APPLICATION OF LAPLACE TRANSFORM TO SOME PROBLEMS OF MAGNETOHYDRODYNAMICS

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CERTIFICATE

This is to certify that the thesis entitle "**APPLICATION OF LAPLACE TRANSFORM TO SOME PROBLEMS OF MAGNETOHYDRODYNAMICS**", which is being submitted by Sri Bhaskar Kalita, Sr. Lecturer, department of Mathematics, T. H. B. College, Jamugurihat, Sonitpur, Assam for the award of the degree of Doctor of philosophy in Mathematical Sciences to the Tezpur University, Tezpur, Assam, is a record of bonafide research work carried out by him under my supervision and guidance.

Sri Kalita has fulfilled all the requirements framed for submitting the thesis for the award of Ph. D. degree.

The results embodied in the work have not been submitted to any other University or Institution for the award of any degree or diploma.

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CONTENTS

LIST OF SYMBOLS

a	thermal diffusivity; sound speed; real part of complex number
b	imaginary part of a complex number
$\vec{B}$	magnetic field intensity
B_0	dimensionless magnetic field
$\mathfrak{C}$	Complex number set
C	species concentration
c	velocity of light
C_A	concentration of the species A
C_p	specific heat of liquid at constant pressure
C_f	skin friction
C_w	species concentration near the plate
D	diffusion coefficient; differential operator
D_{AB}	diffusion coefficient in a binary mixture
d	typical length scale
$\vec{E}_s$	electrostatic field
$\vec{E}$	electric current (field)
$\vec{E}_i$	induced electric field
e	electric charge; specific internal energy
f	electromagnetic radiation frequency
$\vec{F}e$	external force per unit volume
$\vec{F}$	force field vector
$\vec{g}$	gravitational acceleration
h	unit representing length
$\vec{H}$	characteristic magnetic field strength.
$\vec{J}$	current density vector
J_0	dimensionless current density

K	proportionality factor; permeability of the medium
$\mathcal{K}'$	thermal conductivity; overall heat transfer coefficient
L (l/d)	dimensionless parameter; characteristic length
l	characteristic length; depth of the fluid
$\mathcal{L}$	Laplace transform operator
$\mathcal{L}^{-1}$	inverse Laplace transform operator
m	number of molecules per unit volume; mass transfer coefficient in a mixture
m_i	mass of the i[th] species
m_A	mass concentration of the component A
n, n'	temperature decay factor
$\vec{P}$	polarization current
P	dynamic pressure
p	charge on a particle
P_0	reference pressure
Q	amount of heat flux
q	net charge per unit volume
q_w	rate of heat flux at the wall
$\mathcal{R}$	real number set
R_A	molar rate of production of A
s	Laplace transform parameter
T	absolute temperature; dimensionless temperature; temperature in the free stream
T_0	reference temperature
t	time
T_w	temperature at the wall
T_∞	temperature at far distance
T_R	reference temperature
U	free stream velocity; characteristic velocity
$\vec{u}$	fluid velocity; charge carrier velocity vector

$\vec{V}$ velocity of the charge particle

V electrostatic potential

V_0 characteristic velocity; suction constant

(u, v, w) velocity component in Cartesian coordinates

(x, y, z) Cartesian component

[-h, h] space between the plates

GREEK LETTERS

α Volumetric expansion coefficient; heat transfer coefficient

σ electrical conductivity

μ_{eff} effective viscosity

μ_e magnetic permeability

μ viscosity (= $4\pi \times 10^{-7}$) in MKS unit

ν kinematics viscosity (= μ / ρ)

ρ density of fluid

ρ_0 density at some reference temperature T_0

ρ_i local concentration of a given substance

ρ_e electric charge density

ε permeability of the medium

ε_0 permittivity of the free space (= 8.854×10^{-12})

λ electromagnetic wave length

β volume expansion coefficient (thermal)

β^* expansion coefficient connected with concentration

Θ dimensionless temperature

ΔT small temperature difference

Φ dissipation function

τ_w shear stress at the wall

τ shear stress

Ω	angular velocity
ω	angular frequency
erfc	complementary error function

DIMENSIONLESS TERMS

a	thermal diffusivity ($\frac{\kappa}{\rho c_P}$)
B_r	Brinkman number
Da	Darcy number (= $\frac{\kappa}{h^2}$)
Ec	Eckert number
Gr	Grashof number ($Gr_t = g\beta\Delta Th^3/v^2$)
Gr_m	modified Grashof number (= $g\beta^*\Delta Ch^3/v^2$)
H, M	magnetic Hartmann number (= $B_0 d\sqrt{\sigma/\mu}$)
Ma	Mach number (= v/c)
Nu_x	local Nusselt number (= $\alpha l_0/\lambda = xq_w/\kappa(T_w - T_0)$)
Nm_x	local sherwood number (= $h_m x/D_m(C_w - C_0)$)
Pe	Peclet number (= Re.Pr = $w_0 l_0/a$)
Pr	Prandtl number (= $v/a = \rho v C_p/\kappa$)
Re	Reynolds number (= vd/v = hU_0/v)
Rm	magnetic Reynolds number (= $\mu\sigma vd$)
Sc	Schmidt number (= v/D)
X, Y, Z	reference dimensionless terms
Z	viscosity ratio parameter (= v_{eff}/v)

CHAPTER - 1

INTRODUCTION

Magnetohydrodynamics (in short MHD) is an essential science that deals with the mutual interaction between the magnetic fields and moving conducting fluid. Many research work done in this field have revealed that it is immense important for the growth and development of living being. Earlier works done in this field were mostly numerical and steady - state approach. But it is a known fact that the fluid flows, whether Newtonian or Non-Newtonian by and large, are unsteady or / transient. Steady-state assumptions are only due to the fact that it makes the solution of the problem simple and inability of available analytical tools to expressed governing equations of all kinds of flow fields. Laplace transformation is one of the most sophisticated analytical techniques, which can solve many problems arising in the study of fluid mechanics. Also, Laplace transform technique is best fitted for time dependent initial and boundary value problems. Moreover, the solution of magnetohydrodynamics problem using Laplace transform technique is rarely seen in the literature. This is the motivation of this thesis.

In this thesis, we have endeavoured to discuss a few two-dimensional (which circumstances makes one-dimensional) temperature dependent problems in presence of imposed magnetic field. However, two and three-dimensional steady as well as unsteady problems can also be solved under valid restrictions by this method. Basic concept of MHD, its uses; heat and mass transfer rate, transient flow; governing equations; various approximations of MHD; Laplace transform techniques, its uses etc have been discussed in this introductory note in a nutshell.

1.1 Basic concept of MHD:

Fluid dynamics is an important science used to solve many problems arising in aeronautical, chemical, and mechanical and civil engineering field. It also enable us many natural phenomena such as the flying of birds, swimming of fishes and the development of weather conditions to be studied scientifically.

The study of the laws that govern the conversion of energy from one form to another, the direction in which the heat will flow, and the availability of energy to do work is the subject the Thermodynamics. It is based on the concept that in an isolated system, anywhere in the university, there is a measurable quantity of energy called the internal energy (U) of the system. This is the total kinetic and potential energy of the atoms and molecules of the system of all kinds that can be transferred directly as heat; it therefore excludes chemical and nuclear energy.

The study of charge particle in motion, the forces created by electric and magnetic field, and the relationship between them give rise to the subject Electrodynamics.

The combined effects of these three important branches of science namely, Fluid dynamics, Thermodynamics and Electrodynamics give rise to the subject Magneto-fluid dynamics (MFD) which in the form of definition read as "The science of motion of electrically conducting fluid in the presence of a magnetic field''. It has two subtopics: Magnetohydrodynamics (MHD) and Magnetogasdynamics (MGD). MHD deals with electrically conducting liquids whereas MGD deals with ionized compressible gases.

1.2 Magnetohydrodynamics and its uses:

The Magnetohydrodynamics phenomena is a complex situation of the mutual interaction of magnetic field, $\vec{B}$ (say) and fluid velocity field, $\vec{u}$ (say), which arises partially as a result of the laws of Faraday and Ampére, and partially because of the Lorentz force experience by a current carrying body. This situation can comfortably be described splitting the process into three parts –

(i) The relative movement of a conducting fluid and a magnetic field causes an electromagnetic force (of order $|\vec{u} \times \vec{B}|$ in accordance with the Faraday's law of induction. In general, electrical currents will ensue, the current density being of order $\sigma(\vec{u} \times \vec{B})$, σ being the electrical conductivity.

(ii) These induced currents must, according to Ampére's law, give rise to a second magnetic field. This adds to the original magnetic field and the change is such that the fluid appears to 'drag' the magnetic field lines along with it.

(iii) The combined magnetic field (imposed and induced) interacts with the induced currents density, $\vec{J}$, to give rise to a Lorentz force (per unit volume), $\vec{J} \times \vec{B}$. This acts on the conductor and is generally directed so as to inhibit the relative movement of the magnetic field and the fluid.

These last two effects have similar consequences. In both cases the relative movement of fluid and field tends to be reduced. Fluids can 'drag' magnetic field lines, and on the other hand magnetic fields can pull on conducting fluids. It is this partial 'freezing together' of the magnetic medium and the magnetic filed which is the hallmark of MHD. The situation of freezing together is usually strong in Astrophysics, significant in Geophysics, weak in Metallurgical MHD and utterly negligible in electrolytes. However, the influence of $\vec{B}$ on $\vec{u}$ can be important in all four situations [19].

Magnetic field influence many natural and man-made flows. To say the scope of MHD, we should say that MHD operates on every scale from vast to the minute. Let us first look at the heavenly bodies. 'Solar magnetohydrodynamic' (E.R. Priest, 1984) [88], the monograph of Geophysics and Astrophysics, intensively speaks about how MHD is associated with the sun and its different phenomena. In the 'old days' the solar atmosphere was regarded as a static plane parallel structure, heated by the dissipation of sound wave and with its upper layer expanding in spherically symmetric manner as the solar wind. Outside the sunspots the magnetic field was thought to be unimportant with a weak uniform value of a few gauss. Recently, however, there has been a revolution in basic understanding. High-resolution ground-based instrument have revealed a photosphere full of structure and with small-scale magnetic fields that are probably concentrated into intense kilogauss flux tubes. The chromosphere is now known to be of cool jets, and space experiments have shown the corona to be a dynamic, highly complex structure consisting of myriads of hot loops. At small scale in the corona, hundreds of X-ray bright points are seen where new flux is emerging from below the solar surface and causing mini-flares. Also, coronal heating is now thought to be magnetic, either via various wave modes or by direct currents dissipation, and the solar wind has been found to escape primarily from the localized regions knows as coronal holes, where magnetic lines are open. Many of these new features are dominated by the magnetic field. Indeed, much of the detailed structure we now see owes its very existence to the filed and so solar MHD is at a most exciting stage as we attempt to explain and model the magnetic sun. If we look towards the earth we see that the fluid motion in the earth's core maintains the terrestrial magnetic field. Similarly, we see many uses of MHD in cosmic problem. Astrophysical MHD, a branch of science, has developed intensively after Alfvén's ideas used in cosmic problem. MHD is also an intrinsic part of controlled thermonuclear fusion. Here, high plasma temperature is maintained. and magnetic forces are used to confine the hot plasma away from the reactor walls. In the metallurgical industries, magnetic fields are routinely used to heat. pump, stir and levitate liquid metals. The earliest application of MHD is the electromagnetic pump. Now, in fast-breeder nuclear reactor it is used to pump liquid sodium coolant through the reactor core. The most widespread application of MHD in engineering is the use of electromagnetic stirring. Here the liquid metal, which is to be stirred, is placed in a rotating magnetic field. The resulting effect is an induction motor. This is regularly used in casting operation to homogenize the liquid zone of a partial ingot. In another casting operations, magnetic fields are used to dampen the motion of liquid metal. Since the magnetic field is static here, so, it can convert kinetic energy into heat via Joule dissipation. The magnetic levitation or confinement relies on the fact that a high-frequency induction coil repels conducting material by inducing opposite currents in any adjacent conductor. MHD is also important in electrolysis, particularly in those electrolysis cells used to reduce aluminium oxide to aluminium. This process is highly energy intensive. This is due to the fact that electrolyte is high electrical resisting. For example, in the USA, around 3% of all generated electricity is used for aluminium production. There are many other applications of MHD in engineering and metallurgical industry. Theses includes electromagnetic casting of aluminium, vacuum-arc remelting of titanium and nickel-based super alloys, electromagnetic removal of non-metallic inclusions from melts, electromagnetic launchers and the so-called 'cold-crucible' induction melting process in which the melt is protected from the crucible walls by a thin solid crust of its own material. This latter technology is currently finding favour in the nuclear waste. MHD is using in military arena as a propulsion mechanism for submarine. All in all, it would seem that MHD has now found a substantial and permanent place in the world of material processing [19]. MHD principles are using in medical sciences, particularly for the treatment of those diseases, which are related with the blood flow. Hence Biomathematics is the branch of science in which MHD principles would be used for the days to come.

1.2.1 Some other aspects of MHD:

The whole theory of magnetohydrodynamics rests on some fundamental hypothesis in particular, approximation in general. Under these assumptions, the fundamental equations governing the flow field and temperature distribution in MHD can be formulated from the corresponding fundamental equations of motion of ordinary hydrodynamics with suitable modifications using Maxwell's equations and Ohm's law. Some of them are considered below:

(a) Hydrodynamic and Electromagnetic considerations:

(i) The fluid is treated as continuous and homogeneous in all respect, and describable in terms of local properties such as pressure, temperature, velocity, density, viscosity, etc.

(ii) The conducting fluid will be in local equilibrium and transport processes will be isotropic.

(iii) Intermolecular gaps are ignored, and are such that the fluid properties are defined as average over elements. They are large when compare with the microscopic structure of matter and small when compare with the scale of macroscopic phenomenon. Under these considerations the differential equations are used to describe these local fluid properties.

(iv) A relatively collusion free situation is considered.

(v) All velocities are much smaller than c, the velocity of light which is equal to 3×10^{8} m/sec (approx.). Hence non-relativistic electromagnetic theory is considered in MHD, and relativistic corrections are not necessary.

(vi) The acts of expressing the differential equations of Magneto-fluid dynamics in terms of divergences and curls are treated as dangerous because these conceals the essence of electromagnetism whereby charges at rest or in motion (also the magnetic material if present) act upon one another at a distance. Rather one must consider the complete theory of electromagnetism.

(vii) The electrical field, which may be characterized by E is of the same order of magnitude as the induced electric field $\mu_e(V\times H)$. In other words the non-dimensional parameter $R_E = E/[\mu_e(V\times H)]$ is of the order of unity or smaller, where H is the characteristic magnetic field strength. Therefore, it may be shown that the displacement current $\varepsilon_o(\partial E/\partial t)$ and the excess electric charge are negligible in our fundamental equations, and that the energy in the electric field is much smaller than that in the magnetic field. As a result, all the electromagnetic variables may be expressed in terms of magnetic field.

(b) *Electrical properties of the fluid*:

Magnetohydrodynamics differs from ordinary hydrodynamics in the sense that the fluid is electrically conducting. It is not magnetic; it affects a magnetic field not by its mere presence but only by dint of electric current flowing in it. The fluid conducts because it contains free charges (ions or electrons) that can move indefinitely. It can be a dielectric and contain bound charges, which can move only a limited extent under electric fields. This migration of bound charges gives rise to polarization vector. The electrostatic part of the electric field is due to the free and bound charges distributed in and around the fluid.

(c) *The electric and magnetic fields*:

A charged particle such as an electron suffers forces of the following kinds:

(i) It is attracted or repelled by other charged particles. The total force on the particle per unit of its charge due to all the other charges present is the electrostatic field $\vec{E}_s$. From Coulomb's law it follows that $\vec{E}_s$ is irrotational (i.e. $\nabla \times \vec{E}_s = 0$), and $\vec{E}_s = -\nabla V$, where V is the electrostatic potential. This means that $\vec{E}_s$ is solenoid in regions devoid of charge while elsewhere $\nabla \circ \vec{E}_s = q / \varepsilon_o$, where q is the net charge per unit volume and $\varepsilon_o = 8.854 \times 10^{-12}$ in MKS units.

(ii) A vector quantity $\vec{B}$, called the magnetic field intensity, is produced when the charged particles are in motion. It has two effects, forces additional to $\vec{E}_s$

- A charged particle moving with velocity $\vec{V}$ relative to a certain frame of reference suffers a magnetic force $\vec{V} \times \vec{B}$ per unit of its charge. The force is perpendicular to $\vec{V}$ and $\vec{B}$. The direction of $\vec{B}$ is that in which the particle must travel to feel no magnetic field.
- If the magnetic field $\vec{B}$ is changing with time relative to a certain frame of reference, per unit of its charge a particle will suffer an induced electric field $\vec{E}_i$. This is defined by

$$\nabla \circ \vec{E}_i = 0 \qquad \nabla \times \vec{E}_i = -\partial \vec{B} / \partial t \text{ (Faraday's law)} \tag{1.2-1}$$

$$\nabla \circ \vec{B} = 0 \tag{1.2-2}$$

Equation (1.2-2) shows that the magnetic field lines can never end though they do not form closed loops.

It is defined that the sum of $\vec{E}_s$ and $\vec{E}_t$ is $\vec{E}$, the electric field, which states that a charged particle suffers forces of this kinds due to its presence per unit of its charge. Adding with it the later part of (ii), we get the total force experienced by a charged particle per unit of its charge, as

$$(\vec{F} =)\ \vec{E} + \vec{V} \times \vec{B} \tag{1.2-3}$$

This is known as Lorentz force.

(d) Low frequency approximation:

(i) The charge distribution appears unimportant in low-frequency electromagnetism and MHD. The Ampére-Maxwell law relating the magnetic field with the moving charges and the changing electric field is

$$\nabla \times \vec{B} = \mu(\vec{J} + \varepsilon_0 \partial\vec{E} / \partial t) \tag{1.2-4}$$

where, $\vec{J}$ is the current density vector representing the net flow of all charges free or bound, μ is constant and equal to $4\pi \times 10^{-7}$ in MKS unit, $\varepsilon_0(\partial E / \partial t)$ is Maxwell's contribution which states how the change of total electric field $\vec{E}$ affects $\vec{B}$. The magnitude of ratio $(\nabla \times B / \mu)/(\varepsilon_0 \partial E / \partial t)$ is of the order of λ^2 / d^2. This means that the Maxwell term in (1.2-4) is negligible unless frequency is very high. Hence, at low frequency, the Ampére-Maxwell law becomes

$$\nabla \times \vec{B} = \mu\vec{J} \tag{1.2-5}$$

Thus contribution of Maxwell term to $\vec{J}$ is negligible.

(ii) The polarization current $(\partial\vec{P} / \partial t)$ is of the same order as $\varepsilon_0(\partial\vec{E} / \partial t)$, and hence has no contribution to $\vec{J}$.

(iii) The ratio of magnitude of convection current $(q\vec{V})$ to the total current $(\vec{J})$ is v^2 / c^2, which is very small. Hence under low frequency approximation neglecting the convection current, the conduction current is taken as total current. So, the neglecting $\partial\vec{P} / \partial t$ and $q\vec{V}$, the current density $\vec{J}$ can be found as

$$\vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B}) \tag{1.2-6}$$

(iv) The ratio of electric and magnetic parts of the body force '$q\bar{E}+\bar{J}\times\bar{B}$' is v^2/c^2, which is very small. As a result the effect of q, and consequently of $q\bar{E}$ is omitted in MHD.

From these assumptions, it appears that the charge distribution has no importance in MHD under low frequency approximations.

1.3 Transient Flows:

By a transient is meant a solution of a differential equation when there is no force present, but when the system is not simply at rest. Suppose oscillation starts in a way such that it is driven by a force for a while, and then turned the force off. The system what we called the flow, will be non-uniform. It will be time dependent, oscillatory surviving for a small interval of time. All the 'start-up' flows from rest and 'shut-down' flows where the flows dies away in time are transient flows. It is also knows as unsteady flow. It can also be produced by periodic boundary conditions (oscillating wall, periodic conditions for the velocity or pressure). When a wall is suddenly set into motion with a constant velocity, the flows close to the wall will become unsteady by 'start-up' nature.

Again turbulent flows are by nature unsteady [106]. Hence it leads to 'unsteady turbulent flows'. It is made up of two parts – one part is time dependent and other part is time independent. The former one is due to fluctuations, which varies in time and latter one is due to decomposition of turbulent flows into the flow obtained after time averaging. The turbulent flows with time dependent 'mean' motion occur very frequently in practice. All start-up and shutdown processes belong to this group, as do the transition from one steady flow to another. These are called transient flows. Concentration diffusion also makes the flow phenomena unsteady, which give rise to phenomena of unsteady free convection flow.

1.4 Transient Solution:

In literature, transient solution is rare and perhaps relatively a new approach. The reason for existence of such solution is the existence of unsteady problems together with initial and boundary conditions. The complete solution for transient or unsteady problems requires two parts of solutions: one steady-state solution and other transient solution. Though Stokes forwarded the exact solutions for his unsteady plane problems, it was not the complete solutions. After the gap of nearly hundred years, R. Panton [77] got the complete solution by finding the transient solution. However, in practice, it is happen that the transient solutions become unimportant after a short amount of time. For this reason, it is the steady-state solution that is most important in many applications and researchers seem to give more important on this solution. But, still there are other many applications like; heating (or cooling) of various blanks and articles, glass manufacture, bricks burning, vulcanization of rubber, and during starting and shaping of various heat-exchangers, power installations etc., where transient solution is a must. One prerequisite is that for transient solution there must be given the initial condition together with boundary conditions that comes along with partial differential equation/ equations governing the problem under consideration.

1.5 Heat Transfer Processes:

Heat Transfer, is gaining ever-greater importance in many branches of engineering and technology. In the design of heat exchangers such as boilers, condensers, radiators, etc., for example, heat transfer analysis is essential for sizing such equipment. In the design of nuclear reactor cores, heat transfer analysis is important for proper sizing of fuel elements to prevent burnout. In aerospace technology, the temperature distribution and heat transfer problems are crucial because of weight limitations and safety considerations. In heating and air conditioning applications for buildings, a proper heat transfer analysis is necessary to estimate the amount of insulation needed to prevent excessive heat losses or gains.

So, in the light of these shinning approaches, we are very much anxious to discuss about how heat transfer processes takes place in the following few lines.

The science of heat transfer is concerned with the analysis of the rate of heat transfer taking place in a system. The energy transfer by heat flow cannot be measured directly, but the concept has physical meaning because it is related to measurable quantity called temperature. It has long been established by observations that, when there is temperature distribution in a system, heat flows from the region of high temperature to the region of low temperature. If the temperature distribution in the flow is known, the amount of heat transfer per unit area per unit time is readily determined.

It is the spontaneous irreversible process of heat propagation in space. By a process of heat propagation is meant the exchange of internal energy between the individual elements, regions of the medium considered. In this process, we have three modes of heat transfer, in general. They are – conduction, convection and radiation.

In a solid body the flow of heat is the result of the transfer of internal energy from one molecule to another. This process is called conduction. The same process takes place for liquids and gases. In these substances, however, the molecules are no longer confined to a certain point but constantly change their position, even if the substance is in a state of rest. For an incompressible fluid at rest, the transfer of energy takes place entirely by thermal conduction. The flow of heat in the relatively stagnant boundary layer, which adheres to the wall, is therefore by conduction only, and can be calculated from (Fourier's law)

$$q = -K_f(\partial T / \partial n)_w \qquad (1.5\text{-}1)$$

where $(\partial T / \partial n)_w$ is the temperature gradient in the fluid immediately adjacent to the wall K_f is the thermal conductivity of the fluid and q is the heat flux ?

Heating or cooling of the walls of a building is one of the examples of this type of heat transmission.

Convection occurs when volumes of liquids or gases (of fluid medium) moves from regions of one temperature to those of another temperature. The transport of heat is inseparably linked here with the movement of the medium itself. Convection is possible only in a fluid medium. It is well known that a hot plate will cool faster when it is placed in front of a fan than exposed to still air. Convection involves a movement of fluid masses, and the buoyancy and gravity forces are the key factors for fluid motion. It has long been observed that heat transfer by convection occurs between a fluid and a solid boundary, when there contains a temperature difference between them.

Radiation, often known as thermal radiation, is the process of heat propagation by means of electromagnetic waves, depending only on the temperature, and on the optical properties of an emitter, with its internal energy being converted into radiation energy. Heat transfer by radiation is significant at high temperature.

Theses various basic process of heat transfer are often combined both in nature and in engineering applications. Heat transfer by convection is always accompanied by conduction. The combined processes of heat transfer by convection and conduction is referred to as *convective heat transfer*. There are other combined processes of heat transfer, namely, *radiation-conduction*, *radiation-convection*; but these are not important in our problems. So, we keep it off from our discussion.

Processes of heat transfer may occur in various media, in pure substances and in mixture, with and without changes of phase of the working medium, etc, and accordingly will differ in character and be described by different equations.

Transfer of mass accompanies many of the processes of heat transfer. As water evaporates, for instances, heat transfer is accompanied by transport of the vapor formed through an air-vapor mixture. The transport of steam generally occurs both through molecular interaction and convection. The combined molecular and convective transport of mass is called *convective mass transfer*. With mass transfer the process of heat transfer becomes more complicated. In addition, heat may be transported together with the mass of diffusing substances.

1.6 Free Convective Flow:

In section 1.5, we have outline about Heat transfer. In this section, we turn our attention to free convection flow, a subtopic of convective heat transfer. We have emphasized on this topic due to the fact that the problems that we have considered for thesis are of about transient or unsteady free convective flows. The convective heat transfer is of two types: *forced convection* and *free convection.* In case of convection of both types, we have the following expression:

$$\rho = \rho_0[1-\beta(T-T_0)] \tag{1.6-1}$$

where the density ρ varies slightly from point to point because of the variation in the temperature T. Here β is the coefficient of thermal expansion and ρ_0 is the density at some reference temperature T_0.

Free convection originates due to the non-uniform distribution of mass (volume) forces in the fluid being considered. Forces of this kind include the force of gravity, centrifugal force and the force appearing when a high-intensity electromagnetic field is induced in the liquid. The best understood is free convection caused by gravity.

Gravitational forces are taken into account in the flow equations by the term $\rho\vec{g}$, whose unit is force reduced to unit volume. Fluid temperature changes during heat transfer, which leads to the appearance of differences in density and, consequently, of differences in gravitational forces representing a buoyant (descending) force.

The salient differences of free convection from forced convection can be found from its behavior. In free convection the flow pattern is determined solely by the buoyant effect of heated fluid, the velocity profiles and temperature profiles are intimately connected, the Nusselt number depends on Grashof number and Prandtl number.

1.7 **Mass Transfer Processes**:

Like the heat transfer, mass transfer is also one of the domains of contemporary science. It is of great practical interest in evaporation, condensation, adsorption, sublimation, etc. In many branches of Modern Technology, particularly. in atomic power engineering, space research, power plant, industrial power engineering. chemical engineering, construction industry, etc. this mass transfer processes (together with heat transfer processes) automatically appears. *Due to this reason, we have the mind to outline briefly the theory of the processes* Mass Transfer.

Many processes of heat transfer encountered in nature and engineering are accompanied by processes of the mass transfer of one component into the other; for instance, in the condensation of vapor from a vapor-gas mixture and the evaporation of liquid into a vapor – gas flow. The evaporated liquid is distributed throughout the vapor-gas flow by diffusion; the process is accompanied by a change in the nature of flow and a variation in heat transfer intensity, and this, in turn, influences the process of diffusion. And diffusion means the spontaneous process of spreading or scattering of matter in a binary medium or two – component system under the influence of concentration. In a mixture, homogeneous in respect of temperature and pressure, diffusion is directed towards equalizing the concentration in the system and is accompanied by transfer of mass from the region of higher concentration to the region of lower concentration. Diffusion is characterized by the flow of the mass of a component, i.e. by the quantity of mass passing per unit time through the given surface in a direction normal to the surface. Mass transfer may be either molecular (microscopic) or molar (macroscopic). In gases, molecular diffusion is due to the thermal motion of molecules.

In a multi-component system, the concentrations of the various species may be expressed in various ways.

With stationary macroscopic two-component system, homogeneous in respect of temperature and pressure, the rate of mass flow of one of the components, due to molecular diffusion, determined by Fick's law, is given as:

$$J_D = -D(\partial \rho_i / \partial n) \tag{1.7-1}$$

$$= -\rho D(\partial m_i / \partial n) \tag{1.7-2}$$

where ρ_i = local concentration of the given substance, equal to the ratio of the mass of the component to the volume of the mixture;

$m_i (= \rho_i / \rho)$ = relative mass concentration of the ith component;

ρ = Mixture density;

D = co-efficient of molecular diffusion of one component in respect to the other, or, in short, the coefficient of diffusion;

n = direction normal to the surface of a similar concentration of the component;

$(\partial \rho_i / \partial n, \partial m_i / \partial n)$ = concentration gradient which is always directed to the side of rising concentration.

The concentration gradient is the motive force determining the transfer of matter. In heat conduction, it is temperature gradient stands for the motive force. The minus sign of (1.7-2) indicates that the mass is being transferred in accordance with Fick's law, in the direction of diminishing concentration. The process described by Fick's law is known as concentration diffusion (see [47], pp. 312).

1.8 Fundamental Equations:

In fluid mechanics we formulate the fundamental equations by considering a control volume fixed in space bounded by an imaginary surface and with the help of physical principles of mass, momentum, and energy of the isolated portion of the fluid. The principle of conservation of mass gives the equation continuity when there are no surfaces of discontinuity present in the region. The conservation of momentum gives the equation of motion and the conservation of energy gives the equation governing the temperature distribution. In case of binary mixture conservation of mass of the two species gives the equation of mass diffusion. These equations are described briefly below in the vector notation.

(a) EQUATION OF CONTINUITY:

The principle of conservation of mass gives the equation of continuity, which, for an incompressible fluid, can be written as –

$$\text{div } \vec{V} \equiv \nabla \circ \vec{V} = 0 \qquad (1.8\text{-}1)$$

where $\vec{V}$ is the velocity vector of a fluid particle and ∇ (nabla or del) is a vector operator which in Cartesian co-ordinates, is given by

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \tag{1.8-2}$$

where, $\hat{i}, \hat{j}, \hat{k}$ are unit vectors in the direction of x, y, and z respectively.

(b) EQUATION OF MOTION:

The equation of motion originates from the Newton's second law of motion, which is known as law of conservation of momentum, and is written for an incompressible viscous fluid in the form

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V} \circ \nabla)\vec{V} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\nabla \circ (\mu \nabla \circ \vec{V}) + \vec{Fe} \tag{1.8-3}$$

where the symbols have their usual meaning.

The equation (1.8-3) subject to the body force must be generalized to include the effects of electrical conductivity of the fluid and imposed electric and magnetic fields. The body force per unit volume $\vec{Fe}$ is replaced by the imposed electromagnetic fields, which is (see Shercliff, 1965)

$$\vec{Fe} = \rho_e \vec{E} + (\vec{J} \times \vec{B}) \tag{1.8-4}$$

The ratio of electric and magnetic parts of this body force is of order (U^2/c^2). Then $\rho_e \vec{E}$ can be omitted. Hence the equation of motion in magnetohydrodynamic is

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\nabla)\vec{V} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\nabla.(\mu \nabla.\vec{V}) + \frac{1}{\rho}(\vec{J} \times \vec{B}) \tag{1.8-5}$$

(c) ELECTROMAGNETIC FIELD EQUATIONS:

In magnetohydrodynamics, we are mainly concern with conducting fluids in motion and hence it is necessary to consider the electrodynamics field equations of moving media. We consider that the velocities occurring in our problem are much smaller than the velocity of light, and therefore, all the non-relativistic assumptions hold good. With these assumptions Maxwell's (electrodynamics field) equations are –

$$\nabla \circ \vec{B} = 0 \tag{1.8-6}$$

$$\nabla \circ \vec{E} = \rho_e / \varepsilon \tag{1.8-7}$$

$$\nabla \times \vec{B} = \mu_e \vec{J} \tag{1.8-8}$$

$$\nabla \times \vec{E} = -(\partial \vec{B} / \partial t) \tag{1.8-9}$$

The buoyancy force $\vec{f}_1$ and Lorentz force $\vec{f}_2$ are respectively, given by

$$\vec{f}_1 = \beta(\Delta T)\vec{g} \qquad (1.8\text{-}10)$$

$$\vec{f}_2 = \vec{J} \times \vec{B} \qquad (1.8\text{-}11)$$

For electromagnetic problems, an equation, namely the law of conduction, is added to the Maxwell's equation. This equation is known as Ohm's law, and is given by

$$\vec{J} = \sigma[\vec{E} + \vec{V} \times \vec{B}] \qquad (1.8\text{-}12)$$

(d) EQUATION OF HEAT TRANSFER:

The equation of heat transfer arises from the principle of conservation of energy, which states that the total time rate of change of Kinetic and Internal energies is equal to the sum of the works done by the external forces per unit time and the sum of other energies supplied per unit time. From this principle the equation of heat transfer can be written as

$$\rho C_p\left(\frac{\partial T}{\partial t} + \vec{V} \circ \nabla T\right) = \nabla \circ (k\nabla T) + \mu\Phi \qquad (1.8\text{-}13)$$

The dissipation function, Φ, which arises from viscous action, in Cartesian co-ordinate for incompressible, it is given as

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 \qquad (1.8\text{-}14)$$

where u, v. w are the components of $\vec{V}$ in the X, Y, Z direction respectively.

To get the energy equation in MHD. one extra term is necessary, and this can be derived as follows:

The charge within a material moves under the action of electromagnetic forces colliding and exchanging energy with the rest of the material. This fact means that electric work is done on or by the material. It has been found that the electromagnetic field puts energy into the material at the rate ($\vec{E} \circ \vec{J}$) per unit volume and time [104], where $\vec{J}$ can have three possible forms – conduction, convection, and polarization. The contribution of convection and polarization current on the work done is negligible in MHD, only that of the conduction current plays a significant role.

Using Ohm's law in the form of (1.8-12), the rate of electric work done on is given by

$$\vec{E}.\vec{J} = \vec{J}^2/\sigma - \vec{J} \circ (\vec{V} \times \vec{B}) \qquad (1.8\text{-}15)$$

The first term on the right hand side of (1.8-15) represents heat dissipation while the second term can be written as

$$- \vec{J} \circ (\vec{V} \times \vec{B}) = \vec{V} \circ (\vec{J} \times \vec{B}) \qquad (1.8\text{-}16)$$

This describes the phenomena of electromagnetic energy conversion. The term ($\vec{V} \circ (\vec{J} \times \vec{B})$) is the rate at which the magnetic force ($\vec{J} \times \vec{B}$) does work on the conduction as a whole; the term ($\vec{V} \circ (\vec{J} \times \vec{B})$) pushes the fluid either creating kinetic energy or helping it to overcome other forces on the reverse if the term is negative. The term $\vec{J}^2 / \sigma$ is positive and dissipated in the form of heat.

Hence the energy equation in MHD can be written by adding the term $\vec{J}^2 / \sigma$ to the right hand side of the equation (1.8-13) which is given by

$$\rho C_p \left(\partial T / \partial t + \vec{V} \circ \nabla T \right) = \nabla \circ (k \nabla T) + \mu \Phi + \vec{J}^2 / \sigma \qquad (1.8\text{-}17)$$

(e) MASS TRANSFER EQUATION:

Though mass diffusion is concern with the conductivity of the medium, the magnetic field has little effect on this process. So, ordinary diffusion equation can be applied for MHD problems under suitable assumptions. The differential equation describing the distribution of any component in a moving binary mixture, when the fluid is incompressible and has no inner mass source, can be had by Fick's law as

$$J_A = -\rho D (\nabla m_A) \qquad (1.8\text{-}18)$$

where, A and B represents two individual species of the mixture and $D_{AB} = D_{BA} = D$. This binary diffusion coefficient D is a physical property of the mixture. The equation (1.8-18) states that the species A diffuses in the direction of decreasing mole fraction of A, just as heat flows by conduction in the direction of decreasing temperature.

For a multi-component system under the assumption of negligible effects of thermal and pressure diffusion, and of constant ρ and D, the Fick's law (1.7-1) is written as

$$\partial \rho / \partial t + \rho_A (\nabla \circ \vec{V}) + (\vec{V} \circ \nabla \rho_A) = D(\nabla^2 \rho_A) + R_A \qquad (1.8\text{-}19)$$

where R_A is the molar rate of production of A per unit volume.

Using the continuity equation $\nabla \circ \vec{V} = 0$ and dividing the equation by M_A, we get

$$\partial C_A / \partial t + (\vec{V} \circ \nabla C_A) = D(\nabla^2 C_A) + R_A \qquad (1.8\text{-}20)$$

This equation is usually used for diffusion in dilute solution at constant temperature and pressure.

For $R_A = 0$, the equation becomes

$$\partial C_A / \partial t + (\vec{V} \circ \nabla C_A) = D(\nabla^2 C_A) \tag{1.8-21}$$

This equation is similar to the energy equation for a fluid motion when ρ is independent of t. This similarity is the basis for the analogous that are frequently drawn between heat and mass transport in flowing fluids with constant ρ.

(f) EQUATION OF STATE:

In solving fluid dynamical problems, in addition to the equation of continuity, motion, and energy, one should consider a thermodynamic relation of the form

$$e = e(T, p) \tag{1.8-22}$$

where, e: specific internal energy. T: temperature. p: pressure

This equation for hydrodynamic case takes the form

$$\rho = \rho(p, T) \tag{1.8-23}$$

This relation are known as the equation of state.

We have considered, in this thesis, problems in which Boussinesq approximations are valid [19]. It suggests that ρ is constant in all terms in the equation of motion except that one in the external force; therefore, we write

$$\rho = \rho_0[1 - \alpha(T - T_0)] \tag{1.8-24}$$

where, α is the volumetric expansion coefficient of the fluid and the subscript $\circ$ denotes the unheated no flow state.

1.9 Dimensionless Groups:

Due to the complex form of the governing equations of conventional fluid mechanics and MHD. it is extremely difficult to solve convective heat and mass transfer problems except for idealized, simple situation. Therefore, for most cases of practical interest the convective heat and mass transfer is studied experimentally, and the results are presented in the form of empirical equation that involve dimensionless groups. The utility of using dimensionless groups in such correlation is that several variables are combined into a few dimensionless parameters. Consequently, the number of variables to be studied is reduced. Therefore, establishment of such non-dimensional parameters that are appropriate for a given heat and mass transfer problem is most important. We discuss this below in a nutshell through equations of motion, energy and mass diffusion.

In vector notation these equations for an incompressible electrically conducting viscous fluid are –

$$\frac{\partial \vec{V}}{\partial t}+(\vec{V}\circ\nabla)\vec{V}=-\frac{1}{\rho}\nabla p+\nu\nabla^2\vec{V}+g\alpha(T-T_0)+\frac{1}{\rho}(\vec{J}\times\vec{B}) \tag{1.9-1}$$

$$\rho C_p\left[\ \frac{\partial T}{\partial t}+(\vec{V}\circ\nabla)T\ \right]=k\nabla^2 T+\mu\Phi+\frac{\vec{J}^2}{\sigma} \tag{1.9-2}$$

$$\frac{\partial C_A}{\partial t}+(\vec{V}\circ\nabla)C_A=D\nabla^2 C_A \tag{1.9-3}$$

where the symbols have their usual meaning.

Here, the *coupling effects* that arise between heat transfer and mass transfer processes have been dropped from (1.9-2) and (1.9-3) as being very small effects they can produce compared to the effects of diffusion and heat conduction.

We make the quantities non-dimensional with the help of $V_0, T_0, L, \rho_0, p_0, B_0, t_0, C_0, J_0$ and put

$$p^*=\frac{p}{\rho_0 V_0^2},\ \rho^*=\frac{\rho}{\rho_0},\ T^*=\frac{T}{T_0},\ \vec{V}^*=\frac{\vec{V}}{V_0},\ x_i^*=\frac{x_i}{L},\ t^*=\frac{tV_0}{L},\ T^*=\frac{T-T_0}{T_1-T_0}$$

$$\Phi^*=\frac{\Phi L^2}{\mu_R V_0^2},\ \nabla^*=L\nabla,\ \vec{B}^*=\frac{\vec{B}}{B_0},\ \vec{J}=\frac{\vec{J}}{\sigma V_0 B_0},\ C_A^*=\frac{C_A}{C_0},\ i=1,2,3 \tag{1.9-4}$$

The subscript "o" refers to the characteristic value of the other quantities. The current density $\vec{J}$ is taken to be of the order $(\sigma\nabla\circ B_0)$.

Substituting (1.9-4) in (1.9-1) – (1.9-3), we get

$$\rho^*\left[\frac{\partial \vec{V}^*}{\partial t^*}+\left(\vec{V}^*\circ\nabla^*\right)\vec{V}^*\right]=-\nabla^* p^*+\frac{1}{\mathrm{Re}}\nabla^{*2}\vec{V}^*+\frac{Gr}{\mathrm{Re}^2}\rho^* T^*+\frac{M^2}{\mathrm{Re}}\left(\vec{J}^*\times\vec{B}^*\right) \tag{1.9-5}$$

$$\rho^*\left[\frac{\partial T^*}{\partial t^*}+\left(\vec{V}\circ\nabla^*\right)T^*\right]=\frac{1}{\mathrm{Pr}\,\mathrm{Re}}\nabla^{*2}T^*+\frac{Ec}{\mathrm{Re}}\Phi^*+\frac{M^2 Ec}{\mathrm{Re}}\vec{J}^{*2} \tag{1.9-6}$$

$$Sc\,\mathrm{Re}\left[\frac{\partial C_A^*}{\partial t^*}+\left(\vec{V}^*\circ\nabla^*\right)C_A^*\right]=\nabla^{*2}C_A^* \tag{1.9-7}$$

These three equations are the equations of change for free convection in terms of dimensionless variables. For free convection flow only two dimensionless groups, Pr (Prandtl number), Re (Reynolds number) appears in the equation of change. Viscous dissipation term in the equation of energy is dropped being clearly unimportant for free convection. Other non-dimensional groups appearing in these equations are Gr, M, Sc, and Ec, which, respectively, are known as – the Grashof number, the Hartmann number, Schmidt number, and the Eckert number. These dimensionless parameters together with some other constants that appears as co-efficient in the field equations are listed below with physical meaning.

(i) *Reynolds number*: $\text{Re} = \frac{V_0 L}{\nu}$

There are four dimensionless groups, which regularly appear in MHD literature. The first one is Reynolds number. It is ratio of inertia force $((\vec{V} \circ \nabla)\vec{V})$ to viscous force $\nu\nabla^2\vec{V}$, which appears in equation of motion. To specify the circumstances in which different types of fluid occur, we need to introduce the concept of Reynolds number. Reynolds number determines the diffusion of vortices along the streamlines. In general, it is very difficult to get a solution of the Navier-Stokes equation; hence weightage is given to the consideration of limiting cases of very large and very small viscous forces, which are known as the very small Reynolds number and very large Reynolds number. In that case the Navier- Stokes equation becomes tractable. We have also magnetic Reynolds number $R_m = V_0 L/\lambda = \mu\sigma V_0 L$, known as 4th dimensional group in MHD literature. This is being discussed as another topic in a nutshell.

(ii) *Prandtl number*: $\text{Pr} = \frac{\rho\nu c_p}{k} = \frac{\nu}{k}$

The relative importance of viscosity and heat conduction may be indicated by the Prandtl number, which is defined as the ratio of *viscous diffusivity* or momentum diffusivity to the thermal diffusivity. The value of ν shows the effect of viscosity of a fluid. If other things remain the same, the smaller the value of ν, the narrower is the region affected by viscosity. The *thermal diffusivity* shows the effect of heat conduction of a fluid. If other things remains the same, the smaller the value of k, the narrower will be the region affected by heat conduction. Thus Pr gives the relative magnitude of the thermal boundary layer compared to the viscous boundary layer. The Prandtl number is just a constant of the material and does not depend on the property of the fluid. For gases, it is always of the order of unity and for liquid it may vary in a wide range. For air Pr = 0.7, and for water at $60^0 F, \text{Pr} = 7.0$ while for glycerin $\text{Pr} \approx 7250.0$.

(iii) Grashof number: $Gr = \frac{g\beta T_w L^3}{\nu^2}$

This number generally arises in analytical and empirical considerations of free convective heat transfer processes. Gr is the ratio of buoyancy force to viscous force and gives the relative importance to viscous and inertial effects. For a given fluid Gr indicates the type of flow to be expected in which dynamical processes are dominant, whether the flow is laminar or turbulent and so on as the Prandtl does for forced flow. When Gr is large, the viscous force is negligible compared to the buoyancy and inertia forces. On the other hand it tells nothing in the case of small Gr as the apparent prediction that the inertia force is small is in contradiction with the original assumption that the inertia force is comparable to the buoyancy force.

(iv) Hartmann number:

It is the ratio of electromagnetic body force known as the Lorentz force to the viscous force (shear force), and is a hybrid of Re and N, the interaction parameter defined by $N = \sigma B_0^2 L / \rho V_0$. It is $Ha(= M) = (N\,\mathrm{Re})^{1/2} = B_0 L(\sigma / \rho\nu)^{1/2}$ ([19] pp.96). This number was introduced by Hartmann to describe his experiments with viscous Magneto-fluid dynamics channel flow. He got that when $Ha \to 0$, the channel flow is parabolic, when $Ha \to \infty$, the exponential Hartmann layer form on both walls. Here, it is assumed that the Lorentz force $\vec{J} \times \vec{B} \approx (\sigma \nu B_0^2)$, which is true for small or moderate conductivities. Thus the magnitude of the Hartmann number M indicates the relative effects of magnetic and viscous drag force. This is the third dimensionless group, which regularly appear in the MHD literature.

(v) Schmidt number: $Sc = \frac{\nu}{D}$

This parameter frequently appears in the problems of diffusion just as we encounter repeatedly the Prandtl number in problems of heat conduction in flow systems. It is the ratio of viscous diffusion $(\nu = \frac{\mu}{\rho})$ to mass diffusion $(D = D_{AB})$ in case of binary mixture. It measures the relative magnitude of the viscous boundary layer compared to mass diffusion layer. This ratio lies between 0.2 and 5.0 for most gas pairs. For gases Sc is independent of pressure. It varies with temperature and sometimes also with concentration. When $Sc = \Pr$, the profiles for mass and heat transfer are similar.

(vi) <u>*Eckert number*</u>: $Ec = \dfrac{V_0}{c_p \Delta T}$, $\Delta T = T_1 - T_0$

The ratio of Brinkman number $(Br = \mu V_0^2 / k\Delta T)$ to the Prandtl number is a non – dimensional group, called the Eckert number. It is directly related to the temperature increase through adiabatic compression. The quantity Ec can be retained in incompressible flow also, but the interpretation with reference to the adiabatic compression ceases to be valid. By observing the magnitude to Ec, it is to derive a conclusion that the frictional heat and that due to compression are important for calculation of the temperature field when the free stream velocity V_0 is so large that the adiabatic temperature increases is of the same magnitude as the prescribed temperature difference between the body and the main stream.

In addition to these, other characteristic numbers also appears in the conventional fluid mechanics as well as in MHD. They are

$(T_W - T_R)/T_R$, the temperature difference that appears in heat transfer problems.

Pe, the Peclet number which is the product of Re and $\Pr$,

Ma, the Mack number, which is related to Eckert number, and is defined by

$$Ec = Ma^2 \frac{C_R^{\;2}}{c_p T_R}$$

c_f and Nu, the skin-friction coefficient and Nusselt number, which are related to the wall shear stress τ_W and the heat flux at the wall q_w, where

$$c_f = \frac{2\tau_W}{\rho_R V_0^2} \quad \text{and} \quad Nu = \frac{q_W L}{\lambda (T_W - T_\infty)}$$

(vii) <u>*Magnetic Reynolds Number*</u>: $R_m = \dfrac{V_0 L}{\eta} = \mu\sigma V_0 L$

It gives the ratio of the convective fluid flux to the diffusive magnetic flux. Also, it is a measure of the magnitude of the induced magnetic field compared with the total magnetic field associated with the MHD flow problem. The magnitude of R_m determines diffusion of magnetic field along the streamlines, and hence it is a measure of the effects of the flow on the magnetic field. A small $R_m (<< 1)$ indicates that the induced magnetic field is small compared to the total or the applied magnetic field. So, induced magnetic field is neglected, where magnetic Reynolds is small. For most of the MFD flow problems where $\vec{E} \approx 0(\vec{V} \times \vec{B})$, R_m<< 1, usually. That is, the magnetic field is not distorted by the flow. But, when $R_m << 1$, the magnetic field lines move with the fluid, and the phenomena is known as *frozen-in-fluid.*

1.10 **Initial and Boundary Value Problems (IBVPs) and Laplace Transforms**:

This thesis is about the solution of a few *Initial and Boundary Value Problems* involving the linear partial differential equations that appear in modeling many natural and artifacts phenomena in engineering and the physical sciences. In particular, it is about the development and application of *Laplace transformation* and related techniques to the solution of such problems. It does not. however, present any results on Laplace transformation that are directly applicable to the solution of IBVP, nor does it includes some additional techniques to solve such problems. Rather, it is an attempt to show that this analytical method can be used suitably to solve MHD problems. On the other hand the study of any physical problem automatically leads to the boundary value problem. Hence it is immense important to know about IBVP and Laplace Transformation method. Following are some concepts of both of these two.

1.10.1 *Initial and Boundary Value Problem in Fluid Mechanics*:

A problem consisting of finding solutions of a partial differential equation subject to some initial and boundary conditions is normally referred to as a *boundary value problem.*
Let us consider the temperature distribution in a thin bar (of some conducting material), represented by one-dimensional heat equation or diffusion equation, given by

$$u_t(x,t) = ku_{xx}(x,t) \qquad (1.10\text{-}1)$$

Besides the trivial solution, $u \equiv 0$, it is easy to see that it has other four solutions (see [34] pp. 6). Moreover, it is easily verified that the sum of two solutions and the product of one of them by a constant are also solutions. But odd thing is that, none of them able to give the actual temperature distribution in the bar. It is due to the fact that the temperature on the bar depends on several additional conditions. For instance, from a certain time on these temperatures, will depend on the initial temperature distribution at the given instant and, on any amount of heat that may enter or leave at the endpoints. Therefore, we see that both initial and boundary conditions will affect the actual temperature distribution. These conditions can be specified in a variety of ways. For example, the temperature may be given at $t = 0$ as a function of x

$$u(x,0) = f(x)$$

and it may be further specified that the temperatures at the endpoints $u(0,t)$ and $u(a,t)$, will remain fixed for $t > 0$.

Sometimes the boundary and initial conditions are given as limits as the boundary of the domain of definition of the equation. The problem of finding a function $u : D \to R$ that is of class C^2 in D and such that

$$u_t = ku_{xx} \quad \text{in } D \qquad \text{where } \begin{array}{l} D = \{(x,t) \in R^2 : 0 < x < a, t > 0\} \\ \overline{D} = \{(x,t) \in R^2 : 0 \le x \le a, t \ge 0\} \end{array}$$

$$u(x,t) \to 0 \quad \text{as } x \to 0, t > 0$$

$$u(x,t) \to 0 \quad \text{as } t \to 0, 0 < x < a$$

have the solution

$$u(x,t) = (x / t\sqrt{t}) \exp(-x^2 / 4kt)$$

Here, u is not defined $t = 0$ but has the required limit.

Boundary conditions are specified under careful observations. To expect a well-posed physical problem the solution must be unique. More than one solution implies the insufficient boundary conditions.

There is still another requirement that physical considerations impose on the solution of a boundary value problem. Physical measurements or observations of the initial temperature of the bar would result in only approximate values. In a similar manner, the temperature at the endpoints cannot be maintained with perfect accuracy. Thus, the mathematical formulation of a problem will contain small errors in the initial and boundary values, and the corresponding solution can only approximate the true one. What we must require is that, it be a good approximation. That is, if the initial or boundary values change by a small amount the solution of a well-posed problem should also change only by a small amount.

Thus, we see that a good solution of boundary value problem always depends on the following requirements:

(i) there exists a solution

(ii) the solution is unique, and

(iii) the solution is stable; that is, it depends continuously on the boundary conditions in the sense that a small change in the initial and boundary values results in only a small change in the solution.

A boundary value problem, which satisfies these three requirements, is said to be well posed in the sense of *Hadamard* (1865 - 1963), who introduced this concept in his 1920 lectures at *Yale University* ([34] pp.8).

1.10.2 *Laplace Transform Technique* (LTT) :

Many of the concepts of classical analysis had their origin in the study of physical problems leading to the boundary value problems. The search for a solution of this IBVP leads to the discovery of new mathematical tool – tools that are today of immense use in pure and applied mathematics, and other engineering branches, is the *Laplace Transformation.*

(i) ***Laplace Transform Technique! Why*** ?

The branches of science in which Laplace transformations, in use, are – in solving linear partial differential equations with constant coefficients, some ordinary differential equations in which the coefficients are variables, two or more simultaneous ordinary differential equations, in mechanics (dynamics and statics), in telegraphy and electrical circuits, to analysis the characteristic of beam, various partial differential equations subject to boundary conditions, to integral and difference equations etc. Thus, we see that the Laplace transform has its tremendous applications in many branches of pure and applied mathematics, physics and engineering science. However, its application to MHD problems are relatively new, is in monograph, yet to take concrete shape.

(ii) ***Advantages over other methods***:

This we want to discus, in a nutshell, point-wise as follows:

- Inability of all available analytical tools to solve the governing equations of flows of all kinds.
- Analytical tools are more accurate then numerical methods.
- Less time consuming and labour.
- It provides the convenience and effective results.
- Easily portable.
- Most sophisticated and well equipped method.

- For time dependent problems this method is best suitable.
- Open half-plane and closed half-plane problems are full in nature and different laboratory experiments.

Some elementary functions, such as a constant, the exponential, the sine, and the cosine functions do not have Fourier transforms (one of the most popular analytical methods) because they are not integrable on R. The easiest (suitable) way to overcome these sever limitations is the use of Laplace transform technique. In particular, one can apply it to deal with problems in which one of the variables is time.

(iii) *Laplace transforms technique in MHD! Why* ?

The physical aspects of any fluid flow are given by the following fundamental principles:

- Mass is conserved
- $F = ma$ (Newton's second law)
- Energy is conserved

These fundamental principles are expressed in terms of mathematical equations (partial differential equations), in which Laplace transform technique can be used suitably; as it is the art of replacing the governing equations of fluid flow with numbers and advancing these numbers in space and/ or time into an ordinary differential equation, which can be solved by already established rules and, then Inverse Laplace Transforms techniques are applied to get the required results. This method is best fitted for MHD problems, particularly for unsteady problems. The high-speed digital computers and inventions of many algorithms ([79], [51], [42]) together with Computational Methods have allowed the practical growth of LTT in MHD.

(iv) *Laplace Transform Technique! What it is ?*

Laplace transform is an operator that transforms functions into functions. An outstanding example is the differential operator D, which transforms each function of a large class (those possessing a derivative) into another function. Laplace transform can also be called as a mapping. Generally, the operator L is used to represent the Laplace transform operator.

One class of transformations, which are integral transformation, may be defined by

$$T\{F(t)\} = \int_{-\infty}^{\infty} K(s,t)F(t)dt = f(s) \tag{1.10-2}$$

Given a function $K(s,t)$, called the *Kernel* of the transformation, equation (1.10-2) associates with each $F(t)$, of the class of functions for which the above integral exists, a function $f(s)$ defined by (1.10-2). If the kernel $K(s,t)$ is defined by

$K(s,t) = 0$ for $t < 0$

$K(s,t) = e^{-st}$ for $t \geq 0$ [94], then (1.10-2)

$$f(s) = \int_{0}^{\infty} e^{-st} F(s)dt \tag{1.10-3}$$

The function $f(s)$ defined by the integral (1.10-3), is called the Laplace transform of the function $F(t)$, and is denoted by $L\ \{ F(t) \}$ or $\bar{f}(s)$. Thus Laplace transform is a function of a new variable or parameter s given by (1.10-3).

Again Laplace transform can be viewed as a modified form of Fourier Transform. One such Fourier Transform is

$$\frac{1}{2\pi}\int_{0}^{\infty} f(t)\ e^{-(\sigma+i\omega)t} dt \tag{1.10-4}$$

Omitting the factor $1/2\pi$, adopting the letter t for the variable, and denoting $(\sigma + i\omega)$ by s, lead to following modified transformation [34].

<u>*Def*.1.</u>: Let $f : [0,\infty) \to C$ be such that $f(t)e^{-\sigma t}$ is integrable on $[0,\infty)$ for some σ in R. Then the function $F : C \longrightarrow C$ defined by

$$F(s) = \int_{0}^{\infty} f(s)e^{-st} dt \tag{1.10-5}$$

with domain $D_F = \{s \in C : f(t)e^{-st}$ is integrable$\}$,

is called the Laplace transform of f. It is also denoted by $L\ [f]$. If the Laplace transform of an objective function $F(t)$ is $f(s)$ i.e. $L\ \{F(t)\} = f(s)$, then $F(t)$ is called the inverse L.T. of $f(s)$, and it is written as $F(t) = L^{-1}\{f(s)\}$. It is quite easy to compute the Laplace transforms of some elementary functions.

Example1. If $f \equiv a$ on $[0,\infty)$, where a is a real or complex constant, then

$$F(s)=\int_0^\infty ae^{-st}dt=\lim_{\in\to\infty}\int_0^\in ae^{-st}dt=\lim_{\in\to 0}[\frac{ae^{-st}}{-s}]_0^\in=\frac{a}{s}$$

if the real part of s, which we denote by $\mathrm{Re}(s)$, is greater than zero.

Example2. If $f(t)=e^{at}$ on $[0,\infty)$, where a is a real or complex constant, then

$$F(s)=\int_0^\infty e^{(a-s)t}dt=\lim_{\in\to\infty}\left[\frac{e^{(a-s)t}}{a-s}\right]_0^\in=\frac{1}{s-a}\qquad \text{for } \mathrm{Re}(s)>\mathrm{Re}(a)$$

It is to be noted that the operator L, like the differential operator D, is a linear operator. If $F_1(t)$ and $F_2(t)$ have Laplace transforms, and if C_1 and C_2 are any two constant, then

$$L\{C_1F_1(t)+C_2F_2(t)\}=C_1L\{F_1(t)+C_2L\{F_2(t)\}$$

Using elementary properties of definite integral one can easily prove it.

The Laplace transform of $F(t)$ is said to exit if the integral (1.10-3) or (1.10-5) converges for some values of s, otherwise it does not exist. Since e^{-st} is an integral function of both s and t, it is sufficient for the existence of $\int_\varepsilon^T e^{-st}F(t)dt$, that $f(t)$ being intergable over interval $\varepsilon\le t\le T$, where $(\varepsilon\to 0, T\to\infty)$. In the neighborhood of $t=0$, and for fixed s, e^{-st} is absolutely bounded and so the behavior of the Laplace integral in the limit as $\varepsilon\to 0$ is essentially the same as the behavior of $\lim_{\varepsilon\to 0}\int_\varepsilon^T F(t)dt$.

If, as $t\to\infty$, the behavior of $f(t)$ is worse than e^{Kt} for some real constant K, then we could find a t_0 so that $|F(t)|<Me^{Kt}$ for $t>t_0$, and a constant M.

Again we can get a function, known as Inverse Laplace Transform, whose Laplace transform is known. Theoretically it is an easy task, because, according to the definition 1, $F(s)$ is actually a Fourier transform – that of the function that vanishes for $t<0$ and equals $2\pi f(t)e^{-\sigma t}$ for $t\ge 0$, where $\sigma=\mathrm{Re}(s)$. But practically it is not so. The following definition and theorem is sufficient to have an Inverse formula.

Def.2.: Let L be the set of all locally integrable functions $f:R\to C$ such that f vanishes for $t<o$ and $f(t)e^{-\sigma t}$ is integrable for some σ in R.

Theorem: Let $f \in L$ have Laplace transform F, and let σ be the real number such that $f(t)e^{-\sigma t}$ is integrable on $[0,\infty)$. Then

$$\frac{1}{2\pi} PV \int_{-\infty}^{\infty} F(\sigma + i\omega)e^{(\sigma + i\omega)t} d\omega = \frac{1}{2}[f(t+) + f(t-)]$$

at every point on $[0,\infty)$ such that f has right-hand and left-hand derivatives at t and f, is piece-wise continuous on an arbitrary small interval centered at t.

Thus the inverse formula is

$$f(t) = \int_{C-i\infty}^{C+i\infty} f(s)e^{st} ds, \quad C > 0.$$

1.11 A brief report on use of Laplace Transforms in MHD:

The use of the Laplace transform technique in solving hydrodynamical problems has its beginning in the application of *Operational Calculus* derived by **Heavisides** (see historical epilogue in Appendix) to such problems.

Employing Laplace transform, *Čekmarev* [1960a] has obtained an exact solution for the particular case of non-stationary flow of a conducting viscous fluid between parallel infinite conducting walls in the presence of a transverse magnetic field. Using the same transform *Čekmarev* [1960b] again discussed the motion of an electrically conducting viscous liquid over an infinite conducting plate in the presence of uniform magnetic field and obtained the complete solution. Bhatnagar and Kumar [1960, 1964] have applied Laplace and finite and infinite Hankel transforms to study the propagation of small disturbances in a viscous and electrically conducting liquid in the presence of a magnetic field. Gupta [1960] has used Laplace transform to investigate the effect of transverse magnetic field on the flow of liquid near a plate, which moves with velocity proportional to t^n. Rayleigh's problem in magnetohydrodynamics has been studied by Rossow [1960] with the help of Laplace transform. Ulfjand [1961] investigated the unsteady hydromagnetic flows in a channel of rectangular cross-section. Yen and Chang [1961] have studied the fluid flows between two parallel planes with transverse magnetic field under time dependent pressure gradient. Chang and Atabek [1962] have worked on laminar flow between two co-axial tubes in the entrance region. Katagiri [1962] discussed Couette motion in magnetohydrodynamics. Laplace and finite Hankel transforms have been employed by Kumar [1963] to obtain the complete solution of the problem of propagation of small disturbances in a viscous and electrically conducting fluid between two infinite co-axial circular cylinders in the presence of uniform magnetic field. Muhuri [1963] worked on the paper of formulation of Couette flow in magnetohydrodynamics with suction. Both, Rathy [1963] and Shukla [1963] have studied the hydromagnetic flow between two parallel plates, the plates being porous in the later case. Finite Hankel and Laplace transforms have been used by Singh [1965] to deal with the impulsive motion of a viscous conducting liquid contained between two porous concentric circular cylinders in the presence of radial magnetic field. Hydromagnetic flow of a viscous incompressible fluid due to uniformly accelerated motion of an infinite plate in the presence of transverse magnetic field has been discussed by Soundalgekar [1965]. Synder [1965] has also applied Laplace transform technique to deal with the flow of viscous and electrically conducting liquid in the entrance region of parallel plates in the presence of a transverse magnetic field. Datta [1966] investigated the slip flow of an electrically conducting viscous liquid over a porous flat plate under a uniform transverse magnetic field. Both finite Hankel and Laplace transforms have been employed by Groves [1966] to solve the problem of unsteady motion of an electrically conducting viscous liquid in a cylindrical vessel in the presence of axially symmetric magnetic field of constant strength. Dube and Khan [1968] have analyzed the flow of viscous conducting liquid over an infinite harmonically oscillating and conducting plate. Kulshrestha and Puri [1969] have got the exact solution of hydromagnetic rotating flow.

The difficulties in obtaining the inverse of Laplace transforms that appear in problems of physics and engineering decreases considerably when *Hetnarski*'s the classic papers – "On inverting the Laplace transform connecting with the error function" in 1964 and "An algorithm for generating some inverse Laplace transforms of exponential form " in 1975, had appeared for scholars to read and use. He mentioned that such inverses are needed in problems of coupled thermoelasticity and heat conduction in solids of the type considered by Gridamo [1974]. Puri and Kythe [1969, 1974, 1976] have used similar techniques to develop such formulas for inverses of the above class of functions in problems encountered in hydromagnetic rotating flows [58], [84], and in viscoelastic rotating flows [85]. In the problem of heat transfer and visco-elastic flows Puri, *et al.* have encountered two other classes of functions whose inverses were not available in [32], [75], [95], which leads to the publication of the classic paper "Some inverse Laplace transforms of exponential form" in

1988. in which there contains fourteenth new formulas. These formulas are useful in problems of fluid mechanics, particularly in Magnetohydrodynamics.

Pathak [1974] has used integral transform in the case of unsteady hydromagnetic flow along a circular pipe. Das [1975] has employed Laplace transform technique to study the flow of a viscous fluid over a rigid plane base. Using Laplace transform method, Nye [1977] found the approximate solution for unsteady magnetohydrodynamic channel flows. Ram and Mishra [1977] studied the unsteady magnetohydrodynamic flow through a porous medium between two parallel plates, and in a circular pipe. Srinivasan and Bathaiah [1978] have discussed the flow of conducting viscous liquid between parallel plates. Soundalgekar and Uplekar [1979] have applied Laplace transformation to derive an exact solution of the flow of viscous incompressible fluid past an infinite porous plate. Revankar and Korwar [1980] have studied the problem of unsteady MHD flow past a porous infinite plate. Kishore, Tejpal and Tiwari [1981] have worked on hydromagnetic flow past an accelerated porous plate in a rotating system. Tokis and Pande [1981] have carried out an investigated on unsteady magnetohydrodynamic flow of a viscous liquid near a moving porous plate. Devi Singh [1983] presented certain problems of MHD flows employing Laplace transform technique in his Ph. D. thesis. An exact analysis of MHD stagnation point flow with suction havebeen carried out in two and three dimensions by Soundalgekar and Vighnesam [1985]. Tokis [1986] discussed the unsteady MHD free-convection flows in a rotating disc.

An analysis of MHD heat transfer in hyperbolic time-variation flow near a stagnation point of a heated blunt-nosed cylinder whose wall temperature varies as Ax^N was presented by Soundalgekar, Ramana, Murty and Takhar [1990]. Gourla and Katoch [1991] have discussed about the result of unsteady viscous incompressible free convection flow of an electrically conducting fluid between two heated vertical plates in the force field of gravity and applied magnetic field acting in the horizontal direction and perpendicular to the plate. By application of the Laplace transform technique, Srivastava *et al.* [1994] presented a mathematical model of blood flow in single arteries and arterioles subject to both the pulsatile pressure gradient due to normal heart action and a single cycle of body acceleration. Soundalgekar, Das and Deka [1997] have studied the free convection effects on MHD flow past an infinite vertical oscillating plate with constant heat flux. Rathod and Shrikanth [1998] have derived the solution of MHD flow of Rivlin-Ericksen fluid through an inclined channel.

Jordon, Puri and Boros [2000] have presented the valuable paper "A new class of Laplace Inverse and their applications". This paper is valuable in the sense that inverse

Laplace transforms involving nested (double) square roots arise in many areas of applied mathematics, specially, in fluid mechanics.

An exact solution for the transient for MHD Stokes's oscillating plate was presented by Deka, Das and Soundalgekar [2001]. Deka and Soundalgekar [2002] have obtained an exact solution to transient free convection flow through homogeneous porous medium bounded by an infinite vertical isothermal plate, in the presence of temperature gradient heat source. In a theoretical paper, prepared by Soundalgekar, Deka and Das [2003], there contains the generation of flow caused by transient free convection of a viscous incompressible and electrically conducting fluid past an infinite vertical plate in the presence of periodic heat flux.

1.12 **Motivation, Extent and Scope of this thesis:**

As the powerful instrument, the Laplace transform technique, has a lot of scopes in applying it stimulate us to extent its use in Fluid Mechanics, particularly in magneto-hydrodynamic problems. In this thesis, we proposed to study a few problem of electrically conducting as well as free convective incompressible viscous liquid with heat transfer rate employing Laplace transform technique.

Effects of heat and mass transfer due to unsteady free convection flow between two heated vertical plates together with skin friction have been discussed in chapter 2. Exact solutions of the fluid velocity $\bar{u}(y,t)$, the temperature distribution $T(y,t)$ and mass diffusion $C(y,t)$, has been obtained with the help of Laplace transformation, where y is the distance measured between the two plates and t is the time. Using this method, similar cases can be discussed for the problems of heat and mass transfer for various geometries. In many engineering problems, it has been observed that the mass transfer accompanies heat transfer and hence our research may be useful in metallurgical industry,

In chapter 3, we have discussed the effect of magnetic field with heat transfer rate on hydromagnetic flow between two parallel plates – one adiabatic and other isothermal. The magnetic field is placed at angles $0^0, 30^0, 45^0, 60^0, 90^0$ to the vertical walls and resulting natures of the fluid flow have been discussed successively. We feel the need of proper choice of the combinations of the values of Pr, n, and t for flow analysis. It has been observed that, the effect of the magnetic field slowly decreases as the angle between the directions of the fluid velocity field and the direction of the magnetic field decreases from $\pi/2$ to 0. This kind of investigation has practical importance for the study of those fluid mechanical problems where magnetic field is introduced.

In chapter 4, we have investigated the flow and heat transfer characteristic of a viscous incompressible and electrically conducting fluid through a porous medium bounded by two long vertical parallel plates. The effective viscosity of the porous medium (material) is larger than the viscosity of the fluid. Such material has a Darcy number and viscosity ratio parameter of order 10. The effects of the four parameters, namely, Darcy number, Viscosity ratio parameter, magnetic Hartmann number, and Prandtl number on temperature and velocity field together with skin friction have been discussed through tabular values and graphs. It has been observed that this kind of transient free convection problem is found favor in metallurgical industry and many others engineering branches. The problem can be extended further for investigated by considering both Brinkman and Forchheimer terms that appear for permeability of the medium.

The steady and starting phase velocity profiles has been derived for Stokes's second problem in chapter 5. Here the fluid is electrically conducting and viscous incompressible, the plate is porous. A uniform magnetic field perpendicular to the plate is applied. The effect of magnetic field and suction velocity of the plate is greatly observed in this problem. As this problem is of fundamental character may perhaps be useful for future study in MHD and may have a good number of applications.

We have discussed in chapter 6, the unsteady magnetohydrodynamic plane Couette flow and heat transfer with temperature dependent heat generating source and heat absorbing sink. Exact solutions are found for temperature distribution and fluid velocity field with effects of magnetic Hartmann number, Reynolds number, Grashof number, Peclet number, Prandtl number. We have considered the case of moving the horizontal plates parallely in opposite direction. It has been observed that both heat generating source and heat-absorbing sink has small effects on flow field, but large effects on temperature distribution. As these problems are fundamental in nature, have many applications in different branches of physiology and engineering. So, outcome of this wok may be fruitful both theoretically and practically.

An exact solution for unsteady free convection MHD flow and heat transfer rate between two heated vertical plates with heat generating source and heat absorbing sink has been derived in chapter 7. Here a uniform magnetic field has been applied in a direction perpendicular to the flow. It is seen that the magnetic field has no influence on temperature distribution but has significant effect on fluid velocity field. It is also observed that heat-generating source has effects on both temperature distribution and flow field but no effect of heat absorbing sink. This problem can be taken for further study. The study of these types of problems can be helpful in metallurgical processes.

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CHAPTER - 2

EFFECTS OF MASS TRANSFER ON UNSTEADY FREE CONVECTION MHD FLOW BETWEEN TWO HEATED VERTICAL PLATES IN THE PRESENCE OF TRANSVERSE MAGNETIC FIELD

2.1 Introduction:

It has been seen by long experience that, in many engineering activities, especially in chemical engineering, that some processes are considered to be the mass transfer processes, which are sometimes accompanied by many other processes like heat transfer, rotation of fluids, electromagnetic forces, etc. The random movement of the molecules, which by their mixing tend to equalize existing differences in their energy, causes the heat conduction in a gas. By the same movement local differences in concentration of a gas mixture diminished in time even if no macroscopic mixing occurs. This processes is known as diffusion. By diffusion or convection, in a mixture of local concentration differences, a component is transported from one location to another. The mass transport through an interface between various phases of the same medium is found to be a special important in engineering sciences.

The range of free-convection flows that occur in nature and in engineering practices is vast and significant. So far, many papers, both theoretical and experimental, has been published on free convection heat transfer in view of their interest in astrophysics, geophysics, engineering and medical sciences. However, the flow of a fluid is caused not only by the temperature differences but also by concentration differences. These concentration differences also affect the flow and temperature near the surface of a body embedded in a fluid. In engineering applications, the concentration differences are created by either injecting the foreign gases or by coating the surface with evaporating material, which evaporates due to the heat of the surface. These mass transfer differences do affect the rate of heat transfer. In practice H_2, 0_2, CO_2 etc. are the foreign gases, which are injected in the air, flowing past bodies. Thus, for flows past vertical surfaces. there is buoyancy force, which arises due to temperature differences and concentration differences.

In recent years, many research workers have presented analytical as well as numerical solutions to such problems of fluid over vertical surfaces. However. unsteady free convection flows received little attention. Illingworth published the first paper on unsteady laminar flow of gas near an infinite flat plate. in 1950. But the results were published for unit Prandtl number. Siegel (1958) studied unsteady free convection near a semi-infinite vertical plate under uniform wall temperature. Goldstien and Eckert (1960) derived experimentally the one dimensional unsteady free convection flow past a semi-infinite vertical plate. Many papers relating to unsteady free convection flow past an infinite vertical plate were published. These are by Schetz and Eichhorn (1962). Goldstein and Briggs (1964) etc.

Gebhart and Pera [1971] have obtained the solution of the vertical natural convective flows resulting from the combined buoyancy effects of thermal and mass diffusion. Soundalgekar [1979] derived the solution of the effect of mass transfer and free convection currents on the flow past an impulsively started plate. An analysis of the fluid flow, through a porous medium confined between two vertical walls. maintained at different temperature and concentration levels. were presented by Trevisan and Bejan [1985]. Yucel [1990] studied the natural convection heat and mass transfer along a vertical cylindrical surface embedded in a porous medium, and which is maintained at a uniform temperature and concentration. A study on unsteady free convection MHD flow between two heated vertical parallel plates was presented by Gourla and Katoch [1991]. The effects of mass transfer on free convection flow past a vertical isothermal cone surface was studied by Kafoussias [1992].

In this paper. we have discussed the effects of mass transfer together with skin friction on the unsteady free convection flow between two heated vertical parallel plates. In section 2.2. the mathematical formulation of the problem under consideration is presented and in section 2.3, the analytical solutions are set out. The obtained results are shown graphically and a quantitative discussion is given in section 2.4. More importance has been given on the dimensionless parameters Gr_l and Gr_m on the velocity. temperature and concentration profiles as well as on the skin friction and rate of heat transfer. It is hoped that the results obtained not only provide useful information for applications but also serve as a complement for farther studies.

2.2 Mathematical Formulation:

In order to formulate the problem mathematically, we consider that the local properties of the fluid are not affected by the temperature differences except that of the density variation in the body force term. Also the influence of the density variations in other terms of the momentum, energy and concentration equations and the variation of the expansion coefficients β, β^* with temperature is negligible. The boundary layer is supposed to be thin. The level of the species concentration in the fluid is assumed to be so low that Soret and Dufour effects can be neglected. The fluid is supposed to be Newtonian, viscous and incompressible. The viscous dissipation, the induced magnetic field, the Hall effect, electrical effect and polarization effects are neglected.

We consider a vertical channel bounded by two fixed vertical parallel infinite plates and both are at the same temperature T_0', initially. At time t'>o, the plates are supplied heat at constant rate, thereby causing the presence of free convection currents in the fluid near the plates. As the plates are infinite in extent, the flow-variables are functions of y' and t'. The x' – axis is taken along the plates in the vertically upward direction and the y' – axis is taken normal to the plates. The uniform magnetic field B_0 is applied along horizontal direction, i.e. in a direction perpendicular to the fluid motion.

Under the above assumptions following Boussinesq's approximation, the flow fields are seem to be governed by the following equations:

Equation of mass conservation:

$$\frac{\partial v'}{\partial y'} = 0 \tag{2-1}$$

Equation of momentum:

$$\frac{\partial u'}{\partial t'} = \frac{\mu}{\rho}\frac{\partial}{\partial y'}\left(\mu\frac{\partial u'}{\partial y'}\right) + g\beta(T' - T_0) + g\beta^*(C' - C_0) - \frac{\sigma}{\rho}B_0^2 u' \tag{2-2}$$

Equation of energy:

$$\rho C_p \frac{\partial T'}{\partial t'} = K\frac{\partial^2 T'}{\partial y'^2} \tag{2-3}$$

Equation of diffusion:

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{2-4}$$

At time $t'>0$, the temperature of the plates ($y = \pm h$) changes according to $T' = T_0' + (T_w' - T_0')(1 - e^{-n't'})$, where n' is a decay factor,

The concentration of the fluid changes according as $C' = C_0' + (C_w' - C_0')(1 - e^{-n't'})$.

At any time t', the velocity, the temperature and the concentration are given by (u', o, o), T' and C', respectively.

The initial and boundary conditions are given by-

$$u' = 0,\quad T' = T_0',\quad C' = C_0' \qquad \text{for all } y \in [-h, h],\ t' = 0$$

$$u' = 0,\quad T' = T_0' + (T_w' - T_0')(1 - e^{-n't'}) \qquad \text{for } y = \pm h$$

$$C' = C_0' + (C_w' - C_0')(1 - e^{-n't'}) \qquad \text{for } y = \pm h \tag{2-5}$$

The dimensionless quantities which we used are-

$$u = \frac{u'h}{\upsilon}, T = \frac{T' - T_0'}{T_w' - T_0'}, C = \frac{C' - C_0'}{C_w' - C_0'}, t = \frac{t'\upsilon}{h^2}, y = \frac{y'}{h}, n = \frac{n'h^2}{\upsilon}$$

$$\Pr = \frac{\rho\upsilon C_p}{K} \qquad Gr_t = \frac{g\beta h^{3(\,)}(T_w' - T_0')}{\upsilon^2} \qquad Gr_m = \frac{g\beta^* h^3 (C_w' - C_0')}{\upsilon^2}$$

$$\mathrm{M} = B_0 h\sqrt{\frac{\sigma}{\mu}} \qquad \mathrm{Sc} = \frac{\upsilon}{D} \tag{2-6}$$

Using the dimensionless quantities (2-6), the equations (2-2) - (2-4) together with the boundary conditions (2-5), are found as follows:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr_t T + Gr_m C - M^2 u \tag{2-7}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} \tag{2-8}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{2-9}$$

$$\mathrm{u} = 0,\quad \mathrm{T} = 0,\quad \mathrm{C} = 0 \qquad \text{for all } y \in [-1, +1],\ t = 0$$

$$\mathrm{u} = 0,\quad \mathrm{T} = 1 - e^{-nt},\quad \mathrm{C} = 1 - e^{-nt} \qquad \text{for } y = \pm 1 \tag{2-10}$$

2.3 Solution of the equations:

Taking the Laplace Transform of equations (2-7) - (2-10), we get

$$\frac{d^2\bar{u}}{dy^2} - \left(M^2 + s\right)\bar{u} = -\left(Gr_t\bar{T} + Gr_m\bar{C}\right) \tag{2-11}$$

$$\frac{d^2\bar{T}}{dy^2} - \text{Pr}.s.\bar{T} = 0 \tag{2-12}$$

$$\frac{d^2\bar{C}}{dy^2} - Sc.s.\bar{C} = 0 \tag{2-13}$$

where

$$\bar{F}(y,s) = \int_0^\infty e^{-st}F(y,t)dt$$

together with boundary conditions

$$\bar{u}(\pm 1,s) = 0, \quad \bar{T}(\pm 1,s) = \frac{n}{s(s+n)}, \quad \bar{C}(\pm 1,s) = \frac{n}{s(s+n)} \tag{2-14}$$

Since, the equations (2-11) – (2-13) are of 2nd order ordinary differential equations in $\bar{u}, \bar{T}$ and $\bar{C}$, the solutions of the equations by use of boundary conditions (2-14), are found as -

$$\bar{T} = \frac{n}{s(s+n)} \cdot \frac{\cosh\sqrt{s\,\text{Pr}}.y}{\cosh\sqrt{\text{Pr}.s}} \tag{2-15}$$

$$\bar{C} = \frac{n}{s(s+n)} \cdot \frac{\cosh\sqrt{s.Sc}\,y}{\cosh\sqrt{s.Sc}} \tag{2-16}$$

$$\bar{u} = -\frac{n}{s(s+n)}\left[\frac{Gr_t}{s(1-\text{Pr})+M^2} + \frac{Gr_m}{s(1-Sc)+M^2}\right]\frac{\cosh\sqrt{M^2+s}\,y}{\cosh\sqrt{M^2+s}}$$

$$+\frac{n}{s(s+n)}\left[\frac{Gr_t}{s(1-\text{Pr})+M^2}\frac{\cosh\sqrt{s\,\text{Pr}}\,y}{\cosh\sqrt{s\,\text{Pr}}} + \frac{Gr_m}{s(1-Sc)+M^2}\cdot\frac{\cosh\sqrt{Sc.s}\,y}{\cosh\sqrt{Sc.s}}\right] \tag{2-17}$$

The inverse Laplace Transform of (2-15) – (2-17), gives the actual solution as -

$$T = 1 - \frac{\cos\sqrt{n\Pr}y.e^{-nt}}{\cos\sqrt{n\Pr}} + \frac{4n}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k\cos\frac{(2k+1)\pi y}{2}e^{-\frac{(2k+1)^2\pi^2}{4\Pr}t}}{(2k+1)\left\{\frac{(2k+1)^2\pi^2}{4\Pr}-n\right\}} \tag{2-18}$$

$$C = 1 - \frac{\cos\sqrt{nSc}y\exp(-nt)}{\cos\sqrt{nSc}} + \frac{4n}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k\cos\frac{(2k+1)\pi y}{2}.\exp(-(2k+1)^2\pi^2 t/4Sc)}{(2k+1)\left\{\frac{(2k+1)^2\pi^2}{4Sc}-n\right\}} \tag{2-19}$$

$$\begin{aligned}
u = & \frac{Gr_t+Gr_m}{M^2}\left[1-\frac{\cosh My}{\cosh M}\right] - \frac{Gr_t\exp(-nt)}{n(1-\Pr)-M^2}\left[\frac{\cos\sqrt{n-M^2}y}{\cos\sqrt{n-M^2}} - \frac{\cos\sqrt{n\Pr}y}{\cos\sqrt{n\Pr}}\right] \\
& - \frac{Gr_m\exp(-nt)}{n(1-Sc)-M^2}\left[\frac{\cos\sqrt{n-M^2}y}{\cos\sqrt{n-M^2}} - \frac{\cos\sqrt{nSc}y}{\cos\sqrt{nSc}}\right] + 4n\pi \times \\
& \sum_{=0}^{\infty}\frac{(-1)^k(2k+1)\cos((2k+1)\pi y/2)\exp(-(M^2+(2k+1)^2\pi^2/4)t)}{\{M^2+((2k+1)^2\pi^2/4)\}\{M^2+((2k+1)^2\pi^2/4)-n\}} \times \\
& \left[\frac{Gr_t}{(1-\Pr)-\{M^2/(M^2+(2k+1)^2\pi^2/4)\}} + \frac{Gr_m}{(1-Sc)-\{M^2/(M^2+(2k+1)^2\pi^2/4)\}}\right] + \\
& \frac{4n}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k\cos((2k+1)\pi y/2)}{2k+1} \times \left[\frac{Gr_t\exp(-(2k+1)^2\pi^2 t/4\Pr)}{\{(2k+1)^2\pi^2/4\Pr-n\}\{M^2-(1-\Pr)(2k+1)^2\pi^2/4\Pr\}} + \right. \\
& \left.\frac{Gr_m\exp(-(2k+1)^2\pi^2 t/4Sc)}{\{(2k+1)^2\pi^2/4Sc-n\}\{M^2-(1-Sc)(2k+1)^2\pi^2/4Sc\}}\right]
\end{aligned} \tag{2-20}$$

2.4 Results and Discussion:

In this section, we summarize the most important findings uncovered in this investigation and present the supporting results through graphs and tabular values. All figures and tables appearing in this work were generated directly from the exact solution / expressions given in § 2.3 using scientific calculator first, and later on by C – program. Both the obtained results and data have almost same in all cases. To simplify our discussion, we choose the decay factor n and Magnetic Hartmann number M in such a way that there does not come negative number under the square roots. Hence, based on certain and standard values of different parameters and numbers, we state the following.

Figure 2-1 has been obtained by plotting the temperature distribution T against y at different times when n = 1, Pr = .025. This figure shows that the temperature at any point increases with the increase of t. It is seen that the difference of distribution of temperature in between t =1 and t = 2 is large while in between t = 2 and t = 3 is small. It seems that there will be no increase of temperature distribution though time would be increased.

The temperature profiles have been drawn for t = .1, Pr = .025, and for different values of temperature decay factor n, in figure 2-2. It is found from this figure that the temperature at any point inside the vertical channel increases with the increase of n, and it flows with higher values at and near the walls than at the middle of the channel. This values of T uniformly decreases from the wall and minimum value occurs at y = 0.

Figure 2-3 has been drawn to show the effect of Prandtl number (Pr) on temperature distribution. This investigation shows that for any value of Pr, at the closed region of the walls, the values of the temperature distribution is the same; but at the middle of the channel it varies significantly. Towards the middle of the channel, the temperature distribution decreases as Prandtl number increases.

In figure 2-4, we have investigated the mass diffusion (C) against y inside the channel in the presence of temperature decay factor n (=1) and Schmidt number Sc (=. 22) with respect to small time (t = .1) and large time (t = 1, 2, 3). A clear difference of mass diffusion for small and large time ha been noticed. The difference is about to end after time t = 3, also diffusion rate is slow at the middle of the channel.

The figure 2-5, has been drawn to show the effect of temperature factor (n) on mass diffusion at time t = .5 and Schmidt number Sc = .22. The graph obtained for n = 5, shows that the diffusion difference is very high, highest near the walls. This means that at the center of the channel the diffusion rate is very slow. Most intriguingly, however, for n = 15, the diffusion processes is same at all regions of the channel. Perhaps the mass diffusing processes come to an end for values of n greater than 15.

The figure 2-6 has been drawn to show how various species diffuses at same time (t = .5, here), and temperature factor (n = 1, here). It has been observed that as the values of the Schmidt number, Sc, increases the diffusion difference increases. For Sc = 0, the rate of mass diffusion is similar in all regions of the channel.

We have considered the figure 2-7 to show the effect on velocity profiles caused at different times with respect to standard fixed values of the parameters considered. From the figure, it is observed that for smaller times the flow distribution differs greatly than the larger times inside the channel; even for t = 2 and t = 3, the differences in values of u are very negligible. Moreover, the difference appears only after 3 digits of decimal place, which as a result ca' not be seen any difference in the figure.

In figure 2-8, we have shown the effect of magnetic field parameter M on velocity profiles for standard fixed values of different parameters as shown in figure. It is seen that as M increases u decreases. However, this is interesting to note that the flow field neither depends solely on M but also on n. This can be seen in analytical result in § 2.3, under radical sign and on graphical vision in the figure.

Figure 2-9 has been drawn to show the effect of temperature factor (n) on velocity profiles at small time as well as at large time for fixed values of different parameters that appears under assumption. It is clearly observed that for t = .1 and n = 5, the flow distribution differs greatly than for t = 1 and n = 5. From this we confer that as time advances the flow distribution difference decreases. Again we see the effect of temperature decay factor n on velocity field. As the values of n increases, the corresponding values of u first increases, later on decreases; even at the middle of the channel. It is also the sign of stream -lines flow situation.

The velocity profiles have been plotted against y for n = 1, Pr = .025, Sc = .22, M = .5 and for various values of Grashof number (Gr_m) in figure 2-10. The profiles are studied at two different times. It is seen that as the values of Gr_m increases the values of velocity field also increases at time t = .5; but at time t = 2 and for $Gr_m = 4$, it seems to decrease. Thereby it means that the flow field seems to attain the fully developed situation after time t = 2. For small time the calculation shows that the increase of concentration also increase the difference of diffusion.

The view taken in figure 2-11 is one looking down upon the two different values of Prandtl number (Pr) at two different times. The plotted graphs clearly show that as Pr increases in turn the velocity field decreases. If we look down upon the time factor, it clearly suggest the idea that as time increases, the difference of flow distribution decreases irrespective of the kinds of electrically conducting fluid. It gives hints of matured mixture and fully developed situation at large time.

Figure 2-12 illustrates the temporal evaluation of the flow pattern caused by varying values of Prandtl number (Pr) and magnetic Hartman number (M), simultaneously at fixed values of other parameters and time. It is seen that as Pr and M increases simultaneously, the values of u decreases at the center of the channel. However, any change of Prandtl and Hartmann number do not affect on the velocity distribution near the two walls.

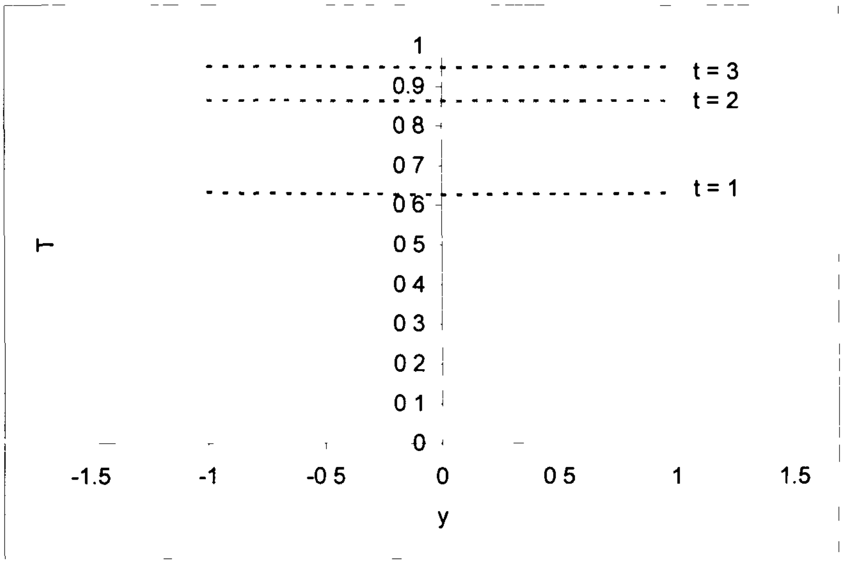

Figure 2-1: T versus y for Pr = .025, n = 1 at time t = 1, 2, 3

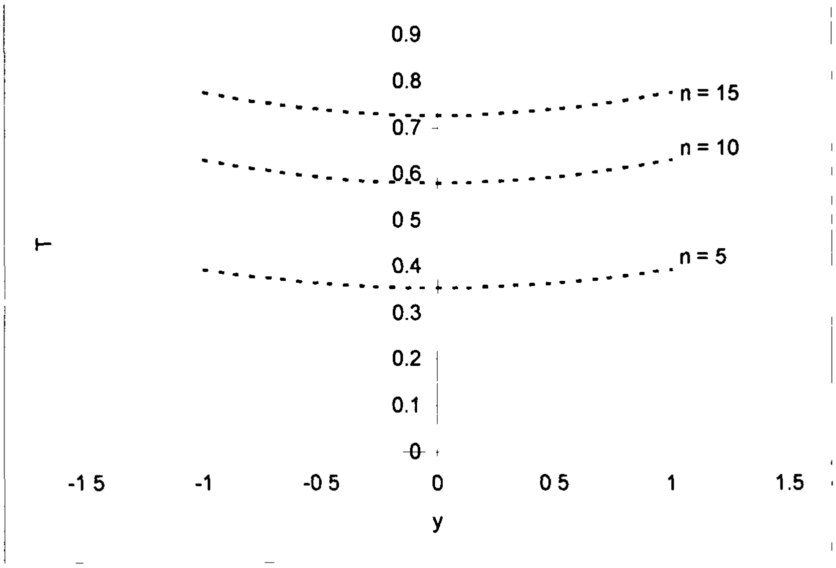

Figure 2-2: T versus y for Pr = .025, t = .1, at n = 5, 10, 15

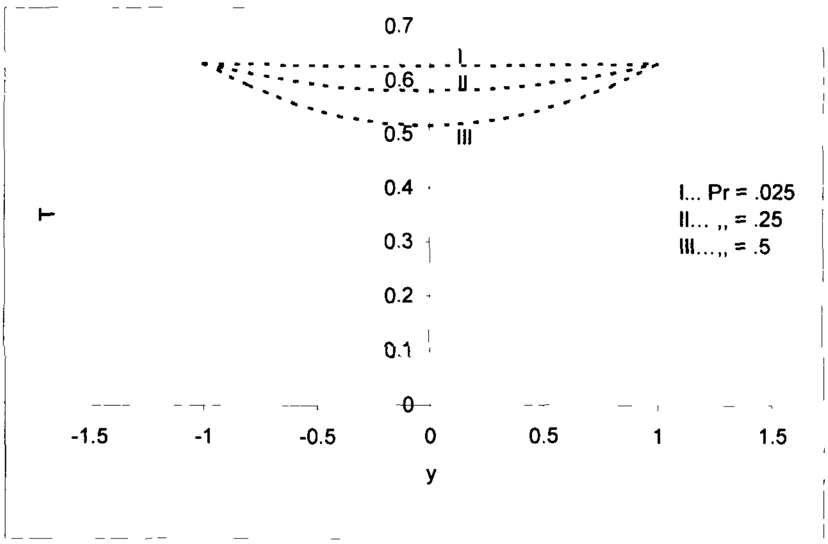

Figure2-3: T versus y for Pr = .025, .25, .5 at t = 1, n = 1

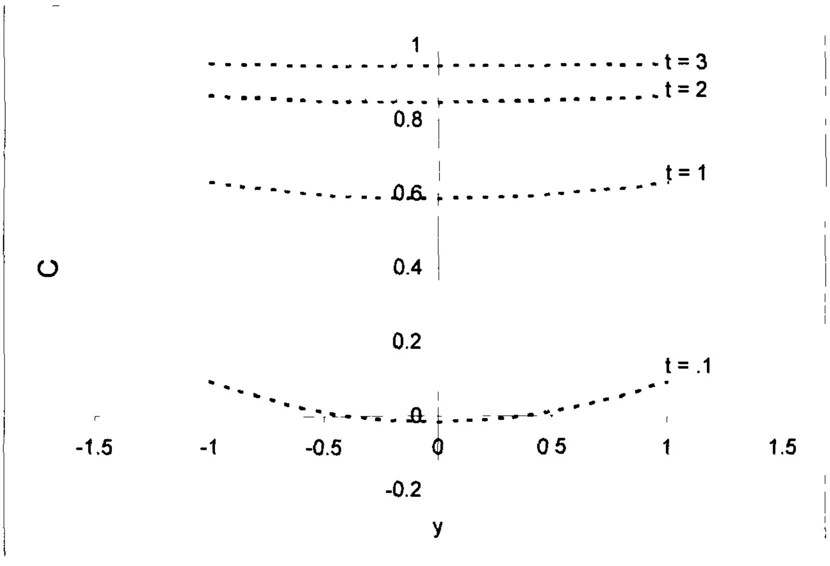

Figure 2-4: C versus y for n = 1, Sc = .22 at t = .1, 1, 2, 3

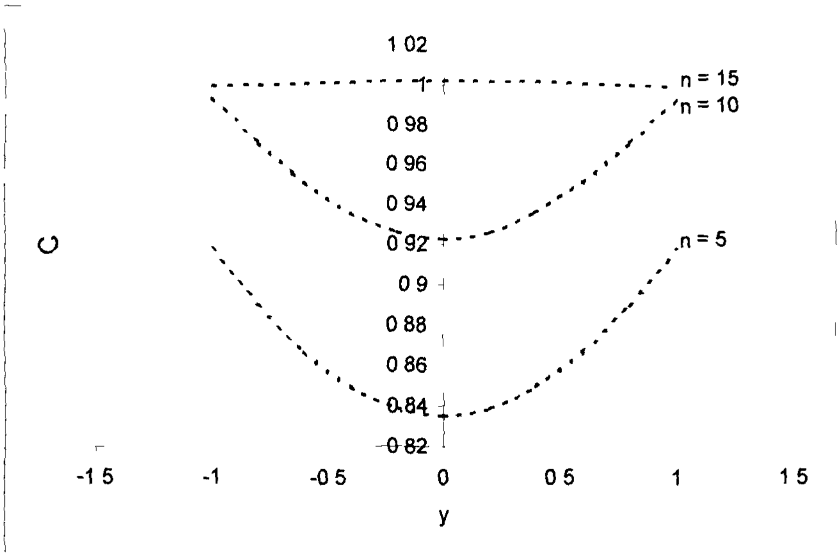

Figure 2-5: C versus y for t = .5, Sc = .22, n = 5, 10, 15

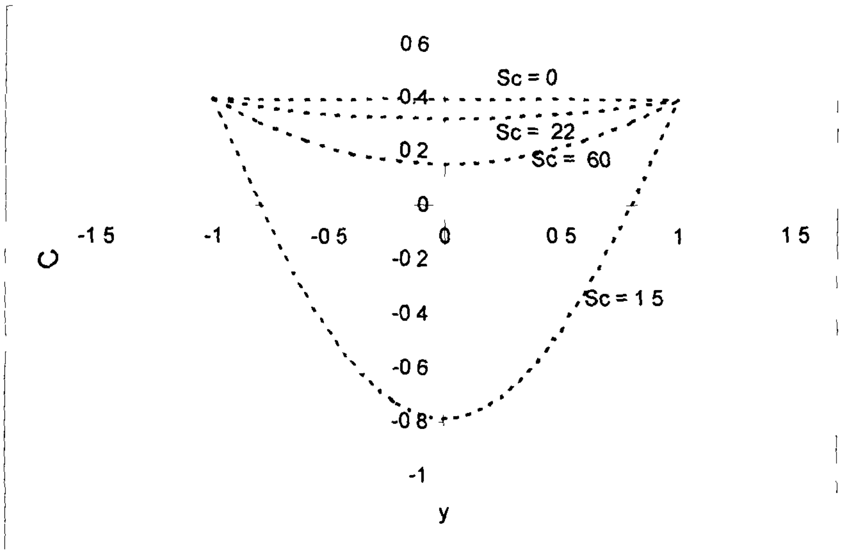

Figure2- 6: C versus y for t = .5, n = 1 at different values of Sc

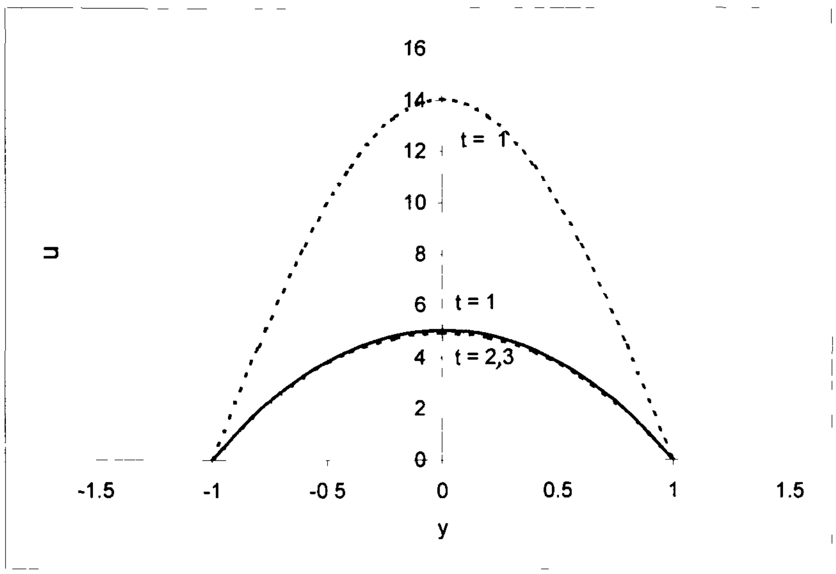

Figure 2-7: u versus y for n = 5, M = 1, Sc = .22, Pr = .025, $Gr_t = 10, Gr_m = 4$

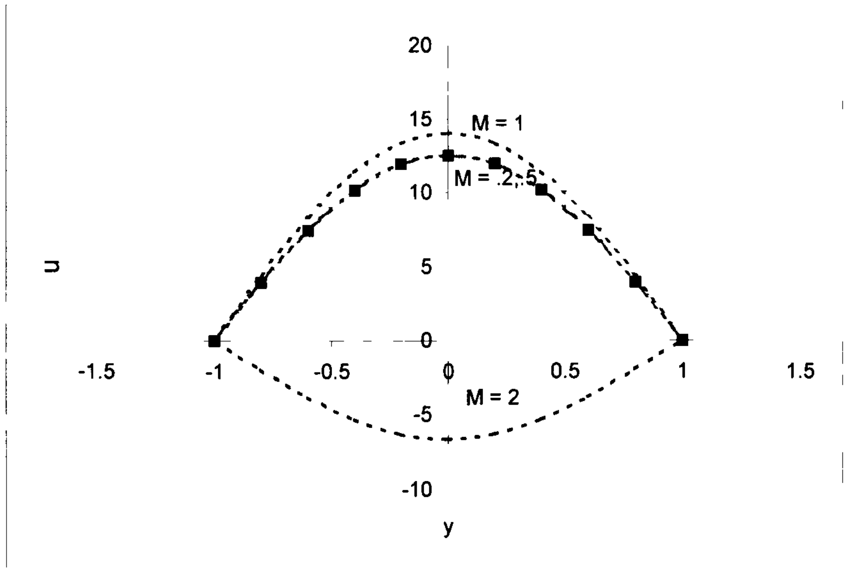

Figure2- 8: u versus for n = .5, t = .1, Pr = .025, Sc = .22, $Gr_t = 10, Gr_m = 4$

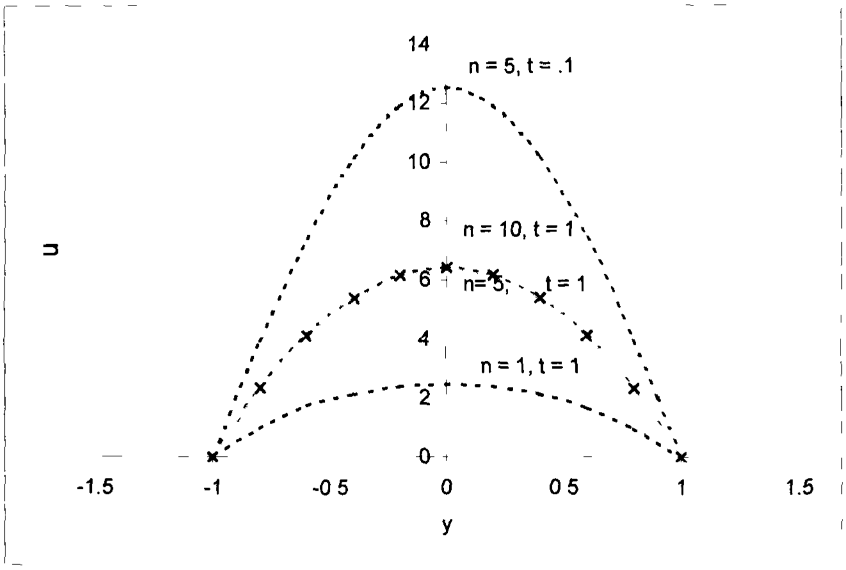

Figure 2-9: u versus y for Pr = .025, M = .5, Sc = .22, $Gr_t = 10, Gr_m = 4$

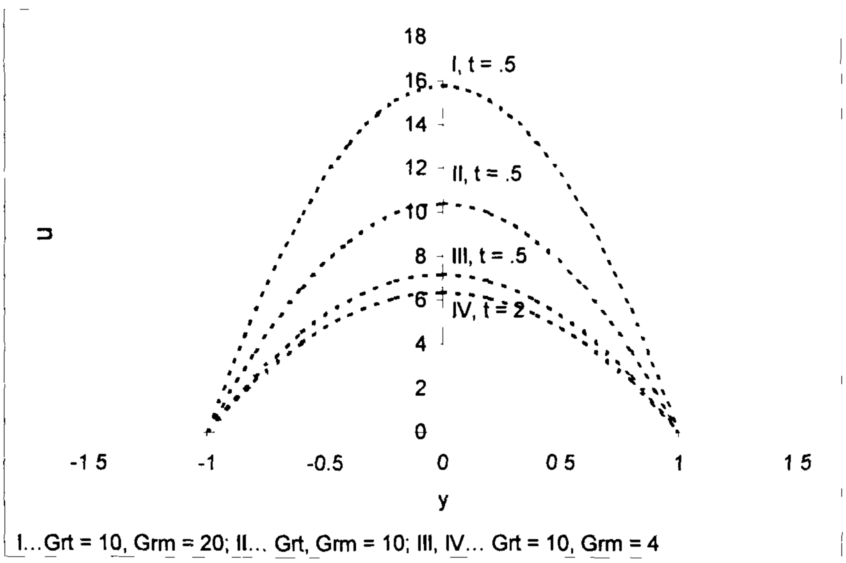

Figure2-10: u versus y for n = 1, Pr = .025, Sc = .22, M = .5

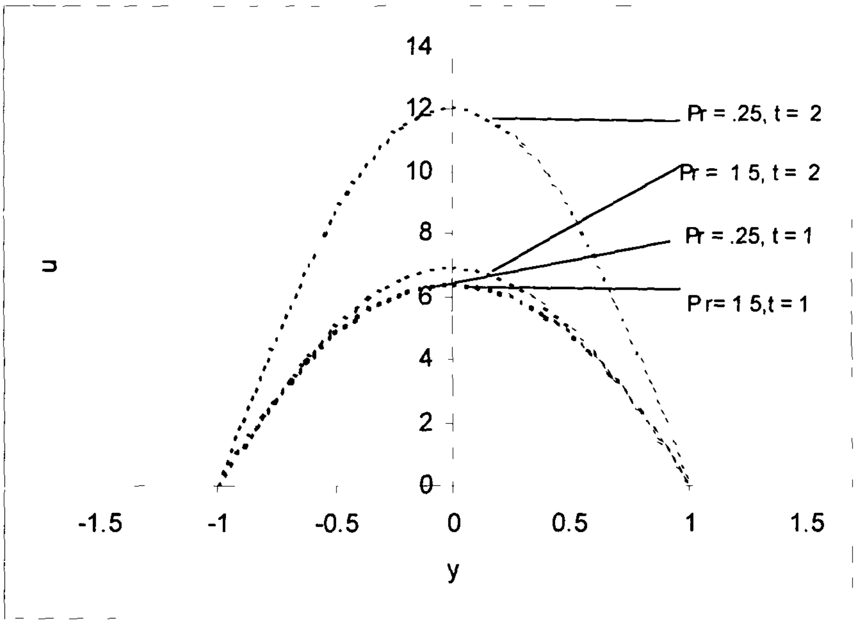

Figure2-11: u versus y for n =5, M = .5, Sc = .22 $Gr_t = 10, Gr_m = 4$

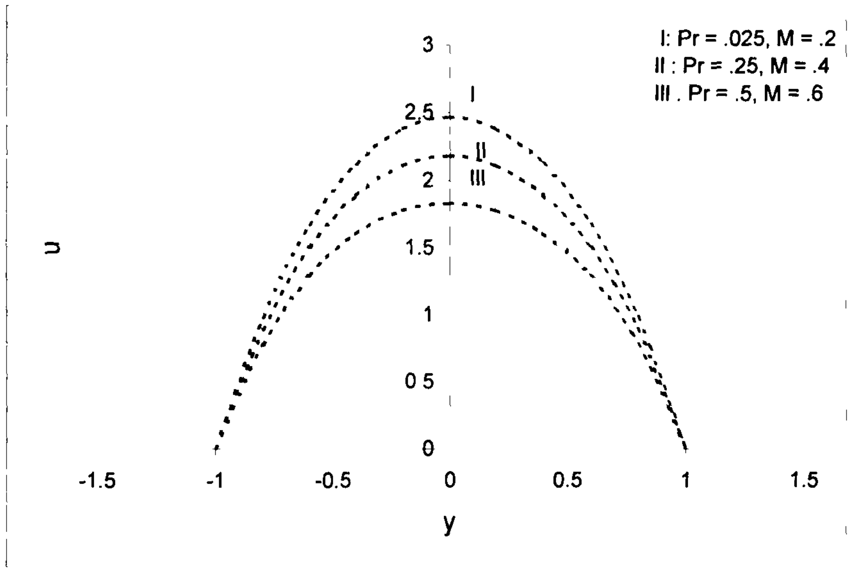

Figure 2-12: u versus y for n = 1, t = 1, Sc = .22, $Gr_t = 10, Gr_m = 4$

2.5 Skin Friction (C_f):

For engineering purposes, one is usually less interested in the shape of the velocity, temperature or concentration profiles than on the values of Skin friction, Heat transfer or Mass transfer parameter. All the values of these letter ones are conventionally described by appropriate coefficients. The following relations define them

$$C_f = \frac{2\tau_w}{\rho U_0^2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_0)}, \quad Nm_x = \frac{h_m x}{D_m(C_w - C_0)} \tag{2-21}$$

$$\text{where } \tau_w = \left[\mu\frac{\partial u'}{\partial y'}\right]_{y'=\pm h}, \quad q_w = -k\left[\frac{\partial T'}{\partial y'}\right]_{y=\pm h}, \quad h_m = -D_m\left[\frac{\partial C'}{\partial y'}\right]_{y=\pm h} \tag{2-22}$$

But, we have only the skin friction here, as Nu_x (the local Nusselt number) and Nm_x (the local Sherwood number) become zero for T and C being the function of t instead of being function of y.

Using (2-6), (2-20) and (2-22), we have (2-21) as

$$C_f = \frac{-2}{\mathrm{Re}^2}\left[\frac{Gr_t + Grm}{M}\tanh(M) + \frac{Gr_t e^{-nt}}{n(1-\mathrm{Pr}) - M^2}(\sqrt{n\mathrm{Pr}}\tan\sqrt{n\mathrm{Pr}} - \sqrt{n-M^2}\tan\sqrt{n-M^2})\right.$$

$$+ \frac{Gr_m e^{-nt}}{n(1-Sc) - M^2}(\sqrt{nSc}\tan\sqrt{nSc} - \sqrt{n-M^2}\tan\sqrt{n-M^2})$$

$$+ \sum_{k=0}^{\infty}\frac{8n(2k+1)^2\pi^2 e^{-\frac{1}{4}\{4M^2+(2k+1)^2\pi^2\}t}}{\{4M^2+(2k+1)^2\pi^2\}\{\frac{4M^2+(2k+1)^2\pi^2}{4} - n\}}\left\{\frac{Gr_t}{(1-\mathrm{Pr}) - \frac{4M^2}{4M^2+(2k+1)^2\pi^2}}\right.$$

$$\left. + \frac{Gr_m}{(1-Sc) - \frac{4M^2}{4M^2+(2k+1)^2\pi^2}}\right\} + 2\sum_{k=0}^{\infty}\frac{nGr_t e^{-\frac{(2k+1)^2\pi^2}{4\mathrm{Pr}}t}}{\left(\frac{(2k+1)^2\pi^2}{4\mathrm{Pr}} - n\right)\left(M^2 - (1-\mathrm{Pr})\frac{(2k+1)^2\pi^2}{4\mathrm{Pr}}\right)}$$

$$\left. + 2\sum_{k=0}^{\infty}\frac{nGr_m e^{-\frac{(2k+1)^2\pi^2}{4Sc}t}}{\left(\frac{(2k+1)^2\pi^2}{4Sc} - n\right)\left(M^2 - (1-Sc)\frac{(2k+1)^2\pi^2}{4Sc}\right)}\right], \quad \mathrm{Re} = \frac{hU_0}{\nu} \tag{2-23}$$

We have obtained the result of skin friction for the plate at h = + 1. All the results that would be found for h = -1 would have been seemed to be opposite to the results found in tables (1) – (5). Here we have observed the effects of the following dimensionless parameters and decay factor.

(a) The effect of Re: From the table 1, it is seen that the effects of the Reynolds number Re, the dimensionless parameter of the ratio of the inertial motion to the viscous resistance, is prominent in the skin friction. For Re = 1, skin friction is the highest, and then it decreases as Reynolds number increases. Other parameters are assumed fixed.

(b) The effect of n: Table 2 has been obtained for various values of n, the temperature decay factor, starting from 1 to 25. It is seen that n plays an important role in the increase or decrease of skin friction. It is difficult to predict the situation that for increasing values of n there would be any increase or decrease in skin friction. This situation particularly depends on magnetic parameter M.

(c) The effect on M: In the table 3 the effect of M on the skin friction has been shown. It is seen that as M increases from .025 to .5, skin friction decreases. However, for other greater values this kind of prediction cannot be done; even we will face domain error if we would consider the values, which are greater than square root values of n.

(d) The effects of Pr and Sc: We have deduced the values of skin friction for different values of Pr and Sc at two values of Re and fixed values of other parameters in table 4. It is obvious from the table that for Reynolds number = 1300 skin friction is negligible, while for Re = 1, it is significant. Moreover, for different pair values of Pr and Sc, we see different skin friction. For higher values of these two, C_f is negative. Alternately, for smaller and standard values, it is positive and remarkable.

(e) The effects of Gr_t and Gr_m: Lastly, in table 5, we have given the variation in Gr_t and Gr_m to show its effect on skin friction. For high Reynolds number, the effects of these parameters are not significant, but for small Reynolds numbers these parameters are countable.

Table 1. Values of $\underline{C_f}$ for different values of Reynolds number (Re) when

$n = 5$, t = .1, $Gr_t = 10, Gr_m = 20$, Pr = .025, Sc = .22, M = .5

Re	$\underline{C_f}$	Re	$\underline{C_f}$
1	758.79895	10	7.588047
100	0.075880	200	0.01897
500	0.003035	1000	0.000759
1200	0.000527	1300	0.000449
1500	0.000337	2000	0.00019
2200	0.000157	2500	0.000121

Table 2. Values of $\underline{C_f}$ for different values of decay factor (n) at

$Gr_t = 10$, $Gr_m = 20$, M = .5, Re = 10, Sc = .22, t = 1, Pr = .025

n	$\underline{C_f}$	n	$\underline{C_f}$
1	- 0.437598	2	- 1.237877
3	2.843106	4	0.498329
5	0.195368	7	0.008061
10	- 0.081594	15	- 0.133889
20	- 0.155934	25	- 0.168086

Table 3. Variation of skin friction $\underline{C_f}$ for different values of M at

$Gr_t = 10$, $Gr_m = 20$, Pr = .025, Sc = .22, t = 1, n = 5, Re = 1300

M	$\underline{C_f}$	M	$\underline{C_f}$
.025	0.357097	.1	0.349901
.25	0.311365	.5	0.195368
1.0	- 0.048251	1.5	0.069489
2.0	- 0.291442	2.2	- 0.262698

Table 4. Skin friction C_f for various values of Pr and Sc when

$Gr_t = 10,\ Gr_m = 20,\ M = .5,\ n = 5,\ t = 1, Re = 1, 1300$

Pr	Sc	C_f(Re = 1)	C_f(Re = 1300)
0.025	0.22	19.536825	0.000012
0.25	0.60	73.327301	0.000043
0.50	0.60	86.569122	0.000051
0.71	0.22	65.498520	0.000039
0.71	1.00	- 2722013.5	- 1.610659
1.00	1.00	- 4083093.5	- 3.756418
2.00	1.50	- 128.387253	- .000076

Table 5. Variation of skin friction C_f for different values of Gr_t and Gr_m at

Pr = .025, t = 1, M = .5, n = 5, Sc = .22

Gr_t	Gr_m	C_f (Re=10)	C_f (Re=1300)
2.0	2.0	0.022515	0.000001
2.0	4.0	0.039074	0.000002
10	4.0	0.062896	0.000004
20	10	0.142352	0.000008
10	20	0.195368	0.000012
10	100	0.857728	0.000051

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CHAPTER - 3

MAGNETIC FIELD EFFECTS ON UNSTEADY FREE CONVECTION MHD FLOW BETWEEN TWO HEATED VERTICAL PLATES (ONE ADIABATIC)

3.1 Introduction:

During the last few decades intensive research works, both theoretical and experimental, have been devoted to problems of free convection heat and mass transfer in view not only of their own interest but also of their application to astrophysics, geophysics, engineering and medical sciences.

Moreover free convection flows play an important role in different technological processes. The flow of a fluid is not only steady but it is also unsteady. Transient free convection occurs in a fluid when the temperature changes caused density variation which give rise to buoyancy forces.

Many papers were published on steady free convection flows past a semi-infinite vertical plate under different physical conditions during 1960 – 1970s. However, unsteady free convection flows received little attention. Illingworth published the first paper on unsteady free convection in 1950 for a uniform plate temperature, but the results were presented for unit Prandtl number. Siegel [1958], later on, studied unsteady free convection flow near a semi-infinite vertical plate under uniform wall temperature or constant heat flux conditions. The problem was solved by momentum – integral method and he showed for the first time that the initial behavior of temperature and velocity fields for a semi-infinite vertical plate is the same as for a doubly infinite vertical plate in which case temperature field is given by unsteady one-dimensional heat conduction equation. Hence, it was concluded that the transition to convection begins only when some effect from the leading edge has propagated up the plate to a particular point depending upon the physical circumstances. These findings were confirmed experimentally by Goldstein and Eckert [1960]. Later on many papers were published in 1960's on unsteady free convection flow past an infinite vertical plate. These are by Schetz and Eichhorn [1962], Goldstein and Briggs [1964]. The unsteady free convection flow past a semi-infinite vertical plate was studied by Chung and Anderson [1961], Sparrow and Gregg [1960], Hellums and Churchill [1961,1962], where boundary layer concept was utilized. The fluid considered in all these studies was a Newtonian fluid like air or water. Gebhart and Pera [1971] studied the transient free convection problem and they found excellent results on it. The unsteady free convection laminar flow past an infinite plate in the presence of a uniform magnetic field has been studied by Brar. Sreekant *et al.* [2001] investigated the unsteady free convection flow of an incompressible dissipative viscous fluid past an infinite plate under the influence of a uniform transverse magnetic field.

On the other hand, many authors have presented the unsteady flow between two plates, horizontal or vertical. Sharma and Kumar [1998] investigated the unsteady flow and heat transfer which arises in fluids due to buoyancy forces and temperature differences in the presence of transverse magnetic field between two horizontal plates, lower plate being a stretching sheet and upper being porous. Borkakati and Bhattacharjee [1984] studied the heat transfer in the flow of a conducting fluid between two non-conducting porous disk-one rotating and other at rest in the presence of a uniform magnetic field, the lower disk being adiabatic. Gourla and Katoch [1991] studied an unsteady free convection MHD flow between two heated vertical plates in the presence of the force field of gravity and applied magnetic field acting in the horizontal direction and perpendicular to the flow.

The general MHD flow problems are studied considering the imposed magnetic field in a direction perpendicular to the direction of the flow. But here we are interested to investigate the nature of a problem of electrically conducting fluid past between two vertical plates where one of the plates is adiabatic and other plate is of variable temperature, in the presence of a magnetic field placed at different angles θ(where θvaries from 0 to $\pi/2$) to the motion of the fluid. The analytical solution for velocity and temperature distributions for different time t, decay factor n, Magnetic number M and Prandtl number Pr have obtained and shown through graphs and discussed in detailed.

3.2 Formulation of the Problem:

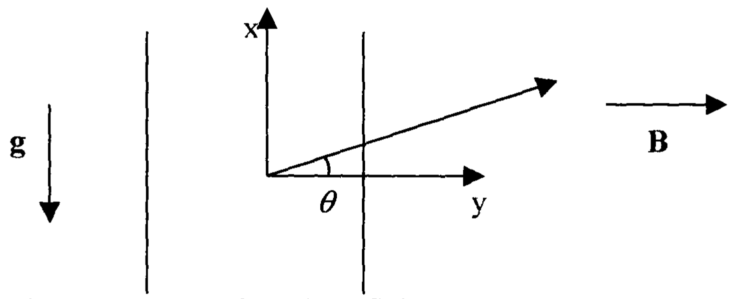

Fig-3.1: Geometry of the flow field

The physical configuration illustrating the problem under consideration is shown in fig. 3.1. The vertical plates are fixed and the fluid flows between the two plates pointing to vertical upward direction. The x-axis, taken as the axis of the channel, is pointing to vertical upward direction through the middle of the two plates. The y-axis is along the horizontal direction. The plates of the channel are kept at y = $\pm h$. $\vec{g}$ acts in the vertical downward directions while B_o acts at an angle $\theta (0 \le \theta \le \pi/2)$, to the y-axis. The velocity components u' and v' are respectively, along x-axis and y-axis. Consequently u' is a function of y' and t' only, but v' is independent of y'. In formulating the problem mathematically, we assume that the fluid properties are not affected by the temperature differences except that of the density of the body force term.

The basic equations of magnetohydrodynamics and ordinary fluid dynamics are different by only additional body force term due to electromagnetic field in the momentum equation and a term due to Joule heating in the energy equations.

In order to derive the fundamental equations we assume that

(i) the fluid is Newtonian, viscous, incompressible.

(ii) the Hall effect, electrical effect and polarization effect, are neglected

(iii) the variation of expansion coefficient β and thermal conductivity K with respect to temperature difference are considered negligible,

(iv) the boundary layer is assumed to be thin relative to the distance between the two plates,

(v) the pressure gradient across the boundary layer is neglected,

(vi) the induced magnetic field is assumed to be very small,

(vii) the viscous dissipation is neglected.

Under the above assumptions the unsteady free convection flow of an incompressible viscous fluid through a vertical channel is formulated by the following equations (3-1) - (3-3) with the boundary conditions (3-4).

$$\frac{\partial v'}{\partial y'} = 0 \qquad \text{(continuity equation)} \qquad (3\text{-}1)$$

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_0) - \frac{\sigma B_0^2 \cos^2\theta}{\rho} u' \qquad \text{(momentum equation)} \qquad (3\text{-}2)$$

$$\frac{\partial T'}{\partial t'} = \frac{K}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} \qquad \text{(energy equation)} \qquad (3\text{-}3)$$

$$\begin{aligned} &\text{when} \quad t' = 0; \quad u' = 0, \quad T' = T_0 && \forall y' \in [-h, h] \\ &\text{when} \quad t' > 0; \quad u' = 0, \quad T' = T_0 + (T_w - T_0)(1 - e^{-n't'}) && \text{for } y = -h \\ &\qquad\qquad u' = 0, \quad \frac{\partial T'}{\partial y'} = 0 && \text{for } y = +h \end{aligned} \qquad (3\text{-}4)$$

At time t > 0, the temperature of the plate at y = - h changes accordingly as $T = T_0 + (T_w - T_0)(1 - e^{-n't'})$, where, T_w and T_0 are the temperature at the plates y = - h and y = + h respectively, and $n'(> 0)$, a real number, denotes the decay factor.

On introduction of the dimensionless variables

$$u = \frac{\upsilon u'}{\beta g h^2 (T_w - T_0)}, \quad y = \frac{y'}{h}, \quad T = \frac{T' - T_0}{T_w - T_0}, \quad t = \frac{\nu t'}{h^2}, \quad n = \frac{h^2 n'}{\nu}$$

$$P_r = \frac{\mu C_p}{K}, \qquad M = \frac{\sigma B_0^2 \cos^2\theta h^2}{\upsilon \rho}, \qquad B_0 = |\overline{B}|$$

the system of equations (3-1) - (3-3) and boundary conditions (3-4) become

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - M^2 u + T \qquad (3\text{-}5)$$

$$\frac{\partial T}{\partial t} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2} \qquad (3\text{-}6)$$

$$\begin{aligned} &\text{when } t = 0; \; u = 0, \quad T = 0 && \forall y \in [-1, +1] \\ &\quad t > 0; \; u = 0, \quad T = 1 - e^{-nt} && \text{for } y = -1, \\ &\qquad\quad u = 0, \quad \frac{\partial T}{\partial y} = 0 && \text{for } y = +1. \end{aligned} \qquad (3\text{-}7)$$

3.3 Solution of the Problem:

Taking the Laplace Transform of equations (3-5) and (3-6), we get

$$\frac{d^2\bar{u}}{dy^2}-(M+s)\bar{u}=-\bar{T} \tag{3-8}$$

$$\frac{d^2\bar{T}}{dy^2}-P_r s\bar{T}=0 \tag{3-9}$$

where $\bar{F}(y,s)=\int_0^\infty e^{-st}F(y,s)dt$

Similarly, using Laplace Transform on the boundary conditions (3-7), we obtain -

$$\bar{u}(\pm 1,s)=0,\quad \bar{T}(-1,s)=\frac{n}{s(s+n)}\quad \& \quad \frac{d\bar{T}}{dy}(1,s)=0 \tag{3-10}$$

Since the equations (3-8) and (3-9) are 2[nd] order differential equations in $\bar{u}$ and $\bar{T}$, the solution of the equations by use of the conditions (3-10) are given by,

$$\bar{T}=\frac{n\cosh\sqrt{P_r s}(y-1)}{s(s+n)\cosh 2\sqrt{P_r s}} \tag{3-11}$$

$$\bar{u}=\frac{-n\sinh\sqrt{M+s}(y+1)}{s(s+n)\{M+s(1-P_r)\}\sinh 2\sqrt{M+s}}+\frac{n\sinh\sqrt{M+s}(y-1)}{s(s+n)\{M+s(1-P_r)\}\sinh 2\sqrt{M+s}}$$

$$+\frac{n\cosh\sqrt{P_r s}(y-1)}{s(s+n)\{M+s(1-P_r)\}\cosh 2\sqrt{P_r s}} \tag{3-12}$$

Taking Inverse Laplace Transform on both sides of (3-11) and (3-12), we get,

$$T=1-\frac{\cos\sqrt{nP_r}(y-1)e^{-nt}}{\cos 2\sqrt{nP_r}}+\frac{4}{\pi}\sum_{k+0}^{\infty}\frac{(-1)^k\cos\frac{(2k+1)\pi(y-1)}{4}e^{-\frac{(2k+1)^2\pi^2}{16P_r}t}}{(2k+1)\left\{\frac{(2k+1)^2\pi^2}{16P_r n}-1\right\}} \tag{3-13}$$

$$u=\frac{1}{M}\left[1-\frac{\cosh\sqrt{M}y}{\cosh\sqrt{M}}\right]-\frac{e^{-nt}}{\{n(1-P_r)-M\}}\times$$

$$\left[\frac{\sin\sqrt{n-M}(y+1)}{\sin 2\sqrt{n-M}\cos 2\sqrt{nP_r}}-\frac{\sin\sqrt{n-M}(y-1)}{\sin 2\sqrt{n-M}}-\frac{\cos\sqrt{nP_r}(y-1)}{\cos 2\sqrt{nP_r}}\right]$$

$$+\frac{4}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k e^{-\frac{(2k+1)^2\pi^2}{16P_r}t}}{(2k+1)\left\{1-\frac{(2k+1)^2\pi^2}{16P_r n}\right\}\left\{M-(1-P_r)\frac{(2k+1)^2\pi^2}{16P_r}\right\}}$$

$$\times\left[\frac{\sin\sqrt{\frac{(2k+1)^2\pi^2}{16P_r}-M}(y+1)}{\sin 2\sqrt{\frac{(2k+1)^2\pi^2-16P_rM}{16P_r}}}-\cos\frac{(2k+1)\pi(y-1)}{4}\right]$$

$$+\frac{\pi}{2}\sum_{k=0}^{\infty}\frac{(-1)^k k e^{-\left(M+\frac{k^2\pi^2}{4}\right)t}}{\left(M+\frac{k^2\pi^2}{4}\right)\left(1-\frac{4M+k^2\pi}{4n}\right)\left(M-(1-P_r)\frac{4M+k^2\pi^2}{4}\right)}$$

$$\times\left[\sin\frac{k\pi}{2}(y-1)-\frac{\sin\frac{k\pi}{2}(y+1)}{\cos\sqrt{P_r\left(4M+k^2\pi^2\right)}}\right] \qquad (3\text{-}14)$$

3.4 Results and Discussion:

The solutions obtained in (3-13) and (3-14) describing the present problem are discussed graphically considering $\frac{\sigma B_0^2 h^2}{\upsilon\rho}$=2(fixed), a part of Magnetic Hartmann number $M=\frac{\sigma B_0^2\cos^2\theta h^2}{\upsilon\rho}$ and supposing $\theta=0,\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{3},\frac{\pi}{2}$. When $\theta=0$, the magnetic field is perpendicular to the direction of the flow of the fluid and when $\theta=\frac{\pi}{2}$ the magnetic field is parallel to the flow field, and in this case magnetic field has no effect on temperature field. The variation of temperature field in respect of changing time t, the Prandtl number P_r and decay factor n (> 0) has been shown through the figures (3-1) - (3-3). As $\theta=\frac{\pi}{2}$ i.e. M = 0, makes the values of the velocity profile infinite, it is not shown in the figure. For other standard values of θ, the velocity distribution profiles are shown for different values of t, P_r, and n. Here, we have assumed the adiabatic plate at $y'=+h(i.e. y=1)$ and temperature function plate at. $y'=-h(i.e. y=-1)$. A quantitative discussion has been given below in a nutshell for each of the values of θ and through every figure.

Figure (3-1) has been obtained by plotting the temperature distribution T against y for t = .1, .5, 1, 4 at n = 1, P_r = 0.025 (fixed). It is observed that as t increases the values of T increases. For small time the temperature function plate receives higher temperature than the adiabatic plate, whereas for large time each plate acquires same temperature. Though not shown in the picture, the calculated value obtained for T, are almost similar for t = 3, 4, 5.

Figure (3-2) of temperature field is obtained for various values of decay factor n (n =1, 5, 10, 15) when t = 0.1 and P_r = 0.025. It is seen that the values of temperature distribution decreases towards the adiabatic plate. An increase in the values of n leads to the increase in the values of T to a certain limit. After that it ceases to increases. In this case the effect of adiabatic plate is significant.

Figure (3-3) has been obtained by plotting the temperature distribution T against y at the Prandtl number P_r=1, 0.5, 0.25, 0.125 and at n = 5 and t = 0.1. This figure shows the significant effect of the Prandtl and adiabatic plate. For the fluid whose Prandtl number is .025, the temperature distribution is less affected, though it has a tendency towards the adiabatic plate. For other fluids that have been considered in this work, the effect of temperature distribution inside the channel increases as P_r increases towards the temperature function plate. But towards the adiabatic plate all values tends to 0.

The results for the velocity u for $\theta = 0^0$ (M = 2), are shown in figures (3-4) to (3-6). Figure (3-4) has been obtained for different time t = 0.1, 0.5, 1, 4 at P_r = 0.025 and n = 5. It is seen that for small time u increases when t increases with indication of no rise of temperature towards the adiabatic plate. But for large time this increasing rate becomes negligible, and also no difference of rise of temperature between the two plates. The flow distribution is highest at the middle of the channel at large time. Figure (3-5) has been obtained for different values of n, keeping t and P_r fixed at 1 and 0.025 respectively. Here we find that though there cannot be seen any effect of the adiabatic plate the effect of increase of n can be observed. But, it cannot go further. For values of n greater than 15, the values of u are almost equal leading to fully developed situation. Considering n = 5, t = 1 and varying P_r at 0.025, 0.125, 0.25, we obtained the graphs for velocity profiles as shown in the figure (3-6). It is seen that as P_r increases, the values of u decreases. The flow distribution for any fluid at and near the plate is almost same, but different at the middle of the channel only. We failed to get the flow distribution for the fluid whose P_r = .71, 1 due to the presence of the square root domain error.

The results for the flow field u for $\theta = 30^0$ are shown through the figures (3-7) - (3-9). Here, it is found that the velocity profiles are similar to the figure (3-4) for t = 0.1, 0.5, 1, 4, and P_r = 0.025 and n = 5. However, in each case the values of u are greater than got in figure

(3-4) for $\theta = 0^0$. This shows the effect of M. Supposing n = 1, 10, 15, 25, and $P_r = 0.025$, t = 1, we obtained the figure (3-8). It is seen that the velocity profiles found in the figures (3-5) and (3-8) have their similarity in their origin. Exception is that these values are greater than the values got in (3-5). Figure (3-9) has been obtained considering t = 1, n = 5, and P_r = .025, .125, .25. This figure is similar to the figure (3-6). But these values are higher than the values got in figure (3-6). Moreover, the effect of adiabatic plate can be observed for high Prandtl number. The investigation cannot be carried out for $P_r = .71$ and 1 due to the domain error appeared in square roots terms. Also clear effect of Prandtl number is depicted rather than the effect of isomorphic plate.

Figures (3-10) - (312) have been obtained by placing the magnetic field at $\theta = 45^0$. In all these three figures an excellent effect of magnetic number has been seen. The velocity distribution found in each figure is parabolic at positive quadrant of the rectangular axes, for different values of t, P_r and n though the variation of t, n and P_r at different stages can - not be neglected. However, in this case each obtained values are more higher than the values got earlier for M = 2, 1.5.

Considering the magnetic field at an angle $\theta = 60^0$ to the horizontal direction, we have found the velocity distribution as shown in figures (3-13) - (3-15). Figure (3-13) has been obtained by assuming P_r = 0.025, n = 5 and t = 0.1, 0.5, 1, 3. In the figure it is found that the velocity profiles are parabolic in the positive quadrant of rectangular axes. The values are more higher than the values got in figures (3-4) and (3-10). For small times the flow distribution near the two plates are different. The figure (3-14) is for n = 1, 5, 10, 20, and for fixed values of P_r = 0.025 and t = 1. Here, it is seen that the fluid velocity is maximum at y = 0, and as n increases u also increases up to a certain limit. The flow distribution differs slightly for large n. The figure (3-15) has been obtained varying only P_r, and assuming t = 1 & n = 1. It is significant to note that for $P_r = .71, \&\ 1$, the velocity fields can be drawn comfortably, which we failed to have for $\theta = 0^0, 30^0, 45^0$. The effect of adiabatic plate is clearly visible in this case. The values of velocity distribution are higher than in each case than got for $\theta = 0^0, 30^0, 45^0$.

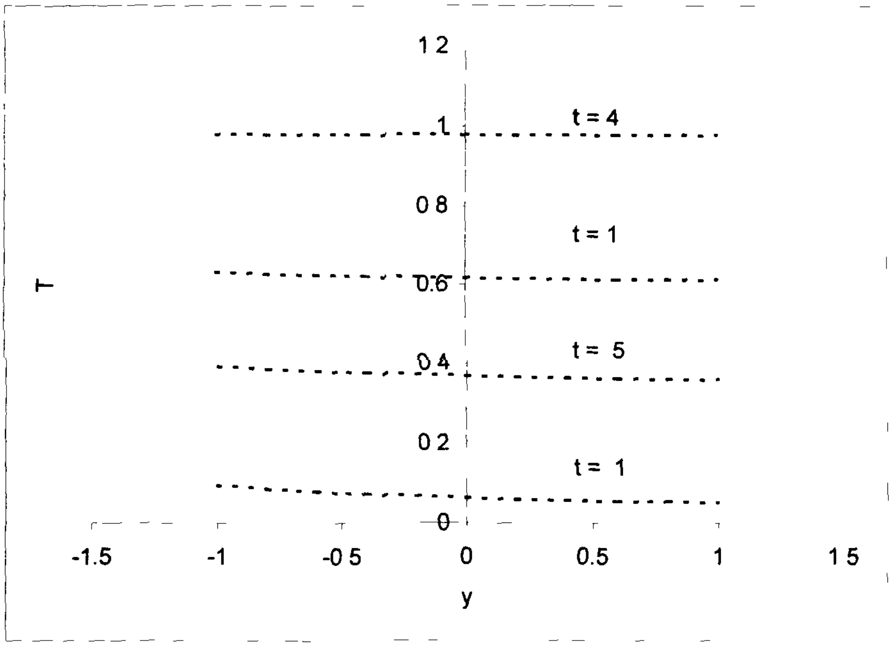

Figure 3-1: T versus y for n = 1, Pr = .025 at different t

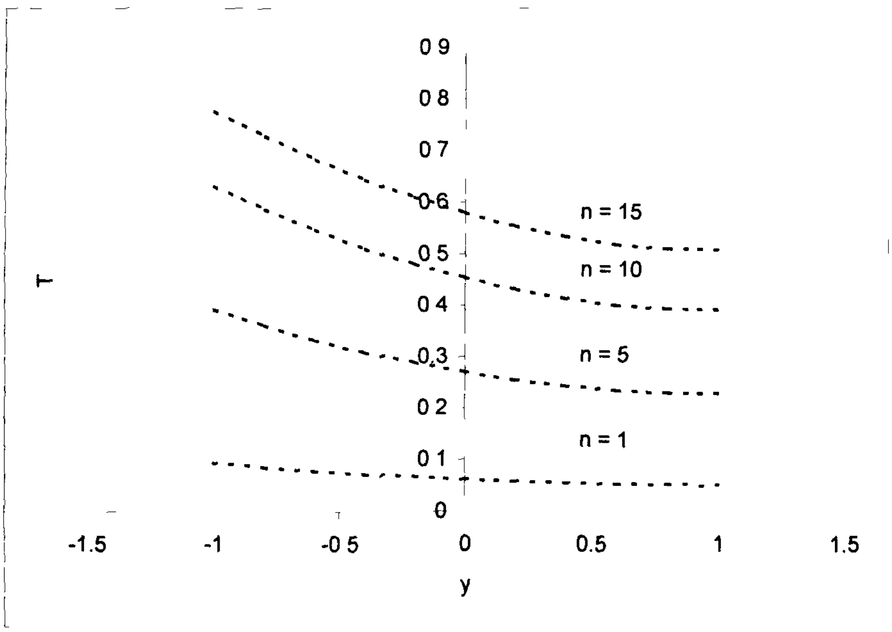

Figure 3-2: T versus y for Pr = .025. t = .1 at different n

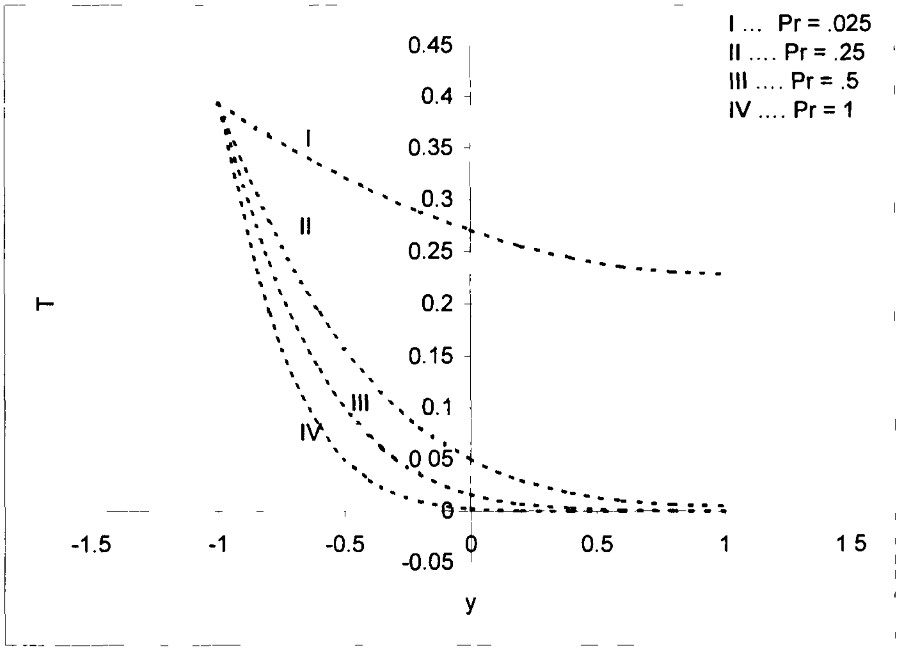

Figure 3-3: T versus y for n = 5, t = .1 at different Pr

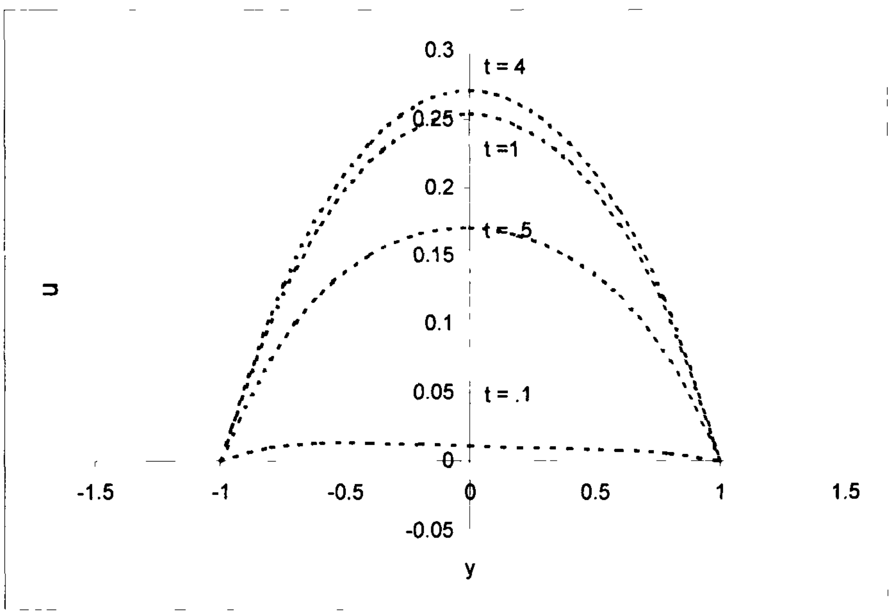

Figure 3-4: u versus y for M = 2 ($\theta = 0^0$), n = 5, Pr = .025; at t = .1, .5,1,4

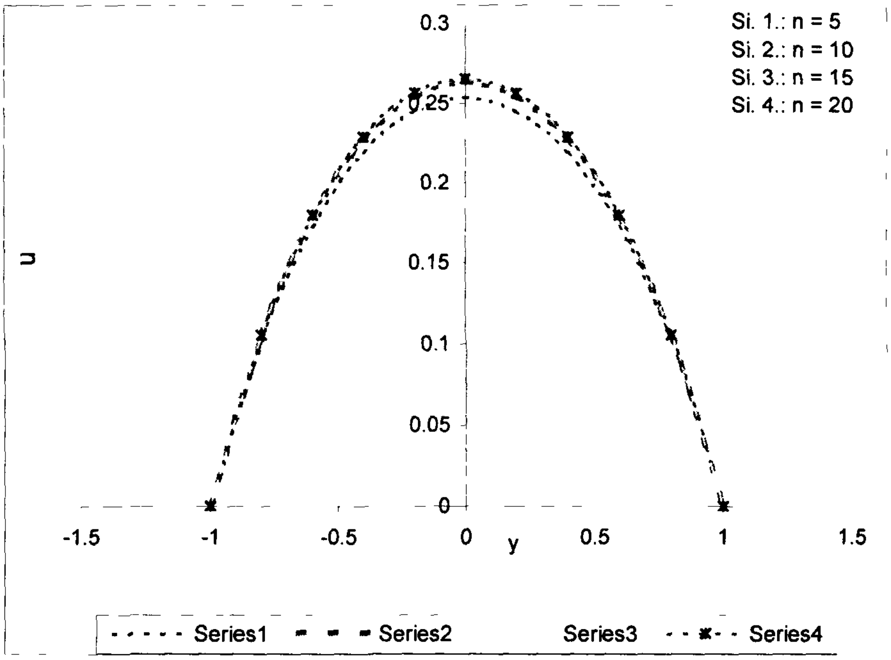

Figure 3-5: u versus y for M = 2 ($\theta = 0^0$), Pr = .025, t = 1, n = 5, 10, 15, 20

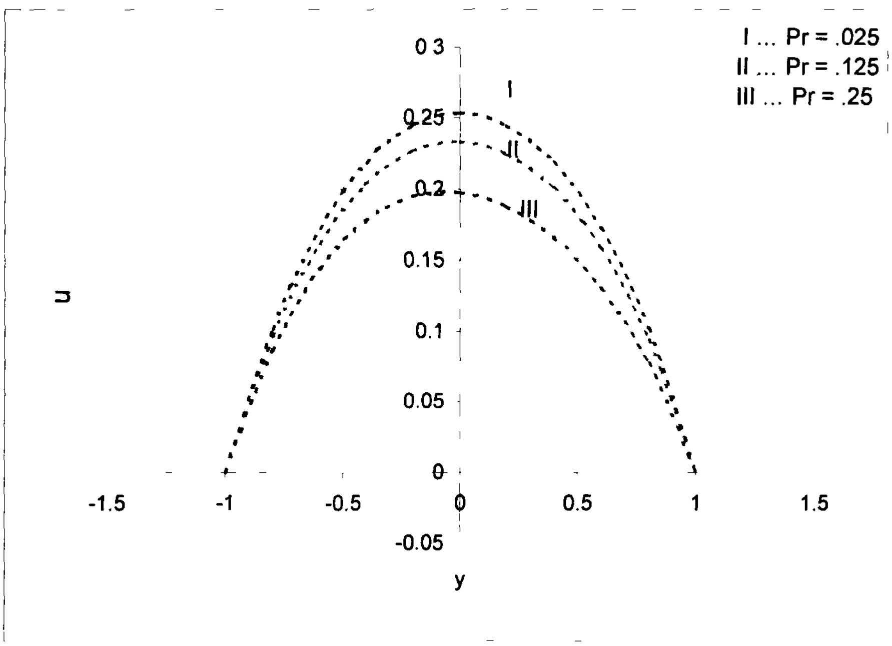

Figure 3-6: u versus y for M = 2 ($\theta = 0^0$), n = 5, t = 1at Pr = .025, .125, .25

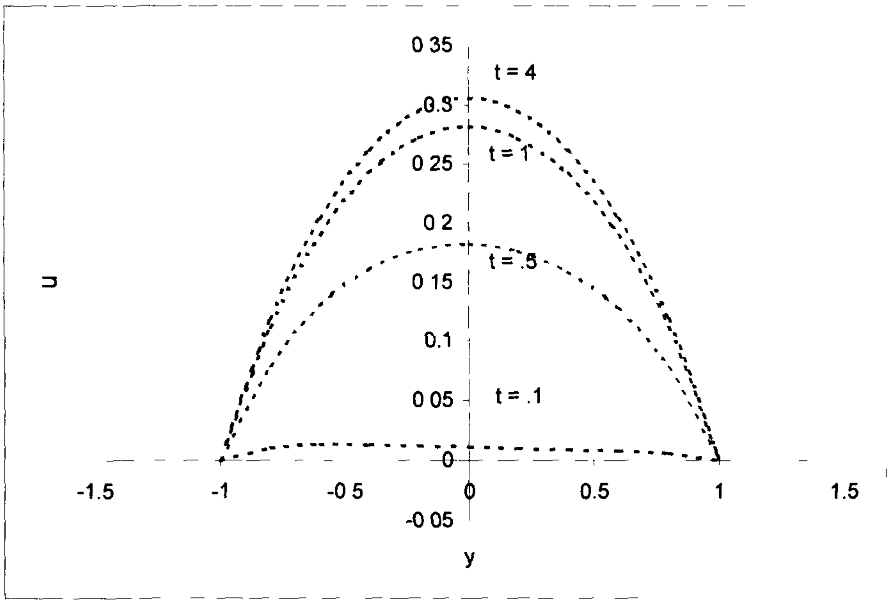

Figure 3-7: u versus y for M = 1.5 ($\theta = 30^0$), Pr = .025, n = 5 at different time

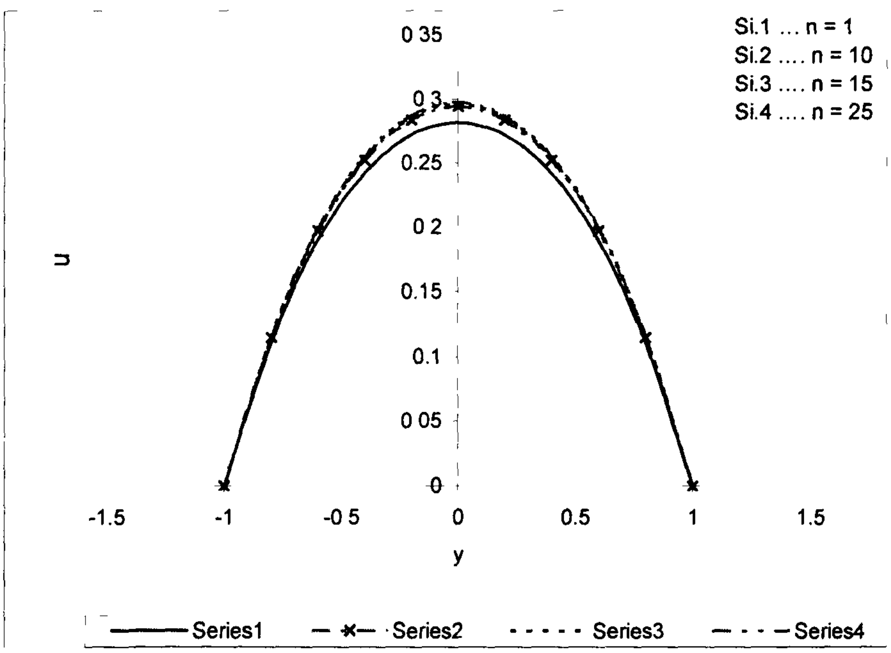

Figure 3-8: u versus y for M = 1.5 ($\theta = 30^0$), Pr = .025. t = 1, n = 5,10, 15, 20

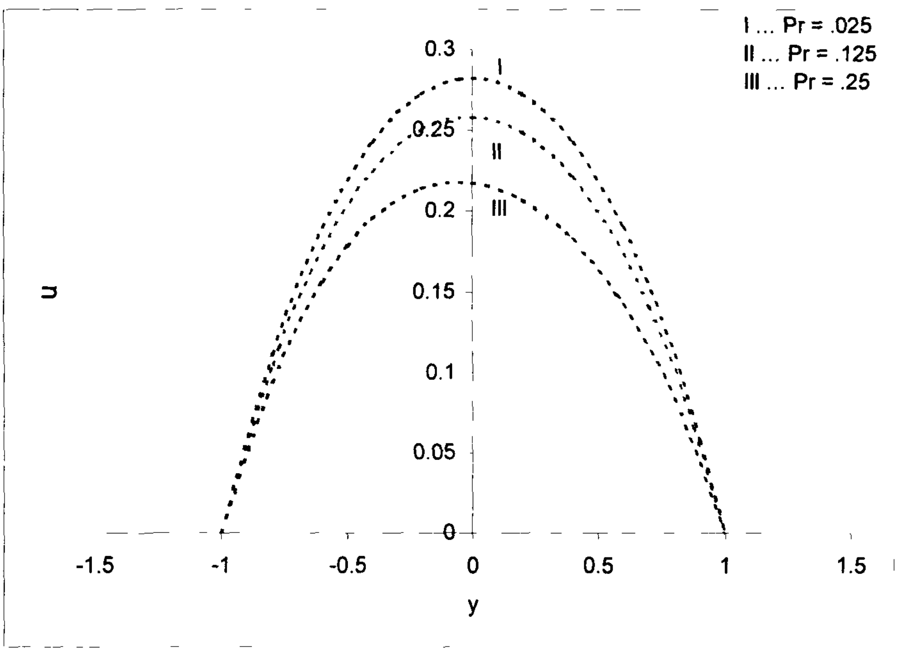

Figure 3-9: u versus y for M = 1.5 ($\theta = 30^0$), t = 1, n = 5, Pr = .025, .125, .25

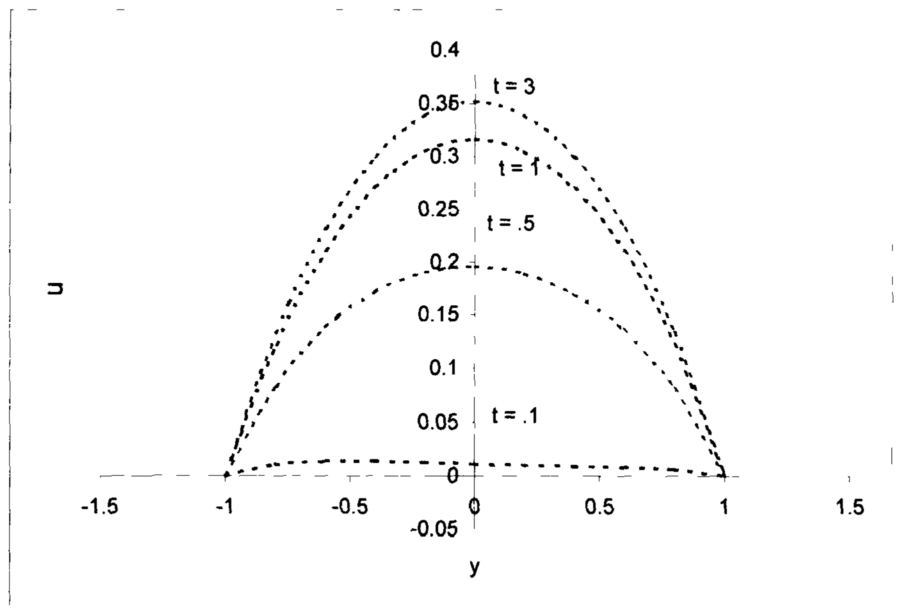

Figure 3-10: u versus y for M = 1 ($\theta = 45^0$), Pr = .025, n = 5, t = .1, .5, 1, 3

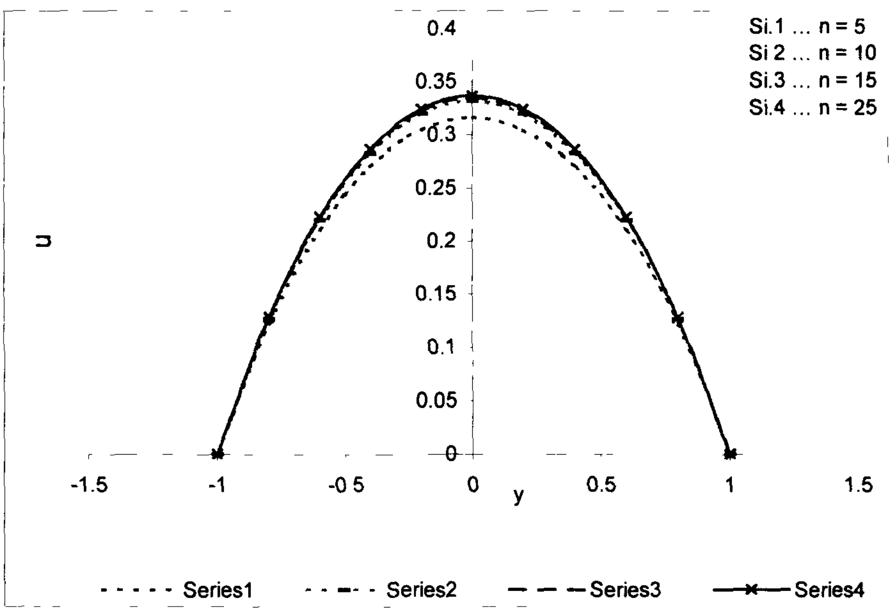

Figure 3-11.:u versus y for M = 1 ($\theta = 45^0$), Pr = .025. t = 1, n =5, 10, 15, 25

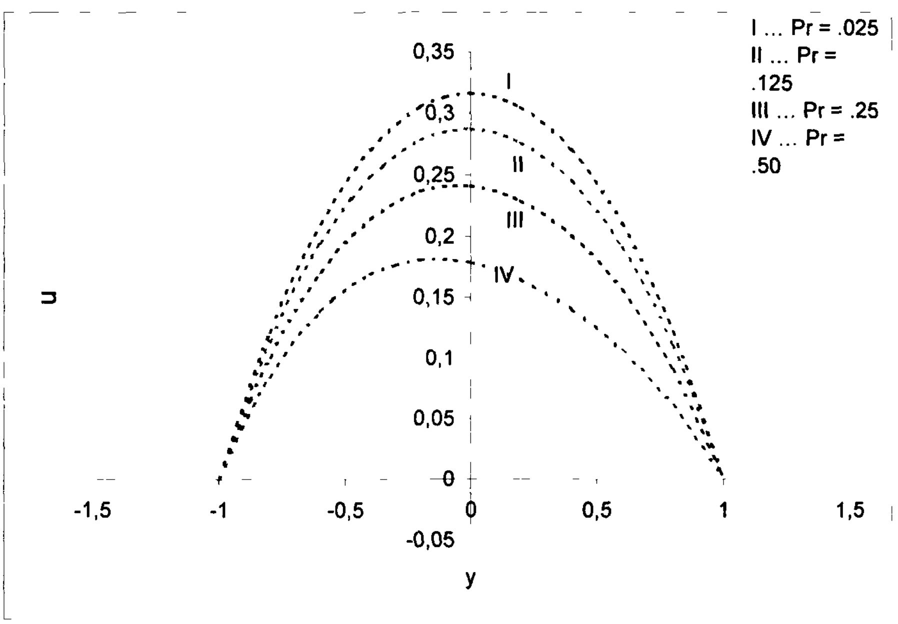

Figure 3-12: u versus y for M = 1 ($\theta = 45^0$), t = 1, n = 5, Pr = .025, .125, .25, .50

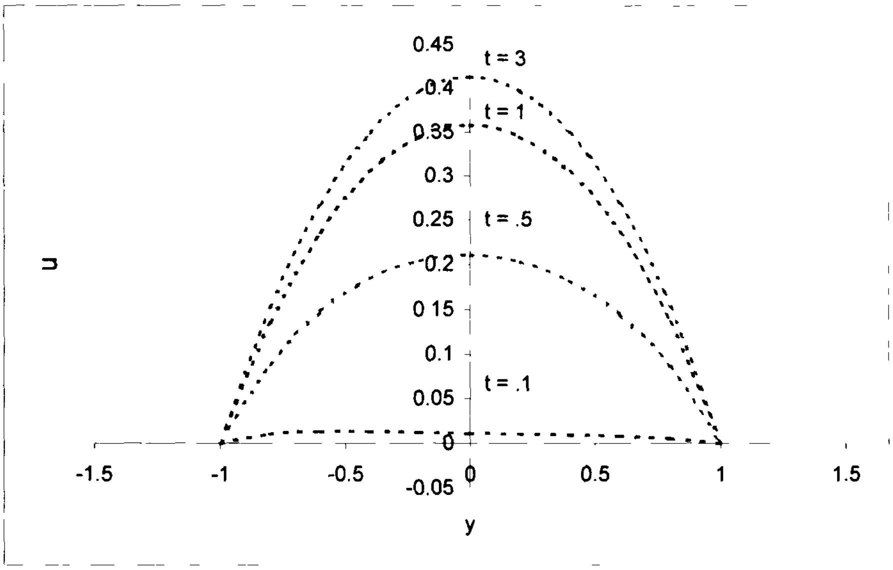

Figure 3-13: u versus y for M = .5 ($\theta = 60^0$), Pr = .025, n = 5, t = .1, .5, 1, 3

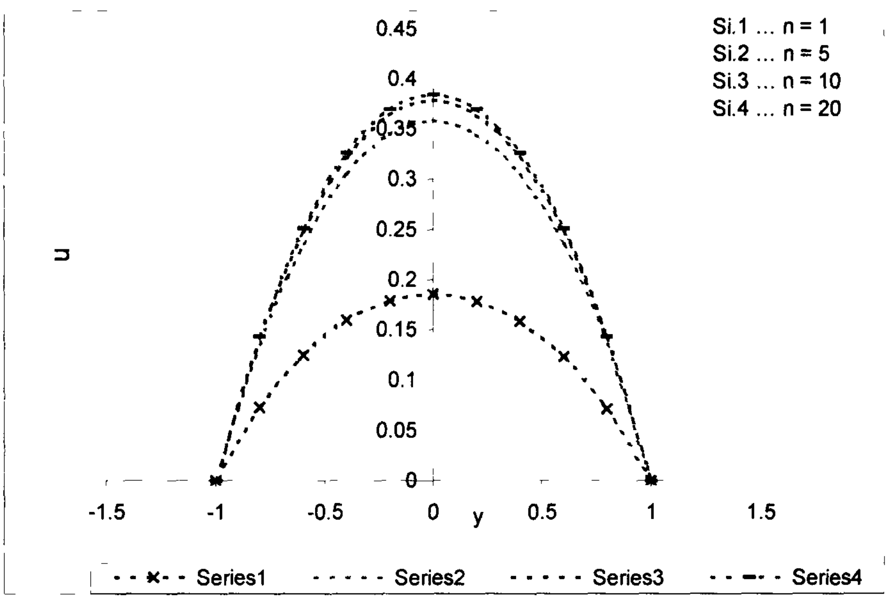

Figure 3-14: u versus y for M = .5 ($\theta = 60^0$), n = 1, 5, 10, 20, t = 1, Pr = .025

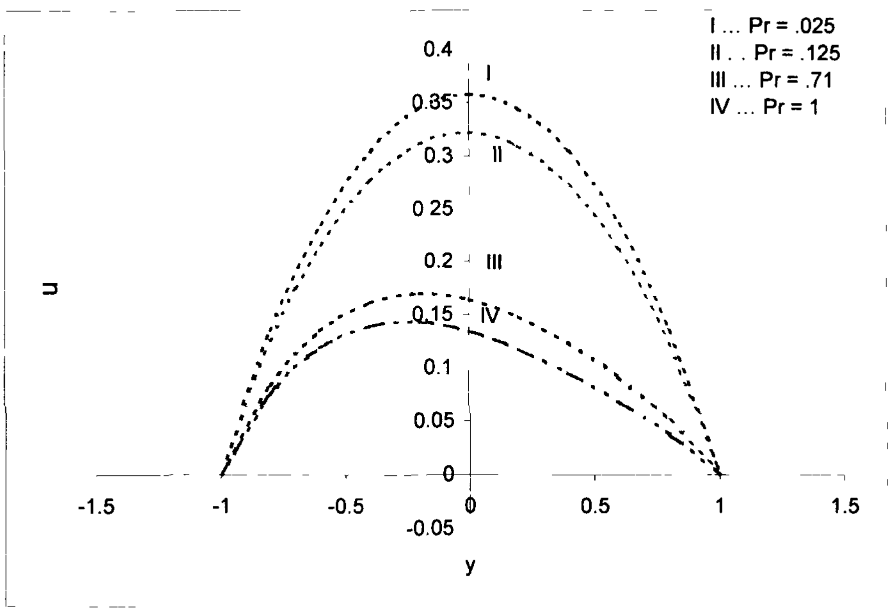

Figure 3-15: u versus y for M = .5 (θ = 60^0), n = 5, t = 1, Pr = .025, .125, .71, 1.0

3.5 Conclusion:

The above analysis shows that if the same amount of magnetic field is applied at different angles to the direction of the fluid velocity, the different nature of velocity profiles are obtained. The effect of the magnetic field is stronger if it is placed at $\theta = 90^0$ to the directions of the fluid than that if it is placed at any other angle. This effect slowly decreases as the angle between the direction of the fluid velocity and direction of the magnetic field decreases from $\frac{\pi}{2}$ to 0. When θ = 60^0 (i.e. M= .5), we are able to get the velocity distribution for the electrolyte solution. The present problem is best fitted for Copper sulphate solution, though it is not a good conductor. For M = 2, 1.5, 1, the flow analysis cannot be under taken for the fluid whose $P_r = .71, 1$. At the onset of free convection flow the effect of the adiabatic plate is significant. But as time increases or decay factor increases, the effects becomes unclear. Perhaps this lead to fully developed situation. The above analysis also indicates that there should be a proper combination of the choice of values of M, P_r, n and t to have a good result.

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CHAPTER - 4

TRANSIENT FREE CONVECTION MHD FLOW THROUGH A POROUS MEDIUM BETWEEN TWO VERTICAL PLATES

4.1 Introduction:

Many researchers studied the transient free convection flows past an infinite vertical plate in 1960,s because of their industrial applications in cooling process. These are by Seigel (1958), Gebhart (1961), Chung and Anderson (1961), Schertz and Eichhorn (1962), Goldstein and Briggs (1964) etc. Siegel studied for the first time that the initial behavior of temperature and velocity fields for a semi-infinite plate is the same as for a doubly infinite vertical plate and here the temperature field is given by the solution of unsteady one-dimensional heat conduction equation. Goldstein and Eckert (1990) later on confirmed these theoretical results through experiments.

Singh *et al.* (1996) have studied similar type of transient free convection flow between two long vertical parallel plates. But transient free convection flow through a porous medium bounded by two long vertical parallel plates has received very little attention. As it has good application in geothermal system, thermal insulation in buildings, heat exchangers, many authors have extensively studied this topic of steady free convection or transient free convection past different types of bodies. Nakayama *et al.* (1993) studied the transient free convection flow through porous medium bounded by two long vertical parallel plates. They considered Brinkman-Forchheimer extended Darcy momentum equation following Vafai and Tien's (1989) model. They presented the solutions for small and large time approximation analytically whereas at intermediate times, finite difference solution was presented. Vafai and Tien's model is based on normal permeability. However, recently, Lage's research group (1996) has discovered porous material with high permeability. Rajasubramanium *et al.* (1994) have developed a material of high permeability. It is based on a biocompatible material, which is a blend of poly-lactic acid and poly-capralacton. Nakayama *et al.* (1993) have studied transient free convection flow through a porous medium such that the viscosities of porous medium and the fluid are same. For small time, they neglected the effects of both Brinkman and Forchheimer terms and for long time, these two terms were considered, and the equations were solved numerically.

The main objective of this chapter is to study the flow of a viscous incompressible and electrically conducting fluid through a porous medium whose effective viscosity is larger than the viscosity of the fluid and bounded by two long vertical parallel plates, in the presence of a uniform magnetic field applied transversely to the plates. Such material has a Darcy number and viscosity ratio parameter of order 10. Such an analysis for a horizontal channel flow through high permeability porous medium, on taking into account both the Brinkman and the Forchheimer terms was recently presented by Nield *et al.* (1996). We have neglected Forchheimer term and retain only the Darcy term. Series solution is presented and the results are shown graphically.

4.2 Formulation of the Problem:

We consider here, the flow of the fluid through a porous medium whose effective viscosity (μ_{eff}) is far greater than the viscosity of the fluid flowing in the vertical upward direction through the channel, which is bounded by two long vertical parallel plates. The plates are maintained at same temperature. One plate is considered at y = 0 along which x' – axis is taken, and the other plate is at y = h, in the vertical upward direction. The y' - axis is taken normal to the plate. Here, $y' \in [o, h]$. Here B_0 acts in a direction normal to the flow. To write down the governing equations following assumptions are made:

(a) the plates are infinitely long. So, flow variables are functions of y' and t' only,

(b) Hall effects, Polarization effect and Induced Magnetic field are neglected,

(c) the external electric field is zero,

(d) the pressure gradient term and gravity term are entirely expressed by buoyancy force term,

(e) the viscous dissipative heat and the effects of the thermal and longitudinal dispersion are neglected.

(f) the flow motion is very slow, and non – fully developed.

Under the above assumptions the governing equations are as follows:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T_h') + \upsilon_{eff}\frac{\partial^2 u'}{\partial y'^2} - \frac{\upsilon}{K}u' - \frac{\sigma B_0^2}{\rho}u', \quad \frac{\mu}{\rho} = \upsilon \qquad (4\text{-}1)$$

$$\frac{\partial T'}{\partial t'} = \frac{1}{\rho C_p}\frac{\partial}{\partial y'}\left(k\frac{\partial T'}{\partial y'}\right) \tag{4 -2}$$

the initial and boundary conditions are

$$\begin{array}{llll} u' = 0, \quad T' = T_h' & \text{for all} & 0 \le y' \le h, & t' \le 0 \\ u' = 0, \quad T' = T_w' & \text{at} & y' = 0, & t' > 0 \\ u' = 0, \quad T' = T_h' & \text{at} & y' = h, & t' > 0 \end{array} \tag{4 -3}$$

The velocity and magnetic field distributions are $\vec{q} = [u(y,t),0,0]$ and $\vec{B} = [0, B_0, 0]$.

We now introduce the following non-dimensional quantities:

$$y = \frac{y'}{h}, \quad t = \frac{t'\mu}{\rho h^2}, \quad u = \frac{u'\mu}{\beta\rho g h^2 (T_w' - T_h')}, \quad Da = \frac{K}{h^2}, \quad Z = \frac{\upsilon_{eff}}{\upsilon}$$

$$\Pr = \frac{\mu C_p}{\rho k}, \quad T = \frac{T' - T_h'}{T_w' - T_h'}, \quad M = B_0 h\sqrt{\frac{\sigma}{\mu}} \tag{4 -4}$$

Then in view of (4-4), equations (4-1) and (4-2) reduce to the following form:

$$\frac{\partial u}{\partial t} = T + Z\frac{\partial^2 u}{\partial y^2} - \left(\frac{1}{Da} + M\right)u \tag{4 -5}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} \tag{4 -6}$$

with the following initial and boundary conditions

$$\begin{array}{llll} \mathrm{u} = 0, & \mathrm{T} = 0 & \text{for all} & 0 \le y \le 1, \ t \le 0 \\ \mathrm{u} = 0, & \mathrm{T} = 1 & \text{at} & \mathrm{y} = 0, \mathrm{t} > 0 \\ \mathrm{u} = 0, & \mathrm{T} = 0 & \text{at} & \mathrm{y} = 1, \mathrm{t} > 0 \end{array} \tag{4 -7}$$

4 .3 **Solution of equations**:

Taking the Laplace transform of (4-5) and (4-6), we get

$$Z\frac{d^2\bar{u}}{dy^2} - (A + s)\bar{u} = -\bar{T} \tag{4 -8}$$

$$\frac{d^2\bar{T}}{dy^2} - \Pr s\bar{T} = 0 \tag{4 -9}$$

Similarly, using Laplace transformation on the boundary conditions (4-7), we obtain

$$\bar{u}(0,s) = \bar{u}(1,s) = 0, \quad \bar{T}(0,s) = 1, \text{ and } \bar{T}(1,s) = 0 \tag{4 -10}$$

Since, the equations (4-8) and (4-9) are 2[nd] order differential equations in $\bar{u}$ and $\bar{T}$, the solutions of the equations by use of conditions (4-10) are

$$\bar{T} = \frac{\sinh\sqrt{s\Pr}(1-y)}{s.\sinh\sqrt{s\Pr}} \tag{4-11}$$

$$\bar{u} = \frac{\sinh\sqrt{B(A+s)}(1-y)}{s\{s(Z\Pr-1)-A\}\sinh\sqrt{B(A+s)}} - \frac{\sinh\sqrt{s\Pr}(1-y)}{s\{s(Z\Pr-1)-A\}\sinh\sqrt{s\Pr}} \tag{4-12}$$

Again, taking inverse Laplace transform of (4-11) and (4-12), we get

$$T(y,t) = 1 - y + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n \sin\{n\pi(1-y)\}e^{-\frac{n^2\pi^2}{\Pr}t}}{n} \tag{4-13}$$

$$u(y,t) = -\frac{\sinh\sqrt{AB}(1-y)}{A\sinh\sqrt{AB}} + \frac{1-y}{A} - 2B\pi\sum_{n=1}^{\infty}\frac{n(-1)^n \sin\{n\pi(1-y)\}e^{-\left(A+\frac{n^2\pi^2}{B}\right)t}}{\left(AB+n^2\pi^2\right)^2\left(Z\Pr-1-\frac{AB}{AB+n^2\pi^2}\right)}$$

$$+\frac{2\Pr}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n \sin\{n\pi(1-y)\}e^{-\frac{n^2\pi^2}{\Pr}t}}{n\{n^2\pi^2(Z\Pr-1)-A\Pr\}} \tag{4-14}$$

where $A = \frac{1}{Da} + M,\ B = \frac{1}{Z}$

4.4 Results and Discussion:

We have computed numerical values of T and u, and these are shown through the figures (4-1) – (4-6). In figure (4-1), the temperature profiles are obtained for Pr = .71, 7.0, 100, and we see that it decreases for increasing values of Prandtl number. In figure (4-2), the velocity profiles are shown for different values of Darcy number keeping viscosity ratio parameter, magnetic field parameter and Prandtl number fixed, respectively, at 0.1, 01, and 0.71. Here, we consider t = 0.5. In this figure we see that as Darcy number increases the fluid velocity also increases. Figure (4-3) is drawn for variable viscosity ratio (Z) at fixed Darcy number (Da = .01), magnetic field parameter (M = 01), Prandtl number (Pr = .71) and time t = 0.5 to give velocity distribution. It is observed that an increase in viscosity ratio parameter leads to a decrease in the velocity. Figure (4-4) is drawn for different values of Prandtl number at (fixed) Da = 0.1, M = 01, Z = 0.1, and t = 0.5 to give the nature of the velocity distribution curve. It is seen that Prandtl number remarkably influences the fluid velocity field. For Pr = 100, the velocity curve fluctuates greatly. Figure (4-5) is obtained for different Hartmann number (M) and for Pr = .71, Z = 0.1, Da = 0.1, t = 0.5, and we observed that an increase in Hartmann number leads to a decrease in the velocity. There is a curve in figure (4-5), which is free from magnetic field. If, we go through this two set of curves, we see that there is the influence of magnetic field on velocity profiles. Figure (4-6) is drawn to show the effect of Darcy number and viscosity ratio parameter simultaneously at constant values of Prandtl number (Pr = .71) and time (t = 0.2) in the absence of magnetic field. We observed that the obtained all numerical values of the velocity are lesser than unity for small time (t = 0.5), and for 4 - parameter values, which we have considered. This is the significance of transient free convection flow. Again the values of the velocity are highest at the middle position of the channel, except slight deflection towards the plate situated at y = 0. We think that this is due to presence of the magnetic field. All investigations undertaken here are for small time.

4.5 Skin friction:

From the velocity field, we now study the skin – friction, which is given by

$$\tau_1 = -\mu \left| \frac{\partial u'}{\partial y'} \right|_{y'=0} \tag{4-15}$$

Now, in view of equation (4-4), (4-15) reduces to

$$\tau = \left| \frac{\partial u}{\partial y} \right|_{y=0} \quad \text{where} \quad \tau = -\frac{\tau_1}{\beta \rho g h (T'_w - T'_h)}$$

Which is finally obtained as (by use of (4-14)) follows

$$\tau = \sqrt{\frac{B}{A}} \coth \sqrt{AB} - \frac{1}{A} + 2B\pi^2 \sum_{n=1}^{\infty} \frac{n^2 e^{-(A+n^2\pi^2/B)}}{(AB + n^2\pi^2)^2 \left(Z\Pr - 1 - AB/(AB + n^2\pi^2)\right)}$$

$$-2\Pr \sum_{n=1}^{\infty} \frac{e^{-\frac{n^2\pi^2}{\Pr}t}}{n^2\pi^2 \{(Z\Pr - 1) - A\Pr\}} \tag{4-16}$$

TABLE-I

Values of τ

t	Z	Da	M	τ /Pr = .71	7	100
0.2	0.1	0.01	0.5	0.306584	0.328508	0.363646
0.2	0.1	0.01	0	0.307327	0.329329	0.364679
0.2	0.1	0.1	0.5	0.878991	1.079423	1.522438
0.2	0.1	2.0	1.0	1.702279	2.430044	3.795205
0.2	1.0	0.01	0.5	0.090988	0.121745	0.233834
0.2	1.0	0.1	1.0	0.218794	0.774409	1.699466
0.2	1.0	2.0	2.5	0.281581	-0.016733	-0.096510
0.2	5.0	0.01	0.5	0.035774	0.055764	0.045106
0.2	5.0	0.1	01	0.053496	0.013741	-0.010259
0.2	5.0	5.0	02	0.061059	0.027994	0.006108
0.2	10.0	0.01	0.5	0.029604	0.488630	1.041958
0.2	10.0	0.1	01	0.029439	0.011727	0.000138
0.2	10.0	05	02	0.031353	0.015121	0.004053

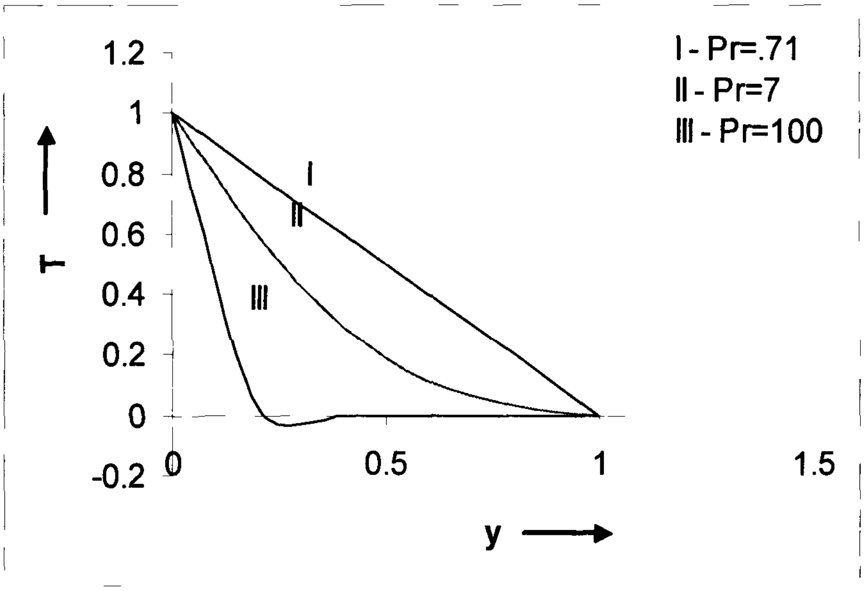

Fig. 4-1: Variation of temperature profiles for t = .5

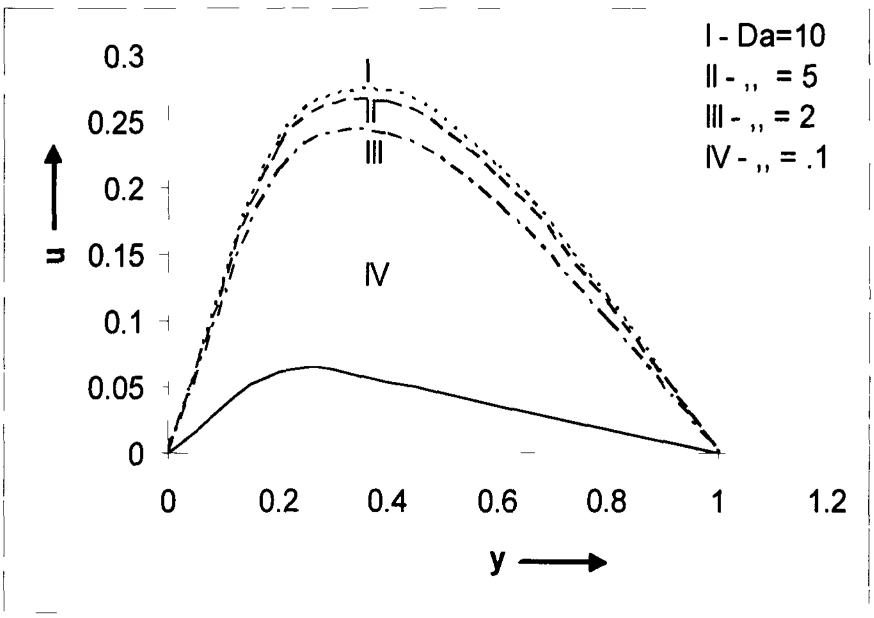

Fig.4-2: Variation of velocity profiles for Z = .1, M = 1, Pr = .71, t = .5

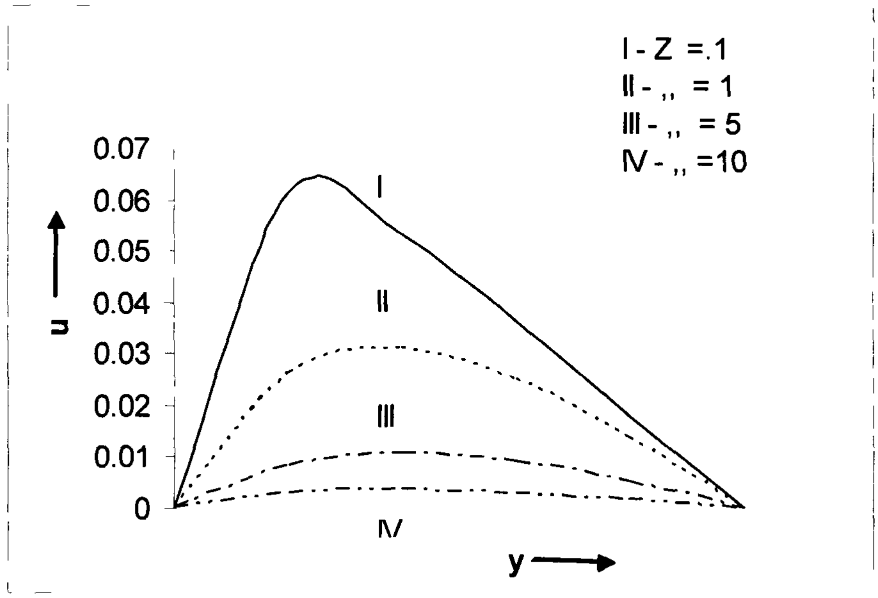

Fig. 4-3: Variation of velocity profiles for Da = .01, Pr = .71, t = .5

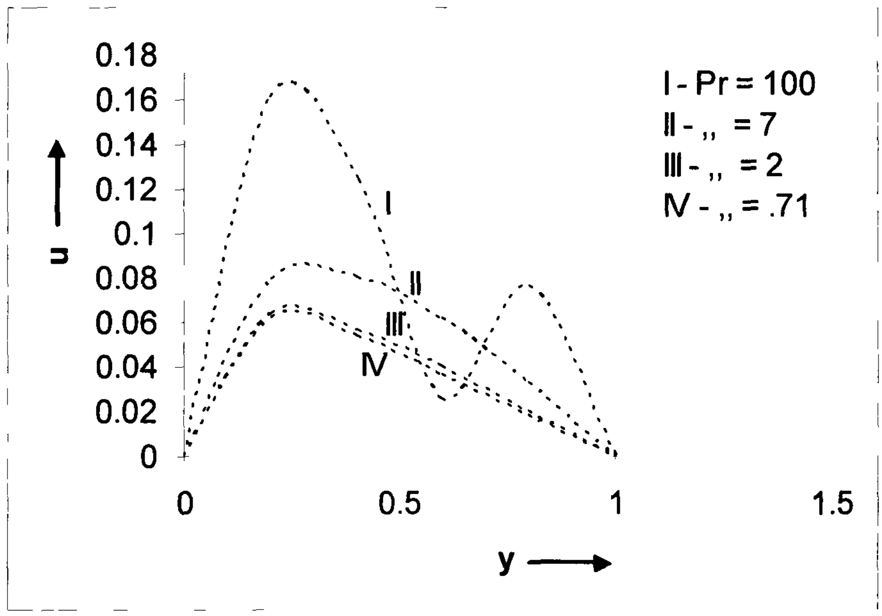

Fig. 4-4: Variation of velocity profiles for Da = .1, M = 1, Z = .1, t = .5

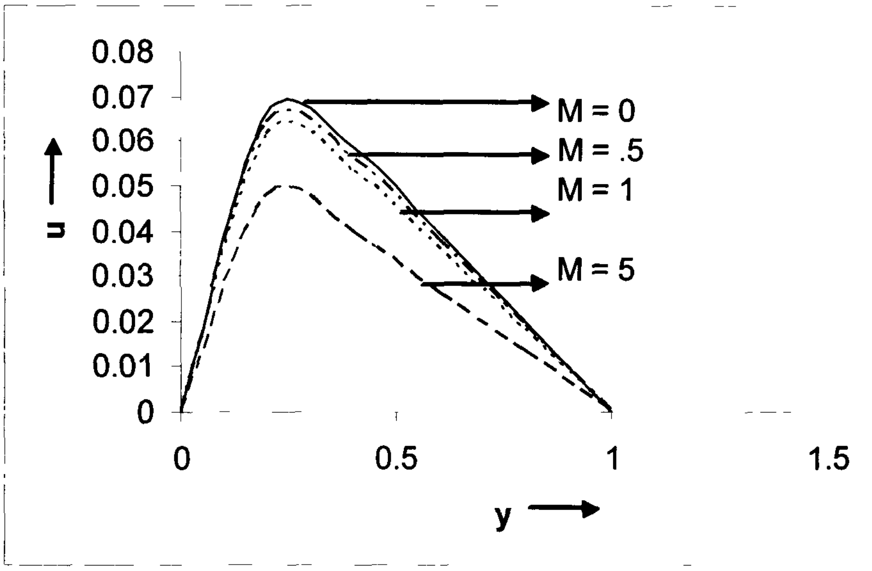

Fig. 4-5: Variation of velocity profiles for Pr = .71, Z = .1, Da = .1, t = .5

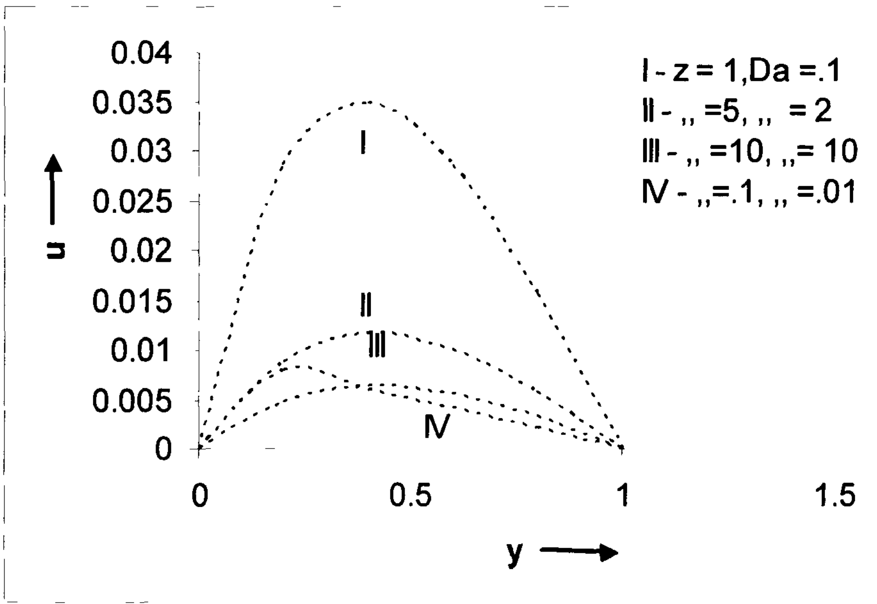

Fig. 4-6: Variation of velocity profiles for Pr = .71, M = 0, t = .2

4.6 Conclusion:

In this paper we have analyzed thc effect of Darcy number, viscosity ratio parameter and the Prandtl number on free convection flow of viscous incompressible fluid through a porous medium bounded by two long vertical parallel plates whose effective viscosity is larger than the viscosity of the fluid. Series solutions are provided for velocity and temperature distributions in terms of the Darcy number, viscosity ratio parameter, the prandtl number and the Hartmann number. Graphs drawn for velocity and temperature profiles show that these parameters have influence on these profiles. So, in order to predict accurately the flow behavior of the electrically conducting fluid, all this parameters must be taken into consideration. In table I, a series of values of shear stress have been given for different values of magnetic field parameter, viscosity ratio parameter, Darcy number and the Prandtl number. It is observed that as the values of the Prandtl number increases, the skin friction also increases for fixed values of Z (= 0.1) and for smaller values of Da and M. Here, for M = 0, (when Da = .01) the values of skin friction increases. For increasing values of Prandtl number, we get the highest values of τ for Z = 0.1, Da = 2, M = 1. As Pr increases, we see the decreasing values of skin friction when magnetic Hartmann number is maximum (in table I) and Z = 1, Da = 2. When Z takes the maximum value (Z = 10 in table I), skin friction decreases for increasing values of Pr. The first two values of table I shows that as M increases from zero (at fixed values of Z and Da) onwards, the values of skin friction decreases. Thus, it shows that the effects of increasing M decrease the skin friction. For standard combination of values of these four parameters, we can get an expected skin friction. On the other hand as the values of all these parameters increases the skin friction decreases. Hence, the porosity of the medium and magnetic field are the factors that can influence a great deal the flow field of fluid.

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CHAPTER - 5

THE TRANSIENT FOR MHD STOKES OSCILLATING POROUS PLATE: A SOLUTION IN TERMS OF TABULATED FUNCTIONS

5.1 Introduction:

Stokes first studied the problem of unsteady free convection flow of a viscous incompressible fluid past an infinite horizontal flat plate. The plate was of two characters – one of impulsively starting of suddenly set into motion which creates a start-up flow to the fluid and other one was of oscillating, oscillating in its own plane. H. Schlichting (2000) named the former one as 'Stokes's First problem' and later one as 'Stokes's Second problem'. Stokes presented the exact solutions to both the problems. Stokes's result for an oscillating plate was the steady- state solution, which applies after the effect of any initial velocity profile has died out. But this solution was not a complete solution since it does not satisfy the initial condition. When the plate starts from rest in a still fluid a transient solution must be added to Stokes's well known steady-state result. Panton (1968) first presented a closed form expression for the transient solution, which contains exponentials and error functions of a complex argument. He presented the transient and starting phase velocity distributions for the plate either oscillating as sin (T) or – cos (T). Later on, Deka *et al* (2001) studied this problem of a semi-infinite incompressible viscous fluid bounded by a flat plate in the presence of a uniform magnetic field applied transversely to the plate.

Stokes's second problem is not only of fundamental theoretical interest but it also occurs in many contexts of applied problems. It arises in acoustic streaming around an oscillating body. It is important in unsteady boundary layers, in the imposed fluctuation in the free stream velocity, on the boundary layer flow past a body etc. In this case Stokes's result is considered as a perturbation in the high frequency limit. This is because for an incompressible fluid flow, it is immaterial whether the plate oscillates in a stagnant fluid or the plate is fixed and the fluid oscillates.

The magnetohydrodynamic transient free convection flow of a viscous incompressible fluid caused by the sinusoidal oscillation of a plane flat porous plate has been studied in this chapter. The constitutive equations of continuity and mass conservation of electrically conducting liquid are obtained in Cartesian co-ordinates. The well-known Laplace transform technique is used to solve the equation. The solution has been obtained in exact form. The answer presented herein contains exponential and error function of complex arguments. These functions are readily available in newer mathematical tables. Graphs of the transient solution are presented for sin (T) boundary conditions. Velocity distributions in the fluid are also plotted. It is seen that magnetic parameter (M) and small effect of the porosity of the plate largely affects the fluid velocity field.

5.2 Mathematical formulation of the Problem:

The fluid is taken to occupy the upper half-plane with the plate. The plate is porous and semi-infinite horizontal in extent. The X'-axis is taken along the plate while the Y'-axis is taken normal to the plate. u' and v' are the components of fluid velocity along X' - and Y'-axis respectively. Since the plate is semi-infinite in extent, u' is a function of Y' and t' only while v' is independent of Y'. Suppose the fluid is electrically conducting and the plate is non-conducting. Let a uniform magnetic field H_o be applied in a direction perpendicular to $X'-$axis. The fluid is assumed to be of low conductivity so that the induced magnetic field can be neglected. The Lorentz force is $-\sigma H_0^2 u'$. At time $t' \leq 0$, the plate and the fluid are in a state of rest. At t'>0, the plate starts oscillating in its own plane. For boundary condition it is assumed that there is no slip at the wall.

Under these assumptions, the flow field governing equations (equations of continuity and mass conservation) can be written as-

$$\frac{\partial v'}{\partial y'} = 0 \tag{5-1}$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma H_0^2 u'}{\rho} \tag{5-2}$$

And the initial and boundary conditions are

$$\begin{aligned} &u'(y',0) = 0 \\ &u'(\infty,t') < \infty \\ &u'(0,t') = U \sin(w't') \end{aligned} \tag{5-3}$$

All the physical variables are defined in *List of symbols*.

We now introduce the following non-dimensional variables and parameters in order to transform Equations (5-1) and (5-2) and the boundary conditions (5-3) into dimensionless form:

$$Y=\frac{y'U'}{\nu},\quad U=\frac{u'}{U'},\quad T=\frac{t'U'^2}{\nu}$$

$$M=\frac{\sigma H_0^2\nu}{\rho U'^2},\quad w'=\frac{U'^2}{\nu},\quad V=\frac{v'}{U'} \tag{5-4}$$

Hence, the equations of continuity, mass conservation and boundary conditions reduces to

$$\frac{U'}{\nu}\frac{\partial V}{\partial Y}=0 \tag{5-5}$$

$$\frac{\partial U}{\partial T}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}-MU,\qquad M=\frac{\sigma\nu H_0^2}{\rho U'^2} \tag{5-6}$$

$$U(Y,0)=0 \tag{5-7a}$$

$$U(0,T)=\sin(T) \tag{5-7b}$$

$$U(\infty,T)\prec\infty \tag{5-7c}$$

5.3 Solution of the equations:

Solving equation (5-5), we obtain

V = constant.

For constant suction we consider $V=-V_0$ (5-8)

The negative sign appeared in (5-8) indicates that the suction is towards the plate. Hence the equation (5-6) becomes

$$\frac{\partial U}{\partial T}-V_0\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}-MU \tag{5-9}$$

The velocity may be decomposed into a steady state and a transient component satisfying equation (5-9) as-

$$U=U^s+U^t \tag{5-10}$$

The steady state component is found in the following form

$$U^s=\exp\left(-aY/\sqrt{2}\right)\sin\left(T-bY/\sqrt{2}\right) \tag{5-11}$$

Where $a=\sqrt{M+\sqrt{1+M^2}}, b=\frac{1}{a}$

This solution (5-11) satisfies the boundary conditions (5-7b) and (5-7c) but not the initial condition (5-7a). For transient solution we require additional boundary conditions. The ultimate conditions are as follows:

$$U'(Y,0) = -\exp\left(-aY/\sqrt{2}\right)\sin\left(-bY/\sqrt{2}\right) = \operatorname{Im}\exp\left(-CY/\sqrt{2}\right) \quad (5\text{-}12a)$$

$$U'(\infty,T) < \infty \quad (5\text{-}12b)$$

$$U'(0,T) = 0 \quad (5\text{-}12c)$$

Where C is the complex constant defined by C = a - i b.

The composition of both transient and steady state solutions completely satisfies the equations (5-9) and (5-7a) – (5-7c).

Now we use the Laplace transform technique to have the transient solution of equation (5-9) and in the boundary conditions (5-12a) – (5-12c). Hence the solution of the equation (5-9) subject to the boundary conditions (5-12a) – (5-12c) is

$$\overline{U}'(Y,s) = \operatorname{Im}\left[\frac{e^{-\frac{V_0+\sqrt{V_0^2+4(s+M)}}{2}Y}}{-\left(\frac{V_0C}{\sqrt{2}}+s+i\right)} + \frac{e^{-\frac{CY}{\sqrt{2}}}}{\frac{V_0C}{\sqrt{2}}+s+i}\right] \quad (5\text{-}13)$$

Where $\overline{U} = \int_0^\infty U e^{-pt}dt$, Re (p)>0

Taking inverse Laplace transform of (5-13), we get

$$U'(Y,T) = -\operatorname{Im} e^{-\frac{YV_0}{2}} L^{-1}\left\{\frac{e^{-\frac{\sqrt{V_0^2+4(s+M)}Y}{2}}}{s+l}\right\} + \operatorname{Im} e^{-\frac{CY}{\sqrt{2}}} L^{-1}\{\,\}\left\{\frac{1}{s+l}\right\} \quad (5\text{-}14)$$

Where $l = \frac{V_0C}{\sqrt{2}} + i$ and L^{-1} is the inverse Laplace transformation operator.

The equation (5-14), after some tough calculations found in this following form

$$U'(Y,T) = \operatorname{Im}[\frac{1}{2}e^{-\left(\frac{V_0C}{\sqrt{2}}+i\right)T}\ \{e^{-\frac{CY}{\sqrt{2}}}\operatorname{erfc}\left(-\frac{Y}{2\sqrt{T}}+\sqrt{T}\left(\frac{C}{\sqrt{2}}-\frac{V_0}{2}\right)\right)$$

$$-e^{-YV_0+\frac{CY}{\sqrt{2}}}\operatorname{erfc}\left(\frac{Y}{2\sqrt{T}}+\sqrt{T}\left(\frac{C}{\sqrt{2}}-\frac{V_0}{2}\right)\right)\}] \quad (5\text{-}15)$$

It is to be noted that each term in (5-15) satisfies the differential equation (5-9) separately. To obtain the answer in real variables, equation (5-15) should be separated into its real and imaginary parts. But it is very difficult due to the presence of the complementary error function in complex argument form in equation (5-15). However, the standard mathematical functions given in Abramowitz and Stegum's Handbook (1964) [1] made it possible to be separated into real and imaginary parts.

For this the auxiliary function, which allows accurate interpolation, given below, is used:

$$w(z) = \exp(-z^2)erfc(-iz) \tag{5-16}$$

Making use of (5-16), the equation (5-15), finally found as-

$$U'(Y,T) = \text{Im}\left[\frac{1}{2}e^{\frac{T}{2}(b^2-a^2)-\left(\frac{Y^2}{4T}+\frac{V_0^2T}{4}\right)}\left\{w(z_1) - e^{-\frac{YV_0}{2}}w(z_2)\right\}\right] \tag{5-17}$$

Where

$$z_1 = \sqrt{\frac{T}{2}}b + i\left(\sqrt{\frac{T}{2}}a - \frac{V_0\sqrt{T}}{2} - \frac{Y}{2\sqrt{T}}\right)$$

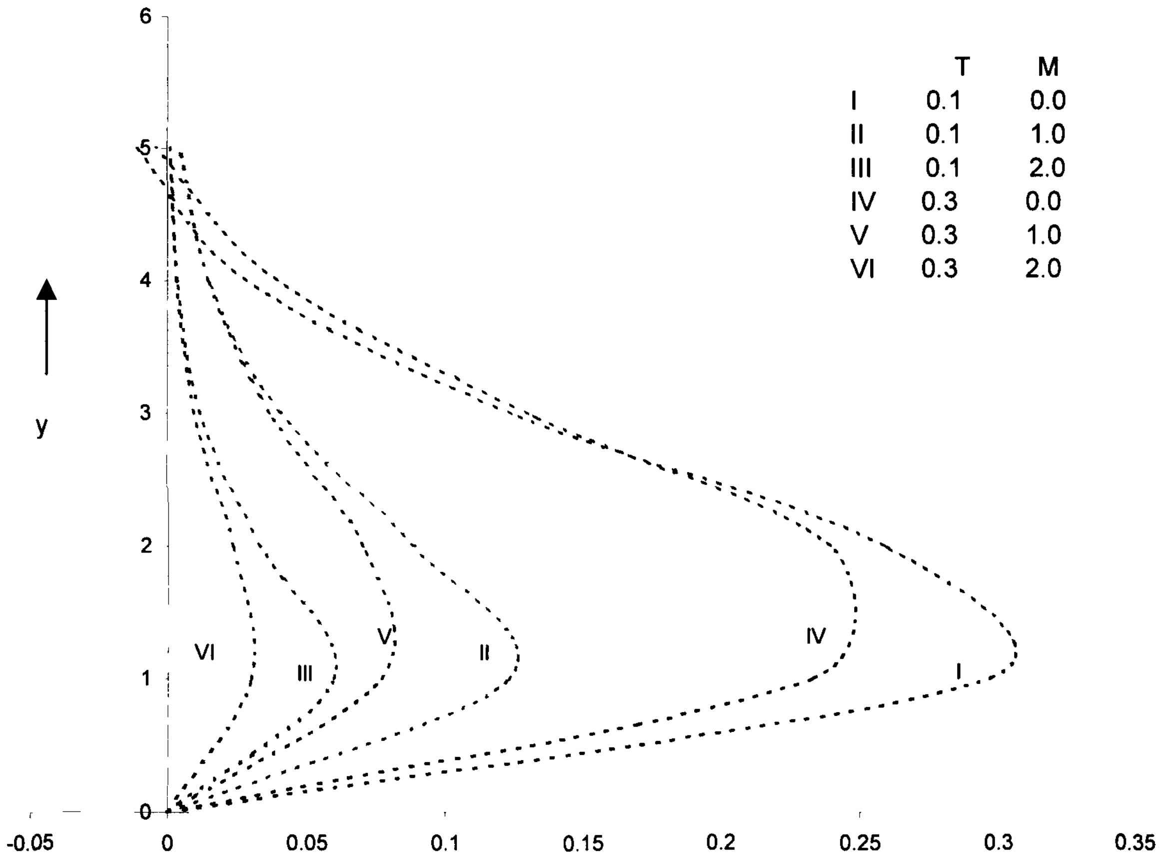
$$z_2 = \sqrt{\frac{T}{2}}b + i\left(\sqrt{\frac{T}{2}}a - \frac{V_0\sqrt{T}}{2} + \frac{Y}{2\sqrt{T}}\right)$$

The imaginary part of (5-17) applies when the boundary condition is sin (T) while the real part is for – cos (T) boundary condition.

5.4 Characteristic of the Solution:

It is easy to see how much the transient is affected due to the oscillating porous plate, and that is what we have investigated here. The transient dies out so rapidly that its every character shown in the graphs is clearly visible. The transient component of the velocity for the case when the wall velocity varies as sin (T), has been given in figure (5-1). The maximum velocity in this figure is slightly less than 0.3 and occurs in between Y = 1 and Y = 2. We have investigated the transient component of velocity for the time T=. 1 and T=. 3, and for magnetic parameter M = 0.0, 1.0 and 2.0 while the suction parameter V_0 is assumed as 0.1. It is seen that as T increases, the fluid velocity decreases. Again, as M increases, the value of the fluid decreases. On the other hand as Y increases from 0 to 5, transient velocity dies out rapidly.

The complete starting phase velocity profiles are shown in the figure (5-2). In order not to confuse between the steady-state profiles and starting phase profiles, the steady-state profiles are plotted as continuous lines while dashed lines represents the starting phase profiles. This velocity profiles are for the case when the fluid is initially still and the plate is moved so that the velocity varies as sin (T). The first curve at T = 0.3 shows the viscous 'wave' has penetrated only slightly into the fluid. On the other hand the steady- state velocity profiles shows that the fluid has negative velocities. It is seen that the penetration is lesser when magnetic field parameter has higher values. The second curve is drawn at T = 1.5. It is seen that a deeper penetration and decay in the difference between the starting and the steady-state solutions occur. The difference is only remarkable when M = 0.0. In case of M = 2.0, the starting and the steady-state profiles coincides. The third curve is drawn at T = 5.0. In this case, it is seen that the plate velocity reverse and penetrates from negative to positive into the fluid, the result of which is just like that it's completing a positive half circle. For M = 0, a difference between the transient and steady-state curves is visible. But for M = 2.0, the difference is small and often this two curves superimposed. It is observed that both the transient and steady- state curves dies out rapidly from around the boundary of the plate as Y varies from 0 to 5. On the other hand the difference between the steady-state curve and transient increases. We think that this difference is due to presence of the suction parameter.



$u' \rightarrow$

Figure 5-1: Transient velocity distribution. Plate velocity sin (T)

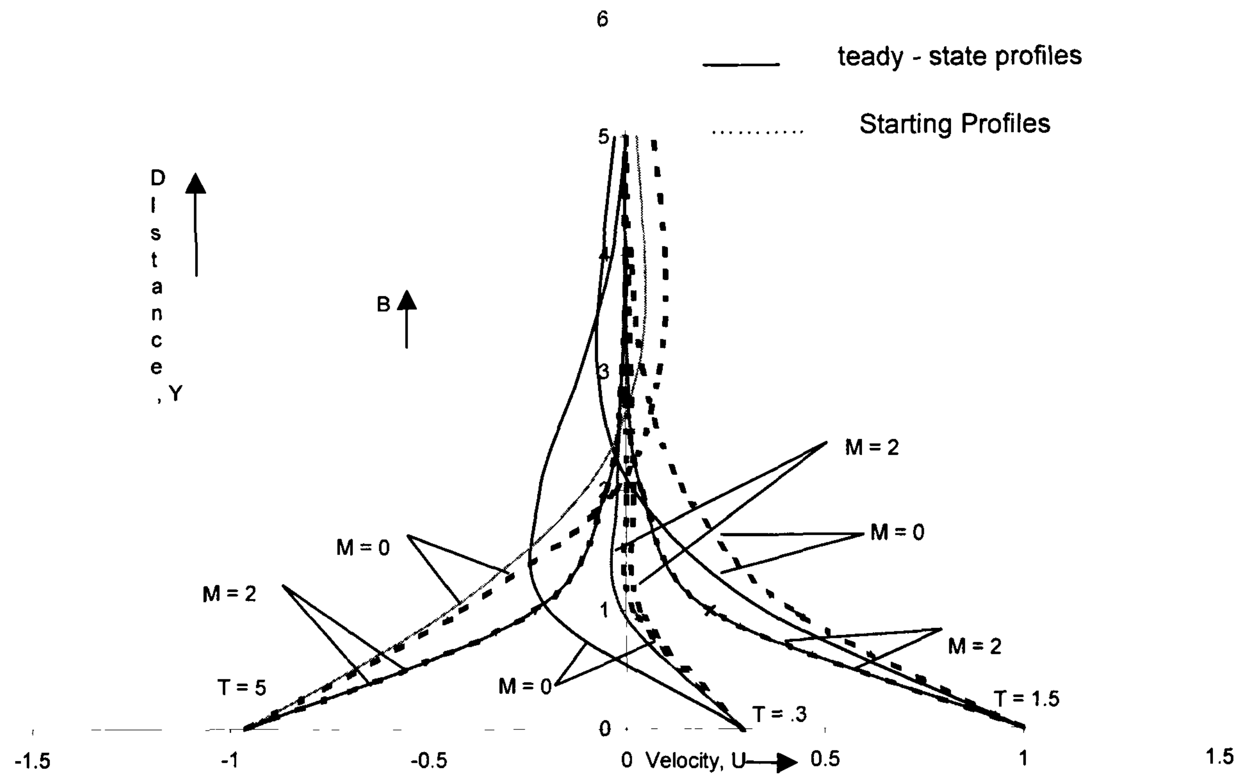


Figure 5-1. Starting phase velocity profiles, plate velocity sin (T)

5.5 Conclusion:

If we go through, closely. at the figures presented by Panton (1968) [77] and Deka *et al.* (2001) [21] in their respective valuable papers with our obtained figures, a clear difference can be seen. We can see the effect of the magnetic parameter and the porosity of the plate on the flow field. However, it is seen that all the figures presented in the already published papers and our paper have their similarity in shape and characteristic. It is easy to see in our paper that all the velocity profiles have its depletion towards the plate. We feel that this is due to the suction velocity of the plate. We think that if the value of suction parameter would increase, the depletion rate of velocity profiles towards the plate would have also increase, and the steady- state profiles would be more effectible than the transient flow.

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CHAPTER - 6

EXACT SOLUTION FOR UNSTEADY PLANE MHD COUETTE FLOW AND HEAT TRANSFER WITH TEMPERATURE DEPENDENT HEAT SOURCE / SINK

6.1 Introduction:

R. L. Panton (1984) [78] first presented a series expansion of the solution to the Navier – Stokes equation for Couette flow for large time approximation. This solution is for the flow of a viscous incompressible fluid between two horizontal parallel plane walls, when one of this two is fixed. Exact solutions of unsteady Couette flow have been computed by J. Steinheuer (1965)[see pp. 131 of [106]] for the case where the walls at rest in the steady state is abruptly brought to a constant velocity. A special case of these solutions is the case of the sudden halting of the moving wall. These problems being of fundamental nature, these are referred in all the textbooks of viscous flow. e.g. Schlichting & Gersten (2000). Bharali and Borkakati (1983) have discussed the flow and heat transfer between two horizontal parallel plates, where the lower plate is a stretching sheet and the upper one is a porous solid plate subjected to a transverse magnetic field. The above-cited problem was solved by numerical method. The exact solutions for the unsteady plane Couette flow of a dipolar fluid was presented by P. M. Jordan and P. Puri (2002) [48] analytically by application of Laplace Transform technique.

This chapter considers the problem of unsteady, one-dimensional MHD plane Couette flow between two infinite horizontal parallel plates. This problem is of fundamental interest, and it has many useful applications in different branches of fluid Mechanics particularly for Magnetohydrodynamic problems; also the presence of heat generating sources/ absorbing sinks in the fluid influences the flow field to a great extent as well as produces remarkable effects on the rate of heat transfer. So, we make up our mind to study this Initial and Boundary Value Problem (IBVP), analytically.

The layout of the present article is as follows. In section (6-2), the problem is formulated with assumptions and boundary conditions. In section (6-3), the solutions are derived through use of Laplace transform technique. In section (6-4), the results are given through graphs and tables following a quantitative discussion. Finally, in section (6-5), the findings are shorted out through concluding note.

6.2 Statement of the Problem:

Taking the positive y-axis of a Cartesian coordinate system in the vertical upward direction let an incompressible viscous and electrically conducting fluid be fill the space between two infinite horizontal parallel plates. The x-axis is then considered along the horizontal direction through the central line between the plates. Here, the plates are at a distance 2h apart, where $-h \le y \le h$. As the plates move parallelly in opposite direction with velocity U_0, the relative velocity of the plates is $2U_0$. We assume that the pressure in the entire fluid is constant. So, the pressure gradient term appearing in the equation of motion becomes zero (Bird *et al.* 1994) [7]. We apply a uniform magnetic field B_0 in the vertical upward direction. So, it is perpendicular to the flow as well as to the walls. The heated wall at y = h is maintained at a temperature T_2, and the cooled wall at y = - h is maintained at a temperature $T_1(< T_2)$. Let the plates be electrically non-conducting. The above assumptions give the components of velocity and magnetic field as:

$$u' = \{u, v, w\} = \{u(y,t), 0, 0\} \qquad B' = \{B_x, B_y, B_z\} = \{0, B_0, 0\}$$

We have assumed that the fluid is of low conductivity so that the induced magnetic field can be neglected. The viscous dissipation, the Hall and Polarization effects are ignored. The fluid and medium properties are assumed to be isotropic and constant.

Without writing the equation of continuity (as it is automatically satisfied), the governing equations of motion based on the above assumptions are found as –

$$\rho \frac{\partial u'}{\partial t'} = \mu \frac{\partial^2 u'}{\partial y'^2} + \vec{J} \times \vec{B} + \rho g \beta (T' - T_1) \quad \text{(momentum equation)} \qquad (6\text{-}1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + S'(T' - T_1) \quad \text{(energy equation)} \qquad (6\text{-}2)$$

Here the second term in the right hand side of (6-1) is the magnetic body force term which whence simplified gives $(-\sigma B_0^2 u')$ [p35 of (P. A. Davidson, 2001)], and the third term is due to buoyancy force which absorbed the pressure gradient term [p320 of [6]]. The second term in the right hand side of equation (6-2) is due to heat generating source or heat absorbing sink. The corresponding initial and boundary conditions are

$$t' \le 0; \quad u' = 0 \quad T' = T_1 \quad \forall y \in [-h, h]$$
$$t' > 0; \quad u' = u_0 \quad T' = T_2 \quad \text{at } y' = +h$$
$$u' = -u_0 \quad T' = T_1 \quad \text{at } y' = -h \tag{6-3}$$

Introduction of non-dimensional variables

$$u = \frac{u'}{u_0}, \quad y = \frac{y'}{h}, \quad t = \frac{t'u_0}{h}, \quad T = \frac{T' - T_1}{T_2 - T_1},$$

and parameters

$$\mathrm{Re} = \frac{hu_0}{\nu} \quad Ha = \frac{\sigma B_0^2 \nu}{\rho u_0^2} \quad Gr = \frac{g\beta h^3 (T_2 - T_1)}{\nu^2} \quad a = \frac{k}{\rho C_p}$$
$$\mathrm{Pr} = \frac{\nu}{a} \quad S = \frac{S'h}{\rho u_0 C_p} \quad Pe = \mathrm{Pr}\,\mathrm{Re} \tag{6-4}$$

in (6-1) – (6-3) yields

$$\frac{\partial u}{\partial t} = \frac{1}{\mathrm{Re}} \frac{\partial^2 u}{\partial y^2} - Ha\,\mathrm{Re}\,u + \frac{Gr}{\mathrm{Re}^2} T \tag{6-5}$$

$$\frac{\partial T}{\partial t} = \frac{1}{Pe} \frac{\partial^2 T}{\partial y^2} + ST \tag{6-6}$$

$$t \le 0; \quad \mathrm{u} = 0, \quad \mathrm{T} = 0 \quad \forall y \in [-1, 1]$$
$$t > 0; \quad \mathrm{u} = 1, \quad \mathrm{T} = 1 \quad \text{at } \mathrm{y} = +1$$
$$\mathrm{u} = -1, \quad \mathrm{T} = 0 \quad \text{at } \mathrm{y} = -1 \tag{6-7}$$

Here the symbols have their usual meaning.

6.3 Exact solution of the problem:

Assuming homogeneous initial conditions and applying the Laplace transform technique to equations (6-5) & (6-6), we obtain

$$p\bar{u} - 0 = \frac{1}{\mathrm{Re}} \frac{d^2 \bar{u}}{dy^2} - Ha\,\mathrm{Re}\,\bar{u} + \frac{Gr}{\mathrm{Re}^2} \bar{T}$$

or $$\frac{d^2 \bar{u}}{dy^2} - (A + p\,\mathrm{Re})\bar{u} = -B\bar{T} \tag{6-8}$$

$$p\bar{T} - 0 = \frac{1}{Pe}\frac{d^2\bar{T}}{dy^2} + S\bar{T}$$

or $$\frac{d^2\bar{T}}{dy^2} + (S - p)Pe\bar{T} = 0 \qquad (6\text{-}9)$$

The corresponding boundary conditions are

$$\bar{T}(1,p) = 1/p \quad \bar{T}(-1,p) = 0 \quad \bar{u}(1,p) = 1/p \quad \bar{u}(-1,p) = -(1/p) \qquad (6\text{-}10)$$

where p is the parameter of the Laplace transform and a bar over a quantity denotes its image in the transform domain.

Subject to the boundary conditions (6-10), the general solutions of equations (6-8) & (6-9) are

$$\bar{T}(y,p) = \frac{\sin\left(\sqrt{Pe(S-p)}(1+y)\right)}{p\sin\left(2\sqrt{Pe(S-p)}\right)} \qquad (6\text{-}11)$$

$$\bar{u}(y,p) = \frac{\sinh(\sqrt{A+p\mathrm{Re}}(1+y))}{p\sinh(2\sqrt{A+p\mathrm{Re}})} - \frac{\sinh(\sqrt{A+p\mathrm{Re}}(1-y))}{p\sinh(2\sqrt{A+p\mathrm{Re}})} -$$

$$\frac{B\sinh\sqrt{A+p\mathrm{Re}}(1+y)}{p\{Pe(S-p)+(A+p\mathrm{Re})\}\sinh 2\sqrt{A+p\mathrm{Re}}} + \frac{B\sin\sqrt{Pe(S-p)}(1+y)}{p\{Pe(S-p)+(A+p\mathrm{Re})\}\sin 2\sqrt{Pe(S-p)}}$$

(6-12)

Applying the inversion formula for the Laplace transform, eqns. (6-11) and (6-12) yields

$$T(y,t) = \frac{\sin\sqrt{PeS}(1+y)}{\sin 2\sqrt{PeS}} - \frac{\pi}{2Pe}\sum_{n=1}^{\infty}\frac{(-1)^n n\sin\left(\frac{n\pi}{2}(1+y)\right)}{\left(S - \frac{n^2\pi^2}{4Pe}\right)} e^{\left(S-\frac{n^2\pi^2}{4Pe}\right)t} \qquad (6\text{-}13)$$

$$u(y,t) = \frac{\sinh\sqrt{A}y}{\sinh\sqrt{A}} - \frac{B\sinh(\sqrt{A}(1+y))}{(PeS+A)\sinh(2\sqrt{A})} + \frac{B\sin(\sqrt{PeS}(1+y))}{(A+PeS)\sinh(2\sqrt{PeS})} +$$

$$\sum_{n=1}^{\infty}\frac{(-1)^n n\pi\cos(\frac{n\pi}{2})\sin(\frac{n\pi}{2}y)}{\left(A+\frac{n^2\pi^2}{4}\right)} e^{-\frac{1}{\mathrm{Re}}\left(A+\frac{n^2\pi^2}{4}\right)t} + \sum_{n=1}^{\infty}\frac{(-1)^n n\pi B\sin(\frac{n\pi}{2}(1+y)).e^{-\frac{1}{\mathrm{Re}}\left(A+\frac{n^2\pi^2}{4}\right)t}}{2\left(A+\frac{n^2\pi^2}{4}\right)\left\{(1-\mathrm{Pr})\left(A+\frac{n^2\pi^2}{4}\right)-(PeS+A)\right\}}$$

$$-\sum_{n=1}^{\infty}\frac{(-1)^n n\pi B\sin(\frac{n\pi}{2}(1+y)).e^{-\left(S-\frac{n^2\pi^2}{4pe}\right)t}}{2Pe\left(S-\frac{n^2\pi^2}{4Pe}\right)\left\{\left(S-\frac{n^2\pi^2}{4Pe}\right)(\mathrm{Re}-Pe)+(A+PeS)\right\}} \qquad (6\text{-}14)$$

6.4 Results and discussions:

In this section, we summarize the most important findings uncovered in this investigation and present the supporting numerical results through graphs and tables. All figures appearing in this work were generated directly from exact solutions/ expressions and programming C^{++} language. For plane Couette flow the lower limit of critical Reynolds number is 1300 and upper limit of which is 3000 [pp.104 Of (106)]. Hence, based on this analysis and the values of the parameters considered, we state the following.

(1) In figure 6-1, we have shown the effect of Reynolds number (Re) varying from Re = 1300 to Re = 3000 on temperature profiles. This is a T versus y profile. As y varies from –1 to .8, the values of T are very small, nearing zero. As a result the corresponding temperature profiles is seen flowing along y-axis (i.e. vertical according to the construction). But, as y tends to 1 from .6, the curve seems taking a right angle turn. When the values of T vary from .000014 and .000046 (at y = .6) to 1 (at y = 1), at the turning point the curves are seen going down y-axis. This is the same case as happened in the case of air, dust particle moving behind the car, bus or supersonic flight when it takes a right angle turn suddenly. In all the cases the values of T are under boundary conditions.

(2) In figure 6-2, we have studied the effect of Prandtl number (Pr) on temperature profiles keeping all other parameters fixed (fixed parameters are given just below the figure). It is seen that as Pr increases the amplitudes of vibration also increases on the x-axis. Also, as y varies from –1 to +1, the values of T increase from 0 to 1. T satisfies the boundary conditions in every case.

(3) We have investigated the temperature profiles against y for various values of source parameter (S), in figure 6-3. It is seen that T = 0 for y = -1 and T = 1 for y = 1 in all cases. As y varies from –1 to .8, the vibrating amplitudes slowly increases from 0 to -.094882, and as y approaches 1, the values of T coincides at 1. It is seen that there is the small variation of the values of T for varying values of S from .01 to 1.5. It is also seen that if one value of T is positive, the next neighboring value is negative for corresponding values of y.

(4) Figure 6-4 shows the effects of Re and Pr on temperature profiles against y. An exception of vibrating temperature profile is seen for Re = 10 and Pr = 100. For Re = 100, Pr = 7 and Re = 1000, Pr = .71, we have seen the same curve in the figure. Nevertheless there are slight differences in values of T for corresponding values of y. But, for Re = 3000 and Pr =

.025, the values of T's are zeros from y = -1 to .4. As y approaches 1 from y = .4 (approx.), the values of T increases to 1.

(5) In figure 6-5, we have investigated the nature of temperature profile against y for varying values of t in the presence of other parameters. For all three values of t, the T values are nearly zeros, though there are alternately positive and negative values for t = .01. It is seen as if it is the vibrating string. The string starts at 0 and ends at 1. The time t = .5 gives non-vibrating curve.

(6) Figure 6-6 illustrates the temporal evaluation of the value of velocity profiles against y for varying values of Re. It is seen that for Re = 3000, the values of u are approximately equal to zero when y varies from -.8 to +. 8 (approx.) But, when y tends to 1 from this assumed point, $u \rightarrow -1$. Similar is the case when y approaches 1 from .8. We have alternately positive-negative values of u for Re = 1300 and 2000, which gives vibrating curves as shown in the picture.

(7) In figure 6-7, it is seen that the flow field is coincident for t = .01 and .1. However, it is different and oscillating type in the region between y = -.8 (approx.) and y = .8 (approx.). Moreover, though it is not shown in the figure, we can have (already verified) conformable velocity profiles for the time varying from .001 to .5. For t > .5, the velocity profiles do not satisfy the boundary conditions (i.e. $u >> 1$ or $u << -1$).

(8) Figure 6-8 is drawn to show the effect of Pr on velocity field against y. We can see no difference of values of u corresponding to every value of y for Pr = 7, .71, .025. Only in the difference of the values of u are for Pr = 100. It is not clearly visible in the picture. Thus it is seen that variation of Prandtl number is not prominent for this kind of flow when other parameters are present, or when both the horizontal plates moves parallel in opposite directions with same velocity.

(9) In the presence of four physical parameters (i.e. Pr, Re, Gr, Ha), we see from figure 6-9 that the behavior of velocity field for t = .1 is exactly the same for S = .01, .2, 1.5, 10. The cause may be for the parallel motion of two horizontal plates in opposite directions with equal velocity. Perhaps it may be different in the absence of any one or two of the above four parameters. The non-distinct character may also be for higher values of Re and Pr.

(10) Each curve shown in figure 6-10 has been plotted from a data-set found by changing Pr and Re simultaneously, and for fixed values of other parameters (i.e. Gr, Ha, S) at t = .5. For Pr = 100 and Re = 10, the values of u increases from –1 to +1 gradually for increasing values of y from –1 to +1. It is apparent that there are differences in values of u for

(Pr = 7, Re = 100), (Pr = .71, Re = 1000), and (Pr = .025, Re = 3000), but nature is the same in each case.

(11) Of the five physical parameters (i.e. S, Pr, Re, Gr, Ha), we use Ha for table I that gives of values of the Newtonian fluid. It changes alternately positive to negative, and negative to positive for different values of Ha. For Ha = .001, these values are higher (or lower) than the values when Ha = .1. We have assumed here S = .2, Pr = .025, Re = 1000 and Gr = .5. For Ha = .5, we get an overflow error in sine hyperbolic function. So, it is not shown in the figure. Here t = .1. This investigation shows that the Magnetic Hartmann number must be less than .5, to have the value of u satisfying the self-sufficient boundary conditions.

(12) In order to show the effect of Grashof number, the table II is given below table I. Of the five parameters (i.e. S, Pr, Re, Ha), we have used the variation of Gr and Re. It is seen that for Re = 1300, and for Gr = .001, .01, the same values of u (up to six decimal places) for every value of y are occurring. Again for Re = 100 and for Gr = .001, .01, we get another set of same values (up to six decimal places) corresponding to every value of y. These letter values of u are smaller than the former values.

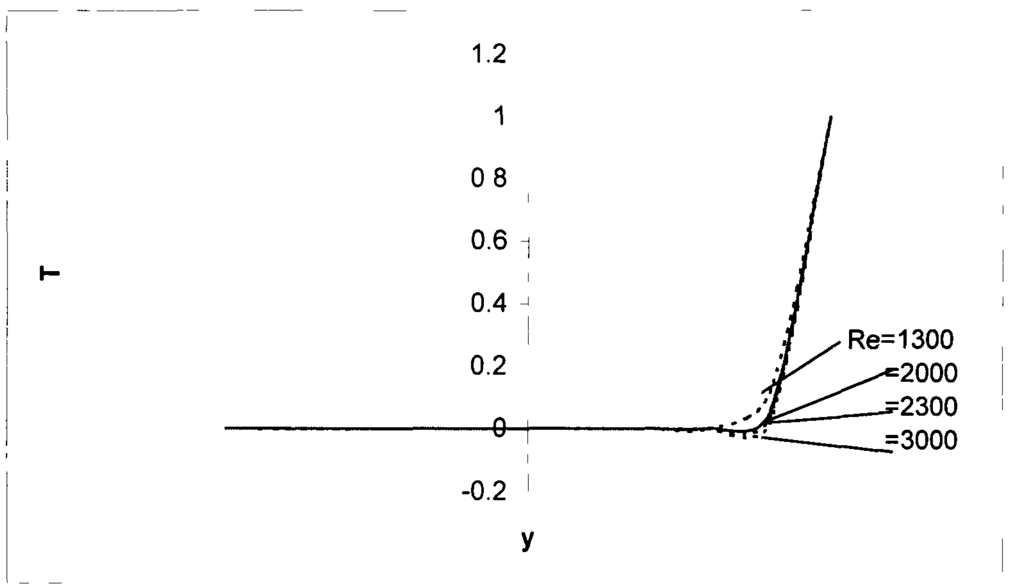

Figure 6-1: T versus y for S = .2, t = .5, Pr = .05, Ha = .01 and for various values of Re

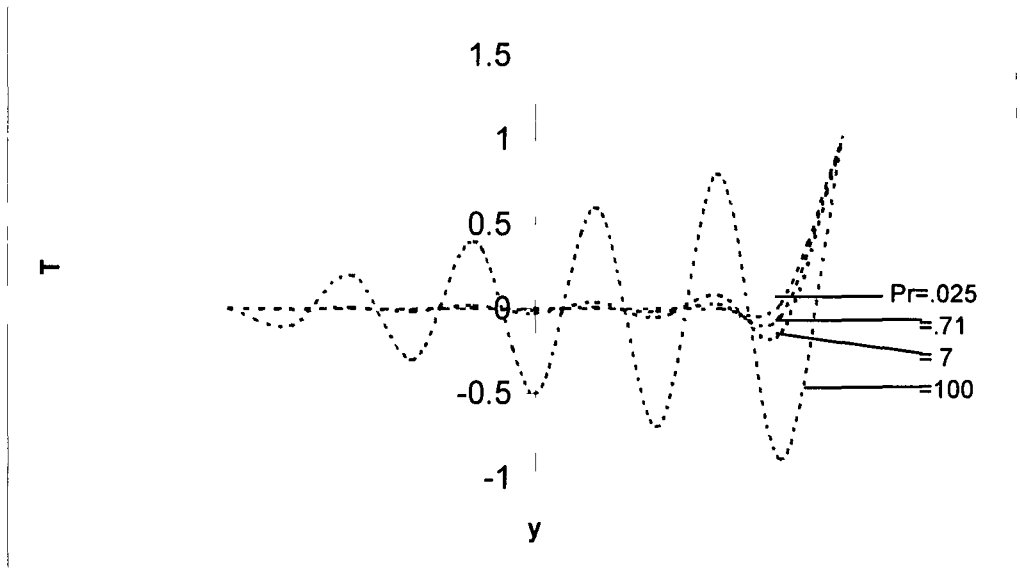

Figure 6-2: T versus y for t = .1, Re = 1000, S = .2, Gr = 5, Ha = .1 and for various values of Pr

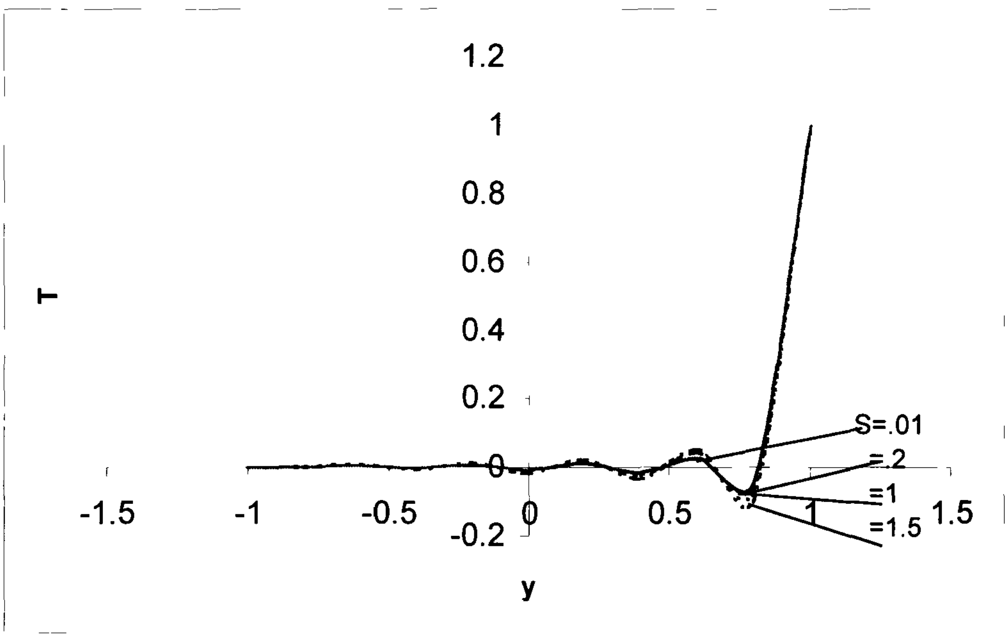

Figure 6-3: T versus y for t = .1, Re = 1000, Pr = .71, Gr = 5, Ha = .1, and for various values of S

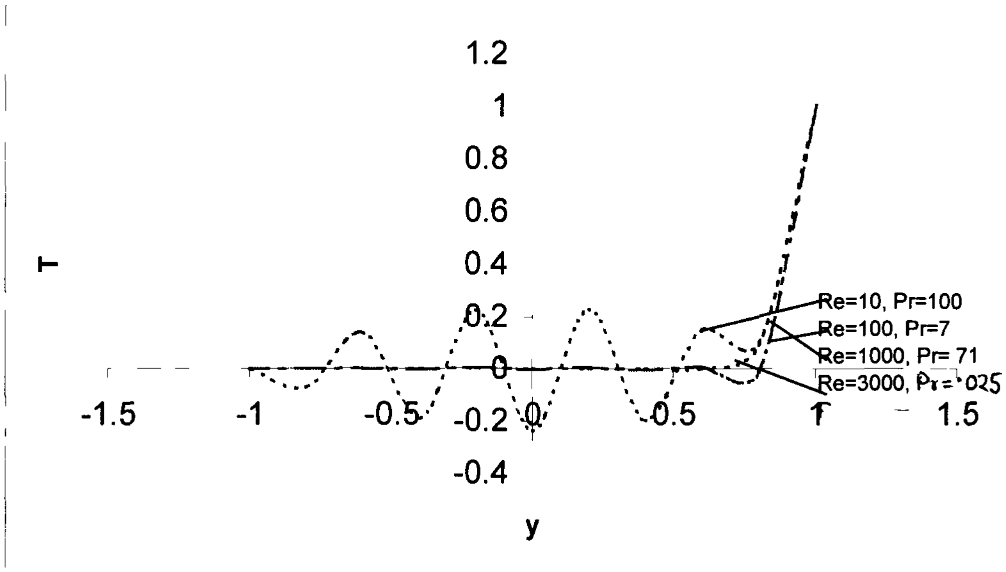

Figure 6-4: T versus y for t = .5, S = .2, Gr = 5, Ha = .01 and for different values of Pr and Re

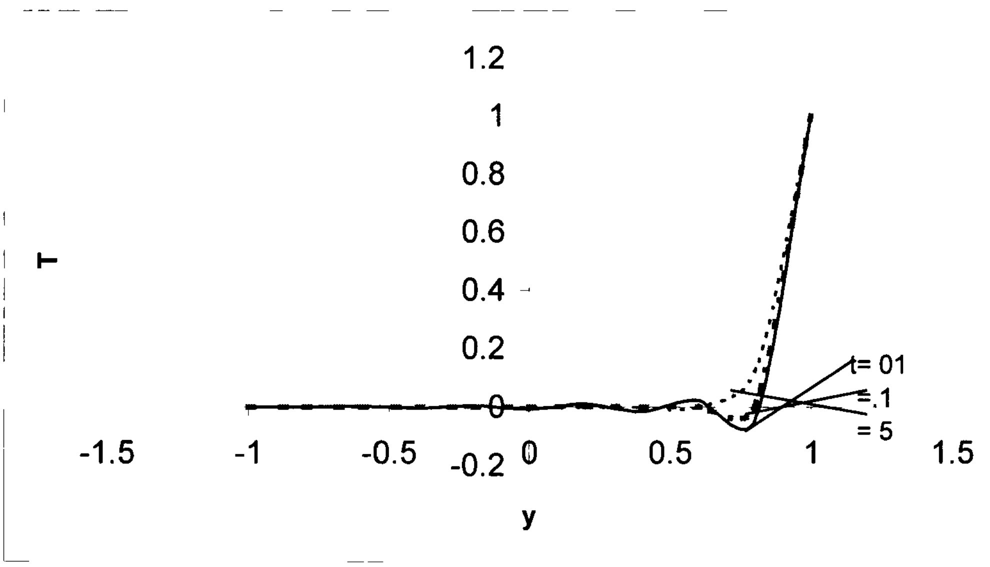

Figure 6-5: T versus y for t = .01, .1, .5; S = .2, Pr = .025, Re = 2500, Gr = 5, Ha = .01

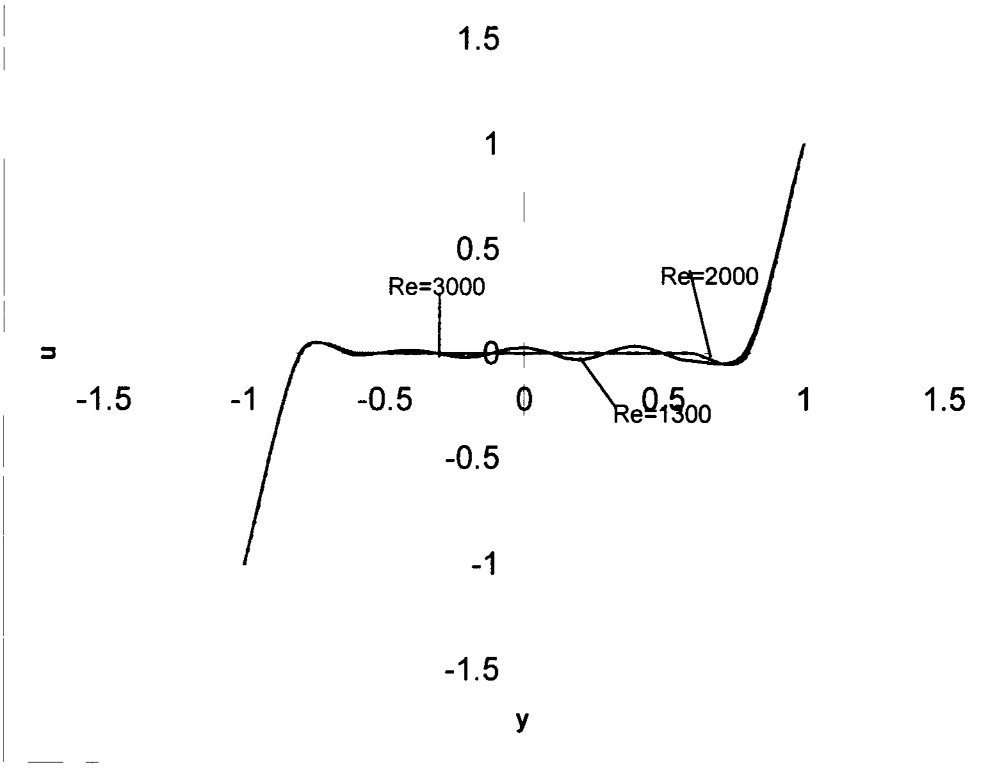

Figure 6-6: u versus y for Re = 1300, 2000, 3000, S = .2, t = .5, Pr = .05, Ha = .01, Gr = 5

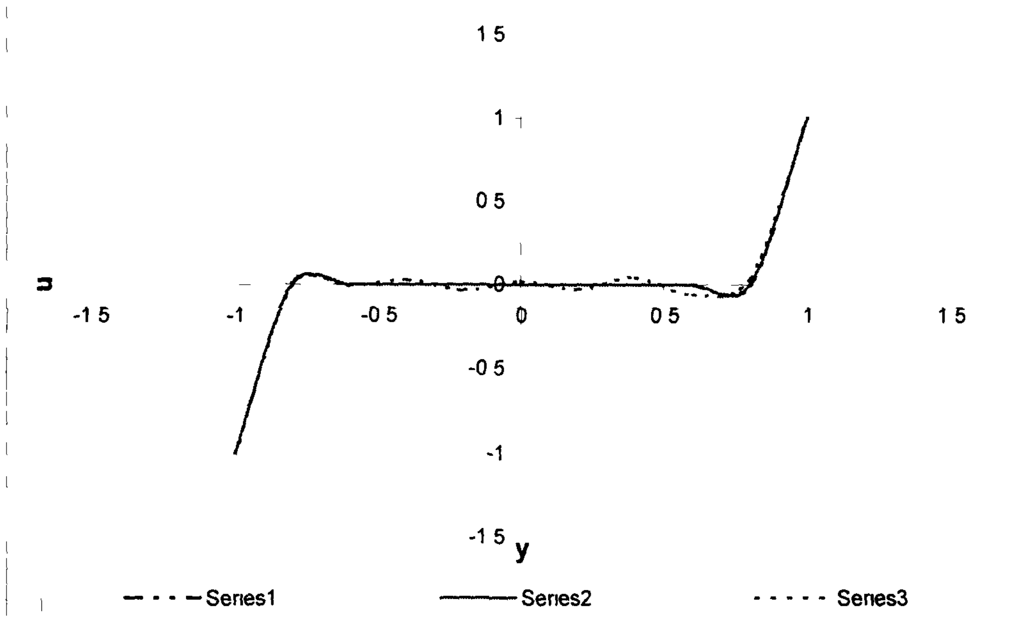

Figure 6-7: Re = 2500, Pr = .025, Gr = 5, S = .2 Ha = .01, in Si.1. t = .01, in Si. 2. t = .1, in Si. 3. t =.5

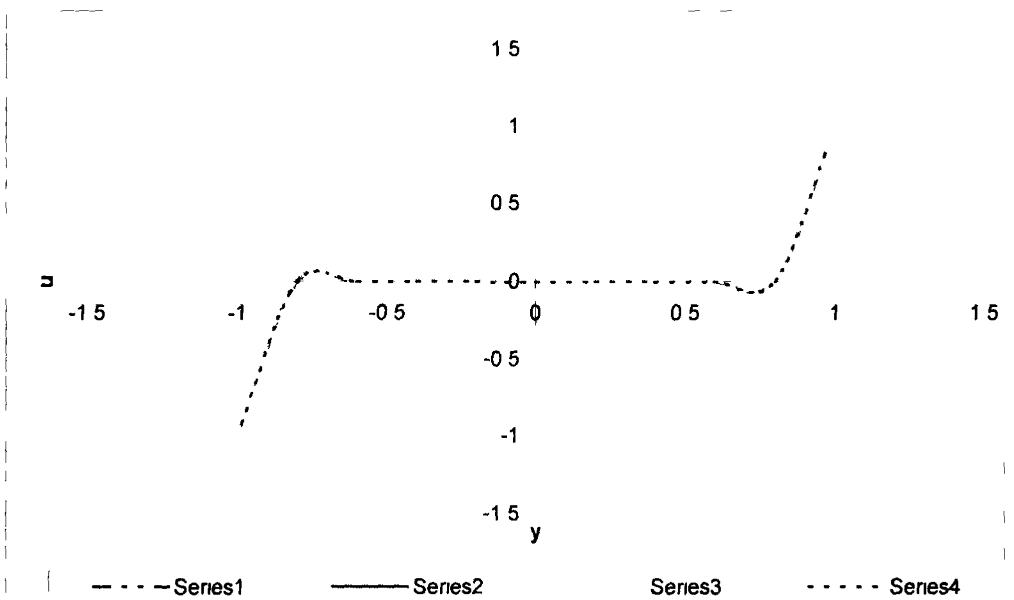

Figure 6-8: u versus y for S = .2, t = .1, Gr = 5, Ha = .1, Re = 1000 in Si. 1. Pr = 100, in Si. 2. Pr = 7, in Si. 3. Pr = .71, in Si. 4. Pr = .025

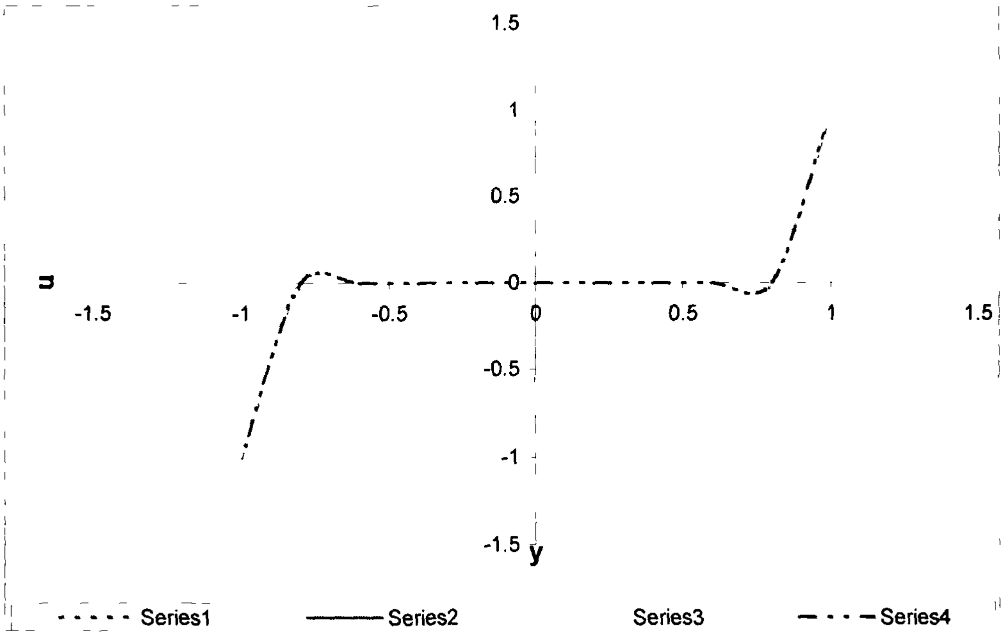

Figure 6-9: u versus y for t = .1, Pr = .71, Gr = 5, Ha = .1, Re = 1000, and in Si.1. S = .2, in Si 2. S = .01, in Si.3. S = 1.5, in Si.4. S = 10

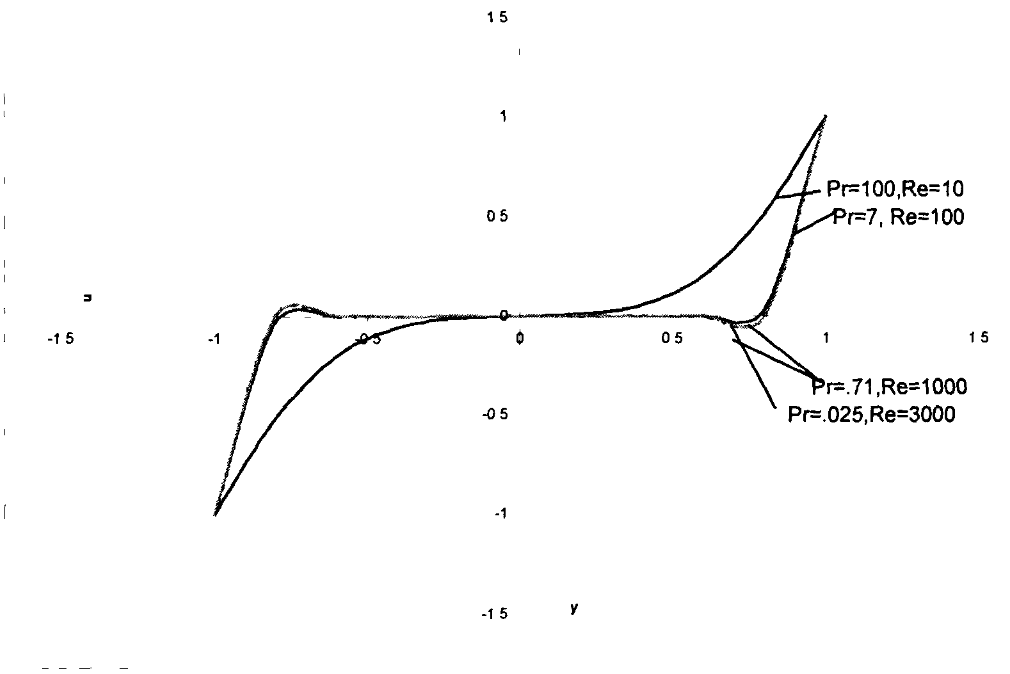

Figure 6-10: u versus y for t = .5, S = .2, Gr = 5, Ha = .01 and for different values of Pr, Re

Table I

y	Ha = .001/	Ha = .01/	Ha = .1
-1	-0.999986	-0.999999	-1
-0.8	0.032655	0.003565	-0.000003
-0.6	-0.014605	-0.001573	0.000007
-0.4	0.007708	0.00083	-0.000011
-0.2	-0.003448	-0.000373	0.000014
0	0.000005	0.000004	-0.000019
0.2	0.003441	0.000365	0.000023
0.4	-0.007701	-0.000821	-0.000027
0.6	0.014594	0.001564	0.000028
0.8	0.032646	-0.003556	-0.000023
1	0.999986	0.999999	1

Table I for S = .2, t = .1, Pr = .025, Gr = 5, Re = 1300

Table II

	Re = 1300		Re = 100	
y	Gr = .001	Gr = .01	Gr = .001	Gr = .01
-1	-1	-1	-1	-1
-0.8	0.00177	0.00177	0.000073	0.000073
-0.6	-0.000783	-0.000783	-0.000033	-0.000033
-0.4	0.000412	0.000412	0.000017	0.000017
-0.2	-0.000184	-0.000184	-0.000008	-0.000008
0	0	0	0	0
0.2	0.000184	0.000184	0.000008	0.000008
0.4	-0.000412	-0.000412	-0.000017	-0.000017
0.6	0.000783	0.000783	0.000033	0.000033
0.8	-0.00177	-0.00177	-0.000073	-0.000073
1	1	1	1	1

Table II for t = .1, Ha = .01, Pr = .025, S = .2

6.5 Concluding Note:

In all the figures the variation of y is seen to be horizontal though it is vertical according to the construction. Also the plates are horizontal moving parallel in opposite directions at y = 1 and –1.

For time t > .5, both the temperature and velocity profiles does not satisfy the boundary conditions, i.e. gives very large and small values outside the [-1,1].

In all cases, we observed the effects of Reynolds number, Prandtl number, Grashof number, Magnetic Hartman number and Heat source parameters. The heat-absorbing sink is not valid for this problem.

Under these five parameters heat transfer rate is prominent for MHD plane Couette flow problem.

The investigation is carried out for small time (< .5). Large time consideration leads to the overflow error, not satisfying the boundary conditions of u. The problem is clearly time-dependent and time-restricted.

In every case considered drawn to show the velocity profiles against y, it is observed that the two plates has its influence on velocity profiles. If the plates would have moved in the same parallel direction, we would be able to see other types of the profiles. This situation is left for further study.

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CHAPTER - 7

UNSTEADY FREE CONVECTION MHD FLOW AND HEAT TRANSFER BETWEEN TWO HEATED VERTICAL PALTES WITH HEAT SOURCE: AN EXACT SOLUTION

7.1 Introduction:

Transient free convection occurs in a fluid when the temperature changes cause density variations which gives rise to buoyancy forces. A lot of free convection heat transfer problems can be seen in literature. It is due to its numerous applications in metallurgical engineering such as magnetic levitation or confinement, thermonuclear fusion etc. In the metallurgical industries magnetic fields are routinely [19] used to heat pump, stir and levitate the liquid metals. Free convection flows with heat transfer rates have found a substantial and permanent place in the world of material processing through MHD processes. Moreover, this type of flows has parallel applications in Astrophysics, Medical sciences, Geophysics, and Aerodynamics. Researchers notably Brar, Borkakati [8], Biswal [12], Choudhury *et al.* [17], Deka [21, 22], Kafoussias [55], Merkin [66], Soundelgekar [102, 103, 107], Teipel [113] etc, did work on transient as well as on steady free convection flows. Many of them studied this type of flows in the presence of a magnetic field.

Datta *et al.* [20] studied the problem of Magnetohydrodynamic unsteady free convection flow and heat transfer of a visco-elastic fluid past an impulsively started porous flat plate with heat sources/ sinks. Ojha and Singh [76] analyzed the heat source / sink effects on free convection flow and mass transfer of visco-elastic fluid past an infinite vertical porous flat plate. Their studies have shown that the presence of heat generating sources or heat absorbing sinks in the fluid influence the flow field to a great extent as well as produce remarkable effects on the rate of heat transfer. Hence, owing to its numerous applications, in the paper of Gourla and Katoch [37], we have considered the effect of heat source with heat transfer rate for farther study.

7.2 Mathematical Formulation:

We assume that a viscous incompressible and electrically conducting fluid flows between two heated vertical long non-conducting plates. At time $t \le 0$, the fluid is at rest and the plates are also at temperature T_0 (reference temperature). At time $t > 0$, the motion of the fluids takes place and the temperature of the plate's changes according as

$T' = T_0 + (T_w' - T_0)(1 - e^{-n't'})$, where n' is the decay factor.

Here, x' – axis is taken along each plate, which in the vertical upward direction and y' – axis is taken normal to the plate. We consider the origin of the axes at the middle point between the plates. A uniform magnetic field of strength B_0 is applied in a direction transverse to the direction of the vertical plates. Therefore, action of the magnetic field is in the horizontal direction and thus perpendicular to the flow while of the fluid velocity field is in the vertical upward direction.

We make the following assumptions to derive the governing equations of motion:

- The fluid is assumed to be of low conductivity, so that the induced magnetic field is negligible.
- The fluid is isotropic and Newtonian.
- The strength of the magnetic field is not very large such that the generalized Ohm's law is negligible.
- For the boundary condition it is assured that there is no-slip at the wall.
- Viscous dissipation and Polarization effects are neglected.
- The viscous fluid flows with constant physical properties (ρ, μ, k, C_p) in between two vertical walls, a distance 2h ($-h < y < h$) apart.
- It is assumed that the plates are very long in the x-direction so the temperature (T') and velocity field (u') are functions of y' and t' alone, and velocity components v' and w' are zero.
- The pressure term is balanced by gravity force term to give rise buoyancy force term.

Under the above assumptions the governing equations of motion are found as follows:

$$\nabla.\vec{q} = 0 \qquad \text{(continuity equation)} \qquad (7\text{-}1)$$

$$\rho \frac{\partial u'}{\partial t'} = \mu \frac{\partial^2 u'}{\partial y'^2} + \rho\beta g(T' - T_0) - \sigma B_0^2 u' \qquad \text{(momentum equation)} \qquad (7\text{-}2)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + S'(T' - T_0) \qquad \text{(energy equation)} \qquad (7\text{-}3)$$

In equation (7-2), the gradient of temperature is due to the weight of the fluid in the slit $\left(\frac{dp}{dx} = -\rho g\right)$ and viscous forces are just balanced by the buoyancy forces [7] only. These equations are to be solved with the following initial and boundary conditions:

$$\text{At} \quad t' = 0: \quad u' = 0, \quad T' = T_0 \qquad \forall \ y' \in [-h, h]$$

$$\text{At} \quad t' > 0: \quad u' = 0, \quad T' = T_0 + (T_w - T_0)(1 - e^{-n't'}) \quad \text{for} \ y' = \mp h \qquad (7\text{-}4)$$

We now introduce the following dimensionless quantities

$$y = \frac{y'}{h}, \quad u = \frac{\mu u'}{\rho \beta g h^2 (T_w - T_0)}, \quad t = \frac{\mu t'}{\rho h^2}, \quad n = \frac{\rho h^2 n'}{\mu}, \quad T = \frac{T' - T_0}{T_w - T_0}$$

and dimensionless characteristic numbers [106]

$$\Pr = \frac{\mu C_p}{k} \qquad M = \frac{B_0^2 \sigma h^2}{\mu} \qquad S = \frac{S' h^2}{\mu C_p} \qquad (7\text{-}5)$$

in equations (7-2) - (7-3) and boundary conditions (7-4), and have the following dimensionless forms of them:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + T - Mu \qquad (7\text{-}6)$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 T}{\partial y^2} + ST \qquad (7\text{-}7)$$

$$t \le 0: \quad \mathrm{u} = 0, \quad \mathrm{T} = 0 \qquad \forall \ y \in [-1, +1]$$

$$t > 0: \quad \mathrm{u} = 0, \quad T = 1 - e^{-nt} \quad \text{for} \ y = \mp 1 \qquad (7\text{-}8)$$

7.3 Solution of the equations:

Taking Laplace transform of equations (7-6) & (7-7), and boundary conditions, (7-8), we have the following equations:

$$\frac{d^2 \bar{u}}{dy^2} - (M + p)\bar{u} = \bar{T} \qquad (7\text{-}9)$$

$$\frac{d^2 \bar{T}}{dy^2} + \Pr(S - p)\bar{T} = 0 \qquad (7\text{-}10)$$

$$\mathrm{t} = 0: \quad \bar{u}(\pm 1, p) = 0, \quad \bar{T}(\pm 1.p) = 0$$

$$t = 0: \quad \bar{u}(\pm 1, p) = 0 \quad \bar{T}(\pm 1, p) = \frac{n}{p(n+p)} \tag{7-11}$$

where $\quad \bar{F}(y,p) = \int_0^\infty e^{-pt} F(y,t) dt$.

The solution of the equation (7-10) subject to the boundary conditions (7-11) is

$$\bar{T} = \frac{n}{p(p+n)} \frac{\cos\sqrt{\Pr(S-p)}y}{\cos\sqrt{\Pr(S-p)}} \tag{7-12}$$

Again, the solution of the equation (7-9) with the help of eqns. (7-11) and (7-12) becomes

$$\bar{u} = \frac{n\cos\sqrt{\Pr(S-p)}y}{(\Pr(S-p)+M+p)p(p+n)\cos\sqrt{\Pr(S-p)}} - \frac{n\cosh\sqrt{M+p}y}{(\Pr(S-p)+M+p)p(p+n)\cosh\sqrt{M+p}} \tag{7-13}$$

Inverting (7-12) and (7-13), we get

$$T(y,t) = \frac{\cos\sqrt{S\Pr}y}{\cos\sqrt{S\Pr}} - \frac{\cos\sqrt{\Pr(S+n)}y}{\cos\sqrt{\Pr(S+n)}} e^{-nt}$$
$$+ \sum_{k=0}^{\infty} \frac{(-1)^k n\pi(2k+1)\cos\left(\frac{\pi}{2}(2k+1)y\right).e^{\left(S - \frac{\pi^2(2k+1)^2}{4\Pr}\right)t}}{\Pr\left(S - \frac{\pi^2(2k+1)^2}{4\Pr}\right)\left(S+n-\frac{\pi^2(2k+1)^2}{4\Pr}\right)} \tag{7-14}$$

$$u(y,t) = \frac{2n\cos\sqrt{\frac{\Pr(S+M)}{1-\Pr}}y.e^{-\frac{S\Pr+M}{1-\Pr}t}}{(S\Pr+M)\left(\frac{S\Pr+M}{1-\Pr}-n\right)\cos\sqrt{\frac{\Pr(S+M)}{1-\Pr}}} + \frac{1}{S\Pr+M}\left[\frac{\cos\sqrt{S\Pr}y}{\cos\sqrt{S\Pr}} + \frac{\cosh\sqrt{M}y}{\cosh\sqrt{M}}\right]$$
$$+ \frac{e^{-nt}}{\{n(1-\Pr)-(S\Pr+M)\}}\left[\frac{\cos\sqrt{\Pr(S+n)}y}{\cos\sqrt{\Pr(S+n)}} - \frac{\cos\sqrt{n-M}y}{\cos\sqrt{n-M}}\right]$$
$$+ \sum_{k=0}^{\infty} \frac{(-1)^k n(2k+1)\pi\cos\left(\frac{2k+1}{2}\pi y\right).e^{\left(S-\frac{\pi^2(2k+1)^2}{4\Pr}\right)t}}{\Pr\left\{\left(S-\frac{\pi^2(2k+1)^2}{4\Pr}\right)^2 + n\left(S-\frac{\pi^2(2k+1)^2}{4\Pr}\right)\right\}\left\{M+S-(1-\Pr)\frac{\pi^2(2k+1)^2}{4\Pr}\right\}}$$

$$+\sum_{k=0}^{\infty}\frac{n\pi(2k+1)\cos\left(\frac{2k+1}{2}\pi y\right).e^{-\left(M+\frac{\pi^2(2k+1)^2}{4}\right)t}(-1)^k}{\left\{\Pr(S+M)-(1-\Pr)\frac{\pi^2(2k+1)^2}{4}\right\}\left\{\left(M+\frac{\pi^2(2k+1)^2}{4}\right)^2-n\left(M+\frac{\pi^2(2k+1)^2}{4}\right)\right\}}$$

(7-15)

7.4 Results and Discussion:

Figure 7-1 has been obtained by plotting the temperature distribution T against y at different values of the heat source parameter S for fixed time (t = .2), decay factor (n =1), and Prandtl number (Pr = .025). As the values of S increases from 0 to 1, the values of T also increases at x-axis (i.e. the axis of parabola as the figure shows) from .607866 (approx) to .615325 (approx) (corrected up to six decimal places). Nevertheless, the curves are homogeneous parabolic with x-axis as its axis in each case. This shows that the temperature distribution is uniform – highest near the walls, and lowest in between the plates.

Figure 7-2 has been drawn to show the effect caused by Pr at different values for fixed values of t (= .2), n (= 5), S (= .5) on temperature distribution. It is seen that temperature distribution changes and gets its new shape according as the change of Prandtl number. For small values of Prandtl number the temperature distribution changes negligibly. But for Pr = 7, T changes remarkably, and it distributes with small change away from the plates. For Pr = 1, we have a fine parabolic curve.

Figure 7-3 depicts the temperature profiles for different values of n when t = .2, S = .5, Pr = .025. It is found form this figure that the temperature at any point inside the vertical channel increase uniformly with increase of n.

We have considered the figure 7-4 to show the temperature distribution T against y for different time at fixed values of Pr (= .025), n (= 5), S (= .5). It is seen that there is negligible change of distribution of temperature between the channel for time t = 1, 2, 3. But for t = .2, though this distribution curve is similar with earlier three, there is difference with earlier three values. In each of the above case all value of T is nearly equal to 1 and all T – curves are parallel to y-axis.

Figure 7-5 has been obtained by plotting the value of the velocity u against y at different magnetic Hartmann number. Series1 & 2 are for t = .1 and n = 1. Series3 & 4 are for t = .5 and n = 10. In every case we have considered Pr = .025, S = .05. It is seen that for small time (t = .1) the curves 1 & 2 are almost parallel to the y-axis between the channel, and the values of u are positive zeros. For time t = .5 and n = 10, the distribution of velocity is as if it is a right circular arc. At y = 0, the value of u is minimum, and at $y = \pm 1$, it is maximum.

Figure 7-6 is drawn for different values of heat source parameter S and for fixed n, Pr, M. Series1 & 2 for t = .1, and series3 & 4 for t = .5. It is seen that for small time the flow of the velocity is not fully developed, but for time t = .5, the velocity field is seemed to be developed, and its shape are homogeneous right circular arc. In each case, near the walls the velocity is the highest while at the center of the channel the fluid velocity is lowest.

The velocity profiles have been plotted against y for M = 1, S = .05, t = .1, n = 1 and for various values of Prandtl number as shown in figure 7-7. This figure shows that the velocity profiles take the shape of positive parabolic curves with x-axis as its axis in each case. It is seen that as Prandtl number increases the velocity curves turns from just forming parabolic to fine uniform and symmetric parabolic curves. Theses are symmetric in x-axis. This signifies the effect caused by different values of Prandtl number.

Figure 7-8 has been drawn for different values of decay factor n and at fixed values of Pr (= .025), S (= .05) and M (= .5). In series1 & 2, we have considered t = .1 whereas in series 3 & 4, this value is .5. It is seen that for small time the value of the velocity is slightly greater than zero. But for time t = .5, the velocity distribution curve is right circular arc. Again, in each case, the value of the velocity is highest at the walls while it is lowest at the center of the channel.

Figure 7-9 depicts the velocity profiles for different combinations of values of Prandtl number, magnetic Hartmann number and Heat Source Parameter at fixed values of t (= .5) and n (= 10). It is seen that in each case the curve is a right circular arc. This means that the velocity is highest near the walls and lowest at the center position of the walls. This figure shows, how different fluids behaves in different environment.

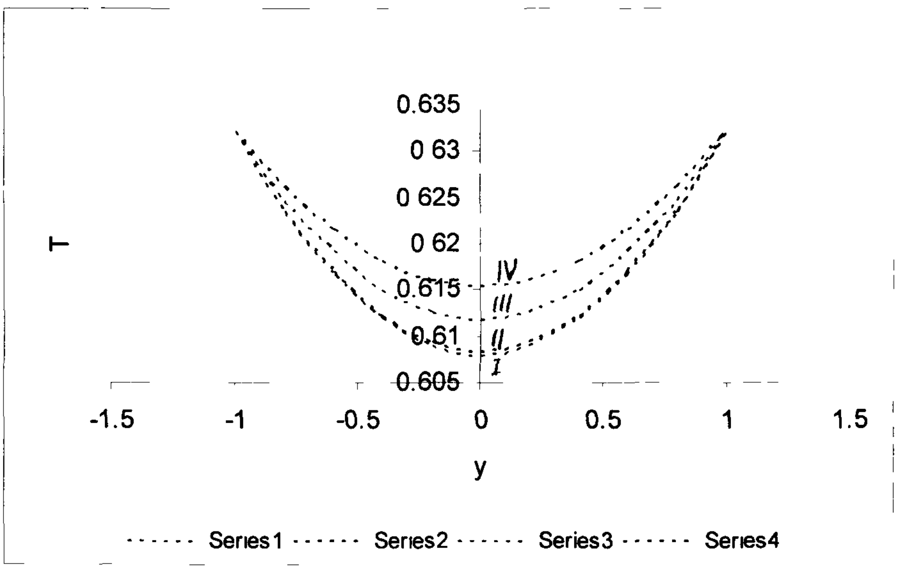

Figure 7-1: T vs. y series1 for S=0, series2 for S= .05, series3 for S=.5, series4 for S=1 at t = .2, n = 5, Pr = .025

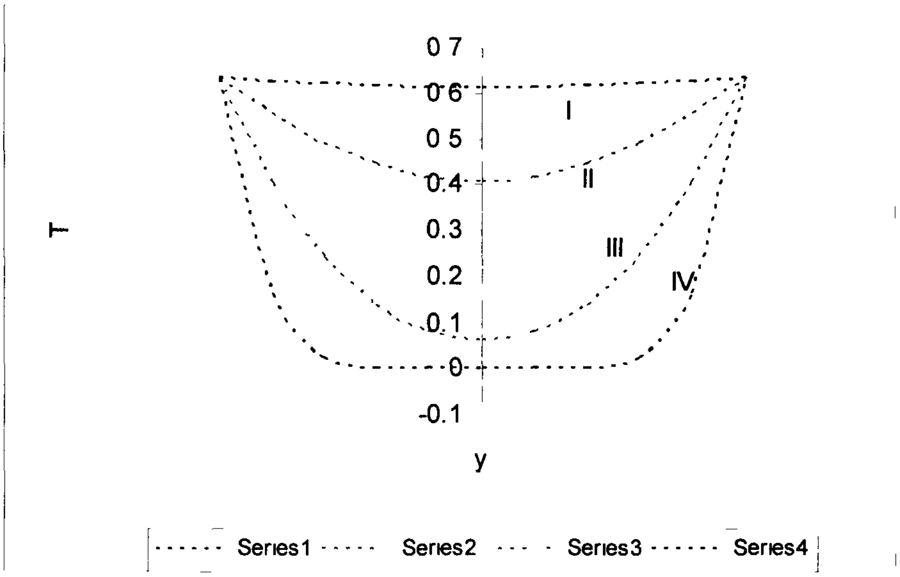

Figure 7-2: T vs. y I for Pr = .025, II for Pr = .25, III for Pr = 1, IV for Pr = 7 at t = .2, n = 5, S = .5

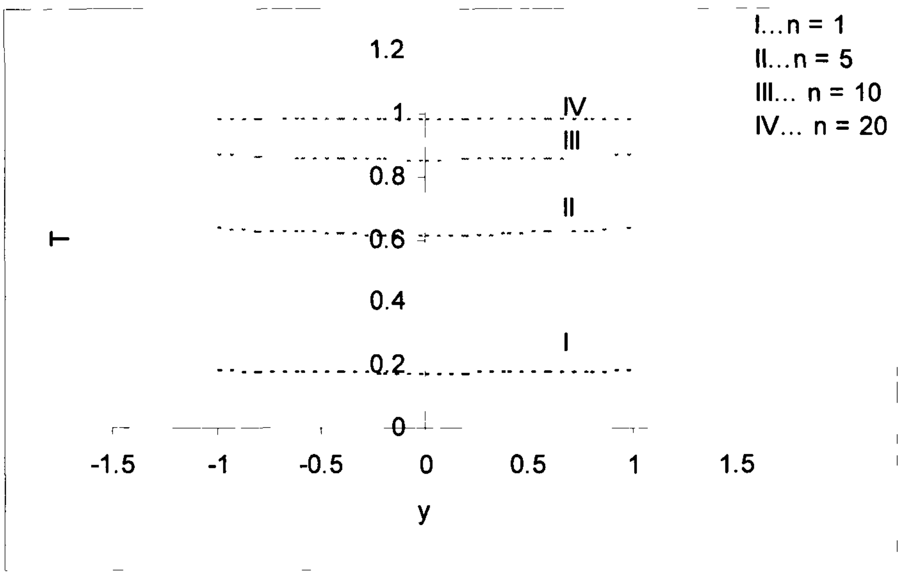

Figure 7-3: T vs. y for n = 1, 5, 10, 20 at Pr = .025, S = .5, t = .2

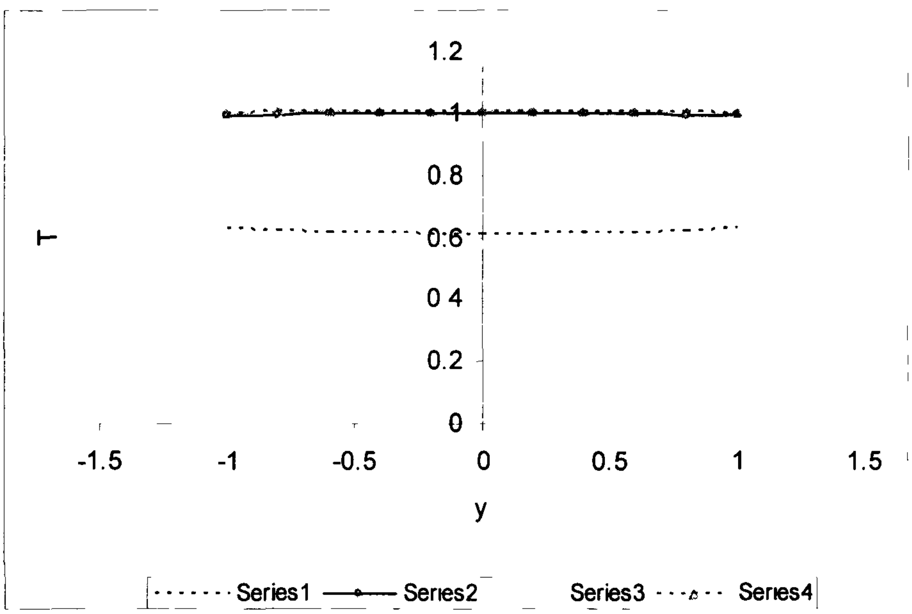

Figure 7-4: T vs. y: series1 for t = .2, series2 for t = 1, series3 for t = 2, series 4 for t=3 at Pr = .025, S = .5, n = 5

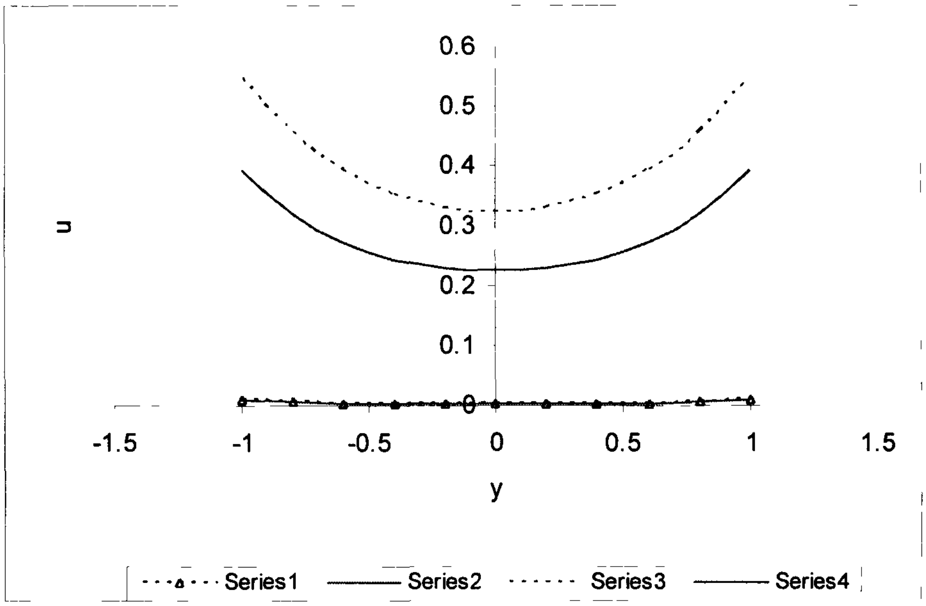

Figure 7-5: u vs. y; series1 for M = .5, series2 for M = 1 at t = .1, n = 1, S = .05, Pr = .025, series3 for M = 2, series4 for M = 4 at t = .5, n =10, S = .05, Pr = .025

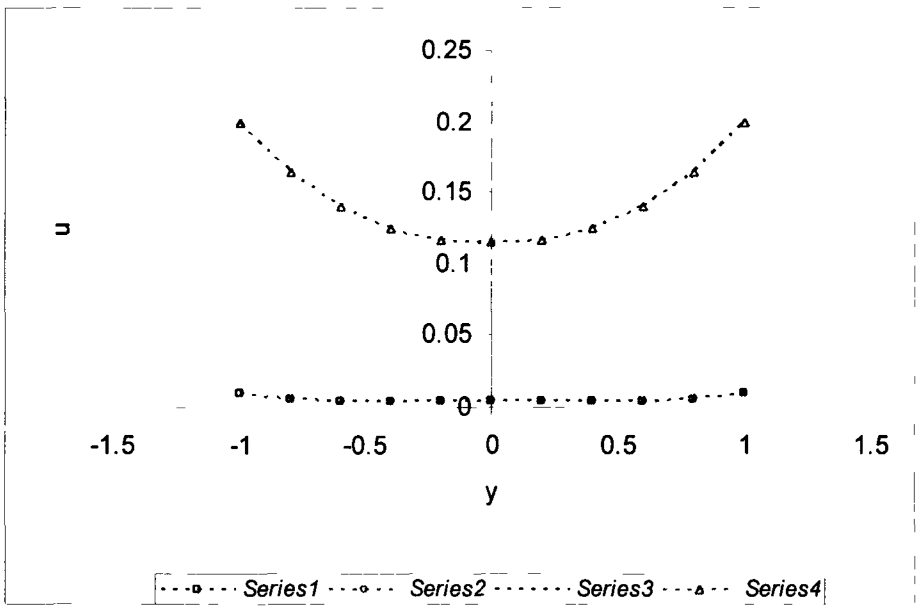

Figure 7-6: u vs. y; series1 for S = 0, series2 for S = .05 at t = .1, M = .5, n = 1, Pr = .025, series3 for S = .5, series4 for S = 1 at t = .5, M = .5, n = 1, Pr = .025

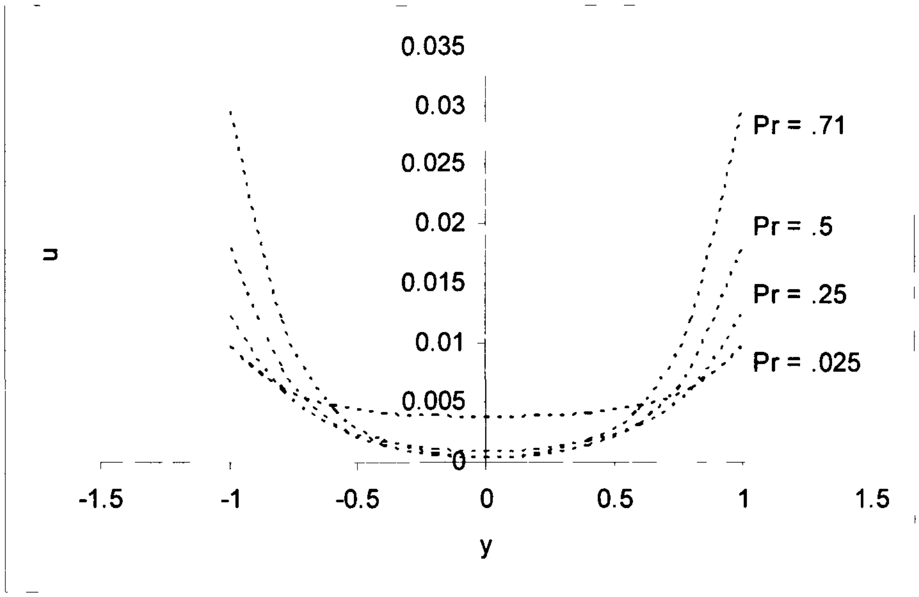

Figure 7-7: u vs. y for Pr = .025, .25, .5, .71 at M = 1, S = .05, t = .1, n = 1

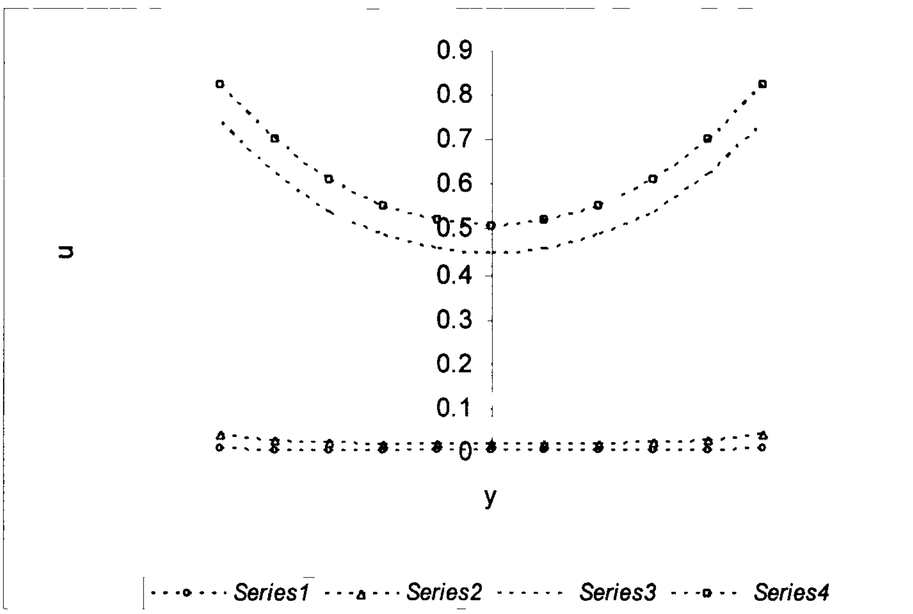

Figure 7-8: u vs. y; series1 for n = 1, series2 for n = 5 at t = .1, M = .5, Pr = .025, S = .05, series3 for n = 10, series4 for n = 20 at t = .5, M = .5, Pr = .025, S = .05

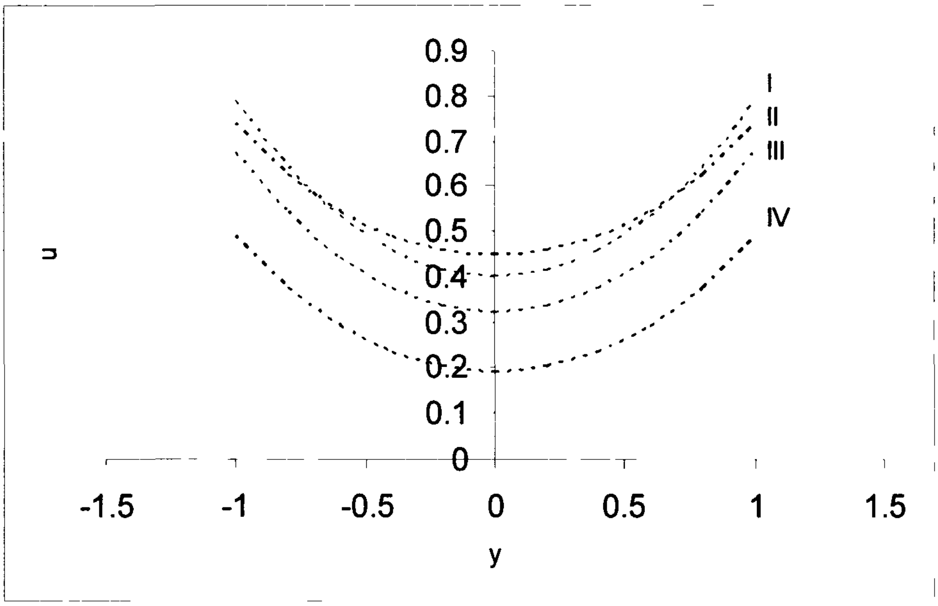

Figure 7-9: u vs. y; I for Pr = .25, M = 1, S = .5; II for Pr = .025, M = .5, S = .05 III for Pr = .5, M = 2, S = 1 IV for Pr = .71, M = 4, S = 0 at t =.5, n = 10

7.5 Observations:

(1) Decay factor 'n' and magnetic field parameter 'M' has a balancing relation. The graphs drown in figure 7-5 for n = 1 and M = .5, 1, and also for n = 10 and M = 2, 4; has shown this characteristic. When n is fixed and at the same time M increases, the values of the velocity decreases. We can also get the values of the velocity for equal values of n and M. But we cannot get a real value of u for the values of M which is higher than n. So, this is a restriction in our problem.

(2) Naturally, the fluids are to flow in such a way that its velocity is the highest at the middle position of the channel. But in our case this phenomena is completely opposite. For ready reference the paper of Gourla and Katoch (that we have investigated), can be cited. We think that this is due to the Heat Source applied at the plate.

(3) The velocity distribution is sharp near the two plates than at the center between the plates.

(4) As time increases the temperature is also increases. As a result the value of the velocity also increases. This is seen in all figures.

(5) In each case, the investigation is carried out for small time (t = .1, .5). So, the solution can be thought of as the onset of free convection.

(6) We have studied the problem for the fluid with prandtl number less than unity.

(7) This problem left scope for further study.

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LIST OF PAPERS PUBLISHED / ACCEPTED / SENT FOR PUBLICATION

Published Papers:

1. *Transient free convection MHD flow through a porous medium between two vertical parallel plates*: "IJAME" Poland, vol. 10, No.2, 2005.
2. *Transient free convection flow through a porous medium between two vertical parallel plates*: "Far-East Journal of Applied Mathematics", Allahabad University, Pushpa Publishing House, Special Volume, ISSN 0972-0960.
3. *Effects of Mass Transfer on unsteady free convection flow between two heated vertical parallel plates in the presence of transverse magnetic field*: International Journal "GUMA", Department of Mathematics, G. U. Dec. 2003 issue.

Papers accepted:

4. *The transient for MHD stockes's oscillating porous plate: a solution in terms of tabulated functions*: International Journal " Theoretical Applied Mechanics" Belgrade, Yugoslavia.
5. *Exact solution for the unsteady plane MHD Couette flow and heat transfer with temperature dependent heat source/ Sink*: An exact solution: "International Journal of Heat and Technology", Engineering Faculty, University of Bologna (Italy).

Paper Presented:

6. *Unsteady free convection MHD flow and Heat Transfer between two heated vertical plates with heat source: An exact solution*: "International Conference of Mathematical Fluid Dynamics" (2 – 7 December, 2004) University of Hyderabad.

Papers communicated:

7. *Magnetic field effects on unsteady free convection MHD flow between two heated vertical plates one of which is adiabatic*: "Indian Journal of Pure and Applied Mathematics", New Delhi.
8. *The transient for MHD flow past an infinite vertical oscillating plate with constant heat flux: An exact solution in terms of tabulated function*: "Indian Journal of Physics", Jadavpur, Kolkata.
9. *Unsteady free convection MHD flow between two heated vertical plates when one is adiabatic*: "Ganita", Bharata Ganita Parisad, Department of Mathematics & Astronomy, University of Lucknow, India.
10. *Unsteady flow of a dusty conducting viscous liquid between two parallel plates in presence of a transverse magnetic field:* "IJAME", Tech. Univ. of Zielona Gora , Poland.

Participation in Conference / Lecturers:

1. *Turn of the Millennium Lecturers in Mathematics* organized by the department of Mathematics of Indian Institute of Technology, Guwahati during 20 – 24 Dec. 1999.
2. *International Conference on Environmental Fluid Mechanics* organized by the department of Mathematics of Indian Institute of Technology, Guwahati , during 3 – 5 Dec. 2005.
3. *Some other day long Seminars organized by various institute of North – East region.*

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Appendix

Historical Epilogue:

In § 1.10.2 of the chapter 1, we have discussed briefly about Laplace Transformation. Here we are narrating a short history about how Laplace Transformation originate and of those genius scholars of who's contributions made it possible to have it in the present form.

The name "Laplace Transformation" used for the integral

$$F(s) = \int_0^\infty f(t)e^{-st}dt$$

was first appeared in Laplace's *Théorie analytique des probabilities* of 1812. However, its re-introduction and importance in solving differential equations and boundary value problems only go back to the 1920s when it replaced **Oliver Heaviside's** *operational calculus* in applications to electrical engineering. Heaviside had developed a strong interest in long distance telegraphy when he worked as an operator. To solve many problems posed by telegraphy, he perfected a method to solve ordinary and partial differential equations that regards differentiation as an operator. For instance, to solve the simple differential equation $y'' - y = 1$ for t > 0 subject to initial conditions $y(0) = y'(0) = 0$, Heaviside used p for the differential operator d/dt and obtained the equation $p^2y - y = \mathbf{1}$, where **1** denote the function that vanishes for t < 0, and has value 1 for $t \geq 0$.

Hence, treating p as an algebraic quantity, one finds

$$y = \frac{1}{p-1}\mathbf{1} \tag{1}$$

This cannot be the end of the line, of course. To obtained the actual solution from this operational calculus the geometric series expansion is assumed as valid for this operator and then

$$\frac{1}{p^2-1}\mathbf{1} = \left[\frac{1}{p^2} + \frac{1}{p^4} + \frac{1}{p^6} + \ldots\ldots\ldots\ldots\right]\mathbf{1}$$

Since p represents differentiation, Heaviside regarded $1/p$ as integration from zero to t. In this manner

$$\frac{1}{p}\mathbf{1} = \text{t and } \frac{1}{p^n}\mathbf{1} = \frac{t^n}{n!}$$

for any positive integer n, leading to the actual solution

$$y(t) = \frac{t^2}{2!} + \frac{t^4}{4!} + \frac{t^6}{6!} + \dots\dots\dots = \frac{e^t + e^{-t}}{2} - 1$$

which, perhaps, surprisingly, was the correct solution.

Heaviside's methods were unrigorous and even more confusing when applied to partial differential equations. His pioneering works, both practical and theoretical, was the object of frequent and serious opposition, but his genius was eventually recognized by Lord Kelvin and Sir Oliver Lodge. Attempts were made by other researchers to justify Heaviside's *Operational Calculus* in solving a differential equation with **1** on the right-hand side. As early as 1915, Thomas John I'A. Bromwich (1875-1929), an English mathematician, did it by resorting to the theory of Complex integration, but in 1927 he admitted that Heaviside's method is easier to use. However, at that time he developed a second method, soon favored by many engineers, in which he followed Heaviside to obtain the operational solution, but then used complex integration to recover the actual solution from it. Meanwhile, John R. Carson, of the American Telephone and Telegraph Company, had come up with some idea of his own in 1917. He also obtained the operational solution first à la Heaviside, call it $G(p)$, and then showed that the actual solution $y(t)$ is a solution of the integral equation

$$G(p) = p\int_0^\infty y(t)e^{-pt}dt\,.$$

The right-hand side is the Laplace transform of y except for the notation and the extra p in front of the integral. The first one to regard this equation as a transformation of $y(t)$ into $G(p)$, and then Bromwich's complex integration as the inverse transformation that obtains $y(t)$ from $G(p)$, was Balthasar van der Pol (1889-1959), of the *Philips Gloeilampenfabriken* in Holland, in 1929. Harry Bateman (1882-1946) was the first to use the Laplace Transform method in its present form in a 1910 paper on the equation of radioactive disintegration, but it seems that this publication didn't make an impact at the time. Later on the book named 'Tables of Integral Transforms', published in 1954. This was the outcome of Betaman's Manuscript Project.

Bromwich, Carson and several other workers (in the field) followed Heaviside's method to find the operational solution. For this they were to be taken to task by the German Mathematician Gustav Doetsch (1892-)-who had already been using the Laplace transform for some time in solving integral and differential equations-in his 1930 review of Carson's 1926 book *Electrical Circuit Theory and Operational Calculus.* Doetsch view was that Carson should have transformed the equations from the start instead of using Heaviside's unrigorous method to obtain the operational solution. Doetsch also stressed the importance of the corresponding inversion formula - already anticipated by Poisson in 1823 - and went on to develop the properties and applications of the transform in his 1937 book *Theorie und Anwengdun der Laplace-Transformation*. the first of several he wrote on the subject. He also switched to the letter s as the variable for the transform, stating as a reason that p looks like a positive constant while s looks like a variable and just next to t, the variable for f, in the alphabet.

Here is how Doetsch's method replaces Heaviside's operators in solving the initial value problem $y'' - y = 1$, $y(0) = y'(0) = 0$. First multiply the equation by e^{-st}, where s is real and positive, and then assume that $y(t)e^{-st}$ and $y'(t)e^{-st}$ approach zero as $t \to \infty$, and integrate from 0 to ∞. Assuming also that the integrals below converge, integrating by parts, and denotes the transform of y by Y, we obtain

$$\int_0^\infty y'(t)e^{-st}dt = \left[y(t)e^{-st}\right]_0^\infty + s\int_0^\infty y(t)e^{-st}dt = -y(0) + sY(s) = sY(s)$$

$$\int_0^\infty y''(t)e^{-st}dt = \left[y'(t)e^{-st}\right]_0^\infty + s\int_0^\infty y'(t)e^{-st}dt = -y'(0) + s^2Y(s) = s^2Y(s)$$

Then, the given initial value problem is transformed into $s^2Y(s) - Y(s) = \mathcal{L}[1]\ (s)$, that is

$$Y(s) = \frac{1}{s^2 - 1}\mathcal{L}[1]\ (s) \qquad (2)$$

Which is a more palatable version of (1), and this is what we at present get. It is seen that there are some differences in computing the Laplace Transforms of new functions from those of old ones. Several properties of this method were then known as generating function as proposed by Abel.

Life and Work of Oliver Heaviside (1850-1925)

Oliver Heaviside, the miracle-ironed man who was behind such marvelous discovery, had to spent a miserable life during the time of new discoveries and other part of his life. How he had faced obstacles in the discoveries, are the following some sketch.

He had no university degree, and worked as an operator at the Great Northern Telegraph Company in Newcastle from 1868 until his resignation in 1874. This was the only employment he ever held in his life, which he largely spent in dire poverty and seclusion. On the basis of his pioneering work, he was elected Fellow of the Royal Society in 1891. Heaviside published two papers in the *Proceeding of the Royal Society*, but when no more of them were accepted because his methods were unrigorous – although he always obtained correct results. Heaviside had a natural scientific habit. So, though his works were not recognized at the time, he went on doing research works and resumed publishing in the *Electrician.* He was made referee of the Royal Society and other scienticulists guilty of looking at the gift horse in the mouth. His numerous journal articles – which he started publishing at the age of 22 – were eventually gathered in two collections: **Electrical papers** (EP), in two volumes, in 1892 and **Electromagnetic theory** (EMT), in three volumes, in 1899 and 1912. Second volume of EMT starts with an assertion of Heaviside's belief that mathematics is an experimental science – the only Mathematical work that he truly admired was Fourier's – and then he put his view of mathematical rigor in a nutshell by asking: 'Shall I refuse my dinner because I do not fully understand the process of digestion ?'

Oliver Heaviside's legacy to mathematics and electromagnetism is impressive. In addition to perfecting the operational calculus that later inspired the Laplace Transform method, he developed vector calculus in 1885, starting with the definition of scalar and vector products as used today. In the same year he formulated what has become the cornerstone of electromagnetic theory. Heaviside refers to his discovery as follows:

I here introduce a new method of treating the subject [Maxwell's theory of Electromagnetism], which may perhaps be appropriately named the Duplex method, since its main characteristic is the exhibition of the electric, magnetic, electromagnetic equations in a duplex form.

This was the first appearance in print of the famous Maxwell's equations of electromagnetism theory.

Heaviside's contribution to telegraphy and telephony was invaluable but for the longest time they fell on deaf ears in his own country. He found a formidable obstacle in William Henry Preece, *Electrician to the post Office*. Heaviside was a very strong, brave, deterministic, and clear-cut man. The following remarks put against W. H. Preece showed this characteristic.

Either, firstly, the accepted theory of electromagnetism must be most profoundly *modified; or, secondly, the view expressed by Mr. Preece in his paper are profoundly erroneous .... Mr. Preece is wrong, not merely in some points of detail, but radically wrong, generally speaking, in methods, reasoning, results, and conclusions.*

W. H. Preece and Sir William did not accept Heaviside's comments on "Inductance". Even they gave trouble in publishing his papers. But Mihajlo Idvorsky Pupin, a Serbian immigrant from the Australia village, a professor of mathematics at Columbia University, acknowledged the Heaviside's contribution by stating the following:

Mr. Oliver Heaviside of England, to whose profound researches most of the existing mathematical theory of electrical wave propagation is due, was the originator and most ardent advocate of wave conductors of high inductance.

Soon afterwards, the American Telephone and Telegraph Company succeeded in establishing coast telephone communication by using increased inductance.

Heaviside was good with words in many ways. To him we owe, for instance, the terms inductance, attenuation, and magnetic reluctance and the use of voltage for electromagnetic force. He was a colorful, entertaining and opinionated writer, as shown by the following additional quotations:

Self-inductance is salvation; As critics can not always find time to read more than the preface, ...; Electric and magnetic force. May they live for ever and never be forgot......; Different men have different opinions – some like apples, some like onions.

Mathematician at large, electrician, philosopher, acid humorist, iconoclast *extraordinaire*, he was awarded – but declined – the Hughes Medal of the Royal Society in 1904, received a Ph. D. *honoris causa* from the University of Göttingen in 1906, was made an honorary member of the Institution of Electrical Engineers of Great Britain in 1908 and of the American Institute of Electrical Engineers in 1918, and was awarded the first Faraday Medal of the Institute of Electrical Engineers in 1923.

At his country home in Torquay, where he spent the last seventeen years of his life, mostly alone and in great financial trouble. He got a small government pension and that too accepted on the condition that it must be in recognition of his scientific work – things were less rosy. For lack of payment the bank was after his home, and the gas company cut off his gas. A victim of lumbago and rheumatic gout, he had to eat cold food and live in a cold house.

On arriving at his door in the winter of 1921, a prominent visitor found a note stating that Heaviside had gone to bed to keep warm. Stuffed in the cracks of the door, to prevent any cold drafts, there was an assortment of papers: some a advertisements, an invitation by the President of the Royal Society, threats from the gas company about cutting off gas ... . The following Spring Headiside wrote:

> *Could not wear boots at all. Could not get proper bed socks to walk about in.*
> *Buried under all the blankets I have. Now and then I scribbled a sort of diary*
> *about my persecution by the Poor the Gas and others.*

Irrepressible in his writing, he continued working on his scientific papers, many of which were found posthumously. He died in a nursing home on February 3, 1925.*

* Courtesy of *"Fourier Analysis and Boundary Value Problems" by Enrique A. González-Velasco.*

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