator which is a weighted linear combination of order statistics (see [52]). Dowd in 2001 proposed a VaR estimator based on sample quantile (see [32]). In 2000, Gouriérox et al. introduced nonparametric kernel VaR estimators (see [48]). Kernel based VaR estimators were also discussed by Chen and Tang in 2005 and Charpentier and Oulidi in 2010 (see [23], [21]). Sheather and Marron in 1990 and Sfakianakis and Verginis in 2008 discussed the L -estimators (see [95], [93]). There are several nonparametric ES estimators, the empirical estimator and the ES estimators discussed by Brazauskas et al., Chen, Scaillet, Yamai and Yoshioka, Magadia, Hill, Inui and Kijima and Peracchi and Tanase (see [17], [22], [91], [107], [71], [53], [60], [85]). Swanepoel and Grann [100] introduced the idea of distribution function estimation based on a nonparametric transformation of the data. An improved distribution function estimator can lead to improved quantile estimator, and estimators of the above mentioned risk measures. This issue is addressed in the Chapters 2 and 3.

1.4 Asset returns

1.4.1 Empirical properties of asset returns

Cont studied statistical properties of prices of stocks, commodities and market indices, using data from various markets and instruments for more than half a century (see [26]). Properties, common across a wide range of instruments, markets and time periods are called stylized empirical facts. Stylized facts are obtained by taking a shared factor among the properties observed in investigation of different markets and instruments. Stylized statistical properties of asset return data observed by Cont [26] are as follows.

- **Absence of autocorrelations:** (linear) autocorrelations of asset returns are often insignificant, except for very small intraday time scales ($\simeq 20$ minutes) for which microstructure effects come into play.
- **Heavy tails:** the (unconditional) distribution of returns seem to display a power-law or Pareto-like tail, with a tail index which is finite, higher than two and less than five. In particular this excludes stable laws with infinite variance and the normal distribution. However the precise form of the tails is difficult to determine.
- **Gain/loss asymmetry:** one observes large drawdowns in stock prices and stock index values but not equally large upward movements.³
- **Aggregational Gaussianity:** as one increases the time scale Δt over which returns are calculated, their distribution looks more and more like a normal distribution. In particular, the shape of the distribution is not the same at different time scales.
- **Intermittency:** returns display, at any time scale, a high degree of variability. This is quantified by the presence of irregular bursts in time series of a wide variety of volatility estimators.
- **Volatility clustering:** different measures of volatility display a positive autocorrelation over several days, which quantifies the fact that high-volatility events tend to cluster in time.
- **Conditional heavy tails:** even after correcting returns for volatility clustering (e.g. via GARCH-type models), the residual time series still exhibit heavy tails. However, the tails are less heavy than in the unconditional distribution of returns.

³This property is not true for exchange rates where there is a higher symmetry in up/down moves.

- **Slow decay of autocorrelation in absolute returns:** the autocorrelation function of absolute returns decays slowly as a function of the time lag, roughly as a power law with an exponent $\beta \in [0.2, 0.4]$. This is sometimes interpreted as a sign of long-range dependence.
- **Leverage effect:** it refers to the observed tendency of an asset's volatility to be negatively correlated with the asset's return.
- **Volume/volatility correlation:** trading volume is correlated with all measures of volatility.

No parametric model is known to capture all or most of these stylized properties of asset return data. Hence, we do not specify any particular model. Instead we assume that the asset return time series is a stationary α -mixing process.

A stationary process $\{X_t\}$ is a stochastic process where the finite dimensional distributions of the vectors $(X_{t_1}, X_{t_2}, \dots, X_{t_k})$ and $(X_{t_1+h}, X_{t_2+h}, \dots, X_{t_k+h})$ are identical for any choice of $t_1, \dots, t_k$ and h . Stationarity is necessary to ensure that the marginal distribution of X_t does not depend on t . But it is not sufficient to ensure that the empirical averages indeed converges to the desired expectation. One needs an ergodic property which ensures that the averages converge to the expectation. In the sequel we assume that the time series $\{X_t\}$ is a stationary α -mixing process. Let, F_k^l denote the σ -algebra of events generated by $\{X_t, k \leq t \leq l\}$ for $l > k$. The α -mixing coefficient introduced by Rosenblatt[88] is defined as

$$\alpha(k) = \sup_i \sup_{A \in F_1^i, B \in F_{i+k}^\infty} |P(AB) - P(A)P(B)|.$$

The series $\{X_t\}_{t \in \mathbb{N}}$ is said to be α -mixing if $\lim_{k \rightarrow \infty} \alpha(k) = 0$. Mixing implies ergodicity (Fristedt and Gray [43], Theorem 6 in Section 28.5).

The estimators obtained under such general assumptions are nonparametric in nature, i.e. they are not dependent on exact specification of a model for the stochastic process. While these nonparametric estimators are in general robust against model risk (i.e. misspecification of the underlying model), but a major concern about them is the sample size required to estimate the parameters accurately (see [23]). Therefore studying the accuracy of the various nonparametric estimators is an important problem in finance.

1.4.2 Data

We have collected the data on the daily and monthly NAV of twenty one index funds and the closing price of the underlying benchmark indices are collected for the duration 1st April,

2007 to 31st March, 2015. We have also collected the data on the daily and monthly NAV of 20 balanced funds, 36 small & mid cap funds and 45 large cap funds and closing price of the NSE S & P Nifty index in National Stock Exchange, India, from 1st April, 2007 to 31st March, 2015. These data are collected from AMFI and NSE websites (see Historical Index Data, 2015 and NAV History Report, 2015). During this period the Indian equity market witnessed lot of volatility. For instance, the S & P Nifty index crashed from above 6000 in 2007 to below 2600 in 2008, and again bounced back above 6000 level in 2010. After that the Nifty index remained below 6000 level up to 2012, but again rallied above 6000 level in 2013. In 2014 and 2015 the Nifty index held the 6000 level, moved up even above 8000 level. For each fund, the daily and the monthly returns during this period are computed. In the Chapters 4 and 5, these data are used to compare the performances of the Indian index, balanced, small & mid cap and large cap funds during 1st April, 2007 to 31st March, 2015. In Appendix A, we reported the returns of 122 mutual funds and two benchmark indices for monthly NAV and the closing prices during 1st April, 2007 to 31st March, 2015.

1.5 Monte Carlo simulation

Monte Carlo simulation has become an important tool in risk management. Monte Carlo methods are based on the analogy between probability and volume (see [47]). Monte Carlo method means sampling randomly from a universe of possible outcomes and taking the fraction random draws that fall in a given set as an estimate of the set's volume (see [47]). The law of large numbers ensure that this estimate converges to the correct value as the number of draws increases. The central limit theorem provides information about the likely magnitude of the error in the estimate after a finite number of draws (see [47]). Let us consider the problem of estimating the integral $\alpha = \int_0^1 f(x)dx$. The integral α can be re-written as

$$\alpha = \int_0^1 f(x)dx = E[f(U)],$$

where U follows uniform distribution on $[0, 1]$. Let $U_1, U_2, \dots, U_B$ be independent random variables following uniform distribution on $[0, 1]$. The Monte Carlo estimator of α is defined as

$$\hat{\alpha}_B = \frac{1}{B} \sum_{i=1}^B f(U_i).$$

If f is integrable over $[0, 1]$ then, by strong law of large numbers, $\hat{\alpha}_B \rightarrow \alpha$ with probability 1 as $B \rightarrow \infty$ (see [47]). If f is square integrable on $[0, 1]$, then by CLT $\sqrt{B}(\hat{\alpha}_B - \alpha)$ is asymptotically normal, as B is increased. i.e. $\hat{\alpha}_B$ is $\sqrt{B}$ -consistent. This idea can be extended estimate the bias or mean squared error of an estimator.

Let $T_n \equiv T_n(X_1, \dots, X_n)$ be an estimator of a parameter θ , where $\{X_i\}_{i=1,2,\dots}$ is a stationary process. The Monte Carlo estimate of the MSE of any estimator T_n is defined as

$$\frac{1}{B} \sum_{j=1}^B (T_{nj} - \theta)^2,$$

where B is the number of Monte Carlo samples each of size n drawn from a given process and T_{nj} is the estimate based on the j th Monte Carlo sample, $j = 1, \dots, B$.

In the Chapters 2 and 3, the accuracy (in terms of MSE) of the various nonparametric estimators are compared using Monte Carlo simulation. In the simulation study we consider the following ten time series models

- (i) $\{X_i\}_{i=1,2,\dots}$ is an i.i.d. process, marginal distribution GPD with $\xi = 1/3$.
- (ii) $\{X_i\}_{i=1,2,\dots}$ is an i.i.d. process, marginal distribution Student's t with 4 df.
- (iii) $\{X_i\}_{i=1,2,\dots}$ is an i.i.d. process, marginal distribution $N(0, 1)$.

To study the effect of dependence on the quantile estimators we consider the following ARMA(1,1) models in Drees [33]

$$\begin{aligned} X_i - \phi X_{i-1} &= Z_i + \theta Z_{i-1}, \\ (iv) \quad \phi &= 0.95, \quad \theta = -0.6, \\ (v) \quad \phi &= 0.95, \quad \theta = -0.9, \\ (vi) \quad \phi &= 0.3, \quad \theta = 0.9. \end{aligned}$$

In addition the following GARCH(1,1) models are also considered

$$\begin{aligned} X_t &= \sigma_t Z_t, \\ (vii) \quad \sigma_t^2 &= 0.0001 + 0.9 X_{t-1}^2, \\ (viii) \quad \sigma_t^2 &= 0.0001 + 0.4 X_{t-1}^2 + 0.5 \sigma_{t-1}^2, \\ (ix) \quad \sigma_t^2 &= 0.0751 X_{t-1}^2 + 0.9194 \sigma_{t-1}^2. \end{aligned}$$

The first two GARCH models are used in the simulation study in Drees (see [33]). The GARCH model (ix) is the GARCH model fitted to the CNX Nifty daily loss data for the duration 1st April 2009 to 31st March 2013 (sample size is 995).

The last model in our simulation study is the following model

$$(x) \quad X_t = I(D_t = 1)(E_{1t} + E_{2t})^+ - I(D_t = 0)(E_{1t} + E_{2t}),$$

where $\{(E_{1t}, E_{2t})\}$ is an i.i.d. Gaussian process, with $m_1 = 10$, $m_2 = -1$, $\rho = 0.89$ and $\sigma_i = 1, i = 1, 2$. And we take $P(D_t = 1) = 0.20$, i.e. the chance of default is assumed to be twenty percent. X_t represents the loss under a netting arrangement.

The term netting is used to describe the process of offsetting mutual positions or obligations between two parties (see [41]). To study the effect of netting on VaR, MS and ES estimation we consider a simple portfolio made of a long position in one asset and a short position in another one with the same counter party. Let E_{1t} , E_{2t} denote the gains in the long and short positions respectively. The vector (E_{1t}, E_{2t}) is assumed to be Gaussian. m_i and σ_i are mean and standard deviation of E_{it} , $i = 1, 2$ and ρ is the correlation coefficient. Since E_{1t} and E_{2t} are long and short position gains, we assume that ρ is negative. Let D_t be a Bernoulli random variable, independent of (E_{1t}, E_{2t}) , such that $D_t = 1$ represents a credit event that causes default at time t (and hence initiation of a netting agreement). In case of default, without any netting arrangement, the loss at time t equals $(E_{1t}^+ + E_{2t}^+)$. However under netting arrangement, the loss due to default at time t equals $(E_{1t} + E_{2t})^+$ (see [41], page 937). Therefore under this netting arrangement, the loss at time t equals $I(D_t = 1)(E_{1t} + E_{2t})^+ - I(D_t = 0)(E_{1t} + E_{2t})$.