SOME PROBLEMS OF FLOW AND HEAT TRANSFER IN MAGNETOHYDRODYNAMICS

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BY

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DEDICATED

TO MY

PARENTS

CERTIFICATE

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Dated: 26-9-1998

This is to certify that the thesis entitled " **SOME PROBLEMS OF FLOW AND HEAT TRANSFER IN MAGNETOHYDRODYNAMICS"** *which is being submitted by Sri Shyamanta Chakraborty, Lecturer Department of Physics, Darrang College, Tezpur, Assam for the award of the degree of Doctor of Philosophy to the Tezpur University, Tezpur, Assam, is a record bonafied research work carried out by him under my supervision and guidance.*

Sri Shyamanta Chakraborty has worked for more than three and a half years and has fulfilled all the requirements for submitting the thesis for the degree of Doctor of Philosophy under Tezpur University.

The results embodied in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

26th , September, 1998
Tezpur , Assam, India.

(Atul Kr. Borkakati)

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(Shyamanta Chakraborty)

CONTENTS

SYNOPSIS

The study of flow problems of electrically conducting fluids is currently receiving considerable interest. Such studies have been made for many years in connection with astrophysical and geophysical problems. Recently various engineering and industrial problems need studies of the flow of an electrically conducting fluid.

The phenomenon generated in the electrically conducting fluid due to the interaction of electromagnetic field with the velocity field is known as Magnetohydrodynamics (MHD) . In MHD , the flow of conducting fluid in presence of an applied magnetic field is considered. The magnetic field induces a current due to the motion of the conducting fluids which in turn modifies the applied magnetic field. The modified magnetic field with current produces an electromagnetic Lorentz force that resists the fluid motion. This interaction between mass motion and electro-magnetic field sets up magneto-hydrodynamic phenomenon .

For a fluid in motion, the energy balance is considered by the internal energy , conduction and convection of heat with the stream, the generation of heat through the friction and the Joule heat due the presence of magnetic field. In compressible fluids, there is an additional term due to the expansion or compression when volume changes . In these cases , generally the effect of radiation is assumed to be negligible

For the motion of conducting fluid the Fourier's law gives that the heat flux {(J / m^2) per unit area and time} is proportional to the temperature gradient ,

$\{ (1/A). dQ/dt = -q = -k\, dT/dn \}$, k being the thermal conductivity of the fluid. The change of total energy is the change in the sum of internal and kinetic energy. The work done per unit time is determined from the contribution of components of normal and shearing stresses.

In recent years, MHD problems under different geometries have been given special attentions because of their practical importances in different industrial and scientific activities. Therefore, our objective in this research work is to discuss MHD problems in various geometries for pure and impure (i.e., Newtonian and non-Newtonian) fluids under the action of uniform magnetic field. These results have practical applications in different engineering and industrial fields.

A few problems of MHD flow and heat transfer in various geometries have been discussed in this thesis. The primary objective of this study is to know the nature of flow and heat transfer due to the effects of different physical properties and parameters of fluid motion, and field, such as (i) effect of variable (temperature dependent) viscosity in MHD flow, (ii) effects of induced magnetic field on steady and unsteady MHD flow of Newtonian and non-Newtonian fluids, (iii) effect of an exponentially decay source in MHD flow, (iv)heat and mass transfer of an unsteady MHD flow, (v) mass and thermal diffusion in MHD flow, etc.

The problems we have studied, have various engineering and industrial applications such as in oil industry, paper industry, rubber industry, nuclear reactor, power transformation etc. The influence of dust particles on visco-elastic fluid flow has its importance in many applications such as extrusion of plastics, in the manufacture of Rayon and Nylon, purification of Crude oil, pulp, paper industry, textile industry and in different geo-physical situations etc.

The thesis consist of seven chapters. Chapter I deals with the introduction of the thesis. The outline of the subject Magnetohydrodynamics , its development and applications , fundamental equations of flow and heat transfer in Magnetohydrodynamics has been discussed in this chapter. The mass transfer processes which are sometimes accompanied by other processes like heat transfer , rotation of fluids , electromagnetic forces etc. have also been discussed briefly in this chapter. During the past two decades, a number of significant experiments have been carried out revealing non-Newtonian characteristics of liquids where a number of new phenomena have been observed in a large number of liquids, of great technological and industrial importances. A brief description of these liquids is also given in this chapter. The shooting method for solution of simultaneous non-linear higher order differential equations which is gaining popularity , has also been stated in this chapter. Lastly, a brief review of earlier workers and scope of this work have also been explained in this chapter.

In chapter II , the flow of a viscous incompressible electrically conducting fluid on a moving flat plate in presence of uniform transverse magnetic field has been discussed. The flat plate which is continuously moving in its own plane with a constant speed is considered to be isothermally heated . Assuming the fluid viscosity as an inverse linear function of temperature velocity and temperature distribution under the field are plotted for different layers of the medium for various constant values of viscosity parameter . Numerical solutions are obtained by using Runga-Kutta and Shooting method . The coefficient of friction and the rate of heat transfer are calculated . The theory of Shooting method , for the solution non-linear higher order simultaneous differential equations has also been stated in this chapter.

In chapter III, we have discussed an unsteady flow of a viscous incompressible electrically conducting fluid between two infinite parallel stationary disks composed of non-conducting material in presence of uniform magnetic field applied transversely to the direction of flow. The flow is due to the source whose strength decays exponentially. The aim of this study is to investigate the effect of Hartmann number and the decay factor on the laminar radial flow due to the source for different values of reduced Reynolds number. Solutions are obtained for the radial velocity and pressure distribution . The skin frictions at the boundary layer flow are calculated and their variations with Hartmann number and (decay factor x time) are shown graphically.

The problem of steady laminar flow of a viscous incompressible fluid between two parallel plates in presence of a uniform magnetic field applied in the direction making angle θ with the vertical axis has been discussed in chapter IV. Assuming that the two plates are maintained at a constant temperature gradient, the expressions for the velocity components , induced magnetic field , temperature distribution , skin friction and the rate of heat transfer has been obtained and calculated numerically. Their natures are shown graphically for different values of Hartmann number and the inclination of the field θ . Perturbation method is used to solve the problem.

An unsteady free convection and mass transfer flow of an incompressible electrically conducting viscous fluid past a steadily moving infinite vertical porous plate under the action of uniform magnetic field has been discussed in chapter V. The magnetic field which is applied transversely to the fluid motion, induces a magnetic field along the line of motion that varies perpendicularly to it. Similarity equations are derived for fluxes of momentum , magnetic field, energy and mass considering thermal diffusion effect (Soret effect), by introducing time dependent length scale. The equation for mass concentration and energy, are solved analytically but due to the complexities, the momentum equation and magnetic field equation are solved numerically using Runga - Kutta method . To find the initial missing values of the boundary conditions of the problem shooting method is used. The effects of various magnetic field parameters, mass diffusion and thermal diffusion parameter on flow are discussed graphically.

In chapter VI , we have discussed a viscous incompressible free convective flow of an electrically conducting fluid between two heated vertical parallel plates in porous medium in presence of a uniform magnetic field, applied transversely to the flow. Maintaining the plates at two different temperatures and considering the dissipation of energy due to flow and porous medium, the numerical values of skin friction and the rate of heat transfer are calculated and figures are plotted for fluid velocity and the temperature distribution for different values of physical parameters. To solve the equations Runga-Kutta and Shootoing method are used

The unsteady laminar flow of an incompressible electrically conducting second order Rivlin-Ericksen fluid in porous medium down a parallel plate channel inclined at an angle θ to the horizontal surface in presence of uniform magnetic field has been discussed in chapter VII. The magnetic field applies transversely to the flow direction, in turn , induces a magnetic field along the line of flow. Assuming the plates are maintained at constant temperature the exact solutions for fluid velocity , particle velocity , induced field distribution and the temperature distribution within the channel have been obtained and are plotted graphically for different magnetic field parameters. Expressions for flow flux of fluid and particle , viscous drag and the rate of heat transfer have been derived.

CHAPTER I

INTRODUCTION

1.1 MAGNETOHYDRODYNAMICS (MHD) :

The phenomenon generated in the electrically conducting fluid due to the interaction of electromagnetic field with the velocity field is known as Magneto-hydrodynamics (MHD). The term "Hydro" implies that the subject pertains to applications in water, or at best , in incompressible fluid (i.e. the medium in which the compressibility effect is negligible). Therefore, MHD is the electromagnetic phenomenon in the flow of an incompressible electrically conducting fluid.

The term " MHD " originally came from the field of fusion. Since most liquids and gases are poor conductors of electricity, their motions can be normally be treated by the principles of fluid dynamics. However, it is possible to make some gases very highly conducting by ionizing them, called Plasma. When studies were made on different plasma phenomena , the equations of motion were found very similar to those which were used in studying hydrodynamic phenomena for many years. The study of plasma or hydrodynamics were carried out under the action of magnetic field using hydrodynamic equations, and is now used to describe the whole field of plasma studies i.e. the study of microscopic interaction of electrically conducting liquids and gases with magnetic field. The subject can further be divided into two branches ; Magneto-hydrodynamics and Magneto-gasdynamics.

Faraday and his contemporaries observed that a solid or fluid material moving in a magnetic field experiences an emf when the material is electrically conducting , and if the current path is available an electric current develops. Alternately an electrical current may be induced when magnetic field changes with time. There may be two basic effects.

(I) An induced magnetic field is associated with the currents perturbing the original magnetic field.

(II) The current interacts with the magnetic field to produce an electromagnetic body force known as pondermotive force or Lorentz force, perturbing the original fluid motion. Moreover the induced current which has its own magnetic field, also added on to the electromagnetic body force perturbing the fluid motion largely.

Thus the interlocking between the mass motion (i.e., the motion of conductor) and the electromagnetic fields, is the Magneto-hydrodynamic phenomenon. It is the science of motion of electrically conduction fluids , and essentially the mutual interaction between the fluid velocity field and the electromagnetic field. Therefore, the flow behaviour in MHD may be examined by combining the electromagnetic field equations with those of fluid dynamics. The mutual interactions occur in MHD between the mass motion with the electromagnetic fields are more pronounced manner in liquids and gases than in solids. This is due to the freedom of movement of the molecules in the former types of the conductors enjoy.

The idea of MHD in fact was pre-Maxwellian. The electrical pioneers of the 1830's perceived that MHD might explain certain natural phenomena . Faraday thought that motions of the sea might account for the observed perturbation of the earth's magnetic field (by effect (I)) , an idea that has recently gained new support among the geophysicists. Ritchie speculated whether ocean movements might be propelled by effect (II), the electric current being of unknown origin. Within the nineteenth century various minor artifacts depending on MHD principles were invented. For example Leduc's magnetometer in 1887, Ritchie's electromagnetic pump in 1832. The applications of MHD to natural events received a belated stimulus when astrophysicist came to realize how prevalent throughout the universe are conducting ionized gases (plasmas) and significantly strong magnetic fields. Bigelow in 1889 supposed that there were magnetic fields on the sun which later on confirmed by Hale and Babcocks. They were of the view that MHD processes must dominate most areas of astrophysics. Larmor in 1918 suggested that the magnetic fields of the sun and the other heavenly bodies might be due to dynamo-action where the conducting materials of the star acted as the armature and stator of a self exciting dynamo. Williams and Hartmann performed various simple experiments on the flow of conducting liquids in the laboratory. Hartmann in 1937, designed a magnetic pump to put mercury in motion for his experiments on the behaviour of conducting fluids in the presence of a magnetic field.

At last the joint consequence of the effects (I) and (II) was clearly realized when engineer -astrophysicist Alfvén in 1942 published the classic paper which marks the emergence of full fledged MHD. According to this idea if a highly conducting fluid moves in a magnetic field, the induced current will tend, which means interaction between relative motion of the fluid with the electromagnetic field so that the field is convected by the fluid motion. Alfven describe it as freezing of the field with the fluid motion. The field is deformed to follow the fluid motion (effect (I)) while the relative motion is opposed by the electromagnetic forces (effect (II)), which Alfven thought of in terms of the Faraday tensions in the field lines. This idea now known as Alfven waves or Magnetohydrodynamics waves and is confirmed by mathematical analysis. The magnetic lines of forces, under apparent tension and inertia of the fluid, frozen together which undergoes a transverse oscillations, and transmit waves just like elastic strings. Alfven successfully applied his ideas into cosmic and astrophysical problems. Thereafter, there has been many important applications of MHD in various fields both experimental as well as theoretical.

Applications :

The study of flow problems of electrically conducting fluids is currently receiving considerable attentions. Such studies have been made for many years in connection with astrophysical and geophysical problems such as Sun spot theory, motion of the interstellar gas , origin of earth magnetism etc. Only recently some engineering problems for instance, in controlled fusion research , reentry problem of intercontinenta ballistic missiles, Plasma jet, Power generation (magnetohydrodynamic generator) etc need the studies of the MHD flow. Because of the engineering applications, in recen years many engineers and aerodynamicists with the astrophysicists have studiec extensively this subject. The applications of the phenomena of MHD have beer concerned with the problems such as stirring of molten metals in eddy currents, stirring and levitation in metallurgical industry, the motion of liquid metal brushes in high curren electrical machinery, the design and operation of electromagnetic pumps electromagnetically pumping of liquid metals, coolant in nuclear reactors anc electromagnetically operated ramjet. The Induction flow meter, which depends on th potential difference in the fluid in the direction perpendicular to the motion and the magnetic field. All these devices utilize the electromagnetic body force (Lorentz force, μ_e (J x H) per unit volume) which arises due to the flow of electric current J in presence of magnetic field H, where μ_e represents permeability of the medium.

One important application of MHD is in the problems of controlled thermo-nuclear reactors for large scale power generation. The cooling of a thermo-nuclear reactor is generally carried out by MHD flow of liquid sodium. The thermo-nuclear fusion could be achieved in controlled manner by confining hot ionized deuterium away from all walls by MHD forces. This led intensive research on MHD in this branch. It is also important to investigate the effects of electromotive forces arising in MHD flow problems on the phenomenon of transition the steady state to turbulence. Post (1956) and Bishop(1958) studied the use of MHD principles in controlled fusion research. The study of MHD has also importance in the field of aeronautics especially missile aerodynamics, since the temperature generated in such high speed flights are sufficient to ionize the air appreciably. In such flights, Joule-heating (i.e., the heating due to the flow of electric current) plays a very important role. For example, when a high speed missile re-enters the earth's atmosphere, a very large amount of heat is generated due to the friction of the air molecules. This viscous heating may sometimes be so considerable as to ionize the air near the forward stagnation region which is electrically conducting. A magnetic field may be applied to it so as to induce electromagnetic body forces in the air which in turn will be retarded. As a result the velocity gradient decreases near the wall implying a reduction of skin friction which automatically implies reduction in heat transfer. A related application is the use of MHD acceleration to shoot plasma into fusion device or to produce high energy wind tunnels for simulating hypersonic flight. MHD effects can also arise from the passage of bodies or waves through the ionosphere in the presence of the earth's magnetic field.

The various applications of MHD in engineering and technical problems have been investigated by Karman(1950) . The feasibility of MHD power generation has been studied theoretically and experimentally by Sutton (1959), Curgen et al. (1960) and Mannaland Mather (1962). The old idea of MHD generation from ionized gas streams has been revived and developed intensively. It offers the prospect of improved power-station efficiency and also cheap, lightweight sources of power for space vehicles. Other potential applications of MHD include electromagnets with fluid conductors , various energy conversion or storage devices, magnetically-controlled lubrication by conducting fluids etc.

1.2 FUNDAMENTAL EQUATIONS OF MAGNETOHYDRODYNAMICS :

In order to derived equations for MHD flow following postulates are considered.

Hydrodynamic and Electromagnetic considerations :

(i)The fluid is treated as continuous and describable in terms of local properties, such as, pressure, velocity, temperature, density, viscosity etc.

(ii) The system under our investigation is considered strictly large compared to the microscopic structure of matter but small enough compared to the macroscopic phenomenon to permit the use of differential equations to describe them.

(iii) A relatively collusion free situations are considered.

(iv) All velocities are much smaller than C, the velocity of light (3×10^8 m / sec approx.), hence non-relativistic electromagnetic theory is considered in MHD and relativistic corrections are not necessary.

(v) A purely local view is meaningless, this means that the charges at rest or in motion, and the magnetic materials are act one upon another at a distance.

(vi) The electrical field which may be characterized by a value E is of the same order of magnitude as the induced electric field $\mu_e (v \times H)$. In other words, the non -dimensional parameter $R_E = E / \{ \mu_e (v \times H)\}$ is of the order of unity or smaller, where H is the characteristic magnetic field strength. Therefore, it may be shown that the displacement current $\varepsilon_0 (\partial E / \partial t)$ and the excess electric charge are negligible in our fundamental equations and that the energy in the electric field is much smaller than that in the magnetic field. As a result, all the electromagnetic variables may be expressed in terms of magnetic field.

Low frequency approximations :

(i) The charge distribution appears unimportant in low-frequency electromagnetism and MHD. The Ampère-Maxwell law relating magnetic field to the electric field due to moving charges is

$$\text{Curl}\,(B / \mu_e) = J + \varepsilon_0 (\partial E / \partial t) \qquad 1.01$$

where J is called current density due to the net flow of charges free or bound, and the last term is known as Maxwell's contribution. The magnitude of ratio

[Curl B / μ_e / (ε_0 $\partial E/\partial t$)] is of the order of λ^2 / d^2 where, d is the length scale and λ is the wavelength (C/ f) of electromagnetic radiation of frequency f. This means that the Maxwell term ε_0 ($\partial E/\partial t$) is negligible unless frequency is high enough. Thus under low frequency, the Ampère-Maxwell law becomes

$$\text{Curl } B = \mu_e J \qquad 1.02$$

Thus no contribution of Maxwell term to J.

(ii)The polarization current ($\partial P/\partial t$) of the same order as ε_0 ($\partial E/\partial t$) and hence no contribution to J.

(iii) The ratio of magnitude of convection current to the total current

i.e., [ε_0 (B v^2 /d) / (B / μ d)] = v^2 / C^2 , which is very small unless frequency is very high. Therefore, under low frequency the convection current is negligible and conduction current is taken as the total current. So that neglecting the convection current (qv) and the polarization current ($\partial P/\partial t$), the current density is written as

$$J = \sigma (E + v \times B) \qquad 1.03$$

(iv) The ratio of magnitude of electric and magnetic part of the body force is

$$[\{(\varepsilon_0 E^2 d) \text{ or } (\varepsilon_0 B^2 v^2 d)\} / \{B^2 / (\mu d) \}] = v^2 / C^2, \qquad 1.04$$

which is very small unless frequency is very high. This means that electric body force (q E) is negligible in MHD.

From these comparisons, it appears that the charge distribution in MHD has no importance under low frequency approximation.

Under above considerations, the fundamental equations governing the flow field and the temperature in MHD can be obtained from the corresponding equations in ordinary hydrodynamics with suitable modifications by using Maxwell's equations and Ohm's law.

(a) *MAXWELL'S ELECTROMAGNETIC EQUATIONS* :

In MHD, we are mainly concerned with conducting fluids in motion and hence it is necessary to consider the electrodynamics equations of moving media. When charges are in motion, the electric and magnetic fields will be associated with this motion which will have space and time variation. The phenomenon is called electromagnetism and we study the electromagnetic wave motion. The study will involve time dependent properties of the electric and magnetic fields. The behaviour of which will be described by a set of equations called Maxwell's equations. These equations under non-relativistic assumptions are :

$$\text{Curl } E = -(\partial B / \partial t), \qquad 1.05$$

$$\text{Curl } H = J + (\partial D / \partial t), \qquad 1.06$$

$$\text{div } B = 0 \qquad 1.07$$

$$\text{div } D = \rho_e \qquad 1.08$$

$$B = \mu_e H \qquad 1.09$$

$$D = \varepsilon E \qquad 1.10$$

where E, B, H, J, D, μ_m, ε and ρ_e are respectively the electric field intensity , the magnetic flux density, the magnetic field intensity, the current density , the electric displacement , the magnetic permeability, the die-electric constant and the electric charge density. In addition to these equations, we have current conservation equations , which is also referred to as the equation of continuity in MHD. This current conservation equation derived from (1.06) and (1.08), is

$$\text{div } J + (\partial \rho_e / \partial t) = 0 \qquad 1.11$$

In electromagnetism and MHD , the displacement current $\partial D/\partial t$ is neglected as compared to J in a good conductor. Because the fluid is in motion with a velocity very small compared to that of the velocity of light. Also for fluids which are almost neutral i.e., in case of stationary currents, the charge density ρ_e at any point within the region remains constant. Therefore ρ_e is negligible, which implies that $\partial \rho_e / \partial t$ must be omitted from (1.11).

Now the Maxwell 's equations under MHD approximation take the form

$$\text{Curl } E = -(\partial B/\partial t), \qquad 1.12$$

$$\text{Curl } H = J \qquad 1.13$$

$$\text{div } B = 0 \qquad 1.14$$

$$\text{div } J = 0 \qquad 1.15$$

$$B = \mu_e H \qquad 1.16$$

$$D = \varepsilon E \qquad 1.17$$

(b) *OHM'S LAW* :

For electromagnetic problems, an equation, namely the law of conduction, is added to the Maxwell's equation. The conduction current density J in stationary conductor is formulated mathematically as $J = \sigma E$, where E is the electric field intensity and σ is the electrical conductivity of the medium. If a charged particle moves with velocity v in a magnetic field B , it suffers a magnetic force v x B per unit of its charge. That is the induced electric field is given by v x B . This force is perpendicular to v and B . Again the Lorentz force for a unit charge moving locally in the medium with velocity v i.e., the total electric field is = E + (v x B).

Hence under a non-relativistic approximation the current density is written as

$$J = \sigma (E + v \times B) \qquad 1.18$$

This equation is known as Ohm's law.

(c) *HALL CURRENT* ;

We know that the Lorentz force on a particle (in a conductor) per unit of its charge due to its motion with the velocity v in presence of a transverse magnetic field B is E + (v x B) (see, Shercliff, 1965). Let free charges of negligible inertia be drifting through it under the action of this Lorentz force . The right conclusions emerge if it is supposed that each drifting particle also suffers a drag force due to collisions equal on the average Kv , where K is a constant for each particle. This represents the dissipate phenomenon of resistivity. Neglecting the inertia of the free charge , we have

$$\sigma (E + v \times B) = K v \qquad 1.19$$

Summing over the free charges in the element of conductor, we get

$$\rho_e'' E + (J_c \times B) = \Sigma(k v / \delta) \text{ per unit volume}$$

$$\text{i.e.,} \quad E + (J_c \times B) / \rho_e'' = \Sigma \{ k v / (\delta \rho_e'')\} \qquad 1.20$$

where, J_c is the conduction current $\Sigma(\rho v / \delta)$, due to the drift of free charges and ρ_e'' is the net free charges per unit volume. The experiments show that the right hand side is proportional to J_c. Hence we have

$$E + (J_c \times B) / \rho_e'' = J_c / \sigma \qquad 1.21$$

where σ is the electrical conductivity. The extra term $(J_c \times B) / \rho_e''$ due to B is known as Hall effect. If the free charges are electrons, of charge $-e$, and number density is n then, $$E - (J_c \times B) / \rho_e'' = J_c / \sigma \qquad 1.22$$

Hall effect is due merely to the sideways magnetic force on the drifting free charges. In liquid conductors, Hall effects are negligible being the number of free charges infinite. When the conductor is moving at a velocity u locally, the velocity of a charge is $u + v$ if v is its velocity relative to the conductor . Summing over all charges, free or bound, we have total current

$$J = \Sigma \{ e (u + v) / \delta \}$$

$$= \rho_e u + \Sigma \{ e v / \delta \} \qquad 1.23$$

in which the term $\rho_e u$ is the convection current , a non-dissipative effect. The term $\Sigma \{ e v / \delta \}$ can be split into, (i) the conduction current J_c due to the motion of free charges relative to fluid which is a dissipative effect, and (ii) the polarization current due to the motion of bound charges relative to fluid. The balance of forces on a free charge is :

$$e \{ E + (v + u) \times B \} = k v \qquad 1.24$$

This leads to the result that Ohm's law is

$$(E + u \times B) = J_c / \sigma \qquad 1.25$$

if the Hall term due to $\sum \{e\, v \times B\}$ is neglected .

With the Hall term , the Ohm's law can be written as

$$J_c = \sigma [E + (u \times B)] - [\sigma / (n e) (J_c \times B)] \qquad 1.26$$

(d) *MASS CONSERVATION EQUATION* :

This equation is developed by writing a mass balance over a stationary volume element $\nabla x\ \nabla y\ \nabla z$ through which the fluid is flowing.

i.e., (Rate of mass accumulation) = (Rate of mass in) - (Rate of mass out)

Let us consider a fluid of density ρ, moving with a velocity v. Then the hydrodynamic equation of conservation of mass is written as

$$(\partial \rho / \partial t) + \mathrm{div} (\rho v) = 0 \qquad 1.27$$

The vector quantity ρ v means the mass flux and its divergence (i.e., div (ρv)) is the net rate of mass efflux per unit volume . Equation 1.27 may be abbreviated as

$$D \rho / Dt = - \rho (\nabla . v) \qquad 1.28$$

where D / Dt is the substantial time derivative. The equation 1.27 describe the rate of change of density as seen by an observer " floating along " with the fluid. The equation remains unchanged for a conducting medium also .

For an incompressible fluid, the density of any particle is invariable with time, so that

$$(\partial \rho / \partial t) = 0$$

Therefore, for an incompressible fluid , the equation 1.27 reduces to the form,

$$\text{div } v = 0$$

(e) *MOMENTUM CONSERVATION EQUATION* :

In MHD , the fluid is electrically conducting, therefore, the magnetic field affects mass motion not because of its mere presence but only by virtue of electric current with a velocity u in presence of a magnetic field B , then the force per unit volume can be written as (sce Shercliff, 1965)

$$f = \rho_e E + J \times B \qquad 1.29$$

The ratio of electric and magnetic parts is this body force is of the order $(u / C)^2$ where u is the characteristic velocity and C is the velocity of light. Thus $\rho_e E$ can be omitted, so that the magnetic body force is (J x B). Introducing this, we write for a volume element $\nabla x\ \nabla y\ \nabla z$ the momentum balance for steady magnetohydrodynamics flow in this form :

(Rate of momentum accumulation) = (Rate of momentum in) - (Rate of momentum out) + (magnetic body force) + (sum of forces acting on the system)

In a single vector equation it takes the form :

$$\{\partial / \partial t(\rho v) \} = - \nabla . (\rho v\, v) - \nabla p - (\nabla . \tau) + (J \times B) + \rho g + X \qquad 1.30$$

where ρ is the fluid density , v is the fluid velocity, B is the magnetic field , p is the hydrostatic pressure i.e., fluid pressure. $\nu = \mu / \rho$, is the kinematic shear viscosity, μ is the coefficient of viscosity.

The first term in the equation 1.30 means the rate of increase of momentum per unit volume . Second term means the rate of momentum gain by convection per unit volume . Third term means the pressure force on element per unit volume . Fourth term is the rate of momentum gain by viscous transfer per unit volume i.e., the viscous force on element per unit volume. Fifth term is the magnetic body force as given in 1.13. The sixth term represents the gravitational force on element per unit volume, while the last term represents the sum of all other forces acting on the system. Considering the mass conservation equation, we get,

$$\rho \{D v / Dt\} = -(\nabla . p) - (\nabla . \tau) + (J \times B) + \rho g + X \qquad 1.31$$

The expressions for the various stresses in terms of velocity gradients and fluid properties for Newtonian fluids are

$\tau_{xx} = -2\mu (\partial v_x / \partial x) + (2/3)\, \mu (\nabla . v)$; $\tau_{yy} = -2\mu (\partial v_y / \partial y) + (2/3)\, \mu (\nabla . v)$

$\tau_{zz} = -2\mu (\partial v_z / \partial z) + (2/3)\, \mu (\nabla . v)$; $\tau_{xy} = \tau_{yx} = -\mu (\partial v_x / \partial y) + (\partial v_y / \partial x)$

$\tau_{yz} = \tau_{zy} = -\mu (\partial v_y / \partial z) + (\partial v_z / \partial y)$; $\tau_{zx} = \tau_{xz} = -\mu (\partial v_z / \partial x) + (\partial v_x / \partial z)$

In case of electrically conducting , incompressible and viscous fluid, the equation 1.41 may be simplified by means of the equation of continuity [$(\nabla . v) = 0$] to give

$$\rho \{D v / Dt\} = -(\nabla . p) - \nabla .(\mu \nabla . v) + (J \times B) + \rho g + X \qquad 1.32$$

The equation 1.32 except the term (J x B) , is celebrated Navier -Stokes equation, first developed by Navier in France in 1822.

For an invisid fluid , $(\nabla . \tau) = 0$, hence the equation 1.32 reduces to

$$\rho \{D v / Dt\} = -(\nabla . p) + (J \times B) + \rho g + X \qquad 1.33$$

(f) *MAGNETIC DIFFUSION EQUATION* :

Combining the equations 1.13 & 1:18 , we obtained that

$$\nabla \times H = \sigma [E + v \times B] \qquad 1.34$$

Eliminating the electric field E by Curl operation on equation 1.17 and using 1.12, we obtain for a fluid of uniform electrical conductivity σ and constant magnetic permeability μ_e

$$(\partial B / \partial t) = \nabla \times (v \times B) - \nabla (\nabla \times B) / (\sigma \mu_e) \qquad 1.35$$

which is the induction equation of the magnetic field.

Since div v = 0 , the equation 1.35 will become

$$(\partial B / \partial t) = \nabla \times (v \times B) + \nu_m (\nabla^2 B) \qquad 1.36$$

where $\nu_m = 1 / (\sigma \mu_e)$ 1.37

ν_m is called magnetic diffusivity or the magnetic viscosity. The equation 1.36 is called the magnetohydrodynamic diffusion equation.

When the magnetic Reynolds number { $R_m = (v\ d / \nu_m)$ } is very small compared with unity, then neglecting the term $\nabla \times (v \times B)$, the equation 1.36 written as

$$(\partial B / \partial t) = \nu_m (\nabla^2 B) \qquad 1.38$$

This is the equation of diffusion of a magnetic field in a stationary conductor, resulting in decay of the field.

When the magnetic Reynolds number R_m is large compared with unity, the equation 1.36 reduces approximatcly to

$$(\partial B / \partial t) = \nabla \times (v \times B) \qquad 1.39$$

(g) *EQUATION OF ENERGY*:

The charge within a material moves under the action of electromagnetic forces colliding and exchanging energy with the rest of the material. This fact means that electric work is done on or by the material. It has been found that the electromagnetic field puts energy into the material at the rate E.J per unit volume and time (see Shercliff, 1966). The current density J can have three possible forms - conduction, convection, and polarization. The contribution of convection and polarization on the work done is negligible in MHD, only that of the conduction current plays a significant role.

Ohm's law, without Hall current, is given by equation 1.25, hence

$$E \cdot J = J^2 / \sigma - J \cdot (v \times B) \qquad 1.40$$

The first term on the right hand side represents the Ohmic dissipation while the second term can be written as

$$-J \cdot (v \times B) = v \cdot (J \times B) \qquad 1.41$$

This describes the phenomenon of electromechanical energy conversion. The term { v . (J x B)] is the rate at which the magnetic force J x B does work on the conduction as a whole. The term { v. (Jx B)} pushes the fluid - either creating kinetic energy or helping it to overcome other forces or the reverse if the term is negative. The term $\{J^2/\sigma\}$ is positive and the dissipated part in the form of heat. The principle of conservation of energy states that the total time rate of change of kinetic and internal energies is equal to the sum of the work done by the external forces per unit time and the sum of other energies supplied per unit time. Therefore, the equation of energy in MHD is written as (Shercliff, 1965).

$$\rho\,(D h_t / Dt) = (\partial p / \partial t) + \nabla.(v.\tau) + \nabla.(K\,\nabla T)$$

$$+ (1/\mu_e)(\nabla \times B).\{\nu_m(\nabla \times B) - (v \times B) + W \qquad 1.42$$

where, $h_t = C_p T + v^2/2$, stagnation enthalpy ; C_p is the specific heat at constant pressure. k , is the thermal conductivity of the fluid.

τ is the stress tensor and $(\nabla.\tau)^i = (\partial \tau^{ij} / \partial x^j)$ 1.43

The equation 1.42 enjoys general validity, but in most practical cases it is possible to simplify it still further. In doing so, it is necessary to distinguish between the perfect gas and that of an incompressible fluid. The thermodynamical properties of the latter do not constitute a limiting case of the properties of the former. In fact, the variation in the internal energy of a perfect gas is $de = C_v\,dT$, whereas that of its enthalpy is $dh_t = C_p\,dT$. The corresponding variations for an incompressible fluid are $de = C_p\,dT$ and $dh_t = C_p\,dT + (1/\rho)\,dp$.

Thus for an incompressible fluid the equation of energy in MHD is

$$\rho\,C_p\,(DT/Dt) = k\nabla^2 T + \mu\,\phi + (J^2/\sigma) + W \qquad 1.44$$

Here ϕ represents the dissipation function given by

$$\phi = 2\{(\partial u/\partial x)^2 + (\partial v/\partial y)^2 + (\partial w/\partial x)^2\} + [\{(\partial v/\partial x) + (\partial u/\partial y)\}^2$$

$$+ \{(\partial w/\partial y) + (\partial v/\partial z)\}^2 + \{(\partial u/\partial z) + (\partial w/\partial x)\}^2]$$

$$- 2/3\ \{(\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z)\} \qquad 1.45$$

1.3 NON-DIMENSIONAL PARAMETERS IN MHD FLOW :

For a steady flow of incompressible, viscous, electrically conducting fluid the equations of momentum, magnetic diffusion and energy are as follows.

$$\rho\,(Du/Dt) = -\nabla p + \nabla\{(\mu)\nabla v\} + (J \times B) + (\nabla\psi) \qquad 1.46$$

$$(\partial B/\partial t) = \nabla \times (v \times B) + \nu_m\,(\nabla^2 B) \qquad 1.47$$

$$\rho\, C_p\,(DT/Dt) = k\,\nabla^2 T + \mu\,\phi + (J^2/\sigma) \qquad 1.48$$

where, ψ is the gravitational potential.

Let us introduce the non-dimensional quantities represented with an asterisk mark in the following way.

$$x_i^* = x_i/L,\ v^* = v/u_0,\ t^* = u_0\,t/L,\ T^* = T/T_0,\ \nabla^* = \nabla L,\ p^* = p/(\rho_0 u_0^2),$$

$$v^* = v/u_0,\ J^* = J/J_0,\ B^* = B/B_0 = H/H_0 = H^*,\ D/Dt = u_0/L,\ \psi^* = \psi/(gL) \qquad 1.49$$

where, i = 1 , 2 , 3 ; and the subscript o refers to a characteristic value.

Substituting 1.49 in 1.46 - 1.48, we get -

$$\{D^*/Dt^*(u^*)\} = -\nabla^*(p^*) + \{(1/Re)\,\nabla^{*2}(u)\} - \{(1/\mu_e^2)\,H^* \times (\nabla^* \times H^*)\} - (1/F_r)\,\nabla^*(\psi^*) \qquad 1.50$$

$$(\partial B^*/\partial t^*) = \nabla^* \times (v^* \times B^*) + (1/R_m)\,\nabla^{*2}(B^*) \qquad 1.51$$

$$\{D^*/Dt^*(T^*)\} = \{1/(Pr\,Re)\}\,\nabla^{*2}(T^*) + (E/R)\,\phi^* + (M^2 E/Re)\,J^{*2} \qquad 1.52$$

The dimensionless parameters which are appeared as

$Re = u_0 L/\nu$, Reynolds number ; $R_m = u_0 L/\nu_m$, Magnetic Reynolds number

$Pr = C_p\,\mu/k$, Prandtl number , $Pm = \nu/\nu_m$, Magnetic Prandtl number ,

$M = B_0 L \sqrt{(\sigma / \rho\nu)}$, Hartmann number, $F_r = u_0^2 / (g L)$, Froude number.

$M_a = u_0 / v_s$, Magnetic Mach number ; $v_s = \mu_e u_0^2 / \rho$, is the Alfven velocity.

The magnetic diffusion equation 1.36 has an analogy with the equation governing the diffusion of vorticity ω of an incompressible non-conducting viscous fluid given by

$$\partial \omega / \partial t = \nabla \times (v \times \omega) + \nu \nabla^2 \omega \qquad 1.53$$

where ν is the kinematic viscosity. The imperfection in the analogy is that ω is intimately related to v (i.e., $\omega = \nabla \times v$) in a way that B is not , but it turns out that this does not prevent the use of the analogy to suggest results concerning B. From the equation 1.36 and 1.53 , we can make the same kind of statement namely that the local rate of change of B or ω results from the net effect of (i) convection (i.e., the term, $\{ \nabla \times (v \times B) \}$) and (ii) diffusion (i.e., the term $\nu_m (\nabla^2 B)$).

(a) _Large magnetic Reynolds number_ :

In any region of length scale δ , where convection and diffusion are equally important , the two terms on the right hand side of the equation 1.36 must be comparable . Thus

$$\{ \nabla \times (v \times B) \} / \{ \nu_m (\nabla^2 B) \} \cong (u_0 \delta / \nu_m) = R_m \qquad 1.54$$

Therefore, δ must be of order ν_m / u_0 . If the whole field of interest has a length scale L such that $R_m >> 1$ then $L >> \delta$, R_m being based on L . Only within a limited region of length δ , where B changes significantly, gradients can be high enough for diffusion and only dissipation matters much ; elsewhere it can be neglected . Thus for large R_m, convection dominates and magnetic boundary layer approximations are expected to work near sources of field and elsewhere the approximations of perfect infinite conductivity would be valid , the diffusivity being zero. So $E + v \times B = 0$ and convection alone holds away. Again, if the characteristic time is t, then the equation (neglecting the diffusion term) is :

$$(\partial B / \partial t) = \nabla \times (v \times B) \qquad 1.55$$

We have, $$(\partial B / \partial t) \cong (B_0 / t) \cong (u_0 B_0 / L) \qquad 1.56$$

i.e., $$t \cong L / u_0 \qquad 1.57$$

Thus the characteristic time in the flow problem, is the transit time (L / u_0) during which a field disturbance diffuses a distance of order $\sqrt{(\nu_m L / u_0)}$ which is much less than L if $R_m >> 1$. Hence diffusion is negligible.

(b) *Small magnetic Reynolds number :*

This is the other extreme case which occurs when the diffusion is dominant and any imposed field B_0 is hardly affected by the fluid motion. It diffuses as if the fluid is stationary where there is no induced current. In absence of induced currents, the field is equal to the imposed field. Due to the absence of these currents , from equations 1.13 and 1.16 we get Curl $B_0 = 0$. From Ohm's law, we get that the induced current J_i is of order $(\sigma u_0 B_0)$. The induced field B_i is determined by

$$\mu_e J_i = \text{Curl } B_i \qquad 1.58$$

and therefore of order ($\mu_e \sigma u_0 L$) ,

thus $$|B_i / B_0| \cong \mu_e \sigma u_0 L \;\; (= R_m) \qquad 1.59$$

When R_m is low , the induced field can be neglected entirely to replace B by the known imposed field B_0 in all the magnetodydrodynamic equations . In this case μ_e J = Curl B can be ignored but div J = 0 must still be retained however. As the magnetic Prandtl number ν / ν_m is equal to R_m / Re , one can arrive at a better appreciation of dissipation phenomena, in actually the ratio of heat generated by viscous effects to the heat generation due to joule heat. When it is small as it is in liquid matels and low-temperature plasmas, magnetic field diffuses much more rapidly than vorticity and magnetic boundary layers are much thicker than viscous ones . This makes for simplification such as the neglect of viscosity in the magnetic boundary layer. Thus when R_m is small , the magnetic field decays by Ohmic dissipation . Omitting the term $\{ \nabla \times (v \times B) \}$ which is small, the induction equation becomes

$$\partial B / \partial t = \mu_e \ (\nabla^2 B) \qquad 1.60$$

From the above equation, it has been seen that since the magnetic field B always decays, it tends to vanish in a characteristic time t which is given by

$$t \cong L^2 / \mu_e \, . \qquad 1.61$$

In mathematical treatments , it is convenient frequently to assume $R_m \to 0$. This approximation gives the idea of some real situations and in this we have solved a few problems with this approximation.

1.4 HEAT TRANSFER IN FLUID MOTION :

Heat transfer in a medium takes place according to three processes which are known as conduction, convection and radiation. In conduction, the flow of heat is the result of the transfer of internal energy from one molecule to another. The flow of heat in solids takes place exclusively by the processes of conduction, convection and radiation which occur simultaneously. In cases of liquid and gases, where heat exchange by convection is prevented and that by radiation is minimized, the principles of heat conduction can be applied to liquids and gases as well . In these substances, however, each molecule no longer confined to certain point but constantly changes its relative position even if the substance is in state of rest. The heat transfer by convection has been seen generally in liquids and gases. By this process , heat may be transported from one point to another by the movement of the macroparticles of the substance in space from a region of one temperature to that of another. Thus the heat is being carried along as internal energy with the flowing medium . Hence the velocity field and the temperature field mutually interact. This means that the temperature distribution depends on the velocity distribution, and conversely, the velocity distribution depends on the temperature distribution. If enthalpy ($J / (m^2 s)$) is transported together with the fluid of mass per unit time ρv ($kg / (m^2 s)$) where v is the velocity of flow and ρ is the density of the fluid , so that the heat convection is

$$\dot{Q}_{conv} = \rho v_i \qquad 1.62$$

The heat convection is always accompanied by conduction. When a gas or liquid is in motion individual particles which are at different temperatures come inevitably into contact with one another. As a result, heat transfer by convection is described by equation

$$Q = Q_{cond} + Q_{conv} = -k \Delta T + \rho v_i \qquad 1.63$$

In special cases, when buoyancy forces are disregarded and the fluid properties are independent of temperature, the velocity field does not depend on the temperature field while the dependence of temperature field on the velocity field persists. Such flows are termed as forced flow and the process of heat transfer in such flows is described as forced convection. Flows in which buoyancy forces are dominant are called natural flow and corresponding heat transfer is known as natural convection. If the natural convection is not constrained to a finite region by boundaries, it is called free convection.

The third mode of heat transfer is that of radiation. Solid bodies as well as liquids and gases, are capable of radiating thermal energy in the form of electromagnetic waves and of picking up such energy by absorption. All heat transfer processes are, therefore, more or less accompanied by a heat exchange by radiation. In this thesis we have not considered the radiation effects.

For constant fluid properties, under free convection flow, the equation of motion can be expressed as (i.e., the equation 1.44 after considering of fluid buoyancy)

$$\rho \{D v / Dt\} = -(\nabla . p) - \nabla .(\mu \nabla . v) + (J \times B) + (\beta \theta \rho g_i) + X \qquad 1.64$$

where ρ is the fluid density, β is the coefficient of buoyancy and $\theta = T - T_0$ is the temperature difference between the fluid medium to the reference temperature.

The law of conservation of energy requires that the difference in the rate of supply of energy to a volume V fixed in space with a surface S and the rate at which energy goes out through S must be equal to the net rate of increase of energy in this volume . Thus the law of conservation of energy gives the following equation where the summation convention is used with i , j = 1, 2, and 3.

$$\int_s u_i (\tau_{ij} n_j) \, ds - \int_s E_t \, \rho u_j n_j \, ds + \int_v F_i u_i \, dv + \int_s k (\partial T / \partial x_j) n_j \, ds = \partial / \partial t \int_v \rho E_t \, dv \qquad 1.65$$

where, ρ is the fluid of density , E_f ($= 1/2 \, u_i u_j + \rho_0 + E$) , u_i are respectively the total energy per unit mass (i.e., sum of kinetic energy, potential energy and internal energy) and the ith component of the velocity ; τ_{ij} and n_{ij} are the ijth components of the viscous stress and jth component of the outer normal of the surfaces respectively ; F_i is the ith component of the external conservative force and k is the coefficient of heat conductivity.

The first term on the left hand side of the equation 1.65 is the rate of heat produced by viscous stresses in contact with outside; the second term represents the energy loss by convection; the third term is the work done by the external forces and the fourth term is the energy loss by the heat conduction. The loss due to radiations assumed to be negligible. The right hand side is the net rate of change of energy in the volume V. Transforming the surface integration to volume integration and the volume V being arbitrary , we get

$$\partial / \partial x_j (u_i \tau_{ij}) - \partial / \partial x_j (\rho E_t \, u_j) + F_i u_i + \partial / \partial x_j (k \, \partial T / \partial x_j) - \partial / \partial t (E_t \, \rho) = 0 \qquad 1.66$$

Using the equation of continuity 1.28 and simplifying , we get the equation 1.66 as

$$\rho \{ DE / Dt + p \; D / Dt (1/ \rho) \} = \{ \partial / \partial x_j \, (k \, \partial T / \partial x_j) \} + \phi \qquad 1.67$$

Using the equation of continuity 1.28 and simplifying , we get the equation 1.66 as

$$\rho \{ DE/Dt + p\, D/Dt(1/\rho) \} = \{ \partial/\partial x_j \, (k\, \partial T/\partial x_j) \} + \phi \qquad 1.67$$

where the dissipation function ϕ can be written as

$$\phi = [\, \mu (\partial u_i/\partial x_j + \partial u_j/\partial x_i) - 2/3\, \mu (\partial u_k/\partial x_k)\, \delta_{ij}]\, \partial u_i/\partial x_j \qquad 1.68$$

For perfect gas, $DE/Dt = C_v\, DT/Dt$, $Dh/Dt = C_p\, DT/Dt$ = enthalpy

where h is the internal energy of the system.

and $C_p\, DT/Dt = C_v\, DT/Dt + D/Dt(p/\rho)$, which reduce the equation 1.67 to

$$\rho\, D/Dt(C_p T) = Dp/Dt + \partial/\partial x_i \, (k\, \partial T/\partial x_i) + \phi \qquad 1.69$$

For incompressible fluid , the above equation simplifies to

$$\rho\, D/Dt(C_p T) = k \{ \partial/\partial x_i \, (\partial T/\partial x_i) \} + \phi \qquad 1.70$$

Equation of state :

In solving a hydrodynamic problem, together with the equations of continuity, motion and energy, we should consider an equation of state

$$\rho = \rho(p, T) \qquad 1.71$$

A few problems in this thesis have been considered with Boussinesq approximations (Chandraskekhar , 1961). It suggests that ρ is constant in all terms in the equation of motion except that one in the external force; therefore, we have

$$\rho = \rho_0 \{ 1 - \alpha (T - T_0) \} \qquad 1.72$$

where, α is the volumetric expansion coefficient of the fluid and the subscript o denotes the unheated no flow state.

Non-dimensional parameter in heat transfer :

In order to understand the phenomenon of heat transfer , we should discuss the non-dimensional parameters which govern the process. For simplicity we take Cartesian coordinates x_j (j = 1, 2, 3) and suppose that the fluid properties are independent of temperature. The momentum equation and energy equations in Cartesian tensors with usual summation conventions are :

$$\rho(D u_i / Dt) = - \partial p / \partial x_i + \rho g_i \beta \theta + \mu [\partial /\partial x_j (\partial u_i / \partial x_j + \partial u_i /\partial x_i)$$
$$- 2 / 3 \{ \partial / \partial x_i (\partial u_i /\partial x_i)\}] \qquad 1.73$$

$$\rho C_p (D\theta / Dt) = k (\partial^2 \theta / \partial x_i \partial x_i) + u_i \partial p / \partial x_i + \mu \theta \qquad 1.74$$

where $\theta = (\partial u_i / \partial x_j) (\partial u_i / \partial x_j + \partial u_i /\partial x_i) - 2 / 3 (\partial u_i / \partial x_j)^2$ 1.75

We make the quantities non-dimensional as follows

$$u_i^* = u_i / u_0 , \quad \theta^* = \theta / \theta_w , \quad x_i^* = x_i / d , \quad t^* = u_0 t / d , \quad p^* = p / \rho u_0^2 \qquad 1.76$$

where $\theta = T - T_0$, d is the characteristic dimension of length, u_0 denotes a unique velocity that characterizes the flow and the subscript w denotes the wall conditions. Substituting 1.76 in the equations 1.73 and 1.74, we get -

$$\{(\partial u_i^* / \partial t^*) + u_j^* (\partial u_i^* / \partial x_j^*)\} = (Gr / Re^2)\theta^* - (\partial p^* / \partial x_i^*)$$
$$- (1 / Re) [[\partial /\partial x_j^* (\partial u_i^* / \partial x_j^*) + (\partial u_j^* / \partial x_i^*) - 2 / 3 \{ \partial / \partial x_i^* (\partial u_j^* / \partial x_i^*)\}]$$

1.77

$$(\partial \theta^* / \partial t^*) + u_i^*(\partial \theta^* / \partial x_i^*) = 1 / (Re\, Pr)(\partial^2 \theta^* / \partial x_i^* \partial x_i^*) + E\, u_i^* (\partial p^* / \partial x_i^*)$$
$$(E / Re)[(\partial u_i^* / \partial x_j^*) \{(\partial u_i^* / \partial x_j^*) + (\partial u_j^* / \partial x_i^*)\} - 2 / 3 (\partial u_i^* / \partial x_j^*)^2]$$

1.78

Apart from the dimensionless terms and dimensionless coordinates , composed of homogeneous physical quantities as stated above , the convection equations contain dimensionless terms with dissimilar physical parameters . These terms referred to as the criteria of the development of hydrodynamics and heat transfer. These are as follows.

$Re = u_0 d / \nu$,Reynolds number ; $Gr = (g_1 \beta \theta_w d^3) / \nu^2$,Grashof number ;

$Pr = C_p \mu / k$, Prandtl number ; $E = u_0^2 / (C_p \theta_w)$, Eckert number .

$Ar = \{ g_1 (\rho_0 - \rho) d^3)\} / (\rho\nu^2)$, Archimedean number;

where, $\nu = \mu / \rho$, the kinematic viscosity.

Reynolds number characterizes the relation between the forces of inertia and viscosity. It is the ratio of inertia force to the frictional force. Reynolds number is a very important characteristic of both isothermal and non-isothermal processes of fluid flow. Grashof number is the ratio of the buoyancy force to viscous force. It characterizes the buoyancy force appearing in the fluid due to differences in density. If we assume the flow such that $\beta \theta_w = (\rho_0 - \rho) / \rho$, the Grashof number Gr may be replaced with its general modification which is known as Archimedean number (Ar). It is identical with the Grashof number on condition that β = constant. The Prandtl number (Pr) depends only on the properties of the medium. It is possible to conclude from the Eckert number (E) that frictional heat and heat due to compression are important for calculation of the temperature field when the free stream velocity v is quite large that the adiabatic temperature increases is of the same order of magnitude as that of the prescribed temperature difference between the body and the stream. The product Pr R = Pe is called Peclet number. We obtain the Peclet number when we divide the convection term by the conduction term of the energy equation. The ratio R^2 / Gr is called Froude number, it compares the inertia and the body force.

Coefficient of heat transfer :

According to Fourier's law the heat flow per unit area is proportional to the temperature decrease in the distance d is given as

$$Q / A = - k (\nabla T) / d = q \qquad 1.79$$

where q represents the flux of thermal energy relative to the local fluid velocity.

Engineers have often to deal with heat transfer from a wall to the surroundings or from the latter to a wall. Therefore, the transport of heat by convection inside the fluid medium which affects local heat transfer , i.e., heat transfer from a wall to a medium or vice versa, may be of indirect interest.

In a flow system with the fluid may be flowing either in or around a solid boundary. If the solid surface is warmer than the fluid, heat is transferred from the solid to the fluid. Then the rate of heat flow across the solid-fluid interface would be expected to depend on the area of the outface and the temperature difference between fluid and solid. Practical calculations are based on Newton's law , $Q = \alpha A \nabla T$

According to this relation, the amount of heat Q transferred from the fluid to an element of area A of the wall exposed to the fluid (or from the surface element A to the fluid) is directly proportional to A and the temperature difference ΔT , where $\Delta T = (t_w - t_f)$, t_w is the surface temperature of the wall and t_f is the temperature of the surrounding liquid or gaseous medium. The temperature difference is also referred to as the temperature drop. The proportionality factor α is known as the heat-transfer coefficient. If the wall-temperature distribution is initially unknown or the fluid properties change appreciably along the pipe , it is difficult to predict the heat transfer coefficients defined as above. Under these conditions , it is customary to rewrite the equation 1.79 in the differential form (see " Transport phenomenon", Bird et al. (1960)):

$$dQ = \alpha_{loc} (\pi D dz) (T_0 - T_b) \qquad 1.80$$

Here dQ is the heat added to the fluid in the distance dz along the pipe , $(T_0 - T_b)$ is the local temperature difference . α_{loc} is the local heat transfer coefficient (measured in W / (m^2 K)), it accounts for the condition under which a practical process of heat transfer occurs, affecting its intensity. This equation is widely used in engineering design.

Thus the coefficient of heat transfer (α) expresses the quantity of heat exchange between the body and the stream . It is defined either as a local quantity or as a mean quantity over the surface of the body under consideration , and referred to the difference between the temperature of the wall and that of the fluid. , the latter being taken at a large distance from the wall. If q (r) denotes the quantity of heat exchanged per unit area and time at a distance r , then according to Newton's law of cooling , it is assumed that

$$Q (r) = \alpha (r) (T_w - T_0) = \alpha (r) \theta_w \qquad 1.81$$

At boundary between a solid body and a fluid , the transfer of heat is solely due to conduction. In accordance with Fourier's law , the absolute value of the heat flux is given as

$$Q(r) = - k (\partial T / \partial \eta)_{\eta = 0} \qquad 1.82$$

Nusselt number (N) : The dimensionless term denoted as

$$Nu(r) = \alpha (r) d / k \tag{1.83}$$

is called the Nusselt number, or criterion of heat transfer . It characterizes the process of heat transfer at the "wall-fluid " boundary .

i.e.,
$$Nu (r) = - d / \theta_w (\partial T / \partial \eta)_{\eta = 0} \tag{1.84}$$

It is usually an unknown in the problems of convection, since it includes the heat transfer coefficient α which is being determined.

The heat flux in terms of Nusslet number is

$$Q (r) = - (k / d) Nu\, \theta_w \tag{1.85}$$

1.5 MASS DIFFUSION IN FLUID MOTION :

Many processes of heat transfer encountered in nature are accompanied by processes of mass transfer of one component into the other ; for instance, the condensation of vapour - gas from a vapour - gas mixture and the evaporation of liquid into a vapour-gas flow . The evaporated liquid is distributed throughout the vapour gas flow by diffusion ; the process accompanied by a change in the nature of flow and a variation in heat transfer intensity, and this, in turn, influences the process of diffusion.

Diffusion means the spontaneous process of spreading or scattering of matter in binary medium or two component system under the influence of concentration. In a mixture homogeneous in respect of temperature and pressure , diffusion is directed towards equalizing the concentration in the system and is accompanied by transfer of mass from the region of higher concentration to the region of lower concentration. By analogy with heat transfer, mass diffusion may be either molecular (microscopic) or molar (macroscopic). In gases molecular diffusion is due to the thermal motion of molecules. Diffusion is characterized by the flow of the mass of a component , i.e., by the quantity of mass passing through the given surface per unit time in a direction normal to the surface .

In a multicompent system, the concentrations of the various species may be expressed in various ways (also see chapter V).

With stationary macroscopic two-component system, homogeneous as regards temperature and pressure , the rate of mass flow of one of the components , due to molecular diffusion, is determined by Fick's law, given as :

$$J_D = -D\,(\partial \rho_i / \partial n) \qquad 1.86$$

$$= -\rho D\,(\partial w_i / \partial n) \qquad 1.87$$

where ρ_i is the local concentration of the given substance or component , equal to the ratio of the mass of the component to the volume of the mixture, ρ is the mixture density; $w_i = \rho_i / \rho$ is the relative mass concentration of the ith component;

D is called coefficient of molecular diffusion of one component in respect to the other (usually in short , the coefficient of diffusion). n is the normal direction to the surface of a similar concentration of the compenent; $(\partial \rho_i / \partial n)$ is the concentration gradient which is always directed to the rise of concentration in the normal direction. The concentration gradient is the motive force determining the transfer of matter. In heat conduction, it is the temperature gradient which is the motive force equivalent to this .

The minus sign of equation 1.86 indicates that the mass is being transferred , in accordance with Fick's law , in the direction of diminishing concentration. The process described by Fick's law is known as concentration diffusion.

For a two component system with level of the species A & B and $D_{AB} = D_{BA} = D$ the Fick's law of diffusion written in vector form as

$$J_D = -\rho D (\nabla w_A) \qquad 1.88$$

This equation states that species A diffuses in the direction of decreasing mole fraction of A , just as heat flows by conduction in the direction of decreasing temperature.

For a muticomponent systems under the assumption of constant ρ and D , the Fick's law 1.86 is written as

$$\partial \rho / \partial t + \rho_A (\nabla . v) + (v . \nabla \rho_A) = D (\nabla^2 \rho_A) + R_A \qquad 1.89$$

where R_A is the molar rate of production of A per unit volume.

Using the continuity equation { (∇ . v) = 0 } and dividing the equation by M_A we get

$$\partial c_A / \partial t + (v . \nabla c_A) = D (\nabla^2 c_A) + R_A \qquad 1.90$$

The equation is usually used for diffusion in dilute solutions at constant temperature and pressure.

For $R_A = 0$ the equation becomes

$$\partial c_A / \partial t + (v . \nabla c_A) = D (\nabla^2 c_A) \qquad 1.91$$

This equation is similar to the energy equation for a fluid motion when ρ is independent of T ; this similarity is the basis for the analogies that are frequently drawn between heat and mass transport in flowing fluids with constant ρ .

Non dimensional mass diffusion equation :

Let us consider a isothermal binary fluid mixture of constant viscosity μ and constant diffusivity D . In addition we assume the range of composition to be small enough that both mass density ρ and molar density c are essentially constant. Writing the mass diffusion equation 1.91

$$D c_A / D t = D(\nabla^2 c_A) \qquad 1.92$$

We now consider the following non-dimensional parameters

$$v^* = (v / u_0), \qquad p^* = \{(p - p_0) / (\rho u_0^2)\}, \quad t^* = (u_0 t / d),$$

$$c_A^* = \{ (c_A - c_{A0}) / c_{A1} - c_{A0}) \}; \qquad 1.93$$

Substituting 1.93 in 1.92 the non-dimensional mass diffusion equation is

$$D c_A^* / D t^* = 1 / (Re\,Sc)(\nabla^{*2} c_A^*) \qquad 1.94$$

where, $Re = (d u_0 \rho / \mu)$, the Reynolds number

$Sc = \mu / (\rho D_{AB})$, is the Schmidt number .

For isothermal mass transfer , the Schmidth number plays a role analogous to that of the Prandtl number in heat transfer.

Proceeding from the analogy between the processes of heat and mass transfer we can write :

$N_D = \beta d / D_{AB}$, called Nusselt number for diffusion or sometimes simply as Sherwood number , and $Pr_D = \nu / D_{AB}$, called Prandtl number for diffusion.

These numbers are analogous to the numbers Nu (Nusslet number) and Pr (Prandtl number) of heat transfer . With the analogy between heat and mass if the like reference dimensionless terms are equivalent, Nu and N_D may also be considered equivalent.

For instance, it is possible to conduct investigations of heat transfer and from this using the derived dimensionless formulae one can investigate the mass transfer, replacing Nu and Pr by N_D and Pr_D respectively . If mass transfer proceeds at a low rate , its effect on heat transfer can often be ignored with the accuracy sufficient for practical applications.

1.6 NON-NEWTONIAN FLUIDS :

The physical property that characterizes the flow resistance of simple fluids is the viscosity . All real fluids are viscous ; a force of internal friction , offering resistance to the flow that always arises between the layers of a fluid moving at different velocities in relation to one another. According to Newtonian law , the tangential force acting at any point of the flow in the plane oriented in the direction of flow is proportional to the negative of the local velocity gradient.

$$\tau_{ij} = -\mu \left(\partial v_i / \partial x_j \right) \qquad 1.95$$

μ is known as the dynamic viscosity or simple viscosity. Kinds of fluids that behave in this fashion are termed Newtonian fluids. There is no obvious reason why real fluids should obey equation 1.6. All gases and most simple liquids are this types of fluid . The three most abundant fluids air, water and petroleum obey 1.95 quite closely.

Equation 1.95 which defines a Newtonian fluid can be applied unidirectional flows only. However, the definition of Newtonian fluid in which the stress depends linearly on the rate of deformation may be generalized to three dimensional flows using the rate of deformation tensor

$$\varepsilon_{ij} = 1/2 \ (\partial q_i / \partial x_j + \partial q_j / \partial x_i) \qquad 1.96$$

where, q is the local velocity of the fluid particle . We can redefine Newtonian fluid as one that satisfies

$$T_{ij} = -p\,\delta_{ij} + 2\,\varepsilon_{ij} \qquad 1.97$$

where the kronecker delta $\delta_{ij} = 1$ for $i = j$ and $\delta_{ij} = 0$ for $i \neq 0$

There are quite a few industrially important fluids which don't obey the Newton's law. The properties of these fluids are not only function of its state of the substance but also depends on the process parameters, the variation of velocity and temperature; they are known as non-Newtonian fluid. The relation between τ_{ij} and ε_{ij} are non-linear for non-Newtonian fluid. Such fluids are primarily pastes , Slurries, high polymers , Blood , Jellies and similar food product, Polymeric melts etc.

According to the Newtonian law of viscosity , the plot of τ_{ij} versus $-(d v_i / d x_j)$ for a given fluid shows a straight line through the origin, and the slope of this line represents the viscosity of the fluid at a given temperature and pressure. Experiments have shown that τ_{ij} indeed proportional to $-(d v_i / d x_j)$ for all gases and for homogenous non-polymeric liquids.

The non-Newtonian flow of fluids is the " science of deformation and flow " which includes the study of the mechanical properties of gases liquids plastics and crystalline materials. Thus the non-Newtonian flow is the part of science of rheology where both Newtonian fluid mechanics and Hookean elasticity are considered. The steady state rheological behaviour of most fluids can be generalized as

$$\tau_{ij} = -\mu_{app} (d v_i / d x_j) \qquad 1.98$$

where μ_{app} is the apparent viscosity , is not a constant, it may be expressed as a function of either ($d v_i / d x_j$) or τ_{ij}

In order to explain the steady state relation for Newtonian and non-Newtonian fluid between τ_{ij} and ($- d v_i / d x_j$) at constant temperature and pressure several models were proposed , such as Power law model, Bingham model ,Prandtl Erying model, Reiner-Philippoff model etc.

Under unsteady state conditions a number of additional types of non-Newtonian behavior are possible , for example thixotropic , rheopectic , visco-elsatic , etc.

(i) . Time independent fluid that are where the rate of shear at a given point solely dependent upon the instantaneous shear stress at that point. Time independent non-Newtonian fluids are also called non-Newtonian viscous fluid or purely viscous fluid.

(ii) Time dependent fluids are those for which the shear rate is function of both the magnitude and the duration of shear. Time dependent non-Newtonian fluid classified into two groups Thixotropic fluid and Rheopectic fluids depending upon whether the shear stress decreases or increases with time at given shear rate at constant temperature.

Fluids that shows limited decrease in μ with time under a suddenly applied constant stress τ_{ij} called Thxotropic. The thixotropic properties have been found in the material such as some solutions or melts of high polymers , oil well drilling muds , greases printing inks , many food materials , paints, etc.

The fluids that shows limited increase of μ with time under a suddenly applied stress τ_{yx} called Rheopectic fluid. Rheopectic fluids are antithxotyropic fluids that exhibit a reversible increase in shear stress with time at a constant rate of shear under isothermal conditions . Examples of these types are bentonite clay , suspension , vanadium pentoxide suspension, gypsum suspension and certain solutions in many pipe problem etc.

(iii) Visco-elastic fluids are those which show partial elastic recovery upon the removal of a deforming shear stress , such materials posses properties of both fluids and elastic solids. These material exhibit both viscous and elastic properties . In a purely Hookean elastic solid the stress corresponding to a given strain is independent of time whereas for visco-elastic substances the stress will gradually dissipate with time . A part of the deformation of the visco-elastic fluids flow when subjected to stress, gradually recovered on removal of the stress . Examples of this type are Bitumen , flour dough , Naplam and similar jellies , Polymersand, Polymeric melts such as Nylon and many Polymeric solutions.

In order to take account of the mechanism of non-Newtonian fluids number of mathematical models were proposed at different time by different mathematicians . In our research work, we have discussed a problem of flow and heat transfer on Rivlin-Ericksen second order visco-elastic fluid. A brief description of Rivlin-Ericksen second order fluid is mentioned below.

Rivlin-Ericksen fluid :

Rivlin and Ericksen in 1955 considered the theory of isotropic fluid for which the stress depends upon the spatial gradients of velocity and acceleration upto any order n. Using the invariant requirements, they showed that the stress must be given by an isotropic function of the tensor $A_{(N)ij}$ as :

$$\tau_{ij} = f\{A_{(1)kl}, A_{(2)kl}, \text{-------------} A_{(N)kl}\} \qquad 1.99$$

where, f obeys an identity

$$Q f_{ij}\{A_{(1)kl}, A_{(2)kl}, \text{-------------} A_{(N)kl}\} Q^T$$

$$= f_{ij}\{Q A_{(1)kl} Q^T, \; Q A_{(2)kl} Q^T, \text{---------} Q A_{(N)kl} Q^T\} \qquad 1.101$$

The term $A_{(N)ij}$ called nth order Rivlin-Ericksen tensor. $A_{(N)ij}$ is related to the velocity gradient tensor V_{ij} by the formula

$$A_{(N)ij} = A_{(N-1)ik} V_{kj} + A_{(N-1)kj} V_{kj} + \bar{A}_{(N-1)ij} \qquad 1.102$$

where, $A_{(1)ij} = 2 e_{ij}$, and $e_{ij} = 1/2\{V_{ij} + V_{ji}\}$ 1.103

The bar denotes material derivative defined as

$$Dx_i / Dt = \partial x_i / \partial t + V^i x_i \qquad 1.104$$

The fluid govern by the constitutive equation 1.99 is called Rivlin-Erucksen fluid of complexity N .

For isotropic fluids , τ may be considered as a function of $A_{(1)}$ and $A_{(2)}$ only

i.e., $\tau_{ij} = f\{A_{(1)}, A_{(2)}\}$ 1.105

So the equations 1.99 and 1.100 with the help of 1.105 , give

$$\tau_{ij} = \mu_0 U + \mu_1[A_{(1)}] + \mu_2[A_{(2)}] + \mu_3[A_{(1)}]^2 + \mu_4[A_{(2)}]^2$$
$$+ \mu_5\{[A_{(1)}][A_{(2)}] + [A_{(2)}][A_{(1)}]\} + \mu_6\{[A_{(1)}]^2[A_{(2)}]$$
$$+ [A_{(2)}][A_{(1)}]^2\} \mu_7\{[A_{(1)}][A_{(2)}]^2 + [A_{(2)}]^2[A_{(1)}]\}$$
$$+ \mu_8\{[A_{(1)}]^2[A_{(2)}]^2 + [A_{(2)}]^2[A_{(1)}]^2\} \qquad 1.106$$

where, μ_m , m = 0 , 1 , 2 , 3 , ---------- 8. are scalar functions of the nine invariance of tensors [$A_{(1)}$] and [$A_{(2)}$]. The fluid govern by the equation 1.85 is called Rivlin-Ericksen fluid of complexity two. For viscometric flows , all tensors [$A_{(N)}$] except [$A_{(1)}$] and [$A_{(2)}$] vanish. Markoviz observed that μ_m , m = 4,5,- - - 8 may be omitted without affecting the solutions . So then the reduced constitutive equation takes the form,

$$\tau_{ij} = -p\,\delta_{ij} + \mu_1 A_{(1)ij} + \mu_2 A_{(2)ij} + \mu_3 A_{(1)im} A_{(1)mj} \qquad 1.107$$

where, $p = \pi - \mu_0$, is the indeterminate isotropic pressure.

μ_1 = Coefficient of ordinary viscosity ; μ_2 = Coefficient of visco-elasticity.

μ_3 = Coefficient of cross viscosity.

A fluid govern by the equation 1.107 is called an incompressible second order Rivlin - Ericksen fluid. We can also write the constitutive equations of higher orders in this way. All the three material constants can be determined from the viscometric equation of state for any material behaving as a second order fluid. Markovite and Coleman proved that μ_2 is negative (experimentally also, it has been found negative under thermodynamical considerations) .

Although the general Rivlin-Ericksen fluid accounts for shear dependent viscosity and normal stress effects; yet it shares the Newtonian fluid as its special case. The effect of changes in shear rate with time upon the stresses in a visco-elastic fluid were incorporated into the constitutive equations by Rivlin and Ericksen . Rivlin has solved some special problems using the theory stated above.

1.7 SHOOTING METHOD FOR SOLUTION OF ORDINARY DIFFERENTIAL EQUATION :

For an ordinary differential equation (ODE), we need n conditions. For an initial value problem (IVP), all n conditions are specified at one point (say, x_0). In the shooting method, we solve BVP as an IVP by guessing the missing the conditions at x_0. The correctness of the guess is judged by seeing how closely the solution satisfies the final condition at x_L . Obviously, for an arbitrary guess, the boundary condition at x_L will never be satisfied. Thus a root finding algorithm to converge to the correct guess is used. To solve a BVP by shooting method we adopt the following strategy

(i) We develop an IVP solver for the same ODE for the initial conditions are

$$f(x_0) = \alpha_0, \quad f'(x_0) = \beta_0 \quad \text{and} \quad f''(x_0) = \gamma_0 \ .$$

(ii) We then use the bisection method to converge to the correct $f''(x_0)$ value which yields $f'(x_L) = \alpha$, in the solution.

The bisection method is as follows.

Using IVP solver we find two values of $f''(x_0)$, say S_1 & S_2 , which yield values of $f'(x_L)$ as r_1 & r_2 respectively, such that $r_1 < 1$ and $r_2 > 1$. Now as $f'(x_L)$ depends directly on the chosen value of $f''(x_0)$, we expect that $f'(x_L) = \alpha$ condition will be satisfied by some value of $f''(x_0)$ which lies between S_1 & S_2 . So we guess a new value of $f''(x_0)$ as $\{S (S_1 + S_2) / 2\}$ which yields on $f'(x_L)$ a value of say r , if $r < 1$ then we replace the old value of S_1 by S otherwise if $r > 1$, the old value of S_2 is replaced by S . Once again S_1 and S_2 will be such that the correct value of $f''(x_0)$ will lie between them. This process is repeated and at each step the interval between S_1 and S_2 is reduced by half and the correct $f''(x_0)$ is squeezed into this interval . When $f'(x_L)$ is close enough to α for any S value , the process is stopped.

{ The shooting method for simultaneous differential equations of nth order is stated in chapter II }.

1.8 SOME WORKS RELATED TO MHD FLOW AND HEAT TRANSFER :

The steady Poissueille flow of mercury between two parallel walls in presence of an applied cross magnetic field , was considered by Hartmann (1937). MHD flow between two parallel plates under a transverse magnetic field , called Hartmann flow, has been studied by many authors under various conditions e.g., Shercliff (1966) and Cowling (1957). Ospal (1955) has outlined the general principles of the analysis of two-dimensional and three-dimensional ground water flow by electrical analogy and described the practical applications of that method with a new conductive material consisting of gelatin , glycerin , water and salt. Srivastava and Sharma (1961) have discussed the effect of a transverse magnetic field on the flow between two infinite disks, one rotating and the other at rest . The above problem has been extended afterwards by Stephenson (1969). He has obtained asymptotic solutions for $R << M$ and numerical solution for couette flow when one of the plates moves impulsively and the other is at rest. The effect of induced magnetic field on the same problem has been discussed by Gobundarajuly (1970) . The problem of steady flow of an electrically conducting fluid through uniformly porous infinite parallel plates channel in the presence of a transverse magnetic field has been investigated by Rao (1961) , Terril and Shrestha (1963 , 1964) and Terril (1964). Sharma (1962) has discussed the MHD couette flow between non-conducting walls in the presence of an electric field. Agarwal (1962) has discussed the generalized MHD couette flow between two parallel plates with or without porosity . The effect of suction or injection and magnetic field on the MHD flow in a straight channel has been studied by Shrestha (1967) , Reddy and Jain (1967) . Chandrasekhar and Redraiah (1970) have discussed the problem of a two dimensional conducting flow between two porous disks for $R << 1$ where there is uniform suction or injection. This two dimensional problem has been extended to three dimensional flow by the same authors (1971) under the assumption that one of the plate is at rest and the other is rotating . Chang and Yen (1962) have studied the heat transfer aspect between the walls. Srivastava and Sharma (1964) have discussed the heat transfer due to the flow between two infinite plates, one rotating and other at rest , under a transverse magnetic field . Chang and Yen's problem has been extended by Soundalgekar (1969a) . In another paper, Soundalgekar (1969b) has studied the heat transfer aspects in MHD couette flow between conducting walls in the presence of an electric field . Gupta (1969) has studied the effect of combined free and forced convection on the flow of an electrically conducting liquid under a transverse magnetic field a horizontal parallel plates channel subjected to a linear axial temperature variation. Vitazhin (1965) has investigated the hydromagnetic viscous compressible flow with Hall currents, past an infinite wall started impulsively from rest. Pope (1971) has discussed the effect of Hall currents in the flow of an incompressible, viscous and electrically conducting fluid past an accelerated motion of an infinite flat plate in the presence of a transverse magnetic field . Here he has considered that the electric circuit as short i.e., $E = 0$. Hall effects in steady flows of a partially ionized gas between two stationary parallel plates has been studied by Sato (1961), Kusukawa (1962) and Sutton and Sherman (1962) . Nayak (1976) have investigated the flow through a channel whose walls were lined with non-erodible material using Beavers and Joseph (1967) slip boundary condition. It was shown that the effect of porous lining is to increase the mass flow rate and the effect of porous lining is to increase the mass flow rate and to decrease the friction factor. Singh et al. (1986) has considered the unsteady two dimensional free convection flow through a porous medium bounded by an infinite vertical plate when the temperature of the plate was oscillating with time about a constant non-zero mean . Fand and Phan (1987) have reported the results of an experimental study of heat transfer by combined forced and natural convection from a horizontal cylinder embedded in a porous medium composed of randomly packed glass spheres saturated with water. Srivastava and Sharma (1991) have considered the flow of a second-order fluid through a circular pipe and its surrounding porous medium when (i) the surrounding region extends to a large distance and (ii) it is bounded by an impervious co-axial circular cylinder. Padmavathi et al (1993) have considered a general non-axisymmetric Stokes flow past stationary porous sphere (using Brinkman's model) in a viscous, incompressible fluid. They proposed a representation of the velocity and the pressure fields for the Brinkman's equation similar to one suggested by Plalaniappan et al. (1990) for Stokes flow . Narasimha Rao (1994) has studied the steady buoyancy induced boundary layer flow of a non Newtonian fluid over a non-isothermal horizontal flat plate immersed in a porous medium by employing the general similarity transformation procedure and the power law model to characterize the non-Newtonian fluid behavior . Temperature profiles and the heat transfer rate at the wall were presented for different values of the non-Newtonian power law index and the exponent associated with the wall temperature distribution. Chandana and Oku Ukpong (1995) have discussed an unsteady second grade aligned MHD fluid flow which undergoes isochoric motion. Shapakidze (1995) has studied the flow of a viscous electrically conducting fluid between two rotating permeable cylinders in the presence of a magnetic field . Kalis (1995) has shown the development and application of special numerical method for the solution of problems in mathematical Physics, hydrodynamics and magnetohydrodynamics. Pukhnachev (1995) has proposed a model of convective motion under small force of gravity. Vajravelu (1995) has discussed about free convection flow of and electrically conducting fluid at a stretching sheet. Meir (1995) has discussed the thermally coupled MHD flow. He has shown that a steady state may be achieved when the sum of viscous force, convective inertial force, thermal pressure, electromagnetic force and buoyant force vanishes. The buoyancy induced flow adjacent to a periodically heated and cooled horizontal surface in porous media have been discussed by Bradean et al. (1996). A MHD flow of an equal kinematics and magnetic viscosity through parallel porous plates have been studied by Manato and Kuiry (1997). The transient MHD free convection flow past an infinite vertical plate embedded in a porous medium with temperature dependent heat source have studied by Das et al. (1997).

1.9 MOTIVATION AND SCOPE OF THIS THESIS :

The motivation of this thesis is to study a few aspects of the free and forced convective flow of incompressible, viscous, electrically conducting Newtonian and non-Newtonian fluids in presence of uniform magnetic field.

Viscosity (μ) is the fluid property depends on the nature of the fluid and to a great extent on its temperature. In case of liquid, the viscosity is nearly independent of pressure but decreases at a high rate with increasing of temperature. In case of gas, the viscosity can be taken to be independent of pressure to a first approximation, but it increases with temperature. For liquids , the type of dependence of the kinematics viscosity on temperature is same as that of μ , because the density ρ changes only slightly with temperature. Therefore, to know the fluid behavior properly in hydrodymamical problems applied in various engineering problems, it is necessary to consider the temperature dependent viscosity of the fluid. Lai and Kulachi (1990) have stated an inverse relation with the temperature for the viscosity of incompressible fluid. Pop , Goula , and Rashidi (1992) have used this relation to determine flow and heat transfer nature in an quiescent fluid , over a flat plate. In chapter II we have discussed a problem with a temperature dependent viscosity in presence of a uniform magnetic field. The magnetic field is applied transversely to the flow. We have discussed the first degree of magnetic field interaction, that is, the solutions are obtained for the coefficients of m (up to second order), the magnetic interaction parameter in the expansion of ascending power of m . There are scopes to study the coefficients of different higher powers of m to know the exact effect of magnetic field on the fluid motion under the variable viscosity. We have neglected viscous dissipation due to the fluid motion, effect of porous medium, induced magnetic field. But in certain cases , these effects play significant role. Hence the problem may be extended including all these effects.

In chapter III , we have studied the problem with small Reynolds number ($Re < 1$, Creeping motion) due to an exponentially decay source placed between two parallel plates. Gourla (1994) have done the problem without considering the action of magnetic field, on the other hand, while we have extended it in presence of the magnetic filed. The problem may be extended for larger values of Re by successive approximations. Further , the problem may be extended for a sinusoidal source instead of the exponential source. The problem may also be extended for higher value of magnetic field which generates an induced field and the Joule effect. In addition to these, one may also consider the porous medium in motion.

We have discussed the effects of an inclined magnetic field on a laminar convective forced flow in chapter IV. The magnetic field is supposed to be high enough to induce another field. The energy dissipation due to magnetic field and fluid viscosity are also considered simultaneously. It has been observed that the effect of inclination of magnetic field from vertical axis are significant on flow and heat transfer. The problem may be extended for porous medium. The fluid properties for example density, viscosity , thermal diffusivity , etc. are supposed to be constant in our discussion. But in actual practice, especially the fluid viscosity and density vary with temperature.

Therefore, there are opportunities to extend the discussion considering fluid density and viscosity as variable with temperature.

Simultaneous heat and mass transfer in a binary mixture due to uniformly moving vertical porous plate has been discussed in chapter V. The problem has been discussed by Sattar (1995) ignoring the thermal diffusion effcct as well as thc effect of induced magnetic field, which we have included in this chapter. In many engineering problems, it has been observed that heat transfer is accompanied by mass and thermal diffusion ; hence our study may be useful in this regard. Using this method , similar problems may be studied in various geometries. The consideration of temperature dependent density and viscosity in this type of problems are useful for practical problems. The thermal effusion effect which has been neglected in this chapter , may also be included to make it more perfect.

In recent years, considerable interest has been evinced in the study of flow past a porous medium because of its natural occurrence and importance in engineering problems . In chapter VI, we have discussed the effects of porosity and kinematic viscosity in a free convection flow in presence of transverse magnetic field . The results conclude that the effect of porosity depends upon the strength of the magnetic field . The magnetic field when sufficiently high , it generates an induced field . Therefore, we can extend the problem with induced field. In many situations, there are impurities in the fluid, for example, muddy water , Crude oil etc. One may extend the problem by incorporating such an impure fluid whose behavior may be discussed as suggested by Coleman and Noll (1959).

Lastly , the flow of a dusty electrically conducting fluid in presence of a transversed magnetic field in an inclined channel has been discussed in chapter VIII. We have calculated velocities of the fluid and the dust particles, rate of heat transfer and skin friction at the plates for fluid and dust particle , and fluid and particle flux within the channel. Their variations with the magnetic field parameter are shown graphically . It has been observed from the velocity profile that the velocities of fluid and particle decrease with the increase of magnetic field strength. The flow of dusty visco-elastic fluids in porous medium plays an important role in hydrodynamics. Therefore, our discussion in chapter VIII can be extended by taking into account the porosity of the medium. Moreover, when the fluid density and viscosity are variable , the stratification effect becomes prominent . The discussion may further be useful while considering the stratification effect under variable viscosity on the fluid as well as the dust particle.

CHAPTER II

LAMINAR CONVECTION FLOW UNDER TEMPERATURE DEPENDENT VISCOSITY IN PRESENCE OF UNIFORM MAGNETIC FIELD

2.1 INTRODUCTION :

The physical property that characterizes the flow resistance of simple fluid is the viscosity. All real fluids are viscous; a force of internal friction, offering resistance to the flow that always arises between the layers of the fluid moving at different velocities relative to one another. The ratio of viscosity μ to density ρ of a fluid motion is known as kinematics viscosity and is denoted by ν ($= \mu / \rho \ m^2 / s$). The two viscosity coefficients μ and ν are physical parameters which govern the fluid motion and are functions of temperature. The viscosity of liquid is almost independent of pressure but declines significantly with rising of temperature . On the other hand the viscosity (μ) of gases increases with the rise of temperature. Most of the studies in fluid mechanics are based on constant physical' properties, say thermal conductivity specific heat , density , thermal diffusivity and viscosity. However Poiseuille Helmholtz (1860) and Reynolds (1886) practically examined the variation of fluid viscosity μ and kinematic viscosity ν with the rise of temperature. They showed that for all liquids viscosity diminishes rapidly at the rise of temperature. Helmohtz showed the water viscosity is

$$\mu_w = [\ 0.01779 / \ \{ 1 + 0.033368\theta + 0.000220999\theta^2 \} \ ] \qquad 2.1$$

On the other hand for gases, the value of μ is found to be sensibly independent of the pressure within very wide limits, but to increase somewhat with the rise of temperature. An empirical formula for the case of air was with its density $\rho = 0.00129$ and at atmospheric pressure was given by Hyde (1919), as

$$\mu_a = [\ 0.0001702\ \{\ 1 + 0.0329\theta\ + 0.0000070\theta^2\ \}\ ] \qquad 2.2$$

where θ is the temperature in the centigrade scale.

Koch (1881) gave the results for mercury as $\mu = 0.01697$ at 0^0c while $\mu = 0.01633$ at 10^0c. Borthel (1956) measured the static flow resistance through porous materials. The theory of laminar boundary layer flow of a viscous fluid caused by the motion of a rigid surface originates from the work by Sakiadis (1961) . Similar type of problems were studied by Grief et al. (1971) and Gupta et al. (1974). Later on many authors and researchers studied this kind of flow , for instance, Revenkar (1989), Igham and Pop(1990) and so on. Lorentz (1881) has discussed the heat transfer from a hot vertical plate under the assumption that the temperature and velocity at any point depend only on the distance from the plate. Schmidt and Beckmann (1930) have done the experimental works on the same problem and have showed that the assumption was invalid and have indicated an alternative method of solution. The problem of simultaneous heat and mass transfer in free convection about a vertical flat plate with uniform surface temperature and concentration has been considered by Bottemanne (1970). He has taken the two buoyancy effects originating from temperature and mass concentration differences as mutually independent and has given a numerical solutions for the system of boundary layer equations for the steady case . Bottemanne (1971) has also experimentally verified his theoretical results . The experimental results concerning stationary heat and mass transfer in the laminar boundary layer of a vertical cylinder placed in air have been given by Bottemanne (1972). Singh and Gupta (1971a) have solved the problem of a flow past a porous sphere considering the full Navier-Stokes equations outside the sphere and have obtained an expression for the drag on the sphere. Rudraiah and Veerabhadraiah (1974) have investigated the laminar steady plane coquette flow having one permeable boundary wall. They have found that the mass flow rate increased and the friction factor decreased as a result of greater heat addition . Bejan (1983) studied the natural convection in a rectangular porous layer heated and cooled with uniform heat flux along the vertical side wall . He reported that in the boundary layer region, the boundary layer thickness was constant and the core region was motionless while the vertical temperature gradient was maintained constant throughout the medium.

All the above problems have been studied on the basis of constant physical properties of the ambient fluid. But the fluid properties especially viscosity varies with the rise of temperature, therefore, to predict accurately the nature and mechanism of a fluid flow and heat transfer, it is necessary to take account into the variation of fluid viscosity with the change of temperature. For liquids as the viscosity varies linearly and inversely with the temperature, the results are distinctly different from those studied at constant viscosity.

Nahme (1940) extended the hydrodynamic theory of lubrications to include the effect of the variation of viscosity with the temperature. Hausenblas (1950), extended the Poiseuille flow through a channel with flat walls to the case of temperature-dependent viscosity. The corresponding solution for a circular pipe was given by Girgull (1955). In more recent time Pop et al. (1990), have studies the flow bchaviour and heat transfer rates on a continuous moving flat plate considering the variation of fluid viscosity as isothermally heated plate. They analyzed a more accurate picture of the momentum and thermal transfer of the fluid motion over the plate. The results obtained by them are distinctly different for different values of Prandtl number from those obtained by Soundalgekar (1980) and Ingham et al. (1990) considering constant fluid viscosity. The results for fluid drag and heat transfer were in good agreement with those of experimental values at various temperatures.

In this chapter we analyze the nature and behavior of a viscous , incompressible, electrically conducting fluid over a flat plate which is moving with a uniform speed in a quiescent fluid, in presence of a uniform magnetic field . The fluid viscosity is considered to be a function of temperature and varies inversely with it . The uniform magnetic field applied externally in a direction transverse to the fluid motion. Perturbation technique is used to show the effects of magnetic field on the mass motion, while the solutions of the equations of the problem are carried out by similarity transformation. Equations are solved up to second order of the magnetic parameter, they are solved numerically using Runga-Kutta and Shooting methods. The numerical values of skin friction factor at the plate and local heat transfer are calculated. The distribution of fluid velocity and temperature at different values of magnetic field parameter and viscosity-temperature coefficient are shown graphically for different values of Prandtl number. The result obtained here are meant for the temperature dependent fluid viscosity in presence of uniform magnetic field.

2.2 FORMULATION OF THE PROBLEM :

We consider laminar flow of a viscous incompressible electrically conducting fluid on a continuous moving flat plate along x axis . The plate is moving in its own plane with a constant speed U_0 in a quiescent fluid . A uniform magnetic field $\mathbf{B_0}$ is applied transversely i.e. , along y-axis. The fluid properties except fluid viscosity (μ) are assumed to be isotropic and constant, and the viscosity is inverse linear function of temperature as considered by Lai and Kulachi (1990) :

$$1/\mu = [(1/\mu_\infty)\{1 + \gamma(T - T_w)\}] \qquad 2.3$$

$$= [(1/a)(T - T_r)] \qquad 2.4$$

where $a = (\mu_\infty/\gamma)$ and $\gamma = 1/(T_\infty - T_r)$, and $T_r = (T_\infty - 1/\gamma)$ 2.5

μ_∞ , ρ_∞ and T_∞ are the fluid viscosity density and temperature away from the plate. Both a and T_r being constant .Their values depend in the reference state and the thermal property of the fluid (i.e; γ) . In general , $a > 0$ for liquid and $a < 0$ for gasses . In order to derive the governing equations of the problem the following assumptions are made.

(i) The fluid is finitely conducting and the viscous dissipation and the Joule heat are neglected.

(ii) Hall effect and polarization effect are neglected.

(iii) The flat plate which is maintained at a constant temperature (T_w) is moving with uniform velocity and the fluid viscosity varies with temperature only ; therefore , all the physical variables are assumed to be time independent.

(iv) The perturbation technique which is used for small values of the magnetic parameter (m) depending upon the degrees of magnetic field interaction, shows the effect due to the magnetic field by the second order term (i.e. the term containing m).

(v) The value of magnetic Reynolds number is so small that the effect of induced magnetic field is negligible.

Considering u and v as the fluid velocities along x , y axes respectively the fluid velocity and magnetic field components of the problem are V= [u , v , 0] and B = [0 , B_0 , 0] respectively. The magnetic body force using 1.29 and omitting the electric part is written as

$$f = J \times B \qquad 2.6$$

where $$J = \sigma (V \times B) \qquad 2.7$$

This gives $$J_x = 0 , \; J_y = 0 , \; J_z = \sigma (B_0 u) \qquad 2.8$$

and hence from 2.6, $$f_x = - (\sigma B_0^2 u) , \quad f_y = 0 , \quad f_z = 0 \qquad 2.9$$

where σ is the electrical conductivity.

Now the equation of continuity using equation 1.28, is :

$$(\partial u / \partial x) + (\partial v / \partial y) = 0 \qquad 2.10$$

The boundary layer equation, from 1.32, is written as :

$$u(\partial u/\partial x) + v(\partial u/\partial y) - (1/\rho_\infty)[\partial/\partial y(\mu\, \partial u/\partial y)] + (\sigma B_0{}^2 u)/\rho_\infty = 0 \qquad 2.11$$

The energy equation of our problem, using 1.44 and 1.45 , is

$$u(\partial T/\partial x) + v(\partial T/\partial y) - [\alpha(\partial^2 T/\partial y^2)] = 0 \qquad 2.12$$

where $\alpha = \{k/(\rho C_p)\}$, known as thermal diffusivity of the fluid ; k and C_p are the thermal conductivity and specific heat at constant pressure of the fluid respectively.

The boundary conditions of the problem , are as

$$\begin{aligned} &u = u_0\ ,\quad v = 0\ ,\quad T = T_w \qquad \text{at}\quad y = 0 \\ &u \to 0\ ,\quad T \to T_\infty \qquad \text{as}\quad y \to \infty \end{aligned} \qquad 2.13$$

2.3 SOLUTION OF THE GOVERNING EQUATIONS :

In view of the boundary conditions 2.13 , we consider the following similarity transformations for the velocity components and temperature equations 2.11 & 2.12, We introduce the stream function ψ as :

$$u = (\partial\psi/\partial y) \quad \text{and} \quad v = -(\partial\psi/\partial x) \qquad 2.14$$

Substituting 2.14 , in the equations, 2.11 & 2.12, we get -

$$(\partial\psi/\partial y)(\partial^2\psi/\partial x\partial y) - (\partial\psi/\partial x)(\partial^2\psi/\partial y^2) + mu_0(\partial\psi/\partial y)$$
$$- (1/\rho_\infty)(\partial\mu/\partial y)(\partial^2\psi/\partial y^2) - \mu(\partial^3\psi/\partial y^3) = 0 \qquad 2.15$$

$$(\partial\psi/\partial y)(\partial T/\partial x) - (\partial\psi/\partial x)(\partial T/\partial y) - \alpha(\partial^2 T/\partial y^2) = 0 \qquad 2.16$$

where $\{(\sigma B_0{}^2)/\rho_\infty\} = mu_0$, and m is the magnetic parameter showing the strength of the magnetic field applied.

Substituting 2.14, in 2.13, the boundary conditions of the problem are :

$$\psi = 0\ ,\ (\partial\psi/\partial y) = u_0\ ,\ T = T_w \quad \text{at} \quad y = 0$$
$$\text{and} \quad (\partial\psi/\partial y) \to 0,\quad T \to T_\infty \quad \text{as} \quad y \to \infty \qquad 2.17$$

We consider the stream function $\psi(x, y) = \nu_\infty(Re)^{1/2} F(\eta, x)$ 2.18

where $\eta = (y/x)(Re)^{1/2}$, 2.19

$Re = (u_0 x/\nu_\infty)$, Reynolds number.

Substituting the stream function relations 2.18 & 2.19 , equations 2.15 & 2.16 can be written as

$$[\{(\theta - \theta_r)^2/(\theta_r)\}\{x(\partial F/\partial\eta)(\partial^2 F/\partial x\partial\eta)\}] - (1/2)F(\partial^2 F/\partial\eta^2)$$
$$- x(\partial F/\partial x)(\partial^2 F/\partial\eta^2) + mx(\partial F/\partial\eta))$$
$$- (\partial\theta/\partial\eta)(\partial^2 F/\partial\eta^2) - (\theta - \theta_r)(\partial^3 F/\partial\eta^3) = 0 \qquad 2.20$$

$$(\partial F/\partial\eta\,((\partial\theta/\partial\eta)(\partial\eta/\partial x) + (\partial\theta/\partial x)) - (x/2)(\partial\theta/\partial\eta)(F - \eta(\partial F/\partial\eta)$$
$$+ (\partial\theta/\partial\eta)(\partial F/\partial x)) - \{(\alpha/(\gamma_\infty x)(\partial^2\theta/\partial\eta^2)\} = 0 \qquad 2.21$$

where $\theta(\eta, x) = \{(T - T_\infty)/(T_w - T_\infty)\}$,

and $\theta_r = \{(T_r - T_\infty)/(T_w - T_\infty)\}$ 2.22

We define θ_r as Viscosity parameter for temperature variation or viscosity-temperature coefficient.

Using 2.18 & 2.19 in 2.17, the boundary conditions are now

$(\partial F/\partial\eta) = 1 \quad F(\eta, x) = 0 \quad \theta(\eta, x) = 1 \quad \text{at} \quad \eta = 0$ 2.23

$(\partial F/\partial\eta) \to 0 \quad \theta(\eta, x) \to 0 \quad \text{as} \quad \eta \to \infty$ 2.24

In order to define the different degrees of magnetic interaction, on velocity field and the temperature, we express the velocity factor $F(\eta, x)$ and temperature factor $\theta(\eta, x)$ in ascending powers of m, where m is the magnetic field parameter representing the strength of applied magnetic field, so that the following expansions for $F(\eta, x)$ and $\theta(\eta, x)$ are assumed :

$F(\eta, x) = \{ f_0(\eta) + (mx) f_2(\eta) + (mx)^2 f_4(\eta) + \text{-------} \}$ 2.25

$\theta(\eta, x) = \{ (\theta_0(\eta) + (mx)\theta_2(\eta) + (mx)^2\theta_4(\eta) + \text{------} \}$ 2.26

These expansions are valid for small values of magnetic parameter (m), which show the degree of magnetic field interactions to the flow and the temperature of the fluid. The first term of these expansions (i.e., the coefficients of lowest power of m) express the absence of the magnetic field while the terms containing m (i.e., the coefficients of second lowest power of m) first degree interaction of magnetic field on velocity and temperature. The terms for higher orders of m are the magnetic field interactions due to other physical parameters which affect flow and the temperature of the system. The magnitudes of these terms are very small and we are neglecting here.

Substituting the expansions 2.25 & 2.26 and equating the coefficients of like powers of m on both side of equations 2.20 and 2.21, we have different set of non-linear equations according to the degree of magnetic parameter (m) as given below .

System (I) : *(in absence of magnetic field action)*

The first pair of equations which is independent of m , gives the velocity and temperature distribution in absence of magnetic field . These equations are

$$f_0'''(\eta) - \{(\theta_0 - \theta_r)/(2\theta_r)\} f_0(\eta) f_0''(\eta) - \{1/(\theta_0(\eta) - \theta_r)\} f_0''(\eta)' = 0 \qquad 2.27$$

$$\theta_0''' (\eta) - (Pr/2) f_0(\eta) \theta_0'(\eta) = 0 \qquad 2.28$$

where $Pr = \nu / \alpha$, *Prandtl number*

Here the prime denotes differentiation with respect to η.

The corresponding boundary conditions are :

$$\begin{aligned} f_0(\eta) &= 0,\ f_0' = 1, \text{ at } \eta = 0 \\ \theta_0(\eta) &= 1 , \text{ at } \eta = 0 \end{aligned} \qquad 2.29$$

System (II): *(effect of magnetic field i.e., the first degree of magnetic field interaction)*

The second pair of equations for the first degree of magnetic interaction are

$$f_2'''(\eta) + \{\theta_2(\eta)/(\theta_0(\eta) - \theta_r)\} f_0'''(\eta) + \{(\theta_0(\eta) - \theta_r)/\theta_0(\eta)\} \{f_0'(\eta) f_2'(\eta)$$

$$-(1/2) f_0(\eta) f_2''(\eta) - (3/2) f_2(\eta) f_0''(\eta) + f_0'(\eta)\} - \{\theta_2(\eta)/\theta_r\} f_0(\eta) f_0''(\eta)$$

$$- \{1/(\theta_0(\eta) - \theta_r)\} \{\theta_0'(\eta) f_2''(\eta) + \theta_2'(\eta) f_0''(\eta)\} = 0 \qquad 2.30$$

$$(1 / Pr)\, \theta_2''(\eta) + 3/2\, \{\theta_0'(\eta)\, f_2(\eta)\} + 1/2\, \{\theta_2'(\eta) f_0(\eta)\} - f_0'(\eta)\theta_2(\eta) = 0 \qquad 2.31$$

where the prime denotes differentiation with respect to η.

The corresponding boundary conditions are

$$f_2(\eta) = 0, \quad f_2' = 0 \quad \text{at} \quad \eta = 0$$
$$\theta_2(\eta) = 0, \qquad \text{at} \quad \eta = 0 \qquad 2.32$$

Skin Friction and Rate of Heat transfer :

The physical quantities of our interest in this problem are the Skin friction coefficient (C_f) and the Nusselt number (N_u). Using 1.95 , Skin friction which is proportional to the local velocity gradient is defined at the plate as:

$$\tau_w = \mu_w (\partial u / \partial y)_{y=0}. \qquad 2.33$$

substituting stream function given in 2.14 in 2.33 then using 2.18 & 2.19 the non-dimensional form of skin friction coefficient is written as

$$C_f = 2\, \tau_w / (\rho\, u_0^2) \qquad 2.34$$

Again using 1.79 the rate of heat transfer which is proportional to the local temperature gradient , given as

$$q_w = -k\, (\partial T / \partial y)_{y=0} \qquad 2.35$$

Using the relation 2.19 and 2.22 the non-dimensional form of rate of heat transfer in terms of the Nusselt number is written as

$$N_u = x\, q_w / (k\, (T_w - T_\infty)) \qquad 2.36$$

Using relations 2.5, 2.18, 2.19 and 2.22, C_f and N_u are written as

$$C_f = (Re)^{-1/2}\,[\,2\,\theta_r/(\theta_r - 1)\{(f_0'' + (mx)\,f_2'' + \text{-----}\}]$$

$$= [\,C_{f,1} + C_{f,2} + \text{-----}] \qquad 2.37$$

and

$$N_u = -\,(Re)^{1/2}\,\{\,\theta_0' + (mx)\,\theta_2' + \text{-------}\,\}$$

$$= [\,N_{u,1} + N_{u,2} + \text{----------}\,] \qquad 2.38$$

Here in 2.37 and 2.38, the coefficients of lowest power of m i.e., the terms $C_{f,1}$, $N_{u,1}$ are meant for coefficient Skin friction and the rate of heat transfer in absence of magnetic field respectively while the coefficients of next higher power of m i.e., the terms $C_{f,2}$ and $N_{u,2}$ are meant the same in the presence of the field.

2.4 SHOOTING METHOD FOR NUMERICAL SOLUTION OF SIMULTANEOUS DIFFERENTIAL EQUATIONS :

Shooting method for system of equations with two or more initial missing conditions is described below.

Let us consider a system of four equations in four unknowns :

$$p' = f_1(x, p, q, r, s) \qquad q' = f_2(x, p, q, r, s)$$
$$r' = f_3(x, p, q, r, s) \qquad s' = f_4(x, p, q, r, s) \qquad 2.39$$

with two conditions given at x = a (say)

$$p(a) = p_a \quad \text{and} \quad q(a) = q_a \qquad 2.40$$

and two conditions at x = b (say)

$$r(a) = r_b^{\cdot} \quad \text{and} \quad s(b) = s_b \qquad 2.41$$

To determine the missing initial conditions viz. r (a) and s(a) , let us assume α_0 , β_0 as the initial values of r & s at x = a respectively. With these assumptions, the values of r and s are obtained at x = b. Let the values be $r(\alpha_0, \beta_0, b)$ and $s(\alpha_0, \beta_0, b)$. Considering the correct initial values of r(a) and s(a) as α , β respectively, r and s at x = b are functions of α and β ; so $r(\alpha, \beta, b)$ and $s(\alpha, \beta, b)$ can be expanded in Taylor's series:

$$r(\alpha, \beta, b) = r(\alpha_0, \beta_0, b) + (\alpha - \alpha_0)\, \partial r / \partial \alpha\, (\alpha_0, \beta_0, b) + (\beta - \beta_0)\, \partial r / \partial \beta\, (\alpha_0, \beta, b)$$

$$2.42$$

$$s(\alpha, \beta, b) = s(\alpha_0, \beta_0, b) + (\alpha - \alpha_0)\, \partial s / \partial \alpha\, (\alpha_0, \beta_0, b) + (\beta - \beta_0)\, \partial s / \partial \beta\, (\alpha_0, \beta, b)$$

Now $r(\alpha, \beta, b)$ and $s(\alpha, \beta, b)$ may be set to their prescribed values r_b and s_b . To solve the equations 2.42 for corrections $\alpha - \alpha_0$ and $\beta - \beta_0$, we must obtain the partial derivatives 2.42. Since the functions r and s are not known, their derivatives cannot be found analytically. However approximate numerical values can be found for them . To do so , we would integrate equations 2.39 once with initial condition p_a, q_a , α_0 , β_0 and once with the condition p_a, q_a, $\alpha_0 + \Delta\alpha_0$, β_0 and then with p_a , q_a , α_0 , $\beta_0 + \Delta\beta_0$ where $\Delta\alpha_0$ and $\Delta\beta_0$ are small increments to α_0 and β_0 . Omitting the variables p_a , q_a which remains fixed, the difference quotients are formed as:

$$\partial r / \partial \alpha (\alpha_0 , \beta_0 , b) = 1/ \Delta\alpha_0 \; [r (\alpha_0 + \Delta\alpha_0 , \beta_0 , b) - r (\alpha_0 , \beta_0 , b)] \quad 2.43a$$

$$\partial r / \partial \beta (\alpha_0 , \beta_0 , b) = 1/ \Delta\beta_0 \; [r (\alpha_0 , \; \beta + \Delta\beta_0 , b) - r (\alpha_0 , \beta_0 , b)] \quad 2.43b$$

$$\partial s / \partial \alpha (\alpha_0 , \beta_0 , b) = 1/ \Delta\alpha_0 \; [s (\alpha_0 + \Delta\alpha_0 , \beta_0 , b) - s (\alpha_0 , \beta_0 , b)] \quad 2.43c$$

$$\partial s / \partial \beta (\alpha_0 , \beta_0 , b) = 1/ \Delta\beta_0 \; [s (\alpha_0 , \; \beta + \Delta\beta_0 , b) - s (\alpha_0 , \beta_0 , b)] \quad 2.43d$$

After replacing $r(\alpha , \beta , b)$ by r_b and $s(\alpha , \beta , b)$ by s_b , equations 2.42 can be solved for $\partial\alpha_0 = \alpha - \alpha_0$ and $\partial\beta_0 = \beta - \beta_0$ to obtain new estimates $\alpha_1 = \alpha_0 + \partial \alpha_0$ and $\beta_1 = \beta_0 + \partial \beta_0$ for the parameters α , β. The entire process is now repeated with p_a , q_a , α_1 , β_1 as initial conditions . The process is stopped when α_k , β_k for some k agrees with r_b , s_b respectively to desired degree of accuracy. If there are n missing conditions, each iteration will require (n + 1) integration of the original equation. Convergence in this case is not guaranteed unless very good initial approximations are available . These techniques can be applied to problems where the boundary conditions are of different nature.

2.5 RESULTS AND DISCUSSION :

The physical quantities of our interest are f_2 and θ_2 which are the factor representing first degree magnetic field interaction on velocity and temperature, and f_2'' and θ_2' which are the first degree magnetic interaction to the factors representing skin friction and the heat transfer (i.e., $C_{f,2}$ and $N_{u,2}$) respectively. Due to the complexities of the equations 2.27 & 2.28 and 2.30 & 2.31 of system I & II respectively, numerical solutions under the boundary conditions 2.29, are obtained using the Runga-Kutta method for simultaneous solutions of non-linear differential equations for two different values of Prandtl number (Pr = 0.71 & 10.0). To find the missing initial conditions of the equations of system (I & II), we have used Shooting method, as stated in above. The quantity θ_r may be called as viscosity parameter for temperature or viscosity-temperature coefficient . The variation of θ_r means the variation of fluid viscosity with respect to the fluid temperature, and our aim is to show the nature of fluid velocity and temperature in the presence of uniform magnetic field under the action of variable viscosity. Figures (i-iv) are plotted for f_2 and θ_2 against θ_r and f_2'' and θ_2'' against θ_r in figures (v & vi) . Further -ve values of viscosity parameter θ_r make $(T_w - T_\infty)$ -ve , and $(T_w - T_\infty)$ is always -ve for an incompressible fluid. Therefore, we have calculated f_2'' and θ_2'' for -ve values of θ_r varying from (-10.0 to -0.10) and are given in the tables (I & II).

Following are the results obtained from the figures and the tables:

(1)(i) Figures (i & ii) show the variation of f_2 with the increase of θ_r for the different values of η. It is observed that f_2 decreases slowly with the increase of θ_r (= -10.0 to -0.1) and f_2 is minimum at θ_r = -1.0, after which it increases when θ_r changes from = (-1.0 to 0.0). At constant θ_r , f_2 increases with the increase of η.

(ii) Figure (iii) shows variation of f_2 with the increase of η at constant θ_r. It is observed that when θ_r remains unchanged f_2 increases with the increases of η and almost vanishes for $\eta = 0$ (i.e. at the ground layer). For all values of η, the magnitude of f_2 decreases with the increase of θ_r

(2) Figure (iv) shows the variation of θ_2 with η at constant values of θ_r . It is seen that θ_2 rises from a minimum value ($\cong 0$) with the increase of η, attains maximum value and then gradually decreases to minimum .

(3) In figure (v) we have shown the variation of f_2'' which is the factor representing the skin friction at the plate with the viscosity-temperature coefficient θ_r , the figure represents that for negative θ_r with the increase of θ_r , f_2'' increases gradually but it decreases sharply as $\theta_r \cong -1$ to $\theta_r \cong 2$ and than again increases for the higher values of θ_r , for all values of Pr (= 0.71 & 10.0).

(ii) Figure (vi) shows the variation of θ_2' which is the factor representing the rate of heat transfer in terms of Nusslet number (Nu) with θ_r. As θ_r increases from -10 to -1.0 (approx.) it increases very slowly but for $\theta_r \cong -1$ to $\theta_r \cong 0$ (approx.) it increases sharply and then for $\theta_r \cong 0$ to $\theta_r \cong 2$ (approx.) it decreases sharply. For $\theta_r \cong 2$ to $\theta_r \cong 10$, θ_2' increases again slowly for all values of Pr (= 0.71 & 10.0).

(4) The tables (I) & (II) show the values of f_2'' and θ_2' which are the factors for skin friction and rate of heat transfer respectively, for Pr = 0.71 & 10.0 . It has been observed that f_2'' increases with the increase of θ_r ; on the other hand, θ_2' decreases for Pr = 0.71 and increases for Pr = 10.0 as θ_r changes from -10.0 to -0.1 . The variation in the values of θ_2' is negligibly small as Pr changes from 0.71 to 10.0 when θ_r is small ($\theta_r \cong$-10.0)

Chart 1

Fig. (i) , Variation of f_2 with θ_r for Pr = 0.71

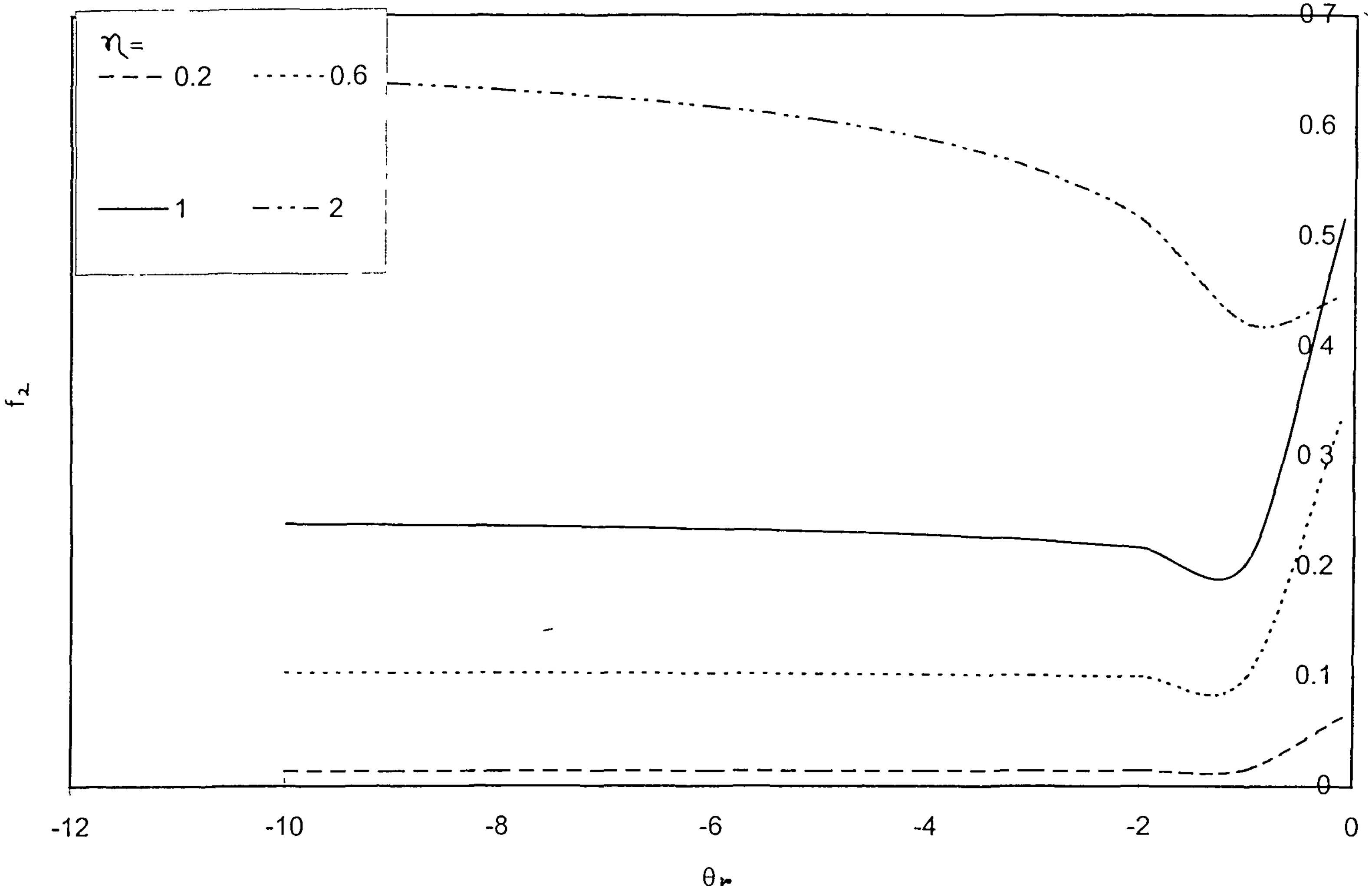


Chart 2

Fig. (ii) , Variation of f_2 with θ_m for $Pr = 10.0$

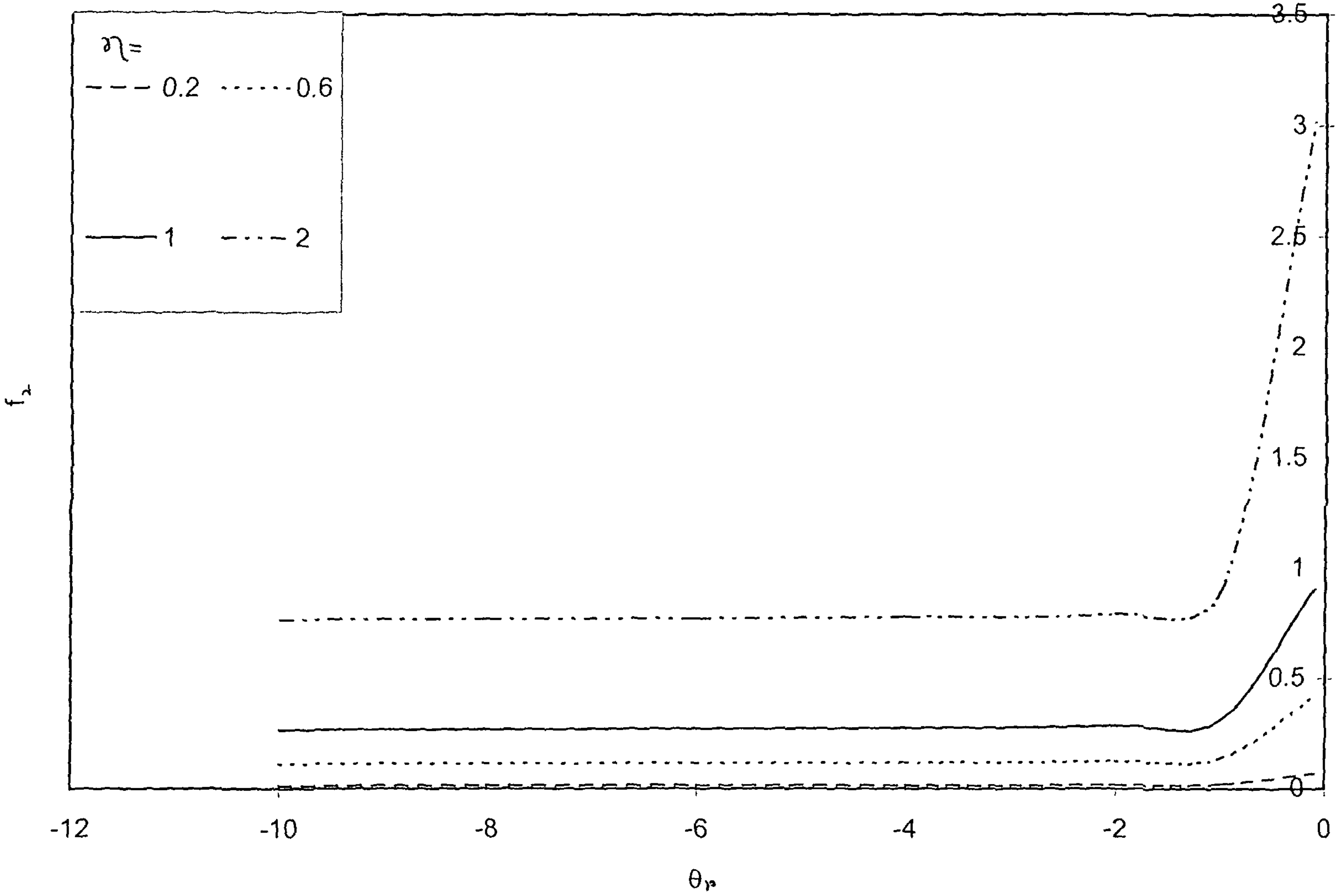


Chart 3

Fig. (iii) , Velocity distribution (f_2) for θ_r= (-1.0 , -5.0 , -10.0)

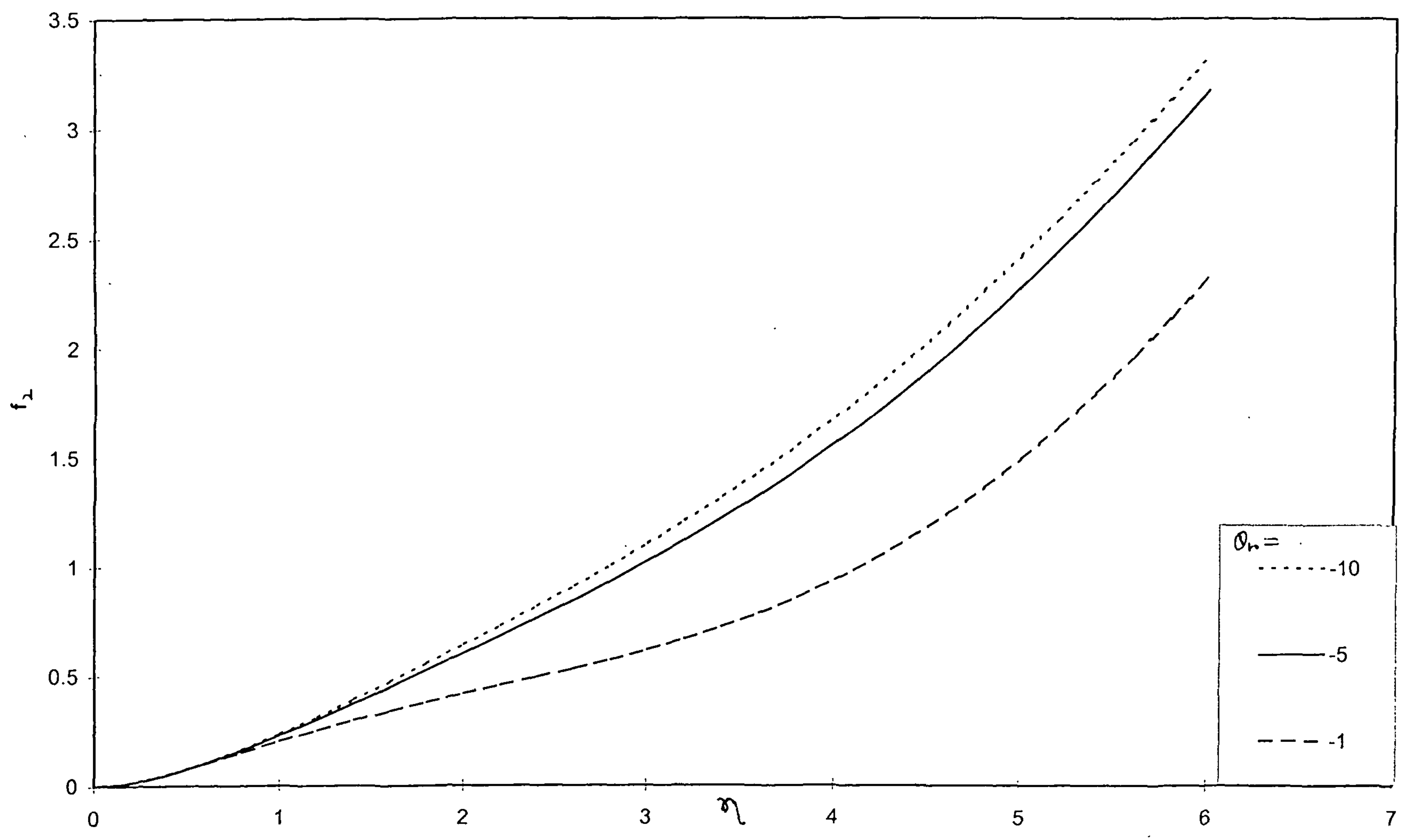
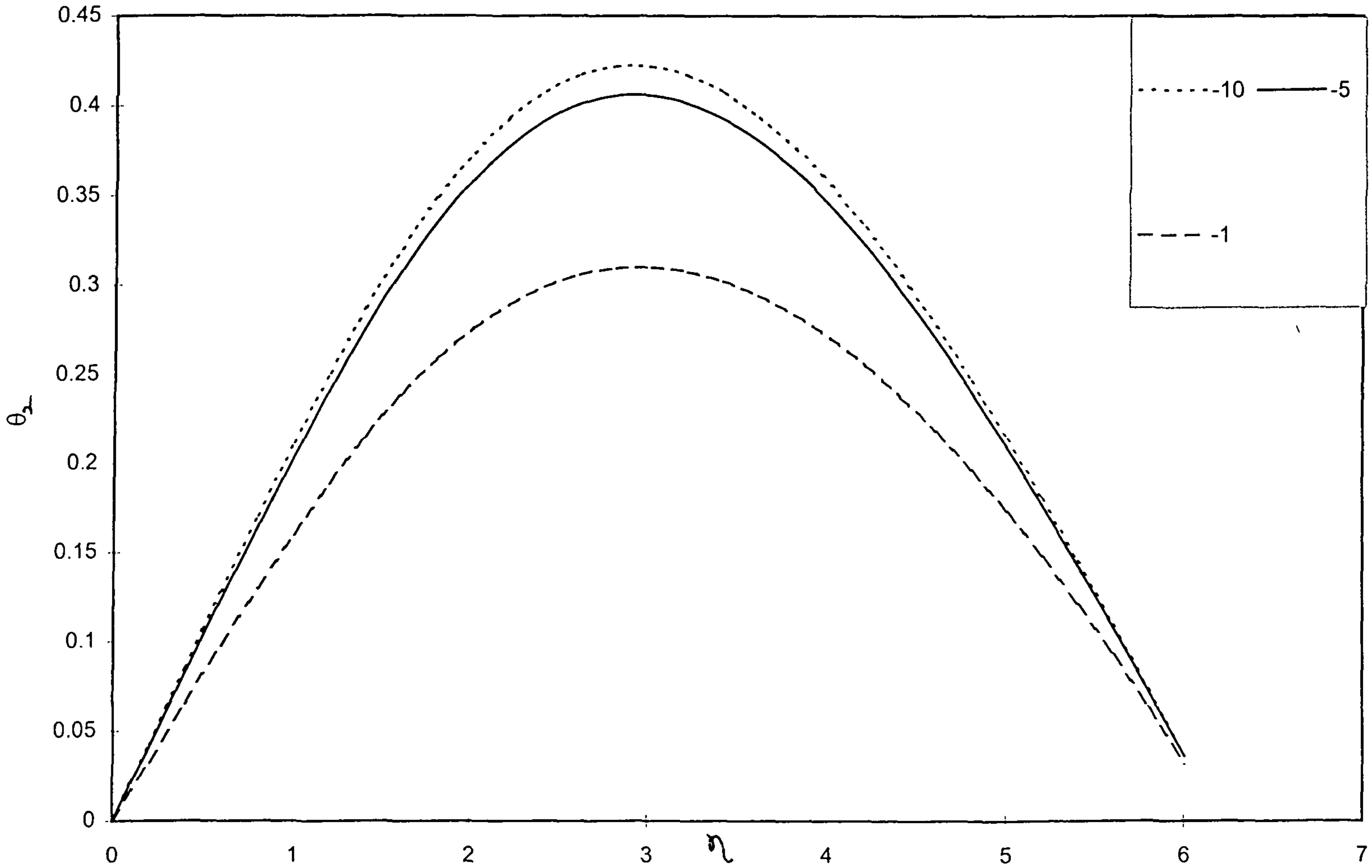


Chart 4

Fig. (iv), Variation of θ_2 with η for $\theta_r = (-10.0, -5.0, -1.0)$ & $Pr = 0.71$



2.6 CONCLUSIONS:

From the above discussions we can draw the following conclusions .

- For all -ve values of the viscosity parameter (i.e. for incompressible fluid) both the fluid velocity and temperature gradually decrease with the increase of viscosity parameter .
- The skin friction increases with the increase of viscosity parameter while the heat transfer decreases with the increase of viscosity parameter at small value of Prandlt number and increases at high value of Prandtl number. At small values of the viscosity parameter, the heat transfer is less dependent on Prandlt number .
- The magnitude of fluid velocities at the lowers layers at constant viscosity are comparatively small to those for variable viscosity parameter.
- The variation (increase or decrease)of magnitude of skin friction and heat transfer for the values of viscosity parameter within -1.0 to 2.0 (approx.) are more sharp. For the variation of values of viscosity parameter, magnitude of skin friction and the rate of heat transfer increase slowly for viscosity parameter varying from -10.0 to -1.0 (approx.), and from 2.0 to 10.0 (approx.).

Table (I)

Values of $f_2''(\eta)$

θ_r	Pr = 0.71	Pr = 10.0
-10.0	0.7679	0.8356
-8.0	0.7700	0.8472
-6.0	0.7737	0.8660
-4.0	0.7821	0.9021
-2.0	0.8132	0.9991
-1.0	0.8934	1.1819
-0.1	3.9084	4.5587

Table (II)

Values of $\theta_2'(\eta)$

θ_r	Pr = 0.71 ,	Pr = 10.0
-10.0	0.2120	0.2129
-8.0	0.2110	0.2151
-6.0	0.2069	0.2188
-4.0	0.2008	0.2259
-2.0	0.1852	0.2454
-1.0	0.1626	0.3020
-0.1	0.0385	0.8446

CHAPTER III

AN UNSTEADY FLOW DUE TO AN EXPONENTIALLY DECAY SOURCE BETWEEN TWO INFINITE PARALLEL DISKS IN PRESENCE OF A UNIFORM MAGNETIC FIELD

3.1 INTRODUCTION :

In a system of flow of viscous fluid, the inertia forces acting, are proportional to the square of the velocity, whereas the viscous forces are only proportional to its first power. Therefore, when the viscous forces are considerably greater than the inertia forces, the popularly used Navier-stokes equations (1.32) are limited under some approximations. It is clear that a flow, for which viscous forces are dominant, is obtained when the velocity is small . This kind of flow is common when the Reynolds number is very small ($Re = u\, d / \nu$ and $Re < 1$), where the inertia terms may be omitted from the equation of motion. The flow sometimes called creeping motion . Although this flow doesn't occur too often in practical applications but sometime occurs in nature ; for example in case of sphere falling in air , the Reynolds number may be smaller than one.

The oldest known problem on creeping motion was studied by Stokes (1851), who investigated the case of parallel flow past a sphere. Later on Prandtl (1935) has discussed the motion in details. An improvement of Stoke's idea was given by Oseen (1910) who took the inertia terms in the Navier-Stokes equation (1.32) partly, into account . He assumed that the velocity components can be represented as the sum of constants and perturbation term i.e., $u = u_\infty + u'$, $v = v'$, $w = w'$; where u' , v' , w' are the perturbation terms which are small with respect to the free stream velocity u_∞ . Oseen's theory was successful up to $Re = 5.0$ (approx.) . Stroke's problem on a sliding surface with respect to the direction of motion was carried out by Reynolds (1886). He also considered the problem of motion between two parallel flat wall with a pressure gradient. He showed that the inertia forces can be neglected with respect to the viscous forces if the reduced Reynolds number Re* $= (u\ d / \nu) (h / d)^2 \} < 1$. The occurrence of high pressure in slow viscous motion is a peculiar property of the type of flow which generally encounter in lubrication. Froessel (1942) calculated the pressure distribution and thrust supported by a slipper of finite width as well as by a spherical and confirmed these calculations by experiments. In many cases when the width of the slipper is finite the assumption made earlier that the flow is one dimensional, is insufficient and the existence of a component w in the z-direction must be taken into account. The relevant theory has on exact two-dimensional theory was developed in great detail by Sommerfield (1904), Guembel (1925) and Vogelpohi (1949). It has also been extended by Bauer (1943) and Michael (1905). Most theoretical calculations have been conducted under the assumptions of constant viscosity. In reality heat is evolved through friction and the temperature of the fluid (e.g., lubricating oil) is increased. Since the viscosity of oil decreases rapidly with the increasing temperature (see also Chapter II), the viscous thrust also decreases. In recent time Nahme (1940) extended the hydrodynamic theory of lubrications to include the effect of the variation of viscosity with the temperature.

This kind of problems where Reynolds number is small enough (Re < 1) has been studied by many authors and researchers. The incompressible radial flow between two parallel stationary disks using the integral approach and the assumption of a parabolic velocity profile was discussed by Livesey (1962). Savage (1964) has obtained the solution by expanding velocity components and pressure in terms of the downstream coordinate , by omitting the no slip condition on the disk. Similar problems were discussed by Peube (1963) , Chen and Peube (1964), Gieger et al. (1964). Elkouh (1975) has given an analysis for a system in which the flow rate varies sinusoidal about a zero mean value. His solution is valid for small values of the reduced Reynolds number and all values of the frequency Reynolds number.

Among the many practical applications , one of the important application is the phenomena which takes place in oil lubricated bearing . At high velocities the clearance between two machine elements which are in relative motion is filled by an oil stream in which extremely large pressure differences are created. Another example of this type of motion is the slide block or slipper moving on a plane guide surface . Another remarkable type of slow motion or Creeping motion is the Hele-Show flow. If a cylindrical body of arbitrary cross-section is inserted between the two plates at right angles so that it completely fills the space between them , the resulting pattern of streamlines is identical with that in potential flow about the same shape. Hele-Show (1898), used this method to obtain experimental patterns of stream lines in potential flow about arbitrary bodies. He has proved that the solution for creeping motion posses the same streamlines as the corresponding potential flow.

Gourla and Mehta (1994), studied for laminar flow due to an exponential source between two parallel stationary infinite disks . They obtained solutions for the motion of liquid in form of an infinite series expanded in terms of reduced Reynolds number Re*.

Their results are valid for small values of Re*. They discussed the significance of convective inertia over the viscous motion and concluded that the effects of non linear-inertia is significant over the motion for small values of the decay factor of the source, while for higher values of the decay factor the effect is less significant.

All these above mentioned problems are limited to the creeping motion and are inherently restricted to very small values of Reynolds number. In principle it is possible to extend the field of application to larger Re by successive approximation. In these cases the calculations are so complicated that it is not practically possible to carry out more than a few steps in the approximation. To these situations of region for which Reynolds number is moderate where inertial and viscous forces are comparable in magnitude throughout the field of flow, has not been investigated by the mathematician.

The motion of a viscous incompressible fluid when the effects of inertia are insensible can be treated in a very general manner in terms of harmonic functions {Lamb(1932)}. In this chapter, we have discussed laminar flow of a viscous incompressible fluid due to an exponentially decay source between two parallel stationery disks in presence of uniform magnetic field. The source we mean a simple source (Lamb , 1932), a point from which fluid is imagined to flow out uniformly in all directions . The total flux outwards across a small closed surface surrounding the point be Q called strength of the source. We have considered the source whose flux (Q) decays with time exponentially. Assuming creeping motion between the parallel infinite disks, solutions are obtained for small values of reduced Reynolds number and large values of r (distance from the source line) which give the effects of linear and non-linear convective inertia on the flow and the pressure under the action of a uniform magnetic field applied transversely to the direction of flow . Considering cylindrical coordinate, the distribution of radial velocities at different Re* are shown graphically for different values of magnetic Hartmann number. The results obtained are meant for simultaneous effect of inertia and magnetic field on fluid velocity, skin friction and the pressure, whose variations with respect to the magnetic field and decay factor of the source are presented graphically .The results observed here are in some cases significantly different from those in absence of the magnetic field {Gourla and Mehta (1994)}, which in turn provides an understanding of the effects of decay factor and reduced Reynolds number on a viscous incompressible flow between two parallel disks under uniform transverse magnetic field.

3.2 FORMULATION OF THE PROBLEM :

We have considered an unsteady axially symmetric flow of a viscous incompressible fluid between two parallel stationary infinite disks . A cylindrical polar coordinate system is considered such that the disks are situated at $z = \pm h$. The line source of the fluid is situated on the z axis at $r = 0$ whose strength varies according to

$$Q(t) = Q_0 e^{-nt} \tag{3.1}$$

A transverse magnetic field B_0 is imposed perpendicular to the disks. u and v are the velocity components along radial and z directions respectively. The uniform magnetic field of strength $\mathbf{B_0}$ is applied along z axis. In order to derive the governing equations of the problem the following assumptions are made.

(i) The fluid is finitely conducting and the viscous dissipation and the Joule heat are neglected.

(ii) Hall effect and polarization effect are neglected.

(iii) The value of magnetic Reynolds number is so small that the effect of induced magnetic field is negligible.

Imposing axial symmetry , and using Curl E = 0 , from the Maxwell's equation, we get $E_\theta = 0$. everywhere . Therefore, the magnetic body force using 1.29 and omitting the electric part is written as

$$f = J \times B \qquad 3.2$$

$$\text{where} \quad J = \sigma (V \times B) \qquad 3.3$$

this gives $J_r = 0$, $J_\theta = -\sigma (B_z u)$, $J_z = \sigma (B_z u)$ 3.4

and hence from 3.1,

$$f_r = - (\sigma B_z^2 u) = (\sigma B_0^2 u), \quad f_\theta = 0 , \quad f_z = 0 \qquad 3.5$$

Governing equations :

The equation of continuity 1.28 and Navier-Stokes equation 1.32 in cylindrical polar coordinate r , θ , z for this problem are

$$\partial u/\partial r + (u / r) + \partial v/\partial z = 0 \qquad 3.6$$

$$\rho\{ (\partial u/\partial t) + u (\partial u/\partial r) + v(\partial u/\partial z) \} = - (\partial p/\partial r) + \mu ((\partial^2 u/\partial r^2) + (1/r)(\partial u/\partial r) - u/r^2 + (\partial^2 u/\partial z^2)) - (\sigma u B_0^2) \quad 3.7$$

$$\rho ((\partial v/\partial t) + u (\partial v/\partial r) + v(\partial v/\partial z)) = - (\partial p/\partial z) + \mu ((\partial^2 v /\partial r^2) + (1/r)(\partial v/\partial r) + (\partial^2 v/\partial z^2)) \quad 3.8$$

The boundary conditions of the problem are

$$u = 0 \ , \ v = 0 \ , \text{ at } \quad z = \pm h \quad 3.9$$

$$\int_{-h}^{+h} 2\pi r \, (udz) = Q_0 \, e^{-nt} \quad 3.10$$

where n is the decay factor of the source .

Introducing the following non-dimensional quantities

$$r' = (r / h) \ , z' = (z / h) , \quad v' = (h v / \nu) , \quad u' = (h u / \nu) ,$$
$$p' = \{(1/ \rho) (h / \nu)^2 p\}, \ n' = \{(h^2 / \nu) n\} \ , \ t' = \{(\nu / h^2) t\} \quad 3.11$$

Substituting 3.11 in equations (3.6- 3.8) and then removing the primes , we get

$$\partial u/\partial r + (u / r) + \partial v/\partial z = 0 \quad 3.12$$

$$\partial u/\partial t + (u\partial u/\partial r) + v(\partial u/\partial z) - \partial^2 u/\partial r^2 - (1/r) \partial u/\partial r + (u / r^2) - \partial^2 u/\partial z^2 + (\partial p/\partial r + M^2 u) = 0 \quad 3.13$$

$$\partial v/\partial t + (u\partial v/\partial r) + v(\partial v/\partial z) - (\partial^2 v / \partial r^2) - (1/ r) \partial v/\partial r - \partial^2 v/\partial z^2 + (\partial p/\partial z) = 0 \quad 3.14$$

where $\{ h^2 B_0^2 \sigma / (\rho\nu) \} = M$ is the magnetic field parameter known as Magnetic Hartmann number .

Using 3.11 in 3.9 & 3.10 , the corresponding boundary conditions are

$$u = 0 \ , \ v = 0 \ , \quad \text{at} \quad z = \pm 1 \qquad 3.15$$

$$\int_{-1}^{+1} (udz) = 2 \, (Re / r) \, e^{-nt} \qquad 3.16$$

where $\{Q_0 / (4\pi \nu h)\} = Re$

Re, which is the Reynolds number of the fluid motion also means the controlling factor of the source .

3.3 SOLUTION OF THE EQUATIONS :

In order solve the equations 3.12 -3.14 under the boundary conditions 3.15 & 3.16, we consider the following expansions :

$$u = (Re / r) [\ f_0'(z , t) + Re^* (f_1'(z , t)) + (Re^*)^2 (f_2'(z , t)) \ + \text{------} \ ] \qquad 3.17$$

$$v = [2 \, (Re^*)^2 \{f_1 (z , t) \} \ + 4 \, (Re^*)^3 \ \{ f_2 (z , t) \} \ + \text{------} \ ] \qquad 3.18$$

$$p = [\, K (z , t) + Re \ \{K_0 (z , t) \log (r) \} \ + (Re^*) \ \{ K_1 (z , t) \} + \text{------} \ ] \qquad 3.19$$

where $Re^* = (Re / r^2)$, reduced Reynolds number .

These expansions are valid for small values of Re^* and large values of r i.e.; at a large distance from the source line and satisfy the equation of continuity. The primes denote partial differentiation with respect to z only .

Using the expansions 3.17 - 3.19, corresponding boundary conditions in terms of $f(z,t)$ and $f'(z,t)$, are given as :

$$f_i(\pm 1, t) = 0 \qquad \text{where } i = 1, 2 \qquad 3.20$$

$$f_i'(\pm 1, t) = 0 \qquad \text{where } i = 0, 1, 2 \qquad 3.21$$

and $$f_0(1, t) - f_0(-1, t) = 2e^{-nt} \qquad 3.22$$

Now for streamline flow , we can consider 3.22 as :

$$f_0(1, t) = e^{-nt} \qquad 3.23$$

and $$f_0(-1, t) = -e^{-nt} \qquad 3.24$$

so that they satisfy 3.22 .

Using the expansions 3.17 & 3.18 and equating the coefficients of like powers of r , we have from equations 3.13 & 3.14 :

$$(\partial^3 f_0 / \partial z^3) - \partial^2 f_0 / (\partial z \partial t) + M^2 (\partial f_0/\partial z) = K_0(z, t) \qquad 3.25$$

$$(\partial k_0 / \partial z) = 0 \qquad 3.26$$

$$(\partial^3 f_1 / \partial z^3) - \{\partial^2 f_1 / (\partial z / \partial t)\} - M^2 (\partial f_1/\partial z) = -\{2k_1(z, t) + (\partial f_0/\partial z)^2\}$$

3.27

$$(\partial k_1 / \partial z) = 0 \qquad 3.28$$

Also, $(\partial k / \partial z) = 0$;

This implies that $k(z, t) = k(t)$

The relations 3.23 & 3.24 suggest that :

$$f_i(z, t) = C_i(z)\, e^{-(i+1)nt} \qquad 3.29$$

and $$k_i = P_i\, e^{-(i+1)nt} \qquad 3.30$$

where $i = 0, 1$

Substituting 3.29 and 3.30 , equations 3.25 & 3.26 reduce to

$$C_0'''(z) + \alpha C_0'(z) = P_0 \qquad 3.31$$

and $$C_1'''(z) + \beta C_1'(z) = -\{2P_1 + (f_0'(z))^2\} \qquad 3.32$$

where $$\alpha = (n - M^2) \quad \text{and} \quad \beta = (2n - M^2) \qquad 3.33$$

The corresponding boundary conditions of the equations 3.31 & 3.32, from 3.20 & 3.21 are given as :

$$C_0(\pm) = \pm 1 \quad , \quad C_0'(\pm 1) = 0 \qquad 3.34$$

$$C_1(\pm 1) = 0 \quad , \quad C_1'(\pm 1) = 0 \qquad 3.35$$

and the solutions of equations 3.25 - 3.28, subject to the boundary conditions 3.34 & 3.35 , are obtained as :

$$f_0(z, t) = \{(z\alpha \cos\sqrt{\alpha} - \sin\sqrt{\alpha}z)/A\} e^{-nt} \qquad 3.36$$

$$K_0(t) = \{(\alpha^{3/2} \cos\sqrt{\alpha})/A\} e^{-nt} \qquad 3.37$$

$$f_1(z, t) = [G \sin(\sqrt{\beta}z) - (2P_1/\beta)z - (\alpha/(2A^2\beta))(2\cos^2\sqrt{\alpha} + 1)z$$
$$+ (\sin 2\sqrt{\alpha}z)/(4\sqrt{\alpha}(\beta - 4\alpha)) - (2\cos\sqrt{\alpha}\sin\sqrt{\alpha}z)/(\sqrt{\alpha}(\beta - \alpha))]\, e^{-2nt}$$

3.38

$$k_1(t) = [\beta/(2A^2)(1/B)\{(3(5\alpha - \beta)\sqrt{\alpha}\sqrt{\beta}\cos\sqrt{\beta}\sin 2\sqrt{\alpha})/(4(\beta - 4\alpha)(\beta - \alpha))$$
$$- (\cos 2\sqrt{\alpha}/(2(\beta - 4\alpha)) - 2\cos^2\sqrt{\alpha}/(\beta - \alpha))\,\alpha \sin\sqrt{\beta}\}$$
$$- \alpha/(4A^2)(2\cos^2\sqrt{\alpha} + 1)]e^{-2nt} \qquad 3.39$$

where $B = (\sin\sqrt{\beta} - \sqrt{\beta}\cos\sqrt{\beta})$, $A = (\sqrt{\alpha}\cos\sqrt{\alpha} - \sin\sqrt{\alpha})$ 3.40

$G = 1/\sin\sqrt{\beta}\ \{\alpha / (2\beta A^2)(2\cos^2\sqrt{\alpha} + 1) + \sqrt{\alpha}/A^2 (3(5\alpha$

$-3)\sin 2\sqrt{\alpha}) / (4(\beta - 4\alpha)(\beta - \alpha)) + 2P_1/\beta\}$ 3.41

3.4 RESULTS AND DISCUSSION :

We define the Radial velocity (u*) and pressure (p*) as

$u^* = [f_0'(z,t) + Re^* \{f_1'(z,t)\} + (Re^*)^2\{f_2'(z,t)\} + \cdots]$ 3.42

$p^* = [K_0(t)\log(r) + (Re^*) K_1(t) + \cdots]$ 3.43

where

$u^* = u(r/Re)$ and $p^* = \{p(r,t) - p(R,t)\}/Re$ 3.44

Here we assume that { p (R , t) } is a known pressure at some cross-section in the flow domain at r = R

The shear stresses (i.e. the Skin friction) at the disks are defined as

$$\tau = -\mu(\partial u/\partial z)_{z=\pm h} \qquad 3.45$$

Substituting the non-dimensional parameters given in 3.11, the shear stresses are given by

$$\tau^* = -(f_0''(\pm 1, t) + Re^*(f_1''(\pm 1, t)) + (Re^*)^2(f_2''(\pm 1, t)) + \cdots)$$

3.46

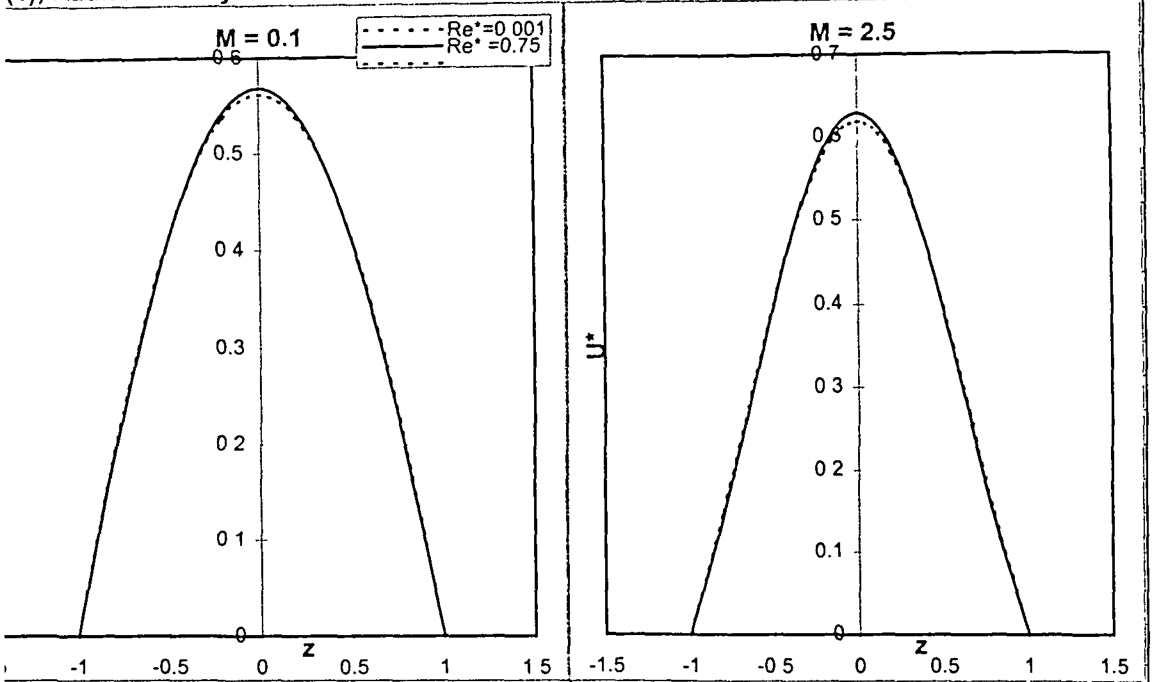
where $\tau^* = [\, \tau \,/\, \{ (\mu Q_0) \,/\, (4\pi\, r\, h^3) \}_{z=\pm 1} \,]$ 3.47

From the relations 3.17- 3.19, it is clear that we have only linear effect of convective inertia on the flow and the pressure at $Re^* \cong 0$, while the non-linear effects of convective inertia are observed for finite values of Re^* . Therefore, we have calculated u^* , τ^* and p^* for the values of reduced Reynolds number Re^* = 0.001 to 0.75 . Their variations with magnetic field parameter (M = 0.0 to 4.0) and the decay factor of the source (n = 0.0 to 10.0) are shown graphically in the figures (1-7) . The results obtained here are in presence of uniform transverse magnetic field which we in some cases significantly different from the results obtained by Gourla et al. (1994) in absence of the field. As the perturbation technique is used for small values of Re^* (= Re / r^2), therefore the results are valid for at a large distance from the source from z-axis while M = 0 in the ordinate axis means in absence of the magnetic field. Following are the observations drawn from the figs. (1-7).

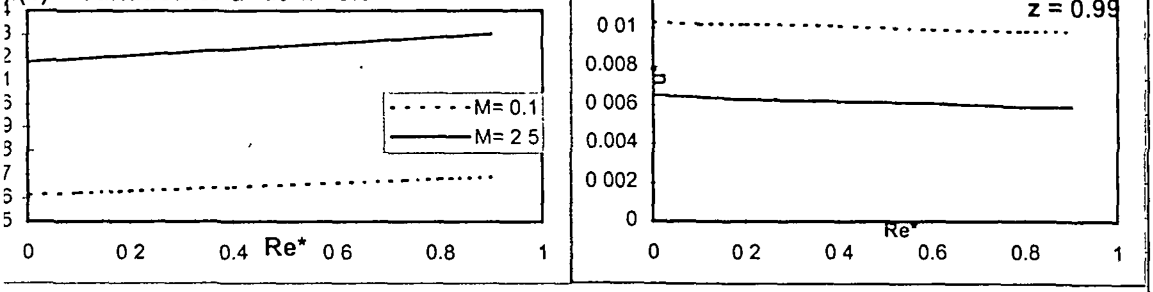
Radial velocity (u^*) :

Effect of Re^*: With the increase of Re^*, radial velocity u^* (i) increases near the middle of the channel{i.e. $z \to 0$ in figs. (1&2)}, and (ii) decreases near the disks ($z \to \pm 1$; figs.1& 2)} at constant M and n. Results are similar in nature to those of Gourla (1994), in absence of the field.

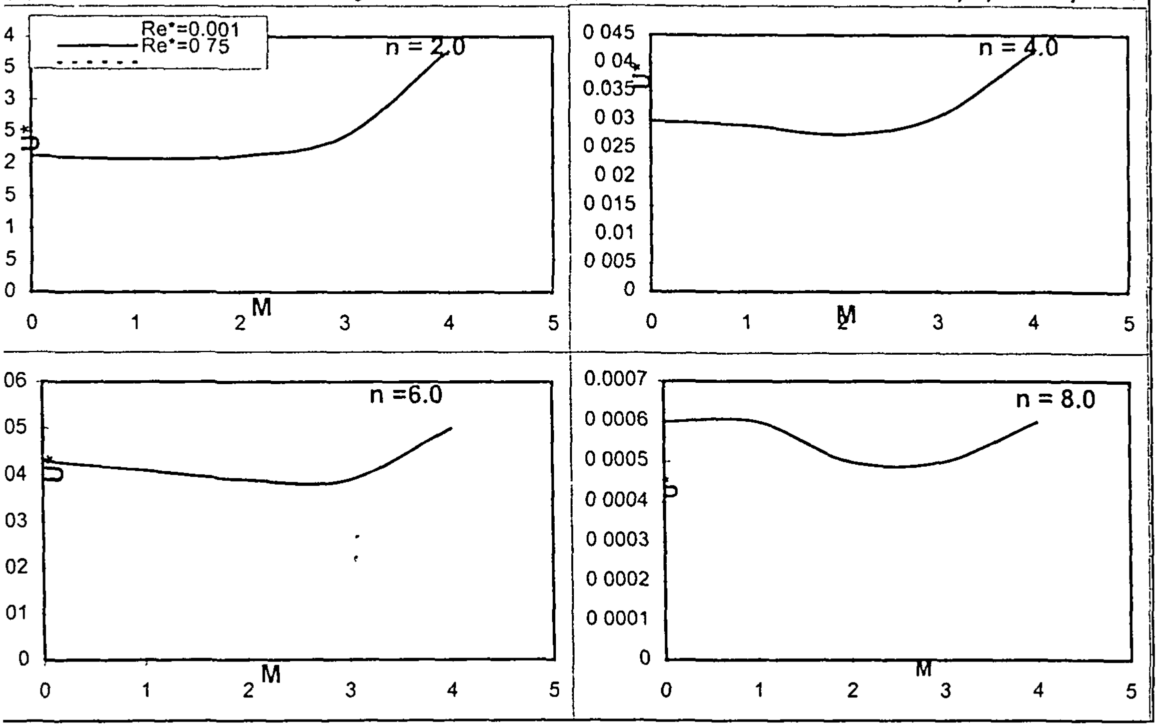
(1), Radial velocity distribution for Re* = 0.001 & 0.75 at M = 0.1 & 2.5 , n =1.0 , t = 1.0 :



(2) Variation of u* at z =0.0



(3),Variation of radial velocity with M at z= 0.0, for Re*= 0.001 & 0.75 at n = 2, 4, 6 & 8 ; t=1.



Fig(4), Variation of skin friction with M at z = 1.0, for Re* = 0.001 & 0.75 and t = 1.0 n=1,

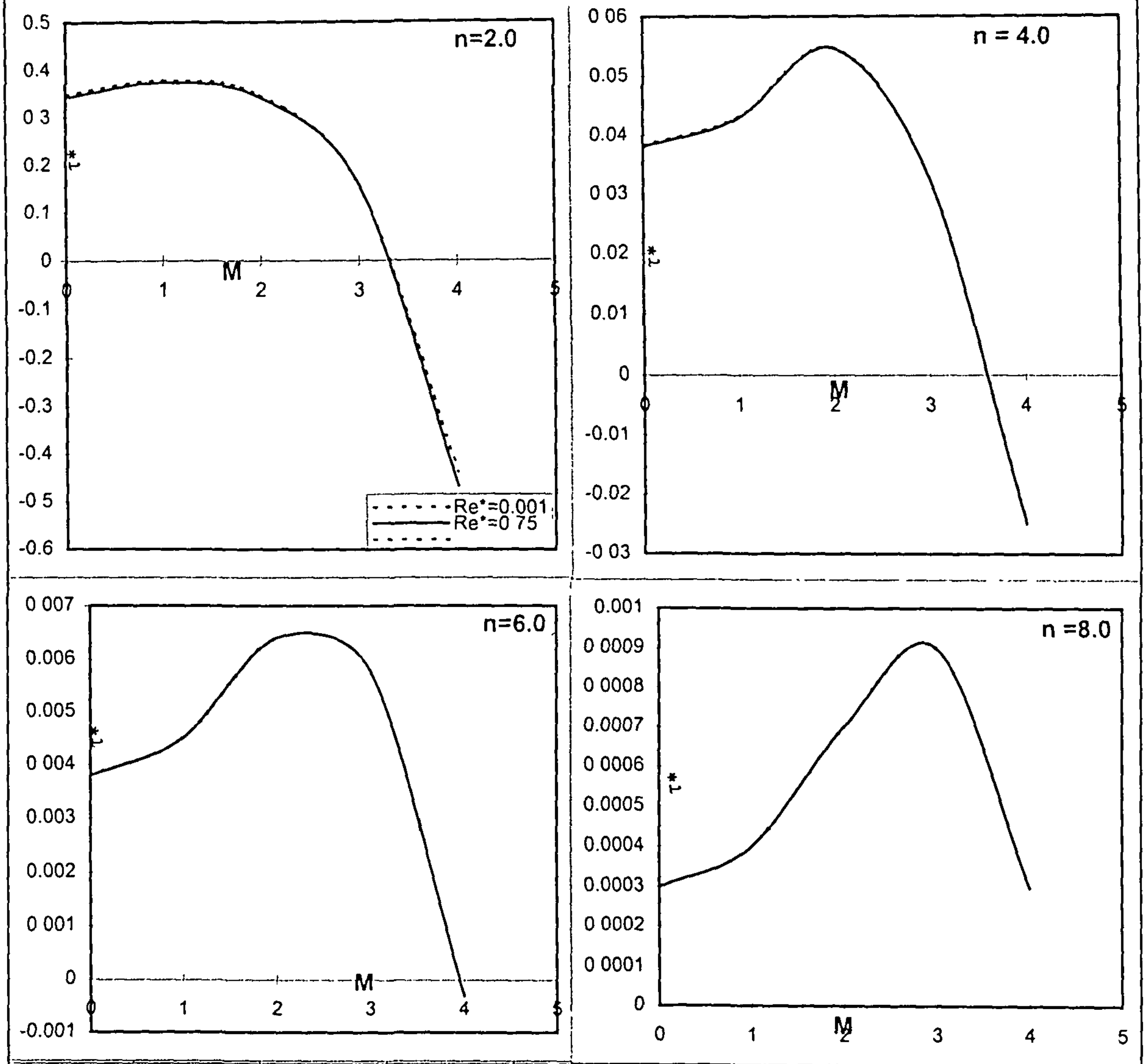
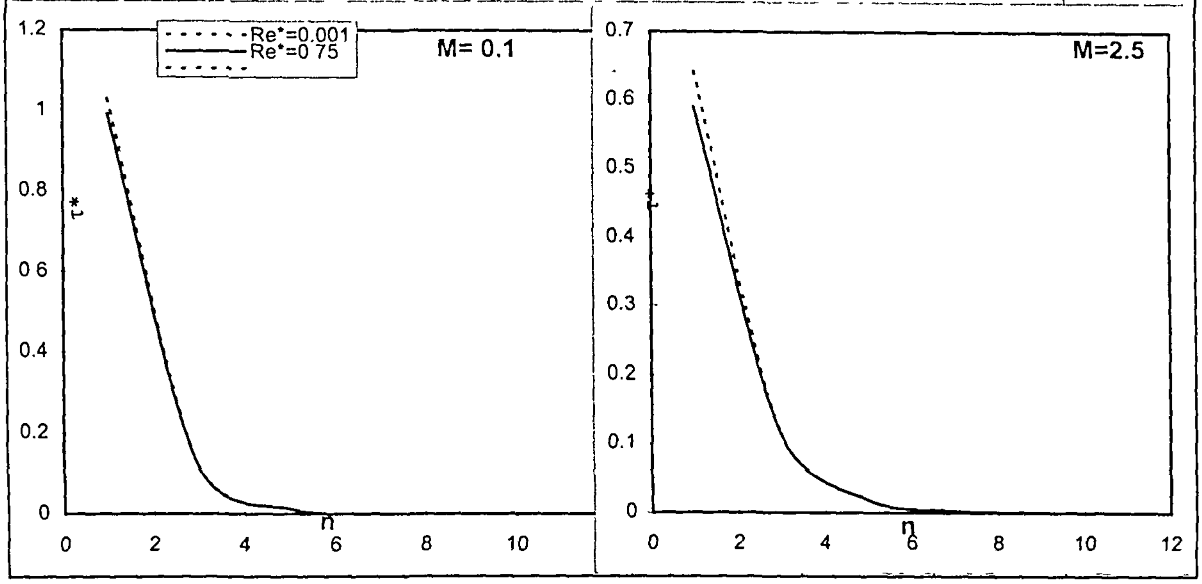


Fig (5),Variation of skin friction with n at z = 1.0 , for Re* =0.001 & 0.75; t =1.0



Sheet 3.

Fig (6),Variation of Pressure with M at z = 0.0, for Re* =0.001 & 0.75, r = 5.0 and t = 1.0

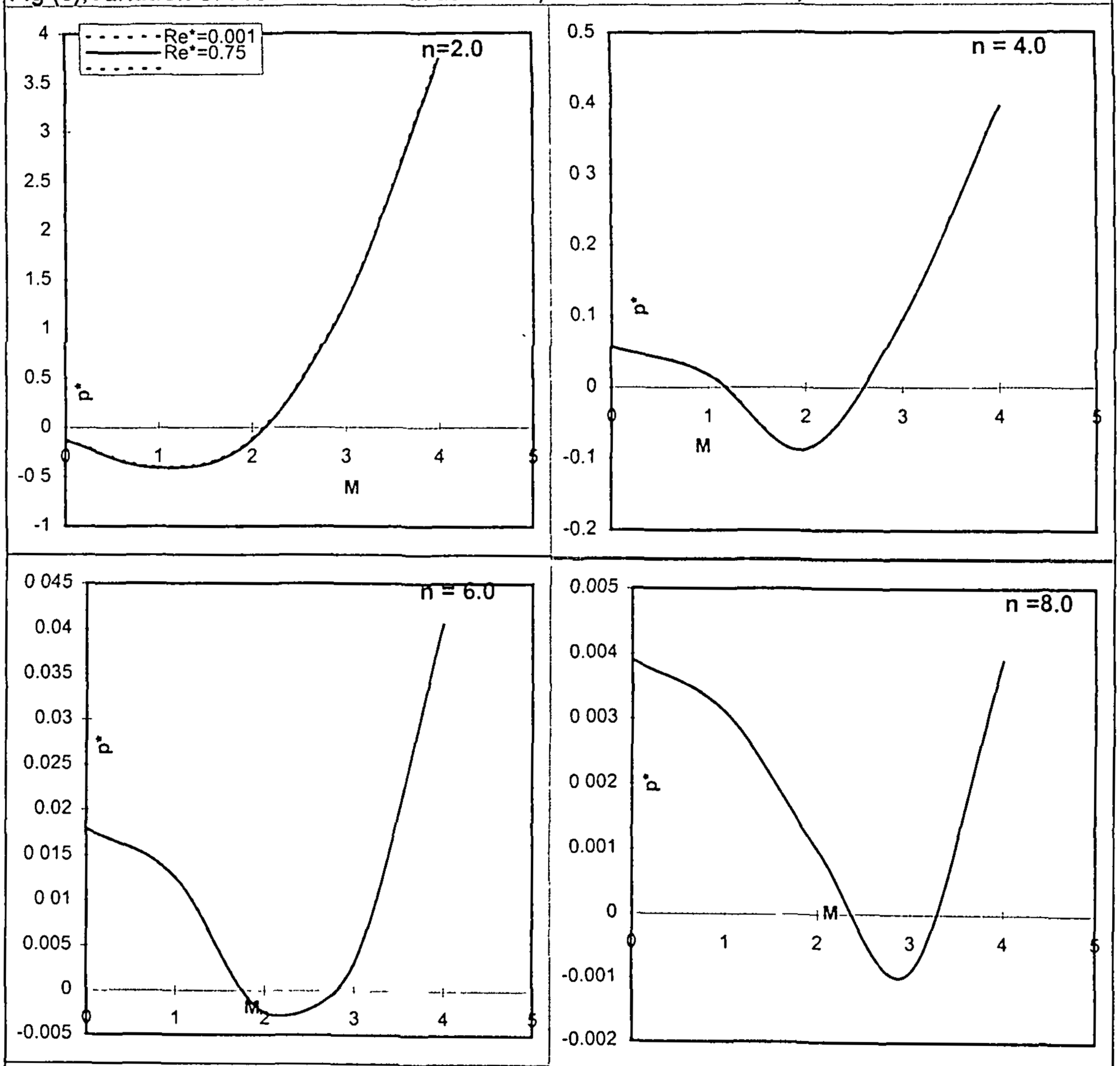
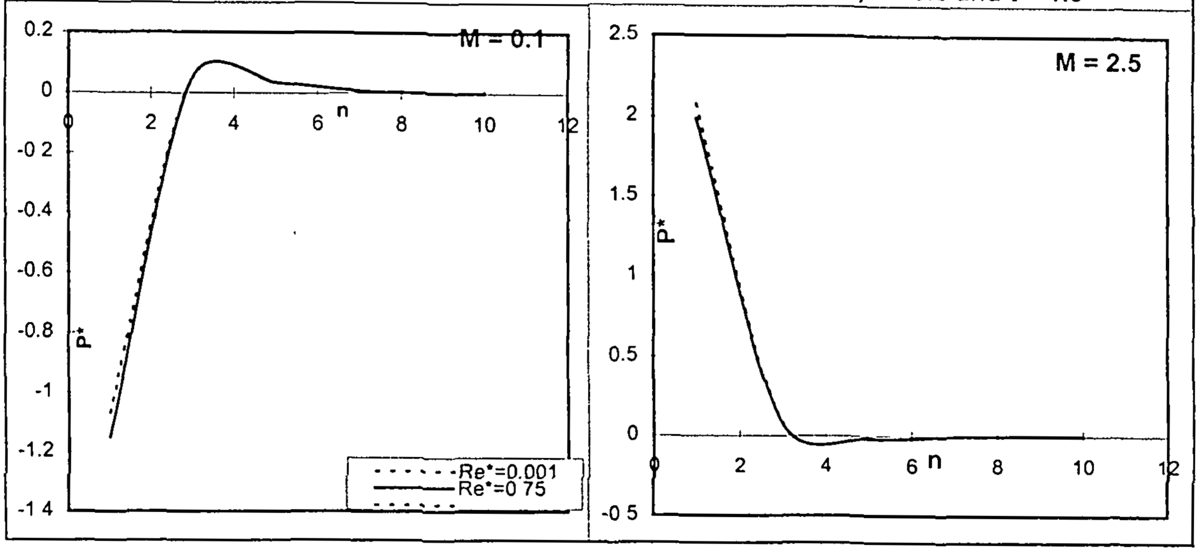


Fig (7),Variation of Pressure with n at z = 0.0 for Re* =0.001 & 0.75 ; r = 5.0 and t = 1.0



Effect of M : For constant n and Re* , u* first decreases slowly with the increase of M, then increases with the increase of M { figs.(3) }. The value of M from where u* increases also increases with the rise of n.

Effect of n : The variations of u* with n at constant values of Re* and M are almost opposite to those of Gourla (1994) in absence of the field . At constant Re* and M , u* decreases appreciably with the increase of n { figs. (3) }.

Skin friction (τ^*) :

(a) Results at the upper plate { z = + 1 , figs. (4 & 5) } :

Effect of Re* : The effect of Re* on τ^* is significant only for small values of n ($\leq$ 2.0) where τ^* decreases with the increase of Re* at constant values of n and M

Effect of M : At constant n & Re* with the increase of M , τ^* first increases slowly then decreases steadily. The value of M from where τ^* decreases also increases with the rise of n. At higher n the rate of increase of τ^* is high.

Effect of n : τ^* decreases with the increase of n at constant Re* and M .

The effects of Re* & n on τ^* are similar in nature to those of Gourla (1994).

(b) Results at the lower plate for τ^* at constant values of Re* , M and nt , are opposite to those obtained in (a).

Pressure (p^*) , {Figs. (6 &7) }:

Effect of Re^* : The effect of Re^* on p^* for all values of M & n, is negligibly small. whereas p^* decreases with the increase of Re^* for all values of M & n in Gourla (1994).

Effect of M : p^* first decreases slowly and then increases steadily with the increase of M at constant n for all values of Re^* . The value of M from where it increases also increases with the rise of n .

Effect of n : At lower value of M ($\cong 0.1$), p^* with the increase of n first increases then decreases slowly to minimum ($\cong 0.0$). At higher value of M ($\cong 2.5$), p^* decreases with the increase of n and becomes negligibly small ($\cong 0.0$) as $n \rightarrow 10.0$ (approx.) . On the other hand in Gourla (1994), p^* decreases appreciably with the increase of n , for all values of n & Re^*.

3.5 CONCLUSIONS :

- Under the uniform magnetic field, with the increase of reduced Reynolds number, the magnitude of radial velocity increases near the central line of the channel but decreases near the disks.
- The variation (increases / decrease) of radial velocity , skin friction and pressure with the increase of magnetic field, depends upon the decay factor .

- The skin friction varies inversely with the rise of decay factor and at high values of the decay factor (n ≅ 6.0 approx.), it almost vanishes.

- Effect of reduced Reynolds number on radial velocity and skin friction and pressure under uniform magnetic field are comparatively distinct for small values of decay factor (n ≤ 1.0 approx.) whereas it is clearly distinct for all values of n in Gourla (1994).

- With the increase of magnetic field, pressure decreases for smaller range of magnetic field , but increases for the higher range of it depending upon the decay factor .

- The effect of reduced Reynolds number on the velocity and skin friction are similar, whereas the effect of decay parameter are significantly different to those in Gourla (1994).

- The effect of reduced Reynolds number on the pressure are negligibly small while it is reasonable in Gourla (1994).

- Non-linear effects of convective inertia on all the physical quantities (radial velocity , skin friction and the pressure) become significant only when the decay factor (n) is small (n ≤ 1 approx.), whereas they are significant at high values of n in Gourla (1994).

CHAPTER IV

LAMINAR CONVECTION FLOW BETWEEN TWO HEATED PARALLEL PLATES IN PRESENCE OF A UNIFORM INCLINED MAGNETIC FIELD

4.1 INTRODUCTION :

The science of fluid motion, very often has to deal with the problem of heat and mass transfer between a solid body and the fluid flow. On the physical fluid motion, two fields interact, where the fluid flow superimposed by the heat flow. Generally speaking, the combined flow of mass motion, heat conduction and convection. If a solid body which is in touch with the fluid is heated so that its temperature is maintained above the surrounding fluid, the temperature of the stream will increase over the layers in the neighborhood of the body. The major part of the transition of temperature of the hot body to that of the colder surrounding, takes place in a thin layer in the immediate neighborhood of the body and over a narrow wake behind it. Thus the flow phenomena interacts with the thermal phenomena to a high degree. This is the reason why in a flow system energy balance for the fluid element in motion has to be considered. In an incompressible fluid, the energy balance is determined by the internal energy, conduction and convection of heat with the stream and the generation of heat through friction. In case of compressible fluid, there is an additional term due to the change of volume change. Although the heat radiation is always there, its contribution is very small at moderate temperature and therefore, we can neglect it in most of our practical problems.

By internal energy, we understand the energy associated with random translation and internal motions of the molecule plus the energy of interaction between the molecules. The internal energy depends on the local temperature and density of the fluid. The potential energy of the fluid as a whole does not appear explicitly. In case of an incompressible fluid the frictional heat plays an important role when the free stream velocity is so large that the adiabatic temperature increase is of the same order of magnitude as the temperature difference between the body and the stream. If this temperature difference is of the same order of magnitude as the absolute temperature of the free stream , Eckert number (E) becomes equivalent to the Mach number (Ma), which is , the case for a rocket at very high altitude. This means the work done due to fluid friction becomes important when the free stream velocity is comparable to that of sound, this occurs in the flight of rockets at very high altitudes. The temperature field, hence the coefficient of heat transfer depends on the Eckert number only when the temperature difference is large (50^0c to 100^0c or 100^0c to 200^0c), and when simultaneously the velocities are also very large (i.e., of the order of the velocity of sound). With moderate velocities , the temperature and velocity fields depends on the Eckert number when temperature differences are small (several degrees). Further, even with the moderate velocities, the buoyancy forces caused by temperature differences, are small compared with the inertia and friction forces. In such cases, the flow is less dependent on the Grashof number (Gr). Such flows are called forced flows. On the other hand if velocities of flow is very small particularly when the motion is caused by buoyancy forces, such as the stream rise along a heated vertical plate, the Grashof number (Gr) becomes important. Such flows are called natural convective flow. In such a flow Reynolds number (Re) is less important. The frictional term, commonly known as viscous dissipation arises due to the mechanical energy generates by the friction which is steadily degraded into thermal energy. It has the magnitude proportional to the local velocity gradient and is important only when $E \cong 1$. Forced flows can be subdivided into two groups with moderate and high velocities depending on whether the heat due to friction (viscous dissipation) need or need not be taken into account. In both cases the temperature field depends on the field of flow. At moderate velocity, when the heat due to friction is neglected , the dependence of the temperature field on the velocity is governed solely by the Prandtl number. At high velocity, work done due to friction must be included. In other words the work done due to friction must be taken into account when the temperature increase due to friction is comparable with the temperature difference between the body and the fluid. If this temperature difference is of the order the mean absolute temperature and flow velocity is comparable with that of sound, the work done due to friction becomes important.

When the hydrodynamic system is brought under a uniform magnetic field, if the fluid is conducting, a number of new phenomena arises. An e.m.f is generated and acts in to the fluid materials. The charges developed within the fluid material move under the action of this e.m.f, colliding and exchanging energy with the rest of the material. This means that electrical work is done on or by the material (i.e., the exchange of electrical energy between the material and the electromagnetic fields). It can be shown that the electromagnetic fields puts energy into the material at a rate of E.J per unit volume and time, where, $E.J = J^2/\sigma - J(V \times B)$, (see also equation 1.40). The first term is the ohmic dissipation and equivalent to I^2R as in electrical case. The second term is the electromechanical energy conversion and has importances in electrotechnology. The term $-J(V\times B) = V.(J\times B)$, is the rate at which the magnetic force $J \times B$ does work on the conductor as a whole. The action of this term is that it pushes the fluid either creating kinetic energy or helping to overcome the other forces or reverse if the term is negative. The term J^2/σ is the wasted part of E.J. The application of this electromechanical energy conversion was successfully demonstrated by William (1925).

Thus in a hydrodynamic system under the action of magnetic field (MHD), the electromechanical conversion commonly known as Joule heat should be considered in addition to the viscous dissipation.

The study of fluid flow problems taking into account of the simultaneous effects of hydromagnetic and coriollis forces is important because of their applications in many geophysical and astrophysical problems. It is generally accepted that the hydromagnetic flow in the earth's liquid core is responsible for the main geomagnetic field. In this chapter, we discuss the effect of heat transfer and magnetic field on the hydromagnetic flow with viscous dissipation and the Joule heat.

It is well known that the magnetohydrodynamic flow between two parallel plates in the presence of an applied transverse magnetic field is one of a few exact solutions of the equations of motion. This problem was first studied by Hartmann(1937). Subsequently, Schercliff (1953), Agarwal (1962), Soundalgekar (1967), Yen (1961), Srivastava (1958 & 1961), Stephenson (1967) have studied the effect of transverse magnetic field in the fluid flows under various geometries.

Gupta (A.S.) (1969) has studied the simultaneous effect of free and forced convection on the flow of an electrically conducting liquid under transverse magnetic field in a parallel plates channel subjected to a linear axial temperature variation. Gupta (P.S.) (1973) has discussed the effect of combined free and forced convection on the flow of a viscous liquid in a parallel plates channel rotating with a uniform angular velocity Ω about an axis perpendicular to the plates where the plates are subjected to a linear temperature variation. Following Chandrasekhar (1961), he has ignored the effect of density variations on the centrifugal force. Such assumptions are justified if the angular velocity of rotation is not too large. The steady flow in a parallel plates channel rotating with an angular velocity Ω and subjected to a constant transverse magnetic field has been analyzed by Nanda and Mohanty (1970). Vitazhin (1965) has investigated the hydromagnetic viscous compressible flow with Hall currents , past an infinite wall started impulsively from the rest. Pop (1971), has discussed the effect of Hall currents on the flow of an incompressible, viscous and electrically conducting fluid past an accelerated motion of an infinite flat plates in the presence of a transverse magnetic field. Gupta (A.S.) (1972), has studied the flow and heat transfer in hydromagnetic coquette flow of a conducting incompressible fluid between two infinite parallel plates, where very strong magnetic field acts transverse to the plates so as to make the Hall effect important. Borkakati and Srivastiava (1976), have discussed the heat transfer in a rotating channel with Hall current under the action of an transverse uniform magnetic field.

Grief et al. (1971) and Soundalgekar and Bhatt (1980) have discussed the laminar convection flow through a porous medium between two vertical plates . Pop et. al (1992) have discussed the laminar boundary layer flow due to a continuously moving flat plate . Recently Das and Sanyal (1995) have studied the laminar convection flow of a conducting incompressible fluid between two vertical porous plates in presence of a uniform transverse magnetic field under different permeabilities of the medium.

In this chapter, we have discussed the laminar convection flow of a viscous electrically conducting incompressible fluid between two parallel plates in presence of a uniform inclined magnetic field. The plates are maintained at constant temperature gradient. A uniform magnetic field is applied in a direction making an angle θ with the vertical axis. The field is consider to be strong enough so that it induces a magnetic field along the flow direction, The analytical expressions for velocity , induced field and the temperature, skin friction and rate of heat transfer at the plates are obtained and their variations are shown graphically for different values of magnetic field parameters and λ (= $\cos\theta$). The fluid viscosity is considered to be remain constant. The problem shows the influence of applied magnetic field and the induced magnetic field. This kind of situation often arises in different practical MHD problems in the laboratory. The problem has its importances in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of Crude oil, pulp, paper industry, textile industry and in different geophysical situations.

4.2 FORMULATION OF THE PROBLEM :

We consider the laminar convection flow of a viscous incompressible and electrically conducting fluid between two parallel plates. Let the x-axis be the central line of the channel along the motion of the fluid and y-axis be perpendicular to it . Let a uniform magnetic field $\mathbf{B}_0$ is applied in the direction making angle (θ) to the vertical line . It induces another magnetic field B along the line of motion. It is assumed that the plates are maintained at a constant temperature gradient (Γ / h) , where 2h is the width of the channel, so that the plate temperature T_w may be considered as

$$T_w = T_0 + (\Gamma / h)\, x \qquad 4.1$$

where T_0 is the temperature at the origin of the channel. The fluid temperature is assume to vary along both horizontal and vertical direction of the channel while all other physical quantities vary along vertical direction only (i.e. along y-direction).

The fluid velocity and magnetic field distributions are

$$\{u(y),\ v_0,\ 0\}$$

$$\text{and} \quad \{B_0\sqrt{(1-\lambda^2)},\ B_0\,\lambda,\ 0\}, \qquad 4.2$$

and the induced magnetic field is $\{\ B(y),\ 0,\ 0\}$

where $\lambda = \cos\theta$, $\mathbf{B}_0$ and $\mathbf{B}$ are applied and induced magnetic field respectively. In order to derive the governing equations of the problem the following assumptions are made.

(i) Hall effect and polarization effect are neglected.

(ii) The fluid within the channel, moves with uniform velocity so that all the physical variables are assumed to be time independent.

The magnetic body force (using 1.29 and omitting the electric part) is written as

$$f = J \times B \qquad 4.3$$

$$\text{where} \quad J = (\nabla \times B) \qquad 4.4$$

This gives $J_x = 0$, $J_y = 0$,

$$J_z = - (\partial B_x / \partial y) \quad 4.5$$

$$= -\{\partial/\partial y (B + B_0\sqrt{(1-\lambda^2)})\} \quad 4.6$$

hence from 4.3, $f_x = [B_0\lambda\{\partial/\partial y (B + B_0\sqrt{(1-\lambda^2)})\}]$, 4.7

$$f_y = [-\{(B + B_0\sqrt{(1-\lambda^2)})\}\lambda\{\partial/\partial y (B + B_0\sqrt{(1-\lambda^2)})\}], \quad 4.8$$

$$f_z = 0 \quad 4.9$$

Under these conditions, the governing equations are as follows :

$$d u / dx = 0 \quad 4.10$$

$$\nu(d^2 u/dy^2) - v_0 (du/dy) + (B_0\lambda/\rho\mu_e)(dB/dy) - k_0 = 0 \quad 4.11$$

$$\nu_m (d^2 B/dy^2) - v_0(dB/dy) + B_0\lambda(du/dy) = 0 \quad 4.12$$

$$\alpha (d^2T/dy^2) + \nu/C_p (du/dy)^2 + [1/(\rho\,\sigma\, C_p\mu^2_e)](dB/dy)^2$$

$$- u(dT/dx) - v_0(dT/dy) = 0 \quad 4.13$$

where k_0 constant, ; T, the temperature ; α , the thermal diffusivity (m^2/sec); ν , the kinematics viscosity ; ρ , density of the fluid medium ; C_p , specific heat at constant pressure ; σ , electrical conductivity, μ_e the magnetic permeability ; $\nu_m = 1/(\sigma\mu_e)$, the magnetic diffusivity .

The boundary conditions of the problem are

$$u = 0 , \quad b = 0 , \quad T = T_w \quad \text{at} \quad y = \pm h \quad 4.14$$

We consider now introduce the following dimensionless quantities

$$x^* = x / h ; \quad y^* = y / h ; \quad u^* = (uh / \alpha) ;$$
$$\phi = (T_w - T) / \Gamma ; \qquad b = (B / B_0) ; \qquad 4.15$$

Substituting 4.15 in equations 4.10 - 4.13 , and then dropping asterisks , the equations 4.11 - 4.13, are as follows .

$$(d^2 u / dy^2) - Re\,(du / dy) + [(M^2 \lambda) / R_m]\,(db / dy) - k_0 = 0 \qquad 4.16$$

$$(d^2 b/dy^2) - R_m\,(db/dy) + (\lambda R_m)(du / dy) = 0 \qquad 4.17$$

$$(d^2\phi / dy^2) - Pr\,[\,Re\,(d\phi / dy) + E\,\{(du/dy)^2 + (M^2 / R_m^2)\,(db/dy)^2\}] + u = 0 \qquad 4.18$$

where $\nu = \mu / \rho$, kinematic viscosity ; $M = B_0 h \sqrt{(\sigma/\nu\rho)}$, Hartmann number ; $R_m = (\alpha\mu_e \sigma)$, the magnetic Reynolds number ; $\nu_m = 1/(\sigma\mu_e)$, the magnetic dffusivity; $Re = (v_0 h / \nu)$, the Reynolds number ; $Pr = (\nu / \alpha)$, the Prandtl number; $E = \alpha^2 / (\Gamma C_p h^2)$, the Eckert number .

Substituting 4.15 in equations 4.14, the non-dimensional boundary conditions are :

$$u = 0, \quad b = 0, \quad \phi = 0 \quad \text{at} \quad y = \pm 1 \qquad 4.19$$

4.3 SOLUTION OF GOVERNING EQUATIONS :

To solutions of the non-linear equations 4.16 -4.18 under the boundary conditions 4.19, are given as

$$u(y) = [k_3 \exp(m_1 y) + k_4 \exp(m_2 y) - (k_0 A_0 + A_1 k_1 + A_2 k_2) - (A_1 k_0 y)] \quad 4.20$$

$$b(y) = (1/SA_2)[k_3 (Re - m_1) \exp(m_1 y) + k_4 (Re - m_2) \exp(m_2 y) - (A_3 y + A_4) k_0 - A_3 k_1 - (Re A_2) k_2)] \quad 4.21$$

$$\phi(y) = A_{18} + A_{19} \exp(Pr\, Re\, y) + E\, Pr\, [A_{20} \exp(2 m_1 y) + A_{21} \exp(2 m_2 y) + A_{22} \exp[(m_1 + m_2) y] - A_{23} \exp(m_1 y) - A_{24} \exp(m_2 y) - A_{25} y^2 - A_{26} y - A_{17} / (Pr\, Re)^2] \quad 4.22$$

where $S = (Re\, Rm - \lambda^2 M^2)$; $A_0 = (1/S)[Rm(Re + Rm)/S - 1]$;

m_1 & m_2 = $[(Re + Rm) \pm \sqrt{\{(Re + Rm)^2 - 4S\}}]/2$; $A_1 = (Rm/S)$;

$A_2 = (\lambda M^2)/(S\, Rm)$; $A_3 = (Re\, A_1 - 1)$; $A_4 = (Rc\, A_0 - A_1)$;

$A_5 = [A_0 (Re - m_2) - A_4]$; $A_6 = [A_1 (Re - m_2) - A_3]$;

$A_7 = [A_0 (Re - m_1) - A_4]$; $A_8 = [A_1 (Re - m_1) - A_3]$;

$A_9 = [(A_6 \coth(m_1)) - A_5]$; $A_{10} = [(A_8 \coth(m_2)) - A_7]$;

$k_1 = k_0 \{(A_9 m_1 - A_{10} m_2)/(A_6 m_1 - A_8 m_2)\}$;

$k_2 = (k_0 / A_2) \{(A_9 A_8 - A_{10} A_6)/(A_6 m_1 - A_8 m_2)\}$;

$A_{11} = (k_3^2 m_1^2)\{1 - (Re - m_1)^2 / (\lambda^2 M^2)\}$;

$A_{12} = (k_4^2 m_2^2)\{1 - (Re - m_2)^2 / (\lambda^2 M^2)\}$;

$A_{13} = (2\,k_3\,k_4\,m_1\,m_2\;)\{\,1 - (\,Re - m_1\,)\,(\,Re - m_2\,)\,/\,(\lambda^2\,M^2\,)\};$

$A_{14} = \;k_3\,[2\,k_0\,m_1\,\{A_1 + A_3\;(\,Re - m_1\,)\,/\,(\lambda^2\,M^2\,)\} + 1/(\,E\,Pr)\,]\,;$

$A_{15} = \;k_4\,[2\,k_0\,m_2\,\{A_1 + A_3\;(\,Re - m_2\,)\,/\,(\lambda^2\,M^2\,)\} + 1/(\,E\,Pr)\,]\,;$

$A_{16} = [A_1\,K_0\,/\,(\,E\,Pr\,)];$

$A_{17} = [\,k_0^2\;\{\,A_1^{\;2} + A_3^{\;2}\,/\,(\lambda^2\,M^2\,)\} + \{(\,k_0\,A_0 + A_1\,k_1 + A_2\,k_2\;)\,/(\,E\,Pr)\}]\,;$

$A_{18} = [E\,Pr\;\{A_{25} + A_{24}\cosh(\,m_2\,) + A_{23}\cosh(\,m_1)) - A_{22}\cosh(\,m_1 + m_2)$

$- A_{21}\cosh(\,2m_2\,) - A_{20}\cosh(\,2m_1\,) + A_{17}\,/\,(\;Re\,Pr)^2\,\}$

$- A_{19}\cosh(\,Re\,Pr\,)]$

$A_{19} = \{E\,Pr\,/\,\sinh(\,Rc\,Pr\,)\,\}\;[\;A_{26} - A_{20}\sinh(\,2m_1\,) - A_{21}\sinh(\,2m_2)$

$- A_{22}\sinh(\,m_1 + m_2) \quad + A_{23}\sinh(\,m_1\,) + A_{24}\sinh(\,m_2\,)\,]\;;$

$A_{20} = [A_{11}\,/\,(\;4m_1^{\;2} - 2\,Re\,Pr\,m_1)]\;; \quad A_{21} = [A_{12}\,/\,(\;4m_2^{\;2} - 2\,Re\,Pr\,m_2)]\,;$

$A_{22} = [A_{13}\,/\,\{(\,m_1 + m_2\,)^{\,2} - Re\,Pr\,(m_1 + m_2\,)\}]\,;$

$A_{23} = [A_{14}/\,(\,m_1^{\;2} - Re\,Pr\,m_1)];$

$A_{24} = [A_{15}\,/\,(\;m_2^{\;2} - \;Re\,Pr\,m_2)];$ $\qquad A_{25} = [A_{16}\,/\,(\,2\,Re\,Pr)]\,;$

$A_{26} = [A_{16}\,/\,(\;Re\,Pr)^2\; + A_{17}\,/\,(\;Re\,Pr)]\,;$

<u>SKIN FRICTION AND HEAT TRANSFER</u> :

The physical quantities of our interest in this problem are the Skin friction coefficient τ and the rate of heat transfer Q at the plates.

Using 1.95, the Skin friction coefficient τ at the plates which is proportional to the local velocity gradient is give as:

$$\tau = (\,-\,\mu du\,/dy\,)_{\,y=\pm 1} \qquad 4.23$$

Again using 1.79 the rate of heat transfer which is proportional to the local temperature gradient, given as:

$$Q = (- k\, dT / dy)_{y=\pm 1} \qquad 4.24$$

where μ , k are viscosity and thermal conductivity of the fluid medium .

Using 4.15, and introducing τ^* and Q^* as non-dimensional parameters for skin friction and rate of heat transfer, and then dropping the asterisks, we get

$$\tau = [-du / dy]_{y=\pm 1} \qquad 4.25$$

$$[\tau]_{y=\pm 1} = - [k_3\, m_1 \exp(\pm m_1) + k_4\, m_2 \exp(\pm m_2\, y) - (\pm A_1\, k_0)] \qquad 4.26$$

and

$$Q = [d\phi/dy]_{y=\pm 1} \qquad 4.27$$

$$[Q]_{y=\pm 1} = (Pr\, Re\, A_{19})\exp(Pr\, Re\, y)+ E\, Pr\, [2m_1 A_{20} \exp(2\, m_1\, y)$$

$$- 2m_2 A_{21}\exp(2m_2\, y) + (m_1 + m_2)A_{22} \exp\{(m_1 + m_2)\, y \}$$

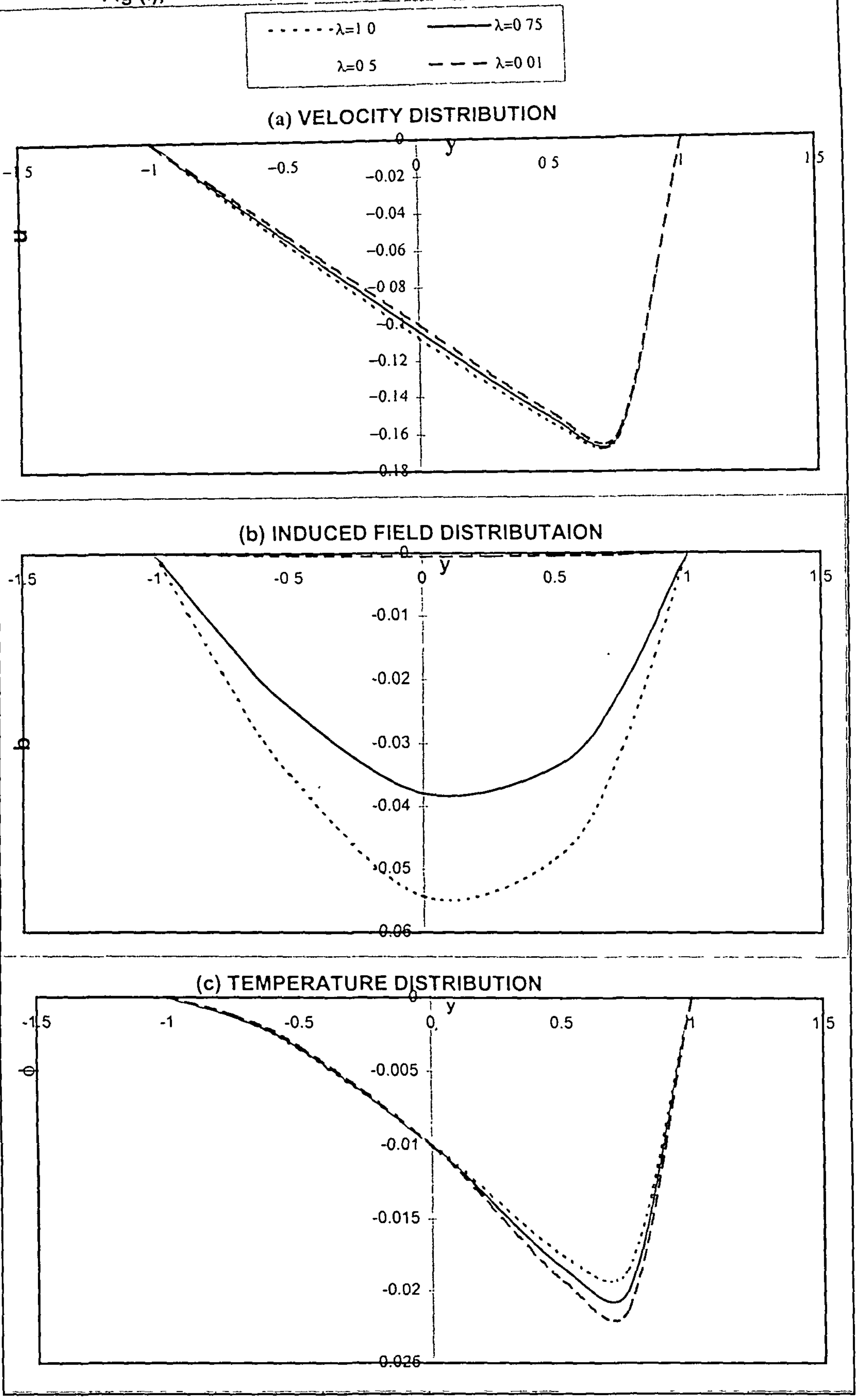
$$- A_{23}\, m_1\exp(m_1\, y) - A_{24}\, m_2\exp (m_2 y) - 2A_{25}\, y - A_{26}]$$

4.28

4.4 RESULTS AND DISCUSSION :

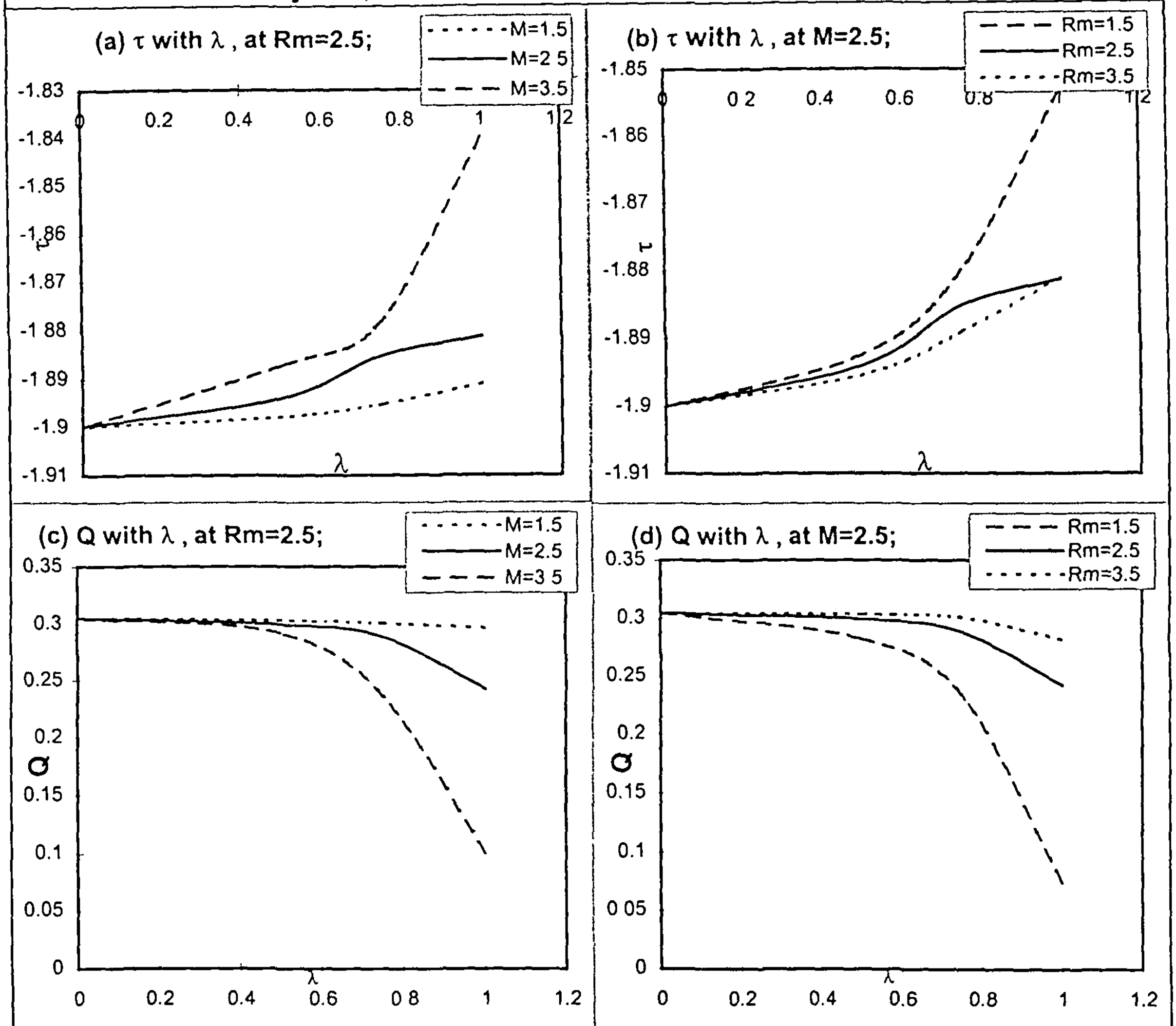
Numerical values for velocity component (u), induced field (b), fluid temperature (ϕ), skin friction coefficient (τ) and rate of heat transfer (Q) are calculated for different constant values of magnetic Hartmann number (M), magnetic Reynolds number (Rm), and at constant values of Reynolds number (Re), Prandlt number (Pr), Eckert number (E). The distributions of u , b and ϕ within the channel are shown in figure (i) while the variations of τ and Q with λ and M are shown in figures {(ii) & (iii)} respectively for constant value of Rm

Fig (i), R=10.0 ; k = 1.0 ; Rm = 1.5 ; M= 1.5 ; E =1.0 ;Pr = 0.71



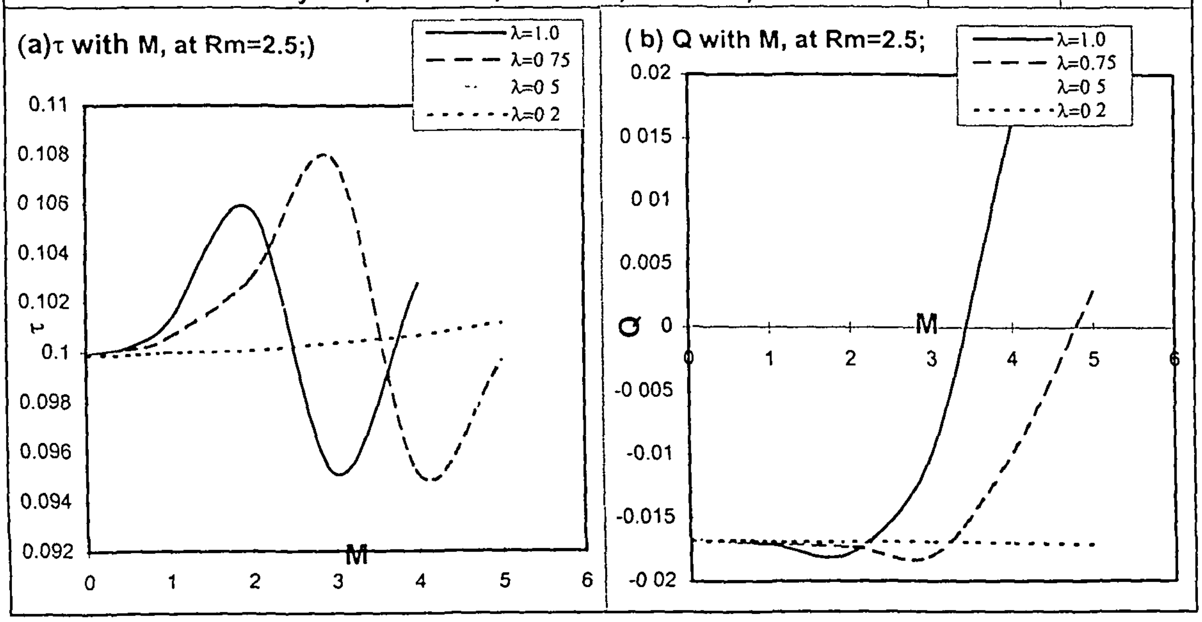
Fig(ii) Variation of SKIN FRICTION (τ)and HEAT TRANSFER (Q) with λ at constant M & Rm

y = +1; R=10.0 ; k = 1.0 ; E =1.0 ; Pr = 0.71



Fig(iii) Variation of SKIN FRICTION (τ)and HEAT TRANSFER (Q) with M at constant λ & Rm

y = 0 ; R=10.0 ; k = 1.0 ; E =1.0 ; Pr = 0.71



(where $\lambda = \cos\theta$, θ is the field inclination from the vertical axis). The viscosity of the fluid is assumed to be constant for all values of temperatures . The numerical values are calculated for Re = 10.0, Pr = 0.71, E = 1.0, k_0 = 1.0, while M & R_m vary from (1.5 to 3.5). Following results are obtained from the figures .

(i) It is observed from fig{(i),a} that for all values of λ, fluid velocity (u) gradually decreases from upper to lower plate and when λ increases (i.e. when θ decreases), velocity gradually decreases.

(ii) For all values of λ , the induced field (b) decreases significantly within the channel. It is minimum near the central line of the channel ($y \cong 0$) and increases gradually towards the plates . With the rise of λ (i.e. decrease of θ), the values of b decrease appreciably and at very small values of λ (i.e. when the field inclination is maximum $\theta \to \theta_{max}$), the induced field is negligibly small (see fig. {(i),b}).

(iii) For all values of λ , ϕ decreases rapidly from upper to lower plate and when λ increases (i.e. when θ decreases), ϕ increases slowly (see fig. {(i),c}).

(iv) For all values of M & Rm , τ increases with the increase of λ , and with the increase of M (at constant Rm), τ increases for all values of λ. Keeping M constant, τ decreases with the increase of Rm for all values of λ (see figs{(ii),a&b}).

(v) For all values of M and Rm , Q decreases steadily with the increase of λ . With the increase of M or Rm , Q decreases appreciably for all values of λ (see figs {(ii),c&d}).

(vi) In figs { (iii) , a & b } variations of τ and Q with M are shown for different values of λ (= 0.2 , 0.5, 0.75 & 1.0) and constant value of Rm(= 2.5). It is seen that with the increase of M, τ rises up to a certain value of M depending upon value of λ after which it declines rapidly and at higher value of M (depending upon value of λ), τ becomes unsteady . On the other hand with the increase of M, Q first gradually decreases for smaller values of M after which it increases rapidly for higher values of M . The nature of increase and decrease of Q with M are different at different values of λ .

4.5 CONCLUSIONS :

- As the inclination of the applied magnetic field to the vertical axis increases the fluid velocity and induced magnetic field increase while the fluid temperature decreases .
- More the inclination of the applied field less is the skin friction acting at the boundary layers of the fluid .
- For the rise of applied field , skin friction increases and the nature of increase largely depends upon the inclination of the field .
- When the applied field increases the rate of heat transfer decreases steadily for smaller values of applied field but increases sharply for higher values of applied field .
- When the applied field remains unchanged, the heat transfer rises up with the rise of inclination of the field to the vertical axis.

- The nature of decrease of heat transfer with the increase of applied field largely depends upon the inclination of the applied field. The decrease of heat transfer with the increase of applied field, is more with the increase of field inclination from the vertical axis.

CHAPTER V

MASS TRANSFER AND THERMAL DIFFUSION EFFECT IN A BINARY MIXTURE PAST AN INFINITE VERTICAL POROUS PLATE IN PRESENCE OF A UNIFORM MAGNETIC FIELD

5.1 INTRODUCTION :

In a moving single-component medium heat is transferred by conduction and convection ; the process is known as convective heat transfer. By analogy, the process of simultaneous molecular and molar transport of matter in a moving multi-component medium is called convective mass transfer. In many processes of flow and heat transfer encountered in nature and engineering are accompanied by the transfer of mass from one component into the other. For instance, the case of condensation of vapour from a vapour-gas mixture and the evaporation of liquid into a vapour gas flow. The process is accompanied by a change in the nature of flow and a variation in heat transfer intensity which in turn influences the process of diffusion. The mass diffusion already we have mentioned in chapter I, is for a binary mixture .

It has been seen in many engineering problems, especially in chemical engineering, the mass transfer processes which are sometimes accompanied by many other process, such as, heat transfer, rotation of fluids, electromagnetic forces etc. The heat conduction in a gas is caused by the random movement of the molecules to equalize existing differences in the energy. By the same movement, local differences in concentration of a gas mixture diminish in time even if no macroscopic mixing occurs. This process is known as diffusion. By diffusion or convection, in a mixture of local concentration differences, a component is transported from one location to another. In a mixture, diffusion is directed towards equalizing the concentration in the system and is accompanied by transfer of mass from the region of higher concentration to the lower concentration. By analogy with the heat transfer, diffusion (mass transfer) may be either molecular (microscopic) or molar (macroscopic). In gases molecular diffusion is due to the thermal motion of molecules. In general the diffusion of a binary gas or liquid mixture are considered from a molecular point of view. Fick's law (equation 1.86) defining the diffusion of binary system has already been explained in chapter I. The mass transport through an interface between various phases of the same medium is found to be of special importance in engineering sciences.

Forced or free mass movement occur in mass transfer. It has also been seen that forced or natural convection also contributes to the mass exchange. Hence, in engineering applications, mass transfer is very complex. When the mass transfer is considered to be from a solid surface into a fluid stream, the transfer process is essentially concentrated in the boundary layer. In most the problems, the heat transfer process is connected with the mass transfer. When we consider the evaporation vapour from a wet surface or condensation on the surface, heat is absorbed or released at the surface by the change of phase, and this process usually creates temperature differences in the fluid. Hence we can consider it to be heat transfer.

A diffusion is characterized by the flow of the mass component i.e., the quantity of mass passing per unit time through the given surface in a direction normal to the surface. In a multicomponent system, the concentration of the various species may be expressed in different ways . For example, the mass concentration ρ_A, is the mass of species A per unit volume of the mixture ; the molar concentration $c_A = \rho_A / M_A$, is the number of moles of species A divided by the total mass density of the solution ; and the mole fraction $x_A = c_A / c$, is the molar concentration of species A divided by the total molar density of the mixture (Bird, et al. 1956).

In a mixture, the velocities of the individual species are different and there are several useful ways of averaging the velocities of the species to get a local velocity for the mixture. It is necessary to choose such a local velocity before the rates of diffusion can be defined. By velocity we don't mean the velocity of an individual molecule of species A, but the sum of the velocities of the molecules of the species divided by the number of molecules within a small volume.

In a mixture, various species are moving at different velocities. If v_A is the velocity of the species A with respect to a stationary coordinates axes, the local mass average velocity for a mixture of n species is defined as

$$v = \sum \rho_A v_A / \sum \rho_A \qquad 5.1$$

where $\rho_A v_A$ is the local rate at which mass of species A passes through a unit cross section placed perpendicular to the velocity v_A . Another way of defining the local molar average velocity v^* is:

$$v^* = \sum c_A v_A / \sum c_A \qquad 5.2$$

where $c_A v_A$ is the local rate at which moles of species A pass through a unit cross section placed perpendicular to the velocity v_A .

In addition to these definitions , some other average velocities are also sometimes used , such as, the volume average velocity.

Diffusion is more complicated than viscous flow or heat conduction and convection because here one has to deal with a mixture. In this chapter , we consider the concentration gradient, that is the motive force determining the diffusion process and the Fick's law (1.86), the law for concentration diffusion. In a mixture, if the temperature is variable, the thermal diffusion (generally known as Soret effect) sets in. In a two component system, thermal diffusion causes the heavier molecule to pass to the colder region. The direction of thermal diffusion may change under definite condition. For example, in an ionized gas, the heavier molecules tend to pass to the hotter region. Thus, the thermal diffusion results the formation of concentration gradient. The steady state is set when the opposing effects of thermal diffusion and concentration diffusion are balanced.

Lorentz (1881) has discussed the heat transfer from a hot vertical plate under the assumption that the temperature and the velocity at any point depend only on the distance from the plate. Schmidt and Beckmann (1930) have done the experimental works on the same problem and have showed that this assumption is invalid and have indicated an alternative method of solution. Ostrach (1953b) has reformulated the theoretical problem in a more general and formal manner starting directly from the basic equations for a compressible, viscous and heat conducting fluid ; but the final equations are same with those of Schmidt and Beckmann (1930).

In practice, mass transfer is mostly coupled with heat transfer. Nusselt (1916) has stated that there exists an analogy between heat and mass transfer. The results of Wilkes, Tobias and Eisenberg (1953) show the best agreement with the boundary layer theory. Their measurements are only valid for large Schmidt number, so that comparison with heat transfer is not possible. The problem of simultaneous heat and mass transfer by free convection about a vertical flat plate with uniform surface temperature and concentration has been considered by Bottemanne (1971). He has taken the two buoyancy effects originating from temperature and mass concentration differences as mutually independent and has given a numerical solution for the system of boundary layer equations for the steady case. Bottemanne (1970) has also experimentally verified his theoretical results . The experimental results concerning stationary heat and mass transfer in the laminar boundary layer of a vertical cylinder placed in still air have been given by Bottemanne (1972) . He has also discussed the combined as well as separate effects of heat and mass transfer.

Several authors have discussed different types of convective mass transfer MHD flow. Gebhart et. al (1971) has studied such an combined heat and mass transfer flow after which many authors like Debnath et al. (1972), Kafoussias et al. (1979), Nanousis et al. (1980), Singh (1982), Raptis et al. (1983), Jha et. al (1990) have also studied the same kind of problem.

These kind of problems become more complicated when discussed under the action of strong magnetic field that induces another magnetic field. The problems have special importance in dealing with astrophysical and geophysical problems where one has to take into account simultaneously both induced field and thermal diffusion (Soret effect), such as those occur in staler region and in the interior part of the Earth. Recently, Sattar et al. (1995) have discussed the heat and mass transfer on

MHD free convection flow past an impulsively started vertical porous plate in a rotating fluid neglecting the induced magnetic field.

In this chapter , we are trying to study the effect of a strong magnetic field that applied normal to the fluid motion which induces a magnetic field when the fluid concentration is changing. We consider a MHD free convection flow of an electrically conducting fluid past an infinite porous plate. The induced magnetic field which is along the line of the motion, varies transversely. The vertical plate is maintained at a constant temperature. The temperature gradient that creates thermal diffusion in the medium results in the formation of concentration gradient which tends to produce movement of the fluid matter with respect to the mean fluid motion . As a result of which a steady state may set in, that is, the opposing effects of thermal diffusion and concentration diffusion will be balanced. The aim of our study is to know how the fluid motion is affected by the mass transfer and thermal diffusion effect (i.e. the Soret effect) in presence of the induced magnetic field . To solve the governing equations, Runga-Kutta and Shooting method are used.

5.2 FORMULATION OF THE PROBLEM :

We have considered an unsteady flow of an electrically conducting incompressible viscous fluid past an infinite porous plate under the action of transverse magnetic field .The flow is assumed to be in the x-direction which is along the plate and y-axis is normal to it . The plate which is vertical and porous, is assumed to be moving steadily in the vertically upward direction along the x -axis. In this discussion, the following considerations are made.

(i) The diffusion is characterized by the flow of mass of component i.e., the quantity of mass passing per unit time through the given surface in a direction normal to the surface. The concentration gradient is the motive force determining the diffusion process and Fick's law.

(ii) The binary mixture system is homogeneous with respect to temperature and pressure so that the variation of coefficient of diffusion with temperature and pressure, is very small and is negligible.

(iii) The coefficient of diffusion is identical for the two mutually diffusing components of a two component mixture.

(iv) The thermal effusion effect, commonly known as Dufour effect which is the resultant effect of temperature difference in the mixing the two components, is neglected .

(v) The component of the mixture don't react chemically with one another.

(vi) The mixture is initially at the same temperature and the transfer of heat resulting from the inter-diffusion of the various species is negligible.

(vii) The fluid is finitely conducting and the viscous dissipation and the Joule heat are neglected.

(viii) Hall effect and polarization effect are neglected.

(ix) The plate temperature and the fluid concentration are maintained at constant T_w and C_w respectively while T_∞ and C_∞ are the same at a large distance from the plate.

The uniform magnetic field B_o is applied along y-axis which induces a magnetic field [B(y)] that varies perpendicular to it. So the fluid velocity and magnetic field components are $v = [\, u(y, t)\, , v_0\, , 0\,]$ and $B = [\, B(y, t),\ B_o\ , 0\,]$ respectively

The magnetic body force using 1.29 and omitting the electric part is written as

$$f = J \times B \qquad 5.3$$

where $J = \sigma\,(\,V \times B\,)$

This gives $J_x = 0\,,\ J_y = 0\,,\ J_z = \sigma\,(\,B_0\, u\,)$ 5.4

and hence from 5.3, $f_x = -\ (\sigma\, B_0^{\,2} u)\,,\quad f_y = 0\ ,\ f_z = 0$ 5.5

where σ is the electrical conductivity.

Under these assumptions, the basic equations of combined free and forced convective flow under Boussinesq's approximation, are given as follows :

$$(\partial u / \partial t) + v_0 (\partial u / \partial y) = B_0 /(\rho \mu_e)(\partial B / \partial y) + g_0 \beta\,(T - T_\infty) + g_0\,\bar{\beta}\,(C - C_\infty) + \nu\, \partial^2 u / \partial y^2 \qquad 5.6$$

$$(\partial B / \partial t) + B_0 (\partial u / \partial y) - v_0 (\partial B / \partial y) + \lambda\ (\partial^2 B / \partial y^2) = 0 \qquad 5.7$$

$$(\partial T / \partial t) + v_0 (\partial T / \partial y) = \{\alpha\,(\partial^2 T / \partial y^2)\} \qquad 5.8$$

$$(\partial C / \partial t) + v_0 (\partial C / \partial y) = D\ (\partial^2 C / \partial y^2) + \{\,\alpha\,(\,\partial^2 T / \partial y^2)\} \qquad 5.9$$

Boundary conditions of the problem are

$$u = u_0 \; ; \quad B = 0 \; ; \quad T = T_w \; ; \quad C = C_w \; ; \qquad \text{at } y = 0 :$$

5.10

$$u = 0 \; ; \quad B = 0 \; ; \quad T = T_\infty \; ; \quad C = C_\infty \; ; \qquad \text{at } y \to \infty :$$

where T and C are the fluid temperature and concentration respectively and ρ is the fluid density ; μ_e = Permeability of the medium ; ρ = Fluid density; α = Thermal diffusivity ; μ = Coefficient of viscosity; λ = Electrical conductivity; $\nu = (\mu / \rho)$, Kinematic viscosity ; $\nu_m = \{1/ (\lambda \; \mu_e)\}$,Magnetic diffusivity (viscosity); D = Molecular diffusivity; β = Volumetric coefficient of thermal expansion ; $\bar{\beta}$ =Volumetric coefficient of mass transfer ;

Introducing the non-dimensional parameters as

$$u = v_0 f(\eta) \; ; \; B = B_0 g(\eta) \; ; T = T_\infty + (T_w - T_\infty)\,\theta(\eta) \; ;$$

5.11

$$C = C_\infty + (C_w - C_\infty)\,\varphi(\eta) \; ;$$

where, $\eta = \{y / \sigma(t)\}$; $\sigma(t)$ is measured in length scale .

Substituting (6) in (1- 4), we have the non-dimensional equations as follows.

$$\{ d^2 f(\eta) / d\eta^2 \} + [\{(\eta\,\sigma / \nu)(d\sigma(t) / dt)\}\, d f(\eta) / d\eta] - Re\,\{d f(\eta) / d\eta\}$$

$$+ [\{M^2 / (Pe\,Rm)\}\, dg(\eta) / d\eta] + Gr\,\theta(\eta) + Gm\,\phi(\eta) = 0$$

5.12

$$\{ d^2 g(\eta) / d\eta^2 \} - [\{(\eta\,\sigma / \lambda)(d\sigma(t) / dt)\}\, d g(\eta) / d\eta]$$

$$- Rm\,[\{d f(\eta) / d\eta \} - \{d g(\eta) / d\eta \}] = 0$$

5.13

$$\{ d^2 \theta (\eta) / d \eta^2 \} + [Pr \{(\eta \sigma / \nu) (d \sigma(t) / d t) - Re \} \{d \theta (\eta) / d \eta \}] = 0$$

5.14

$$\{ d^2 \varphi (\eta) / d \eta^2 \} + Sc [\eta \{ \sigma / \nu) (d \sigma(t) / d t)\} - Re] \{d \varphi (\eta) / d \eta \}$$
$$+ \{S_0 Gm / Gr\} \{ d^2 \theta (\eta) / d \eta^2 \} = 0$$

5.15

where Re = $(v_0 \sigma / \nu)$, Reynolds number ; M = $\{B_0 \sigma \sqrt{(\lambda / (\nu\rho))}\}$, Magnetic Hartmann number. Rm = $\{\mu_e \lambda \alpha\}$, Magnetic Reynolds number. Pr = $\{\nu / \alpha\}$, Prandlt number ; Gr = $\{g_0 \beta (T_w - T_\infty) \sigma^2 / (\nu v_0) \}$, Grashof number. Pe = (Pr Re), Peclet number.

Gm = $\{ g_0 \bar{\beta} (C_w - C_\infty) \sigma^2 / (\nu v_0) \}$, Modified Grashof number.

Sc = (ν / D), Schmidth number ; $S_0 = \{\alpha \bar{\beta} / (D\beta)\}$, Soret number.

Now for equal kinematics and magnetic viscosity, the term $\{(\sigma / \nu) (\partial \sigma(t) / \partial t)\}$ where t appears explicitly, becomes common to all equations 5.12 - 5.15. From similarity condition, following the work of Sattar (1995), we can consider it as a constant.

i.e. $\{(\sigma / \nu) (d \sigma(t) / d t)\} = k$, (constant) 5.16

The equations 5.12 - 5.15. are now rewritten as follows .

$$\{ d^2 f (\eta) / d \eta^2 \} + \{(\eta k - Re)\{d f (\eta) / d \eta \}$$
$$+ [\{M^2 / (Pe Rm)\} dg(\eta) / d \eta] + Gr \theta(\eta) + Gm \phi (\eta) = 0$$

5.17

$$\{ d^2 g(\eta) / d\eta^2 \} - \{(\eta k + Rm)\, d g(\eta) / d\eta \}$$

$$+ \ Rm \{d f(\eta) / d\eta \} = 0 \qquad 5.18$$

$$\{ d^2 \theta(\eta) / d\eta^2 \} + [Pr \ (\eta k - Re)\{d \ \theta(\eta) / d\eta \}] = 0 \qquad 5.19$$

$$\{ d^2 \varphi(\eta) / d\eta^2 \} + \ Sc \, [(\eta k - Re)\{d \varphi(\eta) / d\eta \}$$

$$+ \{S_o Gm / Gr\} \{ d^2 \theta(\eta) / d\eta^2 \}] = 0 \qquad 5.20$$

The non- dimensional boundary conditions are now as follows.

$$f(\eta) = 1; \quad g(\eta) = 0; \quad \theta(\eta) = 1; \quad \varphi(\eta) = 1; \quad \text{at} \ \ \eta = 0 \ ;$$

5.21

$$f(\eta) = 0; \quad g(\eta) = 0; \quad \theta(\eta) = 0; \quad \varphi(\eta) = 0; \quad \text{at} \ \ \eta \rightarrow \infty ;$$

5.3 SOLUTION OF GOVERNING EQUATIONS :

The solutions of the equations 5.19 and 5.20 are simple and they are as follows.

$$\theta(\eta) = C_1 \exp\{ Pr\, Re\, \eta \ - \ Pr (k / 2)\, \eta^2 \} / (Pr\, Re - Pr\, k \ \eta) \ + C_2$$

5.22

$$\varphi(\eta) = 1/(Re - \eta k) [- (C_3 / Pr) \exp\{ Pr (Re\, \eta \ - (k / 2)\, \eta^2) \}$$

$$+ \ C_5 \exp\{Sc (Re\, \eta - \eta^2 \ (k / 2))\}] + C_6$$

5.23

where $C_0 = \exp\{ Pr\, Re\, N - N^2 Pr\,(k/2)\} / (Re - N k)$;

N, being a large number.

$C_1 = \{Pr\, Re / (1 - C_0\, Re)\}$; $C_2 = \{1 - C_1/(Pr\, Re)\}$;

$C_3 = [\{S_0\, Gm / Gr\}\, S_c\, Pr\, C_1 / (Pr - Sc)]$; $C_4 = (Re - N^2 k/2)$;

$$C_5 = [(C_3 / Pr) \exp(C_4 Pr) - \{1 + C_3/(Re\, Pr)\}(Re - N k)] / [\exp(C_4 Sc) - \{(Re - N k)/ Re] ;$$

$C_6 = \{1 + C_3 / (Re\, Pr) - C_5 / Re\}$;

Substituting 5.22 & 5.23 into the equation 5.17 and then numerical solutions of equations 5.17 and 5.18 are obtained using Runga-Kutta method. In order to find the missing initial conditions, we have used the Shooting method.

5.4 RESULTS AND DISCUSSION :

Numerical solutions of equations 5.17 and 5.18 are obtained for different values of Magnetic field parameter M & Rm , the mass transfer factor Sc , and the thermal diffusion factor S_0 . The values of physical parameters except M & Rm are taken as those of Sattar et al. (1995). The magnetic field parameter M (Magnetic Hartrmann number) is varied as M = 0.5 to 4.5 , and the induced field parameter Rm (Magnetic Reynolds number) changes as Rm = 0.5 to 2.0. Figures {1 (i & ii)} show nature of fluid velocity for the variation of mass diffusion parameter Sc ; while the figures { 1 (iii & iv)} show the nature of fluid velocity for the variation of thermal diffusivity factor So. In figures {2(i - iv)} the distribution of induced magnetic field are shown for variation of Sc & So. The numerical values of , the factor $-(\partial \varphi / \partial y)_{y=0}$ proportional to Nusslet number for mass transfer , commonly known as Sherwood number (Sh) are given in the table (I) , and the factor $-(\partial f / \partial y)_{y=0}$ representing the shear stress acting at the plate are shown in the table (II) for different values of M , Rm, Sc , & So ; the vales are compared with those obtained by Sattar(1995).

Following are the results drawn from the figures (1& 2) and the tables (I & II).

(i) At constant M & Rm , the rate of mass transfer (Sh) , decreases with the increase of Sc same as those of Sattar (1995), but increases with the increase of thermal diffusivity So .

(ii) The shear stress at the plate increases with the increase of Sc at constant M, Rm & So , but decreases with the increase of So when M , Rm , Sc are remain constant.

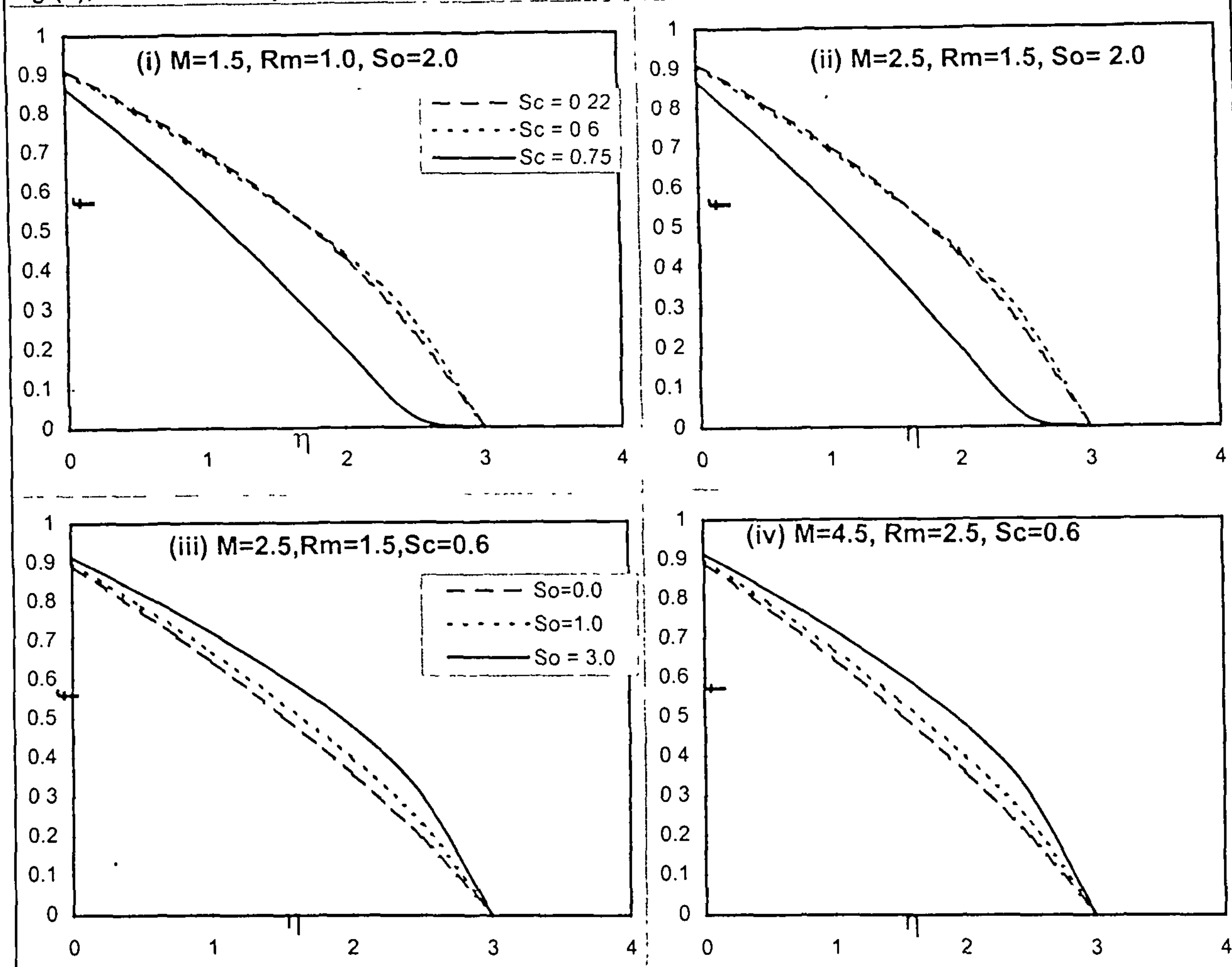
(iii) With the increase of magnetic field the shear stress at the plate increases for constant Sc & So.

(iv) The effect of change of magnetic field (i. e. M & Rm) on the fluid velocity is very small at constant Sc & So.

(v) The induced magnetic field varies inversely with the increase of Sc but varies directly with the increase of So.

(vi) The results obtained in (i) ,(ii) & (iii) are opposite in nature to those of Sattar et al. (1995). The reason of this may be due to the non-consideration of induced magnetic field by Sattar et al. (1995).

Fig (1), Velocity distribution at Re = 10, Gr =10, Gm =100, Pr = 0.71, k =2.0



Fig(2), Magnetic field Profile at Re=10,Gr =10, Gm = 100,Pr = 0.71, k = 2.0

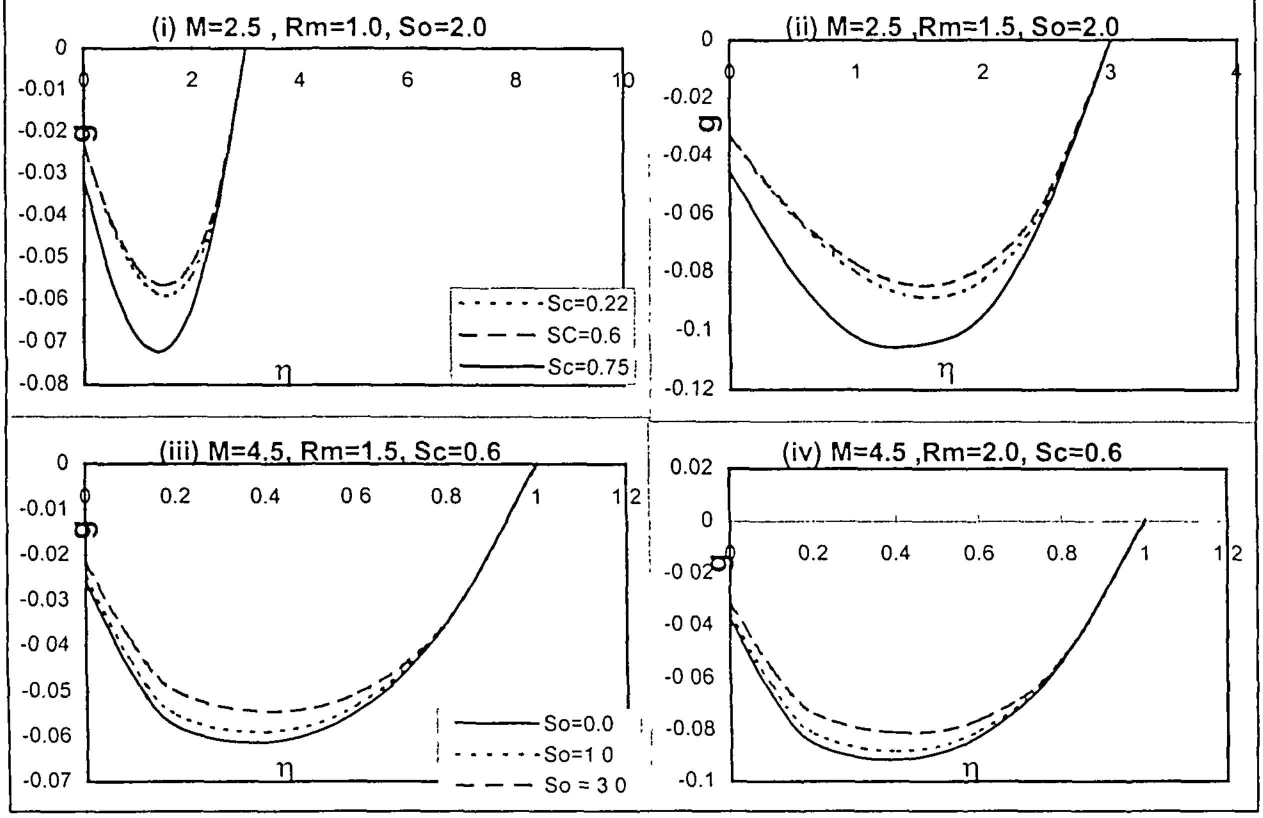


TABLE (I)

(Gr =10.0 , Gm = 4.0, Re = 10.0, M=1.0, R_m= 0.5, Pr = 0.71 , k = 2.0)

Sc	So	Sh
0.22	2.0	2.0464
0.60	2.0	0.2059
0.75	2.0	0.0147
0.60	0.00	0.0302
0.60	1.0	0.1180
0.60	2.0	0.2059

TABLE (II)

(Gr = 10.0 , Gm = 4.0 , Re = 10.0 , Pr = 0.71 , k = 2.0)

M	Rm	Sc	So	τ_{yx}
-0.5	0.1	0.22	1.0	11.7016
0.5	0.1	0.60	1.0	11.7348
0.5	0.1	0.75	1.0	11.8232
0.5	0.1	0.22	0.0	11.7187
0.5	0.1	0.22	1.0	11.7016
0.5	0.1	0.22	2.0	11.6833
0.5	0.1	0.22	3.0	11.6662
0.5	0.1	0.60	1.0	11.7348
0.5	0.1	0.60	2.0	11.7027
0.5	0.1	0.75	1.0	11.8232
0.5	0.1	0.75	2.0	11.8795
1.0	0.5	0.60	1.0	11.7350
1.5	1.0	0.60	1.0	11.7390
2.5	1.5	0.60	1.0	11.7484
4.5	2.0	0.60	1.0	11.7844

CHAPTER VI

FREE CONVECTION FLOW IN POROUS MEDIUM BETWEEN TWO HEATED VERTICAL PARALLEL PLATES IN PRESENCE OF A UNIFORM MAGNETIC FIELD

6.1 INTRODUCTION :

The flow of fluids through porous media plays important roles in hydrology, petroleum engineering, chemical engineering , bio-chemical engineering, medicines, ceramics and paper technology. The production of petroleum and natural gases , well drilling and logging are studied in petroleum engineering. The filtering of gases in liquids, chromatography and gel permeation chromatography etc. are the subjects of study in chemical engineering . The medicine and bio-chemical engineering, biological membranes and filters, the flow of blood and other body fluids and electro-osmosis are a few examples where the role played by porous media is critical.

Flow through porous media has attracted considerable attention in research activity in recent years because of its several important applications, notably in the extraction of energy from the geo-thermal regions.

Porous medium is literally a solid body containing a very large number of pores. It is however much more difficult to give an exact geometrical definition of what is meant by the notion of the pore. Pores are void spaces imbedded in a material . These may be either connected or non-connected, distributed more or less frequently in either a regular or random manner in the material . Interconnected pores are called the effective pores while the non-interconnected are ineffective pores. By ineffective pores, we mean the one through which the fluid can not pass. This may be either due to the surface tension caused by fine holes or the holes may not be interconnected, so that they do no affect the flow directly but affect the compressibility of the medium.

Porous medium may be visualized as an ordinary unconsolidatcd body in which innumerable voids of varying sizes and shapes comprising pores spaces are present. Moreover, each pore connected by channels to other pores forming a completely interconnected network of openings. This forms the channel through which the fluid may flow.

The voids in a porous medium may be classified according to the behavior of the fluid within these spaces. The small void space in which molecular forces between the solid and the fluid are significant, are classified as interstices or capillaries. The large void spaces in which the motion of a liquid is partially affected by the walls of the voids are referred to as caverns. Thus void spaces which are intermediate in size between capillaries and caverns are referred to as pores. Thus the void spaces partially or completely affect the motion of the fluid flowing through these spaces . It is this effect of the minute openings that definitely differentiates this subject from that of the usual hydrodynamics.

A porous medium is not restricted to have the pores belonging to one class. One porous medium may be embedded with the pores of different sizes and shapes. According to this description the term porous medium may encompass a very wide variety of substances. Some of the examples of porous media sand , soil granules, limestone, cement , brick, paper cloth, filter paper etc.

The location, size, shape and the manner in which the pores are interconnected give the nature of the porous media. The behavior of fluid flow through porous media can be determined by two approaches viz. (i) Microscopic and (ii) Macroscopic. The microscopic theory is statistical in nature and largely concerns with the basic physical processes within the porous media, and is obtained when the molecular structure of the fluid is taken into account. In macroscopic theory, the flow is fully determined by specifying the motion of every material point of fluid at any instant. This approach is meaningful only when there are relatively large number of pores. In the present work, we have discussed only from macroscopic point of view. It is worthwhile to characterize porous media by two of its static properties namely porosity and permeability.

The porosity is a quantitative property defined as the fraction of the voids to the total volume. This is a dimensionless quantity expressed either as a fraction of one or in percentage, and is usually define as

$$k_1 = \frac{\text{Volume of the voids of the porous material}}{\text{Bulk volume of the porous material}} \qquad 6.1$$

$$\text{where} \quad 0 < k_1 < 1 \qquad 6.2$$

Porosity which is essentially a static property. It can be classified into groups ; absolute or total porosity and effective porosity. Absolute porosity is the fractional void space with respect to the total volume regardless of pore connection. Effective porosity is that fraction of the total volume constituted by interconnecting pores. Many naturally occurring rocks, such as lava and other igneous rocks, have a high total porosity but essentially no effective porosity. A fluid can flow only through the effective pores and therefore, in our work we have considered the effective porosity. Porosity lies between 0 to 1. In homogeneous isotropic materials porosity is a pure constant but in a non-homogeneous material, it may depends upon position.

Brinkmann (1947) has proposed that , if the permeable medium is a swarm of homogeneous particles which are small spheres and are kept in position by external forces, as in the bed of closely packed particles which support each other by contact, the damping force F_D acting on a volume element is :

$$F_D = (\mu / k_1) V \qquad 6.3$$

There has been numerous studies by hydrologists, petroleum geologists, chemical engineers, geologists and geophysicists of flow in a porous medium. Wyllie and Gregory (1955) has determined experimentally the porosities of various aggregates using spheres, cubes and prisms. They have showed that the Kozeny- Carman (1972a) constant depends on porosity and particle shape, but the shape factor can be empirically assumed from those the experimental data. The surface areas of consolidated porous media can be calculated when the pores were of reasonably uniform shape and size . Borthel (1956) has measured the static flow resistance and porosity of common porous building materials. Jones (1973) has proposed a boundary condition for curved interfaces and has used this condition to solve the problem of slow viscous flow past a spherical porous shell under several limiting conditions. Verma and Bhatt (1976) have investigated the flow past a heterogeneous porous sphere using matched asymptotic expansion procedure as developed by Proudman and Person (1957). Gupta (1980) has solved the problem of two dimensional flow , past a porous circular cylinder, with initial pressure gradient using the method of matched asymptotic expansions. He has shown that drag experienced by the cylinder increases due to initial gradient; although it remains smaller than the drag force experienced by the identical impervious body. Prasad et al. (1984) have reported the numerical studies for steady free-convection in a vertical annulus filled with saturated porous medium whose vertical walls are at constant temperatures while the horizontal wall being insulated. Fand and Phan (1987) have reported the results of an experimental study of heat transfer by combined forced and natural convection from a horizontal cylinder embedded in a porous medium composed of randomly packed glass spheres saturated with water. It has been suggested that the correlation procedure adopted there may yield useful results if applied to other geometries such as for example, forced convection heat transfer in ducks packed with porous media. Srivastava and Sharma (1992) have discussed the flow and heat transfer of an incompressible viscous fluid due to rotating disk at a small distance from the porous medium of finite thickness when the disk and the boundary of the porous medium are maintained at constant temperatures. It has been assumed that the porous medium is fully saturated with the fluid. Mehta and Sood (1994) have analyzed the problem of free convective heat transfer from a non-isothermal axisymmetric body immersed in an inhomogenous porous medium on the basis of boundary layer approximations. Soundalgekar and Bhatt (1990) have considered the laminar convection flow through a porous medium between two vertical plates.

The internal friction in a flowing viscous liquid or gas brings about a process of dissipation of energy. This consists in a fraction of the kinetic energy of the fluid being converted into thermal energy and heating it . The heating, however , may be significant , depending upon the viscosity of the fluid and the flow velocity. This results the increase of heat transfer and the reduction of skin friction. This kind of situation generally arise in various engineering applications such as nuclear reactor, power transformation etc.

In this chapter, we have considered the fully developed free convection laminar flow of an incompressible viscous electrically conducting fluid between two vertical parallel plates in porous medium under the action of a uniform magnetic field applied transversely to the flow . The aim of our study is to know the nature of the flow, and the heat transfer while taking into account the dissipation of energy due to flow and porous medium, under the action of uniform magnetic field. Figures are plotted to show the distribution of fluid velocity and temperature, and numerical values for skin friction and rate of heat transfer are calculated for different values of physical parameters. To solve the equations Runga-Kutta method is used and to find the missing initial conditions Shootoing method is used.

6.2 FORMULATION OF THE PROBLEM :

We are considering laminar connective flow of a viscous incompressible electrically conducting fluid between two vertical parallel plates. Let x - axis be taken along vertically upward direction through the central line of the channel and y - axis is perpendicular to the x- axis . The plates of the channel are at $y = \pm\ b$. A uniform magnetic field B_0 is applied parallel to y - axis. The velocity and magnetic field distributions are $\mathbf{V} = [\ u(y)\ ,\ 0\ ,\ 0\ ]$ and $\mathbf{B} = [0\ ,\ B_0,\ 0]$ respectively In order to derive the governing equations of the problem the following assumptions are made .

(i)Hall effect and polarization effect and the Joule's effect are negligible .

(ii) The value of magnetic Reynolds number is so small that the induced magnetic field is negligible.

(iii) The plates are maintained at two different temperatures T_1 & T_2 while the central plane of the channel is at temperature T_0.

(iv)The plates are considered to be infinite and all the physical quantities are functions of y only .

The equation of continuity is :

$$\nabla . \mathbf{V} = 0 \qquad 6.4$$

Using 6.3 in 1.32, the momentum equation is written as :

$$\nu\nabla^2\mathbf{V} - (\nu/k_1)\mathbf{V} + 1/\rho\,(\mathbf{J} \times \mathbf{B}) + \mathbf{Z}/\rho = 0 \qquad 6.5$$

Using 1.44 and 1.45 , and introducing the term for dissipation due to porous medium, the energy equation of our problem is

$$(1/\rho\, C_p)\, d/dy(\, K dT/dy\,) + \nu / C_p\, (\partial u / \partial y)^2 = \{\, \nu / (k_1\, C_p)\}\, u^2 \qquad 6.6$$

where the third term in the right hand side of equation 6.5, is the magnetic body force and J is the current density due to the magnetic field defined as

$$J = \sigma\,(V \times B) \qquad 6.7$$

$$\text{and} \qquad Z = \beta\, g\,(T - T_0) \qquad 6.8$$

where Z is the force due to buoyancy

The magnetic body force using 1.29 and omitting the electric part is written as

$$f = J \times B \qquad 6.9$$

This gives $J_x = 0$, $J_y = 0$, $J_z = \sigma (B_0 u)$ 6.10

Hence from 6.4, $f_x = -(\sigma B_0^2 u)$, $f_y = 0$, $f_z = 0$ 6.11

where σ is the electrical conductivity.

Using velocity and magnetic field distributions as stated above and the relations 6.8 and 6.11, the equations (6.5- 6.6) are as follows:

$$\nu (d^2 u / d y^2) - (\nu / k_1) u + g\beta (T - T_0) - \sigma B_0^2 u / \rho = 0 \qquad 6.12$$

$$\{ k / (\rho C_p \} d/dy(dT/d y) + (\nu / C_p)(d u / dy)^2 + \{ \nu / (k_1 C_p \} u^2 = 0 \qquad 6.13$$

where σ = Electrical conductivity ; ρ = Fluid density ;

μ = Coefficient of viscosity; $\nu = \mu / \rho$, Kinematic viscosity ;

β = Volumetric coefficient of thermal expansion ;

k_1 = Coefficient of permeability of the porous medium.

The boundary conditions are :

$$u = 0, \text{ at } y = \pm b;$$
$$T = T_2 \text{ at } y = -b; \quad T = T_1 \text{ at } y = +b \qquad 6.14$$

Consider the non- dimensional terms

$y^* = (y / b)$, $u^* = (u / u_0)$ $\theta^* = \{(T - T_0) / (T_1 - T_2)\}$ 6.15

where $u_0 = \{\beta g b^2 (T_1 - T_2)\} / \nu_0\}$. 6.16

Substituting 6.15 and then removing the asterisks, the non-dimensional forms of the equations (6.12 & 6.13) are as follows :

$$M\{ (d^2 u / dy^2) - (Da) u \} + \theta - (M_H^2 u) = 0 \qquad 6.17$$

$$(d^2\theta / dy^2) + Pr\, E \{ (du/dy)^2 + (1/Da) u^2 \} = 0 \qquad 6.18$$

where $E = u_0^2 / \{C_p (T_1 - T_2)\}$, Eckert number ;

$M = (\nu / \nu_0)$, Viscosity parameter ; $Da = (k / b^2)$, porosity parameter.

$M_H = \sqrt{[(B_0^2 b^2 \sigma) / (\rho.\nu_0)]}$, Magnetic Hartmann number ;

K= Thermal conductivity ; $\alpha_1 = k / (\rho C_p)$, Thermal diffusivity ;

$Pr = (\nu / \alpha_1)$, Prandtl number ;

Using 6.15, the boundary conditions 6.14 reduce to

$$u = 0, \text{ at } y = \pm 1;$$
$$\theta = \theta_0 \text{ at } y = -1 \text{ and } \theta = 1 + \theta_0 \text{ at } y = +1; \qquad 6.19$$

SKIN FRICTION AND HEAT TRANSFER :

From Newton's law of viscosity, the Shear stress force per unit area proportional to the local velocity gradient is

$$\tau^* = -\mu (du/dy)_{y=\pm b} \qquad 6.20$$

using 6.15, and dropping the asterisks, the non-dimensional skin friction is written as

$$\tau = -(du/dy)_{y=\pm 1} \qquad 6.21$$

Again using 1.79 the rate of heat transfer which is proportional to the local temperature gradient, is given as :

$$Q^* = -k\,(dT/dy)_{y=\pm b} \qquad 6.22$$

where μ , k are viscosity and thermal conductivity of the fluid medium .

Using 6.15, and dropping asterisks, the non-dimensional rate of heat flow is written as

$$\dot{Q} = -\,(d\theta/dy)_{y=\pm 1} \qquad 6.23$$

6.3 RESULTS AND DISCUSSION :

Numerical solutions of equations 6.17 & 6.18 are obtained for different values of M, Da, Pr & M_H. Figures (1 & 2) are drawn for fluid velocity and temperature distribution while the numerical values of the skin friction $\{(\tau)_{y=-1}\}$ and the rate of heat transfer $\{(\dot{Q})_{y=-1}\}$ are given in the table (I) and table (II), respectively. The values magnetic field parameter M_H is varied from 0.5 to 1.5 ; the viscosity parameter M from 0.1 to 10.0; and the porosity parameter Da from 0.1 to 1.0.

Fig (1), Velocity distribution at E = 0.1 θ = 0.1 , Pr = 0.71 & 10.0, MH = 0.5 & 1.5, Da =0.1 & 1.0

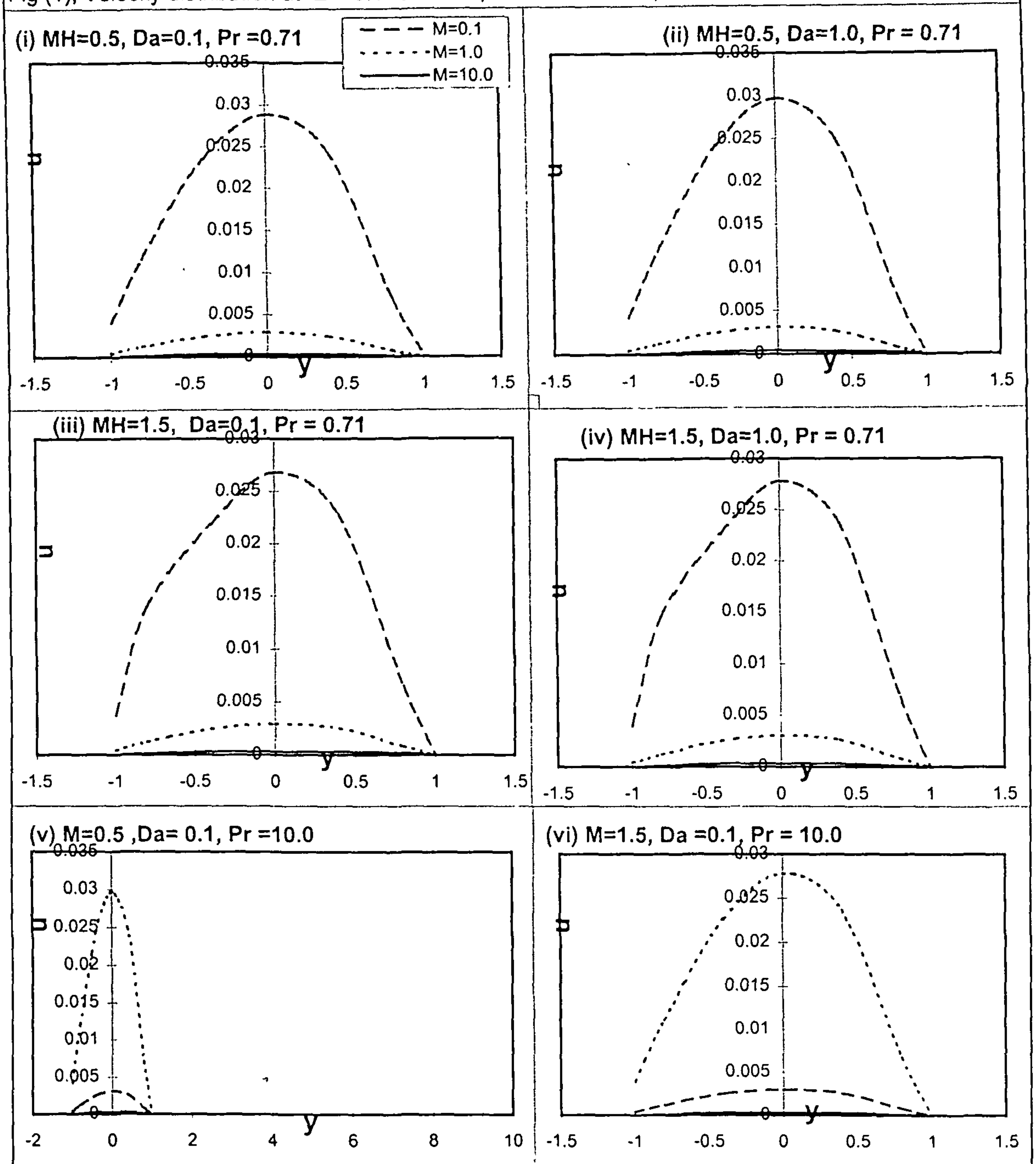


Fig (2), Temperature distribution E = 0.1 θ = 0.1 , Pr = 0.71 MH = 1.5, Da =0.1 & 1.0

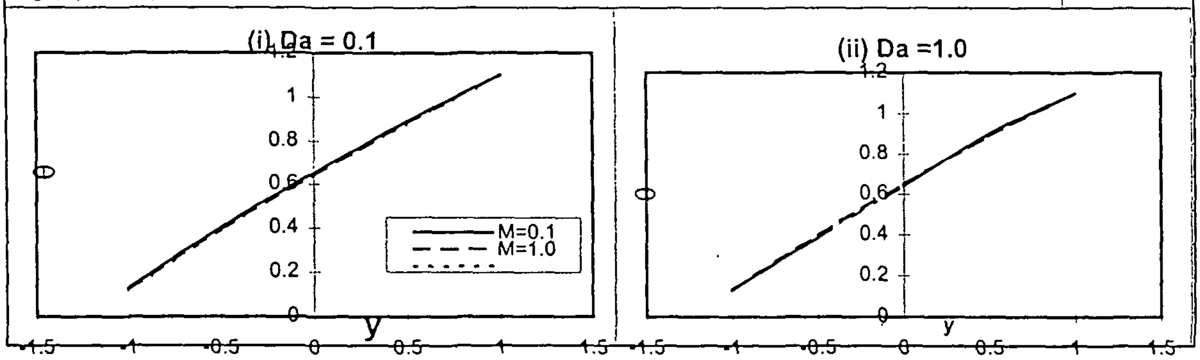


TABLE (I),

SKIN FRICTION AT Pr = 0.71 & 10.0

(E = 0.1 , θ_0 = 0.1)

M_{H}	Da ,	M	$\{\tau\}_{y=-1}$	$\{\tau\}_{y=-1}$
			(at Pr = 0.71)	(at Pr = 10.0)
0.5	0.1	0.1	0.2172	0.2151
		1.0	0.0259	0.0220
		10.0	0.0110	0.0034
	1.0	0.1	0.2200	0.1879
		1.0	0.0231	0.0233
		10.0	0.0034	0.0034
1.5	0.1	0.1	0.2114	0.2073
		1.0	0.0249	0.0220
		10.0	0.0034	0.0034
	1.0	0.1	0.2172	0.2112
		1.0	0.0225	0.0224
		10.0	0.0034	0.0034
2.0	0.1	0.1	0.2075	0.2045
		1.0	0.0220	0.0217
		10.0	0.0034	0.0034

TABLE (II),

HEAT TRANSFER AT Pr = 0.71 & 10.0

(E = 0.1 , θ_0 = 0.1)

M_{II}	$Da,$	M	$\{Q\}_{y=-1}$	$\{Q\}_{y=-1}$
			(at Pr = 0.71)	(at Pr = 10.0)
0.5	0.1	0.1	8.6456	8.8072
		1.0	8.7156	8.9903
		10.0	8.7846	8.8483
	1.0	0.1	8.8033	8.7903
		1.0	8.7202	8.8618
		10.0	9.0956	9.0956
1.5	0.1	0.1	8.7346	8.8462
		1.0	8.8412	8.7798
		10.0	8.8329	8.8470
	1.0	0.1	8.8519	8.7897
		1.0	8.8291	8.8427
		10.0	8.8764	8.8564
2.0	0.1	0.1	8.8182	8.7876
		1.0	8.8261	8.8668
		10.0	8.3457	8.8457

Following are the observations drawn from the figs.(1& 2)and the tables (I & II).

(i) The magnitude of fluid velocity u , decreases with increase of M when all other parameters remain unchanged. For higher value of M ($\cong$ 10.0), u is very small { see fig 1.(i- iv) }.

(ii) With the increase of Da , the fluid velocity u increases slowly for all values of M , M_H , and Pr { see fig 1.(i & ii) }.

(iii) When the M_H increases, the fluid velocity decreases for all values of M , Da and Pr { see fig 1.(i & iii) }.

(iv) With the increase of Pr, fluid velocity increases, the rate of increase depends upon the value of M_H for all values of M , Da and Pr. For small values of M_H ($\cong$ 0.5), the fluid velocity increases steadily with the increase of Pr,{see fig1(i & v)}. As M_H increases, the rate of increase of u , decreases slowly for all values of M and Da { see fig 1. (iii & vi)}.

(v) The fluid temperature (θ), increases linearly within the channel for all values of M , M_H, Da and Pr. With the increase of M , θ decreases slowly for all values of M, M_H , Da and Pr { see fig 2 (i)}.

(vi) With the increase of Da, the change of θ is very small and almost negligible for all values of M, M_H , and Pr { see fig 2 (ii)}.

(vii) With the increase of M, the skin friction τ , gradually decreases but the rate of heat transfer gradually increases for all values of M_H , M & Da { see table(I)}.

(viii) With the increase of magnetic field M_H , the skin friction τ decreases at all values of Pr ; but the rate of heat transfer gradually increases when $Pr = 0.71$ but decreases when $Pr = 10.0$ for all values of M & Da { see table(I)}.

(ix) With the increase of Da, for constant M_H, the skin friction τ at $Pr = 0.71$ increases for $M \cong 1.0$ but decreases for $M \geq 1$. The variation is almost opposite at $Pr = 10.0$ { see table(I)}.

(x) With the rise of Pr, skin friction τ decreases when M_H , Da and M remain unchanged { see table(I)}.

(xi) With the increase of Da, for $M \leq 1.0$, the heat transfer Q increases at $Pr = 0.71$ but decreases at $Pr = 10.0$. For $M > 1.0$, Q increases with the increase of Da at $Pr = 0.71$ & 10.0 { see table(II)}.

(xii) With the increase of Pr (= 0.71 to 10.0) , Q increases for all values of M, Da and M_H { see table(II)}

(xiii) With the increase of M_H , Q increases for all values of M & Da at $Pr = 0.71$ but when $Pr = 10.0$, Q decreases with the increase of M_H for all values of M & Da { see table(II)}.

6.4 CONCLUSIONS :

From the above discussion following points can be concluded .

- Fluid velocity largely depends upon kinematic viscosity, and varies inversely with it when other parameters are unchanged.

- Fluid velocity rises with the increase of porosity of the medium, when other parameters are unchanged.
- Fluid velocity declines with the rise of magnetic field for all values of kinematic viscosity and porosity.
- Fluid velocity rises when Prandtl number changes from 0.71 to 10.0 when other parameters are unchanged.
- Fluid temperature varies inversely with the increase of kinematic viscosity when other parameters are unchanged.
- Fluid temperature is less dependent to the porosity of the medium.
- Skin friction at the plate decreases with the rise of kinematic viscosity.
- Variation of skin friction at the plate with the rise of porosity depends upon kinematic viscosity.
- Rise of magnetic field causes decrease of skin friction when other parameters are unchanged.
- Rate of heat transfer increases with the increase of kinemtatic viscosity.
- The variation of rate of heat transfer, when porosity rises form 0.1 to 1.0, depends upon kinematic viscosity and Prandtl number.
- Rate of heat transfer increases or decreases with the rise of magnetic field depending upon Prandtl number.

CHAPTER VII

EFFECT OF INDUCED MAGNETIC FIELD ON MHD FLOW OF A DUSTY VISCO - ELASTIC FLUID DOWN AN INCLINED CHANNEL IN POROUS MEDIUM

7.1 INTRODUCTION :

In recent years, many authors have studied the flow of immiscible viscous electrically conducting fluids and their different transport phenomena . These fluids also known as non-Newtonian fluids, they are molten plastics , pulps , emulsion etc. and large variety of industrial products having visco-elastic behavior in their motion . Such fluids are often embedded with spherical non-conducting dust particles in the form of impurities . This kind of fluid is then called dusty **Rivlin - Ericksen** second order fluid. The influence of dust particles on visco-elastic fluid flow has its importance in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon , purification of Crude oil , pulp , paper industry , textile industry and in different Geophysical cases.

The two-dimensional incompressible second order steady, laminar flow of visco-elastic liquids through parallel and uniformly porous wall of diffcrent permeabilities are of particular interest and have been studied for their possible applications to the case of cooling , gaseous diffusion etc.

Sproull (1961), has reported that adding dust to air flowing in turbulent motion through a pipe can appreciably reduce the resistance coefficient. A similar report that the aerodynamic resistance of a dusty gas flowing through a system of pipes is less than that of a clean gas, has also been made by Kazakevich and Lrapivin (1958). From their observations, it may be concluded that the pressure difference required to maintain a given volume rate of flow is reduced by the addition of dust, though the increased density of the dusty gas should require a large pressure difference to maintain a given volume rate. A plausible explanation of this is that the addition of dust particles damps the turbulence. The turbulent intensity reduces the Reynolds stresses and the force required to maintain a given flow rate is likewise reduced.

Saffman (1962) has formulated the governing equations of motion of the dusty fluid and has shown that the problem of turbulence is quite related to the stability of laminar flow. Saffman (9162) and Michael (1964) have discussed the stability of the dusty gas flow. Liu (1966) has studied the flow of an incompressible dusty gas induced by the oscillation of an infinite plate in its own plane. Rao (1969) has discussed the laminar flow of an unsteady viscous liquid with uniform distribution of dust particles, through a circular cylinder under the influence of exponential pressure gradient. Michael and Norey (1970) have given an analysis for the slow motion of a sphere in a viscous liquid with dust particles suspension. Nath (1970) has studied the laminar flow of an unsteady incompressible viscous fluid with uniform distribution of dust particles through two rotating coaxial cylinders under the influence of an axial pressure gradient. Reddy (1972) has investigated the laminar flow of an unsteady viscous liquid with uniform distribution of dust particles through a rectangular channel under the influence of exponential pressure gradient with respect to time. Dube and Srivastava (1972) have discussed the flow of a viscous liquid with uniform distribution of dust particles in a channel and a circular pipe under the influence of pressure gradient varying linearly with time.

Dube (9172) has considered the problem associated with the flow of dusty Maxwell liquids near an oscillating plate. Dube and Singh (1972) have studied the laminar flow of a viscous liquid with uniform distribution of dust particles through a channel bounded by two parallel plates under the influence of pressure gradient (i) varying linearly with time , (ii) decreasing exponentially with time. Verma and Mathur (1973) have investigated the unsteady motion of a dusty viscous liquid in a circular pipe. Sharma (1975) has discussed the unsteady flow of a dusty viscous liquid in a channel bounded by two parallel plates. He has found that the liquid and the dust particles which are nearer to the axis of the channel move with greater velocities.

The study of dusty visco-elastic fluids under different physical conditions have been carried out by several authors like Kapur et al. (1964) , Sengupta et al. (1991) , Bagchi (1965) and many others. Gupta and Gupta (1976) have discussed the flow of viscous liquid through a channel with arbitrary time varying pressure gradient.

The study of second order fluid under different conditions in presence of a uniform magnetic field has also been carried out by many authors. Purkait (1984) has studied MHD transient flow of second order Rivlin-Ericksen fluid down an inclined channel. Sisodia and Gupta (1986) have studied an unsteady flow of a dusty viscous flow through a circular and coaxial circular ducts. Lahiri and Ganguli (1986) have also studied the same type of problem. Lal and Johri (1990) have discussed the MHD transient flow of second order Rivlin -Ericksen fluid through porous medium down an inclined channel . Recently Singh and Singh (1995) have studied MHD flow and heat transfer of a dusty visco-elastic liquid down an inclined channel in porous medium. They have shown the nature of fluid and dust particle at different values of magnetic parameter and of the dust particle elasticity under the action of uniform transverse magnetic field.

In this chapter , ~~paper~~ we are trying to investigate about the unsteady flow of dusty visco-elastic electrically conducting fluid down an inclined parallel plate channel in porous medium in presence of uniform magnetic field applied externally transverse to the direction of flow, which in turn induces a magnetic field along the line of flow. The expressions for fluid and dust particle velocity, induced magnetic field , temperature distribution , fluid and dust particle flux , heat transfer , viscous drag at the plates are obtained. The velocity distribution of fluid and dust particle , induced field and the temperature distribution are shown graphically at different magnetic field parameters .

7.2 FORMULATION OF THE PROBLEM :

We consider fully developed flow of an incompressible , dusty Rivlin-Ericksen fluid of electrically conducting material through a parallel plate channel separated by 2h , inclined horizontally by an angle θ . The plates are maintained at two different temperatures which decay exponentially with time . Let the central line of the channel as the x-axis while y-axis is perpendicular to it . The uniform magnetic field B_0 is applied normal to the plates induces a magnetic field B along the line of the flow , which varies perpendicular to it , so that the velocity and magnetic field distributions are $V = [u, 0, 0]$ and $B = [B, B_0, 0]$. The inertial force experienced by fluid due to the motion of the dust particles is equal and opposite to that experienced by the dust particles due to the fluid motion.

To write down the governing equations following assumptions are made .

(i) The plates are infinitely long , so that the fluid velocity (u) and dust particles velocity (v) are functions of y and t only .

(ii) There is neither chemical reactions and mass transfer nor heat radiation among the dust particles .

(iii) The number density of dust particles is constant and has small value throughout the fluid motion.

(iv) Dust particles are solids , elastic spheres , identical and symmetrical in size , electrically non-conducting , and are distributed uniformly within the fluid motion

(v)Hall effect , Polarization effect , and the effect due to buoyancy are negligible .

The magnetic body force (using 1.29 and omitting the electric part) is written as

$$f = J \times B \tag{7.1}$$

where $$J = (\nabla \times B) \tag{7.2}$$

This gives $$J_x = 0 , \; J_y = 0 , \tag{7.3}$$

$$J_z = -(1/\mu_e)(\partial B_x/\partial y) \tag{7.4}$$

$$= -(1/\mu_e)\{\partial B/\partial y\} \tag{7.5}$$

Hence from 7.1, $$f_x = (B_0 / \mu_e)\, \partial B/\partial y \tag{7.6}$$

$$f_y = [B/(\mu_e)]\, \partial B/\partial y \tag{7.7}$$

$$f_z = 0 \tag{7.8}$$

Under these conditions, the governing equations for second order non-Newtonian visco-elastic fluid are as follows (see chapter I) :

$$\partial u/\partial t = -(1/\rho)\ \partial p/\partial x + \nu_1 (\partial^2 u/\partial y^2) + \nu_2 \{\partial/\partial t\,(\partial^2/\partial y^2)\}$$

$$-(\nu_1/k_1)\,u + g\sin\theta + (K N/\rho)(v-u) + (1/\rho)(B_0/\mu_e)\,\partial B/\partial y \qquad 7.9$$

$$(1/\rho)\ \partial p/\partial y + g\cos\theta + [B/(\mu_e\rho)]\,\partial B/\partial y = 0 \qquad 7.10$$

$$m(\partial v/\partial t) - K(u-v) = 0 \qquad 7.11$$

Using 1.44 and 1.45 , and introducing the term for viscous dissipation due to fluid and dust particle, and the Joule heat due to the magnetic field , the energy equation is :

$$\partial T/\partial t = \{\alpha/(\rho C_p)\}(\partial^2 T/\partial y^2) + \nu_1 (\partial u/\partial y)^2 + \nu_2 \{\partial/\partial t\,(\partial u/\partial y)^2\}$$

$$+ \{1/(\sigma\mu_e^2\,\rho\, C_p)\}(\partial B/\partial y)^2 \qquad 7.12$$

From 1.86, the magnetic diffusivity equation is :

$$\partial B/\partial t + B_0(\partial u/\partial y) + \nu_H(\partial^2 B/\partial y^2) = 0 \qquad 7.13$$

where p , Fluid Pressure ; m , mass of the dust particle ; ν_1 , Kinematic coefficient of fluid viscosity ; ν_2 , Kinematic coefficient of visco-elasticity ; k_1 , porosity of the medium ; N , number density of the dust particles ; α , thermal conductivity of the fluid ; C_p , specific heat at constant pressure ; σ , electrical conductivity of the fluid ; μ_e , permeability of the medium ;

$\nu_H = 1/(\sigma\, \mu_e)$, Magnetic diffusivity ; K , Proportionality constant ;

and T, fluid temperature .

we define the pressure p as

$$p = \rho g\{x \sin\theta - y\cos\theta\} + \rho x\, a(t) + \{1/(2\mu_e)\}\, B^2 + A \qquad 7.14$$

where A is a constant and a is any function of time.

Using 7.14 , the equation 7.9, is written as

$$\partial u/\partial t = -a(t) + \nu_1 \partial^2 u/\partial y^2 + \nu_2 \partial/\partial t\,(\partial^2 u/\partial y^2)$$

$$-(\nu_1/k_1)u + (K N/\rho)(v - u) + \{B_0/(\rho\mu_e)\}\,\partial B/\partial y \qquad 7.15$$

The boundary conditions of the problem are

$$u = v = 0\ , \quad B = 0\ , \quad T = T_0\, e^{-2nt}\ , \quad \text{at } y = -h$$

$$u = u_0\, e^{-nt}\ , \quad v = v_0\, e^{-nt}\ , \quad B = 0\ , \quad T = T_1 e^{-2nt}\ , \quad \text{at } y = +h \qquad 7.16$$

where T_0 and T_1 are the temperatures at the plates $y = +h$ & $y = -h$ respectively and n is a real number denoting decay parameter of the plate temperatures.

We consider following non-dimensional parameters

$$u^* = u/u_0\ ;\quad v^* = v/v_0\ ;\quad y^* = y/h\ ;\quad t^* = t\, u_0/h\ ;\quad a^* = ah/u_0^2\ ;$$

$$T^* = T/T_0\ ;\quad b = B/B_0\ ;\quad k^* = h/\sqrt{k_1}\ ;\quad \lambda = (u_0/v_0)\ ; \qquad 7.17$$

Substituting 7.17 in equations 7.10 - 7.14 , and then removing asterisks , we get

$$\partial u/\partial t = -a(t) + (1/R)\partial^2 u/\partial y^2 - \eta\{\partial/\partial t(\partial^2 u/\partial y^2)\}$$

$$-(k^2/R)u + \{C/R_t\}\{(v/\lambda) - u)\} + \{M^2/(R^2 R_m P_r)\}\partial b/\partial y \qquad 7.18$$

$$R_t\ (\partial v/\partial t) - (\lambda u - v) = 0 \qquad 7.19$$

$$\partial^2 T/\partial y^2 = (R P_r)\partial T/\partial t - \{E.P_r(\partial u/\partial y)^2\}$$

$$+[\eta . E . R . P_r\{\partial/\partial t(\partial u/\partial y)^2\}] - [\{M^2 .E/(R_m^2 R^2 P_r)\}(\partial b/\partial y)^2] \qquad 7.20$$

$$\partial b/\partial t + (\partial u/\partial y) + \{1/(R. R_m . P_r)(\partial^2 b/\partial y^2)\} = 0 \qquad 7.21$$

where $R = (u_0\ h / \nu_1)$, Reynolds number ; $\eta = (-\nu_2 / h^2)$, Visco-elastic parameter; $C = (m.N/\rho)$, Dust particle concentration ; $R_t = \{m.u_0 / (K .h)\}$, Relaxation time parameter of dust particles ; $M = \surd\{(B_0^2\ h^2 \sigma) / \rho\nu_1\}$, Magnetic Hartmann number ; $P_r = \nu_1 / \alpha$, Prandlt number ; $R_m = (\mu_e \sigma \alpha)$, Magnetic Reynolds number ; $E = \{u_0^2 / (C_p T_0)\}$, Eckert number :

Using 7.17 in 7.16 , the non-dimensional boundary conditions are

$$u = v = 0, \quad T = e^{-2nt}, \quad b = 0; \quad \text{at } y = -1$$
$$u = e^{-nt}, \quad v = e^{-nt}, \quad T = \chi, \quad b = 0 \quad \text{at } y = +1 \qquad 7.22$$

where $\chi = (T_1 / T_0)$, is a constant temperature .

7.3 SOLUTIONS OF GOVERNING EQUATIONS :

In order to solve the equations 7.18 - 7.21 under the boundary conditions 7.22 , we consider

$$u = f(y)\, e^{-nt} \quad , \quad v = g(y)\, e^{-nt} \, , \; T = F(y)\, e^{-2nt} \, ,$$

$$b = G(y)\, e^{-nt} \, , \; a = a_0\, e^{-nt} \; ; \qquad 7.23$$

Substituting 7.23 in equations 7.18 - 7.21 , we get

$$d^2 f(y) / d y^2 + A_1 f(y) + A_7 (d G(y) / dy) - A_2 = 0 \qquad 7.24$$

$$g(y) = \{\lambda f(y) / (1 - n . R_t)\} \qquad 7.25$$

$$d^2 F(y) / d y^2 + A_4 F(y) + \{A_5 (d f(y) / dy)^2 + A_6 (dG(y) / d y)^2\} = 0 \qquad 7.26$$

$$d^2 G(y) / d y^2 - n A_3 G(y) + A_3 (d f(y) / dy) = 0 \qquad 7.27$$

where , $A_1 = R / (1 + n \eta R) \{ n - K^2 / R + C . n / (1 - n R_t)\}$;

$A_2 = \{ R\, a_0 / (n \eta R + 1)\}$; $A_3 = (R . R_m . P_r)$;

$A_4 = (2.n.R.P_r)$; $A_5 = \{ E.P_r (1 + 2.n. \eta . R)$;

$A_6 = \{(M^2 .E) / (R^2 R_m^2\; P_r)\}$; $A_7 = [1 / (1 + n \eta R) \{M^2 / (R . R_m P_r)\}]$;

The boundary conditions 7.22, are now

$$f(-1) = g(-1) = G(-1) = 0 \quad , \quad F(-1) = 1 \quad \&$$

$$f(+1) = 1 \, , \quad g(+1) = (\lambda / A_{10}) \, , \quad F(+1) = \chi \, , \quad G(+1) = 0 \qquad 7.28$$

The solutions of equations 7.24 - 7.27, subject to the boundary conditions 7.28, are given as

$$f(y) = [C_1 Q_1 e^{m1 y} + C_2 Q_2 e^{m2 y} + C_3 Q_3 e^{m3 y} + C_4 Q_4 e^{m4 y} + C_5] \qquad 7.29$$

$$g(y) = (1/A_{10})[C_1 Q_1 e^{m1 y} + C_2 Q_2 e^{m2 y} + C_3 Q_3 e^{m3 y} + C_4 Q_4 e^{m4 y} + C_5] \qquad 7.30$$

$$G(y) = [C_1 e^{m1 y} + C_2 e^{m2 y} + C_3 e^{m3 y} + C_4 e^{m4 y} + C_5] \qquad 7.31$$

$$F(y) = \{C_6 \cos\sqrt{A_4}.y + C_7 \sin\sqrt{A_4}.y\} - [S_1 e^{2m1 y} + S_2 e^{2m2 y} + S_3 e^{2m3 y}$$
$$+ S_4 e^{2m4 y} + S_5 e^{(m1 + m2)y} + + S_6 e^{(m1 + m3)y} + + S_7 e^{(m1 + m4)y}$$
$$+ S_8 e^{(m3 + m2)y} + S_9 e^{(m4 + m2)y} + S_{10} e^{(m3 + m4)y}] \qquad 7.32$$

where, $A_8 = (A_1 - n A_3 - A_3 A_7)$; $A_9 = (n A_1 . A_3)$; $A_{10} = \lambda / (1 - n R_t)$;

$m_1 = \sqrt{\{-A_8 + \sqrt{(A_8^2 + 4A_9)}\}/2}$; $m_2 = \sqrt{\{-A_8 - \sqrt{(A_8^2 + 4 . A_9)}\}/2}$;

$m_3 = -\sqrt{\{-A_8 + \sqrt{(A_8^2 + 4 . A_9)}\}/2}$; $m_4 = -\sqrt{\{-A_8 + \sqrt{(A_8^2 + 4 A_9)}\}/2}$;

$C_4 = (L_{10} + L_9 C_5) / L_8$; $C_3 = (L_4 C_4 + f_0 - C_5) / L_3$; $C_2 = (L_1 C_3 + L_2 C_4)$

$C_1 = - e^{-m1} [C_2 e^{-m2} + C_3 e^{-m3} + C_4 e^{-m4}]$; $Q_1 = \{n / m_1 - m_1 / A_3\}$;

$Q_2 = \{n / m_2 - m_2 / A_3\}$; $Q_3 = \{n / m_3 - m_3 / A_3\}$; $Q_4 = \{n/ m_4 - m_4 / A_3\}$;

$L_1 = \{e^{-m3} - e^{(m3 - 2m1)}\} / \{e^{(m2 - 2m1)} - e^{-m2}\}$;

$L_2 = \{e^{-m4} - e^{(m4 - 2m1)}\} / \{e^{(m2 - 2m1)} - e^{-m2}\}$;

$$L_3 = \{ Q_2 L_1 e^{m2} + Q_3 e^{m3} - Q_1 (L_1 e^{m2} + e^{m3}) \} ;$$

$$L_4 = \{ Q_1 (L_2 e^{m2} + e^{m4}) - Q_2 L_2 e^{m2} - Q_4 e^{m4} \} ;$$

$$L_5 = \{L_4 (L_1 e^{m2} + e^{m3}) / L_3 + (L_2 e^{m2} + e^{m4}) \} ;$$

$$L_6 = (L_1 e^{m2} + e^{m3})/ L_3 , \ L_7 = (L_6 . f_0) ;$$

$$L_8 = \{ (L_1 L_4 /.L_3 + L_2)Q_2 e^{-m2} + L_4 / L_3 Q_3 e^{-m3}$$
$$+ Q_4 e^{-m4} - L_5 Q_1 e^{-2m1} \} ;$$

$$L_9 = \{ (L_1 /.L_3 Q_2 e^{-m2} + Q_3 L_3 e^{-m3} - L_6 Q_1 e^{-2m1} - 1 \} ;$$

$$L_{10} = \{ L_7 Q_1 e^{-2m1} - .L_1 /.L_3 f_0 Q_2 e^{-m2} - f_0 / L_3 Q_3 e^{-m3} \} ;$$

$$L_{11} = [\{ (L_1 .L_4 /.L_3 + L_2) L_9 /.L_8 - L_1 /.L_3 \} Q_2 e^{m2}$$
$$+ \{ (L_9 .L_4)/ (.L_3 L_8) - 1/ L_3 \} Q_3 e^{m3} + (L_9 / L_8) Q_4 e^{m4}$$
$$- \{(L_9 L_5) / L_8 - L_6 \}Q_1 + 1] ;$$

$$L_{12} = [\{ (L_{10} L_5) / L_8 + L_7 \} Q_1 - \{ (L_1 .L_4 /.L_3 + L_2) (L_{10} / L_8)$$
$$L_1 /.L_3 f_0 \} Q_2 e^{m2} - \{ (L_{10} .L_4)/ (.L_3 L_8) + f_0 / L_3 \} Q_3 e^{m3}$$
$$- (L_{10} / L_8) Q_4 e^{m4}] ;$$

$$S_1 = (A_6 + Q_1^2 A_5) / (4m_1^2 + A_4) m_1^2 C_1^2$$

$$S_2 = (A_6 + Q_2^2 A_5) / (4 m_2^2 + A_4) m_2^2 C_2^2 ;$$

$$S_3 = (A_6 + Q_3^2 A_5) / (4m_3^2 + A_4) m_3^2 C_3^2 ;$$

$$S_4 = (A_6 + Q_4^2 A_5) / (4m_4^2 + A_4) m_4^2 C_4^2 ;$$

$$S_5 = 2 m_1 m_2 C_1 C_2 [(A_6 + A_5 Q_1 Q_2) / \{(m_1 + m_2)^2 + A_4 \}] ;$$

$$S_6 = 2 m_1 m_3 C_1 C_3 [(A_6 + A_5 Q_1 Q_3) / \{(m_1 + m_3)^2 + A_4 \}] ;$$

$$S_7 = 2 m_1 m_4 C_1 C_4 [(A_6 + A_5 Q_1 Q_4) / \{(m_1 + m_4)^2 + A_4 \}] ;$$

$$S_8 = 2\, m_2\, m_3\, C_2\, C_3\, [\, (A_6 + A_5\, Q_2\, Q_3)\ /\ \{(m_2 + m_3)^2 + A_4\}]\quad ;$$

$$S_9 = 2\, m_2\, m_4\, C_2\, C_4\, [\, (A_6 + A_5\, Q_2\, Q_4)\ /\ \{(m_2 + m_4)^2 + A_4\}]\quad ;$$

$$S_{10} = 2\, m_3\, m_4\, C_3\, C_4\, [\, (A_6 + A_5\, Q_3\, Q_4)\ /\ \{(m_3 + m_4)^2 + A_4\}]\quad ;$$

$$C_7 = 1\ /\ (\sin\sqrt{A_4}\,)\ \{\, S_1 \sinh(2m_1) + S_2 \sinh(2m_2)$$
$$+ S_3 \sinh(2\, m_3) + S_4 \sinh(2m_4) + S_5 \sinh(m_1 + m_2)$$
$$+ S_6 \sinh(m_1 + m_3) + S_7 \sinh(m_1 + m_4) + S_8 \sinh(m_3 + m_2)$$
$$+ S_9 \sinh(m_4 + m_2) + S_{10} \sinh(m_3 + m_4) + (\chi - 1)/2\ \}\ ;$$

$$C_6 = (1/\cos\sqrt{A_4}\,)\,[\ \chi - C_7 \sin\sqrt{A_4} + \{\, S_1 e^{2m1} + S_2 e^{2m2} + S_3 e^{2m3}$$
$$+ S_4 e^{2m4} + S_5 e^{(m1+m2)} + S_6 e^{(m1+m3)} + S_7 e^{(m1+m4)}$$
$$+ S_8 e^{(m3+m2)} + S_9 e^{(m4+m2)} + S_{10} e^{(m3+m4)}\ \}\,]\quad ;$$

<u>*SKIN FRICTION AT THE PLATES*</u> :

The viscous drag acting at the plates for fluid (τ_f) and for the particles (τ_p) are defined as

$$\tau_f = [\ \{\ 1/R - \eta\ (\partial/\partial t)\}\ (\partial u/\partial y)\ ]_{y=\pm 1} \qquad 7.33$$

$$= [\ \{\ 1/R - \eta\ (\partial/\partial t)\}\ (e^{-nt}\ \partial f/\partial y)\ ]_{y=\pm 1}$$

$$= (1/R + n\,\eta)\,[C_1 Q_1 m_1 e^{\pm m1} + C_2 Q_2 m_2 e^{\pm m2} + C_3 Q_3 m_3 e^{\pm m3}$$
$$+ C_4 Q_4 m_4 e^{\pm m4}\ ]\, e^{-nt} \qquad 7.34$$

and $$\tau_p = [\ \{\ 1/R - \eta\ (\partial/\partial t)\}\ (\partial v/\partial y)\ ]_{y=\pm 1} \qquad 7.35$$

$$= [\ \{\ 1/R - \eta\ (\partial/\partial t)\}\ (e^{-nt}\ \partial g/\partial y)\ ]_{y=\pm 1}$$

$$= \{ (1/R + n\eta)(\lambda / A_{10}) \} [C_1 Q_1 m_1 e^{\pm m1} + C_2 Q_2 m_2 e^{\pm m2}$$

$$+ C_3 Q_3 m_3 e^{\pm m3} + C_4 Q_4 m_4 e^{\pm m4}] e^{-nt} \qquad 7.36$$

FLOW FLUX FOR FLUID AND PARTICLES :

The flux of flow for fluid (ϕ_f) and the particles (ϕ_p) through the channel are represented as

$$\phi_f = \int_{-1}^{1} u \, dy$$

$$= e^{-nt} \int_{-1}^{1} f(y) \, dy \qquad 7.37$$

$$= 2 [(C_1 Q_1 / m_1) \sinh m_1 + (C_2 Q_2 / m_2) \sinh m_2 +$$

$$(C_3 Q_3 / m_3) \sinh m_3 + (C_4 Q_4 / m_4) \sinh m_4 + C_5] e^{-nt} \qquad 7.38$$

and $$\phi_p = \int_{-1}^{1} v \, dy$$

$$= e^{-nt} \int_{-1} g(y) \, dy \qquad 7.39$$

$$= (2\lambda / A_{10}) [(C_1 Q_1 / m_1) \sinh m_1 + (C_2 Q_2 / m_2) \sinh m_2$$

$$+ (C_3 Q_3 / m_3) \sinh m_3 + (C_4 Q_4 / m_4) \sinh m_4 + C_5] e^{-nt} \qquad 7.40$$

HEAT TRANSFER :

Using 1.79 the rate of heat transfer in terms of the Nusselt number (N_u) at the plates, which is proportional to the local temperature gradient, written as

$$N_u = [\partial T / \partial y]_{y = \pm 1} \qquad 7.41$$

$$= e^{-2nt} [\partial F / \partial y]_{y = \pm 1} \qquad 7.42$$

$$= [\sqrt{A_4}\ \{ C_7 \cos\sqrt{A_4} - (\pm C_6 \sin\sqrt{A_4}\ \} + 2\ \{m_1 S_1 e^{\pm 2 m1} + m_2 S_2 e^{\pm 2 m2}$$

$$+ S_3 m_3 e^{\pm 2 m3} + S_4 m_4 e^{\pm 2 m4}\ \} + \{S_5 (m_1 + m_2)e^{\pm (m1 + m2)} + S_6 (m_1 +$$

$$m_3) e^{\pm (m1 + m3)} + S_7 (m_1 + m_4)e^{\pm (m1 + m4) y} + S_8 (m_2 + m_3) e^{\pm (m3 + m2)}$$

$$+ S_9 (m_4 + m_2)e^{\pm (m4 + m2)} + S_{10} (m_3 + m_4)e^{\pm (m3 + m4)} \} e^{-nt}]$$

7.43

Table (I)

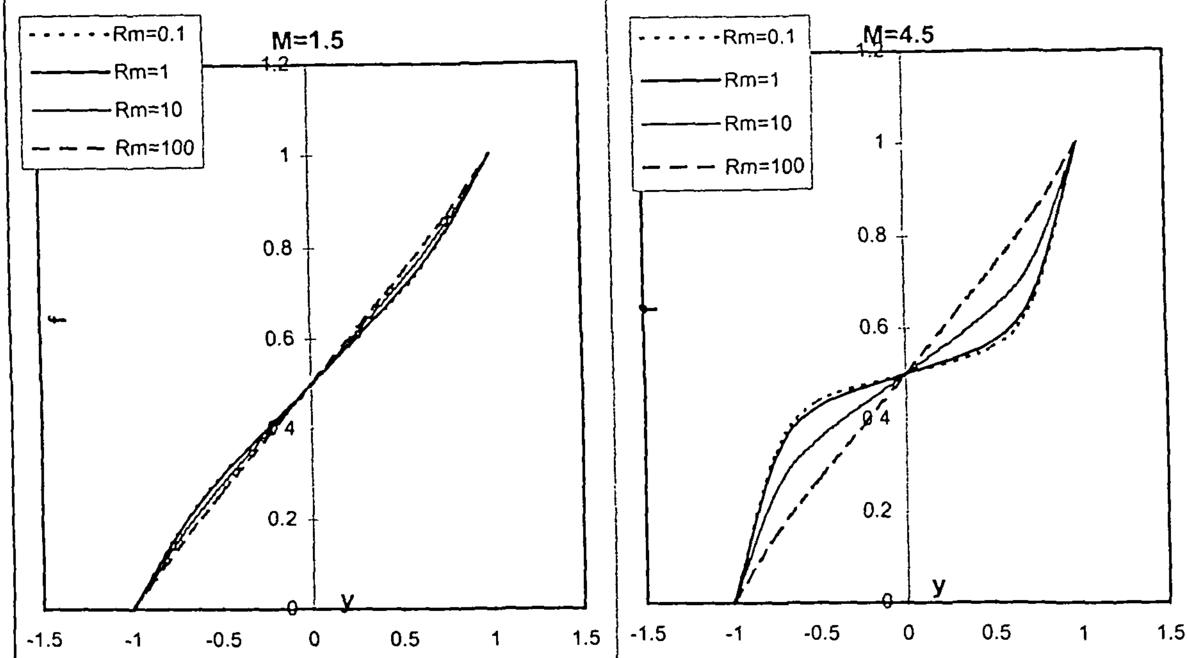
TEMPERATURE DISTRIBUTION (T) WITH R_m AT M = 1.5 & 4.5

C_5 = 0.5, R=0.5, n =1, λ =1.5, E =0.01, η =0.1, C =0.1,
A_0 =1.0 , R_t=0.1, χ =2.0, K= 0.5

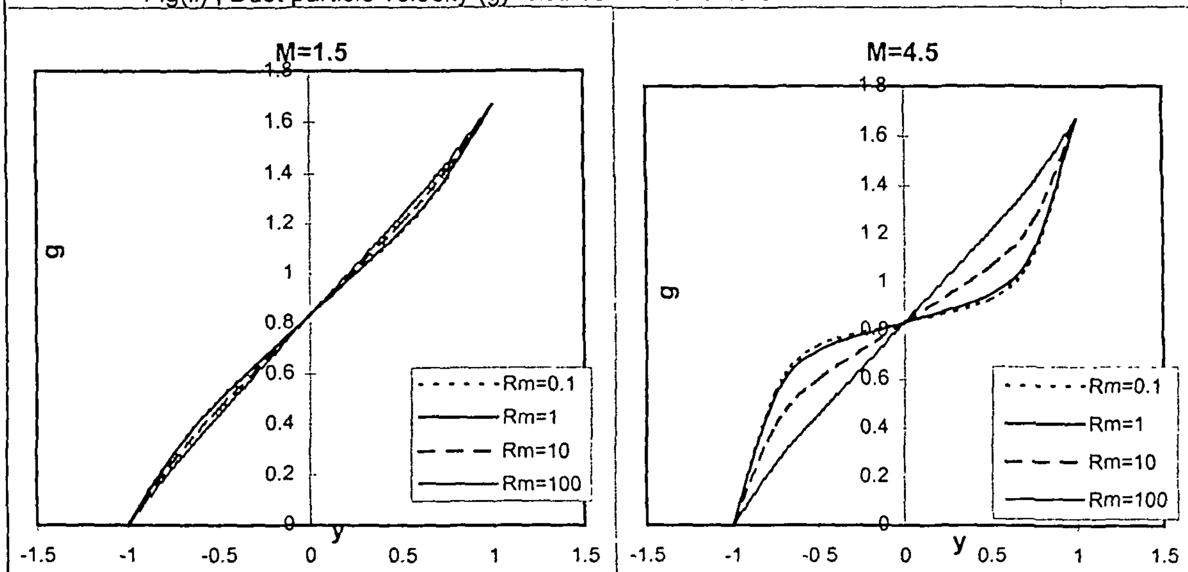
	M = 1.5				M = 4.5			
Y	R_m=0.1	R_m=1.0	R_m=10.0	R_m=100.0	R_m=0.1	R_m=1.0	R_m=10.0	R_m=100.0
-1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
-0.75	1.4238	1.4237	1.4235	1.4234	1.4239	1.4238	1.4235	1.4234
-0.50	1.7843	1.7842	1.7838	1.7838	1.7841	1.7841	1.7837	1.7838
0.00	2.2557	2.2556	2.2552	2.2552	2.2553	2.2552	2.2550	2.2551
0.50	2.3322	2.3321	2.3318	2.3317	2.3321	2.3320	2.3317	2.3317
0.75	2.2152	2.2152	2.2149	2.2149	2.2154	2.2153	2.2150	2.2149
1.00	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000

C5=.5 , R=.5 , n=1 , λ=1.5 , a =1 , Pr=.71 , k=.5 , E=.01 , C=.1 , Rt=.1 , χ =2.0 ,

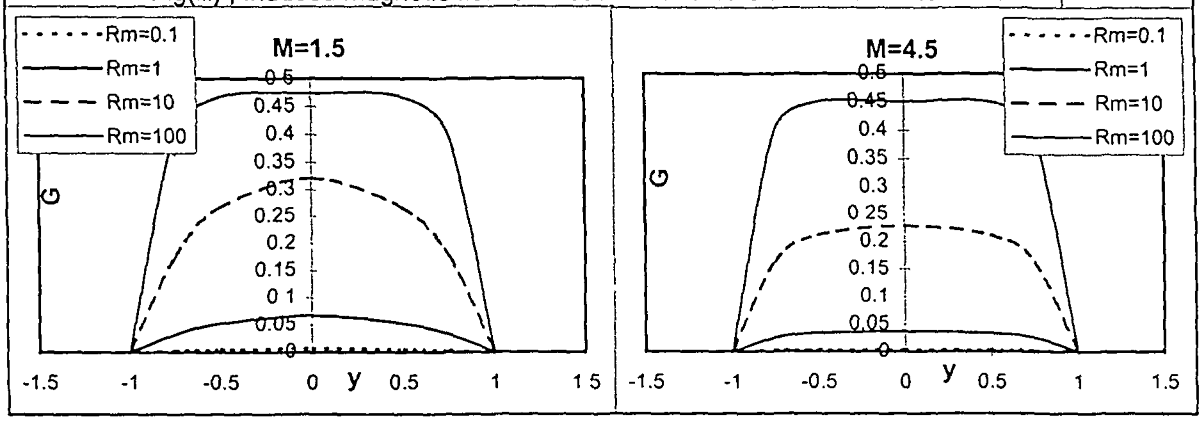
Fig(i) , Fluid velocity (f) distribution for different Rm at M=1.5 & 4.5



'Fig(ii) , Dust particle velocity (g) distribution for different Rm at M=1.5 & 4.5



'Fig(iii) , Induced magnetic field distribution for different Rm at M=1.5 & 4.5



7.4 RESULTS AND DISCUSSION :

The aim of our study is to investigate the effects of magnetic field parameters (i.e., the magnetic Hartmann number , M and the magnetic Reynolds number , R_m) on velocity and temperature distribution for fluid and particles . Numerical results of equations (7.29 - 7.23) are obtained for constant values of R (= 0.5') , n (=1.0) , Pr = 0.71, λ =1.5 , η (= 0.1) , E = (0.01) , C (= 0.1) , a_0 (= 1.0) , χ = 2.0 , K (= 0.5) and C_5 (= 0.5) . Figures (i- iii) show distributions of fluid velocity , particle velocity , induced field and the temperature within the channel for M = 1.5 & 4.5 and magnetic Reynolds number R_m = (0.1 , 1.0 , 10.0 , 100.0) ,

Following results are observed from the figures and the table (I).

1. *Velocity distribution* ; fig (i & ii):

(i) For constant value of M , f & g decrease with the increase of R_m but at the central plane of the channel (y = 0) , f & g remains same for all values of R_m

(ii) At constant R_m , f & g increase gradually with the increase of M , but on the central plane of the channel (y = 0) remains same for all values of M & R_m .

2. *Induced field distribution*: fig (iii):

(i) At constant M , G increases with the increase of R_m throughout the channel, is maximum in the central plane and decreases gradually towards the plates .

(ii) For constant value of R_m , G decreases with the increase of M .

3. *Temperature distribution;[Table (I)]*

(i) While M is constant, T decreases with the increase of R_m and in the higher range ($10 \le R_m \le 100$) rate of decrease is very small .

(ii) At a particular value of R_m and M , T is maximum in the plane (y = 0.5 approx.).

(iiI) When R_m remains constant , T decreases slowly with the increase of M but for higher values of R_m , T almost remains same .

7.5 CONCLUSIONS:

- When the applied magnetic field remains unchanged fluid velocity , particle velocity , and the fluid temperature decrease with the increase of magnetic Reynolds number , and when applied field increases they increase for constant value of magnetic Reynolds number .
- Induced field increases with the increase of magnetic Reynolds number when applied field is remain unchanged whereas induced field decreases for constant value of magnetic Reynolds number when applied field increases.
- At the central plane of the channel both fluid and particle velocity are independent to the variations of magnetic Hartmann number and the magnetic Reynolds number.
- At higher values of magnetic Reynolds number, the temperature variation is almost independent to the variation of applied magnetic field.

LIST OF PAPERS PUBLISHED / ACCEPTED / SENT FOR PUBLICATION

1. " **EFFECT OF UNIFORM MAGNETIC FIELD ON AN UNSTEADY FLOW DUE TO AN EXPONENTIALLY DECAY SOURCE BETWEEN TWO INFINITE PARALLEL DISKS** " has been published in "*JOURNAL OF THEORETICAL AND APPLIED MECHANICS* " Vol.24, pp.13-27, 1998, UDK, 532.517.2, Institute of Mahaniku ; 2100 Navi-sad ; YUGOSLAVIA.

2. "**EFFECT OF VARIABLE VISCOSITY ON LAMINAR CONVECTION FLOW OF AN ELECTRICALLY CONDUCTING FLUID ON A UNIFORM MAGNETIC FIELD** " is accepted for publication in "*JOURNAL OF THEORETICAL AND APPLIED MECHANICS* " Institute of Mahaniku ; 2100 Navi-sad ; YUGOSLAVIA.

3. "**LAMINAR CONVECTION FLOW OF ELECTRICALLY CONDUCTING FLUID BETWEEN TWO PARALLEL PLATES** " is accepted for publication in "*JOURNAL OF THEORETICAL AND APPLIED MECHANICS* " Institute of Mahaniku ; 2100 Navi-sad ; YUGOSLAVIA.

4. " **MHD FLOW AND HEAT TRANSFER OF A DUSTY VISCO-ELASTIC LIQUID DOWN AN INCLINED CHANNEL IN POROUS MEDIUM** " is accepted for publication in " *GANITA* " Indian Association of Mathematics, Lucknow University Lucknow INDIA

5. " **UNSTEADY FREE CONVECTION MHD FLOW BETWEEN TWO HEATED VERTICAL PARALLEL PLATES** " has sent for publication to "*INDIAN JOURNAL OF THEORETICAL PHYSICS* " Institute of theoretical physics, Bignan kutir, Mohan Bagan Lane Calcutta, ; INDIA .

6. " **EFFECT OF INDUCED MAGNETIC FIELD ON AN UNSTEADY FREE CONVECTION MHD FLOW BETWEEN TWO HEATED VERTICAL PARALLEL PLATES** " is accepted for publication in "*INDIAN JOURNAL OF PURE AND APPLIED MATHEMATICS* " Indian Academy of Sciences ; New-Delhi ; INDIA.

7. " **MHD FLOW AND HEAT TRANSFER ON OF A DUSTY VISCO-ELASTIC STRATIFIED FLUID DOWN AN INCLINED CHANNEL IN POROUS MEDIUM UNDER VARIABLE VISCOSITY** " is accepted for publication in "*INDIAN JOURNAL OF THEORETICAL PHYSICS* " Institute of theoretical physics, Bignan kutir, Mohan Bagan Lane Calcutta, ; INDIA .

8. " ***SORET EFFECT* AND MASS TRANSFER PAST AN INFINITE VERTICAL POROUS PLATE** " has been sent for publication to "*PURE AND APPLIED MATHEMATIKA* " Gaziabad , New Delhi ; INDIA .

9. " **INDUCED MAGNETIC FIELD AND HEAT TRANSFER ON MHD FLOW OF A DUSTY VISCO-ELASTIC FLUID DOWN AN INCLINED CHANNEL IN POROUS MEDIUM**" has been sent for publication in "*INDIAN JOURNAL OF THEORETICAL PHYSICS* " Institute of theoretical physics, Bignan kutir, Mohan Bagan Lane Calcutta, ; INDIA .

10. " **MHD FREE CONVECTION AND MASS TRANSFER WITH SORET EFFECT AND INDUCED MAGNETIC FIELD PAST AN INFINITE VERTICAL POROUS PLATE** " has been sent for publication to "*INDIAN JOURNAL OF THEORETICAL PHYSICS* " Institute of theoretical physics, Bignan kutir, Mohan Bagan Lane Calcutta, ; INDIA .

11. " **FREE CONVECTION MHD FLOW THROUGH POROUS MEDIUM BETWEEN TWO HEATED VERTICAL PARALLEL PLATES** " has been sent for publication to "*JOURNAL OF ASSAM SCIENCE SOCIETY* " Lamb Road , Guwahati, Assam, INDIA.

12. " **UNSTEADY FREE CONVECTION MHD FLOW BETWEEN TWO HEATED VERTICAL PARALLEL PLATES IN INDUCED MAGNETIC FIELD**" has been sent for publication to "*BULLETINE OF PURE AND APPLIED SCIENCES* " New Delhi ; INDIA .

PAPER PRESENTED IN CONFERENCE

1. "**A NOTE ON MHD FLOW AND HEAT TRANSFER OF A DUSTY VISCO-ELASTIC LIQUID DOWN AN INCLINED CHANNEL IN POROUS MEDIUM** " has been read out in annual the conference of Indian Association of Mathematics, 1997, Lucknow University, Lucknow .

2. " **INDUCED MAGNETIC FIELD AND HEAT TRANSFER ON MHD FLOW OF A DUSTY VISCO-ELASTIC FLUID DOWN AN INCLINED CHANNEL IN POROUS MEDIUM** " has been read out in annual the conference of Indian Association of Mathematics, 1997, Lucknow University, Lucknow .

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