STUDIES OF COSMOLOGICAL MODELS IN GENERAL RELETIVITY

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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CERTIFICATE

This is to certify that Mr. Naorem Jugeshwor Singh has worked under my supervision for the thesis entitled " STUDIES OF COSMOLOGICAL MODELS IN GENERAL RELATIVITY" which is being submitted to Tezpur University in fulfillment of the requirements for the degree of Doctor of Philosophy. The thesis is Mr. Jugeshwor's own work. It has fulfilled all the requirements under the Ph.D rules and regulations of Tezpur University and to the best of my knowledge, the thesis as a whole or a part thereof has not been submitted in any other University for any degree or diploma.

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ABSTRACT

The thesis will consists of seven chapters. Chapter one will be introductory in nature and will give an account of rotating and non-rotating fluids in general. While describing the background and motivation for taking up the problems, we shall critically review the relevant works of other authors. The topics surveyed, by their very nature, will be centered around the problems we have to tackle. The study of universes will form the subject matter of each chapter.

In chapter II, the dynamics of Lyttleton- Bondi universe involving creation of matter in the presence of a scalar field will be studied considering the Robertson-Walker metric, and their properties will be discussed in detail. We will investigate also the role of scalar field and electromagnetic field in the process of mass creation. Along with finding out of the temporal restrictions and the singularities (which may exist), we will study the importance of such models in the context of modern cosmology.

In chapter III for obtaining exact solutions of charged viscous fluid distribution, we will study the physical and dynamical properties of the model universes emitting radiation and also in special cases of those without emitting radiations. The effects of viscosity and the role played by radiation and their implications on such models will also be studied.

Fourth chapter will focus on standard hot big-bang models involving viscous fluid leading to new interesting solutions. Their properties will be studied from various respective thereby obtaining new idea and information's regarding the evolutions of the universe. Investigation will also be made as to when the Universe ceased to be radiation- dominated and become matter- dominated, pointing out the advantages of our model over the Friedman-model.

Chapter V which is devoted to rotational perturbation of the Robertson-Walker universe will be examined in order to substantiate the possibility of the existence of Massive scalar field. We will study, the exact solutions for metric rotation Ω (r,t) and the matter rotation w (r,t) under different conditions, and also study their nature and role from different angles.

In chapter VI, which is devoted to the vicious fluid distribution cannot be the source for generating gravitational field if a spherically symmetric class one metric is considered. Similarly, the magneto viscous fluid distribution is incompatible with class me metric. And the same result is seen to hold also under certain conditions when the viscous fluid distribution is coupled with scalar field.

In the concluding chapter we will focus on the charged perfect fluid spheres coupled with rotation and their behaviors will be studied as a prelude to knowing minutely the characteristics of pulsars and neutron stars which are great importance in modern cosmology. The effects of the charged field on the rotational motion will be discuss from every possible angles.

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Date : 24. Oct'2005. **(NAOREM JUGESHWOR SINGH)**

STUDIES OF COSMOLOGICAL MODELS IN GENERAL RELATIVITY.

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CHAPTER –I

INTRODUCTION

The investigation in the thesis comprises seven chapters. The first chapter is introductory in nature reflecting the motivation of our work presented in the later chapters. To achieve the main objective of highlighting the important results presented in the thesis. The results of the work of various authors have been presented critically examined relevant to our studies.

It is a well-known fact that no real astrophysical object is composed of a perfect fluid. Despite this, perfect fluid space-time has been studied as models, for instance, Newtron stars. In most astrophysical applications, it seems that perfect fluid models are adequate. However , possibly important changes in properties may occur when dealing with non-perfect fluid sources in highly compact bodies. In this light it has been conjectures [Ellis (1971)] that sometimes during an early phase in the evolution of the Universe when galaxies were formed, the material behaved like a viscous fluid.

In perfect fluid, the mean free paths and times are so short that perfect isotropy is maintained about any point moving with the fluid. But for the imperfect fluids, pressure, density and velocity vary appreciably over the distances of the order of mean free path, or over times of the order of the mean free time or both. In such fluids, thermal equilibrium is not strictly maintained, and the fluid kinetic energy is dissipated as heat. The effect of energy dissipation , occurring during the motion of fluid on that motion itself is studies by various authors. This process is the result of the thermodynamic irreversibility of the motion. This irreversibility always occurs to some extent and is due to internal friction (viscosity) and thermal conduction. The viscosity of a physical distribution of matter is that characteristic of distribution which exhibits a certain resistance to altercations of the form. This aspect has motivated us to take imperfect fluid as the energy-momentum tensor in Einstein's fluid equations and study the consequences in details.

COSMOLOGY :

The word " Cosmology" originates from the Greek word " Kosmos". Hence, cosmology means the scientific study of structure of the universe. The study of the cosmos has been made through the centuries and in the year 1917 Albert Einstein applied his General theory of relativity to the structure of the universe as a whole.

The study of cosmos does not confine to that of our own galaxy but goes as far as exploring further up to the limits of the observable universe by looking back into the past to the moment of initial expansion about twelve thousand million years ago. In general relativity physicists tried to understand the structure of the universe with the help of Einstein's General theory of relativity through Newtonian theory of gravitation. It is the initial concept that gravitation plays a vital role in the structure of the universe because of the involvement of huge masses in the picture. There are, however short range or long range common interactions, not to mention the gravitation alone; this plays an important role in universal scale. Electromagnetic theory, through the only other long range theory, is of little help because the galaxies and the intergalactic medium are believed to be electrically neutral.

Newtonian cosmology as is known at present does not achieve much as it is not successful in making us arrive at a static model of the universe. Einstein , too faced the same problem with his well known relativistic frame work, that the stationery nature of objects in the universe with our knowledge of stars in our own galaxy can be regarded as the complete replacement by the red-shift emitted from the extra-galactic nebulae showing that the objects in the universe are non-static and moving away from each other.

The general theory of relativity with the cosmological principle and Wely's postulate together can lead to the direction of various relativistic cosmological models easily. That overall distribution of masses determines the idea related to inertia was successfully learnt by Einstein from Machian. The principle of equivalence incorporate the combination of the concept of inertia and gravitation.

In 1917 both Einstein and de-sitler studied the application to the cosmological problem on the basis of Einstein field equations. They assume that the universe is, as a whole, spaciously homogenous and isotropic when looked through a large scale view point. The study of cosmological model is firstly based on cosmological principle.

The general theory of relativity is applicable to the whole universe as is shown by Einstein in the following way :-

" On account of our observations on fixed stars we are sufficiently convened that the system of fixed stars does not in the main resemble and islands which floats in infinite empty space, and that there do not exist anything like a centre of gravity of the total amount of existing matter. Rather we feel urged towards the conviction that there exists an average density of matter is space which differs from zero".

The Einstein and de-sitters initial cosmological models can be framed within the theoretical frame-work of general relativity and might correspond to an extent with the initial and final states of actual universe. But Einstein and de-sitter's static cosmological models were replaced by non-static models for the reasons given below :-

(i) Einstein's model permits no shifts in the wave length of light from the nebulae and de-sitter's model permits non-existence of matter or radiation in the space. Hence static model cannot give a satisfactory picture of the present state of actual universe.

(ii) The applicability of relativistic mechanics is increased by removing the assumption of static model. Really, there are some processes in the universe for example, the emission of radiation from the stars that results in the change of gravitation with time which leads to the non-static model.

The non-static line element in the cosmological model was obtained at first by Robertson (1929). Based on the cosmological principle, the fundamental properties of light and definition of substratum including the assumption (Weyl's postulate) that the paths of the particles do not intersect except possibly at one singular point in the past, Roberstson (1935) and Walker (1936) obtained the metric independently for the non-static cosmological model of the universe.

In particular the inclusion of the cosmological term allows the existence of a static cosmological solution for the gravitational field equations. It follows however, that the physical motivation for adding the cosmological term is unfounded and therefore there is no need for it. Einstein assumed that $\Lambda^{-1/2}$ is of the order of magnitude of the radius of the universe, namely $\Lambda^{-1/2} = 10^{10}$P.C.$\approx 10^{28}$ cm. If Λ is taken to be positive, then the cosmological term Λg_{ij} contributes a repulsive force term which varies as the square of the distance between the material bodies producing the gravitational field.

Various arguments have at times been given against the inclusion of the cosmological term in the general form:

1. that it was only an after though of Einstein's (but better discoursed late than never);

2. that Einstein himself eventually rejected it (but authority is no substitute for scientific argument);

3. that with it the well-established theory of special relativity is not a special case of general relativity(but locally the Λ-term is totally unobservable);

4. that it is ad-hoec(but from the formal point of view it belongs to the field equations such as an additive constant belongs to an infinite integral);

5. that similar modifications could be made to Poisson's equation in Newton's theory and Maxwell's equations in electrodynamics (but in general relativity, matter and space are intimately related by the field equations and no mechanical picture is corrected) ;

6. that one should never envisage a more complicated law until a simpler one proves untenable (but in cosmology-especially in the Robertson-Walker case-the technical complication is slight and several recent investigations have suggested that the Λ term indeed may be needed to account for the observations);

7. and more technically that a Λ – term in the geometry would destroy the possibility of quantilizing gravity (but Zeldovich has suggested that an energy tensor $\frac{\Lambda}{k} g_{ij}$ may arise naturally out of quantum fluctuations in vacuum, so that the Λ – term could be regarded as part of the sources rather than part of the geometry.)

These general field equations, then, must be satisfied jointly by the metric of space time and by the energy tensor-relative to the metric of the contents of space-time. In cosmology we are to formulate to be able to restrict the metric considerably by symmetry arguments alone.

COSMOLOGICAL PRINCIPLE :-

The assumption of large scale homogeneity, all together with the assumption of large scale isotropy is called the cosmological principle. The cosmological problem within the frame-work of general relativity consists in finding models of the universe as a whole which is a solution of Einstein equations. The mathematical test of solving the cosmological problem consists in determining a large –scale metric of four dimensional world and a corresponding large-scale mass energy distribution satisfying Einstein's equations. In order to solve such problems one has to build a cosmological theory which is expected to fulfill the following criteria.

1. It must predict an isotropic universe with uniform characteristics everywhere, to conform to the picture of the actual universe uniformally filled with a nearly content matter density.

2. The universe should present the same aspect to all observers, situated in any region of the universe at a given time.

3. A theory of the universe must provide an explanation of the linearly increasing red shift of light for matters of increasing distance from any observer.

Due to the nature of astronomical observations, no means is available to verify the red shift of spectral lines from the distance galaxies. The cosmological red-shift might be assumed to be gravitational in origin but such an assumption violates the first two requirements. The viewpoint adopted in cosmological theory has not assumed a gravitational explanation for the red shift to be a Doppler shift caused by motion of the distant galaxies away from observers on earth.

The most powerful assumption in standard cosmological theory is the second criteria where the universe is spatially homogenous and isotropic, often referred to as the cosmological principle. It can be formulated as statement about the existence of equivalent co-ordinate systems.

Bondi and Gold's (1949) generalization of cosmological principle termed as " perfect cosmological principle" assumes that the universe seems to be the same irrespective of position or time of an observer in natural motion. In this way the perfect cosmological principle is satisfied only by the steady state model of the universe.

COSMOLOGICAL SOLUTIONS :-

In order to find out the cosmological solutions in Einstein's theory one usually chooses the energy-momentum tensor of matter as that due to a perfect fluid. All these models lead to an initial singular state. It is interesting to study cosmological models with a more realistic matter source by taking into account dissipative processes due to viscosity. The introduction of viscosity counteracts gravitational collapse; hence one might expect that the initial state of cosmological models could be change by this dissipative processes.

The main aim to obtain cosmological solution is to study, how-does-the introduction of viscosity effects the singularity problem in Friedmannian cosmology. However, Heller and Klimer (1975) considered the case in which the co-efficient of bulk viscosity is a function of density and has shown that all solutions of the considered class are regular. In 1983, Banerjee and Santosh obtained an exactly solvable Bianchi type-I cosmological model with a viscous fluid. It showed that the role of viscosity is more important in the initial epochs of the universe and , in that same period, pressure is more important than fluid density. Maiti (1982) obtained a cosmological solution of Einstein's equations which admits a transitive group of motion for static spherically symmetric metric with a viscous fluid distribution as source.

ROTATION AND PERTUBATION :-

In the recent years various authors [Silk and Wright (1969), Bayin and Cooperstock (1980)] studied the role played by rotation in stellar models. When any gravitational body is rotating slowly the rotation can be considered as a small perturbation on an already known non-rotating configuration. In Newtonian theory the presence of a massive body does not affect the determination of an inertial frame . But in GRT, a massive body tends to drag the inertial frame along with it. The difference between the angular velocity and the dragging rate (the rate of rotation of the inertial frames inside the massive body) governs the centrifugal forces acting on the star [Thirring (1918)] Axial and reflection symmetries are resulted from slow rotation of configuration, and configuration which minimizes the total mass energy must rotate uniformly. Banerji (1968) considered dynamics of homogenous rotating cosmological models with non-vanishing pressure. It appears from these that such model must in general be associated with shear if there be a linear relation between the pressure and the density except perhaps in the case

$$p = \frac{\rho}{9}.$$

The slowly rotating solid star with a spherical relaxable structure may be treated as a perturbations of a rotating star with some elastic structure. The perturbed state differs from the spherical state by a purely elastic deformation. This approach has the advantage that the equations of the rotating fluid star can be solved instantaneously since they are deduced as a perturbation from the same spherical state. According to Roy Chaudhuri (1979) the study of the fate of perturbation of isotropic models has twofold importance. Firstly, it would be justified to take the isotropic models as affair representation of the actual universe only if the models show reasonable stability. Secondly, the formation of condensation like the observed galaxies is out of homogeneous distribution.

At the first sight these two requirements- stability on one hand and growth of perturbation on the other hand- may appear to be mutually contradictory. However, one may make a compromise: a perturbation on a linear scale much larger than the dimensions and average separation of the galaxies should be smoothed out while those of a certain critical dimension should grow giving to observed galaxies.

GENERAL FORMULAE USE :-

FORMULAE :-

(a) TIMELIKE VECTOR FIELD:- The vector field in four dimensional Riemannian spaces are frequently characterized by the properties of their first derivatives (covariant) and invariants which can be build from these derivatives . One of the most important examples of time like vector field in four dimension is velocity field $U(x^i)$, $U_\alpha U^\alpha = -1$ of a matter distribution.

1. ACCELERATION :- The acceleration $\overset{\circ}{U}_\alpha = U_{\alpha,\beta}U^\beta$ gives the combined effects of the gravitational and inertial force on the fluid. In the absence of non-gravitational interactions U_α would vanish . It is space like as $U^\alpha U_\alpha = -1$ implies that $\dot{U}_\alpha U^\alpha = 0$

2. ROTATIONAL VELOCITY :- The rotational velocity $W_{\alpha\beta}$ is an antisymmetric vorticity tensor which vanishes if the velocity vector is hyper- surface-orthogonal. It determines the rigid rotation of the cluster of galaxies with respect to a local inertial rest frame. We may express vorticity by the vorticity vector W^μ where

$$W^\mu = \frac{1}{2}\eta^{\mu\upsilon\alpha\beta} \cdot U_\upsilon W\alpha_\beta \Leftrightarrow W_{\alpha\beta} = \eta_{\alpha\beta\mu\upsilon} W^\mu U^\upsilon$$

So that $W^\alpha U_\alpha = 0 = W^{\alpha\beta} U_\beta$; $W^\alpha W_{\alpha\beta} = 0$.

The direction of this vector is the axis of the rotation of the matter, since if we choose η_α in the direction of W_α when the vorticity alone is non zero, we find that this direction is left invariant by the rotation . Its magnitude is the vorticity W, where

$$W \; (W^\alpha W_\alpha)^{1/2} = (\frac{1}{2} W^{\alpha\beta} W_{\alpha\beta})^{1/2} \; ;$$

As $W_{\alpha\beta}$ is space like, $W \geq 0$ and $W = 0 \Leftrightarrow W_\alpha = 0 \Leftrightarrow W_{\alpha\beta} = 0$

If the vorticity vanishes once for an element of the fluid, it will vanish always.

3. SHEAR VELOCITY :- The shear velocity $\sigma_{\alpha\beta} = U_{(\alpha;\beta)} + U_{(\alpha u \beta)} - \theta h_{\alpha\beta/3}$

$\sigma_{\alpha\beta} = \frac{1}{2} (U_{\mu;\beta} H^\beta{}_\upsilon + U_{\upsilon;\beta} H^\beta{}_\mu) - 1/3 \; \theta \; H_{\mu\upsilon}$ is a second order symmetric tensor, trace free, orthogonal to velocity vector and it gives the change of shape of space elements orthogonal to U^β .

4. EXPANSION VELOCITY:- The expansion velocity $\theta \equiv U^{\alpha}$; α gives the rate of dilation of a three space element locally orthogonal to the vector U^{α} . Its magnitude is independent of direction ; a volume element is thereby magnified on diminished in size with its form preserved.

5. PROJECTION TENSOR :- The projection tensor $h_{\alpha\beta} = g_{\alpha\beta} + U_{\alpha} U_{\beta}$ is orthogonal to the velocity vector U^{β} .

(b) ENERGY – MOMENTUM TENSOR :-

(i) PERFECT FLUID :-The energy –momentum tensor for perfect fluid is given by

$$T^{ij} = (-\rho + P) U^i U^j - pg^{ij}$$

Where ρ is the possession of a proper density , P is a scalar pressure and U^i is the four velocity vector.

(ii) VISCOUS FLUID :-The energy-momentum tensor for viscous fluid is given by

$$T_{ij} = \rho U_i U_j - (\rho - \varepsilon\theta) H_{ij} + 2\eta\sigma_{ij}.$$

Where ρ and P are the density and isotropic pressure of the distribution respectively and η,ε are the co-efficient of shear and bulk viscosity respectively.

H_{ij} is the projection tensor and σ_{ij} is the shear tensor.

(iii) ELECTROMAGNETIC FIELD:- If we take a flowing field of charged matter which is described by a proper density $\tilde{e}(x^i)$, a four-vector velocity U^i and a proper electric charge density $\varepsilon(x^i)$, then for this free electro-magnetic field the energy-momentum tenser is given by

$$T^{ij} = \frac{1}{4\pi G}\left[\frac{1}{4} g^{ij} F^{lk} F_{lk} - Fl^{i} F^{lj}\right] \text{ along with } F^{ij}{}_{;j} = -\varepsilon U^{i} \,;\, F_{[ij;l]} = 0$$

RADIATION FIELD :- A distribution which is non-statics will radiate energy, and therefore it will be surrounded by an ever-expanding zone of radiation. So for the outside field of a non-static mass, or otherwise for s directed flow of radiation in empty space , we have the energy-momentum tensor in the form

$$T^{ij} = \sigma U^{i} U^{j} \text{ along with } U_j U^{j} = 0 \,;\, U^{i}{}_{;j} U^{j} = 0$$

SCALAR FIELD :- Scalar fields help in explaining the creation of matter in cosmological theories, represent matter fields with quanta (spinless) and can also describe the gravitational fields. For our invertigations we have considered a particular scalar field, namely, the scalar mensen field. The scalar meson fields are of two types; massive scalar field and zero-rest mass scalar fields.

The energy-momentum tensor for massive scalar field (Yukawa field of spin zero) is given by

$$T^{\mu\gamma} = \phi^{\mu} \phi^{\gamma} - ½ g^{\mu\gamma} (\phi_{\alpha}\phi^{\alpha} - M^{2} \phi^{2})$$

Satisfying the relativistically invariant Klein-Gorden equation

$$g^{\mu\gamma} \phi_{;\mu\upsilon} + M^{2} \varphi = \psi$$

where $\phi (x^{i})$ is the potential of the scalar field and $\psi (x^{i})$ is the source density of the scalar field and $\psi (x^{i})$ is the source density of the scalar field, and M is related to the mass of Zero-spin particle by $M = \frac{m}{\hbar}$; where $h = \frac{\hbar}{2\pi}$ (h being the Planck's constant).

For the zero-rest mass scalar field the expression for energy momentum tensor is obtained by putting M =0 in the above expression. In this case Klein-Gorden equation reduces to the form.

$$g^{\mu\upsilon} \phi_{;\,\mu\upsilon} = \psi$$

SPECIAL CASES :- To solve Einstein field equations one often uses the co-moving system , $U^{\alpha} = (0,0,0, U^4)$. If the rotation $W_{\alpha\beta}$ vanishes, then the flow given by U^i is hypersurface orthogonal and the metric can be brought in the form.

$$d s^2 = g_{lm}\, dx^l\, dx^m - U_4^2 dt^2$$

If one writes down the covariant derivative $U_{\alpha\,;\beta}$ explicity in this metric and compares the results with the equations of projection tensor, shear tensor etc., then one can show that [Stephani (1982)];

(I) For $W_{\alpha\beta} = 0$ and $\sigma_{\alpha\beta} = 0$ the metric $g_{\alpha\beta}$ of the three space contains time only in a factor common to all elements.

$$g_{\alpha\beta}(x^{\mu}) = V^2 (x^{\mu}, t)\ \bar{g}_{\alpha\beta}(x^{\mu}).$$

The stress tensor will be that for a perfect fluid, the universe will be completely isotropic and spatially homogeneous.

(II) For $W_{\alpha\beta} = 0$ and $\theta = 0$ the determinant of the three space metric does not depend upon time.

(III) For $W_{\alpha\beta} = 0$ and $U_{\alpha} = 0$ one can transform μ^4 to c.

If the expansion and shear vanish ($\theta = 0$ and $\sigma_{\alpha\beta} = 0$) but not the rotation ($W_{\alpha\beta} \neq 0$) then for the co-moving observer the distances to neighbouring matter elements do not change, and we have a rigid rotation. The dust universes which exhibit the rigid rotation and spatial homogeneity are the Einstein static universe and the Godel stationery model.

Spatially homogeneous universes having non-vanishing rotation and expansion must have a non-vanishing shear. This is also true for dust and perfect fluid universes. In a static star model, we have

$$\theta = \sigma = w = 0$$

In a general fluid flow, both w and σ will be non -zero. Ellis (1971) has shown that there exists a direction left invariant by rotation. Since $W_{\alpha\beta}$, $\sigma_{\alpha\beta}$ and θ determine the relative motion of galaxies in a cosmological model we should like to determine the values of these quantities in the observed universe. However, in principle it is possible to compare the observations with the theoretical expressions for the above quantities at the present time t_0 . The value $\theta_0 = 3\ H_0 = 3 \times (1.3 \times 10^{10}$ years) is probably correct to within a factor 2 [Sciama (1971) and Burbridge (1971)], but we can only obtain rather poor limits on w_0 and σ_0 from direct observation. The condition that the systematic motion of galaxies is away from in all directions (there are no directions in which we see a systematic blue shift effect in galactic spectra) imposes the restriction.

$$\sigma_0 < \frac{1}{3}\,\theta_0.$$

More detailed examination of the direct evidence gives us the limits [Kristian and Sachs (1966)]

$$W_0 \leq \frac{1}{3}\theta_0\ ,\ \sigma_0 \leq \frac{1}{4}\theta_0$$

IDENTIFICATION OF VISCOUS FLUID WITH OTHER $T_{\mu\gamma}$:

Synge (1966) posed a question : Given a metric tensor that does in fact, lead to a $T_{\mu\gamma}$ satisfying the energy conditions, how do we know what type of field this energy tensor represents ? To put it in another way, is it possible that $T_{\mu\gamma}$ is not a uniquely viable energy tensor i.e. can the energy tensors of two apparently different fields be identical in that they have precisely the same components.

Griffiths (1972) has shown that energy tensor of the neutrino field can be identically zero so that the corresponding space-time is also a vacuum solution.

This is the case of the so-called " ghost-neutrinos" ; considered as neutrino solutions they are pathological. The neutrino energy tensor can also be identical with the energy tensor of a null electromagnetic field provided that certain conditions hold. However, the neutrino field is not a classical field, and relating a non-classical field to a classical field seems a somewhat dubious procedure. The problem of identification of different energy tensors can be discussed in the realm of special relativity with a background of Minkowski space-time. However, we are interested in energy tensors which are not only identical, but also each of which forms the right hand side of a set of field equations of the form $G_{\mu\gamma} = k\, T_{\mu\gamma}$ with the identical Einstein tensor $G_{\mu\gamma}$ on the left hand side, and so we shall confine our attention to the general relativistic problem. Tupper (1981) has shown that space-time satisfying the Einstein-Max well equations, either in vacua or coupled with a perfect fluid may also satisfy the field equations for a viscous fluid. For example, the electrovac solutions of Reissner and Nordstrom as well as the Kerr-Newman metric may be alternatively interpreted as due to viscous fluid distribution. With this in mind, Saha (1983) studied viscous fluid motion in the Kerr-Newman black hole in some details and some interesting features of motion in the black hole region are pointed out. However Raychauduri and Saha (1981) investigated the Tupper's (1977,1981) problem and are led to conclude that the viscous fluid interpretation is possible only if the electrovac solution possesses a certain symmetry property. Their results allow them to cite a counter example to Tupper's idea from known electrovac solutions. Novello (1979) noted that Tupper's interpretation allows one to discover some cosmological soliditions having some what unusual properties.

Coley and Tupper (1983 a, b) extended their earlier work and has shown that FRW cosmologies, in particular the zero curvature and Einstein-de-Sitter models do not necessarily represent perfect fluid solutions but also can be exact solutions of the field equations for a viscous fluid, with or without an electro magnetic field and these solution can be physically acceptable.

Recently again Coley and Tupper (1984) gave a solution of Einstein's field equations that represents the collapse of realistic matter distributions. The authors have exploited their earlier ideas that a given energy-momentum tensor may formally represent different types of matter distribution. Obozov (1982 a,b) obtained a necessary and sufficient condition for the gravitational fields of a perfect fluid and a viscous fluid to be conformal to a flat space-time.

CHAPTER – 2

LYTTLETON -BONDI UNIVERSE INVOLVING CREATION OF MATTER COUPLED WITH A SCALAR FIELD

2.1 INTRODUCTION

A cosmological model was developed by Lyttleton and Bondi assuming there is a continuous creation of matter on account of the net imbalance of the electromagnetic charge. In such type of universe the electromagnetic field is modified taking into consideration the process of creation. Lyttleton and Bondi, Burman, and Rao and Panda studied the different properties of such a universe. The dynamics of Lyttleton-Bondi universe were further investigated by Nduka , Reddy, and Reddy and Rao. As it is well known that the creation of particles is connected often with a scalar field, here in this problem the case of Lyttleton- Bondi universe coupled with a scalar field is discussed in detailed . Obtaining some new interesting model solutions we study the dynamics of such a universe and investigate the role of scalar field in the process of mass creation. The temporal restrictions and the conditions for the existence of these solutions are also discussed, taking into account for the singularities which may exist in the course of critically examining these different models. With the study of these models we may be able to have some interesting information's regarding the early universe.

2.2 FIELD EQUATIONS AND THEIR SOLUTIONS:-

For this problem we consider the Robertson-Walker metric.

$$ds^2 = dt^2 - R(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi\right], \qquad \text{.........(1)}$$

The energy-momentum tensor here is

$$T_{\mu\upsilon} = E_{\mu\upsilon} + S_{\mu\upsilon}, \qquad \text{.........(2)}$$

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where $E_{\mu\upsilon}$ is that due to an electromagnetic field and is given by

$$E_{\mu\upsilon} = \left(\frac{1}{4} g_{\mu\upsilon} F_{\mu\upsilon} F^{\mu\upsilon} - F_{\mu j} F_{\upsilon}^{j} \right) + \lambda \left(A_{\mu} A_{\upsilon} - \frac{1}{2} g_{\mu\upsilon} A_{k} A^{k} \right), \ldots\ldots\ldots \quad ..(3)$$

A_u being the four potential vector and $F_{\mu\upsilon}$ the anti-symmetric electromagnetic field tensor. Here in order to incorporate the idea of creation modified form of Maxwell field equations are

$$F_{\mu\upsilon} = A_{\mu,\upsilon} - A_{\upsilon,\mu}, \ldots\ldots\ldots \quad (4)$$

$$F^{\mu\upsilon}_{;\mu} = J^{\mu} + \lambda A^{\mu}, \ldots\ldots\ldots \quad ...(5)$$

$$J^{\mu}_{;\mu} = q, \ldots\ldots\ldots \quad .(6)$$

where J^{μ} is the current density four vector, q the rate of creation of charge per unit proper volume, and λ a constant. $S_{\mu\upsilon}$ is energy-momentum tensor due to the zero-mass scalar field and takes the form

$$S_{\mu\upsilon} = \frac{1}{\phi^{2}} \left(\phi_{\mu} \phi_{\upsilon} - \frac{1}{2} g_{\mu\upsilon} \phi^{k} \phi_{k} \right), \ldots\ldots\ldots \quad ..(7)$$

where ϕ is the scalar potential field which satisfies the relation

$$\frac{\partial}{\partial x^{\upsilon}} \left[\phi_{a} (-g)^{\frac{1}{2}} g^{a\upsilon} \right] - \frac{\phi_{a} \phi_{\upsilon}}{\phi} (-g)^{\frac{1}{2}} g^{a\upsilon} = 0 \ldots\ldots\ldots \quad ...(8)$$

Now for $F_{\mu\upsilon} = 0$ (zero electromagnetic field),

we may write,

$E_{\mu\upsilon} = \lambda(\frac{1}{2} g_{\mu\upsilon} A_k A^k - A_\mu A_\upsilon)$,..........(9)

Here A_μ takes the form

$$A_\mu = (\beta,0,0,\psi) \text{ ,..........(10)}$$

Thus $F_{\mu\upsilon} = 0$ leads to

$$\frac{\partial \beta}{\partial t} = \frac{\partial \Psi}{\partial r} \text{ ,..........(11)}$$

Therefore from relations (1) and (11) we have

$$A^\mu = \left[\left(\frac{1 - kr^2}{R^2} \right) \beta ,0 ,0 , \Psi \right] \text{ ,..........(12)}$$

thereby getting

$$A_\mu A^\mu = \left(\frac{1 - kr^2}{R^2} \right) \beta^2 - \Psi^2 \text{ ,..........(13)}$$

On the assumption that the charge created will not affect the dynamical characteristics of the matric and the mechanical effect of such creation on the energy-momentum tensor is nil, the Einstein's field equations can be used here. Thus corresponding to the field equation.

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R = - 8 \pi G T_{\mu\upsilon} \text{ ,..........(14)}$$

we get four equations

$$\frac{1}{R^2}\left(2R\ddot{R}+\dot{R}^2+k\right)=$$

$$-4\pi G\lambda\left[\left(\frac{1-kr^2}{R^2}\right)\beta^2+\Psi^2\right]-4\pi G\left(\frac{\dot{\phi}}{\phi}\right)^2-4\pi G\left(\frac{1-kr^2}{R^2}\right)\left(\frac{\phi'}{\phi}\right)+\Lambda,\ldots\ldots\ldots\ldots(15)$$

$$\frac{1}{R^2}\left(2R\ddot{R}+\dot{R}^2+k\right)=$$

$$4\pi G\lambda\left[\left(\frac{1-kr^2}{R^2}\right)\beta^2-\Psi^2\right]-4\pi G\left(\frac{\dot{\phi}}{\phi}\right)^2+4\pi G\left(\frac{1-kr^2}{R^2}\right)\left(\frac{\phi'}{\phi}\right)^2+\Lambda,\ldots\ldots\ldots\ldots(16)$$

$$\frac{3}{R^2}\left(\dot{R}^2+k\right)=$$

$$4\pi G\lambda\left[\left(\frac{1-kr^2}{R^2}\right)\beta^2+\Psi^2\right]+4\pi G\left(\frac{\dot{\phi}}{\phi}\right)^2+4\pi G\left(\frac{1-kr^2}{R^2}\right)\left(\frac{\phi'}{\phi}\right)^2+\Lambda,\ldots\ldots\ldots\ldots(17)$$

$$\left(\frac{1-kr^2}{R^2}\right)\lambda\beta\Psi+\frac{\phi'\dot{\phi}}{\phi^2}=0.,\ldots\ldots\ldots\ldots(18)$$

Again from relation (8) we get

$$\left(\ddot{\phi}+3\frac{\dot{R}}{R}\dot{\phi}-\frac{\dot{\phi}^2}{\phi}\right)R^2-\left[\phi''-\frac{\phi'^2}{\phi}+\frac{2-3kr^2}{r\left(1-kr^2\right)}\phi'\right]\left(1-kr^2\right)=0,\ldots\ldots\ldots\ldots(19)$$

which gives

$$\frac{\overset{\circ}{\phi}}{\phi} = aR^{-3}, \qquad (20)$$

and

$$\frac{\phi'}{\phi} = br^{-2}\left(1 - kr^2\right)^{-\frac{1}{2}}, \qquad (21)$$

where a and b are arbitrary constants.

Now equations (15) and (16) gives

$$\lambda\beta^2 + \left(\frac{\phi'}{\phi}\right)^2 = 0, \qquad (22)$$

which implies

$$\beta = 0, \qquad (23)$$

and

$$\frac{\phi'}{\phi} = 0, \qquad (24)$$

It may be noted here that in view of the relations (23) and (24), equation (18) is automatically satisfied and equations (15) and (16) become the same.

Now adding up equations (15) and (17) we get

$$R\overset{\circ\circ}{R} + 2\overset{\circ}{R}^2 + 2k = \Lambda, \qquad (25)$$

which gives

$$R = \pm\left(\frac{\Lambda}{2} - k\right)^{\frac{1}{2}} t + C, \qquad (26)$$

where C is an arbitrary constant. Therefore from (20) , we get

$$\phi = a_0 \exp\left[\pm\left(\frac{a}{2}\right)\left(\frac{\Lambda}{2}-k\right)^{\frac{-1}{2}}\left\{\pm\left(\frac{\Lambda}{2}-k\right)^{\frac{1}{2}}t+C\right\}^{-2}\right], \text{...}(27)$$

where a_0 is an arbitrary constant.

The equation (17) gives

$$\Psi = \pm\frac{1}{2}(\pi G\lambda)^{\frac{-1}{2}}\left[\frac{3\lambda}{2}\left\{\pm\left(\frac{\Lambda}{2}-k\right)^{\frac{1}{2}}t+C\right\}^{-2} - 4\pi G a^2\left\{\pm\left(\frac{\Lambda}{2}-k\right)^{\frac{-1}{2}}t+C\right\}^{-6} - \Lambda\right]^{\frac{1}{2}}, \text{..........}(28)$$

As a special case if $\Lambda = 0$, then for a flat universe from equation (25) , we obtain

$$R = (A_1 t + B_1)^{1/3}, \text{..}(29)$$

$$\phi = a_1 (A_1 t + B_1)^{\frac{a}{A_1}}, \text{..}(30)$$

and

$$\Psi = \pm\left(\frac{1}{2}\right)(\pi G\lambda)^{\frac{-1}{2}}\left(\frac{A_1^{\,2}}{3} - 4\pi G a^2\right)^{\frac{1}{2}}(A_1 t + B_1)^{-1}, \text{..}(31)$$

where a_1 , A_1 and B_1 are arbitrary constant. And with $\Lambda=0$, we obtain for an open universe

$$R = t + B_2 , \text{..}(32)$$

$$\phi = a_2 \exp\left[-\frac{1}{2}a(t+B_2)^{-2}\right], \qquad (33)$$

where B_2 and a_2 are arbitrary constants. In this case equation (17) gives

$$\lambda\Psi^2 + a^2(t+B_2)^{-6} = 0, \qquad (34)$$

2.3 CONCLUSIONS

For the model obtained in equations (26), (27) and (28), two cases arise. The case when $R = \left(\frac{\Lambda}{2} - k\right)^{\frac{1}{2}} t + C$ is taken to be first case here, and that $R = -\left(\frac{\Lambda}{2} - k\right)^{\frac{1}{2}} t + C$ is taken to be second case. It is seen that in both the cases Λ can not be less than 2k. The models obtained here are, in general, non-statie, homogeneous and isotropic. The model of the first case is found to expand indefinitely with time from an initial finite state to that of a universe of infinite radius. In the second case the model universe contracts linearly with time until it reaches a singular state after time $t = C\left(\frac{\Lambda}{2} - k\right)^{\frac{-1}{2}}$ from initial stage of its evolution. If Λ=2k we get a static universe which is not of much interest here. In the model of the first case type the scalar field is found to be increasing uniformly with time if a_0 is a positive quantity, until it finally takes a finite value a_0 ; and thus in this universe the creation of mass goes on constantly. In this model λ can not be zero as in that case the electromagnetic field potential Ψ is undefined. And it is seen that Ψ decreases with time until it becomes zero at infinite time and also it vanishes when the two arbitrary constants a and A_1 so happen that $A_1^2 = 12\ \pi G a^2$. In this case the charge field decreases with the increase of the scalar field which shows that the scalar field has a tendency to decrease the rate of continuous creation of matter which is due to net imbalance of the charge contained in this model. In the model of the second case type the scalar field is found to decrease uniformly with time from a singular state until it finally takes a constant value a_0; thus in this case the rate of creation of mass decreases steadily. Here it is also seen that in the case of static universe the scalar field does not exist. In the first case model the charge potential decreases with time, however it is undefined at infinitely large time from the origin of universe. Thus it indicates that the rate of mass creation due to the charge decreases with time in this case. In the model of the second case the charge potential is found to be negative which is perhaps may be taken as the case of absorbing radiation by the model universe. In both the cases the charge potential ψ if not defined if λ happens to be zero, thus it may be concluded that can not be zero if the model universes here are to be realistic ones.

Next taking up the case of the flat model universe, it is seen that for $A_1>0$ and $B_1>0$ this model expands indefinitely from the initial singular state. And for $B_1 > 0$ and $A_1<0$ the initially finite flat universe counteracts to a singular state in a finite-time. If B, happens to be a negative quality, then in the expansion of this model universe there will be a singularity at a time given by $t=\frac{B_1}{A_1}$ Here the scalar field ϕ increases infinitely with time ; however at time $t=\frac{B_1}{A_1}$ it is undefined. It is found that if a =0 the scalar field does not vanish ; and a =0 implies $\lambda = 0$ and not necessary $\psi = 0$. Similarly $\lambda = 0$ implies a = 0 and not necessarily $\phi = 0$. Again for the electric charge to have a real value, in this model universe, λ is to be a negative quality; and in this case it is seen that as the strength of the scalar field increases, the potentiality of the charge field decreases which indicates that the scalar field has a tendency to decrease the rate of continuous creation of matter which is due to the net imbalance of the charge.

Lastly, concerning the open modél we have obtained here it is seen that it expands linearly with time from an initial finite state reaching to that of a universe of infinite radius in an infinite time. If B_2 happens to be a negative quantity then in the course of expansion of this model universe there will be a singularity after a period of time given by $t = B_2$ from the initial state. Here the strength of the scalar field ϕ increases infinitely with time; however at $t = - B_2$ it is undefined. Also from the relation (33) we see that even if $a = 0$ the scalar field does not vanish, but becomes constant. In this universe the electromagnetic field, and the scalar field are found to be closely interrelated, and it seems that if one of the field also vanishes automatically.

CHAPTER-3

EXACT SOLUTIONS OF RADIATING AND NON-RADIATING VISCOUS FLUID UNIVERSES COUPLED WITH AN ELECTROMAGNETIC FIELD IN GENERAL RELATIVITY.

3.1 INTRODUCTION.

Despite the fact, the cosmological solutions of the Einstain field equations given by Friedmann have successfully incorporated the expansion, homogeneity and isotropy of the universe, and though it is commonly accepted that the Friedmann models represent the present state of the universe quite accurately, and whatever deviations may exist are expected to be small, they do not explain the homogeneization and isotropization of the Universe, which is apparent at scales of the order of 10^8 light-years. Besides this, statistical fluctuations in Freedmann models do not grow fast enough to explain the formation of galaxies, which implies the existence of real in homogeneities at all stages of the evolutions of the universe, and recent observations of voids pose-a-challenging problem to be explained by any responsible cosmological model. Thus here we consider a metric which can give more in right into the minute study of the models.

On the other hand, objects with large energy output, either in the forms of photons or neutrinos or both in some phases of either evolutions, are very much known to exist. A nonstatic distribution would be radiating energy, and so it would be surrounded by an ever expanding zone of radiation. It is widely recognized that in the distant past the universe was dominated by the radiation and the early universe was an undifferentiated soup of matter and radiation in a state of thermal equilibrium.

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During the photon decoupling stage part of the electromagnetic radiation behaved as a perfect fluid co-moving with matter, while another part behaved like a unidirectional stream moving with fundamental velocity. And during the neutrino decoupling stage a similar situation arose in which apart from streaming neutrinos moving with fundamental velocity, there was a past behaving like a viscous fluid co-moving with matter. The discovery of quasi-steller objects and their huge energy requirements motivated the development of a theory of hob, convective, supermassive stars where general relativistic effects are important. It will, therefore be interesting to consider a radiating distribution in trying to explore new results and so that useful information's about the behavior of a realistic Universe can be obtained from such models.

In this we obtain four new solutions and try to study them from various angles. Even though some numerically computed solutions are available in the literature the efficiency of exact solutions for giving a clear understanding of the internal structure of a spherical star cannot be reached. With the help of the exact solutions obtained here we study the physical and dynamical properties of star models emitting and coupled with electromagnetic radiation ; and as special cases, models which have stopped emitting electromagnetic radiation are also discussed and examined. The effects of viscosity on such models are also investigated with great precision. The importance in studying the radiation models lies in the fact that nowadays radiation plays an important role in studying many scientific and astrophysical phenomena. Particularly investigations on electromagnetically charged models are important in connection with the study of the pulsars and the black holes. The models here are also taken to be composed of viscous fluid as so far from the evidences obtained, it is known that no astrophysical object is composed of a purely perfect fluid.

3.2 FIELD EQUATION AND THEIR SOLUTIONS.

The metric considered here is

$$ds^2 = \exp(2\gamma)dt^2 - \exp(2\lambda)dr^2 - y^2 d\Theta^2 - y^2 \sin^2\Theta d\phi^2, \qquad (1)$$

where γ, λ and y are functions of r and t.

The energy-momentum tensor $T_{\mu\gamma}$ is here given by

$$T_{\mu\gamma} = \rho u_\mu u_\gamma + (p - \xi\theta)H_{\mu\gamma} - 2\eta\sigma_{\mu\gamma} + E_{\mu\gamma}, \qquad (2)$$

where p is the isotropic pressure; ρ, the fluid density; η and ξ, the co-efficients of shear and bulk viscosities, respectively; and u_μ, the four vector velocity of flow satisfying the relation.

$$g_{\mu\gamma} u^\mu u^\gamma = 1, \qquad (3)$$

Here

$$E_{\mu\gamma} = \frac{1}{4\pi G}\left[\frac{1}{4} g_{\mu\gamma} F_{\alpha\beta} F^{\alpha\beta} - F_{\mu\alpha} F^\alpha_\gamma\right], \qquad (4)$$

is the energy-momentum tensor due to the electro-magnetic field where $F_{\alpha\beta}$ are electromagnetic field tensors satisfying the relations

$$F^{ij}_{;j} = -\sigma u^i, \qquad (5)$$

and

$$F_{[ij,k]} = 0, \qquad (6)$$

where $\sigma(\mu,t)$ is the charge density of the electro-magnetic field (a semi-colon followed by a subscript denotes covariant differentiation).

The $H_{\mu\gamma}$, s are the projection tensors given by

$$H_{\mu\upsilon} = u_\mu u_\upsilon - g_{\mu\upsilon}$$

While

$$\sigma_{\mu\upsilon} = \frac{1}{2}\left(u_{\mu,\beta}H_\upsilon^\beta + u_{\upsilon,\beta}H_\mu^\upsilon\right) - \frac{1}{3}\theta H_{\mu\upsilon}$$

are the components of the shear tensor where

$$\theta = u^\alpha{}_{;\alpha} = \nabla u$$

is the expansion factor.

Since the only non-zero component of the electro field is F^{41} because of spherical symmetry, Maxwell's equations reduce to

$$\left(e^\upsilon e^\lambda y^2 F^{41}\right)_{,1} = -4\pi y^2 e^\upsilon e^\lambda j^4, \qquad (7)$$

$$\left(e^\upsilon e^\lambda y^2 F^{41}\right)_{,4} = 4\pi y^2 e^\upsilon e^\lambda j^1, \qquad (8)$$

Now, in the rest frame of the fluid, defined by $u_\mu = e^\upsilon \delta^4{}_\mu$ Equations (7) and (8) can be integrated to obtain

$$F^{41} = -e^{-\upsilon-\lambda} y^{-2} Q(r,t), \qquad (9)$$

Where $Q(r,t) = 4\pi \int_0^r j^4 y^2 e^{\upsilon+\lambda} dr, \qquad (10)$

Thus Einstein field equation

$$R_{\mu\upsilon} = \frac{1}{8\pi G}\left(T_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} T\right)$$

gives

$$-y^{-2} + 2y^{-1}e^{-2\lambda}\left(y'' - y'\lambda' + \frac{1}{2}y^{-1}y'^2\right) - 2y^{-1}e^{-2\upsilon}\left(\dot{y}\dot{\lambda} + \frac{1}{2}y^{-1}\dot{y}^2\right)$$

$$= -8\pi\left(e + \frac{1}{8\pi}Q^2 y^{-4}\right), \qquad (11)$$

$$2y^{-1}e^{-2\lambda}\left(-\dot{y}' + y'\dot{\lambda} + \dot{y}\upsilon'\right) = 0, \qquad (12)$$

$$-y^{-2} + 2y^{-1}e^{-2\lambda}\left(y'\upsilon' + \frac{1}{2}y^{-1}y'^{2}\right) - 2y^{-1}e^{-2\upsilon}\left(\ddot{y} + \frac{1}{2}y^{-1}\dot{y}^{2} - \dot{y}\dot{\upsilon}\right)$$

$$= -8\pi\left[-p + \frac{1}{8\pi}Q^{2}y^{-4} - e^{-\upsilon}\left\{\left(-\frac{4}{3}\eta - \xi\right)\dot{\lambda} + \left(\frac{4}{3}\eta - 2\xi\right)\frac{\dot{y}}{y}\right\}\right], \qquad (13)$$

And

$$e^{-2\lambda}\left(y^{-1}y'' + \upsilon'' + \upsilon'^{2} - y^{-1}y'\lambda' - \lambda'\upsilon' + y^{-1}y'\upsilon'\right) + e^{-2\upsilon}\left(-\ddot{\lambda} - \dot{\lambda}^{2} - y^{-1}\ddot{y} + \dot{\lambda}\dot{\upsilon} + y^{-1}\dot{y}\dot{\lambda}\right)$$

$$= 8\pi\left[-p - \frac{1}{8\pi}Q^{2}y^{-4} - e^{-\upsilon}\left\{\left(\frac{2}{3}\eta - \xi\right)\dot{\lambda} + \left(-\frac{2}{3}\eta - 2\xi\right)\frac{\dot{y}}{y}\right\}\right], \qquad (14)$$

Overhead dot and prime respectively denote partial differentiations with respect to 't' and 'r' . We shall now take up four cases.

CASE-1.

From above we see that there are only four equations [equations (11)-(14)] out of which six unknowns are to be solved.

Thus here we assume

$$\dot{\lambda} = -y^{-1}\dot{y}, \qquad (15)$$

Making use of this-relation in equation (12), we get

$$y\dot{y} = \frac{1}{a}e^{\upsilon},\ldots\ldots\ldots\ldots(16)$$

where 'a' is an arbitrary constant.

Now from equation (16) we can conveniently take

$$y = rt,\ldots\ldots\ldots\ldots(17)$$

$$\upsilon = \log(a_1\xi r^2 t),\ldots\ldots\ldots\ldots(18)$$

Taking $a = \xi a_1$

Also from equations (15) and (17), we have

$$\lambda = \log\{t^{-1} f(r)\},\ldots\ldots\ldots\ldots(19)$$

As a particular solution we can take

$$\lambda = \log(b_1\eta\, r\, t^{-1}),\ldots\ldots\ldots\ldots(20)$$

where b_1 is an arbitrary constant.

Now making use of the relations (17), (18) and (20) we see that equations (13) and (14) respectively take the forms.

$$-r^{-2}t^{-2} + 5b_1^{-2}r^{-4}t^2 + a_1^{-2}\xi^{-2}r^{-4}t^{-4}$$
$$= 8\pi p - Q^2 r^{-4}t^{-4} + \frac{64}{3}\pi a_1^{-1}\xi^{-1}\eta r^{-2}t^{-2} - 8\pi a_1^{-1}r^{-2}t^{-2},\ldots\ldots\ldots\ldots(21)$$

And

$$b_1^{-2}\eta^{-2}r^{-4}t^2 + a_1^{-2}\xi^{-2}r^{-4}t^{-4}$$
$$= 8\pi p + Q^2 r^{-4}t^{-4} - \frac{32}{3}\pi\eta a_1^{-1}\xi^{-1}r^{-2}t^{-2} - 8\pi a_1^{-1}r^{-2}t^{-2},\ldots\ldots\ldots\ldots(22)$$

Now from equation (21) and (22) , we get

$$Q^2 = 16\pi a_1^{-1}\xi^{-1}\eta r^2 t^2 + \frac{1}{2}r^2 t^2 - 2b_1^{-2}\eta^{-2}t^6 ,\ldots\ldots(23)$$

And

$$p = \frac{1}{16\pi}\left[6b_1^{-2}\eta^{-2}r^{-4}t^2 + 2a_1^{-2}\xi^{-2}r^{-4}t^{-4} + 16\pi a_1^{-1}r^{-2}t^{-2} - r^{-2}t^{-2} - \frac{32}{3}\pi a_1^{-1}\xi^{-1}\eta r^{-2}t^{-2}\right],\ldots\ldots(24)$$

and equation (11) gives , making use of relation (23

$$e = \frac{1}{8\pi}\left[\frac{1}{2}r^{-2}t^{-2} + \left(3b_1^{-2}\eta^{-2}t^6 - a_1^{-2}\xi^{-2}\right)r^{-4}t^{-4} - 16\pi a_1^{-2}\eta\xi^{-1}r^{-2}t^{-2}\right],\ldots\ldots(25)$$

CASE-II

In this case we take υ to be a function of time only. Then equation (2) gives

$$y' = f e^{\lambda} ,\ldots\ldots(26)$$

where f is an arbitrary function of r.

Now we assume

$$y' = f e^{\lambda} = f' g ,\ldots\ldots(27)$$

where 'g' is an arbitrary function of time only

Then,

$$y = fg, \ldots\ldots(28)$$

and $$\lambda = \log f' + \log g - \log f,\ldots\ldots(29)$$

Therefore, equation (13) and (14) together give

$$Q^2 = \frac{1}{2} f^2 g^2 \text{ ,} \qquad \ldots\ldots(30)$$

$$\text{And } p = \frac{1}{16\pi}\left[2g^{-2} - f^{-2}g^{-2} + 48\pi\xi \frac{\overset{\circ}{g}}{g} e^{-\upsilon} + 2e^{-2\upsilon}\left(\frac{2\overset{\circ}{g}}{g}\overset{\circ}{\upsilon} - 2\frac{\overset{\circ\circ}{g}}{g} - \frac{\overset{\circ}{g}^{2}}{g^{2}}\right)\right], \qquad \ldots\ldots(31)$$

From equation (11), we get

$$\rho = \frac{1}{8\pi}\left[3e^{-2\upsilon}\frac{\overset{\circ}{g}^{2}}{g^{2}} + \frac{1}{2}f^{-2}g^{-2} - 3g^{-2}\right], \qquad \ldots\ldots(32)$$

As a particular example we may take

$$f = r^2 \text{ , } g = b^2 \text{ , } \upsilon = 3 \log t, \qquad \ldots\ldots(33)$$

in which case we get

$$Q^2 = \frac{1}{2} r^4 t^4 \text{ ,,} \qquad \ldots\ldots(34)$$

$$p = \frac{1}{16\pi}\left[2t^{-4} + 8t^{-8} + 96\pi\xi t^{-4} - r^{-4}t^{-4}\right] \qquad \ldots\ldots(35)$$

$$\text{And } \rho = \frac{1}{8\pi}\left[12t^{-8} - \left(\frac{1}{2}r^{-4} - 3\right)t^{-4}\right], \qquad \ldots\ldots(36)$$

CASE –III

Here we take up the case in which the metric assumes the form of the Robertson-Walker type as Robertson-Walker models are possibly most appropriate for a representation of the large-scale structure of the space-time. For that we take.

$\upsilon = 0$, $e^{2\lambda} = f^2(r)\, g^2(t)$, $y = r.g(t)$,..........(37)

where f(r) is a function of r only and g(t) is a function of time only.

Now it is seen that relation (37) satisfy the equation (12) automatically. And equations (13), (14) and (11) give

$$Q^2 = \frac{1}{2}\left[f^{-2}r^2g^2(f-1) - r^3f^{-3}g^2f'\right] \qquad (38)$$

$$p = \frac{1}{16\pi}\left[6\xi\frac{\overset{\circ}{g}}{g} + r^{-2}f^{-2}g^{-2} - \frac{1}{r}g^{-2}f^{-3}f' - r^{-2}g^{-2} - 4\frac{\overset{\circ\circ}{g}}{g} - 2\frac{\overset{\circ}{g}^2}{g^2}\right], \qquad (39)$$

$$\rho = \frac{1}{8\pi}\left[\frac{1}{2}r^{-2}g^{-2} + \frac{5}{2}r^{-1}g^{-2}f^{-3}f' + 3g^{-2}\overset{\circ}{g}^2 - \frac{1}{2}r^{-2}f^{-2}g^{-2}\right], \qquad (40)$$

As a particular case we study the solution when

$f = a_2\eta r^{-1}$, $g = b_2\xi\, t$,..........(41)

where a_2 and b_2 are arbitarary constant.

And here,

$Q = ½\,(b_2\xi\, r\, t)^2$,..........(42)

$$p = \frac{1}{16\pi}\left[6\xi t^{-1} + 2(a_2b_2\eta\xi t)^{-2} - 2t^{-2} - (b_1\xi rt)^{-2}\right] \qquad (43)$$

And

$$\rho = \frac{1}{8\pi}\left[\frac{1}{2}(b_2\xi rt)^{-2} + 3t^{-2} - 3(a_2b_2\eta\xi t)^{-2}\right], \qquad (44)$$

CASE IV :

In this case we take up radiating fluid for which

$$\rho = 3\,p\,, \qquad (45)$$

Here we assume υ to be a function of time only and take

$$\overset{\circ}{\lambda} = -\frac{\overset{\circ}{y}}{y}, \qquad (46)$$

Then from equations (12), (13) and (14) , we obtain

$$2y^{-2} + e^{-2\upsilon}\left(2\frac{\overset{\circ\circ}{y}}{y} + 2\frac{\overset{\circ}{y}^{2}}{y^{2}} - \frac{2\overset{\circ}{y}}{y}\overset{\circ}{\upsilon}\right) = 24\pi\xi e^{-\upsilon}\frac{\overset{\circ}{y}}{y}, \qquad (47)$$

A particular solution of equation (47) is

$$y = \eta c t\,, \qquad (48)$$

$$\upsilon = \log\,(12\pi\xi\eta^{2}\,c^{2}\,t\,), \qquad (49)$$

where c is an arbitrary constant.

Thus Equation (46) gives

$$\lambda = \log\,(d/t), \qquad (50)$$

where d is an arbitrary constant or at most an arbitrary function of r.

Therefore making use of the relation (50) in equation (11) (13) and (14),

we get

$$p = \frac{1}{48\pi}c^{-2}\xi^{-1}\eta^{-2}\left(\xi - \frac{8}{3}\eta\right)t^{-2}, \qquad (51)$$

$$Q=\left[\frac{1}{2}c^{2}\left(1+\frac{8}{3}\xi^{-1}\eta\right)\eta^{2}t^{2}-12\pi\xi\right]^{\frac{1}{2}},\ldots\ldots\ldots\ldots(52)$$

and $$\rho=\frac{1}{16\pi}c^{-2}\xi^{-1}\eta^{-2}\left(\xi-\frac{8}{3}\eta\right)t^{-2},\ldots\ldots\ldots\ldots(53)$$

3.3 DISCUSSION OF THE RESULTS :-

In case I, the fluid pressure as well as the fluid density is found to be a decreasing function of the time and the radial distance both. In this case, viscosity has the tendency to decrease the pressure. On the other hand the bulk viscosity has a tendency to enhance the density while the shear viscosity acts the other way. The charge field on the other hand is an increasing function of the time and the radial distance both ; however the shear viscosity and the bulk viscosity both have tendencies to decrease the strength of the electric charge. Thus the models here will gradually lose the potentiality of emitting radiation and come to that of a dust era, thus modeling a good example which demonstrates the evolution of the Universe. This model will be able to explain the early Universe in some ways.

In case II , if we take f(r) and g(t) to be decreasing functions of r and t respectively, then the pressure and the density are found to be increasing functions of the time and the radial distance both whereas the electric charge is found to be a decreasing function of both. The viscosity has an effect to increase the pressure, while both the density and the electric charge are both unaffected by the viscosity. In the special case obtained when f(r) is an increasing function of the radial distance, and both 'g' and υ are increasing functions of the time, we see that both the fluid pressure and the fluid density are decreasing functions of 'r' and 't' whereas the electric charge is found to be an increasing function of the time and the radial distance both. In this special case also the viscosity has the tendency to increase the fluid pressure, where as the electric charge and the fluid density are both unaffected by viscosity. Concerning the electric charge there is a singularity at the origin of the epoch and also at the center of the model, and at these instances this model will cease to radiate.

Regarding case III , we see that if we take 'f' and 'g' to be respectively increasing functions of 'r' and 't' then the fluid pressure is found to a decreasing function of the time and the radial distance both. The fluid density also here behaves in the same way. The electric charge is an increasing function of the time.

In the particular case where 'f' and 'g' are given by the relations (41), the fluid pressure is found to be a decreasing function of the time but an increasing function of the radial distance. Here the viscosity has a tendency to decrease the pressure and the density both. The density also is a decreasing function of the time and the radial distance both. But on the other hand the electric charge is an increasing function of 'r' and 't' both, and the viscosity accelerates the radiation effect. At $t = 0$ that is at the origin of the epoch the model is seen to have a singularity.

In case IV , we see that the fluid pressure and the fluid density are decreasing functions of the time t and the radial distance r both, whereas the electric charge is an increasing function of the time. Here the viscosity decelerates the pressure and the density both, but it has a tendency to accelerate the electric charge and thereby the radiation. We also see that the relation between the co-efficient of the bulk and shear viscosities are such that 3ξ can not be equal to 8η; for in that case the charge density become imaginary, and both the pressure and the density cannot exist.

CHAPTER - 4

HOT BIG BANG VISCOUS FLUID MODEDL UNIVERSES COUPLED WITH A SCALAR FIELD.

4.1 INTRODUCTION

Einstein thought that for positive equations, $R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = \frac{8\pi G}{C^4}T_{ij}(i,j = 1,2,3,4,)$ had no solutions for $T_{ij} = 0$, that is, for empty space. He was of the opinion that Mach's principle had been incorporated into his theory of gravitation. But this result due to Einstein was shown to be wrong by de sitter(1917) who found a solution of the above equations for empty space. This solution represents an "expanding" universe, in which test particles of negligible mass would continually recede from each other with everincreasing velocity. There upon Einstein abandoned the cosmological tern calling it the biggest blunder of his life.

Big-Bang model is the most commonly accepted model of the universe. With this view in fact , proposing a modification of the cosmological constant introduced by Einstein and assuming Λ as a function of the "Cosmical time" the evolution of the Universe according to the "Standard hot big –bang model" is being studied using the modified field equations obtained therefrom. From such a study it will be possible to get more accurate information about the evolution of the Universe, for example, the prediction of the helium abundance, which has not been predicted accurately by any other theory of gravitation (Ryan et al-1975). Also it is commonly accepted that no real astrophysical object is composed of a perfect fluid. In this context the viscous fluid model is considered for our study here. Moreover it is known that a scalar field plays an important role in the evolution of the Universe and some theories already exist to that effect. We thus introduce here a scalar filed.

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4.2 FORMULATION AND STUDY OF THE DIFFERENT MODELS.

From astronomical observations it is known that the Universe is homogeneous and isotropic on scales of ~ 10^8.light year and larger (Sanadage et al, 1972). By taking such a large-scale viewpoint one can treat galaxies as "particles" of a "gas" that fills the Universe. The energy-momentum tensor for this "cosmic fluid" may be taken as

$$T_{\mu\upsilon} = \rho u_\mu u_\upsilon + (p - \xi\theta)H_{\mu\upsilon} - 2\eta\sigma_{\mu\upsilon} + \phi^{-2}\left(\phi_\mu\phi_\upsilon - \frac{1}{2}g_{\mu\upsilon}\phi^k\phi_k\right), \qquad \text{.....................(1)}$$

where p is the isotropic pressure, ρ the density ϕ, the zero –mass scalar field, η and ξ, the co-efficients of shear and bulk viscosities, respectively, and u_μ. the 4- velocity vector of the cosmic fluid which satisfied the relation

$$g_{\mu\upsilon}\, u^\mu u^\upsilon = 1, \qquad \text{..............................(2)}$$

$H_{\mu\upsilon}$ is the projection tensor defined by

$$H_{\mu\upsilon} = u_\mu u_\upsilon - g_{\mu\upsilon}$$

And $\sigma_{\mu\upsilon} = \frac{1}{2}\left(u_{\mu,\beta}H_\upsilon^\beta + u_{\upsilon,\beta}H_\mu^\beta\right) - \frac{1}{3}\theta H_{\mu\upsilon}$

is the shear tensor where

$$\theta = u^\alpha{}_{,\alpha}$$

is the expansion factor.

Here the scalar field ϕ satisfies the relation

$$\frac{\partial}{\partial x^{\upsilon}}\left(\phi_{\alpha}\sqrt{-g}g^{\alpha\upsilon}\right)-\frac{\phi_{\upsilon}\phi_{\alpha}}{\phi}\sqrt{-g}g^{\alpha\upsilon}=0, \qquad (3)$$

Einstein's field equations in general relativity are given by

$$R_{ij}-\frac{1}{2}Rg_{ij}+\Lambda g_{ij}=-8\pi GT_{ij}, \qquad (4)$$

Where Λ is the cosmological constant which has the dimensions of a space curvature, namely,(length)$^{-2}$. Taking into account two correspondence limits, the Newtonian limits and the special theory limit, Chandra(1977)proposed a modification of the above field equations as

$$R_{ij}-\frac{1}{2}Rg_{ij}+\Lambda_{(ij)}g_{ij}=-8\pi GT_{ij}\left(i,j=1,2,3,4\right), \qquad (5)$$

where $\Lambda_{(ij)}$ are different functions of the "Cosmical time" for i,j = 1,2,3 (space components), and the rest of the components(time components) are constants.

The field equations (5) can be written in contravariant and mixed forms respectively as

$$R^{ij}-\frac{1}{2}Rg^{ij}+\Lambda^{(ij)}g^{(ij)}=-8\pi GT^{ij}\left(i,j=1,2,3,4\right), \qquad (6)$$

$$R^{i}_{j}-\frac{1}{2}Rg^{i}_{j}+\Lambda^{(i)}_{(j)}g^{i}_{j}=-8\pi GT^{i}_{j}\left(i,j=1,2,3,4\right), \qquad (7)$$

The invariant nature of the quantities $\Lambda_{(ij)}$ and symmetric property of the field equations suggest that

$$\Lambda_{(ij)} = \Lambda_{(ji)} = \Lambda^{(i)}_{(j)} = \Lambda^{(j)}_{(i)} = \Lambda^{(ij)} = \Lambda^{(ji)} (i, j = 1,2,3,4), \ldots\ldots (8)$$

Taking into consideration the special theory limit the field equations (5) yield the diagonal values of Λ_{ij} as (Chndra,1977)

$$\Lambda_{(ij)} = A(t) > 0 \ (i = 1,2,3), \ldots\ldots (9)$$

and

$$\Lambda_{(ij)} = -B = constant < 0 \ , \ldots\ldots (10)$$

Under these conditions the Newtonian limit of the field equations (5) is given by

$$\nabla^2\phi + [A(t)+B] = 8\pi G\rho, \ldots\ldots (11)$$

which gives a variable density

$$\rho = \left[\frac{A(t)+B}{8\pi G}\right], \ldots\ldots (12)$$

in the presence of homogeneous matter ,namely ϕ = constant.

In case Mach's principle is satisfied by the modified field equations we cannot have the homogeneous universe that is empty , that is , the metric of a space- time for a homogeneous universe is not possible when T_{ij} = 0. In general the metric of a space-time for a homogeneous universe in a co-moving co-ordinate system (which is always possible in the case of homogeneous universe) is given be

$$ds^2 = dt^2 - g_{ij}\, dx^i\, dx^j \ (i, j = 1,2,3,4), \ldots\ldots (13)$$

where the g_{ij} are functions of the co-ordinates.

Here we consider the Robertson-Walker metric.

$$ds^2 = dt^2 - \frac{R(t)}{(1-\frac{1}{4}kr^2)^2}\left[dr^2 + r^2 d\theta^2 + r^2 \sin\theta d\phi^2\right], \qquad (14)$$

Where k is the curvature index and takes the values +, - or 0, when the universe is closed ,open, on flat, respectively . R(t) is the scale factor for measuring distances in the nonstatic universe. This metric is applicable to all homogeneous and isotropic model universe with the condition that the cosmic fluid is at rest relative to the comoving observer (Robertson, 1935,1936:Walker,1936) whence $u_i = (0,0,0,1)$. Now making use of the relations (1),(9),(10) and (14) into the field equations (5) we get

$$8\pi G(p-\xi\theta) = -4\pi G\left(\frac{\overset{\circ}{\phi}}{\phi}\right)^2 - 4\pi G\left(\frac{1-kr^2}{R^2}\right)\left(\frac{\phi'}{\phi}\right)^2 - 2\frac{\overset{\circ\circ}{R}}{R} - \frac{\overset{\circ}{R}^2}{R} - \frac{k}{R^2} + A(t), \qquad (15)$$

$$8\pi G(p-\xi\theta) = -4\pi G\left(\frac{\overset{\circ}{\phi}^2}{\phi}\right)^2 + 4\pi G\left(\frac{1-kr^2}{R^2}\right)\left(\frac{\phi'}{\phi}\right)^2 - 2\frac{\overset{\circ\circ}{R}}{R} - \frac{\overset{\circ}{R}^2}{R^2} + A(t), \qquad (16)$$

$$8\pi G\rho = -4\pi G\left(\frac{\overset{\circ}{\phi}}{\phi}\right)^2 - 4\pi G\left(\frac{1-kr^2}{R^2}\right)\left(\frac{\phi'}{\phi}\right)^2 + 3\left(\frac{\overset{\circ}{R}^2}{R^2} + \frac{k}{R^2}\right) + B, \qquad (17)$$

Here from Equations (15) and (16) we see that

$$\frac{\phi'}{\phi} = 0, \qquad (18)$$

Again using Equation (18) in relation (3) we have

$$\overset{\circ}{\phi} + 3\frac{\overset{\circ}{R}}{R}\overset{\circ}{\phi} - \frac{\overset{\circ}{\phi}^2}{\phi} = 0$$

which gives

$$\frac{\overset{\circ}{\phi}}{\phi} = CR^{-3}, \qquad (19)$$

where C is an arbitrary constant.

By virtue of (19), Equation (15) and (17) become

$$8\pi G(p-\xi\theta) = -4\pi Gc^2 R^{-6} - 2\frac{\overset{\circ\circ}{R}}{R} - \frac{\overset{\circ}{R}^2}{R^2} - \frac{k}{R^2} + A(t), \qquad (20)$$

$$\frac{8\pi G\rho}{3} = -\frac{4\pi GC^2 R^{-6}}{3} + \frac{\overset{\circ}{R}^2}{R^2} + \frac{k}{R^2} + \frac{B}{3}, \qquad (21)$$

Equations (20) is a dynamic equation that gives the second derivative of the scale factor and thereby governs the dynamic evolution away from initial moment of time .On the other hand an Equation(21) may be taken as an "initial value equation" which gives the relation of $\overset{\circ}{R}$ with R and ρ at the origin of the epoch of time. From these two equations, we get

$$[-8\pi G(p-\xi\theta) + A(t) + B]R^2 \overset{\circ}{R} = \frac{\partial}{\partial t}\left(\frac{8\pi G}{3}\rho R^3\right), \qquad (22)$$

Thus taking for granted that the variation of A(t) and the equations of state for the cosmic fluid are known, Equations (21) and (22) together will give the dynamic evolution of Universe according to the "standard hot big- bang models"

Again the difference of the Equations (20) and (21) gives

$$\overset{\circ\circ}{R} = -\frac{8\pi G}{3}R^{-5} - 4\pi GpR + 12\pi G\xi \overset{\circ}{R} - \frac{4}{3}\pi G\rho R + \frac{1}{2}\left[A(t) + \frac{1}{3}B\right]R, \qquad (23)$$

in which the right hand side is essentially the force producing the acceleration (Milne and Mc Crea, 1934; Schlutor, 1955). The force terms,

$$\frac{8\pi G}{3}R^{-5}, 4\pi GpR, 12\pi G\xi \overset{\circ}{R}, \frac{4}{3}G\pi\rho R \quad \text{and} \quad \frac{\left[\frac{1}{3}B + A(t)\right]}{2}R$$

are due to scalar field pressure, viscosity ,gravity, and the cosmological terms, respectively. Hence the quantities ξ, B and A(t) can be treated as the force parameters counteracting the gravity.

Only when we know the equation of state and the dependence of A(t) on t Equation (22) is integrable, but both are not known. Here it is seen that A(t) is a positive decreasing function of time. Therefore, it is better to take the variation of pressure as a linear function of A(t) to avoid complications. Thus without loss of generality we can assume

$$p = \frac{[A(t) - A(t_0)]}{8\pi G}C^2 + 3\xi\frac{\dot{R}}{R} - \frac{C^2}{2}R^{-6}, \qquad (24)$$

where t_0 is the time at the present epoch from the beginning of the Universe.

Why we assume the pressure in this manner is that with the help of this pressure we will be able to get a differential equation governing uniform model universes, which are best suited with the observations.

Next we take up boundary conditions. Taking t_l as the time when the universe ceased to be radiation dominated and became matter dominated , it is seen that for the radiation-dominated phase of evolutions the boundary conditions can be taken as

$$t = t_l\ ,\ p = p_l\ ,\ \rho = \rho_l\ ,\ R = R_l\ ,\ A(t) = A\,(t_l), \qquad (25)$$

and $T = T_l$ (absolute temperature)

For the radiation-dominated phase of evolutions ($0 \le t \le t_l$) we see that (Misner et al 1973)

$$3p = \rho = \frac{3}{32\pi t^2} = \frac{G\xi\pi^2}{120C^4}x\frac{(kT)^4}{h^3C^3}, \qquad (26)$$

For the matter-dominated phase of evolution the boundary conditions can be taken as

$$t = t_0 \, , \, p = 0 \, , \, \rho = \rho_0 , \, R = R_0 \, , \, A(t) = A(t_0) \, , \ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

The pressure is taken to be zero for the present moment, according to the observations. It may be noted that the pressure equation (24) is also consistent with this conditions . Since if we take two different pressure equations, one for the radiation-dominated phase of evolution and the other for the matter-dominated phase of evolutions, then we cannot follow the boundary conditions(25) strictly; here we assume the validity of the pressure equation (24) in both phase of evolution.

Now making use of Equation (24) into Equation (22) We get

$$-\frac{4\pi GC^2}{3} R^{-3} + \frac{[A(t_0) + B]}{3} R^3 + \alpha = \frac{8\pi G\rho}{3} R^3 , \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

where α is constant of integration which is obtained, using the boundary condition (25) ,as

$$-\frac{4\pi GC^2}{3} R_0^{-3} + \frac{[A(t_0) + B]}{3} R_0^3 + \alpha = \frac{8\pi G\rho}{3} R_0^3 , \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Then Equations (28) and (29) imply

$$\frac{8\pi G\rho}{3} = R^{-3}\left[\frac{8\pi G\rho_0}{3} R_0^3 - \frac{\{A(t_0) + B\}}{3}\left(R_0^3 - R^3\right) + \frac{4\pi G}{3} C^2 \left(R_0^{-3} - R^{-3}\right)\right], \ldots\ldots (30)$$

which expresses the variation of density with scale factor. Equation (30) can also be written in the form

$$\rho - \rho_0 = \frac{\left(R_0^3 - R^3\right)}{R^3}\left[\rho_0 - \frac{\{A(t_0)+B\}}{8\pi G} - \frac{C^2}{2R_0^3R^3}\right], \qquad (31)$$

from which it is seen that form an expanding universe we must have

$$\rho_0 \rangle \frac{[A(t_0)+B]}{8\pi G} + \frac{C^2}{2R_0^3R^3}, \qquad (32)$$

Under this condition Equation (29) implies that the constant of integration α must be a positive quantity.

Making use of Equation (28) into Equation (21) we obtain an equation identical with "Friedmann's differential equation" (Friedmann, 1922,1924)

$$\frac{dR}{\left[\frac{A(t_0)}{3}R^2 + \alpha R^{-1} - k\right]^{\frac{1}{2}}} = dt, \qquad (33)$$

there being a slight change that here we get a positive quantity $A(t_0)$ in place of Λ. The variation of the scale factor with time is given by the solution of this differential equation.

Next we proceed to determine A (to) and B . In terms of the observed parameters $H\left(=\frac{\dot{R}}{R}\right)$ and $q\left(=-R\frac{\ddot{R}}{\dot{R}^2}\right)$ the Equations (20) and (21) can be written as .

$$8\pi Gp = 24\pi G\xi H + H^2(2q-1) - \frac{k}{R^2} + A(t) - 4\pi GC^2R^{-6}, \qquad (34)$$

$$8\pi G\rho = 3H^2 + \frac{3k}{R^2} + B - 4\pi GC^2R^{-6}, \qquad (35)$$

These two equations determine the parameters $A(t_0)$ and B under the conditions (27) as

$$A(t_0) = H_0^2(1-2q_0) + \frac{k}{R_0^2} + 4\pi G C^2 R^{-6} - 24\pi G\xi H, \ldots\ldots(36)$$

and $$B = 8\pi G\rho_0 + 4\pi G C^2 R_0^{-6} - 3H_0^2 - \frac{3k}{R_0^2}, \ldots\ldots(37)$$

where H_0 and q_0 are respectively the values of the parameters H and q at the present epoch.

But since the parameters $H_0, q_0, \frac{k}{R_0^2} and \rho_0$ can be determined numerically from observations (Sandage, 1972), it ism possible to determine the numerical values of A (t_0) and B from the relation (36) and (37).

Again using relations (27) in equation (24) we obtain the value of ξ as

$$\xi = \frac{C^2}{6H_0} R^{-6}, \ldots\ldots(38)$$

Next we determine the numerical value of the constant of integration α by substituting the numerical values of $A(t_0)$, B, ρ_0 and R into Equation (29).And then we substitute the value of $A(t_0)$, k and α into the Equation (33) , which gives the variation of R_1 in terms of t_1 after integration between the limits R =0 and R = R_1. With the help of Equation (30) , this value of R gives the density ρ_1 at time t_1 in the form

$$\rho_1 = \frac{3}{8\pi G} R_1^{-3}\left[\frac{8\pi G\rho_0}{3} R_0^3 - \frac{\{A(t_0)+B\}}{3}\left(R_0^3 - R_1^3\right) + \frac{4\pi G}{3} C^2\left(R_0^3 - R_1^3\right)\right], \ldots\ldots(39)$$

with this value of ρ_1 we obtain, from relation (26) , using the condition (25)

$$p_1 = \frac{1}{8\pi G} R_1^{-3}\left[\frac{8\pi G}{3}\rho_0 R_0^3 - \frac{\{A(t_0)+B\}}{3}\left(R_0^3 - R_1^3\right) + \frac{4\pi G}{3}C^2\left(R_0^3 - R_1^3\right)\right], \qquad (40)$$

$$t_1 = \frac{1}{2}G^{\frac{1}{2}} R_1^{\frac{3}{2}}\left[\frac{8\pi G}{3}\rho_0 R_0^3 - \frac{\{A(t_0)+B\}}{3}\left(R_0^3 - R_1^3\right) + \frac{4\pi G}{3}C^2\left(R_0^3 - R_1^3\right)\right]^{-1/2}, \qquad (41)$$

$$T = \left(\frac{45h^3C^7}{\xi G^2\pi^3 k^4 R_1^3}\right)^{\frac{1}{4}}\left[\frac{8\pi G\rho_0}{3}R_0^3 - \frac{\{A(t_0)+B\}}{3}\left(R_0^3 - R_1^3\right) + \frac{4\pi GC^2}{3}\left(R_0^3 - R_1^3\right)\right]^{\frac{1}{4}}, \qquad (42)$$

with the help of these known parameters use will be able to study the history of the evolution of the universe in the radiation –dominated phase of evolution.

We can get the numerical value of the time t_0 at the present epoch by integrating equation (33) between the limits R=0 and R = R_0. This gives the age of the Universe, and here as a special case if we take the arbitrary constant $\alpha = 0$ then we get the age of the universe as

$$t_0 = \sqrt{\frac{3}{A(t_0)}}\left[\cosh^{-1}\left(\sqrt{\frac{A(t_0)}{3k}}R_0\right) - 1\right], \qquad (43)$$

The history of the evolution of the universe can be studied from the physical quantities thus obtained.

4.3 CONCLUSION :-

Though the Friedmann's model of the 'standard hot big-bang universe' is remarkably powerful and accords well with observations , an exact study of the variation of pressure can not be made in such a model. Though for an expanding universe the pressure must be a decreasing function of time this functional relation is not known in this case. Thus the radiation-dominated phase of evolution and the matter-dominated phase of evolution are studied separately in the Friedmann, cosmology , and therefore in that model it is very difficult to know the time when the universe ceased to be radiation-dominated and become matter-dominated. However in our model these difficulties have been removed, and thus more details about the evolution of the universe can be obtained from it.

CHAPTER-5

ROTATIONAL PERTUBATION OF MASIVE
SCALAR FIELD UNIVERSES

5.1 INTRODUCTION

The studies of rotating astrophysical bodies coupled with gravitational field in presence of other fields are so far done by Bayin (1981,1985) ; Krori it al. (1983); Van den Bergh and Wils (1984); Islam (1985); Tiwari it al (1986) and Koijam (1987, 1988). Many authors have obtained models of rotating objects without expansion and expanding object without rotation. Thus it will be of great interest to find out explicitly solved models of expanding as well as rotating objects so that information's about the behavior of the universe can be obtained from such models. Thus here we investigate rotating as well as expanding models.

The Robertson-Walker models are believed to be appropriate for a representation of large scale structure of the space-time, we consider this type of metric here for our problem. Furthermore, as object of our study, we take up rotational perturbation of Massive scalar field as it will be very stimulating to make investigations on such models in trying to obtain new information's concerning rotating astrophysical objects in this universe and we can draw many conclusions for a realistic universe from such studies. In many respects our problem will be very interesting as, though in most of the Robertson Walker metric can be used for general relativity solutions with rotating and massive scalar field, and many stimulating findings for further research may be obtained from it. The study of the rotational perturbations of these models are also made in order to substantiate the possibility that the universe is endowed with slight rotation in the course of presentation of several analytic solutions.

5.2 FIELD EQUATION

The metric considered here is the perturbation from of the Robertson-Walker metric viz.

$$ds^2 = dt^2 - R(t)\left[\frac{dr2}{1-kr2} + r2d\Theta^2 + r^2 Sin\Theta d\phi^2\right] +$$

$$+ 2\Omega(r,t)R^2(t)r^2\sin^2\Theta' d\phi\, dt, \quad \ldots\ldots(1)$$

where Ω (r,t) is the metric rotation function which is related to the local dragging of the inertial forms.

The energy momentum tensor taken up for this problem is that of the massive scalar field given by

$$T_{ij} = \frac{1}{4\pi}\left[\phi_i\, \phi_j - \frac{1}{2}g_{ij}(\phi_{,}\phi' - M^2\phi)\right], \ldots\ldots(2)$$

Where the scalar potential ϕ satisfied the Klein- Gordon equation.

$$g^{ij}\phi_{,ij} + M^2\phi = \in \quad, \ldots\ldots(3)$$

Here $\in$is the source density of the scalar field and M is related to the mass of the spin particle by.

$$M = {}^{m}\!/_{\hbar} \text{ where } \hbar = \frac{h}{2\pi}$$

(h being the Plank's constant)

Here the spatial velocity distribution is given by

$$v^i = \frac{dx^i}{dt} = (0,0,v^3)$$

and this in this case

$$u_\alpha = (0,0,-g_{33}\,\Omega, 1), \ldots\ldots(4)$$

Now considering terms up to the first order in Ω , the Einstein field equation gives

$$2\frac{\overset{\circ\circ}{R}}{R} + \frac{\overset{\circ}{R}^2}{R^2} + \frac{k}{R^2} = G\left[\frac{M^2}{R^2} - \left(\frac{\overset{\circ}{\phi}}{\phi}\right)^2 - \left(\frac{1-kr^2}{R^2}\right)\left(\frac{\phi'}{\phi}\right)^2\right], \ldots\ldots(5)$$

$$2\frac{\overset{\circ\circ}{R}}{R} + \frac{\overset{\circ}{R}^2}{R^2} + \frac{k}{R^2} = G\left[\frac{M^2}{R^2} - \left(\frac{\overset{\circ}{\phi}}{\phi}\right)^2 + \left(\frac{1-kr^2}{R^2}\right)\left(\frac{\phi'}{\phi}\right)^2\right], \ldots\ldots(6)$$

$$3\left(\frac{\overset{\circ}{R}^2}{R^2} + \frac{k}{R^2}\right) = G\left[\left(\frac{\overset{\circ}{\phi}}{\phi}\right)^2 + \frac{M^2}{R^2} + \left(\frac{1-kr^2}{R^2}\right)\left(\frac{\phi'}{\phi}\right)^2\right], \ldots\ldots(7)$$

$$3\frac{\mathring{R}}{R}\Omega' + \mathring{\Omega}' = 0, \quad \ldots\ldots(8)$$

$$\left(R\mathring{\mathring{R}} + 2\mathring{R}^2 + 2k\right)\Omega = \frac{1}{2}\left(1 - kr^2\right)\Omega'' + \frac{2}{r}\left(1 - \frac{5}{4}kr^2\right)\Omega', \quad \ldots\ldots(9)$$

From Equation (5) and (6) , we have

$$\frac{\phi'}{\phi} = 0, \quad \ldots\ldots(10)$$

Again Equations (6) and (7) gives with the help of relation (10) gives

$$\frac{\mathring{\mathring{R}}}{R} + 2\frac{\mathring{R}^2}{R^2} + \frac{2k}{R^2} = G\frac{M^2}{R^2}$$

$$i.e. R\mathring{\mathring{R}} + 2\mathring{R}^2 + 2k = GM^2, \quad \ldots\ldots(11)$$

Now equation (9) with the help of relation (11) we get

$$\frac{1}{2}\left(1 - kr^2\right)\Omega'' + \frac{2}{r}\left(1 - \frac{5}{4}kr^2\right)\Omega'' = GM^2\Omega, \quad \ldots\ldots(12)$$

Again Equation (8) gives

$$\Omega(r,t) = L(r)R^{-3} + N(t), \quad \ldots\ldots(13)$$

Now making use of the relation (13) in equation (12) , we get

$$\frac{1}{2}\left(1 - kr^2\right)L'' + \frac{2}{r}\left(1 - \frac{5}{4}kr^2\right)L' = GM^2L$$

$$i.e. \left(1 - kr^2\right)\frac{L''}{L} + \left(\frac{4}{r} - 5kr\right)\frac{L'}{L} - 2GM^2 = 0, \quad \ldots\ldots(14)$$

CASE I :-

Now considering k = 1 which corresponding to closed models. Here using the substituting $y = kr^2$ in (14) , we get.

$$y(1-y)L_{yy}+\left(\frac{5}{2}-3y\right)L_y-\frac{2GM^2}{4k}L=0,\qquad(15)$$

We see that equation (15) is similar to the hypergeometric equation

$$y(1-y)F_{yy}+\left[\gamma-(1+\alpha+\beta)y\right]F_y-\alpha\beta F=0,\qquad(16)$$

of which the general solution is given by

F = A_0 F (α,β;υ;υ) + A_1 $y^{1-\upsilon}$ F (1-υ+α, 1-υ+β, z-υ;y), ...(17)

where A_0 and A_1 are arbitrary constants and

$$F(\alpha,\beta;\upsilon;y)=\sum_{n=0}^{\infty}\frac{(\alpha)_n(\beta)_n}{n!(\upsilon)_n}y^n,\qquad(18)$$

Thus in this case we get the general solution of equation (15) as

$$L(r)=A_0\sum_{n=0}^{\infty}\frac{(\alpha)_n(\beta)_n}{n!\left(\frac{5}{2}\right)_n}y^n+A_1y^{-3/2}\sum_{n=0}^{\infty}\frac{\left(\alpha-\frac{3}{2}\right)_n\left(\beta-\frac{3}{2}\right)_n}{n!\left(-\frac{1}{2}\right)_n}y^n,\qquad(19)$$

since the second term is not regular at y = 0
we take $A_1 = 0$, then we get

$$L(r)=A_0\sum_{n=0}^{\infty}\frac{(\alpha)_n(\beta)_n}{n!\left(\frac{5}{2}\right)_n}y^n$$

$$\text{i.e. } L(r)=A_0(1-y)^{\frac{5}{2}-\alpha-\beta}\sum_{n=0}^{\infty}\frac{\left(\frac{5}{2}-\alpha\right)_n\left(\frac{5}{2}-\beta\right)_n}{n!\left(\frac{5}{2}\right)_n}y^n,\qquad(20)$$

Now we give some of the explicit solutions :

(i) If

$$\alpha = -\frac{1}{2}, \beta = \frac{5}{2}, then$$
$$F\left(-\frac{1}{2}, \frac{5}{2}; 5; y\right) = (1-y)^{\frac{1}{2}}$$

In this case we get

$$\Omega(r,t) = A_0\left(1-kr^2\right)^{\frac{1}{2}} R^{-3} + N(t), \qquad \text{.......................................(21)}$$

(ii) If $\alpha = \frac{9}{2}, \beta = -\frac{5}{2}$ then we have

$$F\left(\frac{9}{2}, -\frac{5}{2}; \frac{5}{2}; y\right) = (1-y)^{\frac{1}{2}}\left(1 - 4y + \frac{24}{7}y^2\right)$$

In this case, we get

$$\Omega(r,t) = A_0\left(1-kr^2\right)^{\frac{1}{2}}\left(1 - 4kr^2 + \frac{24}{7}kr^2\right)R^{-3} + N(t), \qquad \text{...........(22)}$$

(iii) $F\left(-1, 3, \frac{5}{2}; y\right) = 1 - \frac{6}{5}y$

In this case α= -1, β = 3

and $\Omega(r,t) = A_0\left(1 - \frac{6}{5}kr^2\right)R^{-3} + N(t),$ ....(23)

(iv) Now $F\left(4, -2; \frac{5}{2}; y\right) = A_0\left(1 - \frac{16}{5}kr^2 + \frac{16}{7}k^2r^4\right)$

for α = 4 , β = -2 ; and here

$$\Omega(r,t) = A_0\left(1 - \frac{16}{5}kr^2 + \frac{16}{7}k^2r^2\right)R^{-3} + N(t), \qquad \text{......................(24)}$$

In case (1) , there , for $z = 2GM^2$, we get

$R = (m_0 t + n_0)^{1/3}$

CASE 2:-

In this case, we consider open models which correspond to k = -1. we obtain here different value of Ω corresponding to different value of M.

If we take $M^2 = \frac{3}{2G}$, then from equation (14) we get

$$L(r) = \frac{C_0}{2} r^{-2}\left(1 - r^2\right)^{\frac{1}{2}} - \frac{C_0}{2} r^{-3} \sinh^{-1} r + C_1 r^{-3}$$

where C_0 and C_1 are arbitrary constants.

Therefore,

$$\Omega(r,t) = \left[\frac{C_0}{2} r^{-3}\left(1 - r^2\right)^{\frac{1}{2}} - \frac{C_0}{2} r^{-3} \sinh^{-1} r + C_1 r^{-3}\right] R^{-3} + N(t), \qquad \text{............(25)}$$

Again if $M^2 = 5/G$, we get

$$L(r) = \left\{C_2\left(1 + r^2\right)^{\frac{1}{2}} - \frac{C_3}{5}\left(8r + 4r^{-1} - r^{-3}\right)\right\}$$

where C_2 and C_3 are arbitrary constant.

Therefore,

$$\Omega(r,t) = \left[C_2\left(1 + r^2\right)^{\frac{1}{2}} - \frac{C_3}{3}\left(8r + 4r^{-1} - r^{-3}\right)\right] R^{-3} + N(t), \qquad \text{..........................(26)}$$

If $M^2 = -\frac{4}{G}$, we have

$$L(r) = r^{-3}\left\{C_4 r - C_4\left(1 + r^2\right)^{\frac{1}{2}} \sin^{-1} r + C_5\left(1 + r^2\right)^{\frac{1}{2}}\right\}, \qquad \text{................................(27)}$$

Therefore

$$\Omega(r,t)=\left[r^{-3}\left\{C_4 r - C_4\left(1+r^2\right)^{\frac{1}{2}}\sin^{-1} r + C_5\left(1+r^2\right)^{\frac{1}{2}}\right\}R^{-3}+N(t)\right], \qquad \text{.........(28)}$$

where C_4 and C_5 are arbitrary constants.

CASE 3 :-

In this case we consider the flat model for which k =0 and here equation (14) becomes

$$\frac{L''}{L}+\frac{4}{r}\frac{L'}{L}=2GM^2, \qquad \text{.........(29)}$$

which gives

$$L(r)=r^{-2}\left[x\left\{A_1\exp\left(x^{\frac{1}{2}}r\right)+A_2\exp\left(-x^{\frac{1}{2}}r\right)\right\}-x^{\frac{1}{2}}r^{-1}\left\{A_1\exp\left(x^{\frac{1}{2}}r\right)-A_2\exp\left(-x^{\frac{1}{2}}r\right)\right\}\right]$$

where A_1 and A_2 are arbitrary constants. and $x = 2\ GM^2$

In this case

$$\Omega(r,t)=r^{-2}\left[x\left\{A_1\exp\left(x^{\frac{1}{2}}r\right)+A_2\exp\left(-x^{\frac{1}{2}}r\right)\right\}-x^{\frac{1}{2}}r^{-1}\left\{A_1\exp\left(x^{\frac{1}{2}}r\right)-A_2\exp\left(-x^{\frac{1}{2}}r\right)\right\}\right]R^{-3}+N(t)$$

$$, \qquad \text{.........(30)}$$

Now if M =0, then we get , from equation (29)

$L(r) = m_1 + m_1 r^{-3}$

where m_1 and n_1 are arbitrary constants .

Thus in this case we have

$\Omega(r,t) = (m_1 + n_1 r^{-3}) R^{-3} + N(t)$,..(31)

which is incidentally corresponding to the case of perfect dragging

Now here R(t) , making use of equation (11) ; is found to be

$R = (m_2 t + n_2)^{1/3}$,..(32)

where m_2 and n_2 are arbitrary constants.

Thus here

$\Omega(r,t) = (m_1 + n_1 r^{-3}) R^{-3} + N(t)$,...(34)

where N(t) is an arbitrary function of time.

5.3 CONCLUSION :

Case I .

In this case, the rotational perturbation decay with the increase of the time for all the models obtained. It is also observed that the smaller the value of A_0 the smaller are the value of Ω (r,t) and w (r,t) which means that the massive scalar field slow down the rotation . In this case the expanding factor is given by

$$\theta = m_0 \left\{ \frac{(m_0 t + n_0)^{-2}}{m_0 t + n_0} \right\}^{\frac{1}{3}}$$

and since it is positive, our model universes here are expanding ones. Thus , the models are rotating as well as expanding one which may be taken as examples of realistic models.

CASE 2:-

In this case, in all the two open models obtained , we find that the rotational perturbations decay as r increases and also with the increase of time if N(t) is a decreasing function of the time. For the model obtained for $M^2 = -4/G$ we get the expansion factor as

$$\theta = 3\, m_0 \coth (m_0 t + n_0)$$

Thus we get a rotating as well as expanding model which can be thought of as one of the value of Ω (r,t) becomes smaller which shows that the presence the massive scalar field decreases the rotational motion. When $M^2 = 5/G$ the solution is restricted within the range $-1 \leq r \leq 1$

CASE 3:-

For all the models obtained in this case the rotational perturbation falls rapidly with the increase of r. In the case of perfect dragging the matter rotation w(r,t) and the rotational velocity Ω (r,t) is independent of the massive scalar field.

CHAPTER 6

ON INCOMPATIBILITY OF VISCOUS AND MAGNETOVISCOUS FLUID DISTRIBUTION WITH CLASS ONE METRIC.

6.1 INTRODUCTION

As most of the astrophysical objects in the universe are composed of viscous fluid, physicists take great interest in studying the viscous fluid models from different angles. Heller and Susycki, Heller and Klimek, and Heller et-al obtained some solutions for imperfect fluid, considering the Robertson-Walker metric. Roy and Rao found that, for the axially symmetric Einstein-Rosen metric, the stress tensor of a scalar meson field associated with meson of rest mass μ cannot be the source term for generating gravitation and also that the same result holds even when this meson field is coupled with and electromagnetic field. A study of the spherically symmetric class one cosmological model based on Lyra's geometry was presented by Bhamra and it was pointed out that the nonstatic model parallels Lemaitri's in Riemannian case , but the law of mass-energy conservation does not hold Maiti derived a cosmological solution of Einstein's equation which admit a transitive group of motions for static spherically symmetric metric with a viscous fluid distribution as source. Banerjee and Santosh obtained an exactly solvable Bianchi type-I viscous fluid cosmological model.

Manihar Singh, Manihar Singh and Bharma, and Manihar Singh and Jugeshwor studied viscous and non-viscous fluid distribution under conditions when the distribution are interacting or not interacting with other fields, and also when they are rotating as well as non-rotating different types of spherically symmetric metrics in Einstein universe. But so far hardly any author found viscous fluid distribution taking into account spherically

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symmetric class one metric. Thus, here, we considering such a metric and investigate the behavior of viscous fluid distribution, and also that of magneto-viscous fluid distribution and viscous distribution coupled with a scalar field respectively. It is found that viscous fluid as well as magneto- viscous fluid distribution cannot be the source term for generating gravitational field for a spherically symmetric class one metric. Also viscous fluid distribution coupled with a scalar field is found to be incompatible with class one metric under certain conditions.

6.2 FIELDS EQUATIONS AND THEIR SOLUTIONS.

The flat metric in spherically polar co-ordinates is given by

$$ds^2 = dt^2 - dr^2 - r^2 d\Theta^2 - r^2 \sin^2 \Theta d\Phi^2 \;, \quad \ldots \ldots \ldots \quad (1)$$

The introduction of a gravitational disturbance function ψ, where ψ is a function of r and t only, converts the flat metric to the form

$$ds^2 = dt^2 - dr^2 - r^2 d\Theta^2 - r^2 \sin^2 \Theta d\Phi^2 - [d\psi\,(r,t)]^2 \;, \quad \ldots \ldots (2)$$

This is a spherically symmetric non-static line element of class one which can be written as

$$ds^2 = (1 - \dot{\Psi}^2)\, dt^2 - (1 + \psi'^2)\, dr^2 - r^2 d\Theta^2 - r^2 \sin\Theta d\Phi^2 - 2\psi' \dot{\Psi}\, dr\, dt \;, \ldots (3)$$

a prime and a dot respectively denoting partial differentiation with respect to r and t.

Here, for this metric, the components of fluid velocity is co-moving co-ordinate system are obtained as

$$u^1 = u^2 = u^3 = 0 \text{ and } u^4 = (1 - \dot{\Psi}^2)^{-1/2} \;, \ldots \ldots \ldots (4)$$

with the relation

$$g^{\mu\nu} u_\mu u_\nu = 1 \;, \ldots \ldots \ldots (5)$$

CASE 1:-

Einstein's field equations for viscous fluid are given by

$$G_{ij} \equiv R_{ij} - \tfrac{1}{2} g_{ij}R$$
$$= \kappa\,[\rho\, u_i\, u_j - (p - \xi\theta)\, H_{ij} + \eta\sigma_{ij}], \qquad \ldots(6)$$

where $k = \frac{1}{8\pi G}$. Here is the fluid pressure, ρ the density η and ξ, the coefficients of shear and bulk viscosities respectively. $H_{\mu\gamma}$ ' s are the projection tensors given by

$$H_{ij} = u_i\, u_j - g_{ij}$$

While $\sigma_{ij} = \tfrac{1}{2}\left(u_{i;\gamma} H^{\gamma}{}_j + u_{j;\gamma} H^{\gamma}{}_i \right) - 1/3\, \theta H_{ij}$ are the components of shear tensors where $\theta = u^{\alpha}{}_{,\alpha} = \nabla_u$ is the expansion factor.

Now for the line element (3) the corresponding field equations are

$$-\frac{\Psi'^2}{r^2 s} + \left\{ \left(1 + \Psi'^2\right)\ddot{\Psi} - \Psi'\,\dot{\Psi}\,\dot{\Psi}' \right\} \frac{2\Psi'}{rs^2} = kp - k\left(\frac{4}{3}\eta + \xi\right)\theta, \qquad \ldots(7)$$

$$-\frac{1}{S^2}\left[\left\{\left(1 - \dot{\Psi}^2\right)\Psi'' - \left(1 + \Psi'^2\right)\ddot{\Psi} + 2\dot{\Psi}'\,\dot{\Psi}\,\dot{\Psi}'\right\}\frac{\Psi'}{r} - \left(\Psi''\,\ddot{\Psi}\,\dot{\Psi}'^2\right)\right] = kp + k\left(\frac{2}{3}\eta - \xi\right)\theta, \qquad \ldots(8)$$

$$-\frac{\Psi'^2}{r^2 s}\left[\left(1 - \dot{\Psi}^2\right)\Psi'' + \Psi'\,\dot{\Psi}\,\dot{\Psi}'\right] = -k\rho, \qquad \ldots(9)$$

$$-\left[\left(1 - \dot{\Psi}^2\right)\Psi'' - \Psi'\,\dot{\Psi}\,\Psi''\right]\frac{2\Psi'}{rs^2} = \frac{k\Psi'\,\dot{\Psi}}{1 - \dot{\Psi}^2}\,p + \rho - \left(\frac{4}{3}\eta + \xi\right)\theta, \qquad \ldots(10)$$

$$\left[\left(1-\overset{\circ}{\Psi}{}^{2}\right)\overset{\circ}{\Psi}{}' + \Psi'\,\overset{\circ}{\Psi}\,\overset{\circ\circ}{\Psi}\right]\frac{2\Psi'}{rs^{2}} = 0, \qquad (11)$$

where $S = 1 + \Psi'^{2} - \overset{\circ}{\Psi}{}^{2}$

Here in this case the expansion factor is given by

$$\theta = \frac{\Psi'}{S\left(1-\overset{\circ}{\Psi}{}^{2}\right)^{\frac{3}{2}}}\left[\overset{\circ}{\Psi}{}' - \overset{\circ}{\Psi}\left(\overset{\circ}{\Psi}\Psi' - \Psi'\,\overset{\circ\circ}{\Psi}\right)\right], \qquad (12)$$

Thus equations (11) and (12) give

$$\theta = 0$$

But this equation (13). Equations (7) –(11) reduce to those corresponding equations for perfect fluid. Hence we can conclude that viscous fluid distribution is incompatible with class one metric.

CASE : 2

In Lichnerowicz Universe, one considers an electrically charged fluid with infinite conductivity so that the electric field vanishes in the rest frame of the fluid but there is a non-vanishing magnetic field. Here we consider magneto-viscous fluid, for which the Einstein's field equations are given by

$$G_{ij} \equiv R_{ij} - \tfrac{1}{2} R\, g_{ij} = -k\,[\,T_{ij} + E_{ij}\,] = -k\,T'_{ij}, \qquad (14)$$

where $T'_{ij} = \rho u_i u_j - (\rho - \xi\theta)\, q_{ij} + \eta\sigma_{ij} + \mu\,|q|^{2}\,(u_i u_j + \tfrac{1}{2} g_{ij}) - q_i q_j$, ...(15)

μ being the magnetic permeability and q_i's are the magnetic field vectors defined by

$$q_i = \frac{1}{\mu} * F_{ij} u^j , \ldots\ldots(16)$$

$$\text{and } |q|^{2\cdot} = q_\alpha q^\alpha \geq 0 , \ldots\ldots(17)$$

where $*F_{ij}$ is the dual electromagnetic field tensor given by

$$*F = \tfrac{1}{2} \eta_{jikm} F^{km} , \ldots\ldots(18)$$

F_{ij} being the skew-symmetric electromagnetic field tensor.

We assume the incident magnetic field to be in the direction of the X-axies. As we consider here magnetoviscous fluid distribution, we find that the only non-vanishing component of electromagnetic field tensor is $F_{23.}$

The first set of Maxwell's equations are

$$F;jij = 4\,\pi J^i , \ldots\ldots(19)$$

$$\text{where } J^i = \varepsilon u^i + \sigma e^i , \ldots\ldots(20)$$

ξ being the energy density of electromagnetic field. In the magnetohydrodynamic universe , the charge density $\sigma \rightarrow \propto$, and the thermal conductivity $e^i \rightarrow 0$ so as the electric current J essentially remains finite. Thus eq. (19) reduces to the form.

$$F_{;j}{}^{ij} = 4\,\pi\xi u^i$$

That is

$$(-g)^{1/2} [\, (-g)^{1/2} F^{ij} \,]_{;j} = 4\,\pi\xi u^i$$

which gives us

$$\frac{\partial F^{23}}{\partial \phi} \, and \, \frac{\partial F^{23}}{\partial \Theta} + F^{23} \cot\Theta = 0 , \ldots\ldots(21)$$

Also from the second set of Maxwell's equations namely,

$$F_{[ij,k]} = 0$$

we get

$$\frac{\partial F_{23}}{\partial r} = \frac{\partial F_{23}}{\partial t} = 0 \text{ ,} \qquad (22)$$

Now from equations (21) and (22) we get

$$F_{23} = l \sin \Theta \text{ ,} \qquad (23)$$

where l is an arbitrary constant.

Thus the only existing component of the magnetic field here is

$$q_{l} = z\, r^2 (1+\Psi'^2 - \overset{\circ}{\Psi}{}^2)^{½} (1- \overset{\circ}{\Psi}{}^2)^{1/2} \text{ ,} \qquad (24)$$

$$\text{where } Z = \frac{\sin \Theta F_{23}}{\mu} \text{ ,} \qquad (25)$$

Therefore the Einstein field equation (14) gives

$$- \frac{\Psi'^2}{r^2 s} + \left[(1+\Psi'^2)\overset{\circ\circ}{\Psi} - \Psi' \overset{\circ}{\Psi} \overset{\circ}{\Psi}{}' \right] \frac{2\Psi'}{rs^2} = kp - k\left(\frac{4}{3}\eta + \xi\right)\theta - \frac{k\mu}{2} z^2 r^4 \text{ ,} \qquad (26)$$

$$- \frac{1}{s^2}\left[\left\{ \left(1-\overset{\circ}{\Psi}{}^2\right)\Psi'' - (1+\Psi'^2)\overset{\circ\circ}{\Psi} + 2\Psi' \overset{\circ}{\Psi} \overset{\circ}{\Psi}{}' \right\} \frac{\Psi'}{r} - \left(\Psi'' \overset{\circ\circ}{\Psi} - \overset{\circ}{\Psi}{}'^2\right) \right]$$

$$= kp + k\left(\frac{2}{3}\eta - \xi\right)\theta + \frac{k}{2}\mu z^2 r^4 \text{ ,} \qquad (27)$$

$$- \frac{\Psi'^2}{r^2 s} - \left[\left(1-\overset{\circ}{\Psi}{}^2\right)\Psi'' + \Psi' \overset{\circ}{\Psi} \overset{\circ}{\Psi}{}' \right] \frac{2\Psi'}{rs^2} = -k\rho - \frac{k\mu}{2} z^2 r^4 \text{ ,} \qquad (28)$$

$$\left[\left(1+\overset{\circ}{\Psi}{}^2\right)\overset{\circ}{\Psi}{}' + \Psi' \overset{\circ}{\Psi} \overset{\circ\circ}{\Psi} \right] \frac{2\Psi'}{rs^2} = 0, \qquad (29)$$

Where

$$S = 1 + \Psi'^2 - \overset{\circ}{\Psi}{}^2 \quad , \qquad (30)$$

But , here the expansion factor

$$\theta = \frac{\Psi'}{\left(1+\Psi'^2-\overset{\circ}{\Psi}\right)\left(1-\overset{\circ}{\Psi}{}^2\right)^{\frac{3}{2}}}\left[\overset{\circ}{\Psi}{}' - \overset{\circ}{\Psi}\left(\overset{\circ}{\Psi}\overset{\circ}{\Psi}{}' - \Psi'\overset{\circ\circ}{\Psi}\right)\right], \qquad (31)$$

Thus comparing Equations (30) and (31) we see that

$$\theta = 0, \qquad (32)$$

Also here we find that

$$\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_{44} = \sigma_{14} = 0, \qquad (33)$$

which implies that the shear tensor is zero.

Thus we can conclude that magneto-viscous fluid distribution is incompatible with class one metric.

CASE 3 :-

Here we consider the case of viscous fluid distribution couple with a zero-mass scalar field whose energy –momentum tensor $T_{\mu\gamma}$ is given by

$$T_{ij} = k\left[\rho u_i u_j - (p - \xi\theta) H_{ij} + \eta\sigma_{ij} + \frac{1}{\phi^2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi^{,k}\phi_{,k}\right)\right], \qquad (34)$$

where ϕ is the scalar potential which satisfies the relation.

$$\frac{\partial}{\partial x^j}\left[\phi_{,i}(-g)^{\frac{1}{2}}g^{ij}\right] - \frac{\phi_{,i}\phi_{,j}}{\phi}(-g)^{\frac{1}{2}}g^{ij} = 0, \qquad (35)$$

Thus Einstein's equation

$$G_{ij} \equiv R_{ij} - \tfrac{1}{2} g_{ij} R = -k\, T_{ij}, \qquad (36).$$

gives in this case

$$-\frac{\Psi'^2}{r^2 s}+\left[\left(1+\Psi'^2\right)\overset{\circ\circ}{\Psi}-\Psi'\mathring{\Psi}\mathring{\Psi}'\right]\frac{2\Psi'}{rs^2}=$$

$$kp-k\left(\frac{4}{3}\eta+\xi\right)\theta+\frac{k}{2s}\left[\left(1-\mathring{\Psi}^2\right)\phi'^2+\left(1+\Psi'^2\right)\mathring{\phi}^2\right], \quad (37)$$

$$-\frac{1}{s^2}\left[\left\{\left(1-\mathring{\Psi}^2\right)\Psi''-\left(1+\Psi'^2\right)+2\Psi'\mathring{\Psi}\mathring{\Psi}'\right\}\frac{\Psi'}{r}-\left(\Psi''\overset{\circ\circ}{\Psi}-\mathring{\Psi}'^2\right)\right]$$

$$=k\rho+k\left(\frac{2}{3\eta}-\xi\right)\theta+\frac{k}{2s}\left(\mathring{\phi}^2-\phi'^2\right)+\frac{k}{2s}\left(\phi'\mathring{\Psi}-\mathring{\phi}\Psi'\right)^2, \quad (38)$$

$$-\frac{\Psi'^2}{r^2 s}-\left[\left(1-\mathring{\Psi}^2\right)\Psi''-\Psi'\mathring{\Psi}\mathring{\Psi}'\right]\frac{2\Psi'}{rs^2}=$$

$$-k\rho-\frac{k}{2s}\left(1-\mathring{\Psi}^2\right)\phi'^2+\left(1+\Psi'^2\right)\phi'^2+\left(1+\Psi'^2\right)\mathring{\phi}^2, \quad (39)$$

$$-\left[\left(1+\Psi'^2\right)\mathring{\Psi}'-\Psi'\mathring{\Psi}\Psi''\right]\frac{2\Psi'}{rs^2}=k\left(\frac{\Psi'\mathring{\Psi}}{1-\mathring{\Psi}^2}\right)\left[p+\xi-\left(\frac{4}{3}\eta+\xi\right)\right]\theta-$$

$$-\frac{k}{s}\left[\left(1+\Psi'^2\right)\phi'\mathring{\phi}-\Psi'\mathring{\Psi}\phi'^2\right], \quad (40)$$

$$\left[\left(1-\mathring{\Psi}^2\right)\Psi'+\Psi'\mathring{\Psi}\overset{\circ\circ}{\Psi}\right]\frac{2\Psi'}{rs^2}=\frac{k}{s}\left[\left(1-\mathring{\Psi}^2\right)\phi'\mathring{\phi}+\Psi'\mathring{\Psi}\mathring{\phi}^2\right], \quad (41)$$

where $S = 1 + \Psi'^2 - \mathring{\Psi}^2$, ...(42)

If we take the scalar potential ϕ to be independent of time and to be a function of position only, then equation (41) gives

$$\left(1-\mathring{\Psi}^2\right)\mathring{\Psi}'+\Psi'\mathring{\Psi}\overset{\circ\circ}{\Psi}=0, \quad (43)$$

But as the expansion factor in this case is given by

$$\theta=\frac{\Psi'}{s\left(1-\mathring{\Psi}^2\right)^{\frac{3}{2}}}\left[\mathring{\Psi}'-\mathring{\Psi}\left(\mathring{\Psi}\mathring{\Psi}'-\Psi'\overset{\circ\circ}{\Psi}\right)\right], \quad (44)$$

we see that here using equation (43),

$$\theta = 0$$

with this relation equations (37) to (41) reduce to those corresponding equations for perfect fluid.

Thus we see that viscous fluid distribution coupled with a zero-mass scalar field is incompatible with class one metric if the scalar field happens to be a function of position only

6.3 CONCLUSIONS :

From the results obtained above it may be concluded that viscous fluid distribution cannot be the source term for generating gravitational field for a spherically symmetric class one metric. Similarly, the magneto-viscous fluid distribution is seen to be incompatible with the gravitational field generated by this type of metric. Further it has been obtained also that there cannot exist any solution for the viscous distribution coupled with a zero-mass scalar field in the case of the above class one metric if the scalar field is a function of position only.

CHAPTER NO -7

ROTATING PERFECT FLUID UNIVERSE COUPLED WITH LECTROMAGNETIC CHARGE INTERECTINGWITH GRAVITATIONAL FIELD.

7.1 INTRODUCTION

It is well known that almost all the astrophysical objects in this Universe has some form of rotation whether differential or uniform. Over the past few years the possibility of the entire universe being endowed with some rotation has attracted many physicists (Birch, Sistero, Bietenholz and Kronberg). From recent observations it is believed that the universe may be rotating at the rate of $\leq 10^{-3}$ rad s^{-1}. The existence of such a small rotation, when taken into consideration during the early stages of the universe, would play a prominent role in the dynamics of the universe as well as in the processes that involve the formation of the galaxies and others cosmological objects. Rotation plays an important role in the structure and equilibrium configuration of elementary particles as well as the astrophysical objects. The equilibrium configuration of rotating fluid can be considered as a small perturbation on a non-rotating configuration. That is why during the last few years there has been considerable effort in introducing rotation in the General Theory of Relativity so that it can be applied to realistic astrophysical situations.

Lense and Thirring were the first to attempt the study of the gravitational field due to a rotating body. Thereafter some physicists investigated on the rotational motion of the cosmological objects. Das etal . Hartle and Sharp Ellis , Stewart and Ellis, Hawking, Silk and Wright, Chandrasekhar and Friedman, Adams et al, Kamini et al , Bayin and Cooperstock, Bayis , Krori et al , Van Den Bergh and Wils, Whiteman, Islam , Tiwari et al , Maniharsingh , Maniharsingh and Bhamra, Maniharsingh and Mohanty studied the rotating fluid distributions under different conditions in trying to understand the structure and equilibrium, the nature and role of rotating astrophysical objects in this universe.

A number of detailed studies of the structure of relativistic stars have been initiated by the discovery of pulsars and their identification with rotating neutron stars. All known pulsars are observed to satisfy the conditions of slow rotation, that is tangential velocity of all fluid elements are much less than the speed of light and the centrifugal force is much less than the gravitational force. The observed frequencies of pulsars and their likely angular velocities during ,most of their lifetime, perhaps with the exception of a very short span immediately after birth, allow a slow rotation approximation to be used. An important feature of pulsar models is the strong magnetic field anchor in the rotating neutron star and they are known to enrich electromagnetic pulses at regular and extremely short intervals. Evidence suggests that pulsars contain extremely strong magnetic fields, among the strongest anywhere in the Universe; so they provide a unique opportunity for the study of complex electromagnetic processes. Thus in trying to obtain new information concerning pulsars in particular and rotating astrophysical objects in general , it is essential and appropriate to take up rotating charged models for our study.

In this problem, we take up slowly rotating charge perfect fluid distribution interacting with gravitational field as the object of our study, as it will be very stimulating to make investigations on such models in trying to obtain new information's concerning rotating astrophysical objects in this universe and we draw many conclusions for a realistic universe from such studies. Though it is believed that Robertson-Walker models are possibly most appropriate for representation of the large-scale structure of the space time, in most of these models are non-accelerating and non-shearing as well as non-rotating. Therefore , one has to investigate more general models than the Robertson-Walker ones. Thus here we consider a special type of metric and with this new metric there is the possibility of obtaining many stimulating results for further research. The study of the rotational perturbations of these models are also made in order to substantiate the possibility that the universe is endowed with some kind of rotation. Some authors had already obtained models of rotating universe without expansion and also model of expanding universe without rotation. Therefore, it will be of great interest to find out explicitly solved models of expanding as well as rotating cosmological objects, so that useful information about the behaviour of the universe can be obtained from such models. Thus we investigate and study here rotating as well as expanding models and it will be very helpful in exploring new ideas and results. Here we obtain some new interesting types of solutions and study their physical and geometrical properties mainly from rotational point of view.

7.2 DERIVATION OF FIELD EQUATIONS :

In this problem we consider the perturbed metric

$ds^2 = dt^2 - \exp[\,h(r) + k(t)\,]\,dr^2 - \exp[k(t)]\,(r^2\,d\Theta^2 + r^2 \sin^2\Theta\,d\phi^2)$

$+ 2\,r^2 \exp[\,k(t) \sin^2\Theta\,\Omega\,(r,t)\,d\phi\,dt$,..(1)

where h(r) is an arbitrary function of r ; k (t) is an arbitrary function of time ; $\Omega(r,t)$, the metric rotation function which is related to the local dragging of inertial frames.

The energy momentum tensor $T_{\mu\gamma}$ is given by

$$T_{\mu\gamma} = P_{\mu\gamma} + E_{\mu\gamma}, \quad \text{.................................(2)}$$

where $P_{\mu\gamma}$ is the energy-momentum tensor due to perfect fluid and takes the form.

$$P_{\mu\gamma} = (p + \rho)\, u_\mu\, u_\gamma - p g_{\mu\gamma}\,, \quad \text{..........................(3)}$$

where p is the isotropic pressure; ρ, the fluid density; and u_μ , the four-flow vector satisfying the relation

$$g_{\mu\gamma}\, u^\mu u^\gamma = 1\,, \quad \text{..(4)}$$

$E_{\mu\gamma}$ is the energy-momentum tensor due to electromagnetic field where $F_{\alpha\beta}$ are the electromagnetic field tensors satisfying the relations.

$$F^{ij}{}_{;j} = -\,\sigma\, u^i\,, \quad \text{..(5)}$$

And $$F_{[ij,k]} = 0, \quad \text{..(6)}$$

where σ(r,t) is the charge density of the electromagnetic field.

Here, in our problem, the only non-zero component of the electric field is F_{14} because of spherical symmetric (Papini and Weiss, 1986).

Therefore, Eistein's field equation

$R_{\mu\gamma} = 8\,\pi G\,(\,T_{\mu\gamma} - \tfrac{1}{2}\, g_{\mu\gamma}\, T)$ gives

$$\frac{\overset{\circ\circ}{k}}{2}\exp(h+k)+\frac{h'}{r}+\frac{3}{4}\overset{\circ}{k}^2\exp(h+k)=8\pi G\left[\frac{1}{2}(\rho-p)\exp(h+k)-\frac{1}{8\pi G}F_{14}^2\right], \quad (7)$$

$$1+\frac{1}{2}rh'\exp(-h)+\frac{1}{2}r^2\overset{\circ\circ}{k}\exp(k)+\frac{3}{4}r^2\overset{\circ}{k}^2\exp(k)-\exp(-h)$$
$$=8\pi G\left[\frac{1}{2}(\rho-p)r^2\exp(k)+\frac{1}{8\pi G}r^2\exp(-h)F_{14}^2\right], \quad (8)$$

$$\frac{3}{2}\overset{\circ\circ}{k}+\frac{3}{4}\overset{\circ}{k}^2=-8\pi G\left[\frac{1}{2}(\rho+3p)+\frac{1}{8\pi G}\exp(-h-k)F_{14}^2\right], \quad (9)$$

$$\frac{3\overset{\circ}{k}}{4}\Omega'+\frac{1}{2}\overset{\circ}{\Omega}'=0, \quad (10)$$

and

$$\left[\left(\frac{r}{2}h'-1\right)\exp(-h)+\frac{1}{2}r^2\left(\overset{\circ\circ}{k}+\frac{3}{2}\overset{\circ}{k}^2\right)\exp(k)\Omega+\Omega+\left(\frac{1}{4}r^2h'-2r\right)\exp(-h)\Omega'-\frac{1}{2}r^2\exp(-h)\Omega''\right]$$
$$\sin^2\Theta=-8\pi G\left[\begin{array}{l}r^2\sin^2\Theta\exp(k)\left\{(p+\rho)(\Omega-\omega)+\frac{1}{2}(p-\rho)\Omega\right\}-\\-\frac{1}{8\pi G}r^2\exp(-h)\sin^2\Theta F_{14}^2\Omega\end{array}\right], \quad (11)$$

Also from relations (5) and (6) we get

$$-\frac{\partial F_{14}}{\partial r}+\left[\frac{1}{2}h'-r\exp(-h)-r\sin^2\Theta\exp(-h)\right]F_{14}=\sigma, \quad (12)$$

The overhead dot and prime denote, respectively, partial differentiations with respect to "t" and "r" and a semi colon followed by a subscript denotes covariant differentiation.

7.3 SOLUTION OF THE FIELD EQUATIONS

Use of Equation (7) in Equation (8) gives

$$F_{14}^2=\frac{1}{2}r^{-2}\left[1-\exp(-h)-\frac{1}{2}rh'\exp(-h)\right]\exp(h), \quad (13)$$

And Equation (7) and (9) together gives

$$2\overset{\circ\circ}{k}+\frac{3}{2}\overset{\circ}{k}^{2}+r^{-1}h'\exp(-h-k)=-16\pi G\rho-2\exp(-h-k)F_{14}^{2}, \qquad (14)$$

and

$$\frac{3}{2}\overset{\circ}{k}^{2}+3r^{-1}h'\exp(-h-k)=-16\pi G\rho-2\exp(-h-k)F_{14}^{2}, \qquad (15)$$

Thus by virtue of Equation (13), Equation(14) and (15) become

$$16\pi Gp=\left(r^{-2}-\frac{1}{2}r^{-1}h'\right)\exp(-h-k)-2\overset{\circ\circ}{k}-\frac{3}{2}\overset{\circ}{k}^{2}-r^{-2}\exp(-k), \qquad (16)$$

and

$$16\pi G\rho=\left(\frac{5}{2}r^{-1}h'-r^{-2}\right)\exp(-h-k)+r^{-2}\exp(-k)+\frac{3}{2}\overset{\circ}{k}^{2}, \qquad (17)$$

By use of Equations (16) and (17) , now (11) takes the form

$$\frac{1}{2}r^{2}\exp(-h)\Omega''-\left(\frac{r^{2}}{4}h-2r\right)\exp(-h)\Omega'$$
$$=\frac{1}{2}r^{2}\left[2r^{-1}\exp(-h)h'-2\exp(k)\overset{\circ\circ}{k}\right](\Omega-\omega), \qquad (18)$$

CASE I:-

In this case we assume

$$\Omega-\omega=[1-\overset{\circ}{Q}(t)]\Omega, \qquad (19)$$

Then equation (19) becomes

$$\frac{1}{2}r^2\exp(-h)\Omega'' - \left(\frac{r^2}{4}h' - 2r\right)\exp(-h)\Omega'$$
$$= \frac{1}{2}r^2\left[2r^{-1}\exp(-h)h' - 2\exp(k)\overset{\circ\circ}{k}\right]\left[1 - \overset{\circ}{Q}(t)\right]\Omega, \qquad (20)$$

But Equation (10) gives

$$\Omega = \exp\left(-\frac{3}{2}k\right)M(r) + N(t), \qquad (21)$$

where M(r) is an arbitrary function of 'r' and N(t) is an arbitrary function of 't' which is set equal to zero for this case.

If we now make use of relation (21) in equation (20)
we get

$$\frac{1}{2}r^2\exp(-h)\frac{M''}{M} - \left(\frac{r^2}{4}h' - 2r\right)\exp(-h)\frac{M'}{M}$$
$$= \frac{1}{2}r^2\left[2r^{-1}\exp(-h)h' - 2\exp(k)\overset{\circ\circ}{k}\right]\left[1 - \overset{\circ}{Q}(t)\right]$$

i.e.

$$\frac{1}{2}\exp(-h)\frac{M''}{M} - \left(\frac{1}{4}h' - \frac{2}{r}\right)\exp(-h)\frac{M'}{M}$$
$$= [r^{-1}\exp(-h)h' - \exp(k)\overset{\circ\circ}{k}][1 - \overset{\circ}{Q}(t)], \qquad (22)$$

Here we see that the left-hand side of equation (22) is a function of 'r' only, therefore the right-hand side must be either a function of 'r' only or a function of 't' only. Thus here we take

$$r^{-1}\exp(-h)\,h' = -a, \qquad (23)$$

where 'a' is an arbitrary constant.

This gives

$$h = -\log\left(\frac{ar^2 - b}{2}\right), \qquad (24)$$

And with this value of 'h' , Equation (22) takes the form

$$\left(1-\frac{a}{b}r^2\right)\frac{M''}{M}+\left(\frac{4}{r}-5\frac{a}{b}r\right)\frac{M'}{M}=-\frac{4}{b}\left[\exp(k)\overset{\circ\circ}{k}+a\right]\left[1-\overset{\circ}{Q}(t)\right]$$

which can be separated as

$$\left(1-\frac{a}{b}r^2\right)\frac{M''}{M}+\left(\frac{4}{r}-\frac{5a}{b}r\right)\frac{M'}{M}=Z_1,\ldots\ldots(25)$$

and $\frac{4}{b}\left[\exp(k)\overset{\circ\circ}{k}+a\right]\left[\overset{\circ}{Q}(t)-1\right]=Z_1,\ldots\ldots(26)$

where Z_1 is an arbitrary constant

A solution of Equation (26) is

$$Q(t)=\left(\frac{b}{4}Z_1+1\right)t+C_1$$

$$k=2\log\left[2^{-1/2}(a-1)^{\frac{1}{2}}t+C_2\right],\ldots\ldots(27)$$

where C_1 and C_2 are arbitrary constants

$$p=\frac{1}{16\pi G}\left[t(a-1)^{\frac{1}{2}}+2^{\frac{1}{2}}C_2\right]^{-2}X$$

$$\left[3a-r^{-2}(b+2)-(a-1)\left\{t(a-1)^{\frac{1}{2}}+2^{\frac{1}{2}}C_2\right\}^{-1}\left[4(2t)^{\frac{1}{2}}-3\left\{t(a-1)^{\frac{1}{2}}+2^{\frac{1}{2}}C_2\right\}^{-1}\right]\right],\ldots\ldots(28)$$

$$\rho=\frac{1}{16\pi G}\left[t(a-1)^{\frac{1}{2}}+2^{\frac{1}{2}}C_2\right]^{-2}\left[r^{-2}(b-2)-11a+3(a-1)\left\{t(a-1)^{\frac{1}{2}}+2^{\frac{1}{2}}C_2\right\}^{-2}\right],\ldots\ldots(29)$$

$$F_{14}^2=\frac{1}{2}r^2(b+2)(ar^2-b)^{-1},\ldots\ldots(30)$$

$$\sigma=2^{\frac{-3}{2}}r^{\frac{-3}{2}}(b+2)^{\frac{1}{2}}(ar^2-b)^{\frac{-1}{2}}\left[2ar(ar^2-b)^{-1}(r-2)-r(ar^2-b)(1+\sin^2\theta)+1\right]\ldots\ldots(31)$$

Using the substitution $y=-\frac{1}{2}a\,r^2$ in equation (25) we have,

$$y(1-y)\frac{d^2M}{dy^2}+\left(\frac{5}{2}-3y\right)\frac{dM}{dy}-\frac{Z_1}{2a}M=0,\ldots\ldots(32)$$

Here we see that Equation (32) is similar to the hypergeometric equation

$$y(1-y)\frac{d^2F}{dy^2}+[y-(1+\alpha+\beta)y]\frac{dF}{dy}-\alpha\beta F=0$$

of which the general solution is given by

$F = A_0F(\alpha,\beta;\gamma;y) + A_1 y^{1-\gamma} F(1-\gamma+\alpha, 1-\gamma+\beta; 2-\gamma;y)$,..................................(33)

where A_0 and A_1 are arbitrary constants and

$$F(\alpha,\beta;\gamma;y)=\sum_{n=0}^{\infty}\frac{(\alpha)_n}{n!(\gamma)_n}y^n$$

Now we get the general solution of Equation (33) as

$$M(r)=A_0\sum_{n=0}^{\alpha}\frac{(\alpha)_n(\beta)_n}{n!\left(\frac{5}{2}\right)_n}y^n+A_1y^{\frac{-3}{2}}\sum_{n=0}^{\alpha}\frac{\left(\alpha-\frac{3}{2}\right)_n\left(\beta-\frac{3}{2}\right)_n}{n!\left(-\frac{1}{2}\right)_n}y^n, \qquad (34)$$

Here since the second term is not regular at y =0

We take $A_1 = 0$

Thus we get $M(r)=A_0\sum_{n=0}^{\infty}\frac{(\alpha)_n(\beta)_n}{n!\left(\frac{5}{2}\right)_n}y^n$

Or $M(r)=A_0(1-y)^{5/2-\alpha-\beta}\sum_{n=0}^{\alpha}\frac{\left(\frac{5}{2}-\alpha\right)_n\left(\frac{5}{2}-\beta\right)_n}{n!\left(\frac{5}{2}\right)_n}y^n$,..................................(35)

Taking different values of α andβ we get different values of M(r) and thereby, different value of Ω. Thus for example, taking α = -1, β = 3 then we get

$Z=\frac{5a}{2}$ and $F=\left(-\frac{1}{2},\frac{5}{2};\frac{5}{2};y\right)=(1-y)^{\frac{1}{2}}$

Thus $M(r)=A_0\left(1+\frac{a}{2}r^2\right)$

In this case we get

$$\Omega(r,t) = A_0\left(1+\frac{a}{2}r^2\right)\left[2^{-1/2}(a-1)^{\frac{1}{2}}t + C_2\right]^{-3}, \qquad (36)$$

and $$w(r,t) = A_0\left(1+\frac{bz_1}{4}\right)\left(1+\frac{a}{2}r^2\right)\left[2^{-1/2}(a-1)^{\frac{1}{2}}t + C_2\right]^{-3}, \qquad (37)$$

Here we see that the solution for M(r) shown in relation (35) above is a series solution. Therefore we try underneath to get some interesting exact solutions by taking some particular relations between 'a' and 'b' and by giving some particular values to Z_1.

CASE I a :-

Here we take up the case of open models by taking a = - and obtain three different expressions for Ω (r,t) corresponding to three different values of Z_1

Taking $Z_1 = 4$ we obtain, from Equation (25),

$M(r) = r^{-3}\,[\,C_3 r - C_3(1+r^2)^{1/2}\sin^{-1} r + C_4(1+r^2)^{1/2}]$,

where C_3 and C_4 are arbitrary constants.

Thus ,here,

$$\Omega(r,t) = \left[r^{-3}\left\{C_3 r - C_3(1+r^2)^{\frac{1}{2}}\sin^{-1} r + C_4(1+r^2)^{\frac{1}{2}}\right\}\right]\left[2^{-1/2}(a-1)^{\frac{1}{2}}t + C_2\right]^{-3}, \qquad (38)$$

and

$$w(r,t) = (1-a)\left[r^{-3}\left\{C_3 r - C_3(1+r^2)^{\frac{1}{2}}\sin^{-1} r + C_4(1+r^2)^{\frac{1}{2}}\right\}\right]\left[2^{-1/2}(a-1)^{\frac{1}{2}}t + C_2\right]^{-3}, \qquad (39)$$

Next, if we take $Z_1 = -3$, we get

$$M(r) = \frac{C_5}{2}r^{-2}(1-r^2)^{\frac{1}{2}} - \frac{C_5}{2}r^{-3}\sinh^{-1} r + C_6 r^{-3}$$

where C_5 and C_6 are arbitrary constants.

Thus here

$$\Omega(r,t) = \left[\frac{C_5}{2}r^{-2}(1-r^2)^{\frac{1}{2}} - \frac{C_5}{2}r^{-6}\sinh^{-1} r + C_6 r^{-3}\right]\left[2^{-1/2}(a-1)^{\frac{1}{2}}t + C_2\right]^{-3}, \qquad (40)$$

and

$$w(r,t)=\left(1+\frac{3a}{4}\right)\left[\frac{C_5}{2}r^{-2}\left(1-r^2\right)^{\frac{1}{2}}-\frac{C_5}{2}r^{-3}\sinh^{-1}r+C_6r^{-3}\right]\left[2^{-1/2}(a-1)^{\frac{1}{2}}t+C_2\right]^{-3} \quad ,......(41)$$

And taking Z_1 = -5 we get

$$M(r)=C_7\left(1+r^2\right)^{\frac{1}{2}}-\frac{C_8}{3}\left(8r-4r^{-1}-r^{-3}\right)$$

where C_7 and C_8 are arbitrary constants.

Therefore, here

$$\Omega(r,t)=\left[C_7\left(1+r^2\right)^{\frac{1}{2}}-\frac{C_8}{3}\left(8r+4r^{-1}-r^{-3}\right)\right]\left[2^{-1/2}(a-1)^{\frac{1}{2}}t+C_2\right]^{-3} \quad ,......................(42)$$

and

$$w(r,t)=\left(1+\frac{5a}{4}\right)\left[C_7\left(1+r^2\right)^{\frac{1}{2}}-\frac{C_8}{3}\left(8r+4r^{-1}-r^{-3}\right)\right]\left[2^{-1/2}(a-1)^{\frac{1}{2}}t+C_2\right]^{-3} \quad ,.......(43)$$

CASE I.b.

Here we consider the case of closed model universes by taking a = b and obtain different values of Ω (r,t) corresponding to different values of Z_1.

Taking Z_1 = -3, Equation (25) gives

$$M(r)=\frac{C_9}{2}r^{-3}\sin^{-1}r-\frac{C_9}{2}r^{-2}\left(1+r^2\right)^{\frac{1}{2}}+C_{10}r^{-3}$$

where C_9 and C_{10} are arbitrary constants.

In this case

$$\Omega(r,t)=\left[\frac{C_9}{2}r^{-3}\sin^{-1}r-\frac{C_9}{2}r^{-2}\left(1+r^2\right)^{\frac{1}{2}}+C_{10}r^{-3}\right]\left[2^{-1/2}(a-1)^{\frac{1}{2}}t+C_2\right],....................(44)$$

and

$$w(r,t)=\left(1-\frac{3a}{4}\right)\left[\frac{C_9}{2}r^{-3}\sin^{-1}r-\frac{C_9}{2}r^{-2}\left(1+r^2\right)^{\frac{1}{2}}+C_{10}r^{-3}\right]\left[2^{-1/2}(a-1)^{\frac{1}{2}}t+C_2\right],..........(45)$$

For $Z_1 = 0$, we have

$$M(r)=\frac{C_{11}}{2}r^{-2}\left(1-r^2\right)^{\frac{1}{2}}-\frac{C_{11}}{2}\log\tan\left[\frac{\pi}{4}+\frac{1}{2}\tan^{-1}\left\{r^{-1}\left(1-r^2\right)^{\frac{1}{2}}\right\}\right]-$$

$$-C_{11}r^{-1}\left(1-r^2\right)^{\frac{1}{2}}-\frac{C_{11}}{3}r^{-3}\left(1-r^2\right)^{3/2}+C_{12},$$

where C_{11} and C_{12} are arbitrary constants.

Thus here

$$\Omega(r,t)=\left[\begin{array}{l}\frac{C_{11}}{2}r^{-2}\left(1-r^2\right)^{1/2}+\frac{C_{11}}{2}\log\tan\left[\frac{\pi}{4}+\frac{1}{2}\tan^{-1}\left\{r^{-1}\left(1-r^2\right)^{1/2}\right\}\right]-C_{11}r^{-1}\left(1-r^2\right)^{\frac{1}{2}}-\\-\frac{C_{11}}{3}r^{-3}\left(1-r^2\right)^{\frac{3}{2}}+C_{12}\end{array}\right]X$$

$$X\left[2^{-\frac{1}{2}}(a-1)^{\frac{1}{2}}t+C_2\right],\ldots\ldots(46)$$

and

$$w(r,t)=\left[\begin{array}{l}\frac{C_{11}}{2}r^{-2}\left(1-r^2\right)^{\frac{1}{2}}+\frac{C_{11}}{2}\log\tan\left[\frac{\pi}{4}+\frac{1}{2}\tan^{-1}\left\{r^{-1}\left(1-r^2\right)^{-\frac{1}{2}}\right\}\right]-C_{11}r^{-1}\left(1-r^2\right)^{\frac{1}{2}}-\\-\frac{C_{11}}{3}r^{-3}\left(1-r^2\right)^{\frac{3}{2}}+C_{12}\end{array}\right]X$$

$$X\left[2^{-1/2}(a-1)^{\frac{1}{2}}t+C_2\right],\ldots\ldots(47)$$

And if $Z_3 = 5$ we obtain

$$M(r)=\frac{C_{13}}{3}\left(r^{-3}+4r^{-1}-8r\right)+C_{14}\left(1-r^2\right)^{\frac{1}{2}}$$

where C_{13} and C_{14} are arbitrary constants

In this case we get

$$\Omega(r,t)=\left[\frac{C_{13}}{3}\left(r^{-3}+4r^{-1}-8r\right)+C_{14}\left(1-r^2\right)^{\frac{1}{2}}\right]\left[2^{-1/2}(a-1)^{\frac{1}{2}}t+C_2\right],\ldots\ldots(48)$$

and

$$w(r,t)=\left(1+\frac{5}{4}a\right)\left[\frac{C_{13}}{3}\left(r^{-3}+4r^{-1}-8r\right)+C_{14}\left(1-r^{2}\right)^{\frac{1}{2}}\right]\left[2^{-1/2}(a-1)^{\frac{1}{2}}t+C_{2}\right]^{-3}, \qquad \text{..........(49)}$$

CASE I.c.

If we take Q(t) = t in equation (19) then it reduces to the case of perfect dragging . In this case equation (20) reduces to the form.

$$\Omega''-\left(\frac{1}{2}h'-\frac{4}{r}\right)\Omega'=0, \qquad \text{.......(50)}$$

But Equation (10) gives

$$\Omega=\exp\left(-\frac{3}{2}k\right)M(r)+N(t)$$

If we make use of this relation in Equation (50) we get

$$M''-\left(\frac{1}{2}h'-\frac{4}{r}\right)M'=0, \qquad \text{.......(51)}$$

which gives

$$M=f(r)+S_1, \qquad \text{.......(52)}$$

S_1 being an arbitrary constant; and

$$f(r)=S_2\int r^{-4}e^{\frac{h}{2}}dr, \qquad \text{.......(53)}$$

where S_2 is an arbitrary constant.

We obtain in this case,

$$\Omega(r,t)=\left[f(r)+S_1\right]\exp\left(-\frac{3}{2}k\right)+N(t), \qquad \text{.......(54)}$$

CASE. 2 .

In this case we assume

$$\Omega-\omega=b_0\,\Omega'', \qquad \text{.......(55)}$$

where b_0 is an arbitrary constant

Then equation (18) takes the form

$$\frac{1}{2}\exp(-h)\Omega'' - \left(\frac{1}{4}h' - \frac{2}{r}\right)\exp(-h)\Omega' = b_0\left[r^{-1}\exp(-h)h' - \exp(k)\ddot{k}\right]\Omega'', \qquad (56)$$

But from Equation (10) we get

$$\Omega = \exp\left(-\frac{3}{2}k\right)M(r) + N(t)$$

Making use of this relation in the above equation (56), we get

$$\frac{1}{2}\exp(-h) - \left(\frac{1}{4}h' - \frac{2}{r}\right)\exp(-h)\frac{M'}{M''} = b_0\left[r^{-1}\exp(-h)h' - \exp(k)\ddot{k}\right], \qquad (57)$$

Here we have

$$h = -\log\left(\frac{ar^2 - b}{2}\right), \qquad (58)$$

Then Equation (56) becomes

$$\left(1 - \frac{a}{b}r^2\right) + \left(\frac{4}{r} - 5\frac{a}{b}r\right)\frac{M'}{M''} = \frac{4b_0}{b}\left[\exp(k)\ddot{k} + a\right], \qquad (59)$$

Since here the left-hand side of Equation (59) is a function of 'r' only whereas the right side is a function of 't' only therefore we can equate both of them to a constant.

Thus here we take

$$\left(1 - \frac{a}{b}r^2\right) + \left(\frac{4}{r} - 5\frac{a}{b}r\right)\frac{M'}{M''} = Z_2, \qquad (60)$$

and

$$\frac{4b_0}{b}\left[\exp(k)\ddot{k} + a\right] = Z_2, \qquad (61)$$

where Z_2 is an arbitrary constant

Here equation (61) gives

$$k = 2\log\left[\left(\frac{4ab - bZ_2}{8b_0}\right)^{\frac{1}{2}} t + b_1\right], \qquad (62)$$

b_1 being an arbitrary constant.

Thus in this case we get

$$\rho = \frac{1}{16\pi G}\left[(4ab_0 - bZ_2)^{\frac{1}{2}}t + 2b_1(2b_0)^{\frac{1}{2}}\right]^{-1} X$$

$$X\left[12b_0r^{-2} - 44ab_0 + 6(4ab_0 - bz_2)\left\{(4ab_0 - bz_2)^{\frac{1}{2}}t + 2b_1(2b_0)^{\frac{1}{2}}\right\}^{-1}\right], \qquad (63)$$

$$p = \frac{1}{16\pi G}\left[(4ab_0 - bz_2)^{\frac{1}{2}} + 2b_1(2b_0)^{\frac{1}{2}}\right]^{-1}\left[36ab_0 - 4r^{-1}b_0(b+2) - 8bz_2 - 6(4ab_0 - bz_2)\right] \qquad (64)$$

$$F_{14} = 2^{\frac{-1}{2}}(b+2)^{\frac{1}{2}}r^{-1}(ar^2 - b)^{-1/2}, \qquad (65)$$

$$\sigma = 2^{-3/2}r^{-3/2}(b+2)^{\frac{1}{2}}(ar^2 - b)^{\frac{-1}{2}}\left[2ar(ar^2 - b)^{-1}(r-2) - r(ar^2 - b)(1 + \sin^2\theta) + 1\right] \qquad (66)$$

Again Equation (60) gives

$$M = b_2\int r^{4/z_2 - 1}\left[\frac{a}{b}r^2 + (Z_2 - 1)\right]^{\frac{17-15Z_2}{6(Z_2-1)}} dr, \qquad (67)$$

where b_2 is an arbitrary constant, thus giving different values of M corresponding to different values of Z_2, a and b. Thus in this case, we get

$$\Omega = N(t) + b_2\left[\left(\frac{bz_2 + 4ab_0}{8b_0}\right)^{\frac{1}{2}}t + b_1\right]^{-3}\int r^{\frac{4}{z_2-1}}\left[\frac{a}{b}r^2 + (z_2 - 1)\right]^{\frac{17-15z_2}{6(z_2-1)}} dr, \qquad (68)$$

and

$$\omega = \left[\frac{b_2\left\{\int r^{\frac{4}{z_{2-1}}}\right\}\left[\frac{a}{b}r^2+(z_2-1)\right]^{\frac{17-17z_2}{6(z_{2-1})}}dr + N(t) - a_0\left(\frac{4}{r}-5\frac{a}{b}r\right)\left(\frac{a}{b}r^2+z_2-1\right)^{-1} r^{\frac{4}{z_{2-1}}}}{\left\{\frac{a}{b}r^2+(z_{2-1})\right\}^{\frac{17-15z_2}{6(z_2-1)}}}\right] X$$

$$X\left[\left(\frac{bz_2+4ab_0}{8b_0}\right)^{\frac{1}{2}}t+b_1\right]^{-3}, \qquad \text{.......(69)}$$

CASE. 3.:-

In this case we assume

$$\omega = d_0 \overset{\circ}{k}(\Omega - g), \qquad \text{.......(70)}$$

so that $\Omega = f(r) + g(t), \omega = -\overset{\circ}{k} f(r),$ ....(71)

where ‘f; is an arbitrary function of ‘r’ and ‘g’ is an arbitrary function of ‘t’.

Then in this case (18) takes the form

$$(d_0 f + 1)^{-1}\left[f'' - \left(\frac{1}{2}h' - \frac{4}{r}\right)f'\right]\exp(-h) = 2\left[r^{-1}\exp(-h)h' - \exp(k)\overset{\circ\circ}{k}\right]g, \qquad \text{.......(72)}$$

where $1 + \overset{\circ}{k} = d_0 g$

d_0 being an arbitrary constant.

Here also since the left-hand side of (72) is a function of ‘r’ only, therefore, the right-hand side must be either a function of r only or a function of ‘t’ only. Thus now in order that the right hand side may a function of time only , we assume.

$$r^{-1}\exp(-h)h' = -a, \qquad \text{.......(73)}$$

where ‘a’ is an arbitrary constant.

This gives

$$h = -\log\left(\frac{ar^2 - d}{2}\right), \qquad (74)$$

where 'd' is an arbitrary constant.

Then Equation (72) assume the form

$$(d_0 f + 1)^{-1}\left[f'' + \left(\frac{2ar}{ar^2 - d} + \frac{4}{r}\right)f'\right] = -2g\left[a + \exp(k)\overset{\circ\circ}{k}\right], \qquad (75)$$

A solution of Equation (75) is of the form

$$f = \frac{1}{3}d^{-2}r^{-2}\left(2ar^2 - d\right)\left(a + dr^{-2}\right)^{\frac{1}{2}}, \qquad (76)$$

$$k = 2\log\left[\left(\frac{a}{2}\right)^{\frac{1}{2}}t + a_1\right],$$

where a_1 is an arbitrary constant.

Thus from equation (71) and (76) we obtain

$$\Omega = \frac{1}{d_0}\left[1 + 2\log\left\{\left(\frac{a}{2}\right)^{\frac{1}{2}}t + a_1\right\}\right] + \left[\frac{1}{3}d^{-2}r^{-2}\left(2ar^2 - d\right)\left(a + dr^{-2}\right)^{\frac{1}{2}}\right], \qquad (77)$$

and

$$\omega = -(2a)^{\frac{1}{2}}\left[\left(\frac{a}{2}\right)^{\frac{1}{2}}t + a_1\right]^{-1}\left[\frac{1}{3}d^{-2}r^{-2}\left(2ar^2 - d\right)\left(a + dr^{-2}\right)^{\frac{1}{2}}\right], \qquad (78)$$

Here in this case we have

$$p = \frac{1}{16\pi G}\left[a - \left\{\left(\frac{a}{2}\right)^{\frac{1}{2}}t + a_1\right\}^{-2}\left\{2a + \frac{r^{-2}}{2}(1 + d)\right\}\right], \qquad (79)$$

$$\rho = \frac{1}{16\pi G}\left[\left(\frac{a}{2}\right)^{\frac{1}{2}} t + a_1\right]\left[\frac{5}{2}\left(a - r^{-1}\right) + r^{-2}\left(1 + \frac{d}{2}\right)\right], \quad (80)$$

$$F_{14}^2 = 2^{-1}(2 + d)r^{-2}\left(ar^2 - d\right)^{-1}, \quad (81)$$

$$\sigma = 2^{\frac{-3}{2}} r^{\frac{-3}{2}} (d + 2)^{\frac{1}{2}}\left(ar^2 - d\right)^{\frac{-1}{2}}\left[2ar\left(ar^2 - d\right)^{-1}(r - 2) - r\left(ar^2 - d\right)\left(1 + \sin^2\Theta\right) + 1\right] \quad (82)$$

7.4 DISCUSSIONS :-

In case 1, it is obtained that the rotational velocities decay with the increase of the time if N(t) is a decreasing function and k(t) is an increasing function of the time. For all the models obtained in this case, the expansion factor found to be

$$\theta = 3x2^{\frac{-1}{2}}(a - 1)^{\frac{1}{2}}\left[2^{\frac{-1}{2}}(a - 1)^{\frac{1}{2}} t + C_2\right]^{-1}, \quad (83)$$

which shows that these model universes are rotating as well as expanding one which may be taken as realistic models useful for studying the properties of rotating cosmological objects here have to satisfy the condition a> 1.

For the model universes obtained in case 1(a) the rotational velocities are not defined at the centers of these models . For the case when $Z_1 = -3$ the solution is restricted within the range $-1 \leq r \leq 1$. Again, also in all the models obtained in case 1(b) which may be taken as closed models, we observes that the solutions are not regular at the origin, and therefore the solution can be considered only in the regions excluding the origin; thus the rotational velocities are found to be arbitrary at the centers of these models. For the case when $Z_1 = 0$ and $Z_1 = 5$, the solutions are not to be valid beyond the range $-1 \leq r \leq 1$.

In case 2, the metric rotation Ω(r,t) as well as the matter rotation ω(r, t) decay with the time if N(t) happen to be a decreasing function of the time; however the matter rotation is found to be arbitrary at the centre of the model . Here the expansion factor is found to be

$$\theta = 3x\left(\frac{4ab_0 - bz_2}{8b_0}\right)^{\frac{1}{2}} x\left[\left(\frac{4ab_0 - bz_2}{8b_0}\right)^{\frac{1}{2}} t + b_1\right]^{-1} \quad ,\ldots\ldots(84)$$

Thus here we see that the model universes are expanding though the rate of expansions decrease with the time and thus in this case we get expanding as well as rotating models which may be taken as good example of real astrophysical situations.

In case 3, the metric rotation is the increasing function of time and decreasing function 'r' and also matter rotation is the decreasing function of both 'r' and 't' . In this case also the expansion factor comes out to be a decreasing function of time . Thus , here also our model universes comes out to be expanding as well as rotating models.

Here in this problem, we find that for all the models, to the first order in Ω, the pressure and the density are unperturbed. In some of the models, the electric field and the source density of the electric field are found to be functions of 'r' only, thereby being in dependent of the time; and they are found to be decreasing functions of 'r'. Thus the effect of the electric field in these cases are independent of the time. Again from the studies made to reveal the intrinsic nature of the rotation and to elucidate the role of Ω, we come to know that although Ω plays a role in the dragging of local inertial frames it is not the angular velocity of these frames except when it concides with 'ω' which is the angular velocity of matter. Even in this case the nature of the rotation is still intrinsic if it is differential. Also the field equations show that if the solutions of charged fluid distribution are known, the solutions of the slowly rotating model universes can be known. Here in each case the integration constant may be determined by matching the interior value of Ω with the exterior one on the boundary.

Again in the absence of the fields the field equations automatically reduce to those of slowly rotating perfect fluid distribution. It may also be noted that the solutions to the field equations (7) –(11) may be useful in understanding the equilibrium structure of the pulsars in particular and the rotating stars in general. Finally to sum up, from the results obtained in connection with our study it may be concluded that there is very much the possibility of the existence of slowly rotating cosmological objects coupled with electromagnetic field models obtained by us in the different cases will have considerable importance in the study of rotating astrophysical objects in this Universe.

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