SOME PROBLEMS OF ELECTRICALLY CONDUCTING FLUID FLOW AND EFFECT OF HEAT TRANSFER IN MAGNETOHYDRODYNAMICS

THESIS

Submitted to Tezpur University in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Mathematics

By

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CERTIFICATE

This is to certify that the thesis entitled "***Some problems of electrically conducting fluid flow and effect of heat transfer in magnetohydrodynamics***" which is being submitted by Mr. Gwjwn Bodosa, Department of Mathematical Sciences, Tezpur University, for the award of the degree of Doctor of Philosophy to the same University, Tezpur, Assam is a record of bonafide research work carried out by him under my supervision and guidance. He has worked for three years and has fulfilled all the requirements for the degree of Doctor of Philosophy of Tezpur University.

The results embodied in the thesis have not been submitted to any other University or Institution for the award of any degree or diploma.

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9.08.04

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THIS DEDICATES TO MY

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SYNOPSIS

We discuss a few problems of the effect of heat transfer and incompressible electrically conducting fluid flows specially paying our attention when a uniform transverse magnetic field is applied. Magnetohydrodynamics is defined as that which deals with the dynamics of an electrically conducting fluid flow (e.g. mercury, copper sulphate solution, etc.) in presence of a magnetic field. The motion of the electrically conducting fluid through the magnetic field experiences electric currents which change the magnetic field, and in the presence of magnetic field on these currents, it gives rise to mechanical forces which modify the flow of the conducting fluid.

Magnetic fields influence many natural and man-made flows. They are routinely used in industry to heat, pump, stir and levitate liquid metals. There is the terrestrial magnetic field which is maintained by fluid motion in the earth's core, the solar magnetic field which generates sunspots and solar flares, and the galactic magnetic field which is thought to influence the formation of stars from interstellar clouds. The study of these flows is called magnetohydrodynamics. Formerly, MHD is concerned with the mutual interaction of the fluid flow and magnetic fields. The fluids must be electrically conducting and non-magnetic, which limits us to liquid metals, hot ionised gases (plasmas) and strong electrolytes. Magnetohydrodynamics (MHD for short) is the study of the interaction between magnetic fields and moving, conducting fluids. It is of importance in connection with many engineering problems as well as in geophysics and astronomy.

The mutual interaction of a magnetic field $\vec{B}$ and a velocity field $\vec{u}$ arises partially as results of the laws of Faraday's and Amp'ere, and partially because of the Lorentz force experienced by a current-carrying body. It is convenient to split the process into the following three parts:

(i) The relative movement of a conducting fluid and magnetic field causes an e.m.f. (of order $|\vec{u} \times \vec{B}|$) to develop in accordance with Faraday's law of induction. In general, when the current density is of order $\sigma(\vec{u} \times \vec{B})$, where σ is the electrically conducting, electric currents will ensue.

(ii) According to Amp'ere's law, these induced currents must give rise to a second, induced magnetic field. This adds to the original magnetic field and the change is usually such that the fluid appears to "drag" the magnetic field lines along with it.

iii) The combined magnetic field (imposed plus induced) interacts with the induced current density, $\vec{J}$ to give rise to a Lorentz force (per unit volume), $(\vec{J} \times \vec{B})$. This acts on the conductor and is generally directed so as to inhibit the relative movement of the magnetic field and the fluid.

If the fluid is non-conducting or the velocity is negligible, there will be no significant induced magnetic field. Conversely, if σ or $\vec{u}$ are large (in some sense), then the induced magnetic field may substantially alter the imposed magnetic field. If it is a poor conductor or moves very slowly, then the induced current and the associated magnetic field will be weak. Conducting fluid usually contains neutral particles and positive and negative charges. So the fluid is neutral in the large and the gaseous fluid referred as plasma. Thus the uniform of the plasma does not constitute the electric current.

The thesis will be dealt with the theoretical investigations of electrically conducting fluid flow and the effect of heat transfer in magnetohydrodynamics problems. In most of the fluid flows, the velocity field and temperature field mutually interact which means that the temperature distribution depends on the velocity distribution. Conversely, the velocity distribution depends on the temperature distribution.

In these cases where the buoyancy forces are disregarded and the properties of the fluid may be assumed to be independent of temperature, the velocity field does not depend on the temperature field, while the dependence of temperature field on the velocity field persists. Such flows are termed as forced flow and the process of heat transfer in such flows is described as forced convection. Flows in which buoyancy forces are dominant are called natural flow and corresponding heat transfer through such natural flow is known as natural convection. If the natural convection is not constrained to a finite region by boundaries, it is called free convection.

In magnetohydrodynamics, the flow of electrically conducting fluid in presence of an applied magnetic field is considered. The magnetic field induces current due to the motion of the conducting fluids which in turn modifies the applied magnetic field, while the electromagnetic Lorentz force resists the fluid motion. The wide application of the subject has been seen in Geophysics, Astrophysics, Aeronautics and many other engineering branches.

The thesis will consist of seven chapters. The chapter-I is going to be dealt with the introduction of the thesis. The outline of the magnetohydrodynamics, its development and applications, fundamental equations of electrically conducting fluid flow and effect of heat transfer in MHD have been discussed in this chapter.

During the past two decades, a number of significant experiments have been carried out revealing non-Newtonian characteristics of liquids where a number of new phenomenon have been observed in a large number of liquids, of great technological and industrial importance. A brief description of these liquids is also given in this chapter. Lastly, a brief review of earlier workers and scope of this work have also been explained in this chapter.

The laminar free convection flow of an incompressible electrically conducting second order fluid under the action of uniform transverse magnetic field over a plate has been discussed in the chapter-II. Exact solutions of the fluid velocity $\bar{u}(y,t)$ and temperature profile $\bar{T}(y,t)$ can be obtained with the help of perturbation technique, where y is the distance measured of the plate and t is the time. It has been observed that this problem is useful in many engineering problems and hence our research may be useful.

The unsteady Couette flow of a viscous incompressible and electrically conducting fluid with the heat transfer between two horizontal parallel plates in the presence of a uniform transverse magnetic field has been discussed in the chapter-III, when in the case-1, the plates are at different temperatures and in the case-2, the upper plate is considered to move with the constant velocity where the lower plate is adiabatic. Our results are useful in geophysical and astrophysical problems as the simultaneous effects of hydromagnetic, buoyancy forces and coriollis forced are observed in various types of problems in these branches of sciences.

A theoretical and numerical analysis of unsteady two dimensional free convection flow of a viscous incompressible electrically conducting fluid through a porous medium due to infinite vertical plate with uniform suction and constant heat flux under the action of a uniform magnetic field has been investigated in the chapter-IV. The effects of Prandtl number, Grashoff number, magnetic parameter and the variable permeability of porous medium on the velocity and temperature profile have been discussed and shown graphically.

In chapter-V, we have discussed the motion of the unsteady MHD flow of an incompressible electrically conducting viscous fluid between two horizontal parallel porous plates on the time-varying motion. The velocity profile and skin-friction are obtained due to the effect of the deflection of a strong magnetic field on the MHD flow past between two parallel plates and the results are obtained and plotted graphically by taking the different values of the magnetic field parameter.

We have discussed in the chapter-VI, the MHD unsteady flow of a visco-elastic (Rivlin-Ericksen) fluid through an inclined channel with two parallel flat plates with heat transfer including heat generating sources or heat absorbing sinks, when the plates are moving with the transient velocity while the one of these two plate is adiabatic. Here the fluid velocity and the temperature profile are obtained by the Perturbation technique and discussed by interpreting the graphs with the help of different values of some appeared non-dimensional parameters.

In the chapter-VII, we have studied the unsteady flow of an incompressible electrically conducting second order fluid through the porous medium due to infinite horizontal plate in the presence of uniform transverse magnetic field which includes the heat generating sources or heat absorbing sinks. Here the plates are maintained at temperatures while one plate is kept at a constant temperature gradient. The values of the velocity and temperature distribution are found out numerically and interpreted with the help of graph.

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Magnetic field effects on the fluid and free convection flow through porous medium due to infinite vertical plate with uniform suction and constant heat flux.

CHAPTER 5

The motion of the electrically conducting fluid with the time-variation through the non-conducting porous plate under the action of magnetic field.

1 INTRODUCTION

1.1 Magnetohydrodynamics (MHD), its flows and applications

We have discussed in this thesis a few problems of the effect of heat transfer and electrically conducting fluid flows specially paying our attention when a transverse magnetic field is applied. Hence in this chapter, we have given a brief account of the effect of heat transfer and magnetohydrodynamics flows. We have also mentioned the works of other Scientists related to the problems attempted in this thesis. In the last article of this chapter, we have given the motivation, extent and scope of our works.

MHD is defined as that which deals with the dynamics of an electrically conducting fluid (e.g. mercury, copper sulphate solution, etc.) in presence of a magnetic field. The motion of the electrically conducting fluid through the magnetic field experiences electric currents which change the magnetic field, and in the presence of magnetic field on these currents, it gives rise to mechanical forces which modify the flow of the conducting fluid.

Magnetohydrodynamics (MHD for short) is the study of the interaction between magnetic fields and moving, conducting fluids. Magnetic fields influence many natural and man-made flows. Formally, MHD is concerned with the mutual interaction of fluid flow and magnetic fields. The fluid must be electrically conducting and non-magnetic, which limits us to liquid metals, hot ionised gases (plasmas) and strong electrolytes.

The mutual interaction of a magnetic field $\vec{B}$ and a velocity field $\vec{u}$ arises partially as a results of the laws of Faraday's and Amp$'$ere, and partially because of the Lorentz force experienced by a current-carrying body. It is convenient to split the process into the following three parts:

(i) The relative movement of a conducting fluid and magnetic field causes an e.m.f. (of order $|\vec{u} \times \vec{B}|$) to develop in accordance with Faraday's law of induction. In general, when the current density is of order $\sigma(\vec{u} \times \vec{B})$, where σ is the electrically conductivity, electrical currents will ensue.

(ii) According to Amp$'$ere's law, these induced currents must give rise to a second, induced magnetic field. This adds to the original magnetic field and the change is usually such that the fluid appears to 'drag' the magnetic field lines along with it.

(iii) The combined magnetic field (imposed plus induced) interacts with the induced current density, $\vec{J}$ to give rise to a Lorentz force (per unit volume), $\vec{J} \times \vec{B}$. This acts on the conductor and is generally directed so as to inhibit the relative movement of the magnetic field and the fluid.

If the fluid is non-conducting or the velocity is negligible, there will be no significant induced magnetic field. Conversely, if σ or $\vec{u}$ are large (in some sense), then the induced magnetic field may substantially alter the imposed magnetic field. If it is a poor conductor or moves very slowly, then the induced current and the associated magnetic field will be weak.

Conducting fluid usually contains neutral particles and positive and negative charges. So the fluid is neutral in the large and the gaseous fluid referred as plasma. Thus the uniform of the plasma does not constitute the electric current.

In 1942, the Engineer-Astrophysicist Alfv′en expressed that if a highly conducting fluid is moving through the magnetic field, the induced electric currents will tend to inhibit the relative motion of the fluid and the magnetic field, so that the magnetic field is convected by the fluid.

1.1a Basic characteristic of MHD

If the solid or fluid materials are moving through the magnetic field, then it experiences electromagnetic forces and also if the materials are electrically conducting and the current path is available, then the electric currents ensue.

Alternatively, currents may be induced by the change of the magnetic field with time. There are two consequences, which are given as follows:

(i) An induced magnetic field associated with the currents appears, perturbing the original magnetic field.

(ii) An electromagnetic force due to the interactions of currents and the field appears, perturbing the original motion.

1.1b MHD approximation

The following postulates are considered to derived equations for MHD flow

(a) **Hydrodynamic and electromagnetic considerations**

(i) The fluid is treated as continuous and describable in terms of local properties such as pressure, velocity, temperature, viscosity, etc.

(ii) The system of our investigation is defined as averages over elements large compared with the microscopic structure of matter but small enough in comparison with the scale of the macroscopic phenomenon to permit the use of the differential calculus to describe them.

(iii) For the good MHD results, relating collision-free situations are considered.

(iv) All velocities are much smaller than the velocity of light, c (3×10^8 m/sec. approx.), hence the non-relative electromagnetic theory is considered in MHD flow and the relative condition is not necessary.

(v) A purely local view can be misleading, because the local statement conceals the essence of electromagnetism where by charges at rest and in motion, and also magnetic materials act upon one another at a distance.

1.1c Electrical properties of the magnetohydrodynamics

If the fluid is electrically conducting, then the MHD will be differed from the ordinary hydrodynamics. It is not magnetic; it effects a magnetic field not by its mere presence but only by virtue of electric currents flowing in it. The fluid conducts because it contains free charges (ions or electrons) that can move indefinitely, but it may also be a dielectric and contain bound charges (e.g. in the form of molecular dipoles), which can only move a limited extent under electric fields. The electrostatic part of the electric field is due to the free and bound charges distributed in and around the field.

1.1d Electric and magnetic field effect on MHD

A charged particle such as an electron suffers as the given forces:

1. A charged particle is repelled or attracted by other charged particles, the total force on the particle per unit of its charge due to all the other charges present being the electrostatic field $\vec{E}$. From the Coulomb's law, it follows that $\vec{E}$ is irrotational (i.e. $curl\vec{E} = 0$) and $\vec{E}$ can be represented by the negative gradient of an electrostatic potential $\vec{v}, i.e. \vec{E} = -grad\vec{v}$.

2. Charged particles in the motion of the fluid and also magnetic materials produce the phenomenon of magnetism, to describe which the economically it is conveniently to invent another magnetic field vector $\vec{B}$. It has the following effects of two forces.

(i) A charged particle moving with the velocity $\vec{v}$ m/sec. relative to a certain frame of reference suffers a magnetic force $\vec{v} \times \vec{B}$ (Newton) per units of its charge. The force is perpendicular to $\vec{v}$ and $\vec{B}$, and the direction of $\vec{B}$ is that in which the particle must travel to feel no magnetic force.

(ii) If the magnetic field $\vec{B}$ is changing with time relative to a certain frame of reference, then a particle will suffer an induced electric force $\vec{E}_t$ per units of its charge,, which is defined by $div\vec{E}_t = 0$ and Faraday's law gives us as

$$curl\vec{E}_t = -\frac{\partial \vec{B}}{\partial t} \tag{1.1.1}$$

But there is a stronger condition on $\vec{B}$, namely $div\vec{B} = 0$. (1.1.2)

This shows that the magnetic field lines can never end; though they do not form closed loops.

1.1e **Low-frequency approximation on MHD**

The Amp′ere-Maxwell law states that the magnetic field is related to the moving charges and changing the electric field, which is defined as

$$\frac{curl\vec{B}}{\mu} = \vec{J} + \varepsilon_o \frac{\partial \vec{E}}{\partial t} \qquad (1.1.3)$$

where $\vec{J}$ is the current density due to the net flow of all charges, free or bound, μ is the permeability of free space and ε_o is the permittivity of the free space. The last term of right side of (1.1.3) is called the Maxwell's condition, which states that the charge of the total electric field $\vec{E}$ affects $\vec{B}$. The charge distribution appears unimportant in low-frequency electromagnetism and MHD. The possible appears are

1. The magnitude of the ratio $\left[curl\vec{B}/\mu)/\left(\varepsilon_o \frac{\partial \vec{E}}{\partial t}\right)\right]$ is of order $(B/\mu d)/\varepsilon_o Ef$ or of order $\frac{1}{\varepsilon_o \mu d^2 f^2}$ or of order $\frac{\lambda^2}{d^2}$, where B and E are of the typical magnitudes, d length scale, f frequency, λ wavelength c/f of electromagnetic radiation of frequency and $c^2 = \frac{1}{\varepsilon_o \mu}$. This ratio is usually very large and the Maxwell term $\varepsilon_o\left(\frac{\partial \vec{E}}{\partial t}\right)$ is negligible unless the frequency is very high. Thus under the low-frequency, the Amp′ere-Maxwell law becomes $curl\vec{B} = \mu\vec{J}$.

2. In dielectrics $\frac{\partial \vec{p}}{\partial t}$ is of the same order as $\varepsilon_o \frac{\partial \vec{E}}{\partial t}$ and its contribution to $\vec{J}$ be neglected.

3. The total charge density q is determined by $div\vec{E}$ and is of order $\varepsilon_o \frac{E}{d}$ or of order $\varepsilon_o \frac{Bv}{d}$. Thus the convection current is taken to be neglected if $(\varepsilon_o Bv^2 / d)/(B/\mu d) = \frac{v^2}{c^2}$, which is very small. Thus neglecting the convection current $(q\vec{v})$ and the polarization current $\frac{\partial \vec{p}}{\partial t}$, the current density can be written as

$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B}). \qquad (1.1.4)$$

4. The ratio of the magnitudes of electric and magnetic parts of the body force $q\vec{E} + \vec{J} \times \vec{B}$ is of order $(\varepsilon_o \frac{E^2}{d}$ or $\varepsilon_o \frac{B^2 v^2}{d}) / \frac{B^2}{\mu d} = \frac{v^2}{c^2}$, which is very small unless the frequency is very high. Thus the effect of q (i.e. q is net charge per unit volume) and the electric body force $q\vec{E}$ is negligible in MHD.

From these above conditions, we have noticed that the charge distribution in MHD has no importance under the low-frequency approximation.

1.1f Applications of MHD

MHD operates on every scale, from the vast to the minute. For example, magnetic fields pervade interstellar space and aid the formation of stars by removing excess angular momentum from the collapsing interstellar clouds. Closer to home, sunspots and solar flares are magnetic in origin, sunspots being caused by buoyant magnetic flux tubes, perhaps 10^4 Km in diameter and 10^5 Km long, erupting from the surface of the sun.

MHD is also an intrinsic part of controlled thermo-nuclear fusion. Here plasma temperatures of around 10^8K must be maintained, and magnetic forces are used to confine the hot plasma away from the reactor walls.

In the metallurgical industries, magnetic fields are routinely used to heat, pump, stir and levitate liquid metals. Perhaps the earliest application of MHD is the electromagnetic pump. This simple device consists of mutually perpendicular magnetic and electric fields arranged normal to the axis of a duct. Provided the duct is filled with a conducting liquid, so that currents can flow, the resulting Lorentz force provides the necessary pumping action. Firstly, it was proposed back in 1832 and the electromagnetic pump has found its ideal application in fast-breeder nuclear reactors, where it is used to pump liquid sodium coolant through the reactor core.

Application of MHD to natural events received a belated stimulus when astrophysicists came to realize how prevalent throughout the universe are conducting, ionized gases (plasmas) and significantly strong magnetic fields. In 1889 Bigelow guessed that there were magnetic fields on the sun and Hale and the Babcocks later confirmed this. The final implication was that MHD processes must dominate most areas.of astrophysics. In 1918 Larmer made the attractive suggestion that the magnetic fields of the sun and other heavenly bodies might be due to dynamo action, whereby the conducting material of the star acted as the armature and stator of a self-exciting-dynamo.

A related application is the use of MHD acceleration to shoot plasma into fusion devices or to produce high-energy wind tunnels for simulating hypersonic flight. Since bodies moving at high speed are preceded by a sock wave which can ionize the air, another possibility is the use of MHD to affect the airstream for purposes of thermal protection, braking, propulsion or control. MHD effects can also arise from the passage of bodies or waves through the ionosphere in the presence of the earth's magnetic field.

Other potential applications for MHD include electromagnets with the fluid conductors, various energy conversion or storage devices, magnetically controlled lubrication by conducting fluids, etc. MHD has a peculiar attraction for aerodynamicists and mechanical engineers; instead of being confined to pushing at the edges of fluid streams, they are enabled by MHD to grab the fluid in midstream.

Perhaps the most widespread application of MHD in engineering is the use of electromagnetic stirring. Here the liquid metal which is to be stirred is placed in a rotating magnetic field. In effect, we have an induction motor, with the liquid metal taking the place of the rotor. This is routinely used in casting operations to homogenise the liquid zone of a partially solidified ingot. The resulting motion has a profound influence on the solidification process, ensuring good mixing of the alloying elements and the continual fragmentation of the snow flake-like crystals which form in the melt. This result is a fine structural, homogeneous ingot.

Another common application of MHD in metallurgy is magnetic levitation or confinement. This relies on the fact that a high-frequency induction coil repels conducting material by inducing opposing currents in any adjacent conductor (opposite currents repel each other).

Thus a 'basket' formed from a high-frequency induction coil can be used to levitate and melt highly reactive metals, or a high-frequency solenoid can be used to form a non-contact magnetic valve which modulates and guides a liquid metal jet.

MHD is also important in electrolysis, particularly in those electrolysis cells used to reduce aluminium oxide to aluminium. These cells consist of broad but shallow layers of electrolyte and liquid aluminium, with the electrolyte lying on top. A large current (perhaps 200K Amps) passes vertically downwards through the two layers, continually reducing the oxide to metal. The process is highly energy intensive, largely because of the high electrical resistance of the electrolyte.

There are many other applications of MHD in engineering and metallurgy. These include electromagnetic (non-contact) casting of aluminium, vacuum-arc remelting of titanium and nickel-based super alloys, electromagnetic removal of non-metallic inclusions from melts, electromagnetic launchers and so-called "cold-crucible" induction melting process, in which the melt is protected from the crucible walls by a thin solid crust of its own material.

1.2 Fundamental equation in MHD

Larmor [1919] has initiated the study of the subject in connection with the astrophysical problems. After Larmor, Cowling [1934], Walen [1944, 1946], Menzel [1951], Dungey [1953] and others have studied the presence of the magnetic field inside the sun and its effects on the sun-spots. Earlier studies of the motion of conducting fluids in the presence of external magnetic fields which deserve mention are those of Hartmann [1937] on the flow of conducting fluid across a magnetic field and the theory of magnetic storms has been developed by

Chapman and Ferraro [1931, 1933, 1940]. Though their works contained a few new ideas but there is no doubt that the development of the subject has followed mainly from Alfv'en's work [1949].

The equilibrium of conducting fluids under the action of the magnetic field on the currents and the fluid pressure is of considerable interest in astrophysics and thermonuclear work. Lundquist [1950] has made the first attempt in this problem of magnetostatic, while Dungey [1958] and Menzel [1951] have considered the application to astrophysical problems.

It is possible to attain equilibrium in a conducting fluid if the current is parallel to the magnetic field. For then, the magnetic forces vanish and the equilibrium of the gas is the same as in the absence of magnetic fields. Such magnetic fields are called force free. They were first postulated by Schluter and Lust [1954]. The existence of force free fields has been firmly established theoretically by Chandrasekhar and Kendall [1957] independently.

Herlofson [1950] and Hulst [1951] have demonstrated that magnetohydrodynamic waves can also be excited in compressible conducting media. Attempts to demonstrate the existence of magnetohydrodynamic waves in the laboratory has been made by Lundquist [1951] and by Lehnert [1954, 1955]. In Lundquist's experiment, it has been seen that because of dissipation, true standing waves can not be excited. Nevertheless, the experiment suffices to demonstrate the existence of magnetohydrodynamic waves. Lehnert, in his experiment, has replaced mercury by liquid sodium and has been able to make more refined measurements.

Teller and Haffman [1950] have discussed the problem of magnetohydrodynamic shock waves. A more detailed discussion for velocities which are small compared with the velocity of liquid has been given by Helfer [1953]. In Geophysical problems, the maintenance of earth's magnetic field and its secular variation has been studied by Bullard [1948, 1949], Elsasser [1950, 1956], Parker [1955], Hide [1965], Vennezian [1967] and others. Karman [1959] has given a review of the work done on the application of magnetohydrodynamics to engineering and technical problems. Sutton [1959], Curzon [1960], Mounel and Mather [1962], Mcgrath [1963] and many others have discussed the feasibility of magnetohydrodynamic principles in controlled thermo-nuclear fusion research. Huges and Elco [1962], Snyder [1962] and others have studied magnetohydrodynamic lubrication problems and found that the application of magnetic field causes the increase of load bearing capacity.

We consider the flow of an incompressible electrically conducting fluid in presence of an applied magnetic field. The fundamental equations governing the flow field and the temperature in MHD can be obtained from the corresponding equations in ordinary hydrodynamics with the suitable modifications. The extra equations occur in MHD are the Maxwell's electromagnetic field equations.

1.2.1 Maxwell's electromagnetic equations

In magnetohydrodynamics, we are mainly concerned with conducting fluid in motion and hence it is necessary to consider the electrodynamics equations of moving media. When charges are in motion, the electric and magnetic field will be associated with the motion of the fluid, which will have the space and the time radiation.

This phenomenon is called the electromagnetism and we study the electromagnetic wave motion. The study will involve time dependent properties of the electric and magnetic fields. The behaviour of which is described by a set of equations called Maxwell's equations. These equations under non-relativistic assumptions are:

$$curl\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{(Faraday's law in differential form)} \tag{1.2.5}$$

$$curl\vec{H} = \vec{J} \quad \text{(Amp}'\text{ere-Maxwell equation)} \tag{1.2.6}$$

$$div\vec{E} = \frac{\rho_e}{\varepsilon} \quad \text{(Gauss' law)} \tag{1.2.7}$$

$$div\vec{B} = 0 \quad \text{(Solenoidal nature of } \vec{B}\text{)} \tag{1.2.8}$$

$$\vec{B} = \mu_m \vec{H} \tag{1.2.9}$$

$$\vec{D} = \varepsilon \vec{E} \tag{1.2.10}$$

where $\vec{E}$, $\vec{B}$, $\vec{H}$, $\vec{J}$, $\vec{D}$ are the electric field, the magnetic field, the magnetic field intensity, the electric current density and the displacement vector respectively; μ_m, ε and ρ_e are the respective permittivity, permeability of the porous medium and the electric charge density.

1.2.2 Ohm's law

For electromagnetic problems, an equation, namely the equation of conduction, is added to the Maxwell's equation. The conduction current density $\vec{J}$ in the stationary condition is formulated mathematically as

$$\vec{J} = \sigma \vec{E} \tag{1.2.11}$$

where $\vec{E}$ is the electric field intensity and σ is the electrical conductivity of the medium.

If a charged particle is moving with the velocity $\vec{u}$ through the magnetic field $\vec{B}$, it suffers a magnetic force $\vec{u} \times \vec{B}$ per unit of its charge. That is the induced electric field is given by $\vec{u} \times \vec{B}$. This force is perpendicular to $\vec{u}$ and $\vec{B}$. Again the total force on a particle per unit of its charge moving locally in the medium with the velocity $\vec{u}$ i.e., the Lorentz force is given by

$$\vec{E} + \vec{u} \times \vec{B}$$

Hence under a non-relativistic approximation, the electric current density can be written as

$$\vec{J} = \sigma(\vec{E} + \vec{u} \times \vec{B})$$

This equation is known as Ohm's law.

1.2.3 **Hall current**

We know that the Lorentz force on a particle (in a conductor) per unit of its charge due to its motion of its velocity $\vec{u}$ under the action of a transverse magnetic field $\vec{B}$ is $\vec{E} + \vec{u} \times \vec{B}$ (see, Shercliff, 1965).

Let free charges of negligible inertia be drifting through it under the action of this Lorentz force. The right conclusions emerge if it is supposed that each drifting particle also suffers a drag force due to collisions equal on the average $k\vec{u}$, where k is a constant for each particle. This represents the dissipative phenomenon of resistivity.

Neglecting the inertia of the free charge, we have-

$$\sigma[\vec{E} + \vec{u} \times \vec{B}] = k\vec{v} \tag{1.2.12}$$

Summing over the free charges in the element of conductor, we get-

$$\rho_e''\vec{E} + \vec{J}_c \times \vec{B} = \sum \frac{k\vec{u}}{\delta} \text{ per unit volume}$$

i.e. $\vec{E}+\frac{\vec{J}_c\times\vec{B}}{\rho_e''}=\sum\frac{k\vec{u}}{\delta\rho_e''}$ (1.2.13)

where $\vec{J}_c$ is the conduction current $\sum\frac{\rho\vec{u}}{\delta}$, due to the drift of the charges and ρ_e'' is the net free charges per unit volume. The experiments show that the right hand side is proportional to $\vec{J}_c$. Hence we have

$$\vec{E}+\frac{\vec{J}_c\times\vec{B}}{\rho_e''}=\frac{\vec{J}_c}{\sigma}$$

where σ is the electrically conductivity of the fluid. The extra term $\frac{\vec{J}_c\times\vec{B}}{\rho_e''}$ due to $\vec{B}$ is known as Hall effect. If the free charges are electrons of charge- e and the number density n, then

$$\vec{E}-\frac{\vec{J}_c\times\vec{B}}{ne}=\frac{\vec{J}_c}{\sigma} \quad (1.2.14)$$

Hall effect is merely due to the sideways magnetic force on the drifting free charges. In liquid conductors, Hall effects are negligible being the number of free charges infinite. When the conductor is moving with the velocity $\vec{u}$ locally, the velocity of a charge is $\vec{u}+\vec{v}$ if $\vec{v}$ is its relative velocity to the conductor. Summing over all charges, free or bound, we have-

Total current $\vec{J}=\sum\frac{e(\vec{u}+\vec{v})}{\delta}$

$$=\rho_e\vec{u}+\sum\frac{e\vec{v}}{\delta} \quad (1.2.15)$$

in which the term $\rho_e\vec{u}$ is the convection current, a non-dissipative effect.

The term $\sum \frac{e\vec{v}}{\delta}$ can split into (i) the convection current $\vec{J}_c$ due to the motion of free charges relative to fluid, which is a dissipative effect and (ii) the polarization current due to the motion of bound charges relative to the fluid.

The balance of forces on a free charge is

$$e[\vec{E} + (\vec{u} + \vec{v}) \times \vec{B}] = k\vec{v} \tag{1.2.16}$$

which leads to the results that Ohm's law is

$$\vec{E} + \vec{u} \times \vec{B} = \frac{\vec{J}_c}{\sigma} \tag{1.2.17}$$

if the hall term due to $\sum e(\vec{v} \times \vec{B})$ is neglected.

With the Hall term, the Ohm's Law can be written as

$$\vec{J}_c = \sigma[\vec{E} + \vec{u} \times \vec{B}] - \frac{\sigma}{ne}(\vec{J}_c \times \vec{B}). \tag{1.2.18}$$

1.2.4 **Equation of continuity**

Let us consider a fluid of density ρ, moving with a velocity $\vec{v}$. Then the mass conservation equation, known as the equation of continuity is

$$\operatorname{div} \rho\vec{v} = -\frac{\partial \rho}{\partial t} \tag{1.2.19}$$

$$\text{or } \rho \operatorname{div} \vec{v} = -\frac{D\rho}{Dt} \tag{1.2.20}$$

where $\frac{D}{Dt}$ denotes the substantive time-derivative.

An incompressible fluid is one where each travelling fluid element charges its density negligible, even though ρ is the non-uniform. Then $\frac{D\rho}{Dt} = 0$ and therefore

$$\operatorname{div} \vec{v} = 0 \tag{1.2.21}$$

1.2.5 Momentum conservation equation

Magnetohydrodynamics differs from ordinary dynamics. In MHD, the fluid is electrically conducting. It is not magnetic; it effects a magnetic field not by its mere presence but only by virtue of electric current flowing in it. If an electrically conducting fluid moves with a velocity $\vec{u}$ in presence of magnetic field $\vec{B}$, then the body force per unit volume can be written as (see Shercliff, 1965)-

$$\vec{F} = \rho_e \vec{E} + \vec{J} \times \vec{B} \tag{1.2.22}$$

The ratio of electric and magnetic parts of this body force is of the order $\frac{u^2}{c^2}$, where u is the characteristic velocity and c is the velocity of light.. Thus $\rho_e \vec{E}$ can be omitted. Hence in the case of the viscous fluid, the equation of the motion in magnetohydrodynamics is

$$\rho \frac{D\vec{u}}{Dt} = -\nabla p + \vec{J} \times \vec{B} + \mu \nabla^2 \vec{u} \tag{1.2.23}$$

And also, in case of an incompressible fluid with μ constant, the equation of the motion of the fluid is

$$\rho \frac{D\vec{u}}{Dt} = -\nabla p + \vec{J} \times \vec{B} + \mu \nabla^2 \vec{u} \tag{1.2.24}$$

1.2.6 Magnetic diffusion equation

The MHD approximations are grouped together below

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \text{(Faradays' law)} \tag{1.2.25}$$

$$\vec{J} = \sigma(\vec{E} + \vec{u} \times \vec{B}) \tag{1.2.26}$$

and $\nabla \times \vec{B} = \mu_m \vec{J}$ (1.2.27)

Eliminating. $\vec{E}$ and $\vec{J}$ from the equations (1.2.25)-(1.2.27), we get the magnetic induction equation as follows

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) - \upsilon_m \nabla \times (\nabla \times \vec{B}), \tag{1.2.28}$$

where $\upsilon_m = \dfrac{1}{\sigma \mu_m}$ is the magnetic diffusivity or the magnetic viscosity.

With the help of (1.2.8), the equation (1.2.28) can be written as

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) + \upsilon_m \nabla^2 \vec{B} \tag{1.2.29}$$

when the magnetic Reynolds number is very small, the equation (1.2.29) is called the magnetohydrodynamics diffusion equation. When the magnetic Reynolds number $(R_m = \dfrac{\upsilon d}{\upsilon_m})$ is very small compared with unity, neglecting the term $\nabla \times (\vec{u} \times \vec{B})$, the equation (1.2.29) becomes as

$$\frac{\partial \vec{B}}{\partial t} = \upsilon_m \nabla^2 \vec{B} \tag{1.2.30}$$

This is the equation of diffusion of a magnetic field in a stationary conductor, resulting in the decay of the field.

When the magnetic Reynold's number R_m is the large compared with the unity, the equation (1.2.29) reduces approximately to

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) \tag{1.2.31}$$

1.2.7 Energy equation in MHD

The charge within a material moves under the action of electromagnetic forces colliding and exchanging energy with the rest of material. This fact means that the electric work can be done on or by the material. It has been found that the electromagnetic field puts energy into the material at the rate $\vec{E}.\vec{J}$ per unit volume and time (see Shercliff, 1965). The current density $\vec{J}$ can have three possible forms – conduction, convection and polarization. The contribution of convection and polarization on the work done is negligible in MHD; only that of the convection current plays a significant part.

Ohm's law, without Hall current, is given by the equation (1.2.17). Hence

$$\vec{E}.\vec{J} = \frac{\vec{J}^2}{\sigma} - \vec{J}.(\vec{u} \times \vec{B}) \tag{1.2.32}$$

The first term on the right side of the above equation (1.2.32) represents the Ohmic dissipation and the second term can be written as

$$-\vec{J}.(\vec{u} \times \vec{B}) = \vec{u}.(\vec{J} \times \vec{B}) \tag{1.2.33}$$

This describes the phenomenon of electromechanical energy conversion. $\vec{u}.(\vec{J} \times \vec{B})$ is the rate at which the magnetic force $\vec{J} \times \vec{B}$ does work on the conductor as a whole. The term $\vec{u}.(\vec{J} \times \vec{B})$ pushes the fluid - either creating kinetic energy or helping to overcome other forces or the reverse if the term is negative. The term $\frac{\vec{J}^2}{\sigma}$ is positive and the dissipated part in the form of heat. Therefore for an incompressible fluid, the equation of energy in MHD is

$$\rho c_p \frac{D\vec{T}}{Dt} = k\nabla^2\vec{T} + \mu\phi + \frac{\vec{J}^2}{\sigma} \tag{1.2.34}$$

where c_p is the specific heat at constant pressure and ϕ is the dissipation function given by

$$\phi = 2\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right\} + \left\{\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2\right\}$$

$$-\frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2. \quad (1.2.35)$$

1.3 Non-dimensional parameters in MHD flow

For an unsteady flow of incompressible electrically conducting viscous fluid, the equations of motion, magnetic diffusion and energy are followed as

$$\rho\frac{D\vec{u}}{Dt} = -\nabla p + \vec{J} \times \vec{B} + \mu\nabla^2\vec{u} + \rho\nabla\psi \quad (1.3.36)$$

$$\frac{\partial\vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) + \upsilon_m\nabla^2\vec{B} \quad (1.3.37)$$

$$\rho c_p\frac{\partial\vec{T}}{\partial t} = k\nabla^2\vec{T} + \mu\phi + \frac{\vec{J}^2}{\sigma} \quad (1.3.38)$$

where ρ is the density of the fluid, σ is the electrically conductivity of the fluid, μ is the permeability of the medium, c_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid, p is the pressure of the fluid, ψ is the gravitational potential and ϕ is the dissipation function which is given by (1.2.35).

Let us introduce the non-dimensional quantities with the help of T_o, L, B_o, t_o, H_o, J_o and put

$$x_i' = \frac{x_i}{L},\ \vec{u}' = \frac{\vec{u}}{u_o},\ v' = \frac{\vec{v}}{u_o},\ t' = \frac{tu_o}{L},\ T' = \frac{\vec{T}}{T_o},\ p' = \frac{p^2}{\rho u_o^2},\ \phi' = \frac{\phi L^2}{u_o^2},$$

$$\nabla' = L\nabla,\ \frac{D}{DT} = \frac{u_o}{L}\left(\frac{D}{Dt'}\right),\ \psi' = \frac{\psi}{gL},\ \vec{B}' = \frac{\vec{B}}{B_o} = \frac{\vec{H}}{H_o} = \vec{H}',\ \vec{J}' = \frac{\vec{J}}{J_o}. \qquad (1.3.39)$$

where u_o and L is the characteristic velocity and length respectively, the subscript 'o' refers to a characteristic value and i = 1, 2, 3.

Substituting the conditions (1.3.39) in (1.3.36)-(1.3.38), we get-

$$\frac{D\vec{u}'}{Dt'} = -\nabla' p' + \frac{1}{\text{Re}}\nabla'^2\vec{u}' - \frac{1}{Fr}\nabla'\psi' - \frac{1}{\mu_e^2}\vec{H}'\times(\nabla'\times\vec{H}') \qquad (1.3.40)$$

$$\frac{\partial\vec{B}'}{\partial t'} = \nabla'\times(\vec{u}'\times\vec{B}') + \frac{1}{Rm}\nabla'^2\vec{B}' \qquad (1.3.41)$$

$$\frac{DT'}{Dt'} = \frac{1}{\Pr\text{Re}}\nabla'^2 T' + \frac{\vec{E}}{\text{Re}}\phi + \frac{M^2\vec{E}}{\text{Re}}\vec{J}'^2 \qquad (1.3.42)$$

The non-dimensional parameters which are appeared as the following co-efficients in these equations:

$\text{Re} = \frac{u_o L}{\upsilon}$ (Reynolds' number), $Rm = \frac{u_o L}{\upsilon_m}$ (Magnetic Reynolds' number),

$\Pr = \frac{\mu c_p}{k}$ (Prandtl number), $p_m = \frac{\upsilon}{\upsilon_m}$ (Magnetic Prandtl number),

$M = B_o L\left(\frac{\sigma}{\rho\upsilon}\right)^{1/2}$ (Hartmann number), $Fr = \frac{u_o^2}{gL}$ (Froude number),

and $M_m = \frac{u_o}{A}$ (Magnetic Mach number), where A is the Alfv'en number, $A^2 = \frac{B_o^2}{\mu\rho}$.

The equation of magnetic diffusion (1.2.29) has an analogy with the equation governing the diffusion of vorticity $\vec{\omega}$ of an incompressible non-conducting viscous fluid given by

$$\frac{\partial\vec{\omega}}{\partial t} = \nabla\times(\vec{u}\times\vec{\omega}) + \upsilon\nabla^2\vec{\omega} \qquad (1.3.43)$$

where υ is the kinematics viscosity of the fluid.

The imperfection in the analogy is that $\vec{\omega}$ is intimately related to $\vec{u}$ (i.e. $\vec{\omega} = \nabla \times \vec{u}$) in a way that $\vec{B}$ is not, but it turns out that this does not prevent the use of the analogy to suggest results concerning $\vec{B}$. From the equations (1.2.29) and (1.3.43), we can make the same kind of statement namely that the local rate of change of $\vec{B}$ or $\vec{\omega}$ results from the local net effect of (i) convection {i.e. the term, $\nabla \times (\vec{u} \times \vec{B})$} and diffusion (i.e. the term $\upsilon_m \nabla^2 \vec{B}$).

1.3.1 Large magnetic Reynold's number

In any region of length scale δ, where the convection and diffusion are equally important, the two terms on the right hand side of the equation (1.2.29) must be comparable. Thus-

$$\frac{\nabla \times (\vec{u} \times \vec{B})}{\upsilon_m \nabla^2 \vec{B}} \cong \frac{u_o \delta}{\upsilon_m} (= Rm) \tag{1.3.44}$$

so that δ must be of order $\frac{\upsilon_m}{u_o}$. If the whole field of interest has a length scale L such that $Rm >> 1$, then $L >> \delta$, Rm being based on L only within a limited region of length δ, where $\vec{B}$ changes significantly, gradients can be high enough for diffusion and only dissipation matters much; elsewhere it can be neglected. Thus for the large Rm, convection dominates and magnetic boundary layer approximations are expected to work near sources of field and elsewhere the approximations of perfect or infinite conductivity would be valid, the diffusivity being zero. So $\vec{E} + \vec{u} \times \vec{B} = 0$ and the convectional one holds away.

Again, if the characteristic time is t, then from the equation (neglecting the diffusion term)

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) \tag{1.3.45}$$

we have, $\frac{\partial \vec{B}}{\partial t} \cong \frac{B_o}{t} \cong \frac{u_o B_o}{L}$ $i.e. t \cong \frac{L}{u_o}$ (1.3.46)

Thus the characteristic time in the flow problem, is the transit time $\left(\frac{L}{u_o}\right)$ during which a field disturbance diffuser a distance of order $\left(\frac{\upsilon_m L}{u_o}\right)^{1/2}$ which is much less than L if $Rm >> 1$. Hence the diffusion is negligible.

1.3.2 Small magnetic Reynold's number

This is the other extreme case, which occurs when the diffusion is dominant and any imposed field $\vec{B}_o$ is hardly affected by the fluid motion. It diffuses as if the fluid is stationary where there is no induced current; the field is equal to the imposed field.

From Maxwell's equations

$$\text{curl}\, \vec{H} = \vec{J} \tag{1.3.47}$$

and $\vec{B} = \mu_m \vec{H}$ (1.3.48)

where μ_m is the permittivity of the medium.

Due to the absence of induced currents, we get-

$$\text{curl}\, \vec{B}_o = 0.$$

From Ohm's law, we have the induced current $\vec{J}_i$ is of order $\sigma u_o B_o$. The induced field $\vec{B}_i$ is determined by

$$\mu_m \vec{J}_i = \text{curl}\, \vec{B}_i$$

and is therefore of order $\mu_m \sigma u_o L$; thus

$$\left|\frac{\vec{B}_i}{B_o}\right| \cong \mu_m \sigma u_o L \; (= Rm)$$

when Rm is low, the induced field can be neglected entirely to replace $\vec{B}$ by the known imposed field $\vec{B}_o$ in all the magnetohydrodynamics equations. In this case, $\mu_m \vec{J} = \text{curl}\,\vec{B}$ can be ignored but $\text{div}\,\vec{J} = 0$ must still be retained however. As the magnetic Prandtl number $\frac{\upsilon}{\upsilon_m}$ is equal to $\frac{Rm}{\text{Rc}}$, one can arrive at a better appreciation of dissipation phenomena in magnetohydrodynamics from this relation. This ratio is actually the ratio of heat generated by viscous effects to the heat generation due to the Joule heat. When it is small, as it is in liquid metals and low temperature plasmas, magnetic field diffuses much more rapidly than the vorticity and magnetic boundary layers are much thicker than viscous ones. This makes for simplifications such as the neglect of viscosity in the magnetic boundary layer. Thus when Rm is small, the magnetic field decays by Ohmic dissipation. Omitting the term $\nabla \times (\vec{u} \times \vec{B})$, which is small, the induction equation becomes

$$\frac{\partial \vec{B}}{\partial t} = \mu_m \nabla^2 \vec{B}.$$

From this equation, it has been noticed that since the magnetic field $\vec{B}$ always decays, it tends to vanish in a characteristic time t, which is given by $t \cong \frac{L^2}{\mu_m}$.

In mathematical analysis, it is convenient frequently to assume $Rm \to 0$. This approximation gives the idea of some real situations and in this we have solved a few problems with approximations.

1.4 Boundary conditions on magnetohydrodynamics

When electrically conducting fluid is in contact with a rigid surface (or with another unmixed fluid), the following boundary condition must be satisfied in order to maintain contact: the fluid and the surface with which the contact is preserved must have the same velocity normal to the surface.

Let $\vec{n}$ denote a normal unit vector drawn at the point of the surface of contact and let $\vec{v}$ denote the fluid velocity at that point. When the rigid surface of contact is at rest, we must have $\vec{v}.\vec{n} = 0$ at each point of the surface. This expresses the condition that the normal velocities are both zeroed and hence the fluid velocity is tangential to the surface at its each point.

Again, if the rigid surface be in motion and $\vec{u}$ is its velocity at the point, then we must have-

$$\vec{v}.\vec{n} = \vec{u}.\vec{n}$$

$$\Rightarrow (\vec{v} - \vec{u}).\vec{n} = 0, \tag{1.4.49}$$

which expresses the fact that there must be no normal velocity at the point between boundary and fluid, that is, the velocity of the fluid relative to the boundary is tangential to the boundary at its each point.

For inviscid fluid, the above condition must be satisfied at the boundary. However, for viscous fluid (in which there is no slip), the fluid and the surface with which contact is maintained must also have the same tangential velocity at the point.

The above mentioned kinematics boundary conditions must hold independently of any particular physical hypothesis. In this case of a non-viscous fluid in contact with rigid boundaries (fluid or moving), the pressure of the fluid must act normal and continuous at the boundary.

The Maxwell's equations (1.2.25) and (1.2.27) or their equivalent equations are valid only for those points in whose neighbourhood the physical properties of the medium vary continuously. On the boundary of the flow field, the physical properties of the medium may exhibit discontinuities. For instance, at a solid boundary, the electromagnetic properties of the MHD will change abruptly to those of the solid. Across such a surface of discontinuity of electromagnetic properties, the following four conditions hold.

1. The transition of the normal component of magnetic induction $\vec{B} = \mu_m \vec{H}$ is continuous, i.e.,

$$(\vec{B}_2 - \vec{B}_1).\vec{n} = 0 \tag{1.4.50}$$

where $\vec{n}$ is the unit vector normal to the surface of discontinuity. Subscripts 1 and 2 refer to the values immediately on each side of the surface.

2. The behaviour of the magnetic field $\vec{H}$ at this boundary is

$$\vec{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s \tag{1.4.51}$$

where $\vec{J}_s$ is the surface current density. For finite electrically conductivity, $\sigma \neq \infty$, $\vec{J}_s$ is zero; whereas for infinite electrical conductivity, $\sigma = \infty$, $\vec{J}_s$ may be different from zero.

3. The transition of the tangential component of the electric field $\vec{E}$ is continuous, i.e.,

$$\vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0 \tag{1.4.52}$$

4. The behaviour of dielectric displacement $\vec{D} = \varepsilon\vec{E}$ at this boundary is

$$\vec{n}.(\vec{D}_2 - \vec{D}_1) = \rho_{es} \tag{1.4.53}$$

where ρ_{es} is the surface free charge density.

For most of our problems of magnetohydrodynamics, we may neglect the surface current density $\vec{J}_s$ and the surface free charge density ρ_{es}. Hence our boundary conditions become that both the tangential components of $\vec{H}$ and $\vec{E}$, and the normal components of $\vec{B}$ and $\vec{D}$ are all continuous across a surface separating a body and a fluid or two fluids. The distributions between $\vec{H}$ and $\vec{B}$, and between $\vec{E}$ and $\vec{D}$ should be noticed here because the values of μ_m and ε may be different on both sides of the boundary.

1.5 Rivlin-Ericksen fluid

Rivlin-Ericksen (1955) considered the theory of isotropic material for which they considered that the stress depends on the spatial gradients of velocity, acceleration, upto an order (N-1)th acceleration. Using the invariant requirements, they showed that the stress must be given by an isotropic function of the tensors $A_{(N)ij}$ as

$$\tau_{ij} = f_{ij}[A_{(1)k1}, A_{(2)k2}, \ldots\ldots\ldots\ldots\ldots, A_{(N)k1}] \tag{1.5.54}$$

where f obeys an identity.

$$Qf_{ij}[A_{(1)k1}, A_{(2)k1}, \ldots\ldots\ldots\ldots\ldots\ldots, A_{(N)k1}]Q^T$$

$$= f_{ij}[QA_{(1)k1}Q^T, QA_{(2)k1}Q^T, \ldots\ldots\ldots\ldots\ldots\ldots, QA_{(N)k1}Q^T] \tag{1.5.55}$$

for all orthogonal tensors Q. The tensors Q^T denotes the transpose of Q.

The tensors $A_{(N)ij}$ are called Rivlin-Ericksen tensors and can be generated successive material differentiation of the squared arc elements ds^2 as

$$\frac{D^N}{Dt^N}(ds^2) = A_{(N)ij}dx^i dx^j \tag{1.5.56}$$

where $\frac{D}{Dt}$ is the material or substantive derivative defined as

$$\frac{D\chi}{Dt} = \frac{\partial \chi}{\partial t} + v^{i} x_{,i} \tag{1.5.57}$$

The recurrence formula for $A_{(N)ij}$ may be written as

$$A_{(1)ij} = v_{i,j} + v_{j,i} = 2e_{ij},$$

$$A_{(2)ij} = a_{i,j} + a_{j,i} + 2v^{m}_{,i} v_{m,j}, \quad (a_{i} = \frac{Dv_{i}}{Dt}) \tag{1.5.58}$$

and $A_{(N)ij} = A_{(N-1)ik} v^{k}_{,j} + A_{(N-1)kj} v^{k}_{,i} + \frac{D}{Dt} A_{(N-1)ij}$

The fluid governed by the constitutive equation (1.5.54) is called Rivlin-Ericksen fluid of complexity N. The next important class of Rivlin-Ericksen fluids have the constitutive equation of the form

$$\tau''_{ij} = f_{ij}[A_{(1)kl}, A_{(2)kl}] \tag{1.5.59}$$

For isotropic fluids, if τ' is considered as a function of $A_{(1)}$ and $A_{(2)}$ only, then the equation (1.5.54) and (1.5.55) with the help of (1.5.59) gives us as

$$\tau'_{ij} = \mu_{o}U + \mu_{1}[A_{(1)}] + \mu_{2}[A_{(2)}] + \mu_{(3)}[A_{(1)}]^{2} + \mu_{4}[A_{(2)}]^{2} + \mu_{5}\{[A_{(1)}][A_{(2)}]$$

$$+[A_{(2)}][A_{(1)}]\} + \mu_{6}\{[A_{(1)}]^{2}[A_{(2)}] + [A_{(2)}][A_{(1)}]^{2}\} + \mu_{7}\{[A_{(1)}][A_{(2)}]^{2} + [A_{(2)}]^{2}[A_{(1)}]\}$$

$$+[A_{(2)}][A_{(1)}]\} + \mu_{6}\{[A_{(1)}]^{2}[A_{(2)}] + [A_{(2)}][A_{(1)}]^{2}\} + \mu_{7}\{[A_{(1)}][A_{(2)}]^{2} + [A_{(2)}]^{2}[A_{(1)}]\}$$

$$+ \mu_{8}\{[A_{(1)}]^{2}[A_{(2)}]^{2} + [A_{(2)}]^{2}[A_{(1)}]^{2}\} \tag{1.5.60}$$

where $\mu_{m}\,(m = 0,1,2,..............,8)$ are scalar functions of the nine invariants of tensors $[A_{(1)}]$ and $[A_{(2)}]$.

For viscometric flows, all tensors $[A_{(N)}]$ except $[A_{(1)}]$ and $[A_{(2)}]$ vanish. Markovitz (1957) observed that μ_m ($m = 4,5, ...,8$) may be omitted without affecting the solutions. So, then the reduced constitutive equation takes form,

$$\tau_{ij} = -p\delta_{ij} + \mu_1 A_{(1)ij} + \mu_2 A_{(2)ij} + \mu_3 A_{(1)im} A_{(1)mj} \quad (1.5.61)$$

where $p = \pi - \mu_o$ is the determinate isotropic pressure,

μ_1 = co-efficient of ordinary viscosity,

μ_2 = co-efficient of visco-elasticity

μ_3 = co-efficient of cross viscosity.

A fluid governed by the equation (1.5.61) is called an incompressible second order Rivlin-Ericksen fluid. We can also write constitute equations of higher orders in this way. All three material constants can be determined from the viscometric equation of state for any material behaving as a second order fluid. Markovitz and Coleman [1964] proved that μ_2 is negative (experimentally also, it has been found negative under thermodynamical considerations).

Although the general Rivlin-Ericksen fluid accounts for shear dependent viscosity and normal stress effects; yet it shares the Newtonian fluid as its special case. The effect of changes in shear rate with time upon the stresses in a visco-elastic fluid were incorporated into the constitutive equations by Rivlin and Ericksen.

When $[A_{(N)}] = 0$, for $N = 1,2, ...,n$) the extra stress on a Rivlin-Ericksen fluid can not change in time, but it does in actual relaxation experiments on visco-elastic materials such as high polymetric fluids.

1.6 Non-Newtonian fluids

The physical property that characteristics of the flow resistance of simple fluids is the viscosity. All real fluids are viscous, a force of internal friction, offering resistance to the flow that always arises between the layers of a fluid moving at different velocities in relation to one another. Fluids which obeys Newton's law of viscosity are known as Newtonian fluids. Common fluids like water, air and mercury are all Newtonian fluids. Fluids which do not obey Newton's law of viscosity are known as non-Newtonian fluids. Thus, for such fluids the shear stress is not proportional to the velocity gradient. Fluids like paints, coal tar and polymer solutions are all non- Newtonian fluids. According to Newtonian law, the tangential force acting at any point of the flow in the plane oriented in the direction of flow is proportional to the negative of the local velocity gradient

$$\tau_{ij} = -\mu \frac{\partial v_i}{\partial x_j} \tag{1.6.62}$$

where μ is known as the dynamic viscosity or simple viscosity. Kinds of fluids that have in this fashion are termed Newtonian fluids.

The equation (1.6.62) which defines a Newtonian fluid can be applied unidirectional flows only. However, the definition of Newtonian fluid in which the stress depends linearity on the rate of deformation may be generated to three-dimensional flows using the rate of deformation tensor.

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial q_i}{\partial x_j} + \frac{\partial q_j}{\partial x_i}\right) \tag{1.6.63}$$

where q is the local velocity of the fluid particle.

We can redefine Newtonian fluid as one that satisfies

$$T_{ij} = -p\delta_{ij} + 2\varepsilon_{ij} \tag{1.6.64}$$

where the Kronecker delta $\delta_{ij} = \begin{cases} 1 & for\, i = j \\ 0 & for\, i = 0 \end{cases}$

There are quite a few industrially important fluids which don't obey the Newton's law. The properties of these fluids are not only function of its state of the substance but also depends on the process parameters, the variation of velocity and temperature, they are known as non-Newtonian fluid. The relation between τ_{ij} and ε_{ij} are non-linear for non-Newtonian fluid (such fluids are primarily pastes, slurries, high polymers, blood, jellies and similar food product, polymeric melts, etc.).

According to the Newtonian law of viscosity, the plot of τ_{ij} versus $\left(-\frac{\partial v_i}{\partial x_j}\right)$ for a given fluid shows a straight line through the origin, and the slope of this line represents the viscosity of the fluid at a given temperature and pressure. Experiments have shown that τ_{ij} indeed proportional to $\left(-\frac{\partial v_i}{\partial x_j}\right)$ for all gases and for homogeneous non-polymeric liquids. The non-Newtonian flow of fluids is the "Science of deformation and flow" which includes the study of the mechanical properties of gases liquids plastics and crystalline materials.

Thus the non-Newtonian fluid flow is the part of science of rheology where both Newtonian fluid mechanics and Hookean elasticity are considered. The steady state rheological behaviour of most fluids can be generalized as

$$\tau_{ij} = -\mu_{app} \frac{\partial v_i}{\partial x_j} \tag{1.6.65}$$

where μ_{app} is the apparent viscosity, is not a constant, it may be expressed as a function of either $\frac{\partial v_i}{\partial x_j}$ or τ_{ij}.

In order to explain the steady state relation for Newtonian and non-Newtonian fluid between τ_{ij} and $-\frac{\partial v_i}{\partial x_j}$ at constant temperature and pressure several models were proposed, such as power law model, Bingham model, Prandtl Erying model, Reiner-Philippoff model, etc.

Under steady state conditions a number of additional types of non-Newtonian behaviour are possible, for example thixotropic, rheopectic, viscoelastic, etc.

i) Time independent fluid that are where the rate of shear at a given point solely dependent upon the instantaneous shear stress at that point. Time independent non-Newtonian fluids are also non-Newtonian viscous fluid or purely viscous fluid.

ii) Time dependent fluids are those for which the shear rate is function of both the magnitude and the duration of shear. Time dependent non-Newtonian fluid classified into two groups: Thixotropic fluid and Rheopectic fluids depending upon whether the shear stress decreases or increases with time at given shear rate at constant temperature. Fluids that show limited decrease in μ with time under a suddenly applied constant stress τ_{ij} called Thixotropic. The Thixotropic properties have been found in the material such as some solutions or melts of high polymers, oil well drilling muds, greases printing inks, many food materials, paints, etc. The fluids that show limited increase of μ with time under a suddenly applied stress τ_{ij} called Rheopectic fluid.

Rheopectic fluids are antithixotropic fluids that exhibit a reversible increase in shear stress with time at a constant rate of shear under isothermal conditions. Examples of these types are bentonite clay, suspension, vanadium pentoxide suspension, gypsum suspension and certain solutions in many pipe problems, etc.

iii) Visco-elastic fluids are those which show partial elastic recovery upon the removal of a deforming shear stress, such materials possess properties of both fluids and elastic solids. These materials exhibit both viscous and elastic properties. In a purely Hookean elastic solid the stress corresponding to a given strain is independent of time whereas for visco-elastic substances the stress will gradually dissipate with time. A part of the deformation of the visco-elastic fluids flow when subjected to stress. Examples of this type are Bitumen, flour dough. Naplam and similar jellies, polymer sand, polymeric melts such as Nylon and many polymeric solutions.

In order to take account of the mechanism of non-Newtonian fluids number of mathematical models were proposed at different time by different mathematicians. In our research working, we have discussed a problem of flow and heat transfer on Rivlin-Ericksen second order visco-elastic fluid. A brief description of Rivlin-Ericksen second order fluid is mentioned above.

1.7 Heat transfer in fluid motion

The heat transfer is devoted for the steady of processes of heat propagation in the solid, liquid and gaseous bodies. Simply it states that heat is a form of energy, which is transferred from one body to another body at a lower temperature by virtue of the temperature difference between the bodies. In this problem, we consider with the rate at which the heat is transferred.

The rate of heat transfer may be constant or variable, depending on whether the conditions are such that the temperatures remain the same or change continually with time. Temperature differences in a body are reduced by heat flowing from regions of higher temperature to those of lower temperature. This process takes place in all substances, which are found in nature-solids, liquids and gases. Heat is transferred in three ways, which are known as conduction, convection and radiation.

In conduction, the flow of heat is the result of the transfer of internal energy from one molecule to another. The flow of heat in solids takes place exclusively by conduction process, while in liquids and gases the processes of conduction, convection and radiation occur simultaneously. In cases, where the heat exchange by convection is prevented and exchange by radiation is minimized, the principles of heat conduction can be applied to liquids and gases as well. In these substances, however, molecules are no longer confined to a certain point but constantly change their relative position even if the substance is a state of rest.

The heat transfer by convection has been seen generally in liquids and gases. By this process, heat may be transported from one noint to another by being carried along as internal energy with the flowing medium

Hence the velocity field and the temperature field mutually interact which means that the temperature distribution depends on the velocity distribution and conversely, the velocity distribution depends on the temperature distribution. In special cases when buoyancy forces are disregarded and the fluid properties are independent of temperature, the velocity field does not depend on the temperature field while the dependence of temperature field on the velocity field persists. Such flows are termed as forced flow and the process of heat transfer in such flows is described as forced convection. Flows in which buoyancy forces are dominant are called natural flow and corresponding heat transfer is known as natural convection. If the natural convection is not constrained to a finite region by boundaries, it is called free connection.

In radiation, solid bodies as well as liquids and gases are capable of radiating thermal energy in the form of electromagnetic waves and of picking up such energy by absorption. All heat transfer processes are, therefore, more or less accompanied by a heat exchange by radiation.

If the working medium begins to move due to the difference between the densities of individual parts of the fluid upon the heating, then mode of heat transfers if referred to as free or natural convection. But if the working medium is put into the motion artificially (by means of a fan, compressor, mixer, etc.) to as forced convection.

1.7a Fundamental equations in heat transfer

We consider a fluid in which the density ρ is a function of the position $x^j (j = 1,2,3)$ and the velocity $u^j (j = 1,2,3)$.

1.7.1 Equation of continuity

The conservation of mass is given by the equation of continuity, which can be written as

$$\frac{D\rho}{Dt} + \rho u^i_{,i} = 0 \qquad (1.7.66)$$

Where a comma denotes a covariant differentiation with respect to x^i and t denotes time. The equation of continuity for the incompressible fluid is

$$u^i_{,i} = 0 \qquad (1.7.67)$$

In this case, the velocity field is therefore, Solenoid.

1.7.2 Equation of motion

The τ_{ij} be the stress tensor acting in the direction of x^{j} per unit area on an element of surface normal to x^{i}. In terms of stresses τ_{ij}, the hydrodynamic equations of motion can be written as

$$\rho \frac{D\rho}{Dt} = \rho F_{i} + \tau_{il,m} g^{lm} \tag{1.7.68}$$

where F_{i} is the ith component of the body force per unit mass and g^{lm} is a component of the metric tensor. The stress tensor is a function of the rate of strain tensor e_{ij}, which is given by

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{1.7.69}$$

The constitutive equation of a fluid gives the relation between τ_{ij} and e_{ij}. The simplest relation between these two tensors is linear and has been proposed on experimental basis by Newton as

$$\tau_{ij} = -p\delta_{ij} + 2\mu e_{ij} - \frac{2}{3}\mu\delta_{ij} I, \tag{1.7.70}$$

where μ is the co-efficient of viscosity, p is the undetermined hydrostatic pressure, $I(= e_{11} + e_{22} + e_{33})$ is the first invariant of the strain rate tensor, δ_{ij} is the Kronecker delta. This constitutive equation is true for most of the fluids as water, air, etc. and these fluids are known as Newtonian fluids. The equation (1.7.70) can not explain the behaviour of many fluids like oil, paint, mud, blood, etc. and hence many nonlinear constitutive equations have been proposed to explain the behaviour of these fluids.

One of the constitutive equations, which can explain many of the behaviours of these fluids and have sound mathematical basis, is that of a second-order fluid proposed by Coleman and Noll (1960).

1.7.3 **Equation of energy**

The law of conservation of energy requires that the difference in the rate of supply of energy to a volume V fixed in space with a surface S and the rate at which energy goes out through S must be equal to the net rate of increase of energy in this volume. Thus the law of conservation of energy gives the following equation where the summation convection is used with $i, j = 1,2,3$.

$$\int_S u_i(\tau_{ij}n_j)ds - \int_S E_i\rho u_j n_j ds + \int_V F_i u_i dV + \int_S k\frac{\partial T}{\partial x_j}n_j ds = \frac{\partial}{\partial t}\int_V \rho E_i dV \qquad (1.7.71)$$

where $E_i(=\frac{1}{2}u_iu_i + \rho_{ii} + E)$, u_i are respectively the total energy (i.e. sum of kinetic energy, potential energy and internal energy) and the ith component of the velocity; τ_{ij} and n_j are the ijth components of the viscous stress and jth component of the outer normal of the surfaces respectively; F_i is the ith component of the external conservative force and k is the co-efficient of heat conductivity. The first term on the left hand side of the equation (1.7.71) is the rate of heat produced by various stresses in contact with outside; the second term represents the energy loss by convection; the third term is the energy loss by the heat conduction. The loss due to the radiation is assumed to be negligible. The right hand side is the net rate of change of energy in the volume V.

Transforming the surface integration to volume integration and the volume V being arbitrary, we get-

$$\frac{\partial}{\partial x_j}(u_i\tau_{ij})-\frac{\partial}{\partial x_j}(\rho E_i u_j)+f_i u_i+\frac{\partial}{\partial x_j}(k\frac{\partial T}{\partial x_j})-\frac{\partial}{\partial t}(E_i\rho)=0 \qquad (1.7.72)$$

Using the equation of continuity (1.7.66) and simplifying, we get the equation (1.7.72) as

$$\rho[\frac{DE}{Dt}+\rho\frac{D}{Dt}(\frac{1}{\rho})]=\frac{\partial}{\partial x_j}(k\frac{\partial T}{\partial x_j})+\phi \qquad (1.7.73)$$

where the dissipation function ϕ can be written as

$$\phi=[\mu(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i})-\frac{2}{3}\mu(\frac{\partial u_k}{\partial x_k})\delta_{ij}]\frac{\partial u_i}{\partial x_j} \qquad (1.7.74)$$

For the perfect gas, $\frac{DE}{Dt}=C_v\frac{DT}{Dt},\frac{DH}{Dt}=C_p\frac{DT}{Dt}=$ enthalpy

and $C_p\frac{DT}{Dt}=C_v\frac{DT}{Dt}+\frac{D}{Dt}(\frac{p}{\rho})$, which reduce the equation (1.7.74) to

$$\rho\frac{D}{Dt}(C_pT)=\frac{Dp}{Dt}+\frac{\partial}{\partial x_i}(k\frac{\partial T}{\partial x_i})+\phi \qquad (1.7.75)$$

For incompressible fluid, the above equation simplifies to

$$\rho\frac{D}{Dt}(C_pT)=k\frac{\partial}{\partial x_i}(\frac{\partial T}{\partial x_i})+\phi \qquad (1.7.76)$$

1.7.4 Equation of state

In solving a hydrodynamic problem together with the equation of continuity, motion and energy, we should consider an equation of state as

$$\rho=\rho(p,T) \qquad (1.7.77)$$

It suggests that ρ is constant in all terms in the equation of motion except that one in the external force; therefore, we have-

$$\rho = \rho_o[1 - \alpha(T - T_o)],$$

where α is the volumetric expansion co-efficient of the fluid and the subscript 'o' denotes the unheated no flow state.

1.7.5 Theoretical similarity of heat transfer in MHD

In the MHD flow where temperature differences bring about differences in density it is necessary to include buoyancy forces in the equations of motion of a viscous fluid and to treat them as imposed body forces. These buoyancy forces are caused by changes in volume, which are associated with the temperature differences. If we denote the co-efficient of expansion by β, that for perfect gases $\beta = \frac{1}{T}$, and denoting the temperature difference between a hotter fluid particle and the colder surroundings by $\theta = T - T_o$, then we can see that the relative change in volume of the hotter particle is $\beta\theta$ so that the lift force per unit volume $= \rho g\beta\theta$, where ρ is the density of the fluid before heating and g is the vector of gravitational acceleration. The components of the latter will be denoted by g_x, g_y, g_z. Introducing these body forces into the momentum conservation equation (1.2.24) for unsteady incompressible flow and assuming that the viscosity is constant, we obtain:

$$\rho \frac{D\vec{u}}{Dt} = -\nabla p + \vec{J} \times \vec{B} + \mu\nabla^2\vec{u} + \rho g\beta\theta \qquad (1.7.78)$$

1.7.6 Thermometric case (i. e. adiabatic wall)

If the transfer of heat by radiation is neglected, then it can occur only through conduction. According to the Fourier's heat conduction law, the flux $q(J/m^2 \sec)$ per unit area and time is proportional to the temperature gradient along the surface, so that

$$q = -k\frac{\partial T}{\partial n} \qquad (1.7.79)$$

where n is the direction of the normal to the surface of the body, k is the thermal conductivity of the fluid and the negative sign signifies that the direction of the flux is opposite to that of the temperature gradient (i. e. the negative sign signifies that the heat flux is reckoned as positive in the direction of the temperature gradient).

It is necessary to mention that the variety of possible sets of boundary conditions is much greater for the temperature field than for the velocity field. The temperature on the surface of the body may be constant or variable but, moreover, it also possible to encounter problems for which the heat flux is prescribed. The equation (1.7.79) shows that the temperature gradient at the wall appears as a boundary condition. This condition is called the adiabatic wall, since there is no heat flux from the wall to the fluid i.e. the boundary condition at the wall is

$$\left(\frac{\partial T}{\partial n}\right)_{n=0} = 0 \text{ (adiabatic wall).}$$

In this case, it visualises that the wall of the body is perfectly insulated against the heat flow. The heat generated by the flow through the friction serves to heat the wall until the condition $\left(\frac{\partial T}{\partial n}\right)_{n=0} = 0$ is reached.

Thus the temperature of the wall which is called the adiabatic wall temperature becomes higher than that of the fluid at some distance from it.

1.8 Non-dimensional parameters in heat transfer

In order to understand, the phenomenon of heat transfer, we should discuss the non-dimensional parameters, which govern the process. For simplicity we take cartesian co-ordinates $x_j (j = 1,2,3)$ and suppose that the fluid properties are independent of temperature. The equation of momentum and energy in cartesian tensors with usual summation conventions are

$$\rho[\frac{Du_i}{Dt}] = -\frac{\partial p}{\partial x_i} + \rho g_i \beta\theta + \mu[\frac{\partial}{\partial x_j}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}) - \frac{2}{3}\frac{\partial}{\partial x_i}(\frac{\partial u_j}{\partial x_i})^2] \qquad (1.8.80)$$

and $$\rho c_p[\frac{D\theta}{Dt}] = k[\frac{\partial^2\theta}{\partial x_i \partial x_i}] + u_i\frac{\partial p}{\partial x_i} + \mu\phi, \qquad (1.8.81)$$

where $\phi = [\frac{\partial}{\partial x_j}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}) - \frac{2}{3}\frac{\partial}{\partial x_i}(\frac{\partial u_j}{\partial x_i})^2]$.

Let us make the non-dimensional quantities with the help of u_o, d, θ_w and put-

$$u_i' = \frac{u_i}{u_o},\ \theta' = \frac{\theta}{\theta_w},\ x_i' = \frac{x_i}{d},\ t' = \frac{tu_o}{d},\ p' = \frac{p}{\rho u_o^2} \qquad (1.8.82)$$

where $\theta = T - T_o$, d is the characteristic dimension, u_o denotes a unique velocity that characterizes the flow, and the subscript w denotes the wall conditions.

Substituting conditions (1.8.82) in (1.8.80) and (1.8.81), we get-

$$[\frac{\partial u'_i}{\partial t'} + u'_j \frac{\partial u'_i}{\partial x'_j}] = \frac{Gr}{\mathrm{Re}^2}\theta' + \frac{1}{\mathrm{Re}}[\frac{\partial}{\partial x'_i}(\frac{\partial u'_i}{\partial x'_i} + \frac{\partial u'_j}{\partial x'_i}) - \frac{2}{3}\frac{\partial}{\partial x'_i}(\frac{\partial u'_j}{\partial x'_i})^2] - \frac{\partial p'}{\partial x'_i} \quad (1.8.83)$$

and $$[\frac{\partial \theta'}{\partial t'} + u'_i \frac{\partial \theta'}{\partial x'_i}] = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\frac{\partial^2 \theta'}{\partial x'_i \partial x'_i} + Eu'_i \frac{\partial p'}{\partial x'_i} + \frac{E}{\mathrm{Re}}[\frac{\partial u'_i}{\partial x'_i}(\frac{\partial u'_i}{\partial x'_i} + \frac{\partial u'_j}{\partial x'_i}) - \frac{2}{3}(\frac{\partial u'_i}{\partial x'_i})] \quad (1.8.84)$$

The product $\mathrm{Pr}\,\mathrm{Re} = Pe$ is called Peclet number. We obtain the Peclet number when we divide the convection term by the conduction term of the energy equation. The ratio $\frac{\mathrm{Re}^2}{Gr}$ is called Froude number, it compares the inertia and the body force.

1.9 Some worked out problems related to MHD flow and heat transfer

The steady Poissuille flow of mercury between two parallel walls in the presence of an applied cross magnetic field, was considered by Hartmann. The MHD flow between two parallel plates under the transverse magnetic field, called Hartmann flow, has been studied by many authors under various conditions e.g., Shercliff (1966) and Cowling (1957). Ospal (1955) has outlined the general principles of the analysis of two-dimensional and three dimensional ground water flow by electrical analogy and described the practical applications of that method with a new conductive material consisting of gelatin, glycerin, water and salt. Srivastava and Sharma (1961) have studied the effect of a transverse magnetic field on the fluid flow between two infinite disks, one rotating and the other at rest. This problem has been extended afterwards by Stephenson (1969). He has obtained the asymptonic solutions for the condition R<<M and the numerical solution for the arbitrary R and M.

Katagiri[1962] discussed the MHD Couette flow when one of the plates moves impulsively and the other is at rest. Muhuri [1963] has generalized Katagiri's [1962] work to include the case of accelerated plate problem. The effect of induced magnetic field on the same problem has been discussed by Gobindarajulu [1970]. The problem of steady flow of an electrically conducting fluid through uniformly porous infinite parallel plates channel in the presence of a transverse magnetic field has been investigated by Rao [1960], Terril and Shrestha [1963, 1964] and Terril [1964]. Sultan and Sharma [1965] have discussed the MHD Couette flow between non-conducting walls in the presence of an electric field which is normal to the applied transverse magnetic field. Agarwal [1962] has discussed the generalized MHD Couette flow between two parallel plates with or without porosity. In the above investigations the plates are assumed to be electrically insulated. The effect of suction or injection and magnetic field on the MHD flow in a straight channel has been studied by Shrestha [1967], Reddy and Jain [1967]. Chandrasekhar and Rudraiah [1970] have discussed the problem of a two dimensional conducting flow between porous disks for R<<1 where there is uniform suction or injection. This two-dimensional flow by the same authors [1971] under the assumption that one of the plate is at rest and the other is rotating. Chang and Yen [1962] have studied the heat transfer aspect between the walls. Srivastava and Sharma [1964] have discussed the heat transfer due to the flow between two infinite plates, one rotating and other at rest, under a transverse magnetic field. Chang and Yen's problem has been extended by Soundalgekar [1969a]. In another paper, Soundalgekar [1969b] has studied the heat transfer aspect in MHD Couette flow between conducting walls in the presence of an electric field.

Gupta[1969] has studied the effect of combined free and forced convection on the flow of an electrically conducting liquid under a transverse magnetic field in a horizontal parallel plates channel subjected to a linear axial temperature variation. A.K. Borkakati and A. Bharali [1979] studied the heat transfer in the flow of a conducting fluid between two non-conducting porous disks (one is rotating and other is stationary) in the presence of a transverse uniform magnetic field and under uniform suction. Here asymptotic solutions are obtained for $R << M^2$ and also the rate of heat flux from the disks and the temperature distribution are investigated. Taking Hall effects into account the steady magnetohydrodynamical flow past an infinite horizontal porous plate is theoretically investigated by A. Bharali and A. K. Borkakati [1980] when a strong magnetic field is imposed in a direction which is perpendicular to the free stream and makes an angle α to the vertical direction. The response of flow and heat transfer to change of direction of the imposed magnetic field in steady magnetohydrodynamic laminar free convection flow past an infinite vertical porous plate is studied by A. Bharali and A. K. Borkakati [1983]. Hydromagnetic flow and heat transfer between two horizontal parallel plates, where the lower one is a stretching sheet and the upper one is a porous solid plate is studied by A. K. Borkakati and A. Bharali [1983] in the presence of a transverse magnetic field. A. K. Borkakati and D.B. Chetri [1989] investigated theoretically the effect of the deflection of a strong magnetic field on the oscillatory MHD flow past an infinite horizontal plate, keeping the Hall parameter constant. In this problem, they made to study theoretically the effect of the deflection on an oscillatory magnetohydrodynamic flow past an infinite horizontal flat plate, considering the plate is insulator and the imposed magnetic field makes an angle α to the free stream velocity.

B. S. Dandapat and A. S. Gupta [1989] discussed the flow of an incompressible second-order fluid due to stretching of a plane elastic surface in the approximation of boundary layer theory. An analysis of MHD heat transfer in hyperbolic time-variation flow near a stagnation point of a heated blunt-nosed cylinder whose wall temperature varies as Ax^N was presented by V. M. Soundalgekar, T. V. Ramana Murty and H. S. Takhar [1990]. The effect of uniform suction or injection on the free convection boundary layer over a cone was theoretically investigated by T. Watanade [1991]. M. G. Gourla and Suaham L. Katoch [1991] discussed about the result of unsteady viscous incompressible free convection flow of an electrically conducting fluid between two heated vertical plates in the presence of the force field of gravity and applied magnetic field acting in the horizontal direction and perpendicular to the flow. Magnetohydrodynamic flow of an electrically conducting power-law fluid over a stretching sheet in the presence of a uniform transverse magnetic field is investigated by H. I. Andersson, K. H. Bech and B. S. Dandapat [1992] by using an exact similarity transformation. T. Watanabe and I. Pop [1993] theoretically studied the main results of the effects of a uniform magnetic field on the free convection flow of an electrically conducting fluid past an isothermal wedge. The effect of an axial magnetic field on the flow and heat transfer about a fluid underlying the axy-symmetric spreading surface is investigated by C. R. Lin and C. K. Chen [1993]. A. Kumar Singh and N. P. Singh [1995] studied the laminar flow and heat transfer of an incompressible, electrically conducting second order Rivlin-Ericksen liquid in porous medium down a parallel plate channel inclined at an angle θ to the horizon in the presence of uniform transverse magnetic field. The above problem has been extended by S. Chakraborty and A. K. Borkakati [1998].

The commencement of the Couette flow in Oldroyd liquid has been studied by S. Biswal and B. K. Pattnaik [1996], in the presence of a uniform transverse magnetic field. S. Biswal and S. Mishra [1998] analysis the combined free and forced convection effects on the MHD flow of a visco-inclastic fluid through a channel without considering dissipation energy. The unsteady flow and heat transfer through a viscous incompressible fluid in the presence of transverse magnetic field between two horizontal parallel plates, the lower plate being a stretching sheet and upper being porous has been investigated by P. R. Sharma and N. Kumar [1998]. The problem of unsteady flow of an elastic-viscous conducting incompressible fluid through porous medium between two infinite parallel plates under uniform transverse magnetic field and a uniform body force has been studied by S. K, Ghosh and S. K. Samad [1998].

N. Datta, S. Biswal and P. K. Sahoo [1998] have been discussed about the magnetohydrodynamics unsteady flow of a visco-elastic liquid (Rivlin-Ericksen) near a porous wall suddenly set in motion with heat transfer including heat generating sources or heat absorbing sinks. Flow of Rivlin-Ericksen incompressible fluid through an inclined channel with two parallel flat walls under the influence of magnetic field has been discussed by V. P. Rathod and H. Shrikanth [1998]. The unsteady flow and heat transfer of a visco-elastic fluid through a circular pipe had been investigated by P. R. Sharma and H. Kumar [1998]. T. K. Mahato and D. R. Kuiry [1999] studied about the flow behavior of a viscous incompressible and electrically non-conducting fluid due to the time-varying acceleration of an infinite porous plate in the presence of a uniform transverse magnetic field.

The transient free convection flow of an incompressible visco-elastic fluid past an infinite vertical plate under uniform surface heat flux conditions has been studied by U. N. Das, R. Deka and V. M. Soundalgekar [1999]. An unsteady viscous incompressible free convection flow of an electrically conducting fluid between two heated vertical parallel plates has been worked out in the presence of a uniform magnetic field applied transversely to the flow, by S. Chakraborty and A. K. Borkakati [2000]. A theoretical analysis of free convective two-dimensional unsteady flow through porous medium of variable permeability, bounded by an infinite vertical porous plate with uniform suction and constant heat flux has been presented by A. Maharshi and S. S. Tak [2000]. M. Acharya, G. C. Dash and L. P. Singh[2000] discussed the analysis of steady two-dimensional free convection and mass transfer flow of a viscous incompressible electrically conducting fluid through a porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in the presence of a uniform magnetic field.

The general problem of impulsive motion of an electrically conducting second order fluid under the transverse magnetic field over a plate has been formulated and solved by R. N. Ray, A. Samad and T. K. Chaudhury [2001]. S. Sreekanth, A. S. Nagaranjan and S. V. Ramana [2001] have discussed the unsteady transient free convection flow of an incompressible dissipative viscous fluid past an infinite vertical plate in considered on taking into account viscous dissipative heat, under the influence of a uniform transverse magnetic field. K. D. Singh and R. Sharma [2002] studied the effect of period permeability on the free convective flow of a viscous incompressible fluid through a highly porous medium, when the porous medium is bounded by an infinite vertical porous plate.

1.10 Motivation, extent and scope of this thesis

The motivation of this thesis is to study a few aspects of the effects of heat transfer in the incompressible viscous fluid as well as in the electrically conducting incompressible viscous fluid. Some problems of magnetohydrodynamics also have been discussed here.

The chapter-1 is going to be dealt with the introduction of the thesis. The outline of the magnetohydrodynamics, its development and applications, fundamental equations of electrically conducting fluid flow and effect of heat transfer in MHD have been discussed in this chapter. During the past two decades, a number of significant experiments have been carried out revealing non-Newtonian characteristics of liquids where a number of new phenomenon have been observed in a large number of liquids, of great technological and industrial importance. A brief description of these liquids and electrically conducting fluids is also given in this chapter. Lastly, a brief review of earlier workers and scope of this work have also been explained in this chapter.

The laminar free convection flow of an incompressible electrically conducting second order fluid under the action of uniform transverse magnetic field over a plate has been discussed in the chapter-2. Exact solutions of the fluid velocity $\bar{u}(y,t)$ and temperature profile $\bar{T}(y,t)$ can be obtained with the help of the perturbation technique, where y is the distance measured between the two plates and t is the time. It has been observed that this problem is useful in many engineering problems and the unsteady magnetohydrodynamics free convection flow of an electrically conducting fluid between two heated vertical parallel plates is of considerable interest to the technical field due to its frequent occurrence in industrial and technical applications.

The unsteady Couette flow of a viscous incompressible and electrically conducting fluid with the heat transfer between two horizontal parallel plates in the presence of a uniform transverse magnetic field has been discussed in the chapter-3, when in the case-I, the plates are at different temperatures and in the case-II the upper plate is considered to move with the constant velocity where the lower plate is adiabatic. Our results are useful in geophysical and astrophysical problems as the simultaneous effects of hydromagnetic buoyancy forces and coriollis forces are observed in various types of problems in these branches of sciences.

A theoretical and numerical analysis of unsteady two-dimensional free convection flow of a viscous incompressible electrically conducting fluid through a porous medium of variable permeability, bounded by an infinite vertical porous plate with uniform suction and constant heat flux under the action of a uniform magnetic field has been investigated in the chapter-4. The effects of Prandtl number, Grashoff number, magnetic field parameter and the permeability parameter of porous medium on the velocity and also the effects of Prandtl number on the temperature profile have been discussed and shown graphically.

In the chapter-5, we have discussed the motion of an unsteady MHD flow of an incompressible electrically conducting viscous fluid between two horizontal parallel porous plates on the time-varying motion. The velocity profile and the skin-friction are obtained due to the effect of the deflection of a strong magnetic field on the MHD flow past between two parallel plates and the results are obtained numerically and plotted graphically by taking the different values of the non-dimensional parameter of magnetic field parameter.

We have discussed in the chapter-6 the MHD unsteady flow of a visco-elastic (Rivlin-Ericksen) fluid through an inclined channel with two parallel flat plates with heat transfer including heat generating sources or heat absorbing sinks, when the plates are moving with the transient velocity while one of these two plates is adiabatic. Here the fluid velocity and temperature profile are obtained by the Perturbation technique and discussed by interpreting the graphs with the help of different values of some appeared non-dimensional parameters.

In the chapter-7, we have studied the unsteady flow of an incompressible electrically conducting second order fluid through the porous medium due to infinite horizontal plate in the presence of uniform transverse magnetic field which includes the heat generating sources or heat absorbing sinks. Here the plates are maintained at temperatures while one plate is kept at a constant temperature gradient. The values of the velocity and temperature distribution are found out numerically and interpreted with the help of graph. The problems of determining the electrically conducting fluid flow and heat transfer through a porous channel driven by a pressure gradient are fundamental with obvious applications in physiology and engineering. So, our research may be useful.

CHAPTER 2

Unsteady MHD free convection flow of a second order fluid between two heated vertical plates.

2.1 Introduction

A.Bharali and A. K. Borkakati [1983] discussed about the response of the flow and heat transfer to the change of direction of the imposed magnetic field in steady magnetohydrodynamics laminar free convection flow past an infinite vertical porous plate by taking Hall effects into account. The magnetohydrodynamics unsteady viscous incompressible free convection flow of an electrically conducting fluid between two heated vertical plates in the presence of the force field of gravity and applied magnetic field acting in the horizontal direction and perpendicular to the flow was discussed by M. G. Gourla and S. L. Katoch [1991]. N. Dutta, S. Biswal and P. K. Sahoo[1998] studied the magnetohydrodynamic unsteady flow of a visco-elastic liquid (Rivlin-Ericksen) near a porous wall suddenly set in motion with the heat transfer including heat generating sources or heat absorbing sinks and they found that the temperature of the fluid is largely affected by the presence of the heat sources or sinks. The transient free convection flow of an incompressible visco-elastic fluid past an infinite vertical plate under the uniform surface heat flux conditions has been studied by U. N. Das, R. Deka and V. M. Soundalgekar [1999].

Also, they discussed about the velocity and length of penetration effect due to leading edge increase with the increasing of the elastic parameter or time t, but decreases when the Prandtl number increases. S. Chakraborty and A. K. Borkakati [2000] investigated the fully developed free convection laminar flow of an incompressible viscous electrically conducting fluid between two heated vertical parallel plates in presence of a uniform magnetic field applied transversely to the flow. The general problem of unsteady parallel flow of an electrically conducting second order fluid under the transverse magnetic field due to the impulsive start of a parallel to itself has been formulated by R. N. Ray, A. Samad and T. K. Chaudhury [2001] and solved by the method of Laplace Transform for the two cases of motion corresponding to the so-called Stoke's first and second problems. The laminar convection flow of a viscous incompressible electrically conducting fluid on a continuous moving flat plate in the presence of uniform transverse magnetic field, was studied by S. Chakraborty and A. K. Borkakati [2002]. Here the flat plate which is continuously moving in its own plane with a constant speed is considered to be isothermally heated.

In this chapter, we analyze about the unsteady free convection flow of a second order viscous, incompressible electrically conducting fluid between two heated vertical plates in the presence of uniform transverse magnetic field. The uniform magnetic field applied externally in the direction normal to the fluid motion. Perturbation technique is used to solve numerically the equations of the problem and the numerical results obtained are shown and discussed graphically for the different values of magnetic field parameter, elastic parameter, Grashof number, Prandtl number.

The unsteady magnetohydrodynamics free convection flow of an electrically conducting fluid between two heated vertical parallel plates is of considerable interest to the technical field due to its frequent occurrence in industrial and technical applications

2.2 Mathematical formulation of the problem

We consider the unsteady free convection flow of an incompressible viscous-elastic second order electrically conducting fluid between two heated vertical parallel plates separated by a distance 2h apart. We now consider the unsteady flow starting from the rest of an electrically conducting second order fluid over a plate in presence of a uniform transverse magnetic field. Let the x'-axis be taken along the plate with the direction of the fluid flow and y'-axis normal to the plate. Let u' and v' be the velocities of the fluid along the x'-axis and y'-axis respectively. Then consequently u' is a function of y' and t' only, but v' is independent of y'. Then the component of the fluid velocity are given by

$$(u'(y', t'), 0, 0).$$

Let u_0 be a constant impulsive velocity to the plate in its own plane and let the uniform magnetic field B_0 be applied in the direction normal to the plate. In order to derive the governing equations of this problem the following assumptions are taken.

(i) The fluid is finitely conducting and non-magnetic.

(ii) The viscous dissipation and the Joule heat are neglected.

(iii) Hall effect and polarization effect are neglected.

(iv) Initially i.e. at time $t' = 0$ the plates and the fluid are at constant temperature (i.e $T' = T_0'$) and there is no flow within the channel.

At time $t' > 0$, the temperature of the plate $y' = +h$ changes to $T' = T'_o$, and the temperature of the plate $y' = -h$ changes according to $T' = T'_o + (T'_w - T'_o)\varepsilon e^{i\omega t'}$, where ω is the frequency of the fluctuations with time t', T'_o is the constant temperature of the fluid and T'_w is the temperature of the fluid at the wall.

(v) The value of magnetic Reynold's number is assumed to be of low conductivity, such that the induced magnetic field is negligible.

Then the Lorentz's force is $-\sigma B_o^2 u'$, where σ is the electrically conductivity of the fluid. The second order approximation of the general constitutive equations given by Rivlin-Ericksen can be written as follows:

$$\tau = -PI + \alpha A_1 + \beta A_1^2 + \gamma A_2 \qquad (2.2.1)$$

where τ is the stress tensor, I is the unit tensor, P is an intermediate pressure and α, β, γ are co-efficients of viscosity, cross-viscosity and viscous-elasticity respectively. A_1 and A_2 are given by the symmetric matrices defined by

$$A_1 = V_{i,j} + V_{j,i} = \frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \qquad (2.2.2)$$

and $A_2 = a_{i,j} + 2V_{m,i}V_{m,j}$

$$\Rightarrow A_2 = \frac{\partial}{\partial x_j}\left(\frac{DV_i}{Dt}\right) + \frac{\partial}{\partial x_j}\left(\frac{\partial V_i}{\partial t}\right) + 2\frac{\partial V_m}{\partial x_i}\frac{\partial V_m}{\partial x_j} \qquad (2.2.3)$$

where a_i's are components of acceleration given by

$$a_i = \frac{\partial V_i}{\partial t} + V_j V_{i,j} \qquad (i, j, m = 1, 2, 3)$$

The equation of continuity is

$$V_{i,i} = 0 \tag{2.2.4}$$

where V_i are components of velocity.

Hence the flow field of the fluid motion is governed by the following equations

Equation of the continuity

$$\frac{\partial v'}{\partial y'} = 0 \tag{2.2.5}$$

In the absence of pressure gradient, the flow field is governed by the third order differential equation which takes in the following form:

Equation of momentum

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} + \frac{k_0}{\rho} \frac{\partial^3 u'}{\partial t' \partial y'^2} - \frac{\sigma B_0^2 u'}{\rho} + \beta g (T' - T'_\infty) \tag{2.2.6}$$

Equation of energy

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} \tag{2.2.7}$$

Where ρ is the density of the fluid, B_0 is uniform magnetic field applied transversely to the plate, υ is the co-efficient of the kinematic viscosity, k is the thermal conductivity of the fluid, η_0 is the co-efficient of viscosity, c_p is the specific heat at constant pressure of the fluid, β is the co-efficient of thermal expansion, g is the acceleration due to gravity, k_0 is the co-efficient of elasticity, T' is the temperature of the fluid.

The initial and boundary conditions are given by

$$t' \le 0 : u' = 0,\ T' = T'_o \qquad \text{for } -h \le y' \le +h$$

$$t' > 0 : u' = u_o,\ v' = -v_o,\ T' = T'_o + (T'_w - T'_o)\varepsilon e^{i\omega t} \qquad \text{at } y' = -h$$

$$: u' \to 0,\ T' \to T'_o \qquad \text{at } y' = +h \qquad (2.2.8)$$

We now introduce the following non-dimensional variables and parameters in order to transform the equations (2.2.5)-(2.2.7) into the non-dimensional form:

$$y = \frac{y'u_o}{\upsilon},\ u = \frac{u'}{u_o},\ t = \frac{t'u_o^2}{\upsilon},\ Rc = \frac{k_o u_o^2}{\eta_o \upsilon},\ Ha = \frac{\sigma B_o^2 \upsilon}{\rho u_o^2},$$

$$w = \frac{\upsilon w'}{u_o^2},\ Gr = \frac{\upsilon g\beta(T'_w - T'_o)}{u_o^3},\ T = \frac{T' - T'_o}{T'_w - T'_o},\ \Pr = \frac{\eta_o c_p}{k}.$$

Consequently, the equation of continuity, motion and energy into the non-dimensional form are

$$\frac{\partial v}{\partial y} = 0 \qquad (2.2.9)$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Rc\frac{\partial^3 u}{\partial t \partial y^2} - Ha\,u + GrT \qquad (2.2.10)$$

and $$\frac{\partial T}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} \qquad (2.2.11)$$

where $\upsilon = \frac{\eta_o}{\rho}$, $\eta_o = \rho\upsilon$, Rc is the elastic parameter, Ha is the magnetic field parameter, Gr is the Grashoff number and $\Pr$ is the Prandtl number.

The initial and boundary conditions of the dimensionless form are given by

$$t \le 0: u = 0, T = 0 \qquad \text{for } -1 \le y \le +1$$

$$t > 0: u = 1, T = \varepsilon e^{i\omega t} \qquad \text{at } y = -1$$

$$: u = 0, T = 0 \qquad \text{at } y = +1 \qquad (2.2.12)$$

2.3 Solution of the equations

In order to solve equations (2.2.10) and (2.2.11), we apply small parameter regular perturbation technique. Consequently, we assume that to solve the equations (2.2.10) and (2.2.11), the solutions of the equations of the motion and energy as

$$u = u_1(y) + u_2(y)\varepsilon e^{i\omega t} \qquad (2.3.13)$$

and $$T = T_1(y) + T_2(y)\varepsilon e^{i\omega t} \qquad (2.3.14)$$

where ω is the frequency of the fluctuations with time t and $\varepsilon(<1)$ constant quantity.

The corresponding boundary conditions (2.2.12) are now modified as

$$t > 0: u_1 = 1, u_2 = 0, T_1 = 0, T_2 = 1 \qquad \text{at } y = -1$$

$$: u_1 = 0, u_2 = 0, T_1 = 0, T_2 = 0 \qquad \text{at } y = +1 \qquad (2.3.15)$$

Now, using the condition (2.3.13) and (2.3.14) in the equations (2.2.10) and (2.2.11), and also separating the time-dependent and time-independent terms, we get-

$$\frac{1}{\Pr} T_1''(y) = 0 \qquad (2.3.16)$$

$$T_2''(y) - \Pr i\omega T_2(y) = 0 \qquad (2.3.17)$$

$$u_1''(y) - Ha\, u_1(y) = -Gr T_1(y) \qquad (2.3.18)$$

and $$(1 + i\omega Rc)u_2''(y) - (Ha + i\omega)u_2(y) = -Gr T_2(y) \qquad (2.3.19)$$

Solving equations (2.3.16)-(2.3.19) with the help of the condition (2.3.15), we get-

$$T_1(y)=0 \tag{2.3.20}$$

$$T_2(y) = -\frac{\sinh(y-1)\sqrt{\Pr i\omega}}{\sinh 2\sqrt{\Pr i\omega}} \tag{2.3.21}$$

$$u_1(y) = -\frac{\sinh(y-1)\sqrt{Ha}}{\sinh 2\sqrt{Ha}} \tag{2.3.22}$$

and $$u_2(y) = \frac{Gr}{\{Rc\Pr\omega^2 - Ha - i\omega(1+\Pr)\}}\left[\frac{\sinh(y-1)\sqrt{\Pr i\omega}}{\sinh 2\sqrt{\Pr i\omega}} - \frac{\sinh(y-1)\sqrt{\dfrac{Ha+i\omega}{1+i\omega Rc}}}{\sinh 2\sqrt{\dfrac{Ha+i\omega}{1+i\omega Rc}}}\right] \tag{2.3.23}$$

Substituting conditions (2.3.20)-(2.3.23) in the relations (2.3.13) and (2.3.14), we get-

$$T = -\varepsilon e^{i\omega t}\frac{\sinh(y-1)\sqrt{\Pr i\omega}}{\sinh 2\sqrt{\Pr i\omega}} \tag{2.3.24}$$

and $$u = -\frac{\sinh(y-1)\sqrt{Ha}}{\sinh 2\sqrt{Ha}} + \varepsilon e^{i\omega t}\frac{Gr}{\{Rc\Pr\omega^2 - Ha - i\omega(1+\Pr)\}}\left[\frac{\sinh(y-1)\sqrt{\Pr i\omega}}{\sinh 2\sqrt{\Pr i\omega}} - \frac{\sinh(y-1)\sqrt{\dfrac{Ha+i\omega}{1+i\omega Rc}}}{\sinh 2\sqrt{\dfrac{Ha+i\omega}{1+i\omega Rc}}}\right] \tag{2.3.25}$$

Now, taking the real parts of the velocity and temperature profile from the equations (2.3.24) and (2.3.25), we get-

$$T = -\varepsilon\cos\omega t\frac{\sinh(y-1)\sqrt{\Pr i\omega}}{\sinh 2\sqrt{\Pr i\omega}} \tag{2.3.26}$$

and $u = Gr\varepsilon[(M_1 \cos\omega t - M_2 \sin\omega t)\{\dfrac{\sinh(y-1)\sqrt{\Pr i\omega}}{\sinh 2\sqrt{\Pr i\omega}} - \dfrac{G_1G_3 + G_2G_4}{G_3^2 - G_4^2}\}$

$$+ (M_1 \sin\omega t + M_2 \cos\omega t)\ (\frac{G_2G_3 - G_1G_4}{G_3^2 - G_4^2})] - \frac{\sinh(y-1)\sqrt{Ha}}{\sinh 2\sqrt{Ha}} \tag{2.3.27}$$

where $M_1 = \dfrac{Rc\Pr\omega^2 - Ha}{(Rc\Pr\omega^2 - Ha)^2 - \omega^2(1+\Pr)^2}$,

$$M_2 = \frac{\omega(1 - HaRc)}{(Rc\Pr\omega^2 - Ha)^2 - \omega^2(1+\Pr)^2},$$

$$\cos\theta = \frac{Ha + \omega^2 Rc}{\omega^2 Rc^2 - 1},$$

$$\sin\theta = \frac{\omega(1 - Ha\,Rc)}{\omega^2 Rc^2 - 1},$$

$$G_1 = \sin\{(y-1)\cos\frac{\theta}{2}\}\cosh\{(y-1)\sin\frac{\theta}{2}\},$$

$$G_2 = \cos\{(y-1)\cos\frac{\theta}{2}\}\sinh\{(y-1)\sin\frac{\theta}{2}\},$$

$$G_3 = \sin(2\cos\frac{\theta}{2})\cosh(2\sin\frac{\theta}{2}),$$

and $G_4 = \cos(2\cos\frac{\theta}{2})\sinh(2\sin\frac{\theta}{2})$.

Skin-friction

The skin-friction at the plates is given by

$$\tau_w = \left[\frac{du}{dy}\right]_{y=\pm 1} + R_c\left[\frac{d^2u}{dy^2}\right]_{y=\pm 1}$$

$$= -\frac{\sqrt{Ha}}{\sinh 2\sqrt{Ha}} + Gr\varepsilon(M_3 + iM_4)[\frac{\sqrt{\Pr\omega}}{\sinh 2\sqrt{\Pr\omega}}$$

$$-\frac{G_5\cos\frac{\theta}{2} - G_6\sin\frac{\theta}{2}) + i(G_5\sin\frac{\theta}{2} + G_6\cos\frac{\theta}{2})}{G_5^2 - G_6^2}], \quad \text{for } y = +1$$

$$= -\sqrt{Ha}\tanh 2\sqrt{Ha} + Gr\varepsilon(M_3 + iM_4)[\sqrt{\Pr\omega}\tanh 2\sqrt{\Pr\omega} - (M_5 + iM_6)]$$

$$+ Rc[Ha + Gr\varepsilon\{M_3M_7 + M_4M_8) + i(M_4M_7 - M_3M_8)\}], \quad \text{for } y = -1$$

Now, taking the real parts of the above skin-friction, we get-

$$\tau_w = -\frac{\sqrt{Ha}}{\sinh 2\sqrt{Ha}} + Gr\varepsilon[\frac{M_3\sqrt{\Pr\omega}}{\sinh 2\sqrt{\Pr\omega}} - \frac{M_3(G_5\cos\frac{\theta}{2} - G_6\sin\frac{\theta}{2})}{G_5^2 - G_6^2}$$

$$+\frac{M_4(G_5\sin\frac{\theta}{2} + G_6\cos\frac{\theta}{2})}{G_5^2 - G_6^2}] \quad \text{for } y = +1 \tag{2.3.28}$$

$$= -\sqrt{Ha}\tanh 2\sqrt{Ha} + Gr\varepsilon[M_3\sqrt{\Pr\omega}\tanh 2\sqrt{\Pr\omega} - M_3M_5 + M_4M_6$$

$$+ Rc(M_3M_7 + M_4M_8)], \quad \text{for } y = -1 \tag{2.3.29}$$

where $M_3 = M_1\cos\omega t - M_2\sin\omega t$,

$$M_4 = M_1\sin\omega t + M_2\cos\omega t,$$

$$G_5 = \cos(2\sin\frac{\theta}{2})\sinh(2\cos\frac{\theta}{2}),$$

$$G_6 = \sin(2\sin\frac{\theta}{2})\cosh(2\cos\frac{\theta}{2}),$$

$$G_7 = \cos(2\sin\frac{\theta}{2})\cosh(2\cos\frac{\theta}{2}),$$

$$G_8 = \sin(2\sin\frac{\theta}{2})\sinh(2\cos\frac{\theta}{2}),$$

$$M_5 = \frac{G_5(G_7 \cos\frac{\theta}{2} - G_8 \sin\frac{\theta}{2}) + G_6(G_7 \sin\frac{\theta}{2} + G_8 \cos\frac{\theta}{2})}{G_5^2 - G_6^2},$$

$$M_6 = \frac{G_5(G_7 \sin\frac{\theta}{2} + G_8 \cos\frac{\theta}{2}) - G_6(G_7 \cos\frac{\theta}{2} - G_8 \sin\frac{\theta}{2})}{G_5^2 - G_6^2},$$

$$M_7 = \frac{Rc\omega^2\{\Pr - Ha - (1 - \Pr)\}}{1 - \omega^2 Rc^2}$$

and $M_8 = \dfrac{\omega\{Rc(Rc\Pr\omega^2 - Ha) + 1 - \Pr\}}{1 - \omega^2 Rc^2}$.

Heat transfer

The heat flux i.e. rate of heat transfer co-efficients in terms of Nusselt number (Nu) at the plates is given by

$$Nu = \left[\frac{dT}{dy}\right]_{y=\pm 1}$$

$$= -\frac{\varepsilon\sqrt{\Pr\omega}\cos\omega t}{\sinh 2\sqrt{\Pr\omega}}, \qquad \text{if } y = +1$$

$$= -\varepsilon\sqrt{\Pr\omega}\cos\omega t \tanh 2\sqrt{\Pr\omega}, \qquad \text{if } y = -1$$

2.4 Results and discussion

The figure-I has obtained by plotting the velocity distribution against the variable y with the various values of Prandtl number Pr = 0.71, 1.0, 2.0, when Ha = 5, Gr = 0.1, Rc = 0.02, ωt = 45°, $\varepsilon = 0.5$. The velocity distributions take the greater values when the variable y has the negative values and less values having the positive values of y variable. This figure shows that the fluid velocity decreases with the increase of the Prandtl number.

The figure-II has been drawn the velocity distribution with the various values of magnetic field parameter Ha = 5.0, 3.5, 0.5, when Pr = 0.71, Gr = 0.1, Rc = 0.02, $\omega t = 45^0$, $\varepsilon = 0.5$. The velocity distributions have the maximum values towards the plate of y<0 and minimum values towards the plate of y>0. Also we see that the velocity distribution decreases due to the increase of the magnetic field parameter Ha.

The figure-III has been plotted the velocity profile against the variable y in the interval [-1, 1] with the different values of the elastic parameter Rc = 0.02, 0.03, 0.06, when Ha = 5, Gr = 0.1, Pr = 0.71, $\omega t = 45^0$, $\varepsilon = 0.5$. The velocity distribution takes the less values in the positive sides of the interval [-1, 1] and greater values in the negative sides in the interval [-1, 1]. In this figure also, the fluid velocity increases gradually with the increase in elastic parameter.

The figure-IV has been found by drawing the velocity distribution with the different values of Grashoff number Gr = 0.1, 0.2, 0.3, when Ha = 5, Pr = 0.71, Rc = 0.02, $\omega t = 45^0$, $\varepsilon = 0.5$. The fluid velocity increases gradually due to the increase in Grashoff number, and also its values take the greater values in the negative side of the interval [-1, 1] and less values in the positive side of the interval [-1, 1].

The figure-V has been obtained by drawing the velocity profile with the various values in phase angles of $\omega t = 45^0, 60^0, 75^0$, when Ha = 5, Pr = 0.71, Rc = 0.02, Gr = 0.1, $\varepsilon = 0.5$. In this figure, we have seen that the velocity profile increases very slowly with the increase in phase angle ωt. The velocity profile takes the greater values when the variable y has the negative values and less values when the variable y has the positive values.

In the figure-VI, the temperature distribution has been drawn against the variable y with the different values of the Prandtl number Pr = 0.71, 1.0, 2.0, when $\omega t = 45^0$, $\varepsilon = 0.5$. In this figure, we have observed that the temperature distribution for the corresponding negative and positive values of the variable y at the interval [-1, 1] decreases very slowly with the increase of the Prandtl number.

The figure-VII has been obtained by drawing the temperature distribution against the variable y with the various values of phase angles $\omega t = 45^0, 60^0, 75^0$, when Pr = 0.71, $\varepsilon = 0.5$. In this figure, here we notice that the temperature distribution takes the greater values for the y<0 than the y>0 in the interval [-1, 1]. Also, it decreases due to the increase in phase angle ωt.

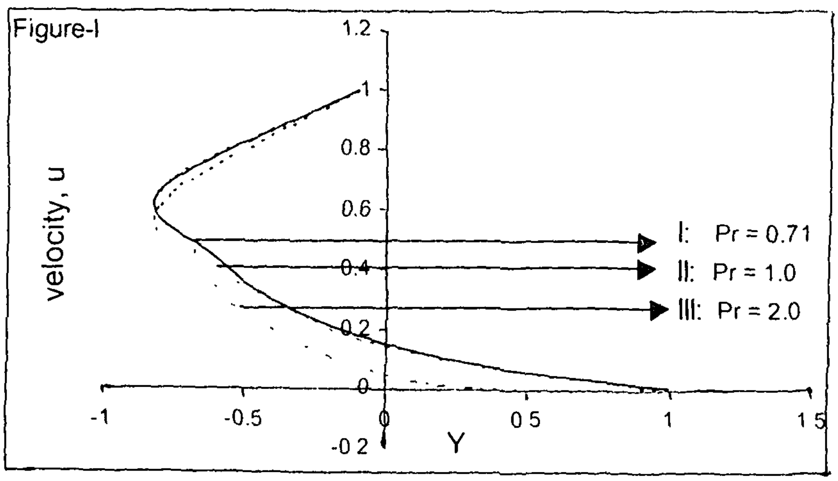

Fig.I: Velocity distribution versus y when $\varepsilon = 0.5$

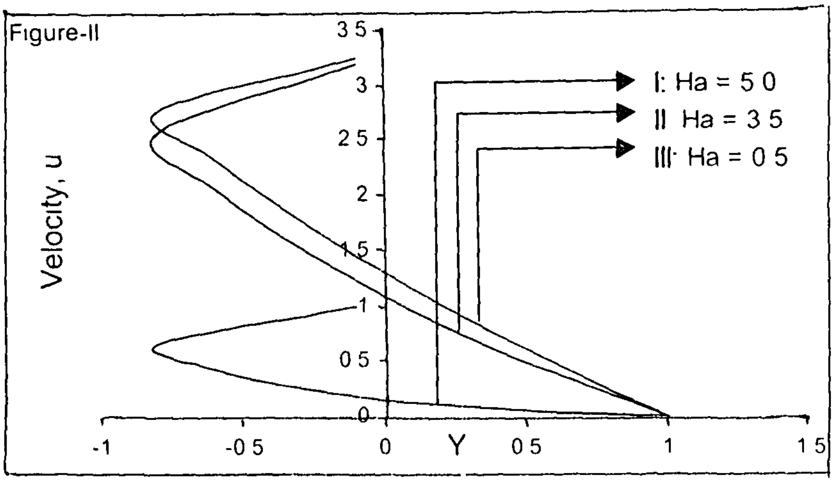

Fig.II: Velocity distribution versus y when $\varepsilon = 0.5$

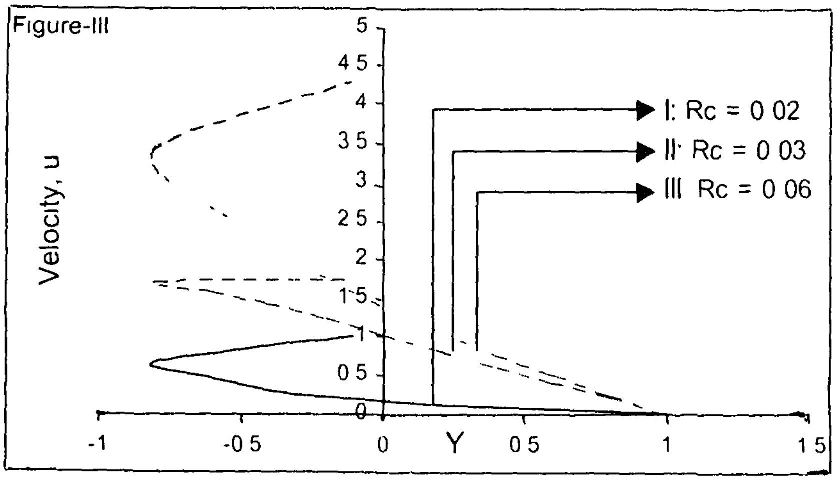

Fig.III: Velocity distribution versus y when $\varepsilon = 0.5$

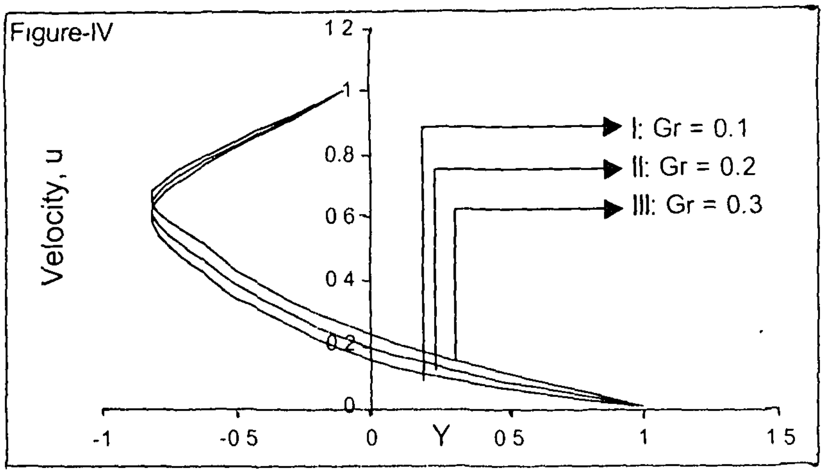

Fig.IV: Velocity distribution versus y when $\varepsilon = 0.5$

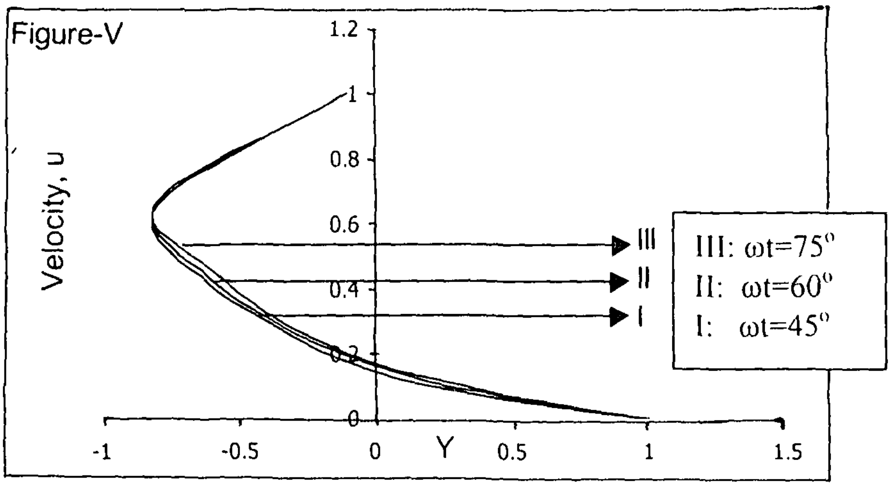

Fig.V: Velocity distribution versus y when $\varepsilon = 0.5$

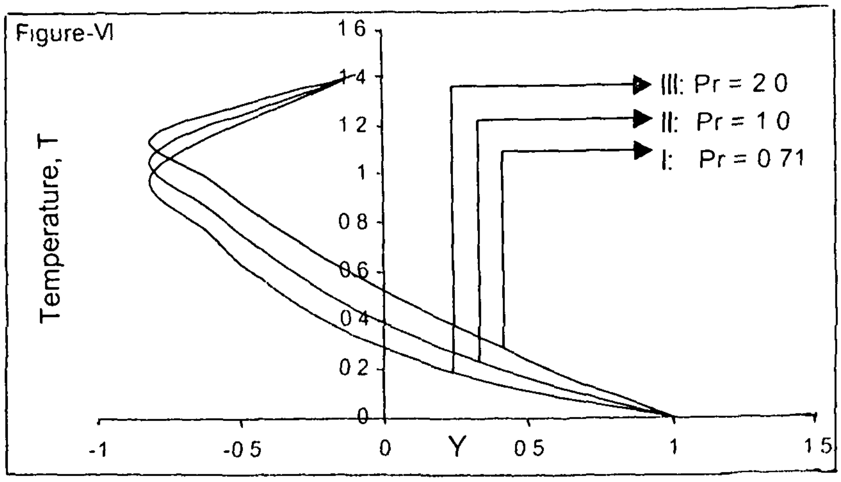

Fig.VI: Temperature distribution versus y when $\varepsilon = 0.5$

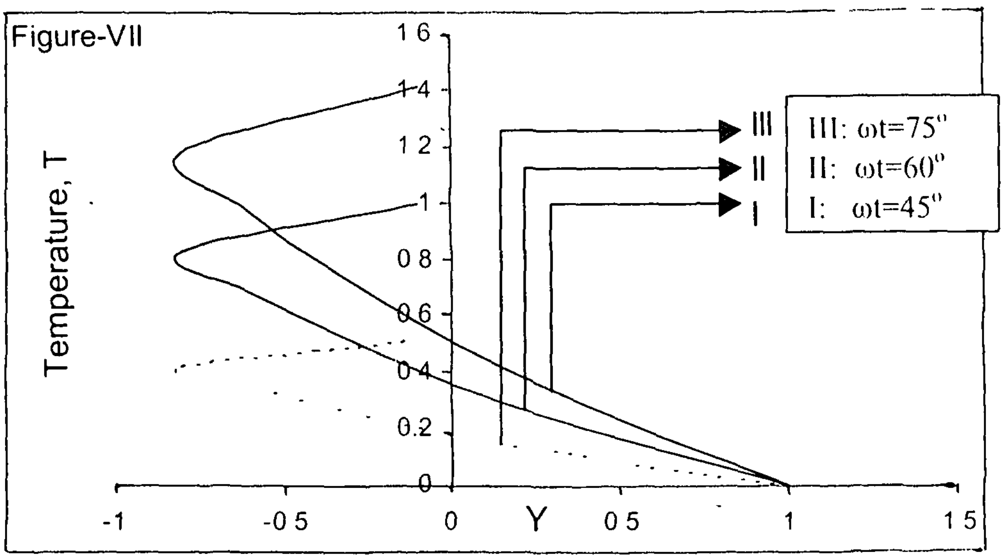

Fig.VII: Temperature distribution versus y when $\varepsilon = 0.5$

CHAPTER 3

Unsteady Couette flow with heat transfer between two horizontal plates in the presence of a uniform transverse magnetic field.

3.1 Introduction

A.K. Borkakati and A. Bharali [1979] has discussed the flow and heat transfer between two horizontal parallel plates, where the lower plate is a stretching sheet and the upper one is a porous solid plate in the presence of a uniform transverse magnetic field. The heat transfer in an axi-symmetric flow between two parallel porous disks under the effect of a transverse magnetic field is studied by A. Bharali and A. K. Borkakati [1983]. Also, they discussed the hydrodynamic flow and heat transfer between two horizontal parallel plates, where the lower one is a stretching sheet and the upper one is a porous solid plate in the presence of a transverse magnetic field. A. K. Borkakati and I. Pop [1979] studied the problem with the effects of Hall currents on the unsteady hydromagnetic flow past an infinite flat plate when a uniform magnetic field acts in a plane which makes an angle θ with the plane transverse along to the plate. Recent studies on the hydromagnetic flows with Hall currents are mainly focussed upon those where the magnetic field is imposed normal to the plate. Taking Hall effects into account the steady magnetohydrodynamical flow past an infinite horizontal porous plate is theoretically investigated by A. Bharali and A. K. Borkakati [1980], when a strong magnetic field is imposed in a direction which is perpendicular to the free stream and makes an angle α to the vertical direction. They discussed the effect of Hall currents on the flow as well as the heat transfer is studied for various values of α. Also, in 1982 they discussed about the response of flow and heat transfer to change of direction of the imposed magnetic field in steady magnetohydrodynamic laminar free convection flow past an infinite vertical porous plate by taking Hall effects into account. The effect of the deflection of a strong magnetic field on the oscillatory MHD flow past an infinite horizontal plate is studied theoretically by A. K. Borkakati and D. B. Chetri [1989] keeping the Hall parameter constant. In this problem, an attempt has been made to study theoretically the effect of the deflection on an oscillatory magnetohydrodynamic flow past an infinite horizontal flat plate. The plate is considered to an insulator and the imposed magnetic field makes an angle α to the free stream velocity. Hall effect is taken into consideration as the applied magnetic field is very strong. Shih-I-Pai [1961] studied an unsteady motion of an infinite flat insulated plate sets impulsively into the uniform motion with velocity in its own plane in the presence of a transverse uniform magnetic field.

In this chapter, the unsteady two-dimensional flow of a viscous incompressible and electrically conducting fluid between two parallel plates in the presence of a uniform transverse magnetic field has been analyzed, when in case-I the plates are at different temperatures and in case-II the upper plate is considered to move with constant velocity where as the lower plate is adiabatic. Fluid velocity and temperature distribution are obtained numerically with the help of perturbation technique and interpreted graphically with the various values in angle θ.

The problem shows the influence of imposed magnetic field and the induced magnetic field. This kind of situation often arises in different practical MHD problems in the laboratory. This problem is very importance in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, pulp, paper industry, textile industry, in different geophysical and astrophysical situation.

3.2 Mathematical formulation of the problem

The unsteady laminar flow of an incompressible viscous electrically conducting fluid between two horizontal parallel non-conducting plates separated at a distance 2h apart is considered under the action of uniform transverse magnetic field. The fluid flow is assumed to be along the X'-axis in the horizontal direction through the central line of the channel and Y'-axis is normal to it. The plates of the channel are at $y' = \pm h$ and that the relative velocity between the two plates is $2u_o$ and also, there is no pressure gradient in the flow field. The uniform magnetic field B_o makes an angle θ with X'-axis induced a magnetic field $B(y)$ or the imposed magnetic field makes an angle θ to the free stream velocity. At the time $t' > 0$, the plate at $y' = -h$ is maintained at temperature T_o, while the other plate $y' = +h$ is kept at temperature $T_1 (T_1 > T_o)$ and the plates are electrically non-conducting.

The components of the velocities and the magnetic field are given as follows:

$$u' = (u(y,t),\ v = 0,\ w = 0),\ B' = (B_x = \lambda B(y,t),\ B_y = \sqrt{1-\lambda^2}\, B_o,\ B_z = 0)$$

and p = constant, where $\lambda = \cos\theta$ is imposed and t is the time.

In order to derive the governing equations of the problem, we are to assume the following conditions:

(i) The fluid is finitely conducting and non-magnetic.

(ii) The viscous dissipation and the Joule heat are neglected

(iii) The Hall effect and polarization effect are negligible.

(iv) The buoyancy force is considered in the equation of motion of the fluid.

Under the above conditions the governing equations are as follows:

$$\rho \frac{D\vec{u}'}{Dt'} = -\nabla p + \mu \nabla^2 \vec{u}' + \vec{J} \times \vec{B} + \vec{X} \tag{3.2.1}$$

and $$\rho c_p \frac{\partial \vec{T}'}{\partial t'} = k \frac{\partial^2 \vec{T}'}{\partial y'^2} \tag{3.2.2}$$

Here the third term in the right hand side of equation (3.2.1) is the magnetic body force and $\vec{J}$ is the current density due to the magnetic field and $\vec{X}$ is the force due to the buoyancy, $\vec{X} = \rho g \beta (\vec{T}' - T_o)$. Where ρ is the density of the fluid, σ is the electrically conductivity, k is the thermal conductivity, $\upsilon = \frac{\mu}{\rho}$ is the kinematics viscosity, μ is the co-efficient of viscosity, c_p is the specific heat at constant pressure and β is the co-efficient of thermal expansion.

Using velocity and magnetic field distributions as stated above, the equations (3.2.1) and (3.2.2) are as followed:

$$\frac{\partial \vec{u}'}{\partial t'} = \upsilon \frac{\partial^2 \vec{u}'}{\partial y'^2} - \frac{\sigma B_o^2}{\rho}(1 - \lambda^2)\vec{u}' + g\beta(\vec{T}' - T_o) \tag{3.2.3}$$

and $$\frac{\partial \vec{T}'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 \vec{T}'}{\partial y'^2} \tag{3.2.4}$$

We consider here two different cases: (i) *when the plates are maintained at different temperatures*; (ii) *when the lower plate is adiabatic and the upper plate is maintained at a constant temperature.*

Case (i): *when the plates are at different temperature*, the initial and boundary conditions are

$$t' = 0 : \bar{u}' = 0,\ \bar{T}' = 0, \qquad \text{for } -h \le y' \le +h$$

$$t' > 0 : \bar{u}' = u_o, \bar{T}' = T_1 \qquad at\ y' = +h$$

$$: \bar{u}' = -u_o,\ \bar{T}' = T_o \qquad at\ y' = -h \tag{3.2.5}$$

Let us consider the non-dimensional variables and parameters as

$$u = \frac{\bar{u}'}{u_o},\ y = \frac{y'}{h},\ t = \frac{t'u_o}{h},\ T = \frac{\bar{T}' - T_o}{T_1 - T_o} \qquad Ha = \frac{\sigma B_o^2 \upsilon}{\rho u_o^2},$$

$$Gr = \frac{g\beta h^3 (T_1 - T_o)}{\upsilon^2},\ \Pr = \frac{\upsilon}{\alpha} \tag{3.2.6}$$

Using the conditions (3.2.6) in the equations (3.2.3) and (3.2.4), we get-

$$\frac{\partial u}{\partial t} = \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2} - Ha\,\mathrm{Re}(1-\lambda^2)u + \frac{Gr}{\mathrm{Re}^2}T \tag{3.2.7}$$

and $$\frac{\partial T}{\partial t} = \frac{1}{Pe}\frac{\partial^2 T}{\partial y^2} \tag{3.2.8}$$

where Ha is the Magnetic field parameter, $\mathrm{Re} = \frac{hu_o}{\upsilon}$ is the Reynolds number, Gr is the Grashoff number, $\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity, $\Pr$ is the Prandtl number and $Pe = \Pr \mathrm{Re}$ is the Peclet number.

For the relation (3.2.6), the initial and boundary conditions (3.2.5) become

$$t = 0 : u = 0,\ T = 0, \qquad \text{for } -1 \le y \le +1$$

$$t > 0 : u = 1,\ T = 1 \qquad at\ y = +1$$

$$: u = -1,\ T = 0 \qquad at\ y = -1 \tag{3.2.9}$$

3.3 Solution of the equations

In order to solve equations (3.2.7) and (3.2.8), we consider

$$u = f(y)e^{-nt} \quad \text{and } T = g(y)e^{-nt}, \tag{3.3.10}$$

where n is the decay constant.

Substituting (3.3.10) in equations (3.2.7) and (3.2.8), the equations become

$$f''(y) - \mathrm{Re}\{Ha\,\mathrm{Re}(1-\lambda^2) - n\}f(y) = -\frac{Gr}{\mathrm{Re}}g(y) \tag{3.3.11}$$

$$\text{and } g''(y) + nPeg(y) = 0 \tag{3.3.12}$$

Therefore the corresponding boundary conditions are given by

$$t > 0 : f = e^{nt},\ g = e^{nt} \qquad at\ y = +1$$

$$: f = -e^{nt},\ g = 0 \qquad at\ y = -1 \tag{3.3.13}$$

Solving the equations (3.3.11) and (3.3.12) with the help of boundary conditions (3.3.13), and substituting in the relations (3.3.10), we get-

$$u = \frac{[2\,\mathrm{Re}(a_1^2 + a_2^2) - Gr]sha_2y}{2\,\mathrm{Re}(a_1^2 + a_2^2)sha_2} - \frac{Grcha_2y}{2\,\mathrm{Re}(a_1^2 + a_2^2)cha_2}$$

$$+ \frac{Gr}{\mathrm{Re}\sin 2a_1}\left[\frac{\sin(1+y)a_1}{a_1^2 + a_2^2}\right] \tag{3.3.14}$$

$$\text{and } T = \frac{\sin(1+y)a_1}{\sin 2a_1} \tag{3.3.15}$$

where $a_1 = \sqrt{nPe}$ and $a_2 = \sqrt{\text{Re}\{Ha\,\text{Re}(1-\lambda^2)-n\}}$.

Case (ii): *when the lower plate is adiabatic*, then the initial and boundary conditions are

$$t' = 0 : u' = 0,\ T' = 0, \qquad \text{for } -h \le y' \le +h$$

$$t' > 0 : u' = u_o,\ T' = T_1 \qquad at\ y' = +h$$

$$: u' = -u_o,\ \frac{\partial T'}{\partial y'} = 0 \qquad at\ y' = -h \tag{3.3.16}$$

For the relation (3.2.6), the initial and boundary conditions (3.3.16) become

$$t = 0 : u = 0,\ T = 0, \qquad \text{for } -1 \le y \le +1$$

$$t > 0 : u = 1,\ T = 1 \qquad at\ y = +1$$

$$: u = -1,\ \frac{\partial T}{\partial y} = 0 \qquad at\ y = -1 \tag{3.3.17}$$

For the relation (3.3.10), the corresponding boundary conditions are given by

$$t > 0 : f = e^{int},\ g = e^{int} \qquad at\ y = +1$$

$$: f = -e^{int},\ \frac{\partial g}{\partial y} = 0 \qquad at\ y = -1 \tag{3.3.18}$$

Solving the equations (3.3.11) and (3.3.12) with the help of (3.3.18), and substituting in the relations (3.3.10), we get-

$$u = \left[\frac{Gr(1-\cos 2a_1)}{2\,\text{Re}(a_1^2 + a_2^2)\cos 2a_1\, sha_2} + \frac{1}{sha_2}\right] sha_2 y$$

$$-\left[\frac{Gr}{2\,\text{Re}(a_1^2 + a_2^2)\cos 2a_1\ cha_2}\right] sha_2 y$$

$$+\frac{Gr}{\mathrm{Re}\cos 2a_1}\left[\frac{\cos(1+y)a_1}{a_1^2+a_2^2}\right]. \tag{3.3.19}$$

and $$T=\frac{\cos(1+y)a_1}{\cos 2a_1}. \tag{3.3.20}$$

3.4 Results and discussion

In the case-I, numerical solutions of equations (3.3.14) and (3.3.15) are obtained for different values of λ, where $\lambda = \cos\theta$ which varies as $\theta = 45^\circ, 60^\circ, 75^\circ$. The figure-I shows that the nature of the fluid velocity with the various values of Reynolds' number Re. The values of the velocity distribution decrease with the increase for the values of Re. The velocity distribution increases for the positive variable y and also decreases for the negative values of the variable y, depending upon the values of Re.

The figure-II is obtained by plotting the velocity distribution against the variable y for different values of Prandtl number Pr, while $\mathrm{Pr} = 0.71$, 1.0 and 2.0. The velocity distribution between the plates decreases gradually with the increase of Pr. But the values of velocity increase towards the plate $y > 0$ and decreases towards the plate $y < 0$. Also the values of the velocity due to the increase of Pr is very closed that is why the plotted graphs are touching among the three curves which are drawn by taking the values of $\mathrm{Pr} = 0.71$, 1.0 and 2.0.

The figure-III is found by plotting the velocity distribution with the different values of magnetic field parameter Ha versus the variable y. The velocity profile decreases due to the increase of Ha. Its values are maximum towards the positive side of plate and minimum towards the negative side of the plate.

In the figure-IV, the fluid velocity is drawn against the variable y with the various values of the Grashoff number Gr. The velocity profile increases with the increase of Gr.

The figure-V is obtained by plotting the velocity profile against the variable y with the various values of θ. The velocity profile decreases due to the increase of the angle θ. In this figure also, the values of the velocity are maximum towards the positive values of the variable y and minimum towards the negative values of the variable y.

The figure-VI is found by plotting the temperature distribution against the variable y with the different values of the Peclet number Pe = 1.07, 1.5, 3.0. The temperature distribution increases very slowly with the increase in Peclet number.

In the case-II, numerical solutions of the equations (3.3.19) and (3.3.20) are obtained for different values of λ, where $\lambda = \cos\theta$ which varies as $\theta = 45^{\circ}, 60^{\circ}, 75^{\circ}$. The figure-VII shows that the nature of the fluid velocity with the various values of Reynold's number and the values of the velocity distribution decrease with the increase of the values of the Reynold's number Re.

The figure-VIII is found by plotting the velocity distribution against the variable y due to the various values of Prandtl number Pr. The velocity distribution between the two plates decreases gradually with the increase of Pr.

By the case-I, in the figure-IX of case-II, the velocity distribution decreases gradually with the increase of the magnetic field parameter Ha. In the figure-X of case-II, the conditions of the velocity distribution are same as the given in the case-I of figure-IV. In the figure-XI of case-II also, the velocity distribution varies same as the given in the figure-V of case-I.

The figure-XII is found by plotting the temperature distribution against the variable y for different values of Peclet number Pe = 1.07, 1.5, 3.0. The temperature distribution between the two plates increases gradually with the increase of Pe.

In the table-I, we have noticed that the values of the Nusselt number at the plate of the variable y = -1 increase gradually with the increase in Peclet number Pe. But the values of the Nusselt number at the plate of the variable y = +1 decrease very slowly with the increase in Peclet number.

In the table-II, we have observed that the values of the skin-friction co-efficient decrease with the increase in Pr at the plates y =± 1. Skin-friction increases with the increase in Re at y =± 1 and it also decreases with the increase in Ha at y =± 1. The skin-friction co-efficient increases due to the increase in Gr at y =± 1 and decreases with the increase in θ at the variable y =± 1.

In the table-III, we notice that the values of the nusselt number at the values of the variable y = -1 become zero with the different values of the Peclet number. But the nusselt numbers decrease at the values of the variable y = +1 with the increase in Peclet number.

In the table-IV, we have seen that the values of the skin-friction co-efficient decrease with the increase in Pr at the plates y =± 1. Skin-friction decreases with the increase in Re at y =± 1 and it also increases with the increase in Ha at y =± 1. The skin-friction co-efficient decreases due to the increase in Gr at y =± 1 and increases with the increase in θ at the variable y =-1, but it decreases at y=+1.

FOR CASE-I

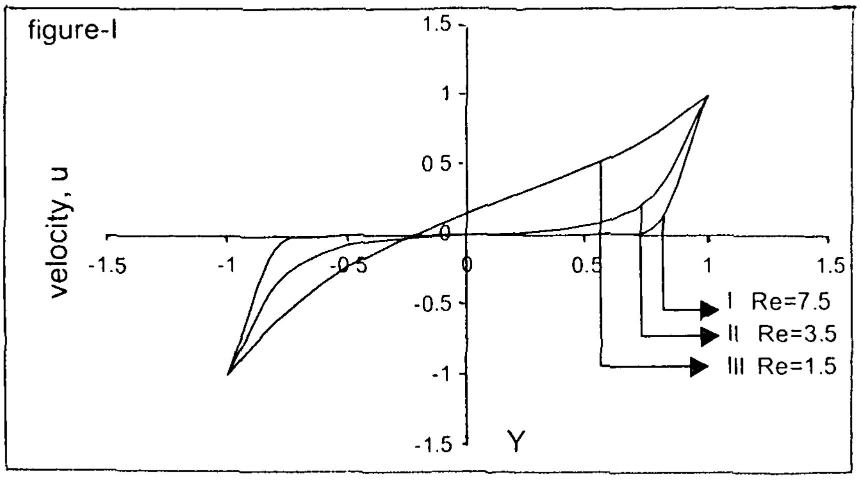

Figure-I: velocity at the plates: I. Re = 7.5, II. Re = 3.5, III. Re=1.5
Pr = 0.71, Ha = 5, Gr = 3, n = 1, θ=45°.

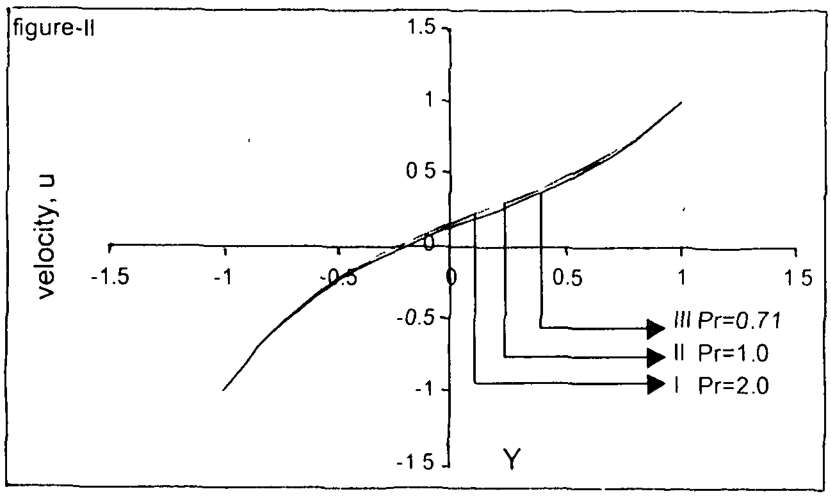

Figure-II: velocity at the plates: I. Pr = 0.71, II. Pr = 1.0, III. Pr = 2.0,
Re = 1.5, Ha = 5, Gr = 3, n = 1, θ=45°.

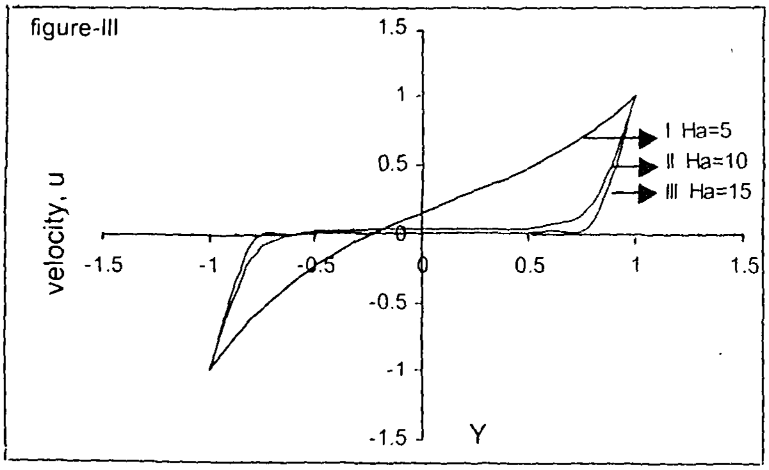

Figure-III: velocity at the plates: I. Ha = 5, II. Ha = 10, III. Ha = 15, Re = 1.5, Pr = 0.71, Gr = 3, n = 1, θ=45°.

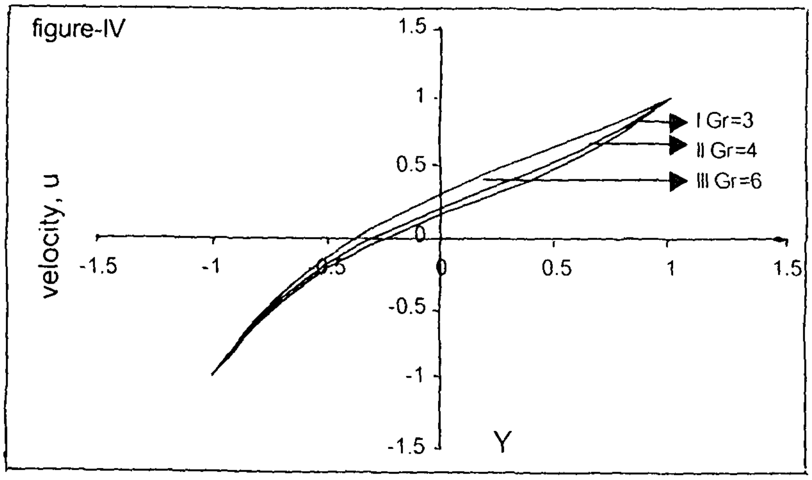

Figure-IV: velocity at the plates: I. Gr = 3, II. Gr = 4, III. Gr = 6, Re = 1.5, Pr = 0.71, Ha = 5, n = 1, θ=45°.

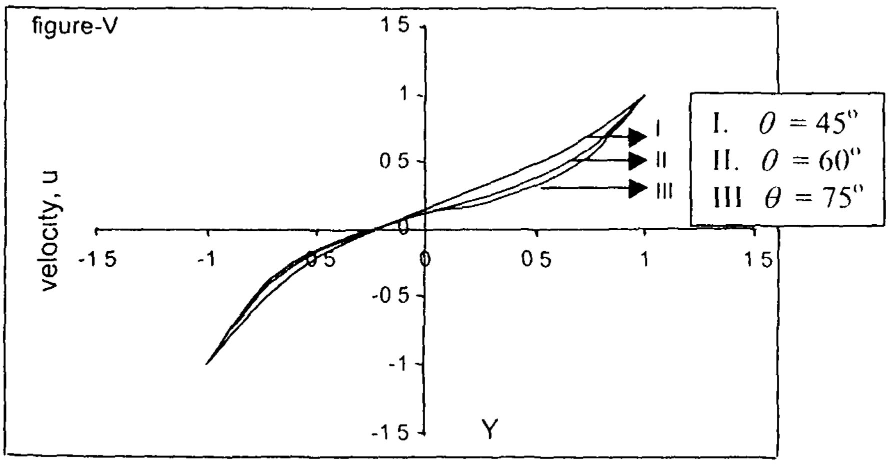

Figure-V: velocity at the plates: I. $\theta = 45^{\circ}$, II. $\theta = 60^{\circ}$, III. $\theta = 75^{\circ}$, Re = 1.5, Pr = 0.71, Ha = 5, n = 1, Gr = 3.

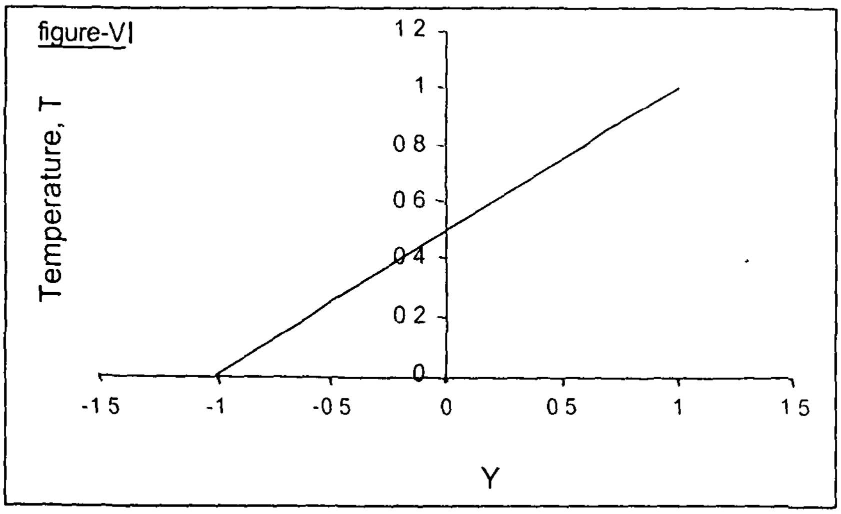

Figure-VI: Temperature at the plates: Pc = 1.07, Pe = 1.5, Pc = 3.0, n = 1.0.

FOR CASE-II

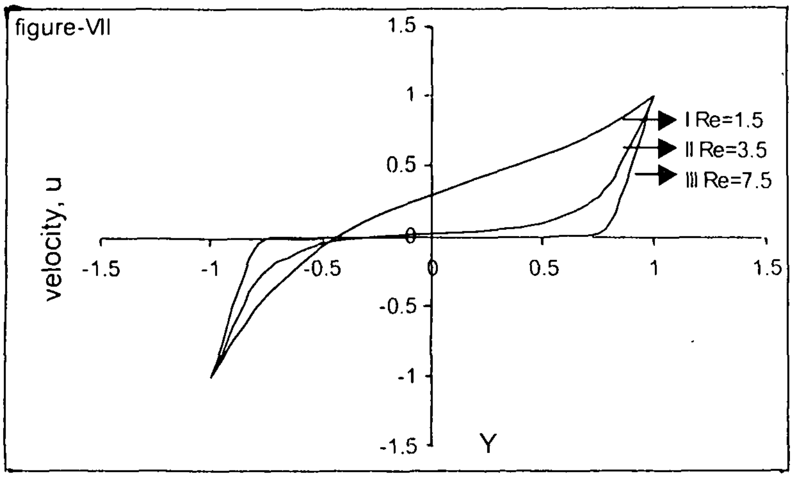

Figure-VII: velocity at the plates: I. Re = 1.5, II. Re = 3.5, III. Re = 7.5, Pr = 0.71, Ha = 5, Gr = 3, n = 1, θ=45°.

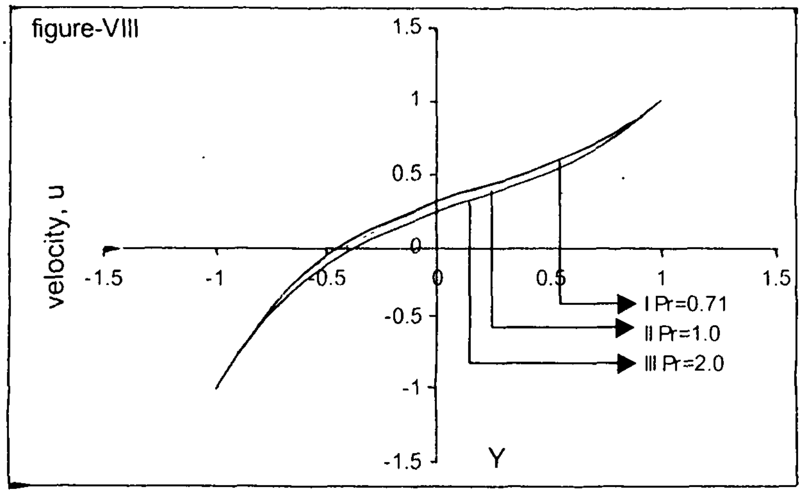

Figure-VIII: velocity at the plates: I. Pr = 0.71, II. Pr = 1.0, III. Pr = 2.0, Re = 1.5, Ha = 5, Gr = 3, n = 1, θ=45°.

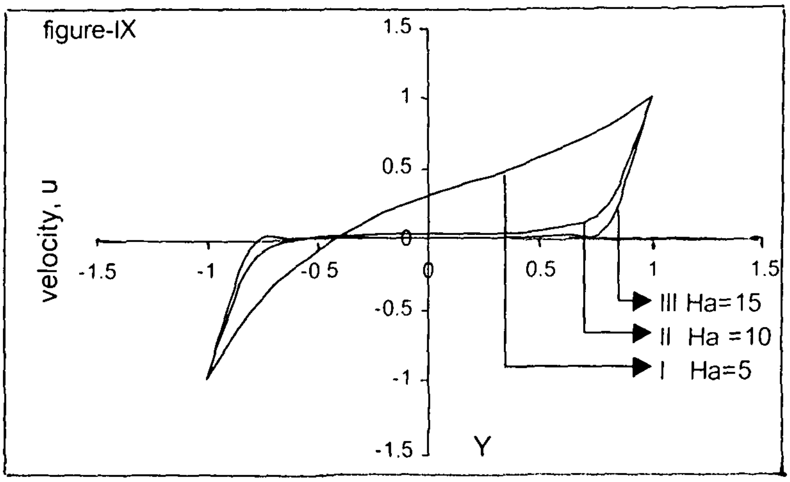

Figure-IX: velocity at the plates: I. Ha = 5, II. Ha = 10, III. Ha = 15, Re = 1.5, Pr = 0.71, Gr = 3, n = 1, θ=45°.

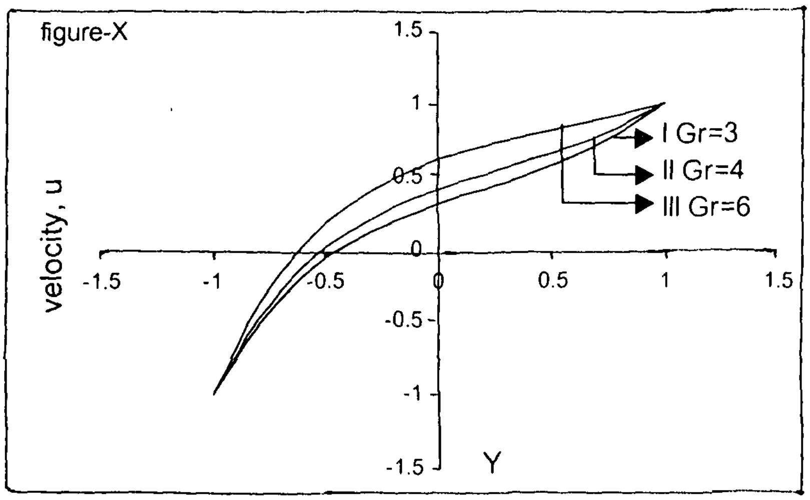

Figure-X: velocity at the plates: I. Gr = 3, II. Gr = 4, III. Gr = 6, Re = 1.5, Pr = 0.71, Ha = 5, n = 1, θ=45°.

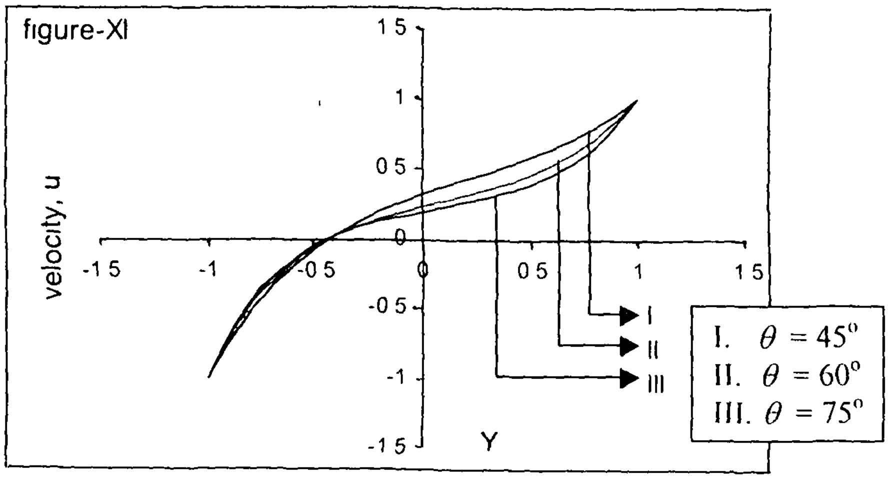

Figure-XI: velocity at the plates: I. $\theta = 45°$, II $\theta = 60°$, III $\theta = 75°$, Rc = 1.5, Pr = 0.71, Ha = 5, n = 1, Gr = 3.

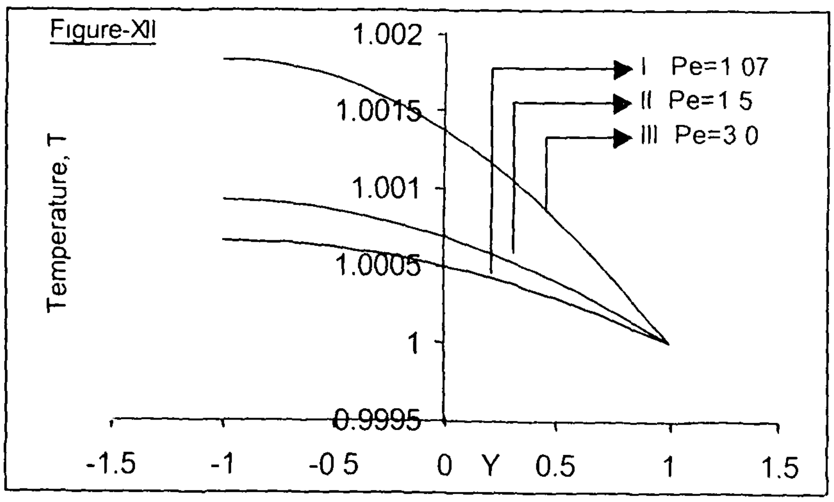

Figure-XII: Temperature at the plates: n = 1.0.

CASE-I

Table-I: values of Nusselt number at the plates:

Pe	n	(Nu)-1	(Nu)+1
1.07	1.0	28.6541164	28.6354393
1.50	1.0	28.6566185	28.6304345
3.00	1.0	28.6653506	28.6129748

Table-II: values of Skin-friction co-efficient at the plates:

Pr	Re	Ha	θ	Gr	$(C_f)_{-1}$	$(C_f)_{+1}$
0.71	1.5	5.0	45°	3	13.25679819	12.48422318
1.00	1.5	5.0	45°	3	13.17025848	12.35343098
2.00	1.5	5.0	45°	3	11.43753306	10.78539197
0.71	1.5	10	45°	3	8.755653085	8.705577072
0.71	1.5	15	45°	3	6.090770886	5.930020109
0.71	3.5	5.0	45°	3	11.62361463	11.58909340
0.71	7.5	5.0	45°	3	12.91562283	12.75686440
0.71	1.5	5.0	60°	3	10.14728014	9.462274368
0.71	1.5	5.0	75°	3	8.914997877	8.297478896
0.71	1.5	5.0	45°	4	17.17686149	16.08776882
0.71	1.5	5.0	45°	6	25.19008350	23.55644450

CASE-II

Table-III: values of Nusselt number at the plates:

Pe	n	(Nu)-1	(Nu)+1
1.07	1.0	0.000000	-0.03736629
1.50	1.0	0.000000	-0.05239180
3.00	1.0	0.000000	-0.10484739

Table-IV: values of Skin-friction co-efficient at the plates:

Pr	Re	Ha	θ	Gr	$(C_f)_{-1}$	$(C_f)_{+1}$
0.71	1.5	5.0	45°	3	2.914250420	1.438632508
1.00	1.5	5.0	45°	3	2.868405574	1.322784632
2.00	1.5	5.0	45°	3	2.743707830	1.300698366
0.71	1.5	10	45°	3	8.746873196	8.249099447
0.71	1.5	15	45°	3	12.71230515	12.249099447
0.71	3.5	5.0	45°	3	11.57240906	11.39471959
0.71	7.5	5.0	45°	3	3.368916613	5.047345910
0.71	1.5	5.0	60°	3	3.629556079	2.394250926
0.71	1.5	5.0	75°	3	3.726341957	1.972990533
0.71	1.5	5.0	45°	4	3.343298483	1.006198890
0.71	1.5	5.0	45°	6	3.184947601	0.458218889

CHAPTER 4

Magnetic field effects on the fluid and free convection flow through porous medium due to infinite vertical plate with uniform suction and constant heat flux.

4.1 Introduction

The study of the electrically conducting fluid flow problems taking into account of the simultaneous effects of the magnetic field on the fluid and free convection flow through porous medium due to infinite vertical plate with uniform suction and constant heat flux is important because of their applications in many problems of geophysical and astrophysical fields.

Acharya, Dash and Sing [2000] studied the steady two-dimensional free convection and mass transfer flow of a viscous incompressible electrically conducting fluid through porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in the presence of a uniform magnetic field. The fluctuating free convection through porous medium due to infinite vertical plate with constant heat flux has been analysed by Maharshi and Tak [2000]. Kumar [2000] studied the stability of two superposed Rivlin-Ericksen elastic-viscous fluids permitted with suspended particles in the porous medium. Sing [1996] devoted to an important study of an unsteady electrically conducting stratified viscous fluid flow through a porous medium between two parallel plates in the presence of transverse exponentially variable magnetic induction when the stream velocity at the lower plate fluctuates with time.

The magnetohydrodynamic unsteady flow of a visco-elastic liquid (Rivlin-Ericksen) near a porous wall suddenly set in motion has been studied by Datta, Biswal and Sahoo [1998] with the heat transfer including heat generating sources or heat absorbing sinks. The transient free convection flow of an incompressible visco-elastic fluid past an infinite vertical plate under uniform surface heat flux conditions is studied by Das, Deka and Soundalgekar [1999].

In this chapter, a theoretical analysis of unsteady two-dimensional free convection flow of a viscous incompressible electrically conducting fluid through a porous medium of variable permeability, bounded by an infinite vertical porous plate with uniform suction and constant heat flux under the action of a uniform magnetic field is studied. The constitutive equations of this problem have been derived by taking all the physical variables dependent on the variable 'y' only. The equation of continuity, the momentum equation and the energy equation are solved by non-dimensionalysing the equations first and then by applying the method of perturbation technique. The expressions for the fluid velocity, temperature profile and skin-friction are obtained. The effects of Prandtl number, Grashof number, magnetic field parameter and the variable permeability of porous medium on the velocity are interpreted graphically, and also, temperature profile are discussed and shown graphically.

4.2 Mathematical formulation of the problem

To formulate the governing equations in this chapter, let us consider a two-dimensional unsteady free convection flow of an incompressible electrically conducting fluid through porous medium bounded by an infinite vertical porous plate in the presence of uniform magnetic filed.

It is assumed that there is a uniform suction velocity of the fluid and the constant heat flux through the porous plate. Here X'-axis is considered to be taken along the plate and Y'-axis is taken normal to it. Let u' be the velocity of the fluid along the X'-axis and let v' be the velocity of the fluid along the Y'-axis. So consequently, u is a function of the variable y' and t' only. But v' is independent of the variable t'.

To derive the governing equations of the problem, the following conditions are considered:

(i) The plates are infinitely long, so that the fluid velocity u' is the function of y' and t' only.

(ii) The buoyancy force is considered in the equation of motion of the fluid.

(iii) The flow between the plates is fully developed.

(iv) The Joule heat and viscous dissipation are assumed to be neglected.

(v) The Hall effect and polarization effect are neglected.

(vi) The fluid is supposed to be of low conductivity, such that the induced magnetic field is negligible.

(vii) Only electro-magnetic body force (Lorentz force) is considered.

Then the Lorentz's force is $-\sigma B_o^2 u'$, when the fluid velocity u' is given to the plate in its own plane and a uniform magnetic field B_o is applied transversely to the plate.

Thus the flow field is governed by the following equations:

Equation of continuity

$$\frac{\partial v'}{\partial y'} = 0 \qquad (4.2.1)$$

Equation of momentum

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \upsilon\frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) - \frac{\upsilon}{k}u' - \frac{\sigma B_o^2}{\rho}u' \qquad (4.2.2)$$

Equation of energy

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{k'}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} \qquad (4.2.3)$$

where ρ is the density of the fluid, υ is the kinematic viscosity of the fluid, B_o is a uniform magnetic field applied transversely to the plate, σ is the electrical conductivity of the fluid, k is the permeability of the porous medium, k' is the thermal conductivity of the fluid, g is an acceleration due to gravity, β is the co-efficient of the thermal expansion, c_p is the specific heat of the fluid at constant pressure, T′ is the temperature of the fluid and T'_∞ is the temperature of the fluid at infinity from the plate.

Let k be of the form $k(t) = k_o(1 + \varepsilon e^{i\omega t})$, where k_o is the mean permeability of the porous medium, ω is the frequency of the fluctuations with time t and $\varepsilon(<1)$ constant quantity.

The relevant initial and boundary conditions are given by

$$t' \le 0 : u' = 0,\ T' = T'_\infty \qquad \text{for all } y'$$

$$t' > 0 : u' = 0,\ \frac{\partial T'}{\partial y'} = -\frac{q}{k'} \qquad at\ y' = 0$$

$$: u' = 0,\ T' = T'_\infty \qquad at\ y' = \infty \qquad (4.2.4)$$

From the equation (4.2.1), we get

v'=constant.

For the constant suction, let us take

$$v' = -v_o \qquad (4.2.5)$$

where the negative sign indicates that the suction towards the plate.

Introducing the following non-dimensional variables and parameter quantities:

$$y = \frac{y'v_o}{\upsilon},\ t = \frac{t'v_o^2}{\upsilon},\ u = \frac{u'}{v_o},\ Ha = \frac{\sigma B_o^2\upsilon}{\rho v_o^2},\ Gr = \frac{g\beta q\upsilon^2}{k'v_o^4},$$

$$T = \frac{(T' - T'_\infty)v_o k'}{q\upsilon},\ \Pr = \frac{\mu c_p}{k'},\ \gamma = \frac{\mu}{\rho},\ \alpha = \frac{v_o^2 k_o}{\upsilon^2} \qquad (4.2.6)$$

then the equations (4.2.2) and (4.2.3) with the help of the conditions (4.2.6), reduce to the following form:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + GrT - \{\frac{1}{\alpha(1+\varepsilon e^{i\omega t})} + Ha\}u \qquad (4.2.7)$$

and $$\frac{\partial T}{\partial t} - \frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} \qquad (4.2.8)$$

where Gr is the Grashof number, Pr is the Prandtl number, α is the permeability parameter and Ha is the magnetic field parameter. But magnetic field parameter is not defined same as the Hartmann number $Hr = B_o h\sqrt{\frac{\sigma}{\eta}}$.

The corresponding initial and boundary conditions are given by

$$t \le 0: u = 0,\ T = 0 \qquad \text{for all } y$$

$$t > 0:\ u = 0,\ \frac{\partial T}{\partial y} = -1 \qquad at\ y = 0$$

$$: u = 0,\ T = 0 \qquad at\ y = \infty \qquad (4.2.9)$$

4.3 Solution of the equations

To solve the equations (4.2.7) and (4.2.8), let us break up the velocity (u) and temperature (T) into two parts, one time-dependent and other time-independent respectively. Thus we assume that the following series expressions for the velocity and temperature profile:

$$u = u_o(y) + \varepsilon u_1(y)e^{i\omega t} + \ldots\ldots\ldots\ldots \tag{4.3.10}$$

and $$T = T_o(y) + \varepsilon T_1(y)e^{i\omega t} + \ldots\ldots\ldots\ldots \tag{4.3.11}$$

Now, substituting the equations (4.3.10) and (4.3.11) in equations (4.2.7) and (4.2.8), and separating the harmonic and non-harmonic terms of like powers of ε to zero, the following partial differential equations are obtained.

$$\frac{1}{\Pr}T_o'' + T_o' = 0 \tag{4.3.12}$$

$$\frac{1}{\Pr}T_1'' + T_1' - i\omega T_1 = 0 \tag{4.3.13}$$

$$u_o'' + u_o' - (\frac{1}{\alpha} + Ha)u_o = -GrT_o \tag{4.3.14}$$

and $$u_1'' + u_1' - (\frac{1}{\alpha} + Ha + i\omega)u_1 = -GrT_1 - \frac{u_o}{\alpha} \tag{4.3.15}$$

The initial and boundary conditions (4.2.9) are now modified as

$t \le 0 : u_o = 0 = u_1,\ T_o = 0 = T_1,$ for all y

$t > 0 :\ u_o = 0,\ u_1 = 0,\ \dfrac{\partial T_o}{\partial y} = -1,\ \dfrac{\partial T_1}{\partial y} = 0$ at $y = 0$

$$: u_o = 0,\ u_1 = 0,\ T_o = 0,\ T_1 = 0 \qquad at\ y = \infty \tag{4.3.16}$$

Now, solving the equations (4.3.12)-(4.3.15) by using the conditions (4.3.16), we get

$$T_o(y) = \frac{1}{\Pr} e^{-\Pr y} \tag{4.3.17}$$

$$T_1(y) = 0 \tag{4.3.18}$$

$$u_o(y) = \frac{Gr}{M_1}[e^{-a_1 y} - e^{-\Pr y}], \tag{4.3.19}$$

and $u_1(y) = \dfrac{GrG_1 e^{-a_2 y}}{\alpha M_1(M_3^2 - M_4^2)}[(\alpha M_1 M_3 \cos\phi_1 y + M_4 \sin\phi_1 y)$

$$- i(\alpha M_1 M_3 \sin\phi_1 y - M_4 \cos\phi_1 y)] \tag{4.3.20}$$

where $a_1 = \dfrac{1 + \sqrt{1 + 4(\frac{1}{\alpha} + Ha)}}{2}$,

$M_1 = \Pr\{\Pr^2 - \Pr - (\frac{1}{\alpha} + Ha)\}$,

$M_2 = \Pr\{a_1^2 - a_1 - (\frac{1}{\alpha} + Ha)\}$,

$M_3 = M_1 M_2 - \omega^2 \Pr^2$,

$M_4 = \omega \Pr(M_1 + M_2)$,

$G_1 = \Pr^2(\Pr - a_1)(\Pr + a_1 - 1)$,

$\cos\theta = \frac{1}{4} + \frac{1}{\alpha} + Ha$, $\sin\theta = \omega$, $\phi_1 = \sin\frac{\theta}{2}$, *and* $a_2 = \frac{1}{2} + \cos\frac{\theta}{2}$.

Thus substituting the solutions (4.3.17)-(4.3.20) in the relations (4.3.10) and (4.3.11), we get

$$T = \frac{1}{\Pr} e^{-\Pr y} \tag{4.3.21}$$

and $u = \frac{Gr}{M_1}[e^{-a_1 y} - e^{-\Pr y}] + \varepsilon e^{i\omega t}\frac{Gr\,G_1 e^{-a_2 y}}{\alpha M_1(M_3^2 - M_4^2)}[(\alpha M_1 M_3 \cos\phi_1 y$

$$+ M_4 \sin\phi_1 y) - i(\alpha M_1 M_3 \sin\phi_1 y - M_4 \cos\phi_1 y)] \qquad (4.3.22)$$

Now, taking the only real parts of the velocity, we get

$$u = \frac{Gr}{M_1}[e^{-a_1 y} - e^{-\Pr y}] + \frac{\varepsilon Gr G_1 e^{-a_2 y}}{\alpha M_1(M_3^2 - M_4^3)}[\alpha M_1 M_3 \cos(\omega t - \phi_1 y)$$

$$+ M_4 \cos(\omega t + \phi_1 y)] \qquad (4.3.23)$$

Skin-friction

The skin-friction at the plates is given by

$$\tau_w = (\frac{\partial u}{\partial y})_{y=0}$$

$$= \frac{Gr}{M_1}[(\Pr - a_1) - \frac{\varepsilon G_1}{\alpha(M_3^2 - M_4^2)}\{\alpha M_1 M_3(a_2 \cos\omega t - \phi_1 \sin\omega t)$$

$$+ M_4(a_2 \cos\omega t + \phi_1 \sin\omega t)\}] \qquad (4.3.24)$$

4.4 Results and discussion

The velocity distributions of the fluid are shown by the curves of figures-I, II, III and IV. In the figure-I, the velocity distribution is obtained by drawing against the variable y for the different values of the magnetic field parameter Ha = 1.5, 3.5, 7.5, when Pr = 0.71, Gr = 5.0, α =1, ω =0.10, ω t = 45^0, ε = 0.2. Here we have noticed that the velocity distribution increases gradually near the plate ($0 \le y < 1$) and then decreases slowly far away from the plate (y>>1). Also, the values of the velocity distribution decrease for the increasing of values of the magnetic field parameter Ha.

The figure-II is obtained by drawing the velocity distribution against the variable y with the various values of Prandtl number Pr = 0.71, 1.2, 2.3, when Ha = 1.5 as the values of Gr, α, ωt, ω, ε remain same as taking on the plotted figure-I. Here the velocity distribution increases gradually near the plate ($0 \le y < 1$) and then decrease slowly far away from the plate (y>>1). Also, the velocity distribution decreases with the increase in Pr.

The figure-III is obtained by plotting the velocity distribution against y with the different values of the Grashoff number Gr = 5, 10, 15, when Pr = 0.71, Ha = 1.5, α, ωt, ω, ε remain same as considering on the plotted figure-I. Here also, we see that the velocity increases gradually towards the plate ($0 \le y < 1$) and the decreases slowly far away from the plate (y>>1). The velocity distribution increases due to the increase in the Grashoff number.

The figure-IV is found by drawing the velocity distribution against the variable y with the various values of the permeability parameter α = 1.0, 2.0, 3.0, when Pr = 0.71, Gr = 5.0, Ha = 1.5 as the values of ωt, ω, ε remain same as considering in the figure-II. In this figure also, we have seen that the velocity distribution increases towards the plate ($0 \le y < 1$) and then decreases slowly far away to the plate (y>>1). The fluid velocity increases gradually with the increase of permeability parameter.

The solution of the temperature distribution is similar to that followed by Maharshi and Tak [2000]. Here the temperature distribution is plotted against the variable y $(0 \le y < \infty)$. From the figure-V, it is observed that the temperature increases near the plate for the different values of the Prandtl number Pr = 0.71, 1.2, 2.3 against the variable y (0<y<1) and decreases far away from the plate for the variable $(1 < y < \infty)$. But the temperature distribution decreases with the increase in Prandtl number Pr.

From the table-I, II and III, we have observed that the skin-friction increases due to the increase in Gr, but decreases gradually due to increasing values of Prandtl number and permeability parameter.

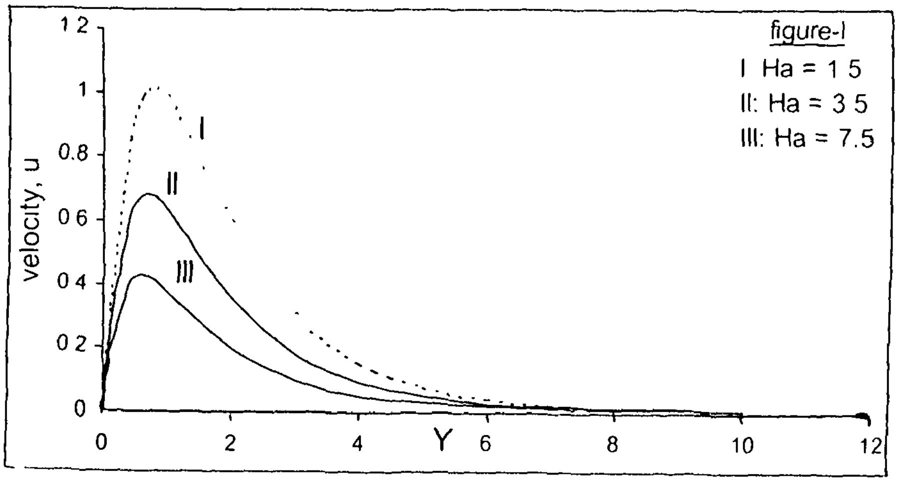

Fig.I: Velocity profile versus y when ω =0.10 and ε =0.2.

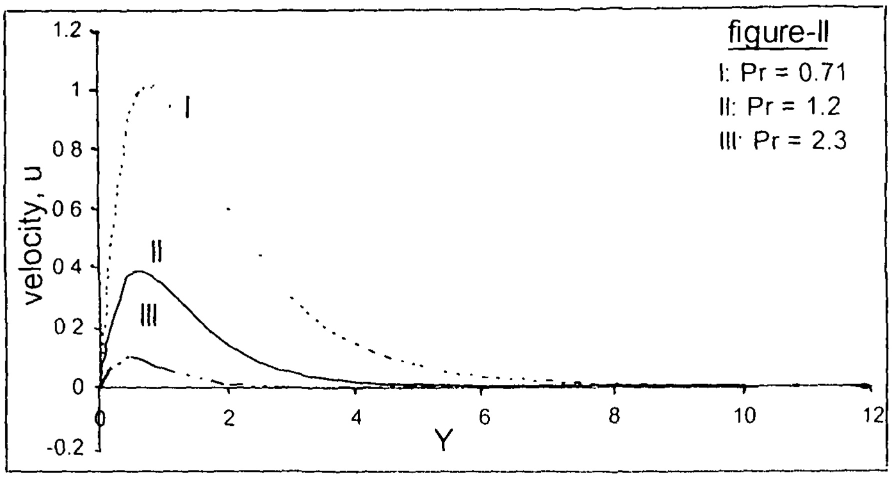

Fig.II: Velocity profile versus y when ω =0.10 and ε =0.2.

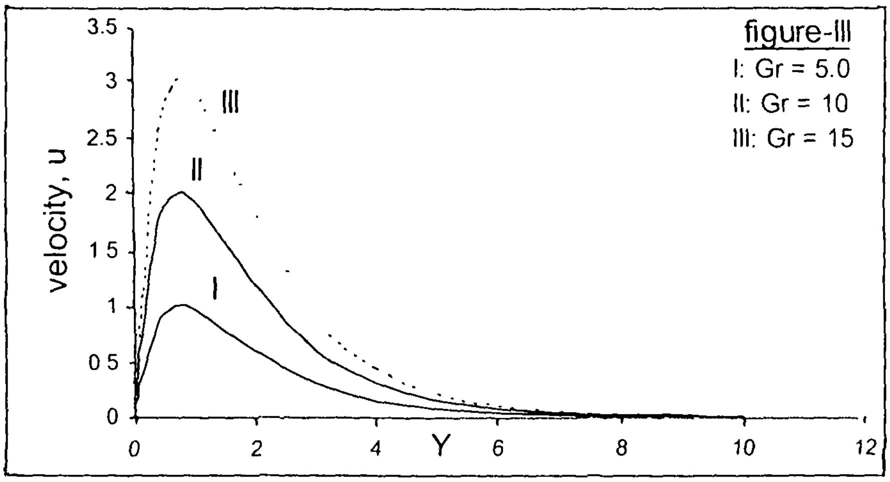

Fig.III: Velocity profile versus y when ω =0.10 and ε =0.2.

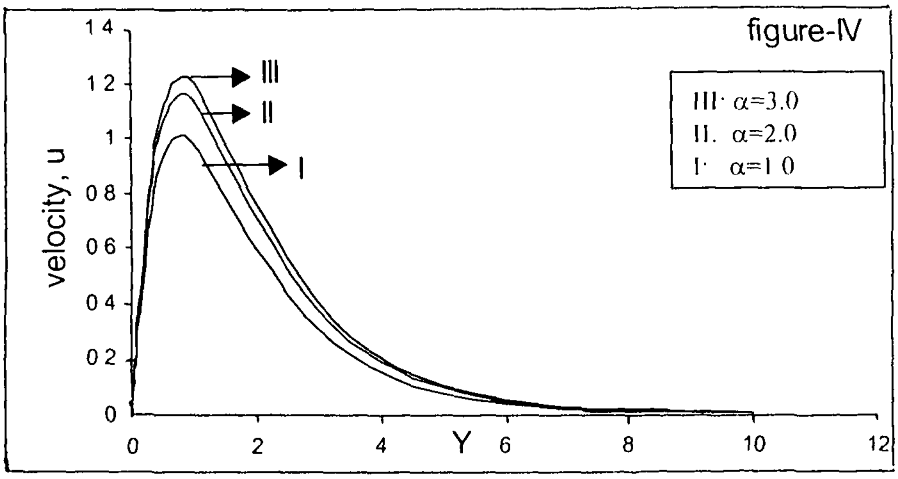

Fig.IV: Velocity profile versus y when ω =0.10 and ε =0.2.

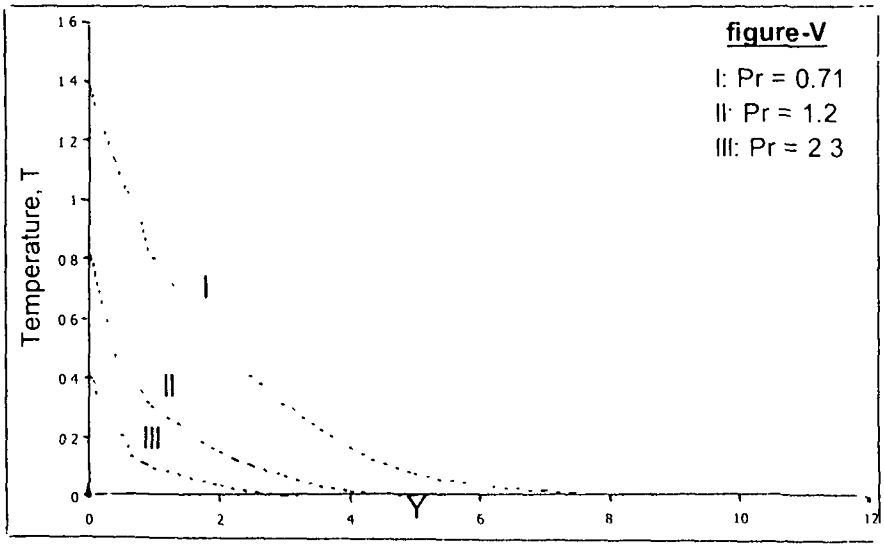

Fig.V: Temperature distribution versus y for various values of Pr as shown in figure.

Table I: Values of Skin-friction τ_w at the plate

Pr	α	Gr	τ_w Ha=1.5	τ_w Ha=7.5
0.71	1.0	1.0	0.7464	0.4433
0.71	1.0	5.0	3.7319	2.2167
0.71	1.0	10	7.4637	4.4447
0.71	1.0	15	11.1956	6.6563

Table II: Values of Skin-friction τ_w at the plate

Pr	α	Gr	τ_w Ha=1.5	τ_w Ha=7.5
1.0	2.0	1.0	0.7336	0.3487
1.0	2.0	5.0	3.6680	1.7434
1.0	2.0	10	7.3360	3.4868
1.0	2.0	15	11.0041	5.2302

Table III: Values of Skin-friction τ_w at the plate

Pr	α	Gr	τ_w Ha=1.5	Ha=7.5
2.0	3.0	1.0	0.1689	0.1353
2.0	3.0	5.0	0.8447	0.6767
2.0	3.0	10	1.6893	1.3534
2.0	3.0	15	2.5340	2.0301

CHAPTER 5

The motion of the electrically conducting fluid with the time-variation through the non-conducting porous plate under the action of magnetic field.

5.1 Introduction

Borkakati and Bharali [1980] discussed the steady magnetohydrodynamics flow past an infinite horizontal porous plate is theoretically investigated by taking Hall effects into account, when a strong magnetic field is imposed in a direction which is a perpendicular to the free stream and makes an angle α to the vertical direction. Borkakati and Chetri [1989] have respectively studied the effect of the deflection of a strong magnetic field on the oscillatory MHD flow past an infinite horizontal plate is studied theoretically, keeping the Hall-parameter constant. Recently, Mahato and Kuiry [1999] have discussed the flow behaviour of a viscous incompressible and electrically conducting fluid due to the time-varying acceleration of an infinite porous plate is analyzed in the presence of a uniform transverse magnetic field. Also, Sharma and Kumar [1998] discussed the unsteady flow and heat transfer through a viscous incompressible fluid in the presence of transverse magnetic filed between two horizontal parallel plates, the lower plate being a stretching sheet and upper plate being porous.

The objective of this chapter is to analyze and discuss about the results of an unsteady viscous incompressible flow of an incompressible electrically conducting fluid between two horizontal parallel porous plates in the presence of a uniform transverse magnetic field of which the direction is deflected. Exact solutions of the governing equations have been obtained and plotted through graphs. The velocity profile and the skin-friction are found due to the effect of the deflection of a strong magnetic field on the MHD flow past between two parallel plates and the results are obtained numerically and plotted graphically. The magnetic field parameter effects on the electrically conducting fluid flow are shown by plotting graphs.

5.2 Mathematical formulation of the problem

Let us consider an unsteady flow of an electrically conducting incompressible viscous fluid between two horizontal porous plates in the presence of a uniform transverse magnetic field. The electrically conducting fluid flow is assumed to be in the X'-axis which is along the plate and Y'-axis is normal to it. We assume that the fluid is finitely conducting and non-magnetic. A strong and uniform magnetic field B_o is imposed on the MHD flow between two parallel plates, when it makes an angle α to the free stream velocity. The components of the magnetic field are given by

$$\vec{B}_o = (B_o\lambda,\ B_o\sqrt{1-\lambda^2},\ 0),$$

where $\lambda = \cos\alpha$ and the velocity distribution is $\vec{v} = (u(y),\ v,\ 0)$.

All physical quantities except pressure are functions of y' and t', as the plate is extended infinitely and the porous plate is moving with time-varying velocity. As the magnetic Reynolds number is small, the induced magnetic field is neglected.

Under these assumptions, the governing equations of the problem are given as follows:

Equation of continuity

$$\frac{\partial v'}{\partial y'} = 0 \qquad (5.2.1)$$

Equation of motion

$$\frac{\partial u'}{\partial t'} + v\frac{\partial u'}{\partial y'} = \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_o^2}{\rho}\sqrt{1-\lambda^2}\,(\sqrt{1-\lambda^2}\,u' - \lambda v) \qquad (5.2.2)$$

where ρ is the density of the fluid, σ is the electrically conductivity of the fluid, υ is the co-efficient of the kinematics viscosity.

Let the fluid velocity change to zero velocity situating the plate $y' = 0$ be at rest and the velocity at the plate $y' = +h$ be moving on the time-varying motion with a constant velocity v_o, for the time $t' > 0$. Then the initial and boundary conditions are given by

$$t' \leq 0 : u' = 0, \qquad \text{for all } y'$$

$$t' > 0 : u' = 0, \qquad for\ y' = 0$$

$$: u' = v_o e^{-nt'}, \qquad for\ y' = +h \qquad (5.2.3)$$

Now, solving the equation (5.2.1), we obtain

v = constant.

For the constant suction, let us consider

$$v = -v_o \qquad (5.2.4)$$

where the negative sign indicates that the suction towards the plate.

Then the equation (5.2.2) in the help of equation (5.2.4) becomes

$$\frac{\partial u'}{\partial t'} - v_o \frac{\partial u'}{\partial y'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_o^2}{\rho}\sqrt{1-\lambda^2}(\sqrt{1-\lambda^2}u' + \lambda v_o) \tag{5.2.5}$$

We now introduce the following non-dimensional variables and parameters in order to transform the equation (5.2.5) into non-dimensional form:

$$y = \frac{y' v_o}{\upsilon}, \quad u = \frac{u'}{v_o}, \quad t = \frac{t' v_o^2}{\upsilon}, \quad Ha = \frac{\sigma B_o^2 \upsilon}{\rho v_o^2}, \quad \frac{h v_o}{\upsilon} = 1 \tag{5.2.6}$$

Using the condition (5.2.6) in the equation (5.2.5), we get-

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - Ha(1-\lambda^2)u - Ha\lambda\sqrt{1-\lambda^2} \tag{5.2.7}$$

The initial and boundary conditions of the non-dimensional form are given by

$t \le 0 : u = 0,$ for all y

$t > 0 : u = 0$ $at\ y = 0$

$: u = e^{-nt}$ $at\ y = +1$ (5.2.8)

5.3 Solution of the equations

To solve the equation (5.2.7), we consider that

$$u = f(y)e^{-nt} \tag{5.3.9}$$

Substituting (5.3.9) in the equation (5.2.7), we get-

$$\frac{\partial^2 f}{\partial y^2} + \frac{\partial f}{\partial y} - \{Ha(1-\lambda^2) - n\} f = e^{nt} Ha\lambda\sqrt{1-\lambda^2} \tag{5.3.10}$$

The corresponding boundary conditions are given by

$f = 0$ $at\ y = 0$

and $f = 1$ $at\ y = +1$ (5.3.11)

Solving the equation (5.3.10) with the help of the condition (5.3.11), we get-

$$f(y) = e^{-\frac{1}{2}y}[M_1 e^{my} \cosh a_1 y] + [\frac{e^{\frac{1}{2}}(1 + M_1 e^{m})}{\sinh a_1} - \frac{e^{m} M_1 \cosh a_1}{\sinh a_1}]\sinh a_1 y$$

$$- M_1 e^{my} \qquad (5.3.12)$$

where $a_1 = \frac{1}{2}\sqrt{1 + 4\{Ha(1 - \lambda^2) - n\}}$

and $M_1 = \frac{Ha\lambda\sqrt{1 - \lambda^2}}{Ha(1 - \lambda^2) - n}$.

Substituting (5.3.12) in the relation (5.3.9), we get-

$$u = e^{-\frac{1}{2}y}[M \cosh a_1 y] + [\frac{e^{\frac{1}{2}}(e^{-m} + M_1)}{\sinh a_1} - \frac{M_1 \cosh a_1}{\sinh a_1}]\sinh a_1 y - M_1 \qquad (5.3.13)$$

5.4 Computation of the skin-friction

The skin-friction C_f at the upper and lower plates are given by

$$C_f = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0,1}$$

$$= \mu[\{\frac{e^{\frac{1}{2}}(e^{-m} + M_1)}{\sinh a_1} - \frac{M_1 \cosh a_1}{\sinh a_1}\}a_1 - \frac{M_1}{2}], \qquad for\ y = 0$$

$$= \mu[\{\frac{e^{\frac{1}{2}}(e^{-m} + M_1)}{\sinh a_1} - \frac{M_1 \cosh a_1}{\sinh a_1}\}a_1 \cosh a_1$$

$$- M_1 e^{-\frac{1}{2}}\{\frac{1}{2}\cosh a_1 - a_1 \sinh a_1\}] \qquad for\ y = +1$$

5.5 Results and discussion

The figure-I has been obtained by plotting the velocity distribution u against the values of the variable y with the different values of the magnetic field parameter Ha = 2.5, 3.5, 4.5, when $\theta = 45^{\circ}$, nt = 1.0, n = 1.0. Here in this figure, we have seen that the velocity distribution decreases due to the increase of the values in Ha against the positive values of the variable y.

The figure-II has been found by drawing the velocity distribution u against variable y with the various values of angle $\theta = 45^{\circ}, 60^{\circ}, 75^{\circ}$, when Ha = 2.5, nt = 1.0, n = 1.0. In this figure, we have noticed that the velocity distribution decreases gradually due to the increase of the angle θ.

From the table, we observe that the values of the skin-friction co-efficient $(C_f)_{y=0}$ increase very slowly with the increase in magnetic field parameter Ha and also increase with the increase of the angle θ. But the skin-friction co-efficient $(C_f)_{y=1}$ decreases with the increase in Ha at the plate y = +1 and also decreases due to the increase of the angle θ.

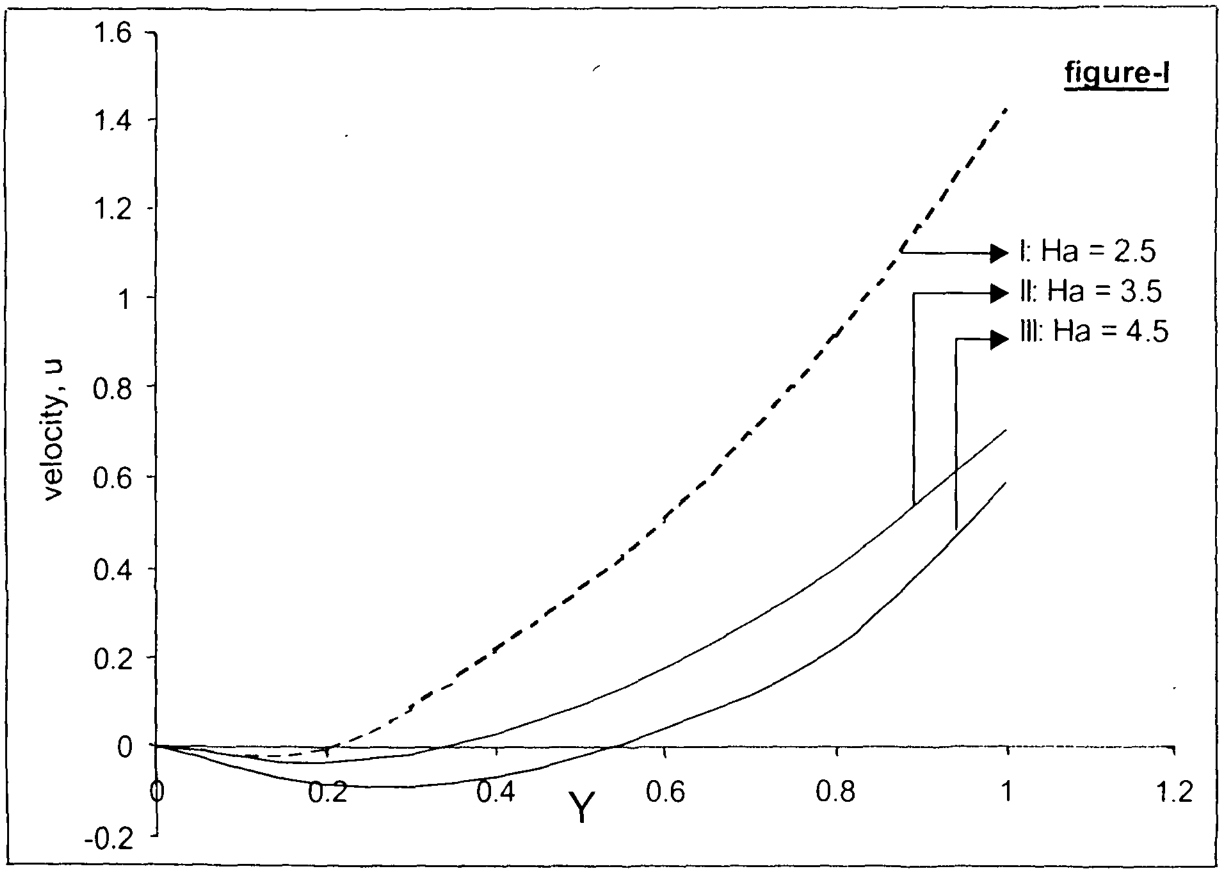

Fig.I: Velocity distribution versus y for the different values of Ha when n=1.0.

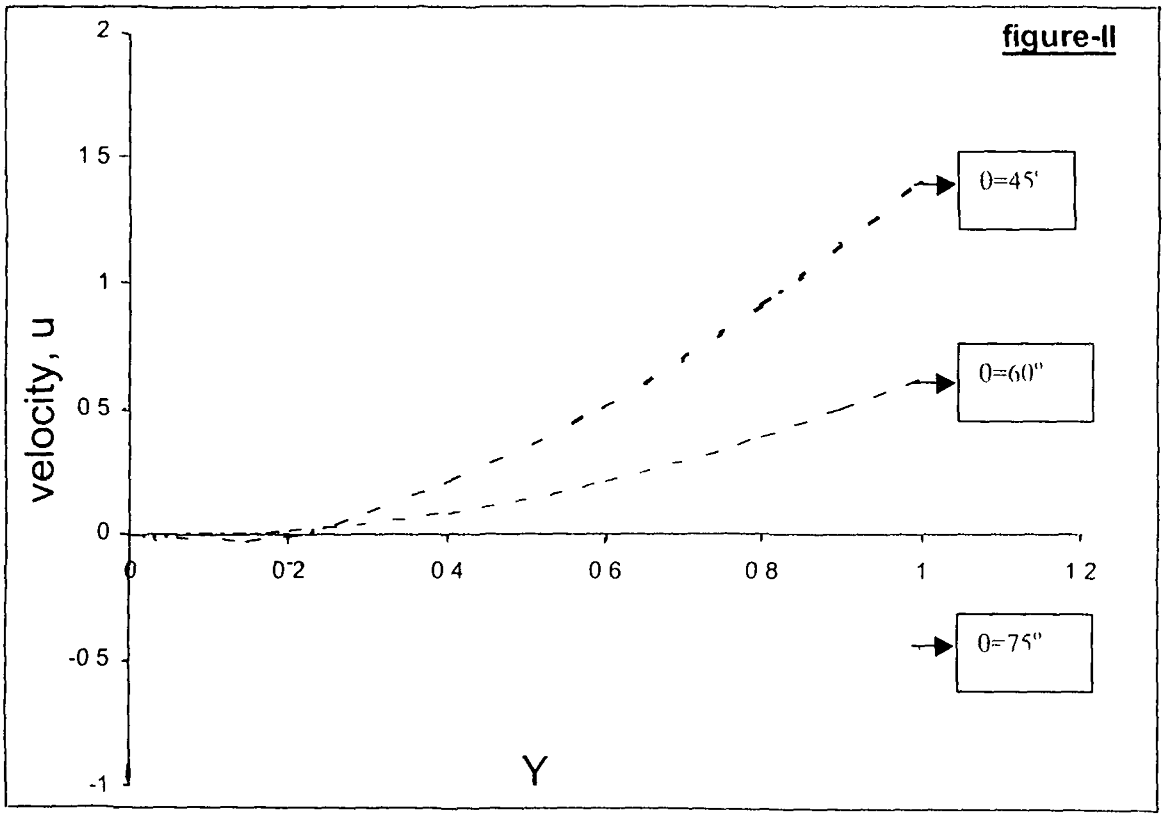

Fig.II Velocity distribution versus y for the different values of θ when n=1.0

Table: Values of skin-friction co-efficient at the plates

Ha	θ	nt	n	μ	$(C_f)_{y=0}$	$(C_f)_{y=1}$
2.5	45^{o}	1.0	1.0	0.2	-0.181178117	0.566074632
3.5	45^{o}	1.0	1.0	0.2	-0.141169949	0.339101223
4.5	45^{o}	1.0	1.0	0.2	-0.089946544	0.288357991
2.5	60^{o}	1.0	1.0	0.2	-0.017022186	0.253520585
2.5	75^{o}	1.0	1.0	0.2	0.028271017	0.205206853

CHAPTER 6

MHD flow and heat transfer of Rivlin-Ericksen fluid through an inclined channel with heat sources or sinks.

6.1 Introduction

The heat transfer in the flow of a conducting fluid between two non-conducting porous disks-one rotating and the other at rest, in the presence of a transverse uniform magnetic field, the lower disk being adiabatic (which is given and well-known by Schlichting [1968]), was studied by Bhattacharjee and Borkakati [1984]. Singh and Singh [1995] discussed the laminar flow and heat transfer of an incompressible, electrically conducting second order Rivlin-Ericksen liquid in porous medium down a parallel plate channel inclined at an angle θ to the horizontal in the presence of uniform transverse magnetic field. The commencement of the Couette flow in Oldroyd liquid in the presence of a uniform transverse magnetic field with heat sources/sinks has been studied by Biswal and Pattnaik [1996]. Rathod and Shrikanth [1998] have studied the unsteady MHD flow of Rivlin-Ericksen incompressible fluid through an inclined channel with two parallel flat walls under the influence of magnetic field. The magnetohydrodynamic unsteady flow of a visco-elastic liquid (Rivlin-Ericksen) near a porous wall suddenly set in motion with the heat transfer including heat generating sources or heat absorbing sinks has been studied by Datta, Biswal and Sahoo [1998].

Chakraborty and Borkakati [1998] have investigated the laminar convection flow of an electrically conducting second order visco-elastic fluid in porous medium down an inclined parallel plate channel in the presence of uniform transverse magnetic field. The unsteady flow and heat transverse magnetic field between two horizontal parallel plates, the lower plate being a stretching sheet and upper being porous was investigated by Sharma and Kumar [1998].

In this chapter, we investigate that the magnetohydrodynamics unsteady flow of a visco-elastic (Rivlin-Ericksen) fluid through an inclined channel with two parallel flat plates under the influence of a uniform magnetic field with heat transfer including heat generating sources or heat absorbing sinks, when the plates are moving with the transient velocity while the one of these two plates is adiabatic. The constitutive equations for continuity, motion and energy of visco-elastic liquid are obtained, and to obtain the numerical expression for the velocity and temperature distribution, the perturbation method is applied. The effects of sources/sinks parameter on the fluid motion and heat transfer of visco-elastic fluid through an inclined channel have not been studied yet. So, our aim here is to analyse the magnetohydrodynamic unsteady flow and heat transfer of an incompressible electrically conducting fluid through an inclined parallel plate channel in the presence of a uniform transverse magnetic field, when the plates are moving with transient velocity while the one plate is adiabatic. The effects of magnetic field parameter, elastic parameter, Reynolds' number, Grashoff number, Froude number, Prandtl number and source sink term on the velocity distribution is discussed with the help of graphs. Also, the effects of Prandtl number and source or sink parameter on the temperature distribution is expressed with the aid of graphs.

6.2 Mathematical formulation of the problem

Let us consider two dimensional incompressible electrically conducting Rivlin-Ericksen fluid flowing through an inclined channel between two parallel flat plates which are at a distance 2h apart under the influence of a uniform transverse magnetic field. We assume that the x'-axis along the straight line mid-way between the two plates, the y'-axis perpendicular to it. A magnetic field of uniform strength B_o is assumed to be applied in the y'-direction. Let u' be the velocity component along the direction of the x'-axis and the other components of the velocity be zero.

To write down the governing equations of the problems, the following conditions are considered:

(i) The plates are infinitely long, so that the fluid velocity u' is the function of y' and t' only.

(ii) The temperature is uniform within the fluid particles and the buoyancy force is considered in the equation of motion of the fluid.

(iii) The flow between the plates is fully developed.

(iv) The conductivity of the fluid is assumed to be very low, so that the induced magnetic field is neglected.

(v) The Polarization effect and heat Joule are neglected.

(vi) The Hall effect and viscous dissipation are assumed to be neglected.

(vii) Only electro-magnetic body force (Lorentz force) is considered.

(viii) Initially i.e. at time $t' = 0$, the plates and the fluid are at constant temperature (i.e. $T' = T_o$) and there is no flow within the channel. Where T_o is constant temperature.

At time $t' > 0$, the temperature of the plate $y' = +h$ changes to $\frac{\partial T'}{\partial y'} = 0$, and the temperature of the plate $y' = -h$ changes according to $T' = T_o + (T_w - T_o)e^{-n't'}$, where T_w is the temperature of the fluid at the wall, and $n' \geq 0$ is a real number, denoting the decay factor.

Under these above assumptions, the governing equations of continuity, motion and energy for the unsteady flow of a visco-elastic incompressible electrically conducting fluid between two non-conducting parallel plates in the presence of magnetic field are

$$\frac{\partial u'}{\partial x'} = 0 \tag{6.2.1}$$

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \upsilon\frac{\partial^2 u'}{\partial y'^2} + \frac{k_o}{\rho}\frac{\partial^3 u'}{\partial t'\partial y'^2} - \frac{\sigma B_o^2}{\rho}u' + g\sin\theta + g\beta(T' - T_o) \tag{6.2.2}$$

and $$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + S'(T' - T_o) \tag{6.2.3}$$

where $\rho =$ density of the fluid,

$B_o =$ uniform magnetic field applied transversely to the plate,

$\sigma =$ electrical conductivity of the fluid,

$\upsilon =$ co-efficient of kinematics viscosity,

$k =$ thermal conductivity of the fluid,

$c_p =$ specific heat at constant pressure of the fluid,

$\beta =$ co-efficient of thermal expansion,

$g =$ acceleration due to gravity,

$p' =$ pressure of the fluid,

k_o = co-efficient of the elasticity,

η_o = co-efficient of viscosity

S' = the heat source or sink term.

The initial and boundary conditions of the problem are given by

$$t' \le 0 : u' = 0, T' = T_o, \qquad \text{for } -h \le y' \le +h$$

$$t' > 0 : u' = -u_o, T' = T_o + (T_w - T_o)e^{-n't'} \qquad \text{at } y' = -h$$

$$: u' = +u_o, \frac{\partial T'}{\partial y'} = 0 \qquad \text{at } y' = +h \qquad (6.2.4)$$

In order to bring out the essential features of the equation of this problem. We now consider the following non-dimensional parameters as given by Shih-I Pai [1961]:

$$x = \frac{x'u_o}{\upsilon},\ y = \frac{y'u_o}{\upsilon},\ u = \frac{u'}{u_o},\ t = \frac{t'u_o^2}{\upsilon},\ Ha = \frac{\sigma B_o^2 \upsilon}{\rho u_o^2},\ T = \frac{T' - T_o}{T_w - T_o},$$

$$\Pr = \frac{\mu c_p}{k},\ n' = \frac{\upsilon n}{u_o^2},\ S = \frac{S'\upsilon}{u_o^2},\ Gr = \frac{\upsilon g \beta (T_w - T_o)}{u_o^3},\ p = \frac{p'}{\rho u_o^2},$$

$$Fr = \frac{u_o^2}{gh},\ Rc = \frac{k_o u_o^2}{\eta_o \upsilon},\ \text{Re} = \frac{u_o h}{\upsilon},\ \frac{h u_o}{\upsilon} = 1 \qquad (6.2.5)$$

Substituting the non-dimensional variables and parameters in the equations (6.2.1)-(6.2.3), we get-

$$\frac{\partial u}{\partial x} = 0 \qquad (6.2.6)$$

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + Rc\frac{\partial^3 u}{\partial t \partial y^2} - Hau + \frac{\sin\theta}{Fr\,\text{Re}} + GrT \qquad (6.2.7)$$

and $$\frac{\partial T}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} + ST \qquad (6.2.8)$$

where Rc is the elastic parameter, Ha is the magnetic field parameter, Fr is the Froude number, Re is the Reynolds number, Gr is the Grashoff number, Pr is the Prandtl number and S is the source or sink term.

The initial and boundary conditions of the non-dimensional form are given by

$$t \le 0 : u = 0, T = 0, \qquad \text{for } -1 \le y \le +1$$

$$t > 0 : u = -1, T = e^{-nt} \qquad \text{at } y = -1$$

$$: u = +1, \frac{\partial T}{\partial y} = 0 \qquad \text{at } y = +1 \qquad (6.2.9)$$

6.3 Solution of the equations

The equation (6.2.6) shows that u is a function of y and t only and constant. Also, the equation (6.2.7) shows that the velocity u is independent of x and therefore u is a function of y and t only. Thus, the term $\frac{\partial p}{\partial x}$ must be a constant or the function of t only.

Let us assume that $\frac{\partial p}{\partial x} = -h(t)$ (6.3.10)

Then the equation (6.2.7) becomes

$$\frac{\partial u}{\partial t} = h(t) + \frac{\partial^2 u}{\partial y^2} + Rc\frac{\partial^3 u}{\partial t \partial y^2} - Hau + \frac{\sin\theta}{Fr\,\mathrm{Rc}} + GrT \qquad (6.3.11)$$

In order to solve the equations (6.2.8) and (6.2.11) under the boundary conditions (6.2.9), we consider-

$$u = f(y)e^{-nt}$$

$$T = g(y)e^{-nt}$$

$$\text{and } h = h_o e^{-nt} \qquad (6.3.12)$$

The corresponding boundary conditions are given by

$$f(-1) = -e^{+nt},\ g(-1) = 1$$

$$\text{and } f(+1) = +e^{+nt},\ g'(+1) = 0 \tag{6.4.13}$$

Substituting (6.3.12) in the equation (6.2.8) and (6.3.11), we get-

$$\frac{\partial^2 g}{\partial y^2} + \Pr(S+n)g = 0 \tag{6.3.14}$$

$$\text{and } (1-nRc)\frac{\partial^2 f}{\partial y^2} - (Ha-n)f = -h_o - \frac{\sin\theta\, e^{-nt}}{Fr\,\mathrm{Rc}} - Gr\, g \tag{6.3.15}$$

Now, solving the equations (6.3.14) and (6.3.15) using the boundary conditions (6.3.13), and substituting in the equations (6.3.12), we get-

$$g(y) = e^{-nt}\frac{\cos a_1(1-y)}{\cos 2a_1} \tag{6.3.16}$$

$$u = \left[\frac{1}{\sinh b_1} + \frac{Gr\, e^{-nt}}{2M_1 \sinh b_1}\left(1 - \frac{1}{\cos 2a_1}\right)\right]\sinh b_1 y$$

$$-\left[\frac{Gr\, e^{-nt}}{2M_1 \cosh b_1}\left(1 + \frac{1}{\cos 2a_1}\right) + \frac{M_2}{(Ha-n)\cosh b_1}\right]\cosh b_1 y$$

$$+\frac{M_2}{Ha-n} + \frac{Gr\, e^{-nt}\cos a_1(1-y)}{M_1 \cos 2a_1} \tag{6.3.17}$$

$$\text{and } T = \frac{\cos a_1(1-y)}{\cos 2a_1} \tag{6.3.18}$$

where $a_1 = \sqrt{\Pr(S+n)}$,

$$b_1 = \sqrt{\frac{Ha-n}{1-nRc}},$$

$$M_1 = \Pr(S+n)(1-nRc) + Ha - n$$

and $M_2 = h_o e^{-nt} + \dfrac{\sin\theta}{Fr\,\mathrm{Re}}$.

5.4 Results and discussion

The figure-I is obtained by plotting the velocity distribution for the different values of magnetic field parameter Ha = 1.5, 2.5, 7.5 against the variable y considering the parameters values as Pr = 0.5, S = 0.5, Rc = 0.3, Fr = 3.0, h_o = 1.0, Rc = 1.0, n = 1.0, t = 1.0, Gr = 5.0 and θ = 30°. In this figure, the velocity decreases with the increase of magnetic field parameter Ha and it is maximum near the plate y = +1 and minimum towards the plate y = - 1.

The figure-II is drawn for the fluid velocity for the different values of Prandtl number Pr = 0.5, 0.25, 0.025 for the values of Ha = 1.5, S = 0.5, Rc = 0.3, Fr = 3.0, h_o=1.0, Re =1.0, n =1.0, t =1.0, Gr = 5.0 and θ = 30° against the variable y. So, it is observed that the velocity of the fluid increases with the decrease of Pr and its value is maximum near the plate y =+1 and minimum towards y = -1.

The figure-III has been obtained by plotting the velocity distribution u against the variable y for various values of source or sink term S, when Ha = 1.5, Pr = 0.5, Rc = 0.3, Fr = 3.0, h_o=1.0, Re =1.0, n =1.0, t =1.0, Gr = 5.0 and θ = 30°. This figure shows that the velocity increases as S increases.

The figure-IV depicts the velocity profiles against the variable y for different values of elastic parameter Rc, when Ha = 1.5, S = 0.5, Fr = 3.0, h_o=1.0, Rc =1.0, n =1.0, t =1.0, Gr = 5.0 and θ = 30°.

From the figure-IV, we observe that the velocity is maximum near the plate y = +1 and minimum towards the plate y = -1, and when Rc increases, the velocity also increases.

The figure-V has been found by drawing the velocity distribution for various values of Grashoff number Gr when Ha = 1.5, Pr = 0.5, Rc = 0.3, Fr = 3.0, h_o=1.0, Re =1.0, n =1.0, t =1.0, S = 0.5 and θ = 30°. So, we notice that the velocity increases due to the increase in Gr.

The figure-VI has been plotted to represent the velocity distribution against the variable y for different values of Froude number Fr, when Ha = 1.5, Pr = 0.5, Rc = 0.3, S = 0.5, h_o=1.0, Re =1.0, n =1.0, t =1.0, Gr = 5.0 and θ = 30°. From this figure, we have seen that the velocity decreases with the increase of Froude number.

The figure-VII has been obtained by plotting the velocity profiles against y for different values of Reynolds' number Re, when Ha = 1.5, Pr = 0.5, Rc = 0.3, Fr = 3.0, h_o=1.0, S = 0.5, n =1.0, t =1.0, Gr = 5.0 and θ = 30°. So, it is seen that the velocity of the fluid decreases due to the increase of Reynolds' number.

The figure-VIII has been obtained by plotting the temperature distribution T against the variable y for the different values of Prandtl number Pr = 0.5, 0.25, 0.025, when n =1.0, t =1.0, S = 0.1. It is found from this figure that the temperature decreases gradually with the decrease of Prandtl number.

In the figure-IX, the temperature distribution has been drawn against the variable y for various values of S, when Pr = 0.5, n =1.0 and t =1.0. From this figure-IX, it is observed that the temperature increases for the increasing of the values of the source or sink term S.

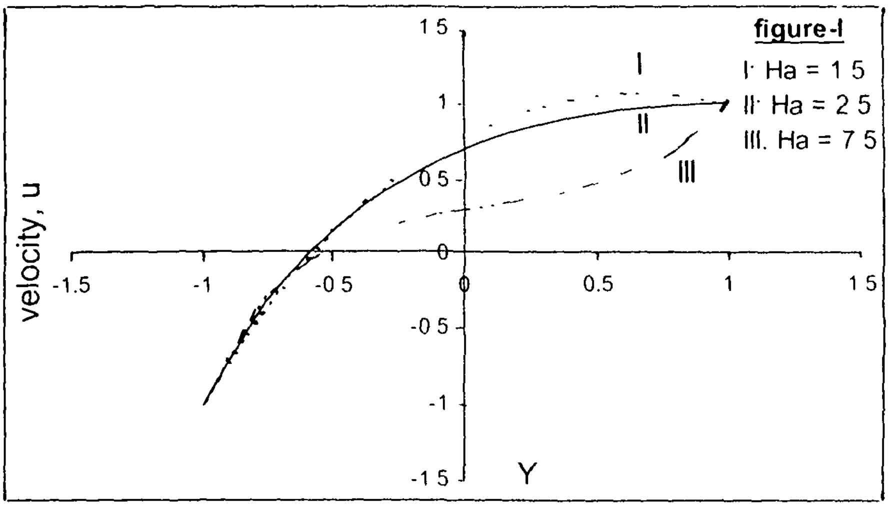

Fig.I: Velocity distribution versus y for different values of Ha.

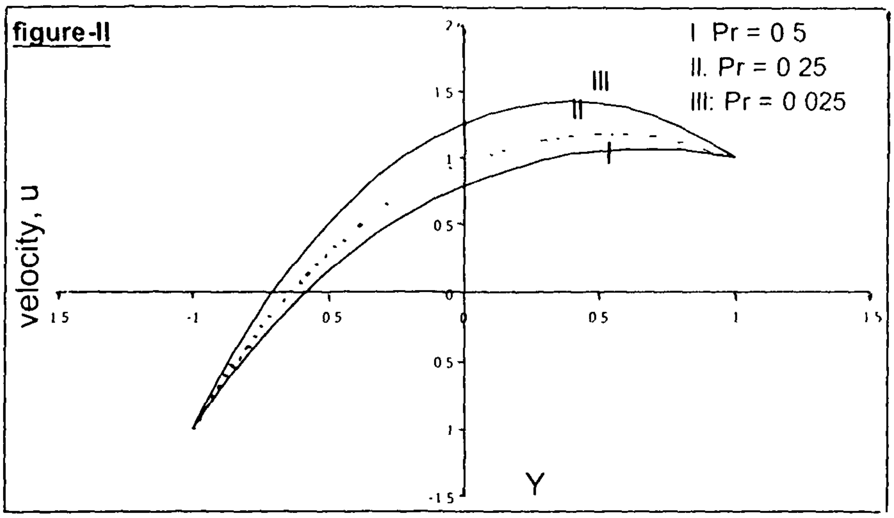

Fig.II: Velocity distribution versus y for different values of Pr.

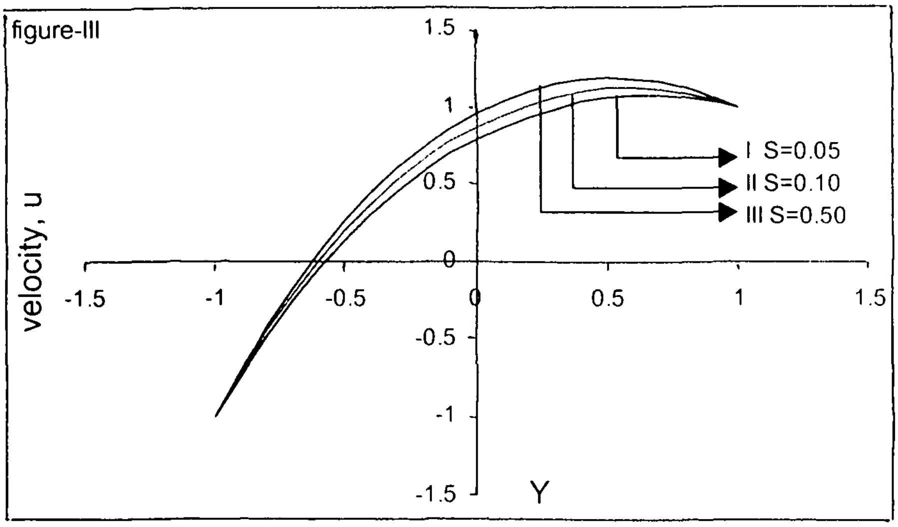

Fig.III: Velocity distribution versus y for different values of S.

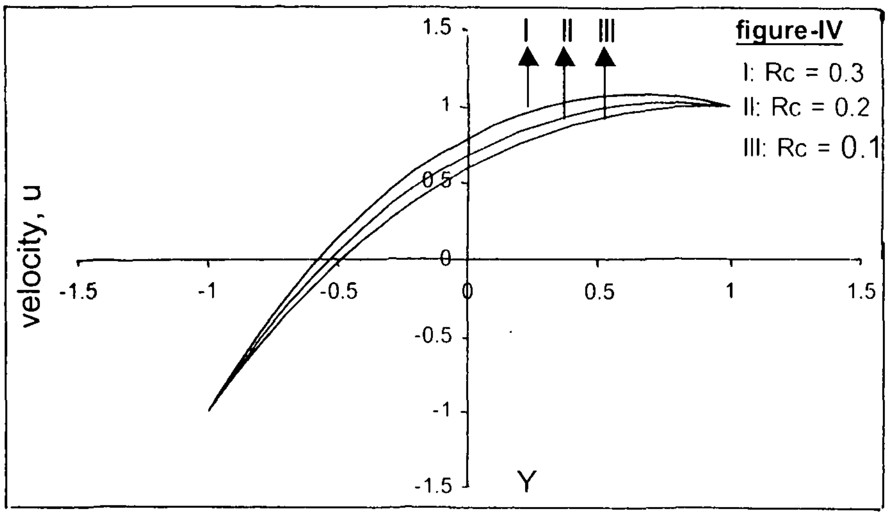

Fig.IV: Velocity distribution versus y for different values of Rc.

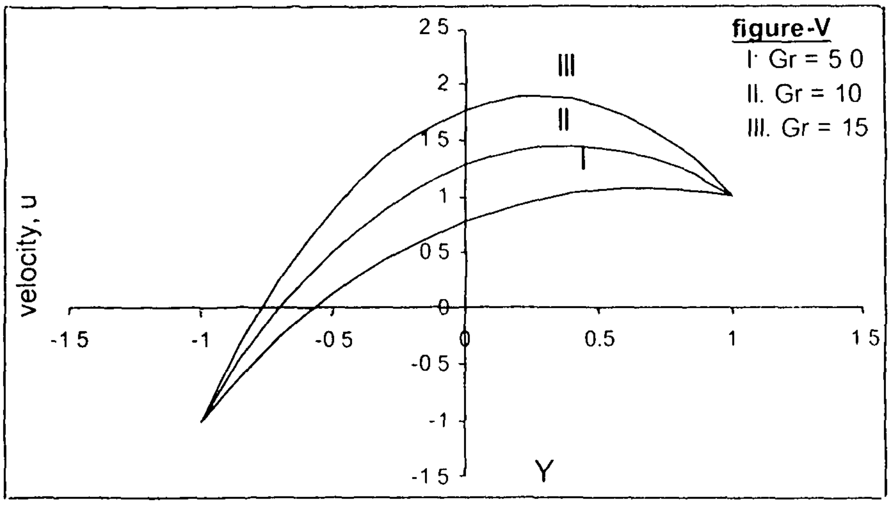

Fig.V: Velocity distribution versus y for different values of Gr.

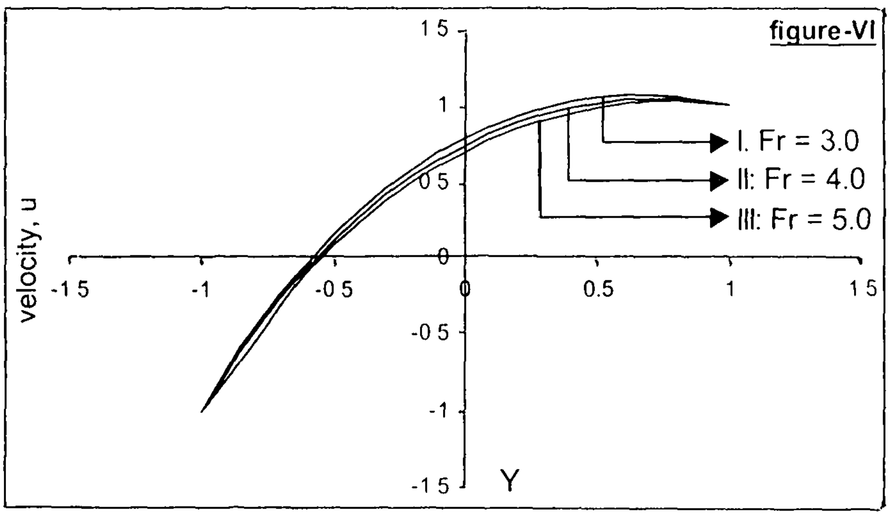

Fig.VI: Velocity distribution against y for different values of Fr

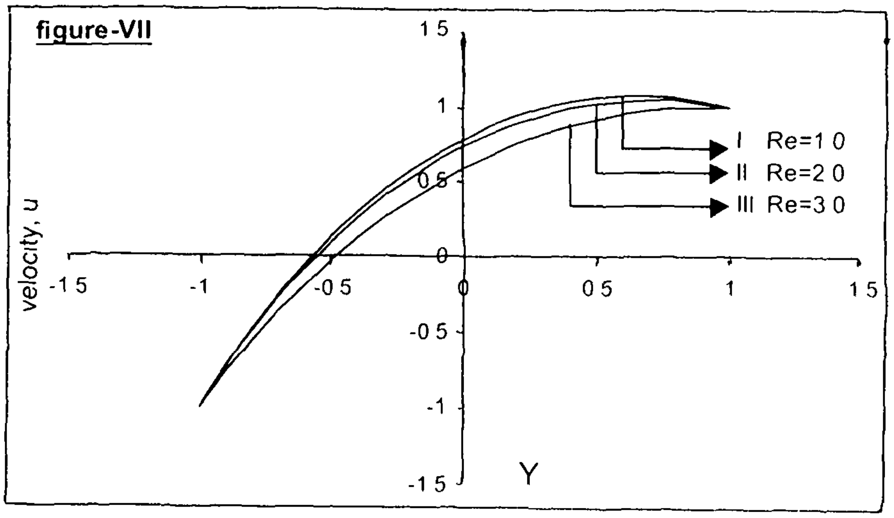

Fig.VII: Velocity distribution against y for different values of Re

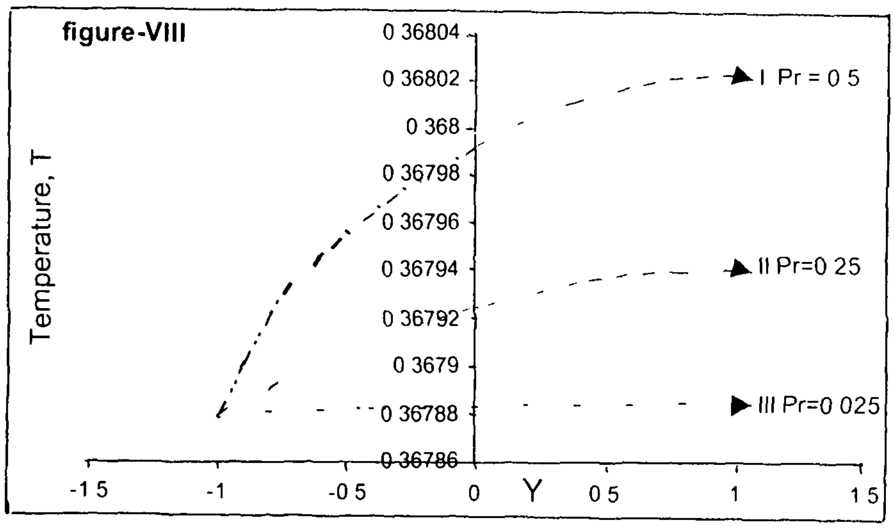

Fig.VIII: Temperature distribution against y for different values of Pr

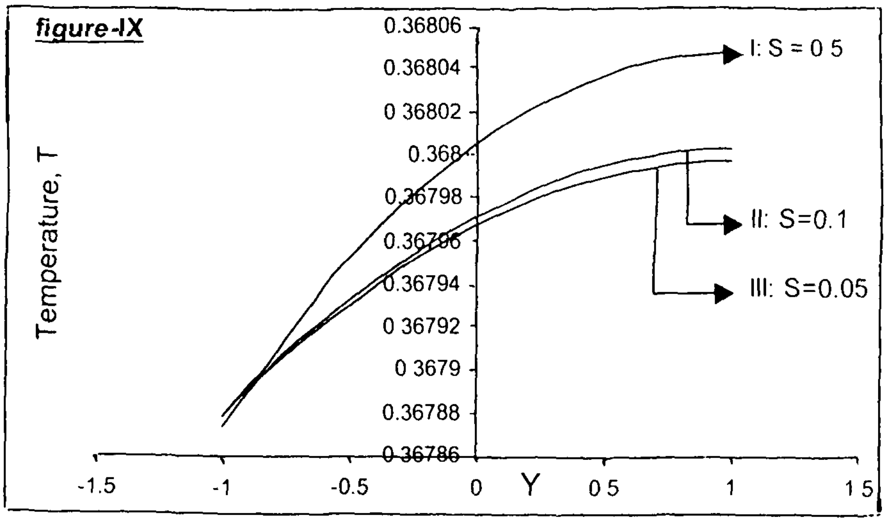

Fig.IX: Temperature distribution against y for different values of S.

CHAPTER 7

Magnetohydrodynamics unsteady free convection flow and heat transfer of a visco-elastic fluid through a porous medium past an impulsively started porous flat plate.

7.1 Introduction

Datta, Biswal and Sahoo [1998] studied the magnetohydrodynamic unsteady flow of a visco-elastic liquid (Rivlin-Ericksen) near a porous wall suddenly set in motion with the heat transfer including heat generating sources or heat absorbing sinks. The commencement of the Couette flow in Oldroyd liquid in the presence of a uniform transverse magnetic field with heat sources or sinks has been studied by Biswal and Pattnaik [1996]. Maharshi and Tak [2000] discussed the theoretical analysis of free convective two-dimensional unsteady flow through porous medium of variable permeability, bounded by an infinite vertical porous plate with uniform suction and constant heat flux. An analysis of steady two-dimensional free convection and mass transfer flow of a viscous incompressible electrically conducting fluid through a porous medium bounded by a vertical infinite surface with the constant suction velocity and constant heat flux in the presence of a uniform magnetic field is presented by Acharya, Dash and Singh [2000]. The flow of Rivlin-Ericksen incompressible fluid through an inclined channel with two parallel flat walls under the influence of magnetic field was studied by Rathod and Shrikanth [1998].

Sharma and Kumar [1998] were made an interesting analysis of the unsteady flow and heat transfer of a visco-elastic fluid through a circular pipe. In this chapter, we have studied that the unsteady flow of an incompressible electrically conducting second order Rivlin-Ericksen fluid through the porous medium due to infinite horizontal plate in the presence of uniform transverse magnetic field which includes the heat generating sources or heat absorbing sinks. The plates are maintained at different temperatures while any one of these two plates is kept at a constant temperature gradient. Using the perturbation technique, the obtained constitutive equations of continuity, motion and energy are solved at which the velocity and temperature distribution are found. The effects of magnetic field parameter, visco-elastic parameter, permeability parameter, Prandtl number, source or sink term and Grashoff number on the velocity distribution are discussed with the help of graphs. Also, the effects of Prandtl number and source or sink parameter on the temperature distribution are expressed with the aid of graphs. So, here the main purpose of this chapter is to analyze the magnetohydrodynamic unsteady flow and heat transfer of an incompressible electrically conducting visco-elastic (Rivlin-Ericksen) fluid through the porous medium due to infinite plates channel, while one plate remains constant of temperature gradient. The problems of determining the electrically conducting fluid flow and heat transfer through a porous channel driven by a pressure gradient are fundamental with obvious applications in physiology and Engineering.

7.2 Mathematical formulation of the problem

Here we consider the unsteady MHD flow and heat transfer of a visco-elastic incompressible electrically conducting fluid through the porous medium bounded by an infinite porous plate. It is assumed that the x'-axis is taken along the plate and y'-axis is taken normal to the plate. Let u' be the velocity of the fluid along the x'-axis and v' the fluid velocity along the y'-axis. Consequently, u' is a function of y' and t' only, but v' is independent of y'. Then the components of the fluid velocity are given by

$$(u'(y, t), v', 0).$$

Let u_o be the constant impulsive velocity along the plate in its own plane and B_o be a uniform magnetic field applied transversely to the plate. The fluid is assumed to be of low conductivity, so that the induced magnetic field is neglected. Thus the Lorentz's force is given by $-\sigma B_o^2 u'$.

To obtain the governing equations of the problem, the following conditions are assumed:

(i) The plates are considered to be infinite and all the physical quantities are functions of y' and t' only.

(ii) The fluid is finitely conducting and the viscous dissipation and the Joule heat is neglected.

(iii) The buoyancy force is considered in the equation of the fluid motion.

(iv) Hall effect and polarization effect are negligible.

(v) Initially (i.e. at time $t' = 0$) the plates and the fluid are at the temperature $T' = T_o'$ and there is no flow within the channel.

(vi) At time $t' > 0$, the temperature of the plate $(y' = 0)$ is kept at a constant temperature gradient $[i,e, \frac{\partial T'}{\partial y'} = \mathrm{A}$ (constant)] and the temperature for $(y' \to \infty)$ changes to T'_∞, where T'_∞ is the temperature of the fluid at infinity.

Under the above assumptions, the flow field is governed by the third order differential equation which takes the non-dimensional form. Hence the fluid flow is governed by the following equations:

Equation of continuity

$$\frac{\partial v'}{\partial y'} = 0 \tag{7.2.1}$$

Equation of motion

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \upsilon\frac{\partial^2 u'}{\partial y'^2} + \frac{k_o}{\rho}\frac{\partial^3 u'}{\partial t'\partial y'^2} - \frac{\mu u'}{\rho k'} - \frac{\sigma B_o^2}{\rho}u' + g\beta(T' - T'_\infty) \tag{7.2.2}$$

Equation of energy

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + S'(T' - T'_\infty) \tag{7.2.3}$$

where $\upsilon = \frac{\mu}{\rho}$ is the co-efficient of the kinematics viscosity,

ρ = density of the fluid,

μ = viscosity of the fluid,

σ = electrical conductivity of the fluid,

k_o = co-efficient of elasticity,

B_o = uniform magnetic field applied transversely to the plate,

k' = permeability of the porous medium,

g = acceleration due to gravity,

β = co-efficient of thermal expansion,

c_p = specific heat at constant pressure of the fluid,

k = thermal conductivity of the fluid,

T' = temperature of the fluid,

S' = the heat source or sink parameter.

The initial and boundary conditions of the problem is given by

$$t' \le 0: \; u' = 0, \; T' = T'_\infty \qquad for \; y' \ge 0$$

$$t' > 0: \; u' = u_o, \; \frac{\partial T'}{\partial y'} = A \text{ (constant)} \qquad for \; y' = 0$$

$$: u' \to 0, \; T' \to T'_\infty \qquad for \; y' \to \infty \tag{4.2.4}$$

We now consider the following non-dimensional variables and parameters in order to transform equations (7.2.1)-(7.2.3) into non-dimensional form:

$$x = \frac{x'u_o}{\upsilon}, \; y = \frac{y'u_o}{\upsilon}, \; u = \frac{u'}{u_o}, \; v = \frac{v'}{u_o}, \; t = \frac{t'u_o^2}{\upsilon}, \; Ha = \frac{\sigma B_o^2 \upsilon}{\rho u_o^2}, \; T = \frac{T' - T'_\infty}{T_w - T'_\infty},$$

$$\Pr = \frac{\mu c_p}{k}, \; S = \frac{S'\upsilon}{u_o^2}, \; Gr = \frac{\upsilon g \beta (T' - T'_\infty)}{u_o^3}, \; p = \frac{p'}{\rho u_o^2}, \; Rc = \frac{k_o u_o^2}{\eta_o \upsilon}, \; \alpha = \frac{k' u_o^2}{\upsilon^2}. \tag{7.2.5}$$

Now, substituting the above non-dimensional parameters in (7.2.1)-(7.2.3), we get-

$$\frac{\partial v}{\partial y} = 0 \tag{7.2.6}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial T}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + Rc\frac{\partial^3 u}{\partial t \partial y^2} - (Ha + \frac{1}{\alpha})u + GrT \tag{7.2.7}$$

and $$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} + ST \tag{7.2.8}$$

Where Rc is the elastic parameter, Ha is the magnetic field parameter, Gr is the Grashoff number, α is the permeability parameter, Pr is the Prandtl number and S is the source or sink parameter.

The boundary conditions of the dimensionless form are given by

$$t > 0 : u = 1, \frac{\partial T}{\partial y} = \chi \qquad for \;\; y = 0$$

$$: u \to 0, T \to 0 \qquad for \; y \to \infty \tag{7.2.9}$$

where $\chi = \dfrac{\upsilon A}{(T'_w - T'_\infty)u_o}$.

Note: *When* $\chi = 0$, *then there is no heat flux from the plate to the fluid i.e. the boundary condition at the plate* $y = 0$ *is adiabatic.*

7.3 Solution of the equations

From the equation (4.2.6), we have-

$v =$ constant.

For the constant suction, let us take

$$v = -v_o \tag{7.3.10}$$

Here the negative sign indicates that the suction towards the plate.

Thus the equations (7.2.7) and (7.2.8) with the aid of condition (7.3.10) becomes

$$\frac{\partial u}{\partial t} - v_o \frac{\partial T}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + Rc\frac{\partial^3 u}{\partial t \partial y^2} - (Ha + \frac{1}{\alpha})u + GrT \tag{7.3.11}$$

and $$\frac{\partial T}{\partial t} - v_o \frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} + ST \tag{7.3.12}$$

The equation (7.3.11) shows that the fluid velocity u is independent of x, and this is equation of the function of y and t only. Hence from the equation (7.3.11), it follows that the term $\frac{\partial p}{\partial x}$ is a function of t alone.

Suppose $\frac{\partial p}{\partial x} = -h(t)$ (7.3.13)

Therefore using the condition (7.3.13) in the equation (7.3.11), we have-

$$\frac{\partial u}{\partial t} - v_o \frac{\partial T}{\partial y} = h(t) + \frac{\partial^2 u}{\partial y^2} + Rc\frac{\partial^3 u}{\partial t \partial y^2} - (Ha + \frac{1}{\alpha})u + GrT \tag{7.3.14}$$

To solve the equation (7.3.12) and (7.3.14) under the boundary condition, let us consider

$$u = f(y)e^{-nt},\ T = g(y)e^{-nt} \ \ and\ h = h_o e^{-nt} \tag{7.3.15}$$

The corresponding boundary conditions are given by-

$$f = e^{nt},\ \frac{\partial g}{\partial y} = \chi e^{nt} \qquad for\ y = 0$$

$$f \to 0,\ \ g \to 0 \qquad for\ y \to \infty \tag{7.3.16}$$

Now, using the conditions (7.3.15) in the equations (7.3.12) and (7.3.14), we have-

$$\frac{\partial^2 g}{\partial y^2} + \Pr v_o \frac{\partial g}{\partial y} + \Pr(S + n)g = 0 \tag{7.3.17}$$

and $$(1 - nRc)\frac{\partial^2 f}{\partial y^2} + v_o \frac{\partial f}{\partial y} - (Ha + \frac{1}{\alpha} - n)f = -h_o + \frac{\chi Gr e^{-a_1 y}}{a_1} \tag{7.3.18}$$

Solving the equations (7.3.17) and (7.3.18) with the help of the boundary conditions (7.3.16), we have-

$$T = -\frac{\chi e^{-a_1 y}}{a_1} \tag{7.3.19}$$

and $u = e^{-b_2 y} + \frac{h_o e^{-nt}}{Ha + \frac{1}{\alpha} - n}\left[1 - e^{-b_2 y}\right] + \frac{\chi Gr e^{-nt}}{M_1}\left[e^{-a_1 y} - e^{-b_2 y}\right]$, (7.3.20)

where $a_1 = \frac{\Pr v_o + \sqrt{\Pr^2 v_o^2 - 4\Pr(S+n)}}{2}$,

$$b_1 = \frac{\sqrt{v_o^2 + 4(1 - nRc)(Ha + \frac{1}{\alpha} - n)}}{2},$$

$$b_2 = \frac{v_o}{2} + b_1$$

and $M_1 = a_1\{(1 - nRc)a_1^2 - v_o a_1 - (Ha + \frac{1}{\alpha} - n)\}$.

7.4 Results and discussion

The figure-I has been plotted to interpret the velocity distribution against the variable y>0 for the different values of the Prandtl number Pr = 0.71, 2.0, 3.0, when Gr = 5, Rc = 0.10, Ha = 2, S = -0.50, n = 0.05, α =1, χ=1, h_o=1, v_o=1 and t = 1. The values of the fluid velocity remain fixed at y = 0 and the fluid velocities decrease near the plate y = 0. Also, it increases slowly for the increase of the variable y as well as the increase of the Prandtl number. But the values of the velocity are very closed among the other for the greater values of the variable y.

The figure-II can be obtained by plotting the velocity against the variable y due to various values of source or sink term S = -0.50, -0.30, -0.10 having Gr = 5, Rc = 0.10, Ha = 2, Pr = 0.71, n = 0.05, α =1, χ=1, h_o=1, v_o=1 and t = 1.

In the figure-II, the fluid velocity decreases first for the greater values of the variable y and increase gradually for the more greater values of the variable y. Also, the fluid velocity distribution increases with the increase of the source or sink parameter.

The figure-III, we have drawn to represent the curves of the fluid velocity for the different values of the magnetic field parameter Ha against y when Gr = 5, Rc = 0.10, S = -0.50, Pr = 0.71, n = 0.05, α =1, χ=1, h_o=1, v_o=1 and t = 1. For the ascending values of the variable y, we notice that the values of the fluid velocity remain fixed at y = 0 and decreases first for the ascending values of y, and also increases for the more ascending values of the variable y. But the fluid velocity decreases with the increasing values of the magnetic field parameter.

The figure-IV has been found by plotting the velocity distribution with the various values of the elastic parameter Rc against the variable y, when Gr = 5, S = -0.50, Ha = 2, Pr = 0.71, n = 0.05, α =1, χ=1, h_o=1, v_o=1 and t = 1. In this figure we can observe that the velocity decreases with the increase of the elastic parameter considering with the different values of variable y.

The figure-V has been drawn to represent the velocity distribution for the various values of Grashoff number Gr against the variable y, when S = -0.50, Rc = 0.10, Ha = 2, Pr = 0.71, n = 0.05, α =1, χ=1, h_o=1, v_o=1 and t = 1. The values of the fluid velocities decrease with the increase of the values of Grashoff number depending upon the variable y.

The figure-VI can be obtained by drawing to show the velocity distribution against the variable y with the various values of permeability parameter α=1, 2, 3, when S = -0.50, Rc = 0.10, Ha = 2, Pr = 0.71, n = 0.05, Gr = 5, χ=1, h_o=1, v_o=1 and t = 1. From this figure-VI, we observe that the velocity decreases with the increase of the permeability parameter depending upon the values of the variable y.

The figure-VII has been obtained by drawing the temperature distribution against the variable y with the various values of the source or sink parameter S = -0.50, -0.30, -0.10, when Pr = 0.71, n = 0.05, $\chi=1$, $v_o=1$. The temperature decreases due to *the increase of the values of the source or sink term S depending upon the values of the* variable y.

In the figure-VIII, we have seen that the temperature distribution increases due to the increase of the Prandtl number Pr = 0.71, 2.0, 3.0 against the variable y, when S = -0.50, n = 0.05, $\chi=1$, $v_o=1$.

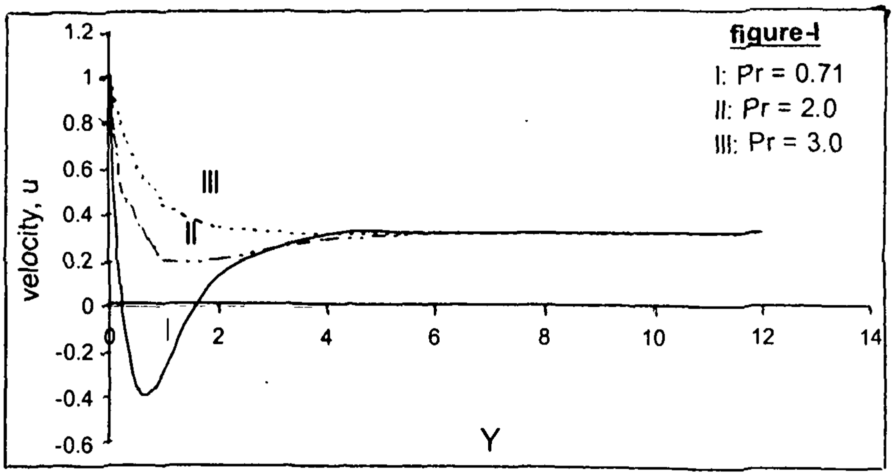

Fig.I: Velocity distribution against y for different values of Pr.

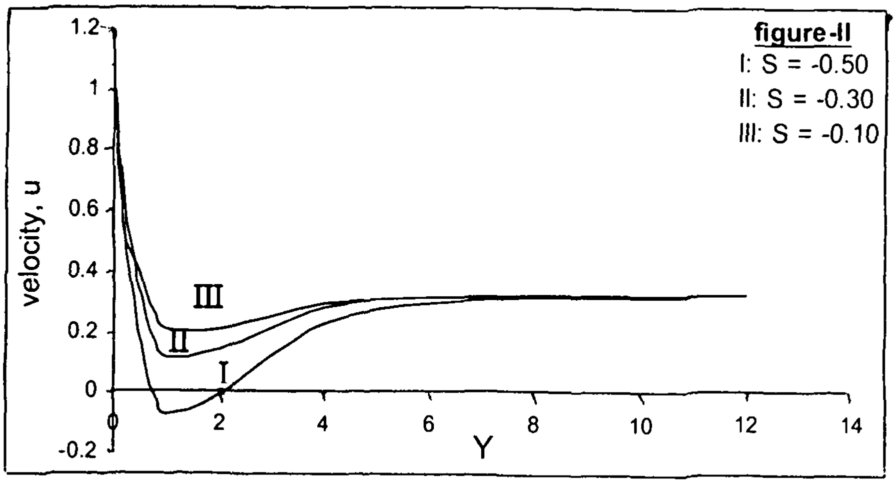

Fig.II: Velocity distribution against y for different values of S.

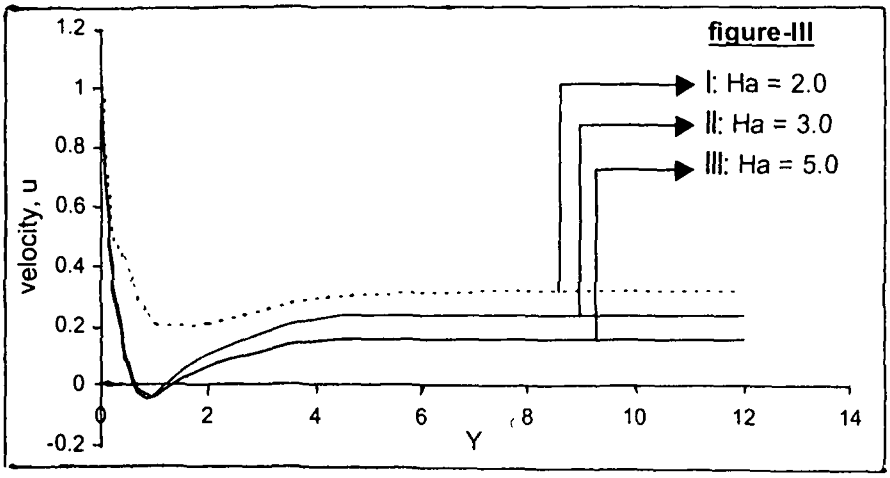

Fig.III: Velocity distribution against y for different values of Ha.

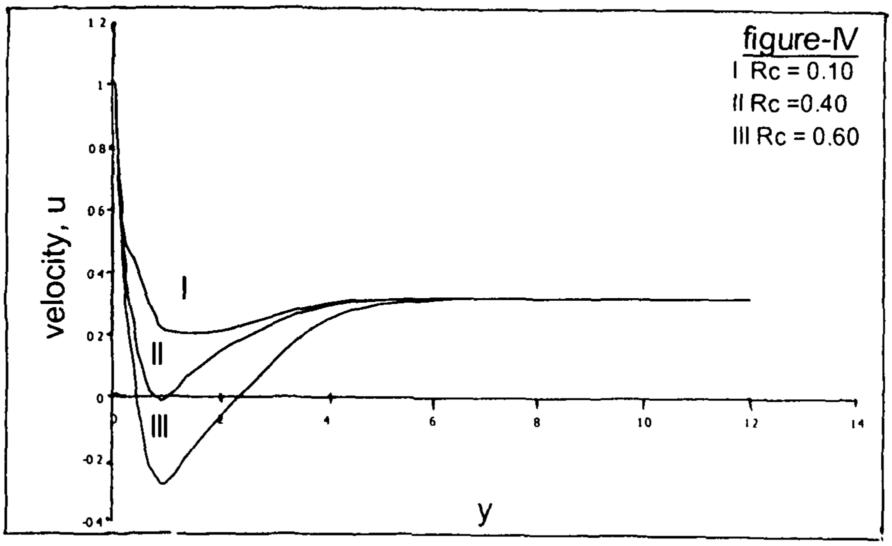

Fig.IV: Velocity distribution against y for different values of Rc.

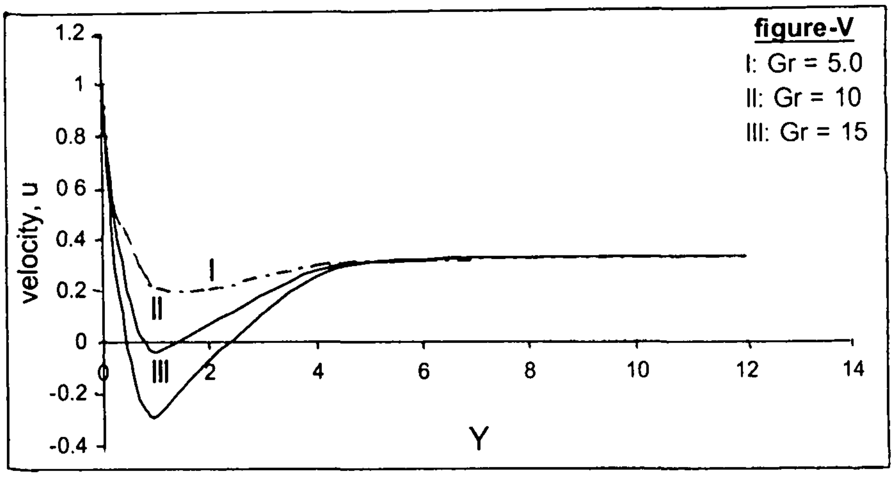

Fig.V: Velocity distribution against y for different values of Gr.

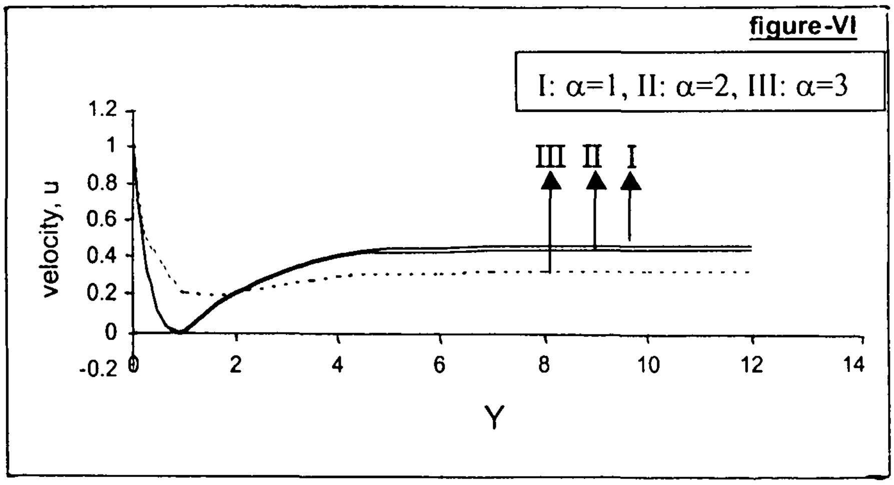

Fig.VI: Velocity distribution against y for different values of α.

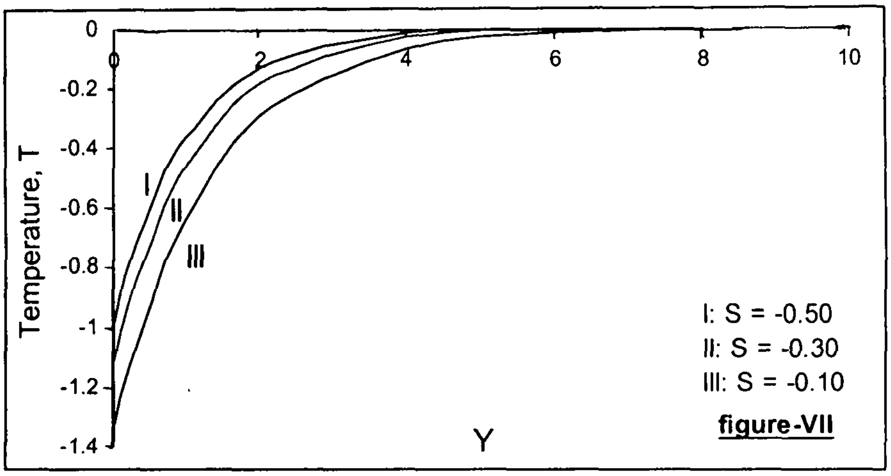

Fig.VII: Temperature distribution against y for different values of S.

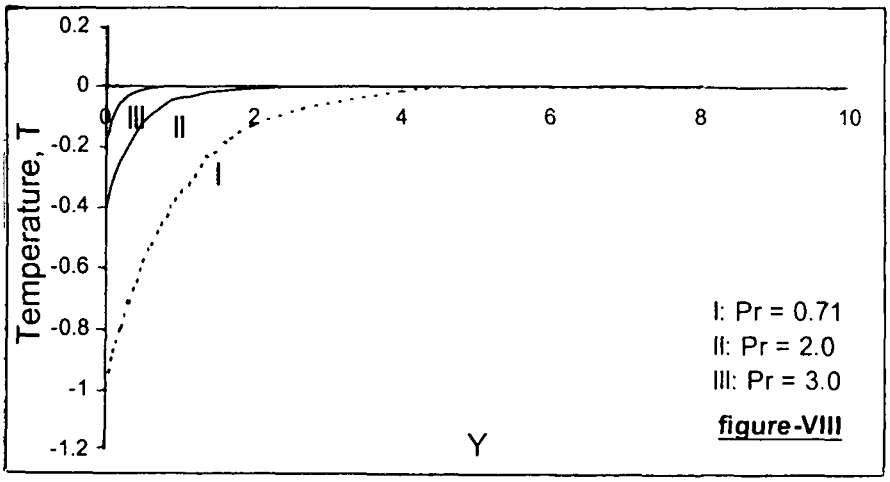

Fig.VIII: Temperature distribution against y for different values of Pr.

LIST OF PAPERS PUBLISHED/ACCEPTED/SENT FOR PUBLICATION

Papers published:

[1] On some unsteady free convection MHD flow of second order fluid between two heated vertical Plates: *Indian Journal of Theoretical Physics*, Vol.50, No.4, p-287-296 [2002], Calcutta.

[2] Magnetic field effects on the free convection flow through porous medium due to infinite vertical plate with uniform suction and constant heat flux: *Journal of the Indian Academy of Mathematics*, Vol.25, No.1, p-145-157 [2003], Indore.

[3] MHD Couette flow with heat transfer between two horizontal plates in the presence of a uniform magnetic field: *Journal of Theoretical & Applied Mechanics*, Vol.30, No.1, p-1-9, Belgrade [2003], Yugoslavia.

[4] Unsteady free convection MHD flow and heat transfer of a visco-elastic fluid past between heated horizontal plates with heat sources/sinks: *Indian Journal of Theoretical Physics*, Vol.51, No.1, p-39-46 [2003], Calcutta.

Papers accepted

[5] The motion of an electrically conducting fluid in presence of a magnetic field on the time-varying motion of a non-conducting porous plate: *Indian Journal of Mathematics*, Allahabad.

[6] MHD flow and heat transfer of Rivlin-Ericksen fluid through an inclined channel with heat sources or sinks: *Bulletin of Pure & Applied Sciences*, New Delhi.

[7] Magnetohydrodynamics unsteady free convection flow and heat transfer of a visco-elastic fluid through a porous medium past an impulsively started porous flat plate: *Indian Journal of Pure and Applied Mathematics*, New Delhi.

Paper presented to conference:

[8] Unsteady MHD flow and heat transfer of Rivlin-Ericksen fluid through an inclined channel in porous medium: *Indian Society of Theoretical & Applied Mechanics (ISTAM)*, IIT, [2002] Guwahati.

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[9] MHD flow and heat transfer of a viscous fluid with a magnetic field relative to the time-varying motion of an infinite porous plate: *Journal of Pure & Applied Mathematika*, Ghaziabad.

[10] Unsteady free convection MHD flow between two heated vertical plates: one plate at rest, when other plate is adiabatic: *Journal of Pure & Applied Mathematika*, Ghaziabad.

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[12] Unsteady MHD flow and heat transfer of Rivlin-Ericksen fluid through an inclined channel: *Indian Journal of Pure & Applied Mathematics*, New Delhi.

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