A Statistical Study on Flood Frequency Analysis of North-East India

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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Dedicated to my beloved father

(Late Gokul Bhuyan)

ABSTRACT

The North-East India lies in Zone 2 out of the 7 hydro-meteorological Zones of India. The three hydro-meteorological subzones of Zone 2 have been considered here for the regional flood frequency analysis. The three parameter estimation methods know as L-moments, LH-moments and LQ-moments are considered for regional flood frequency analysis of these subzones. First of all, the regional frequency analysis by using L-moments has been carried out for these three subzones. The homogeneity of these subzones has been tested by using heterogeneity measures. It has been observed from heterogeneity measures that the subzones 2(b) and 2(c) are found to be homogeneous and the subzone 2(a) is found to be heterogeneous. The K-means cluster analysis technique has been used to divide the subzone 2(a) into two homogeneous subzones i.e. 2(a)A and 2(a)B. The five probability distributions namely generalized extreme value (GEV), generalized logistic (GLO), generalized Pearson (GPA), generalized normal (GNO) and Pearson type 3 (PE3) have been used for our study. The Z-statistic criteria and L-moment ratio diagram have been used as goodness of fit tests for selection of best fitting distributions for these four homogeneous subzones of North-East India. It has been observed from Z-statistic criteria as well as L-moment ratio diagram for these four homogeneous subzones that the PE3, GPA, PE3 and GLO distributions are identified as best fitting distributions for subzones 2(a)A, 2(a)B, 2(b) and 2(c), respectively. The identified regional best fitting distributions are used for developments of regional relationships for gauged and ungauged catchments areas of these four subzones.

For these four homogeneous subzones identified in the method of L-moment, the regional flood frequency analysis has been performed by using another parameter estimation method developed by Wang (1997) known as LH-moments. The four levels of LH-moments i.e. from L_1- to L_4-moments are considered for our study. The homogeneity of these four subzones is tested by using the LH-moments based heterogeneity measures. It has been observed that all the four subzones are found to be homogeneous for all level of LH-moments used in our study. In this study the three distributions namely GEV, GLO and GPA have been used for LH-moments based regional frequency analysis. The Z-statistic criteria and LH-moments ratio diagram have been used as goodness of fit test for identification best fitting distributions for all

the four homogeneous subzones at all level of LH-moments. It has been observed that the GEV distribution with different level of LH-moments attains the minimum Z-statistic values for all the four subzones. The GEV distribution with L_1-moments is best fitting distribution with the method of parameter estimation for subzone 2(a)A. Similarly, for subzones 2(a)B, 2(b) and 2(c) it has been observed that the GEV distribution with L_4-, L_1- and L_2-moments, respectively are identified as best fitting distributions for these subzones. The identified distributions are used for developments of regional flood frequency relationships for gauged and ungauged catchment areas of these four subzones.

The same subzones are again considered for regional frequency analysis by using another method of parameter estimation developed by Mudholkar and Hutson (1998) known as the LQ-moments. The homogeneity of these four subzones is again tested by using the heterogeneity measures based on the method of LQ-moments. It has been observed from the heterogeneity measures that all the four subzones are found to be homogeneous. Therefore, the regional frequency analysis can be performed for all the four subzones by using LQ-moments. The same five probability distributions used for the method of L-moments are used for this method also. The Z-statistic criteria in terms of LQ-moments and LQ-moments ratio diagram have been used as goodness of fit tests for identification of best fitting distributions for each of these subzones. It has been observed from both the goodness of fit tests that the PE3, GPA, PE3 and GLO distributions are identified as best fitting distributions for subzones 2(a)A, 2(a)B, 2(b) and 2(c), respectively. The regional flood frequency relationships have been developed for gauged and ungauged catchments areas of these four homogeneous subzones.

It has been observed that for all the three method of parameter estimations, all four subzones are found to be homogeneous. Therefore, the comparative study between the parameter estimation methods can be performed for these four homogeneous subzones of North-East India. Both the parameter estimation methods i.e. LH-moments and LQ-moments have been compared with the method of L-moments. For comparative study the Z-statistic values are used and the obtained results are again verified by using the Monte Carlo Simulation techniques in terms relative root mean square error (RRMSE) and relative bias (RBIAS). It has been observed from comparative study between the L- and LH-moments that the GEV distribution with level one LH-moment (i.e. L_1-moment) is identified as the best fitting distribution for subzone 2(a)A. Similarly, for subzones 2(a)B, 2(b) and 2(c) it has been observed from comparative study that the GEV with L_4-moment, GEV with L_1-moment and GLO with L-moment, respectively are the best fitting distributions with methods of parameter estimation for these subzones.

Again, from comparative study between the method of L-moment and LQ-moments, it has been observed that the PE3, GPA, PE3 and GLO distributions with L-moment are identified as the best fitting distributions with methods of parameter estimation for subzones 2(a)A, 2(a)B, 2(b) and 2(c), respectively.

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DECLARATION

I, **Abhijit Bhuyan**, hereby declare that the subject matter in this thesis entitled "**A Statistical Study on Flood Frequency Analysis of North-East India**" is the record of work done by me, that the contents of this thesis did not form basis of the award of any previous degree to me or to the best of my knowledge to anybody else, and that the thesis has not been submitted by me for any research degree in any other university/institute.

This thesis is being submitted to the Tezpur University for the degree of Doctor of Philosophy in Mathematical Sciences.

Place: Napam, Tezpur

Date: 21/06/10

(Abhijit Bhuyan)

TEZPUR UNIVERSITY

CERTIFICATE

This is to certify that the thesis entitled "**A Statistical Study on Flood Frequency Analysis of North-East India**" submitted to the School of Science and Technology, Tezpur University in partial fulfillment for the award of the degree of Doctor of Philosophy in Mathematical Sciences is a record of research work carried out by **Mr. Abhijit Bhuyan** under my supervision and guidance.

All help received by him from various sources have been duly acknowledged.

No part of this thesis has been submitted elsewhere for award of any other degree.

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Place: Napam, Tezpur
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(Abhijit Bhuyan)

Chapter 1

Introduction

Floods are natural hazards that cause deaths of humans and other living beings and damages to agricultural, economical and communicational infrastructure etc. in many parts of the world year after year, and more so North-East India. It is an inevitable natural phenomenon occurring from time to time in all rivers and natural drainage systems. Thus, people need protection against these disasters. Protective measures require accurate and reliable quantification of the frequency associated to a given flood stage or discharge.

The approaches to estimate the design flood fall under two categories- deterministic and probabilistic. Flood frequency analysis encompasses the techniques utilized to estimate the magnitude of extreme flood events corresponding to specified probability levels. It is the probabilistic approach to flood estimation.

The magnitude of a flood event is commonly referred to as T-year flood, where T usually called the return period or the recurrence interval, is a measure of the probability level of the event. The T-year flood event is that which can be expected to be exceeded once on an average of every T year. The estimation of the T-year flood typically involves inferences based on n years of flood records. The occurrence of extreme flood events is believed to be a stochastic process, since they tend to occur in an apparently random manner. Thus, one of the fundamental hypothesis of flood frequency analysis is that extreme flood events are random variables.

Due to the probabilistic assumption of flood frequency analysis, it is important that the sequence of flood records used for inferences be representative of a random sample. The popular approach for abstracting a random sample is to compose the sample of instantaneous annual maximum discharges recorded during successive water years. This so called annual maximum (AM) series, while seemingly consisting of occurrences of independent and identically distributed random variables (random samples), utilizes a very small fraction of flood records available.

This study concerns the AM-series models only. When the AM series is being analyzed, the probability of the T-year flood being exceeded in any single water year is $1/T$.

In many instances, the problem of flood frequency analysis boils down to the estimation of the T-year return period flood events Q_T, given the annual maximum series of flood discharges derived from n years of records, where n is usually is much less than T. In these cases, a probability model is used to fit the random sample, which is then extrapolated to the probability level, or equivalently, return period of interest.

While the true or the so-called parent distribution of the real world annual floods is unknown, many of its characteristics derived from a great many real world AM series have been investigated with particular emphasis or the behavior of the distributions right hand tail, since this portion of the distribution more directly affects the bias in the extreme flood quantile estimator. As the sample skewness is a statistic particularly sensitive to the behavior of the right tail of the distribution, the analysis of the skewness of the observed AM series should be especially useful. Matlas et al. (1975) found that within a given geographical region, the skewness of observed AM series exhibited a very high variability (standard deviation) about its regional mean.

1.1 Background of the study

The statistical analysis of flood magnitudes and corresponding frequencies of occurrence is an important area in hydrological research. It is essential for design of various hydraulic structures such as dams, spillways, road and railway bridges, culverts, urban drainage systems, flood plain zoning, economic evaluation of flood protection projects etc. Pilgrim and Cordery (1992) mention that estimation of peak flows on small to medium sized rural drainage basins is probably the most common application of flood estimation as well as being of greatest overall economic importance. As per Indian design criteria, frequency based floods find their applications in estimation of design floods for almost all the types of hydraulic structures, excluding large and intermediate size dams (Kumar et al., 2003). For design of large and intermediate size dams probable maximum flood and standard project flood are adopted, respectively (National Institute of Hydrology, 1992).

Traditionally, design floods of a given frequency have been estimated by fitting a probability distribution to observations at a single site. While this approach is relatively simple, the estimates may suffer from sampling variability especially for estimating return periods that exceed the length of the observed record (Hosking and Wallis, 1993; Cunnane, 1988). Again, at-site estimation of design floods considers only the data available from the specific site under consideration, and the reliability of the estimate is directly related to the amount of information available. For these reason, a number of regional flood frequency estimates have been developed. The biggest advantage of regional estimation is seen to be the increase in record length. Many studies (e.g. Lettenmaier et al., 1987; Hosking, 1990) have shown that flood estimates based on regional information are more accurate (have less absolute error) and are more stable (have less variance) than those based solely on at-site records. Again, the regional flood frequency analysis may be preferable because estimates at a single gauged site can be enhanced by pooling data from other sites confirmed to have similar frequency distributions. Furthermore, some regional methods provide a means of estimating flood frequencies at ungauged sites within a region where observations exist. The index-flood method, developed by the United States Geological Survey (Dalrymple, 1960), is commonly used to develop regional flood frequency models for ungauged sites or gauged sites where hydrologic information is not sufficient for reliable estimation of extreme events. However, information from other sites can be appropriately transferred only within a "homogeneous" region, and thus additional methods are needed to assure reliable identification of regions.

Bobee and Rasmussen (1995) reported that the use of regional information allows a reduction of sampling uncertainty by introducing more data, as well as a reduction of model uncertainty by facilitating a better choice of distribution.

The extreme value distribution in flood frequency analysis was first introduced by Gnedenko (1943), and was later modified by Gumbel (1954). Since than an extensive effort has been put into investigation of suitability of several probability distribution functions (PDFs). Cunnane (1989) provided a comprehensive survey of the alternative distribution functions commonly used for flood frequency analysis, in many parts of the world. Annual flood series were found to be often skewed, which led to the development and use of many skewed distributions, with the most commonly applied distributions now being the Gumbel (EV1), the generalized extreme value (GEV), the log Pearson type III (LP3), and the 3 parameter lognormal (LN3) (Pilon and Harvey, 1994). However, there is no theoretical basis for justifying the use of one specific distribution for modeling flood data and long term flood records show no justification for the adoption of a single type of distribution (Benson, 1968). In this respect development and application of statistical procedures for alternative parameter estimation of the utilized PDFs has received much attention.

There are various parameter estimation techniques available in statistical literature, out of these some of the commonly used methods are the method of moment (MOM), method of maximum likelihood (MML), probability weighted moments (PWMs) etc. However, all the methods are not suitable for all applications and also depend on the situation and data availability. In case of annual maximum flood series, data is not available for longer periods due to some technical problems and measurement error etc. The PWMs parameter estimation method proposed by Greenwood et al. (1979) as an alternative to the more conventional MOM and MLM, yields more accurate parameter estimates from the small samples. Hosking (1990) introduced L-moment parameter estimation method, which is nothing but a linear combination of PWMs. It gives more accurate estimation of parameters of a distribution than PWMs and easier to be used than PWM-based regional analysis (Hosking and Wallis, 1997).

L-moments can be defined for any random variable whose mean exists. They form the basis of a general theory which covers the summarization and description of theoretical probability distributions and observed data samples, and the estimation of parameters and quantiles of probability distributions. L-moments are analogous to conventional moments. However, a distribution may be specified by its L-moments even if some of its conventional moments do not exist.

The L-moment methods are demonstrably superior to those that have been used previously, and are now being adopted by many organizations worldwide (Hosking and Wallis, 1997). The L-moments offer significant advantages over ordinary product moments, especially for environmental data sets, because of the following (Zafirakou-Koulouris et al., 1998).

- L-moment ratio estimators of location, scale and shape are nearly unbiased, regardless of the probability distribution from which the observations arise (Hosking, 1990).
- L-moment ratio estimators such as L-coefficient of variation, L-skewness, and L-kurtosis can exhibit lower bias than conventional product moment ratios, especially for highly skewed distributions.
- The L-moment ratio estimators of L-coefficient of variation and L-skewness do not have bounds which depend on sample size as do the ordinary product moment ratio estimators of coefficient of variation and skewness.
- L-moment estimators are linear combinations of the observations and thus are less sensitive to the largest observations in a sample than product moment estimators, which square or cube the observations.
- L-moment ratio diagrams are particularly good at identifying the distributional properties of highly skewed data, whereas ordinary product moment diagrams are almost useless for this task (Vogel and Fennessey, 1993).

In the field of statistical analysis of extreme events, Wang (1997) introduced a new parameter estimation method known as LH-moments. The LH-moments are nothing but a generalization of the L-moments and are linear combinations of higher-order statistics for characterizing the upper part of distributions and larger events in data (Wang, 1997).

Recently, Mudholkar and Hutson (1998) introduced another parameter estimation method known as LQ-moment, which is an alternative of L-moments. The LQ-moments behaves like L-moments and it is easy to calculate (Mudholkar and Hutson, 1998). In their study they developed the LQ-moments of GEV distribution for flood frequency analysis of Black Stone and Feather rivers of United States. Shabri and Jemain (2007) used LQ-moment for statistical analysis of extreme events and a comparative study between L-moments and LQ-moments are performed for GEV distribution. In another study, LQ-moments are used for finding the best fitting probability distribution for annual maximum rainfall in Peninsular Malaysia performed by Zin, et al. (2008). In all the studies they used LQ-moment parameter estimation method for only the at site flood frequency analysis.

Hosking and Wallis (1993, 1997) reported the advantage of index-flood method together with L-moments for regional frequency analysis. The methodology has been applied successfully in modeling floods in a number of case studies from the United States (Vogel et al., 1993; Vogel and Wilson, 1996), New Zealand (Pearson, 1991, 1995), India (Parida et al., 1998; Kumar et al., 1999; Kumar and Chatterjee, 2005), Australia (Pearson et al., 1991) and Turkey (Saf, 2009).

Meshgi and Khalili (2009a, 2009b) used LH-moments for regional flood frequency analysis for Karkhe watershed located in western Iran. In their study they used same regional flood frequency analysis procedure proposed by Hosking and Wallis (1993, 1997) for L-moment with extension to higher level LH-moments and developed LH-moments parameters for generalized Pareto (GPA) and generalized logistic (GLO) distributions. Recently, Bhuyan et al. (2010) applied the LH-moment for regional flood frequency analysis proposed by Meshgi and Khalili (2009a, 2009b) to north-bank region of river Brahmaputra, Assam. They developed regional flood frequency relationships by using the identified GEV distribution with level one LH-moment for both gauged and ungauged catchments areas of this region.

In India, a number of studies have been carried out for the estimation of design floods for various structures by different organizations. Out of these Central Water commission (CWC), Research Designs and Standards Organization (RSDO), National Institute of Hydrology (NIH) and Flood control department etc. are prominent.

In this study an attempt has been made for regional flood frequency analysis of the entire North-East India region. The three parameter estimation methods i.e. L-moments, LH-moments and LQ-moments are used for regional flood frequency analysis of the study area. A comparative study between the parameter estimations methods have been performed and best fitting probability distributions along with the parameter estimation methods are identified.

1.2 Study area

In India, 26 hydro-meteorological subzones have been identified by the Planning and Coordination Committee of the Government of India to undertake detailed hydro-meteorological studies in each subzone. The location map of these 26 hydro-meteorological subzones of India is shown in Figure 1.1. The shaded portion in Figure 1.1 represents our study area.

In this study, the three subzones 2(a), 2(b) and 2(c) of Zone 2 also known as North-East India have been considered for regional flood frequency analysis. The subzones 2(a) and 2(b) covers the north and the south bank region of the river Brahmaputra, Assam, India. Subzone 2(c) covers the entire Barak valley region of Assam and all rivers of the Tripura state. The brief description of the entire study area and the available annual maximum flood discharge data for our study are discussed below.

The North-East India lies in 22^0N to 29.5^0N latitude and 90^0E to 97.5^0E longitude. The North-East India consists of eight states namely Arunachal Pradesh, Assam, Manipur, Meghalaya, Mizoram, Nagaland, Sikkim, and Tripura. With a total population of 39 million (Census of India, 2001) and covering an area of 262,179 square kilometers, the North Eastern region is relatively sparsely populated compared to much of the rest of India. However, population density varies widely among the North Eastern states. Assam and Tripura are the most densely populated (with over 300 people per square kilometer around the Indian average) while Arunachal Pradesh is the less densely populated (13 persons per square kilometer).

The region has abundant water resources one-third of India's runoff flows from the North-East through the Brahmaputra, the Barak and the rivers of Tripura state and there is a substantial unutilized groundwater resource. There is estimated to be about 60,000 megawatts of economically viable hydropower potential, of which only about 65 megawatts is developed or under construction. It is also clear that the abundant water resources impose severe distress and costs on the region through frequent flooding and that this needs to be managed to improve economic development.

The Brahmaputra river basin extends over an area of 580,000 km^2 and lies in Tibet, Bhutan, India and Bangladesh. The drainage area of the basin lying in India is 194,413 km^2; which forms nearly 5.9% of the total geographical area of the country. The water resources potential of the basin is the highest in the country, while present utilization is the lowest. This basin also holds promises for transfer of water to other deficit basins, which will reduce the flood problem in the valley also. The mean annual rainfall over the basin excluding Tibet and Bhutan is about 2,300mm.

In Assam, the Brahmaputra flows in a highly braided channel marked by the resence of numerous mid-channel and lateral bars and islands. An extremely dominant monsoon interacting with a unique physiographic setting, fragile geological base and active seismo-tectonic instability together with anthropogenic factors have molded the Brahmaputra into one of the world's most dynamic and complex fluvial system (Goswami, 1985; Ives and Messerli, 1989). In the course of its 2880 km. journey, the Brahmaputra receives as many as 22 major tributaries in Tibet, 33 in India- mostly in the North-East India (20 of these coming from the north and 13 from the south) and 3 in Bangladesh. The northern and southern tributaries differ considerably in their hydro geo-morphological behavior owing to different geological and climatic conditions. The north bank tributaries generally flow in shallow braided channels, have steep slopes, carry a heavy silt charge and are flashy in character, whereas the south bank tributaries have a flatter gradient, deep meandering channels with beds and banks composed of fine alluvial soils, marked by a relatively low sediment load. The hydrological regime of the Brahmaputra that responds to the seasonal rhythm of the monsoon and freeze-thaw cycle of the Himalayan snow is characterized by an extremely large and variable flow, enormous rates of sediment discharge, rapid channel aggradations, accelerated rates of basin denudation and unique patterns of river morphology.

The river Barak with its network of tributaries is the second largest river system in the North-East region and is a part of the Ganga-Brahmaputra-Meghna system. It rises in the Manipur hills south of Mao bordering Nagaland and Manipur, south east of mount Japvo. From its origin the river flows in a south-westerly direction through a narrow valley upto Jirighat where it takes a westward turn and after traversing through Manipur hills, Mizoram and Assam-Manipur border emerges from the hills and debouches into the plains, known as the Barak Valley, near Lakhipur. It traverses the valley in a westerly direction up to Karimganj where it bifurcates into two branches known as the Surma and the Kusiyara which reunite near Bhairab Bazar in Bangladesh, the joint stream being called the Meghna which later meets with the Brahmaputra, known locally as the Padma and eventually flows into the Bay of Bengal. The Barak has a total length of 902 km from its origin to its outfall with the Meghna in Bangladesh, of which the Indian reach is 564 km long. Out of the total basin area of 42,455 km2, about 62% lies within India (North East India).The principal tributaries of Barak in Assam are Jiri, Chiri, Madhura, Jatinga in the north and Sonai, Rukni, Dhaleswari, Katakhal, Singla and Longai in the south.

The main rivers of state Tripura are Gumti, Manu, Khowai and Haora. These rivers are often classified into two broad groups; a few rivers of the state follow the north direction and the rest of the rivers follow the west direction. The main rivers at Tripura that flow towards the north are Khowai, Manu, Dhalai, Langai and Juri. The rivers of the state that flow towards the west are Gumti, Feni and Muhuri. The Gumti is the biggest river of the state. It is considered very sacred by the people who live in this region. The entire drainage systems of North-East India, which are used for our study, are shown in Figure 1.2.

1.3 Available data for the study

The data used in this study are obtained from the Brahmaputra Board, Guwahati, Assam, India and National Institute of Hydrology (NIH), Roorkee, India. The annual maximum peak discharge data of 32 stream flow gauging sites of North-East India are available for our study. Out of these 32 stream flow gauging sites, 18 lies in 2(a) subzone and 7 each lies in 2(b) and 2(c) subzone. It has been observed from the data of 18 stream flow gauging sites of subzone 2(a) that the mean annual peak flood varies from 39.85 to 8916.07 m^3/s and record length varying over 11- 37 years. In subzone 2(b), it has been observed from the data of 7 sites that the mean annual peak flood varies from 105.19 to 1160.60 m^3/s and record length varying over 13-29 years. Again for subzone 2(c), it has been observed from data of 7 sites that the mean annual peak floods varies from 186.44 to 3927.59 m^3/s and record length varying over 11-28 years.

1.4 Objectives

The main objective of this thesis is to develop regional flood frequency relationships to estimate the design floods for both gauged and ungauged catchments areas of North-East India. For this purpose, the three hydro-meteorological subzones 2(a), 2(b) and 2(c) of North-East India are considered for regional flood frequency analysis. To achieved our main objective some step by step objectives have to be followed and the details objectives are outlines below.

- To identify homogeneous subzones of study area by using the method of L-moments.
- To identify best fitting probability distributions of the identified homogeneous subzones for the method of L-moment.
- To developed the regional flood frequency relationships for both gauged and ungauged catchments areas of the homogeneous subzones by using the best fitting distributions for the method of L-moment
- Test the homogeneity of the identified homogeneous subzones for L-moment by using the method of LH-moments.
- To identify the best fitting distributions with level of LH-moments of the homogeneous subzones for the method LH-moments.
- To develop the regional flood frequency relationships for gauged and ungauged catchments areas of the homogeneous subzones of North-East India by using best fitting distributions with the level of LH-moments for the method LH-moments.
- To test the homogeneity of the identified homogeneous subzones for L-moments by using the method of LQ-moments.
- To identify best fitting probability distributions of the identified homogeneous subzones for the method of LQ-moment.
- To developed the regional flood frequency relationships for both gauged and ungauged catchments areas of the homogeneous subzones by using the best fitting distributions for the method of LQ-moment.
- To compare the LH-moments and LQ-moment parameter estimation methods with L-moment method for regional flood frequency analysis of the identified homogeneous subzones of the study area.

1.5 Thesis Outlines

This thesis consists of six chapters. **Chapter 1** is the introductory one provides objective of the thesis and detail background of statistical flood frequency analysis procedures for estimating the design floods. It provides the details about the study area and the availability of the data for our study and also provides the overall outline of the thesis.

In **Chapter 2** detail theory of L-moments along with regional flood frequency analysis procedures by using L-moments are covered. The homogeneity of all the three subzones of the North-East India, are tested by using the heterogeneity measures. It also covered the details of the application of regional flood frequency analysis procedure for homogeneous subzones of North-East region of India. In last sections of this chapter provides the regional flood frequency relationships for both gauged and ungauged catchments of all the homogeneous subzones of the North-East India.

In **Chapter 3**, the theory of LH-moments and the regional flood frequency analysis of North-East India by using LH-moments methods are discussed in details. The initial screening of the data and homogeneity test based on LH-moments for all the homogeneous subzones mentioned in chapter 2 of this thesis are carried out before the regional frequency analysis. The regional flood frequency relationships for both gauged and ungauged catchments areas of homogeneous subzones of North-East India are developed by using the identified best fitting distributions with the level of LH-moments.

The **Chapter 4** provides the theory of LQ-moments and LQ-moment based regional flood frequency analysis of North-East India. The homogeneity of the identified homogeneous subzones in chapter 2 of North-East India, are measured in terms of LQ-moment. The $\left|Z_{LQ}^{DIST}\right|$-statistic criteria and LQ-moment ratio diagram have been used as goodness of fit tests for identifying the best fitting distributions for each of the homogeneous subzones of North-East India. The regional flood frequency relationships for both gauged and ungauged catchments areas of all the identified homogeneous subzones of the North-East India are also covered in details.

In **Chapter 5**, two comparative studies one is between L-moments and LH-moments and another is between L-moments and LQ-moment are discussed in details for each of the identified homogeneous subzones of North-East India. For both the comparative studies Z-statistic values of each of the probability distributions for three parameter estimation methods have been used. The obtained results are again verified by using Monte Carlo simulation techniques based on the relative bias (RBIAS) and relative root mean square error (RRMSE) values. The box plots of the RBIAS and RRMSE values are used as another graphical tool for this purpose.

The conclusion and discussion of the results are described in **Chapter 6** of the thesis. In **Chapter 6** some feature recommendation of our study also provided.

There are two appendix chapters provided in the thesis. In appendix-A, there are some tables which are useful for our study. In appendix-B, we are providing, the FORTRAN subroutines used in our study for numerical calculations.

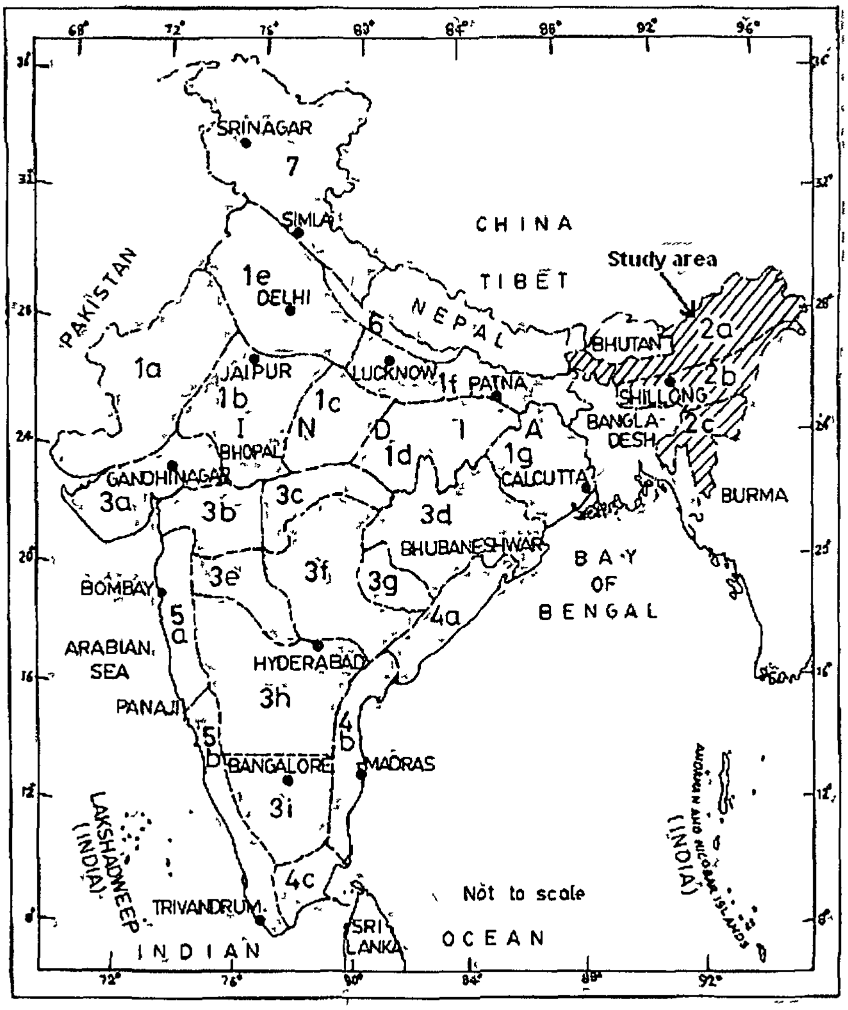

Figure 1.1 Map showing the hydro-meteorological subzones of India

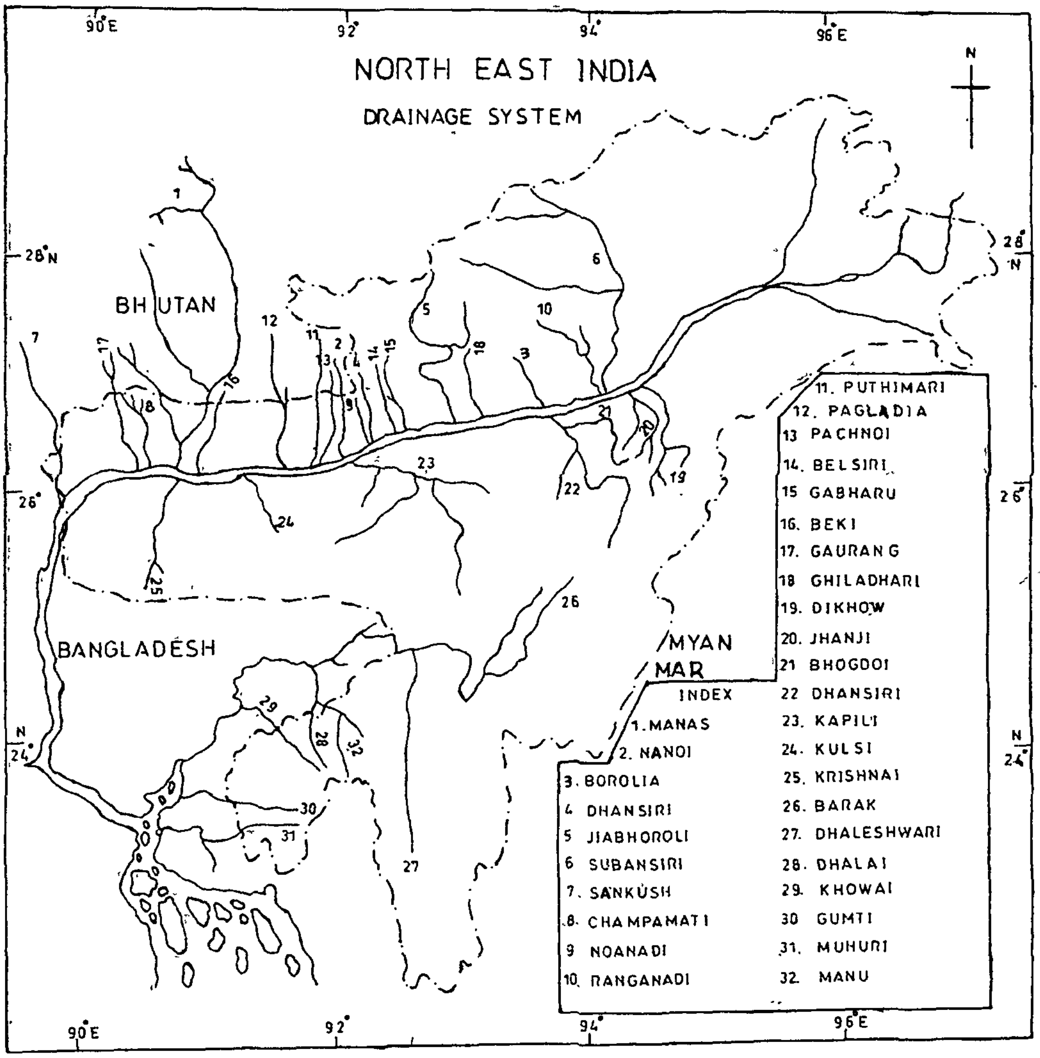

Figure 1.2 Map showing the rivers considered for our study from North-East India

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Chapter 2

Regional Flood Frequency Analysis by Using L-moments

2.1 Introduction

In this Chapter the regional flood frequency analysis by using L-moments has been carried out for three hydro-meteorological subzones 2(a), 2(b) and 2(c) of North-East India. For this purpose, the index flood procedure proposed by Dalrymple (1960) has been used in terms of L-moments. The five probability distributions, namely the generalized extreme value (GEV), generalized logistic (GLO), generalized Pareto (GPA), generalized log-normal (GNO) and Pearson type III (PE3) have been considered for regional flood frequency analysis of our study area.

2.2 Probability distributions used for our study

The probability distributions used for our study can be found from references such as Hosking and Wallis (1997) and Rao and Hamed (2000). The probability density functions (PDFs) and quantile functions for each of the five distributions used for our study are given below, where x denotes the observed values of the random variables representing the events of interest.

2.2.1 GEV distribution

The PDF of the GEV distribution is given by

$$f(x)=\frac{1}{\alpha}\left[1-k\frac{(x-\xi)}{\alpha}\right]^{\frac{1}{k}-1}\exp\left[-\left\{1-k\frac{(x-\xi)}{\alpha}\right\}^{\frac{1}{k}}\right] \quad (2.2.1)$$

where, ξ, α and k are the location, scale and shape parameters of the distribution and the range of x are $-\infty<x\le\xi+\frac{\alpha}{k}$ if $k>0$; $-\infty<x<\infty$ if $k=0$; $\xi+\frac{\alpha}{k}\le x<\infty$ if $k<0$.

The quantile function, $Q(F)$ can be written as

$$Q(F)=\xi+\frac{\alpha}{k}\left[1-(-\ln F)^{k}\right] \quad (2.2.2)$$

2.2.2 GLO distribution

The PDF of the GLO distribution is given by,

$$f(x)=\frac{1}{\alpha}\left\{1-k\frac{(x-\xi)}{\alpha}\right\}^{\frac{1}{k}-1}\left[1+\left\{1-k\frac{(x-\xi)}{\alpha}\right\}^{\frac{1}{k}}\right]^{-2} \tag{2.2.3}$$

where, ξ, α and k are the location, scale and shape parameters of the distribution and the range of x are $-\infty<x\le\xi+\frac{\alpha}{k}$ if $k>0$; $-\infty<x<\infty$ if $k=0$; $\xi+\frac{\alpha}{k}\le x<\infty$ if $k<0$.

The quantile function, $Q(F)$ can be written as

$$Q(F)=\xi+\frac{\alpha}{k}\left[1-\left\{\frac{(1-F)}{F}\right\}^{k}\right] \tag{2.2.4}$$

2.2.3 GPA distribution

The PDF of the GPA distribution is given by

$$f(x)=\frac{1}{\alpha}\left\{1-k\frac{(x-\xi)}{\alpha}\right\}^{\frac{1}{k}-1} \tag{2.2.5}$$

where, ξ, α and k are the location, scale and shape parameters of the distribution and the range of x are $\xi\le x\le\xi+\frac{\alpha}{k}$ if $k>0$; $\xi\le x<\infty$ if $k\le 0$.

The quantile function, $Q(F)$ can be written as

$$Q(F)=\xi+\frac{\alpha}{k}\left\{1-(1-F)^{k}\right\} \tag{2.2.6}$$

2.2.4 GNO distribution

The PDF of the GNO distribution is given by

$$f(x)=\frac{\exp^{-\log\left\{1-k\frac{(x-\xi)}{\alpha}\right\}-\frac{1}{2}\left[-\frac{1}{k}\log\left\{1-k\frac{(x-\xi)}{\alpha}\right\}\right]^{2}}}{\alpha\sqrt{2\pi}} \tag{2.2.7}$$

where, ξ, α and k are the location, scale and shape parameters of the distribution and the range of x are $-\infty<x\le\xi+\frac{\alpha}{k}$ if $k>0$; $-\infty<x<\infty$ if $k=0$; $\xi+\frac{\alpha}{k}\le x<\infty$ if $k<0$.

The quantile function, $Q(F)$ can be written as

$$Q(F)=\xi+\frac{\alpha}{k}\left\{1-\exp(-k\Phi^{-1}(F)\right\} \tag{2.2.8}$$

where, $\Phi^{-1}(F)$ is the quantile function of standard normal distribution.

2.2.5 PE3 distribution

The PDF of the PE3 distribution is given by

$$f(x)=\frac{(x-\xi)^{\alpha-1}\exp\left\{-\frac{(x-\xi)}{\beta}\right\}}{\beta^{\alpha}\Gamma(\alpha)} \tag{2.2.9}$$

where, ξ, α and β are related to the location, scale and shape parameters μ, σ and γ as follows

$$\mu=\xi+\frac{2\sigma}{\gamma},\ \alpha=\frac{4}{\gamma^2}\text{ and }\beta=\frac{1}{2}\sigma|\gamma|$$

The range of x are $\xi\le x<\infty$ if $\gamma>0$; $-\infty<x<\infty$ if $\gamma=0$; $-\infty<x\le\xi$ if $\gamma<0$.

The quantile function for PE3 distribution can be expressed in terms of μ, σ, and γ as follows

$$Q(F)=\mu+\sigma\left[\frac{2}{\gamma}\left\{1+\frac{\gamma}{6}\Phi^{-1}(F)-\frac{\gamma^2}{36}\right\}^3-\frac{2}{\gamma}\right] \tag{2.2.10}$$

The quantile functions for T years return periods for each of the five distributions mentioned above can be obtained by substituting, $F=1-\frac{1}{T}$.

2.3 L-moments

The probability weighted moments (PWMs) of a random variable X with cumulative distribution function (CDF), $F(.)$ were defined by Greenwood et al. (1979) as

$$\beta_r=M_{1,r,0}=\mathrm{E}\left[X\{F(X)\}^r\right] \tag{2.3.1}$$

where,

$$M_{p,r,s}=\mathrm{E}\left[X^p\{F(X)\}^r\{1-F(X)\}^s\right] \tag{2.3.2}$$

and β_r can be rewritten as:

$$\beta_r=\int_0^1 x(F)F^r dF,\ r=0,1,2,..., \tag{2.3.3}$$

where, $x(F)$ is the inverse CDF of x evaluated at the probability F.

The general form of L-moments in terms of PWMs is given by Hosking and Wallis (1997) as

$$\lambda_{r+1} = \sum_{k=0}^{r} p^{*}_{r,k}\beta_k \tag{2.3.4}$$

where, $p^{*}_{r,k}$ defined by Hosking and Wallis (1997) as

$$p^{*}_{r,k} = \frac{(-1)^{r-k}(r'+k)!}{(k!)^2(r-k)!} \tag{2.3.5}$$

The first four L-moments can be defined as:

$$\lambda_1 = \beta_0 \tag{2.3.6}$$

$$\lambda_2 = 2\beta_1 - \beta_0 \tag{2.3.7}$$

$$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0 \tag{2.3.8}$$

$$\lambda_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0 \tag{2.3.9}$$

Hosking and Wallis (1997) defined L-moments ratios (LMRs) as:

Coefficient of L-variation, $\tau = \lambda_2 / \lambda_1$ (2.3.10)

Coefficient of L-skewness, $\tau_3 = \lambda_3 / \lambda_2$ (2.3.11)

Coefficient of L-kurtosis, $\tau_4 = \lambda_4 / \lambda_2$,(2.3.12)

The sample estimation of L-moments can be defined as:

$$\hat{\lambda}_{r+1} = \sum_{k=0}^{r} p^{*}_{r,k} b_k \tag{2.3.13}$$

with

$$b_r = n^{-1} \sum_{j=r+1}^{n} \frac{(j-1)(j-2)...(j-r)}{(n-1)(n-2)...(n-r)} x_j \tag{2.3.14}$$

where x_j, for $j = 1,2,...n$ is the ordered sample and n is the sample size.

The first four sample L-moments can be analogously obtained as equations (2.3.6) to (2.3.9). The sample estimates of β_r given in equation (2.3.14) and λ_r given in equation (2.3.13) are unbiased, while the following estimators of L-moments ratios (LMRs) τ, τ_3 and τ_4 given by

$$\hat{\tau} = \frac{\hat{\lambda}_2}{\hat{\lambda}_1},\ \hat{\tau}_3 = \frac{\hat{\lambda}_3}{\hat{\lambda}_2} \text{ and } \hat{\tau}_4 = \frac{\hat{\lambda}_4}{\hat{\lambda}_2}$$

are consistent but not unbiased (Hosking and Wallis, 1997).

The values of $\hat{\lambda}_1$, $\hat{\tau}$, $\hat{\tau}_3$ and $\hat{\tau}_4$ are useful summary statistics of data sample and can be used to judge which distributions are consistent with a given data sample. They can also be used to estimate paramèters when fitting a distribution to a sample, by equating sample and distribution L-moments (Hosking, 1990).

2.4 Index flood approach based on L-moments

The key assumption of an index flood procedure is that the region is homogeneous, that is, the frequency distributions of the NS sites in a region are identical, apart from a site-specific scaling factor. The distribution common to all sites in the region is called the regional frequency distribution. It is dimensionless and defined by its (regional) quantiles, q_T. It is usually assumed that the form of q_T is known apart from p undetermined parameters $\theta_1, \theta_2, \ldots, \theta_p$. The site-specific scaling factor is called the index flood, denoted μ_i at site i (Hosking and Wallis, 1993). The index flood is usually taken to be the sample mean of the frequency distribution at site i, although any location parameter of the frequency distribution may be used instead.

Thus we can write

$$Q_T^i = \mu_i q_T, \quad i = 1, \ldots, NS \tag{2.4.1}$$

where, Q_T^i is the quantile of return period T at site i.

A standard scaled data approach is the simplest index flood method. This involves dividing each measure by its at-site sample mean, and then treating all the scaled data points as if they were observations from the regional frequency distribution. Parameter estimates are found and the estimated regional flood distribution is then multiplied by the at-site mean of the site under investigation.

A more advanced index flood procedure was outlined by Hosking and Wallis (1993)

1. Estimate the mean at each site, $\hat{\lambda}_1^{(i)}$, by the sample mean at site i.
2. Rescale the data, $x_{ij} = x_{ij} / \hat{\lambda}_1^{(i)}, j = 1, \ldots, n_i, i = 1, \ldots, NS,$ as the basis for estimating q_T. Remember that n_i is the number of years of record at site i and the region consists of NS sites.
3. Estimate the parameters separately at each site. Denote the site i estimate of θ_k by $\hat{\theta}_k^{(i)}$.
4. Combine the at-site estimates to give regional estimates:

$$\hat{\theta}_k^{(R)} = \frac{\sum_{i=1}^{NS} n_i \hat{\theta}_k^{(i)}}{\sum_{i=1}^{NS} n_i} \tag{2.4.2}$$

Each estimated regional parameter is a weighted average. The site i estimate is given weight proportional to n_i, since for regular statistical models the variance of $\hat{\theta}_k^{(i)}$ is inversely proportional to n_i.

5. Substitute estimates $\hat{\theta}_k^{(1)},...,\hat{\theta}_k^{(n)}$ into q_T to give $\hat{q}_T$, the estimated regional quantile or growth factors for return period T.
6. The site i quantile estimates are obtained by combining the estimates of μ_i and q_T:

$$\hat{Q}_T^i = \hat{\lambda}_1^{(i)} \hat{q}_T . \tag{2.4.3}$$

2.5 Regional flood frequency analysis

Hosking and Wallis (1993, 1997) proposed the following steps for regional frequency analysis by using L-moments and are given as

1. Screening of data
2. Identification of homogeneous region
3. Selection of best fitting probability distribution
4. Estimation of regional parameters and quantiles or growth factors of the probability distributions
5. Development of regional relationships for gauged and ungauged catchments of the homogeneous region

All the steps mentioned above are used for regional flood frequency analysis of subzones 2(a), 2(b) and 2(c) of North-East India and are discussed in the following sub-sections.

The L-moments package of Fortran-77 subroutine developed by Hosking (2005) has been used in our study for all calculations.

2.5.1 Screening of data

The screening of data is the first step in any statistical analysis to check that the available data are appropriate for regional frequency analysis. Hosking and Wallis

(1993, 1997) proposed L-moment based discordancy measure (D_i) for initial screening of the data for regional frequency analysis. The aim of discordancy measure is to identify those sites that are grossly discordant with the group as a whole (Hosking and Wallis, 1997).

Hosking and Wallis (1993, 1997) defined the discordancy measure (D_i) by considering the NS sites in a group as follows.

Let $u_i = [\hat{\tau}^{(i)}\ \hat{\tau}_3^{(i)}\ \hat{\tau}_4^{(i)}]^T$ be a vector containing the sample LMRs, $\hat{\tau}^{(i)}$, $\hat{\tau}_3^{(i)}$ and $\hat{\tau}_4^{(i)}$ for the site i; the superscript T denotes transposition of matrix. Let

$$\bar{u} = NS^{-1}\sum_{i=1}^{NS} u_i \quad (2.5.1)$$

be the (unweighted) group average. The matrix of sums of squares and cross product defined as:

$$S = \sum_{i=1}^{NS}(u_i - \bar{u})(u_i - \bar{u})^T \quad (2.5.2)$$

The discordancy measure for site i defined by

$$D_i = \frac{1}{3}NS(u_i - \bar{u})^T S^{-1}(u_i - \bar{u}) \quad (2.5.3)$$

Hosking and Wallis (1997) gave some critical values for the discordancy statistic D_i against the number of sites in a region. Using this critical values of D_i, the site i is declare to be discordant, if D_i is greater than the critical value.

2.5.2 Identification of homogeneous region

Hosking and Wallis (1993) proposed a test statistics, termed as heterogeneity measure for testing the regional homogeneity. It is used to estimate the degree of heterogeneity in a group of sites and to assess whether they might reasonably be treated as homogeneous. It compares the between-site variations in sample L-moments for the group of sites with that expected for a homogeneous region. The formula for H_i, can be given as

$$H_i = \frac{V_i - \mu_{V_i}}{\sigma_{V_i}}, \ i = 1,2,3. \quad (2.5.4)$$

where, V_1, V_2, and V_3 are the three measures of variability and are given as follows

$$V_1 = \sum_{i=1}^{NS} N_i (\hat{\tau}^{(i)} - \hat{\tau}^R)^2 / \sum_{i=1}^{NS} N_i \tag{2.5.5}$$

$$V_2 = \sum_{i=1}^{NS} N_i \{(\hat{\tau}^{(i)} - \hat{\tau}^R)^2 (\hat{\tau}_3^{(i)} - \hat{\tau}_3^R)^2\}^{1/2} / \sum_{i=1}^{NS} N_i \tag{2.5.6}$$

$$V_3 = \sum_{i=1}^{NS} N_i \left\{ \left(\hat{\tau}_3^{(i)} - \hat{\tau}_3^R\right)^2 \left(\hat{\tau}_4^{(i)} - \hat{\tau}_4^R\right)^2 \right\}^{1/2} / \sum_{i=1}^{NS} N_i \tag{2.5.7}$$

where, NS in above equations is the number of sites, N_i is the record length at each site and $\hat{\tau}^R$, $\hat{\tau}_3^R$ and $\hat{\tau}_4^R$ are the regional average value of $\hat{\tau}^{(i)}$, $\hat{\tau}_3^{(i)}$ and $\hat{\tau}_4^{(i)}$ respectively, are given by

$$\hat{\tau}^R = \sum_{i=1}^{NS} N_i \hat{\tau}^{(i)} / \sum_{i=1}^{NS} N_i \tag{2.5.8}$$

$$\hat{\tau}_3^R = \sum_{i=1}^{NS} N_i \hat{\tau}_3^{(i)} / \sum_{i=1}^{NS} N_i \tag{2.5.9}$$

$$\hat{\tau}_4^R = \sum_{i=1}^{NS} N_i \hat{\tau}_4^{(i)} / \sum_{i=1}^{NS} N_i \tag{2.5.10}$$

and μ_{V_i} ($i = 1,2,3$), σ_{V_i} ($i = 1,2,3$) are the mean and standard deviation of V_i, $i = 1,2,3$, respectively.

To evaluate the heterogeneity measures, a Kappa distribution (Hosking, 1988) is fitted to the regional average L-moments $1, \hat{\tau}^R$, $\hat{\tau}_3^R, \hat{\tau}_4^R$. Simulations of a large number of regions, N_{sim}, from this Kappa distribution are performed. The regions are assumed to homogeneous and the data are assumed to have no cross-correlation or serial correlation. The sites are assumed to have the same record lengths as their real world counterparts. For each simulated region, V_i ($i = 1,2,3$) are calculated and the mean μ_{V_i} ($i = 1,2,3$) and standard deviation σ_{V_i} ($i = 1,2,3$) are determined.

As suggested by Hosking and Wallis (1997), a region is acceptably homogeneous if heterogeneity measure $H_i < 1$, possibly heterogeneous if $1 \le H_i < 2$ and definitely heterogeneous if $H_i \ge 2$.

We are mainly concern with only the H_1 value of heterogeneity measure because the other heterogeneity measures H_2 and H_3 has lack power to discriminate between homogeneous and heterogeneous regions than H_1 (Hosking and Wallis 1993).

In this study the heterogeneity measures $H_i, i = 1,2,3$ was computed by carrying out 500 simulations using the four parameters Kappa distribution as mentioned above. The heterogeneity measures of all the three subzones i.e. 2(a), 2(b) and 2(c) are given in Table 2.5. It has been observed from the heterogeneity measures of the three subzones that it is greater than 1 for subzone 2(a) and less than 1 for both the subzones 2(b) and 2(c). Therefore, the subzone 2(a) is found to be heterogeneous and the subzones 2(b) and 2(c) are found to be homogeneous. Since the subzone 2(a) is found to be heterogeneous therefore cluster analysis techniques have been used to divide it into homogeneous subzones.

Cluster analysis techniques have been extensively used in hydrology, as well as in a variety of other disciplines (Theodoridis and Koutroubas, 1999; Jain and Dubes, 1988; Kaufman and Rousseeuw, 1990; Everitt, 1993). Cluster analysis was used by Burn and Goel (2000) to identify regions for regional flood frequency analysis in Canada. Hosking and Wallis (1997) used cluster analysis for regionalization of total annual precipitation in the USA and recommended Ward's and the K-means clustering methods for identifying homogeneous regions. Saf (2009) used K-means clustering method of MacQueen (1967) and divide the data of West Mediterranean region of Turky based on the individual sites' first five L-moment statistics. In this study K-means clustering method based on the first four L-moment ratio statistics of the individual sites have been used to divide the subzone 2(a) into two homogeneous subzones i.e. 2(a)A and 2(a)B. (see heterogeneity measures of these subzones in Table 2.5)

The names, sample statistics, discordancy measures of all sites for 2(a)A, 2(a)B, 2(b) and 2(c) subzones are given in Table 2.1 to Table 2.4. From Table 2.1 to Table 2.4, it has been observed that the D_i values of all the sites for all the subzones are less than the critical value of D_i and an exceptional case has been observed at the site Beki of subzone 2(a)B.

Though, the site Beki is found to be discordant we include it for regional flood frequency analysis of subzone 2(a)B because of the following reasons.

Since, the D_i value of Beki (i.e. 2.24) for subzone 2(a)B is not too far beyond the critical value suggested by Hosking and Wallis (1997) for eight sites i.e. 2.14. Again, the site Beki has the smallest record length as well as lowest skewness and third lowest kurtosis. Furthermore, the site Beki has no such gross errors or incorrect recording of data values. Any site that has D_i value less than equal to three with no inconsistencies or gross errors in the data can be taken for further analysis (Hussain and Pasha, 2009).

Table 2.1 Name of sites, sample size, catchments areas, sample L-moment statistics and discordancy measures of subzone 2(a)A

Site name	Sample size	Catchments area (km^2)	$\hat{\lambda}_1$	$\hat{\tau}$	$\hat{\tau}_3$	$\hat{\tau}_4$	D_i
Manas (1)	17	30,100	6048.51	0.1706	0.2432	0.1458	0.83
Nanoi (2)	11	148	99.60	0.2036	0.2730	0.3059	0.76
Borolia (3)	15	310	190.18	0.2257	0.0450	0.0542	0.66
Dhansiri (4)	21	530	1322.28	0.1953	0.0969	0.1881	0.51
Jiabhoroli (5)	36	11,000	4234.33	0.2256	0.1851	0.0717	0.61
Subansiri (6)	27	25,886	8916.07	0.1763	0.2611	0.1912	0.71
Sankush (7)	12	9,799	1883.45	0.1320	0.0103	0.1682	2.14
Champamati (8)	22	1,142	798.10	0.2214	0.2346	0.2207	0.43
NoaNadi (9)	13	745	39.85	0.1491	-0.0324	-0.1636	2.20
Ranganadi (10)	19	2,941	968.97	0.2405	0.0301	0.0112	1.16

Table 2.2 Name of sites, sample size, catchments areas, sample L-moment statistics and discordancy measures of subzone 2(a)B

Site name	Sample size	Catchments area (km^2)	$\hat{\lambda}_1$	$\hat{\tau}$	$\hat{\tau}_3$	$\hat{\tau}_4$	D_i
Puthimari (11)	37	1,100	583.38	0.3076	0.2554	0.0700	0.26
Pagladia (12)	35	770	659.63	0.3460	0.2036	0.1163	0.69
Pachnoi (13)	22	198	219.61	0.2779	0.2764	0.1661	0.43
Belsiri (14)	23	460	304.66	0.2716	0.2314	0.0378	1.19
Gabharu (15)	15	324	269.76	0.3711	0.1811	-0.0515	1.72
Beki (16)	13	1,331	752.18	0.2554	-0.0372	0.0517	**2.24**
Gaurang (17)	17	1,023	1040.35	0.3216	0.2859	0.2261	0.91
Ghiladhari (18)	20	670	76.65	0.2993	0.3266	0.2064	0.56

The bold figure represents the discordancy measure greater than the critical value

Table 2.3 Name of sites, sample size, catchments areas, sample L-moment statistics and discordancy measures of subzone 2(b)

Site name	Sample size	Catchments area (km^2)	$\hat{\lambda}_1$	$\hat{\tau}$	$\hat{\tau}_3$	$\hat{\tau}_4$	D_i
Dikhow (19)	26	4,022	720.34	0.1750	0.1943	0.1375	0.23
Jhanji (20)	13	1,139	158.86	0.2472	0.1360	-0.0272	0.99
Bhogdoi (21)	13	920	196.08	0.2512	0.2134	0.0464	1.07
Dhansiri (22)	29	10,305	1106.94	0.1943	0.1560	0.1368	0.04
Kapili (23)	26	15,068	1160.60	0.1848	0.3531	0.0852	1.61
Kulsi (24)	24	1,020	105.19	0.1661	0.0790	0.3205	1.65
Krishnai (25)	19	985	482.03	0.1758	0.0990	0.0475	1.42

Table 2.4 Name of sites, sample size, catchments areas, sample L-moment statistics and discordancy measures of subzone 2(c)

Site name	Sample size	Catchments area (km^2)	$\hat{\lambda}_1$	$\hat{\tau}$	$\hat{\tau}_3$	$\hat{\tau}_4$	D_i
Barak (26)	11	26,300	3927.59	0.1628	0.3173	0.2334	1.01
Dhaleshwari (27)	16	1,500	635.76	0.1530	-0.1221	0.4045	1.75
Dhalai (28)	11	630	186.44	0.1718	0.2587	0.1244	0.76
Khowai (29)	19	1,328	283.85	0.2933	0.1874	0.0614	1.04
Gumti (30)	24	2,492	413.83	0.2356	0.0341	0.1194	0.59
Muhuri (31)	28	576	360.14	0.2083	0.1302	0.1045	0.36
Manu (32)	12	2278	762.24	0.2466	0.2221	0.3088	1.48

The number given in bracket (in Table 2.1 to Table 2.4) represent the site number as shown Figure 1.2

The heterogeneity measures of the subzones 2(a), 2(a)A, 2(a)B, 2(b) and 2(c) are given in Table 2.5.

Table 2.5 Heterogeneity measures of all five subzones of North-East India

Name of subzones	H_1	H_2	H_3
2(a)	5.44	1.44	0.46
2(a)A	0.54	0.69	0.87
2(a)B	-0.14	-0.63	-0.36
2(b)	0.31	-0.18	0.10
2(c)	0.64	0.06	0.33

2.5.3 Selection of best fitting probability distribution

(a) Z^{DIST}-Statistic criteria

This measure, decides how well the simulated L-skewness and L-kurtosis of a fitted distribution matches the regional average L-skewness and L-kurtosis values obtained from the observed data. The goodness of fit criterions for each of the five distributions is calculated by using the formula given by Hosking and Wallis (1997) and can be defined as

$$Z^{DIST} = (\tau_4^{DIST} - \hat{\tau}_4^R + B_4)/\sigma_4, \tag{2.5.11}$$

where, $\hat{\tau}_4^R$ = regional average value of $\hat{\tau}_4$

B_4, σ_4 =bias and standard deviation of $\hat{\tau}_4$, respectively, defined as

$$B_4 = N_{sim}^{-1} \sum_{m=1}^{N_{sim}} (\tau_4^{(m)} - \hat{\tau}_4^R), \tag{2.5.12}$$

$$\sigma_4 = \left[(N_{sim} - 1)^{-1} \left\{ \sum_{m=1}^{N_{sim}} (\tau_4^{(m)} - \hat{\tau}_4^R)^2 - N_{sim} B_4^2 \right\} \right]^{1/2}, \tag{2.5.13}$$

where, N_{sim} = number of simulated regional data sets generated by using Kappa distribution,

$m = m^{th}$ Simulated region obtained by using Kappa distribution.

Declare the fit to be adequate if Z^{DIST} is sufficiently close to zero, a reasonable criterion being $|Z^{DIST}| \leq 1.64$. While a number of distributions may qualify the goodness of fit criteria, the most potential will be the one that has the minimum $|Z^{DIST}|$-statistic value.

Like heterogeneity measure here also 500 simulations is used to calculate the $|Z^{DIST}|$-statistic values of various distributions for subzones 2(a)A, 2(a)B, 2(b) and 2(c), and which are given in Table 2.6. For subzone 2(a)A, it has been observed from Table 2.6 that the $|Z^{DIST}|$-statistic values are lower than 1.64 for the three distributions namely GEV, GNO and PE3. So, these three distributions are suitable candidates for this subzone. Out of these three candidate distributions for subzone 2(a)A, the PE3 distribution has the lowest $|Z^{DIST}|$-statistic value i.e. 0.36. Therefore, out of these three distributions the PE3 distribution is the best fitting distribution for the subzone 2(a)A. Similarly, it has been observed from Table 2.6 that the $|Z^{DIST}|$-statistic values of GPA, PE3 and GLO are the lowest among all other distributions for subzones 2(a)B, 2(b) and 2(c) respectively. Therefore, the GPA, PE3 and GLO distributions are best fitting distributions for subzones 2(a)B, 2(b) and 2(c), respectively.

Table 2.6 The $|Z^{DIST}|$-statistics of five distributions for four homogeneous subzones.

Sl. No.	Distributions	$\|Z^{DIST}\|$			
		2(a)A	2(a)B	2(b)	2(c)
1	general Logistic (GLO)	2.40	3.85	2.19	**0.01**
2	generalized extreme value (GEV)	0.83	2.61	0.95	1.07
3	generalized normal (GNO)	0.75	2.17	0.75	1.05
4	Pearson type III (PE3)	**0.36**	1.34	**0.28**	1.23
5	generalized Pareto (GPA)	2.52	**0.33**	1.82	3.32

The bold figures represent the lowest $|Z^{DIST}|$-statistic of each subzones

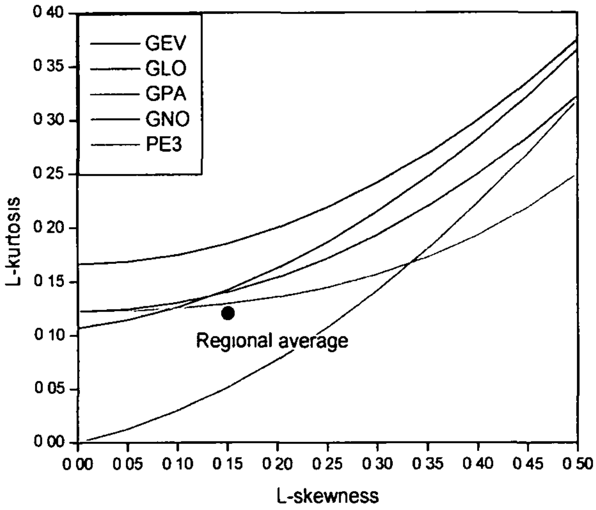


Figure 2.1 L-moment ratio diagram of subzone 2(a)A

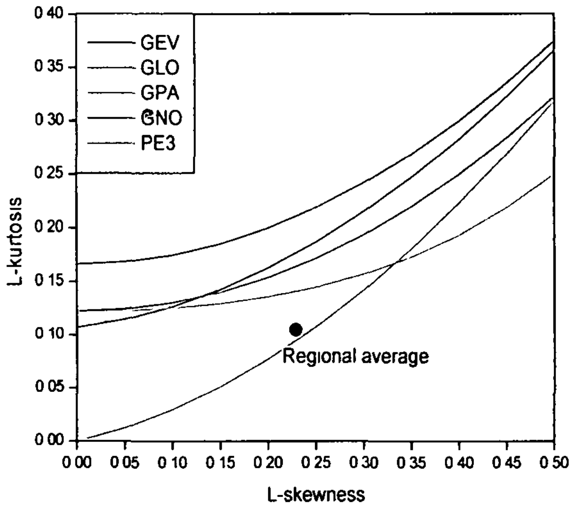


Figure 2.2 L-moment ratio diagram of subzone 2(a)B

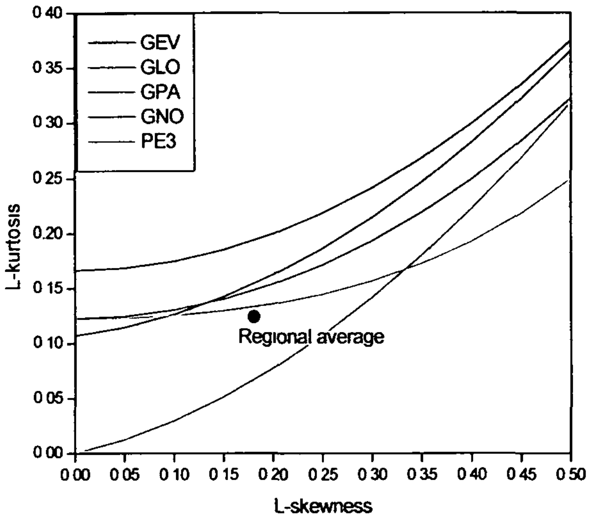


Figure 2.3 L-moment ratio diagram of subzone 2(b)

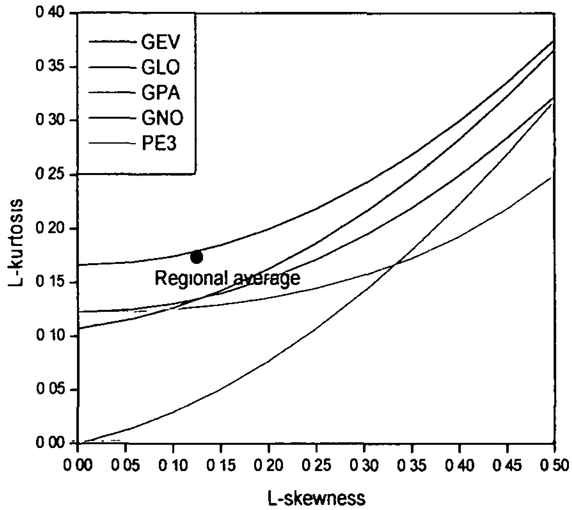


Figure 2.4 L-moment ratio diagram of subzone 2(c)

Table 2.7 Regional average L-moments ratio of subzones 2(a)A, 2(a)B, 2(b) and 2(c)

Name of subzones	$\hat{t}^R$	$\hat{t}_3^R$	$\hat{t}_4^R$
2(a)A	0.1993	0.1502	0.1208
2(a)B	0.3087	0.2286	0.1047
2(b)	0.1920	0.1805	0.1240
2(c)	0.2161	0.1246	0.1741

2.5.4 Estimation of regional parameters and growth factors

The regional parameters of the distributions for a homogeneous region can be obtained by using regional average L-moment ratios $1, \hat{\tau}^R, \hat{\tau}_3^R, \hat{\tau}_4^R, \ldots$.

The regional parameters of each of the probability distributions for all the four homogeneous subzones of the North-East India are calculated and are given in Table 2.8.

The regional quantiles or growth factors $\hat{q}_T$ of the fitted distributions can be estimated from the estimated regional parameters for each of the homogeneous subzones. The regional growth factors $\hat{q}_T$ with return periods T of each of the probability distributions for all the homogeneous subzones of our study area are given in Table 2.9 to Table 2.12. These regional growth factors can be used to draw the regional growth curves of each of the probability distributions for each of the homogeneous subzones.

Table 2.8 Regional parameters of various distributions for subzones 2(a)A, 2(a)B, 2(b) and 2(c)

Name of subzones	Distribution	Regional parameters		
		Location	Scale	Shape
2(a)A	GEV	0.838	0.296	0.031
	GLO	0.951	0.192	-0.150
	GPA	0.506	0.730	0.478
	GNO	0.946	0.340	-0.309
	PE3	**1.000**	**0.363**	**0.914**
2(a)B	GEV	0.726	0.407	-0.089
	GLO	0.887	0.283	-0.229
	GPA	**0.304**	**0.875**	**0.256**
	GNO	0.875	0.498	-0.474
	PE3	1.000	0.580	1.379
2(b)	GEV	0.838	0.273	-0.016
	GLO	0.944	0.182	-0.181
	GPA	0.541	0.637	0.388
	GNO	0.938	0.321	-0.372
	PE3	**1.000**	**0.353**	**1.094**
2(c)	GEV	0.831	0.332	0.072
	GLO	**0.956**	**0.211**	**-0.125**
	GPA	0.448	0.860	0.557
	GNO	0.952	0.373	-0.256
	PE3	1.000	0.390	0.760

The bold figures represent the regional parameters of best fitting distribution

Table 2.9 Regional growth factors of various distributions for subzone 2(a)A

Distribution	Return period in years (T)							
	2	5	10	20	50	100	500	1000
GEV	0.946	1.272	1.481	1.678	1.926	2.107	2.511	2.679
GLO	0.951	1.247	1.451	1.662	1.966	2.221	2.921	3.278
GPA	0.937	1.326	1.525	1.668	1.798	1.864	1.955	1.977
GNO	0.946	1.273	1.481	1.675	1.921	2.104	2.523	2.705
PE3	**0.945**	**1.279**	**1.486**	**1.676**	**1.909**	**2.077**	**2.447**	**2.600**

Table 2.10 Regional growth factors of various distributions for subzone 2(a)B

Distribution	Return period in years (T)							
	2	5	10	20	50	100	500	1000
GEV	0.878	1.379	1.740	2.110	2.625	3.040	4.103	4.609
GLO	0.887	1.349	1.695	2.077	2.664	3.191	4.778	5.661
GPA	**0.860**	**1.458**	**1.826**	**2.135**	**2.466**	**2.671**	**3.026**	**3.139**
GNO	0.875	1.390	1.753	2.116	2.606	2.989	3.935	4.370
PE3	0.871	1.411	1.776	2.123	2.565	2.890	3.627	3.938

Table 2.11 Regional growth factors of various distributions for subzone 2(b)

Distribution	Return period in years (T)							
	2	5	10	20	50	100	500	1000
GEV	0.938	1.252	1.464	1.668	1.937	2.141	2.622	2.832
GLO	0.944	1.231	1.435	1.652	1.972	2.248	3.034	3.449
GPA	0.928	1.304	1.511	1.669	1.823	1.908	2.035	2.070
GNO	0.938	1.255	1.465	1.666	1.928	2.125	2.592	2.799
PE3	**0.937**	**1.263**	**1.473**	**1.668**	**1.912**	**2.088**	**2.482**	**2.647**

Table 2.12 Regional growth factors of various distributions for subzone 2(c)

Distribution	Return period in years (T)							
	2	5	10	20	50	100	500	1000
GEV	0.951	1.303	1.521	1.719	1.960	2.131	2.494	2.638
GLO	**0.956**	**1.275**	**1.490**	**1.707**	**2.014**	**2.266**	**2.938**	**3.270**
GPA	0.943	1.362	1.564	1.701	1.817	1.873	1.944	1.959
GNO	0.952	1.302	1.518	1.715	1.960	2.138	2.539	2.709
PE3	0.951	1.306	1.521	1.714	1.949	2.117	2.483	2.633

The bold figures (in Table 2.9 to Table 2.12) represent growth factors of the best fitting distribution

The growth curves of five probability distributions for each of the four subzones 2(a)A, 2(a)B, 2(b) and 2(c) of North-East India are shown in Figure 2.5 to Figure 2.6. For subzones 2(a)A and 2(b), it has been observed from Figure 2.5 and Figure 2.7 that the growth curve of GPA distribution is lower and GLO distribution is higher than the PE3 distribution after 20 year return period onwards. But the GNO and GEV distributions have higher growth curves than PE3 distribution after 100 years onwards. Again for subzone 2(a)B, it has been observed from Figure 2.6 that the growth curves of GEV, GLO, GNO and PE3 are higher than the GPA distribution after 20 year return period onwards. Similarly, for subzone 2(c), it has been observed from Figure 2.8 that the growth curves of GEV, GNO, PE3 and GPA distributions are lower than the GLO distribution after 20 year return period onwards.

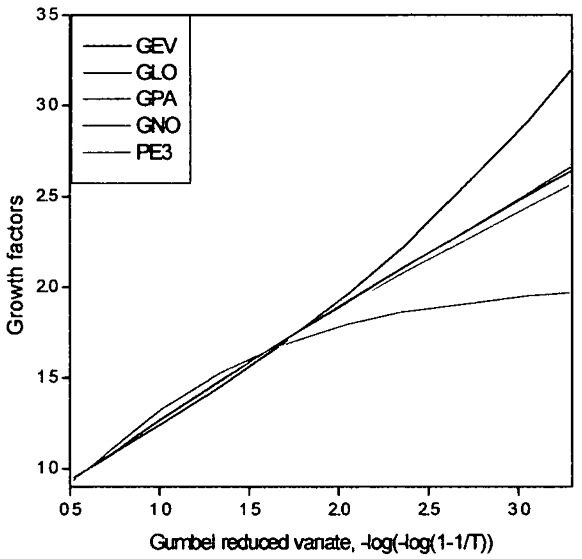


Figure 2.5 Growth curves of various distributions for subzone 2(a)A

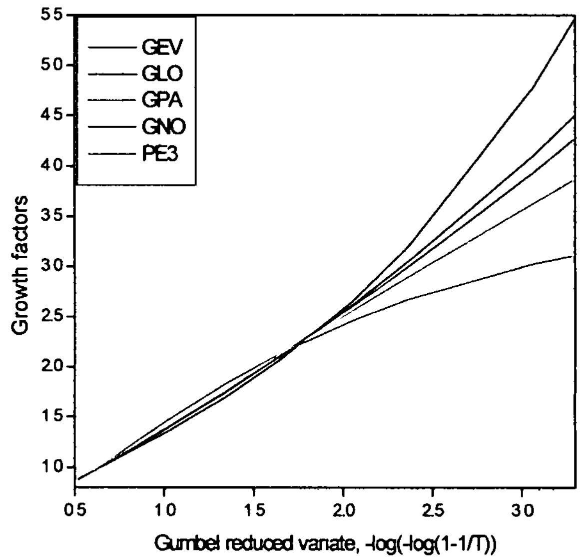


Figure 2.6 Growth curves of various distributions for subzone 2(a)B

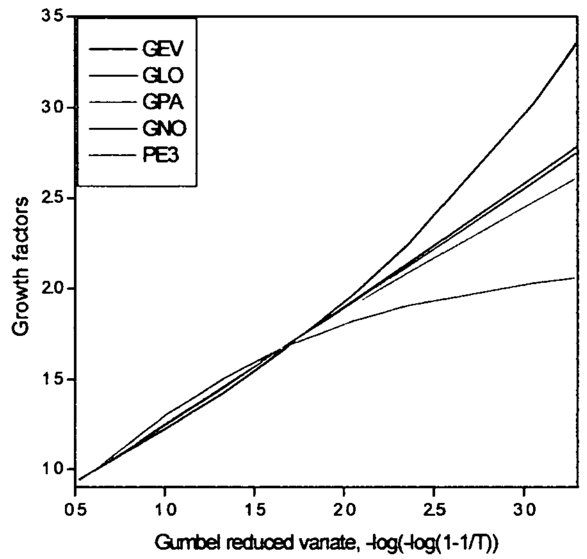


Figure 2.7 Growth curves of various distributions for subzone 2(b)

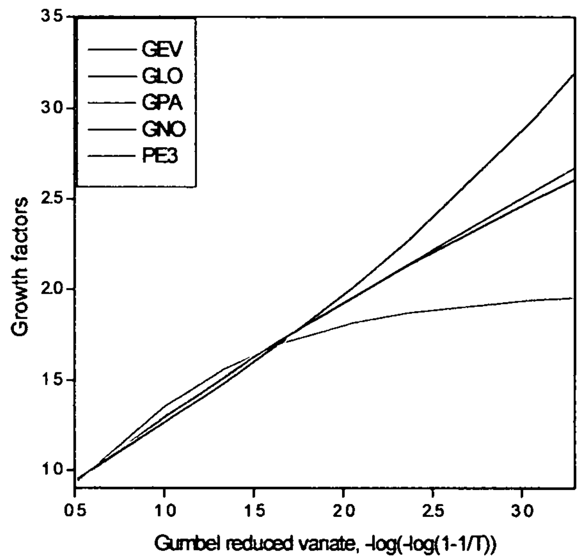


Figure 2.8 Growth curves of various distributions for subzone 2(c)

2.5.5 Development of regional flood frequency relationships

The index flood approach discussed in section 2.4, is used for development of regional flood frequency relationships for all the four homogeneous subzones 2(a)A, 2(a)B, 2(b) and 2(c) of North-East India. The relationships developed for both gauged and ungauged catchments areas of the four homogeneous subzones are given below.

(a) Gauged catchments

For estimation of floods of various return periods for gauged catchments of the homogeneous subzones 2(a)A, 2(a)B, 2(b) and 2(c), the regional flood frequency relationships have been developed by using the identified best fitting distributions. For development of regional relationships for subzones 2(a)A and 2(b) the identified best fitting PE3 distribution is used. Since, the PE3 distribution has no explicit analytical inverse form available in literature. Therefore, the quantile estimates $\hat{Q}_T^i$ with return period T at each gauged catchments of the homogeneous subzones 2(a)A and 2(b), may be computed by multiplying mean annual flood discharge of a catchments of subzone 2(a)A (given in Table 2.1) and subzone 2(b) (given in Table 2.3) by the corresponding values of the growth factors of PE3 distribution given in Table 2.9 and Table 2.11, respectively. Again, for subzones 2(a)B and 2(c) the identified best fitting distributions GPA and GLO have an explicit inverse form therefore, the quantile estimates $\hat{Q}_T^i$ with return period T at each gauged catchments i can be expressed by equation (2.5.14) and (2.5.15), respectively as follows

$$\hat{Q}_T^i = \left\{3.722 - 3.418 * \left(\frac{1}{T}\right)^{0.256}\right\} * \hat{\lambda}_1^i \qquad (2.5.14)$$

$$\hat{Q}_T^i = \left\{-0.732 + 1.688 * \left(\frac{1}{T-1}\right)^{-0.125}\right\} * \hat{\lambda}_1^i \qquad (2.5.15)$$

where, $\hat{\lambda}_1^i$ in above equations is the mean annual floods for site i of subzones 2(a)B and 2(c).

Alternatively, for subzones 2(a)B and 2(c) also, floods of various return periods may also be obtained by multiplying the mean annual maximum discharges of the catchment (Table 2.2 and Table 2.4) by the corresponding growth factors given in Table 2.10 and Table 2.12.

(b) Ungauged catchments

For ungauged catchments the at-site mean cannot be computed in absence of the observed flow data. Hence, a relationship between the mean annual flood discharge of gauged catchments in the region and their physiographic catchments characteristics is developed, which is used to estimate mean annual flood discharge for an ungauged site.

The regional relationships developed between mean annual flood discharge and catchments areas for the four subzones 2(a)A, 2(a)B, 2(b) and 2(c), in log domain using least squares approach are given in equations (2.5.16) to (2.5.19), respectively as follows

$$\hat{\lambda}_1^i = 1.749(A_i)^{0.804} \quad (2.5.16)$$

$$\hat{\lambda}_1^i = 3.670(A_i)^{0.720} \quad (2.5.17)$$

$$\hat{\lambda}_1^i = 1.669(A_i)^{0.697} \quad (2.5.18)$$

$$\hat{\lambda}_1^i = 2.401(A_i)^{0.717} \quad (2.5.19)$$

where, A_i is the catchments area , in km^2 and $\hat{\lambda}_1^i$ is the mean annual flood discharge in m^3/s at ungauged site i for subzones 2(a)A, 2(a)B, 2(b) and 2(c), respectively. For Equations (2.5.16) to (2.5.19), the correlation coefficients are r=0.852, 0.561, 0.864 and 0.929, respectively.

For development of regional flood frequency relationship for ungauged catchments, the regional flood frequency relationship developed for gauged catchments is coupled with regional relationship between mean annual flood discharge and catchments area. The regional flood frequency relationships for ungauged catchments of all the four subzones are given below.

The quantile estimates $\hat{Q}_T^i$ with return period T at ungauged catchments i of four subzones 2(a)A, 2(a)B, 2(b) and 2(c) are given in equations (2.5.20) to (2.5.23), respectively as follows

$$\hat{Q}_T^i = (A_i)^{0.804} * \hat{U}_T \quad (2.5.20)$$

$$\hat{Q}_T^i = \left\{13.660 - 12.544 * \left(\frac{1}{T}\right)^{0.256}\right\} * A_i^{0.720} \quad (2.5.21)$$

$$\hat{Q}_T^i = (A_i)^{0.697} * \hat{U}_T \quad (2.5.22)$$

Chapter 3

Regional Flood Frequency Analysis by Using LH-moments

3.1 Introduction

In this chapter the regional flood frequency analysis of four subzones i.e. 2(a)A, 2(a)B, 2(b) and 2(c), which are found to be homogeneous for L-moments, have been carried out by using LH-moments. The four levels of LH-moments i.e. from level one to four are considered for our study. The homogeneity of these four subzones are tested at all the four levels of LH-moments by using heterogeneity measure. Out of the five probability distributions used in previous chapter, the three distributions namely generalized extreme value (GEV), generalized logistic (GLO) and generalized Pareto (GPA) are used in this chapter. The detail theory of LH-moments as parameter estimation method and the regional flood frequency analysis procedure based on LH-moments are discussed in the following sections.

3.2 LH-moments

Wang (1997) introduced the concept of LH-moments as generalization of the L-moments and defined as:

$$\lambda_1^\eta = E[X_{(\eta+1)(\eta+1)}] \tag{3.2.1}$$

$$\lambda_2^\eta = \frac{1}{2} E[X_{(\eta+2)(\eta+2)} - X_{(\eta+1)(\eta+2)}] \tag{3.2.2}$$

$$\lambda_3^\eta = \frac{1}{3} E[X_{(\eta+3)(\eta+3)} - 2X_{(\eta+2)(\eta+3)} + X_{(\eta+1)(\eta+3)}] \tag{3.2.3}$$

$$\lambda_4^\eta = \frac{1}{4}E[X_{(\eta+4)(\eta+4)} - 3X_{(\eta+3)(\eta+4)} + 3X_{(\eta+2)(\eta+4)} - X_{(\eta+1)(\eta+4)}] \quad (3.2.4)$$

when, $\eta = 0$, LH-moments will be Hosking (1990) L-moments. As η increases, LH-moments reflect more and more the characteristics of the upper part of distributions and larger events in data (Wang, 1997). The LH-moments are denoted as L_1-moments, L_2-moments,... etc. for $\eta = 1,2,...$, respectively. The LH-moments ratios (LHMRs) can be defined as

LH-coefficient of variation, (τ^η)= $\lambda_2^\eta / \lambda_1^\eta$ (3.2.5)

LH-coefficient of skewness, (τ_3^η)= $\lambda_3^\eta / \lambda_2^\eta$ (3.2.6)

LH-coefficient of kurtosis, (τ_4^η)= $\lambda_4^\eta / \lambda_2^\eta$ (3.2.7)

For given a ranked sample, $x_{(1)} \le x_{(2)} \le ... \le x_{(n)}$, the sample estimates of LH-moments defined by Wang (1997) as

$$\hat{\lambda}_1^\eta = \frac{1}{{}^nC_{\eta+1}} \sum_{i=1}^{n} {}^{i-1}C_\eta x_{(i)} \quad (3.2.8)$$

$$\hat{\lambda}_2^\eta = \frac{1}{2}\frac{1}{{}^nC_{\eta+2}} \sum_{i=1}^{n} ({}^{i-1}C_{\eta+1} - {}^{i-1}C_\eta\, {}^{n-i}C_1)x_{(i)} \quad (3.2.9)$$

$$\hat{\lambda}_3^\eta = \frac{1}{3}\frac{1}{{}^nC_{\eta+3}} \sum_{i=1}^{n} ({}^{i-1}C_{\eta+2} - 2\,{}^{i-1}C_{\eta+1}\, {}^{n-i}C_1 + {}^{i-1}C_\eta\, {}^{n-i}C_2)x_{(i)} \quad (3.2.10)$$

$$\hat{\lambda}_4^\eta = \frac{1}{4}\frac{1}{{}^nC_{\eta+4}} \sum_{i=1}^{n} ({}^{i-1}C_{\eta+3} - 3\,{}^{i-1}C_{\eta+2}\, {}^{n-i}C_1 + 3\,{}^{i-1}C_{\eta+1}\, {}^{n-i}C_2 - {}^{i-1}C_\eta\, {}^{n-i}C_3)x_{(i)} \quad (3.2.11)$$

where

$${}^mC_j = \binom{m}{j} = \frac{m!}{j!(m-j)!} \quad (3.2.12)$$

Alternatively, Wang (1997) defined the LH-moments as linear combination of normalized PWMs as:

$$\hat{\lambda}_1^\eta = B_\eta \quad (3.2.13)$$

$$\hat{\lambda}_2^\eta = \frac{1}{2}(\eta+2)\{B_{\eta+1} - B_\eta\} \quad (3.2.14)$$

$$\hat{\lambda}_3^\eta = \frac{1}{3!}(\eta+3)\{(\eta+4)B_{\eta+2} - 2(\eta+3)B_{\eta+1} + (\eta+2)B_\eta\} \quad (3.2.15)$$

$$\hat{\lambda}_4^\eta = \frac{1}{4!}(\eta+4)\{(\eta+6)(\eta+5)B_{\eta+3} - 3(\eta+5)(\eta+4)B_{\eta+2} + 3(\eta+4)(\eta+3)B_{\eta+1} - (\eta+3)(\eta+2)B_\eta\} \quad (3.2.16)$$

where,

$$B_r = \frac{\int_0^1 x(F)F^r dF}{\int_0^1 F^r dF} = (r+1)\int_0^1 x(F)F^r dF = (r+1)\beta_r \tag{3.2.17}$$

The sample LH-moment ratios can be defined as follows

$$\hat{\tau}^\eta = \hat{\lambda}_2^\eta / \hat{\lambda}_1^\eta,\ \hat{\tau}_3^\eta = \hat{\lambda}_3^\eta / \hat{\lambda}_{2\eta} \text{ and } \hat{\tau}_4^\eta = \hat{\lambda}_4^\eta / \hat{\lambda}_2^\eta$$

The LH-moments and parameters based on LH-moments of GEV, GLO and GPA distributions are given below.

3.2.1 GEV Distribution

The probability weighted moments (PWMs) of GEV distribution developed by Hosking et al. (1985) is given by

$$\beta_r = \frac{1}{1+r}\left\{\xi + \frac{\alpha}{k}[1-\Gamma(1+k)(r+1)^{-k}]\right\} \tag{3.2.18}$$

Wang (1997) developed LH-moment for GEV distribution in terms of normalized PWMs as

$$\lambda_1^\eta = \xi + \frac{\alpha}{k}[1-\Gamma(1+k)(\eta+1)^{-k}] \tag{3.2.19}$$

$$\lambda_2^\eta = \frac{(\eta+2)\alpha\Gamma(1+k)}{2!k}[-(\eta+2)^{-k} + (\eta+1)^{-k}] \tag{3.2.20}$$

$$\lambda_3^\eta = \frac{(\eta+3)\alpha\Gamma(1+k)}{3!k}[-(\eta+4)(\eta+3)^{-k} + 2(\eta+3)(\eta+2)^{-k} - (\eta+2)(\eta+1)^{-k}] \tag{3.2.21}$$

$$\lambda_4^\eta = \frac{(\eta+4)\alpha\Gamma(1+k)}{4!k}[-(\eta+6)(\eta+5)(\eta+4)^{-k} + 3(\eta+5)(\eta+4)(\eta+3)^{-k} - 3(\eta+4)(\eta+3)(\eta+2)^{-k} + (\eta+3)(\eta+2)(\eta+1)^{-k}] \tag{3.2.22}$$

At-site parameters

Wang (1997) developed a relation between shape parameter k and τ_3 for different level of LH-moments; the values of coefficients have been shown in Table 3.1.

The parameters of GPA distribution in terms of LH-moments developed by Meshgi and Khalili (2009b) are given as follows

At-site parameters

$$k = \frac{-5 - 2\eta + \dfrac{(\eta+3)\left[(\eta+3)\beta_{\eta+2} - (\eta+1)\beta_\eta\right]}{(\eta+2)\beta_{\eta+1} - (\eta+1)\beta_\eta}}{-1 + \dfrac{(\eta+3)\beta_{\eta+2} - (\eta+1)\beta_\eta}{(\eta+2)\beta_{\eta+1} - (\eta+1)\beta_\eta}} \tag{3.2.34}$$

$$\alpha = -\frac{k\Gamma(\eta+3+k)\Gamma(\eta+2+k)\left[(\eta+2)\beta_{\eta+1} - (\eta+1)\beta_\eta\right]}{(\eta+1)!\Gamma(1+k)\left[(\eta+2)\Gamma(\eta+2+k) - \Gamma(\eta+3+k)\right]} \tag{3.2.35}$$

$$\xi = (\eta+1)\beta_\eta - \frac{\alpha}{k}\left[1 - \frac{(\eta+1)\Gamma(\eta+1)\Gamma(1+k)}{\Gamma(\eta+2+k)}\right] \tag{3.2.36}$$

Regional parameters

$$k = \frac{(\eta+3)(1-3\bar{\tau}_3^\eta)}{3\bar{\tau}_3^\eta + \eta + 3} \tag{3.2.37}$$

$$\alpha = -\frac{2k\Gamma(\eta+3+k)\Gamma(\eta+2+k)\bar{\tau}^\eta}{(\eta+2)(\eta+1)!\Gamma(1+k)\left[(\eta+2)\Gamma(\eta+2+k) - \Gamma(\eta+3+k)\right]} \tag{3.2.38}$$

$$\xi = 1 - \frac{\alpha}{k}\left[1 - \frac{(\eta+1)\Gamma(\eta+1)\Gamma(1+k)}{\Gamma(\eta+2+k)}\right] \tag{3.2.39}$$

3.2.3 GLO distribution

The PWMs of GLO distribution developed by Hosking (1986) is

$$\beta_r = \frac{1}{1+r}\left\{\xi + \frac{\alpha}{k}\left[1 - \frac{\Gamma(1+k)\Gamma(1+r-k)}{\Gamma(1+r)}\right]\right\} \tag{3.2.40}$$

The LH-moments for GLO distribution developed by Meshgi and Khalili (2009a) are given by

$$\lambda_1^\eta = \xi + \frac{\alpha}{k}\left[1 - \frac{\Gamma(1+k)\Gamma(\eta+1-k)}{\eta!}\right] \tag{3.2.41}$$

$$\lambda_2^\eta = \frac{(\eta+2)\alpha\Gamma(1+k)}{2!k}\left[-\frac{\Gamma(\eta+2-k)}{(\eta+1)!} + \frac{\Gamma(\eta+1-k)}{\eta!}\right] \tag{3.2.42}$$

$$\lambda_3^\eta = \frac{(\eta+3)\alpha\Gamma(1+k)}{3!k}\left[-\frac{(\eta+4)\Gamma(\eta+3-k)}{(\eta+2)!} + 2\frac{(\eta+3)\Gamma(\eta+2-k)}{(\eta+1)!} - \frac{(\eta+2)\Gamma(\eta+1-k)}{\eta!}\right] \tag{3.2.43}$$

$$\lambda_4^{\eta} = \frac{(\eta+4)\alpha\Gamma(1+k)}{4!k} \times [-\frac{(\eta+6)(\eta+5)\Gamma(\eta+4-k)}{(\eta+3)!} + 3\frac{(\eta+5)(\eta+4)\Gamma(\eta+3-k)}{(\eta+2)!} - 3\frac{(\eta+4)(\eta+3)\Gamma(\eta+2-k)}{(\eta+1)!} + \frac{(\eta+3)(\eta+2)\Gamma(\eta+1-k)}{\eta!}] \quad (3.2.44)$$

The parameters of GLO distribution for LH-moments developed by Meshgi and Khalili (2009b) are given as follows

At-site parameters

$$k = -\frac{(\eta+3)(\eta+2)\beta_{\eta+2} - \left[(\eta+2)^2 + (\eta+2)(\eta+1)\right]\beta_{\eta+1} + (\eta+1)^2\beta_\eta}{(\eta+2)\beta_{\eta+1} - (\eta+1)\beta_\eta} \quad (3.2.45)$$

$$\alpha = \frac{\Gamma(\eta+2)\left[(\eta+2)\beta_{\eta+1} - (\eta+1)\beta_\eta\right]}{\Gamma(\eta+1-k)\Gamma(1+k)} \quad (3.2.46)$$

$$\xi = (\eta+1)\beta_\eta - \frac{\alpha}{k}\left[1 - \frac{\Gamma(\eta+1-k)\Gamma(1+k)}{\Gamma(\eta+1)}\right] \quad (3.2.47)$$

Regional parameters

$$k = -\frac{3(\eta+2)^2\bar{\tau}_3^{\eta} - \eta(\eta+3)}{(\eta+4)(\eta+3)} \quad (3.2.48)$$

$$\alpha = \frac{2\Gamma(\eta+2)\bar{\tau}^{\eta}}{(\eta+2)\Gamma(\eta+1-k)\Gamma(1+k)} \quad (3.2.49)$$

$$\xi = 1 - \frac{\alpha}{k}\left[1 - \frac{\Gamma(\eta+1-k)\Gamma(1+k)}{\Gamma(\eta+1)}\right] \quad (3.2.50)$$

3.3 Regional flood frequency analysis

The regional flood frequency analysis based on LH-moments by using an index flood procedure involves the same steps proposed by Hosking and Wallis (1993, 1997) for L-moments. For this purpose, the index flood procedure for L-moments (section 2.4) is extended to higher order LH-moments by using the index flood as first at-site LH-moments i.e. $\hat{\lambda}_1^{\eta,i}$ $(\eta = 1,2,3,...)$, instead of at-site mean. The detail procedures are given in the following sections.

For all calculations some computer programs have been developed (in Fortran-77) and the list of subroutines used for these programs are given in Appendix-B.

3.3.1 Screening of data

For screening of data, the discordancy measures ($D_i^\eta, \eta = 1,2,3,...$) for LH-moments is used, and which can be defined similarly as the discordancy measure (D_i) for L-moment (section 2.5.1), by replacing L-moment with LH-moments.

The discordancy measure ($D_i^\eta, \eta = 1,2,3,...$) for LH-moments of a region having NS sites is defined as:

$$D_i^\eta = \frac{1}{3} NS(u_i^\eta - \bar{u}^\eta)^T S_\eta^{-1}(u_i^\eta - \bar{u}^\eta), \eta = 1,2,3,... \qquad (3.3.1)$$

where, $u_i^\eta = [\hat{\tau}^{\eta,i} \quad \hat{\tau}_3^{\eta,i} \quad \hat{\tau}_4^{\eta,i}]^T$ be a vector containing the sample LHMRs for the site i, $\bar{u}^\eta$ and S_η can be defined similarly as equation (2.5.1) and (2.5.2), by replacing u_i with u_i^η.

The critical values for discordancy measure D_i, given by Hosking and Wallis (1997) are used as critical values for $D_i^\eta (\eta = 1,2,3,4,...)$ and consider the site i is declare to be discordant, if $D_i^\eta (\eta = 1,2,3,4,...)$ is greater than the critical value.

The unbiased sample LH-moment,($L_\eta, \eta = 1,2,3,4$) statistics and the discordancy measures $D_i^\eta (\eta = 1,2,3,4)$ for sub-zones 2(a)A, 2(a)B, 2(b) and 2(c) are given in Appendix-A (see Table 3.2 to Table 3.17) . In case of subzone 2(a)A, it has been observed that the D_i^2, D_i^3 and D_i^4 values of all the ten gauging sites are less then the critical value for ten sites i.e. 2.491 (Hosking and Wallis, 1997). Therefore, all the ten sites of 2(a)A can be used for L_2-, L_3- and L_4-moments based regional flood frequency analysis of this subzone. But an exceptional case has been observed in the D_i^1 value of site Nonai i.e 2.68 (see Table 3.2 in Appendix-A), which is higher than the critical value for ten sites, therefore this site can be considered as discordant site for L_1-moments based regional flood frequency analysis of subzone 2(a)A. Though the site Nonai found to be discordant, we include this site for L_1-moments based regional frequency analysis of subzone 2(a)A. The reasons for inclusion of discordance site for regional flood frequency analysis are, the discordancy measure of site is less than three and we have found no gross errors or incorrect recording of data values for this site. For subzones 2(a)B, 2(b) and 2(c), it has been observed that D_i^1, D_i^2, D_i^3 and D_i^4 values of all the sites are less than the critical value for eight and seven sites, respectively. Therefore, all the eight sites of 2(a)B and all the seven sites of 2(b) and 2(c) each, can be considered for regional flood frequency analysis of these subzones for all level of LH-moments i.e. L_η, $\eta = 1,2,3,4$.

3.3.2 Identification of homogeneous region

For testing the regional homogeneity of a region in terms of LH-moments, a heterogeneity measure, H_i^η, $i = 1,2,3$, $\eta = 1,2,3,4,...$ is used. It can be defined in similar way to the heterogeneity measure for the method of L-moments (section 2.5.2). The formula for H_i^η, $i = 1,2,3$, $\eta = 1,2,3,4,...$ can be written as

$$H_i^\eta = \frac{V_i^\eta - \mu_{V_i^\eta}}{\sigma_{V_i^\eta}}, \; i = 1,2,3, \eta = 1,2,3,4,... \qquad (3.3.2)$$

where, the three measures of variability V_1^η, V_2^η and V_3^η can be obtained from equations (2.5.5), (2.5.6) and (2.5.7), by replacing L-moments with LH-moments and $\mu_{V_i^\eta}$ $(i = 1,2,3, \eta = 1,2,3,4,...)$, $\sigma_{V_i^\eta}$ $(i = 1,2,3, \eta = 1,2,3,4,...)$ are the mean and standard deviation of V_i^η $(i = 1,2,3, \eta = 1,2,3,4,...)$, respectively for each of the simulated region.

The heterogeneity measures H_i^η, $(i = 1,2,3, \eta = 1,2,3,4,...)$ have been computed by carrying out 500 simulations using the LH-moments based four parameters Kappa distribution.

The heterogeneity measures $H_i^\eta, (i = 1,2,3, \eta = 1,2,3,4,...)$ of all the four subzones 2(a)A, 2(a)B, 2(b) and 2(c) of North-East regions are given in Table 3.18. From Table 3.18, it has been observed that the $H_1^\eta, (\eta = 1,2,3,4)$ values of all the four subzones are less than 1. Therefore, by using the homogeneity criteria proposed by Hosking and Wallis (1997) for L-moments, we conclude that all the four subzones are found to be homogeneous for all level of the LH-moments ($L_\eta, \eta = 1,2,3,4$).

Table 3.18 Heterogeneity measures of four subzones for different levels of LH-moments $L_\eta, \eta = 1,2,3,4$ i.e. (L_1 to L_4)

Name of subzones	LH-moments	H_1^η	H_2^η	H_3^η
2(a)A	L_1	-0.15	1.16	0.80
	L_2	0.03	1.15	0.92
	L_3	0.18	0.57	0.95
	L_4	0.46	0.45	0.95
2(a)B	L_1	-1.02	-.54	-0.34
	L_2	-0.86	-0.52	-0.86
	L_3	-0.73	-0.96	-0.84
	L_4	-0.64	-1.04	-0.50
2(b)	L_1	-0.48	-1.07	-0.58
	L_2	-1.36	-0.95	-0.15
	L_3	-1.80	-0.58	0.47
	L_4	-2.01	0.18	1.13
2(c)	L_1	0.17	-0.81	0.22
	L_2	-0.22	-0.20	0.64
	L_3	-0.47	0.12	0.92
	L_4	-0.38	0.52	1.01

3.3.3 Selection of best fitting probability distribution

(a) Z_η^{DIST} -Statistic criteria

To obtain the best fitting probability distributions with level of LH-moments the Z_η^{DIST} -statistic criteria has been used as goodness of fit measure for our study. It is defined similarly as the equation (2.5.11), by replacing L-moments with LH-moments and can be written as

$$Z_\eta^{DIST} = (\tau_4^{\eta,DIST} - \hat{\tau}_4^{\eta,R} + B_4^\eta)/\sigma_4^\eta,\ \eta = 1,2,3,4,... \qquad (3.3.3)$$

where, $\hat{\tau}_4^{\eta,R}$ $(\eta = 1,2,3,4,...)$ is the regional average value of $\hat{\tau}_4^{\eta,i}$ $(\eta = 1,2,3,4,...)$ obtained from the data of a given region and B_4^η $(\eta = 1,2,3,4,...)$, σ_4^η $(\eta = 1,2,3,4,...)$ are defined similarly as equations (2.5.12) and (2.5.13) respectively, by replacing L-moments with LH-moments, and $\tau_4^{\eta,DIST}$ $(\eta = 1,2,3,4,...)$ are the LH-kurtosis values for a particular probability distribution at different level of LH-moments for which test statistics applied (the relationships are given below).

The regional average values of $\hat{\tau}^{\eta,i}$, $\hat{\tau}_3^{\eta,i}$ and $\hat{\tau}_4^{\eta,i}$ for, $\eta = 1,2,3,4$, for four sub-zones are calculated and are given in Table 3.19.

Table 3.19 Regional average LH-moments ratio for four subzones

Name of sub-zones	LH-moments	$\hat{\tau}^{\eta,R}$	$\hat{\tau}_3^{\eta,R}$	$\hat{\tau}_4^{\eta,R}$
2(a)A	L_1	0.1431	0.1983	0.1105
	L_2	0.1207	0.2073	0.1179
	L_3	0.1078	0.2132	0.1289
	L_4	0.0992	0.2211	0.1429
2(a)B	L_1	0.2171	0.2354	0.1178
	L_2	0.1789	0.2451	0.1226
	L_3	0.1573	0.2522	0.1209
	L_4	0.1433	0.2557	0.1184
2(b)	L_1	0.1423	0.2278	0.1349
	L_2	0.1213	0.2521	0.1455
	L_3	0.1094	0.2675	0.1487
	L_4	0.1016	0.2742	0.1425
2(c)	L_1	0.1496	0.2326	0.1705
	L_2	0.1270	0.2756	0.1360
	L_3	0.1153	0.2748	0.1045
	L_4	0.1073	0.2570	0.0729

For calculating the Z_η^{DIST}-statistic values also the 500 simulations are carried out by using the LH-moments based four parameter Kappa distribution.

The $|Z_\eta^{DIST}|$,($\eta = 1,2,3,4$)-statistic values from L_1-moments to L_4-moments of GEV, GLO and GEV distributions for 2(a)A, 2(a)B, 2(b) and 2(c) are given in Table 3.20.

It has been observed from Table 3.20 that for subzone 2(a)A, the $|Z_\eta^{DIST}|$-statistic values of GLO, GEV and GPA distributions are less than the critical value 1.64, for all level of LH-moments $L_\eta, \eta = 1,2,3,4$ i.e. (L_1 to L_4). But an exceptional case has been observed for GPA distribution at level four LH-moments i.e, the $|Z_4^{DIST}|$-statistic value of GPA distribution is bigger than the critical value at this level. However for this sub-zone, the GEV distribution for L_2-moments attains the minimum $|Z_\eta^{DIST}|$-statistic value. Therefore, the GEV distribution with L_2-moments is the best fitting probability distribution among the three distributions for subzone 2(a)A.

Similarly, for subzones 2(a)B, 2(b) and 2(c), it has been observed from Table 3.20 that the $\left|Z_{\eta}^{DIST}\right|$-statistics values of different distributions are less than the critical value i.e. 1.64, at different level of LH-moments. However, the GEV distributions attains the minimum $\left|Z_{\eta}^{DIST}\right|$-statistic value at level four, at level one and at level two LH-moments for subzones 2(a)B, 2(b) and 2(c), respectively. Therefore, the GEV distribution with level four LH-moments i.e. L_4-moments, level one LH-moments i.e. L_1-moments and level two LH-moments i.e. L_2-moments are identified as the best fitting distribution for subzones 2(a)B, 2(b) and 2(c), respectively.

Table 3.20 $\left|Z_{\eta}^{DIST}\right|$ ($\eta = 1,2,3,4$)-statistics for all the homogeneous subzones

Sl. No.	Distribution	LH-moments	$\left\|Z_{\eta}^{DIST}\right\|$			
			2(a)A	2(a)B	2(b)	2(c)
1	GLO	L_1	1.44	1.67	0.80	0.26
		L_2	0.41	0.80	0.08	0.50
		L_3	0.29	0.52	0.15	0.91
		L_4	1.10	0.35	0.12	1.02
2	GEV	L_1	0.43	0.78	**0.04**	0.83
		L_2	**0.24**	0.21	0.37	**0.13**
		L_3	0.75	0.11	0.46	0.63
		L_4	1.42	**0.05**	0.35	0.82
3	GPA	L_1	1.14	0.68	1.20	1.74
		L_2	1.13	0.61	1.01	0.40
		L_3	1.30	0.40	0.83	0.30
		L_4	1.83	0.35	0.66	0.54

The bold figures represent lowest $\left|Z_{\eta}^{DIST}\right|$-statistic value for each of the four subzones

(b) LH-moments ratio diagram

The LH-moments ratio diagram is nothing but a graph between LH-kurtosis and LH-skewness like L-moments ratio diagram proposed by Hosking (1991). The theoretical curves of probability distributions for LH-moments as well as the regional average LH-skewness and LH-kurtosis are plotted on the same graphs to select the best fitting distributions. The LH-moments ratio diagrams for all the four level of LH-moment, $L_{\eta}, \eta = 1,2,3,4$ i.e. (L_1 to L_4) are drawn for the subzones 2(a)A, 2(a)B, 2(b) and 2(c) and are shown in Figure 3.1 to Figure 3.4. For this, the relationships between LH-kurtosis and LH-skewness of three distributions viz. GEV, GLO and GPA for

LH-moments, $L_\eta, \eta = 1,2,3,4$ i.e. (L_1 to L_4) developed by Meshgi and Khalili (2009a) are used and are given as follows

For L_1-moments

$$\tau_4^{1,GEV} = 0.0666 + 0.1208\tau_3^1 + 0.8711(\tau_3^1)^2 - 0.0484(\tau_3^1)^3 + 0.0084(\tau_3^1)^4$$

$$\tau_4^{1,GLO} = 0.1167 + 0.0187\tau_3^1 + 0.8859(\tau_3^1)^2$$

$$\tau_4^{1,GPA} = 0.2083\tau_3^1 + 0.9115(\tau_3^1)^2 - 0.1134(\tau_3^1)^3 + 0.0124(\tau_3^1)^4$$

For L_2-moments

$$\tau_4^{2,GEV} = 0.0483 + 0.1357\tau_3^2 + 0.8710(\tau_3^2)^2 - 0.0317(\tau_3^2)^3 + 0.0045(\tau_3^2)^4$$

$$\tau_4^{2,GLO} = 0.0889 + 0.0467\tau_3^2 + 0.8960(\tau_3^2)^2$$

$$\tau_4^{2,GPA} = 0.2143\tau_3^2 + 0.8816(\tau_3^2)^2 - 0.0754(\tau_3^2)^3 + 0.0059(\tau_3^2)^4$$

For L_3-moments

$$\tau_4^{3,GEV} = 0.0378 + 0.1491\tau_3^3 + 0.8644(\tau_3^3)^2 - 0.0222(\tau_3^3)^3 + 0.0026(\tau_3^3)^4$$

$$\tau_4^{3,GLO} = 0.0714 + 0.0714\tau_3^3 + 0.8929(\tau_3^3)^2$$

$$\tau_4^{3,GPA} = 0.2187\tau_3^3 + 0.8813(\tau_3^3)^2 - 0.0538(\tau_3^3)^3 + 0.0031(\tau_3^3)^4$$

For L_4-moments

$$\tau_4^{4,GEV} = 0.0310 + 0.1602\tau_3^4 + 0.8564(\tau_3^4)^2 - 0.0163(\tau_3^4)^3 + 0.0017(\tau_3^4)^4$$

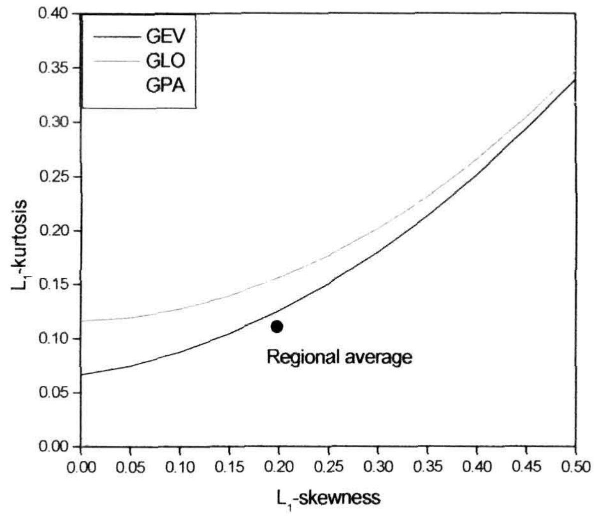
$$\tau_4^{4,GLO} = 0.0595 + 0.0918\tau_3^4 + 0.8856(\tau_3^4)^2$$

$$\tau_4^{4,GPA} = 0.2212\tau_3^4 + 0.8374(\tau_3^4)^2 - 0.0665(\tau_3^4)^3 - 0.0112(\tau_3^4)^4$$

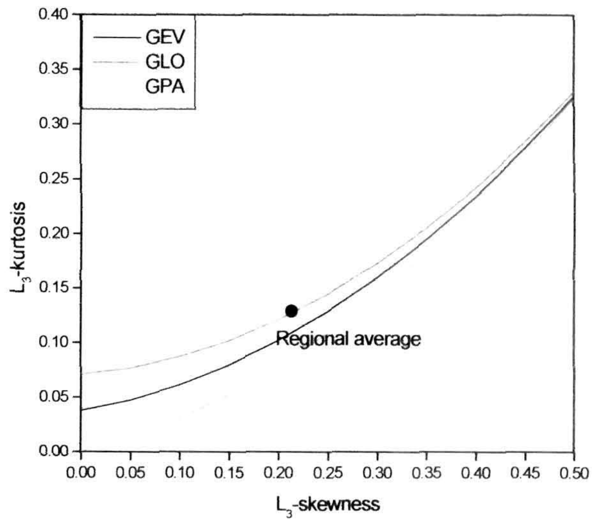
It has been observed from Figure 3.1 that the regional average values for L_1- and L_2-moments lies near to the GEV distribution and the same are lies near to the GLO distribution for L_3- and L_4-moments. Therefore, the GEV distribution for L_1- and L_2-moments and GLO distribution for L_3- and L_4-moments are identified as the best fitting distributions for subzone 2(a)A.

Similarly, from Figure 3.2 to Figure 3.4, it has been observed that different distributions are identified as the best fitting distributions for subzones 2(a)B, 2(b) and 2(c), respectively, at different level of LH-moments.

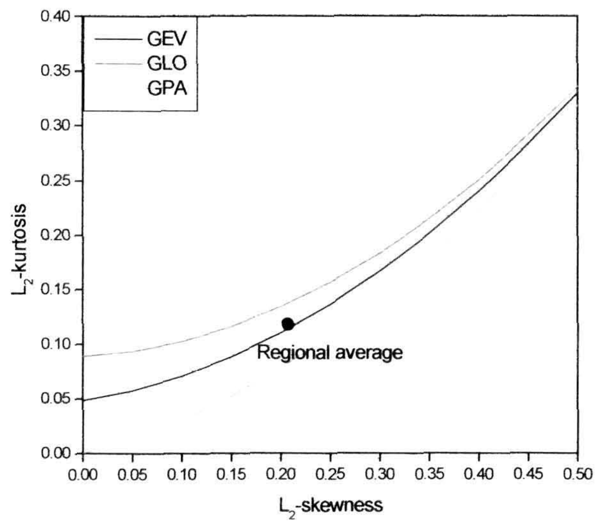
Therefore, it is not possible to identify a single probability distribution with level of LH-moments for a homogeneous region by using the LH-moments ratio diagrams. Hence, only the $|Z_\eta^{DIST}|$-statistic criteria mentioned above is used to identify a single best fitting distribution with level of LH-moments for all the four homogeneous subzones.



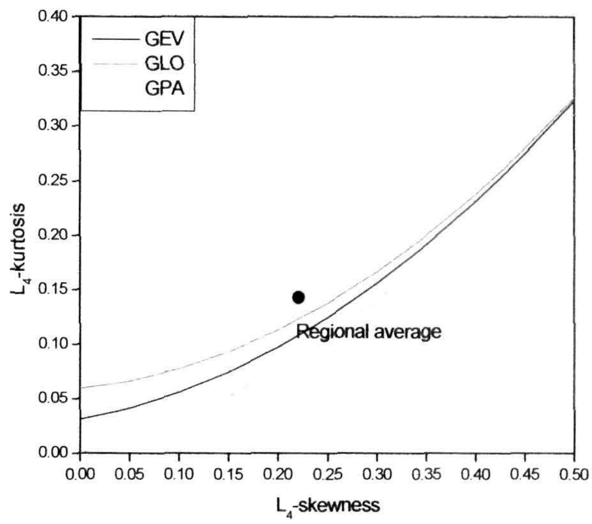
(a) L_1-moment ratio diagram



(c) L_3-moment ratio diagram

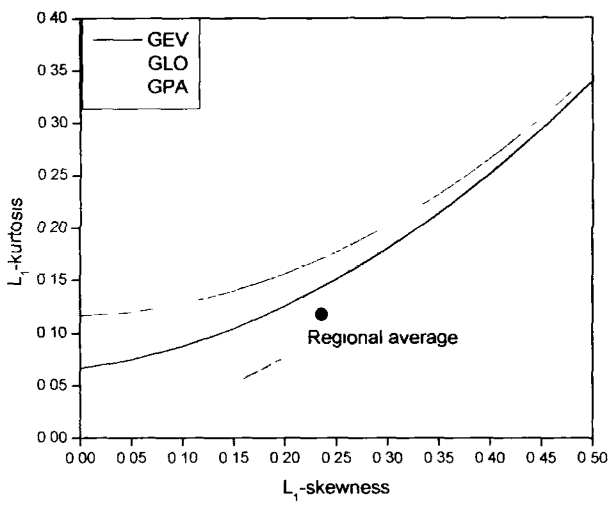


(b) L_2-moment ratio diagram

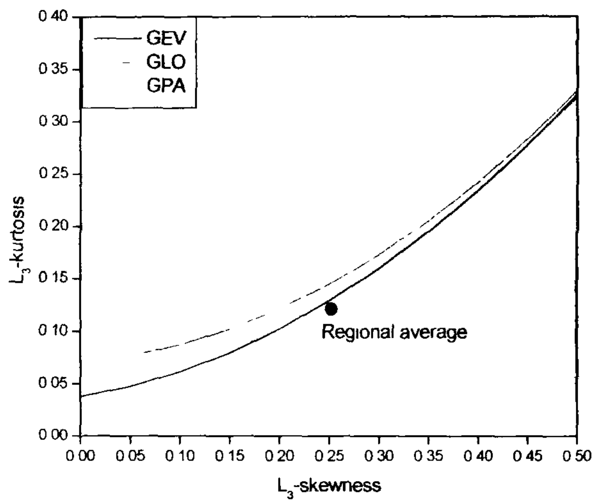


(d) L_4-moment ratio diagram

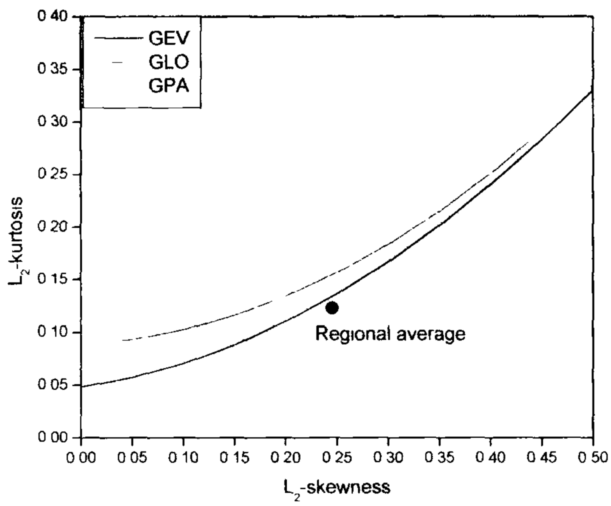
Figure 3.1 LH-moments ratio diagrams for subzone 2(a)A



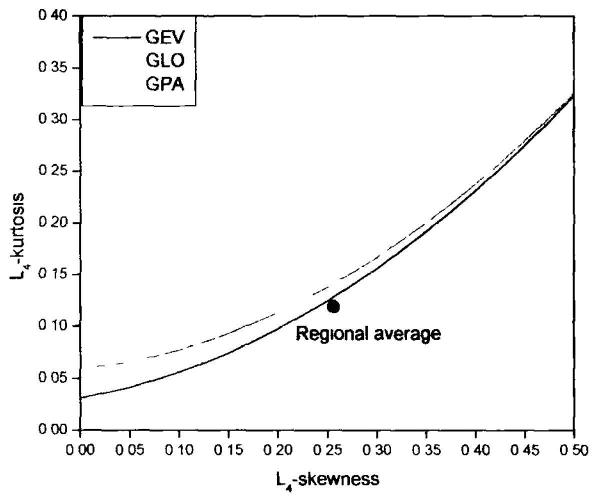
(a) L_1-moment ratio diagram



(c) L_3-moment ratio diagram

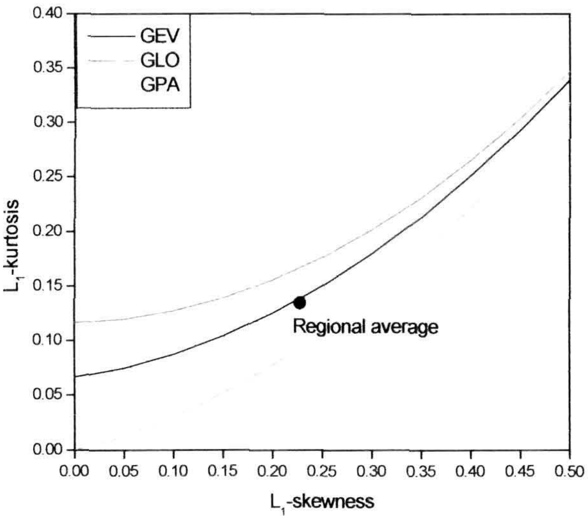


(b) L_2-moment ratio diagram

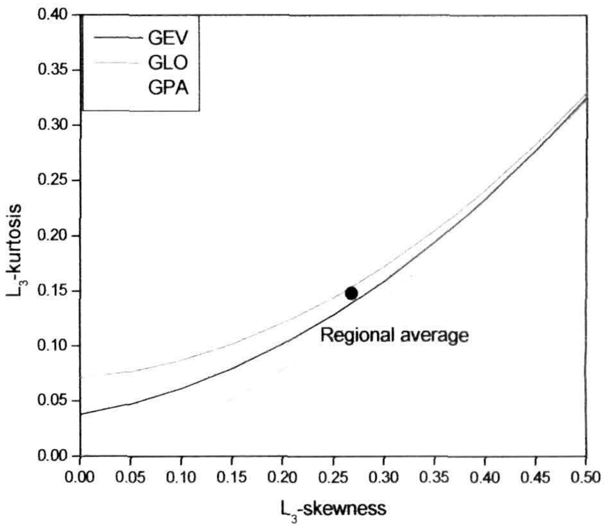


(d) L_4-moment ratio diagram

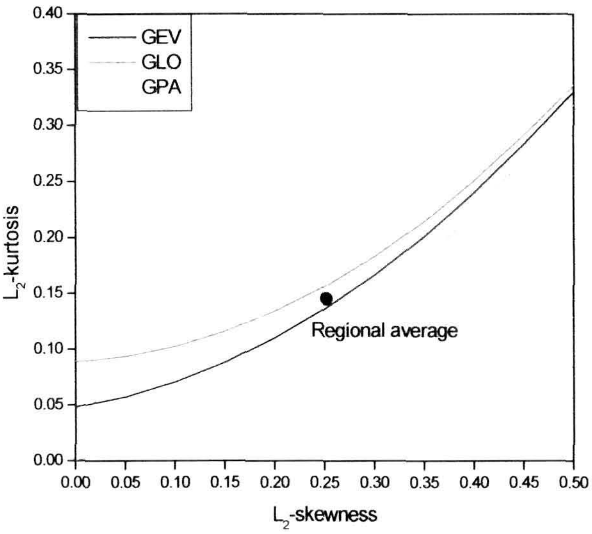
Figure 3.2 LH-moments ratio diagrams for subzone 2(a)B



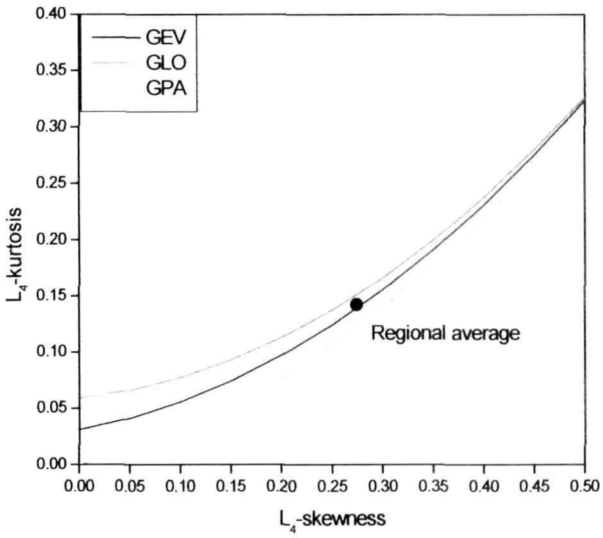
(a) L_1-moment ratio diagram



(c) L_3-moment ratio diagram

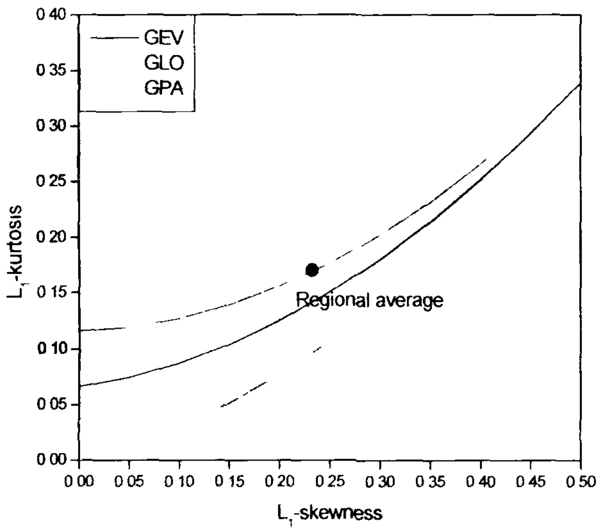


(b) L_2-moment ratio diagram

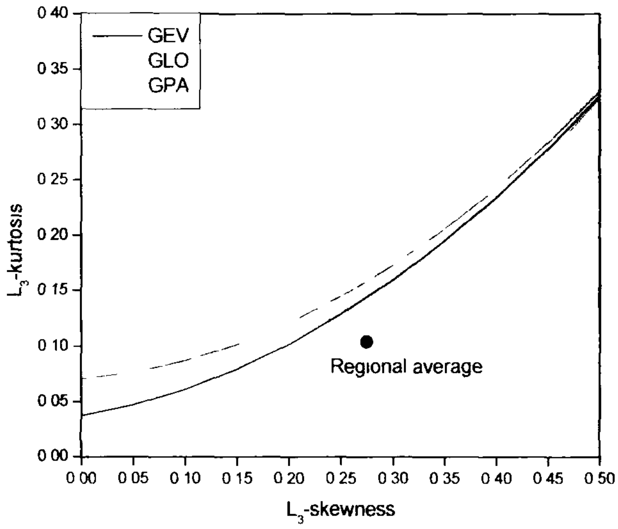


(d) L_4-moment ratio diagram

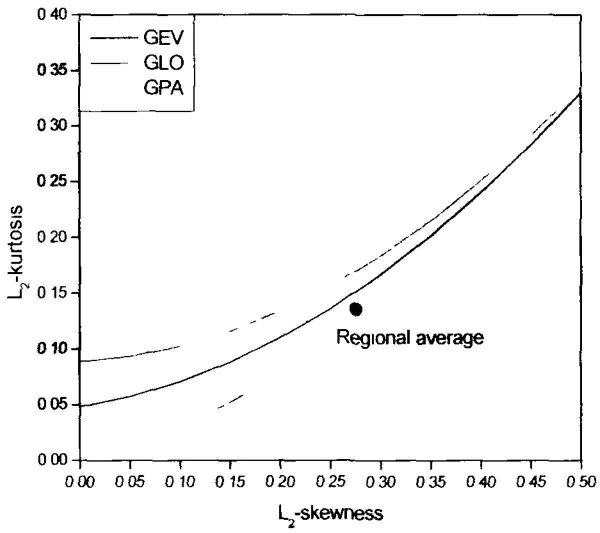
Figure 3.3 LH-moments ratio diagrams for subzone 2(b)



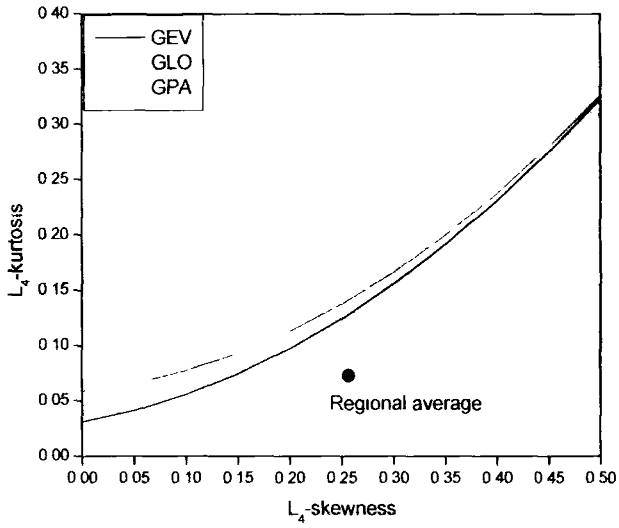
(a) L_1-moment ratio diagram



(c) L_3-moment ratio diagram



(b) L_2-moment ratio diagram



(d) L_4-moment ratio diagram

Figure 3.4 LH-moments ratio diagrams for subzone 2(c)

3.3.4 Estimation of regional parameters and growth factors

The regional parameters of a distribution for a homogeneous region can be obtained by using regional average LH-moments ratios i.e. $\hat{\tau}^{\eta,R}$ and $\hat{\tau}_3^{\eta,R}$, together with $\hat{\lambda}_1^{\eta} = 1$.

The regional parameters of each of the three probability distributions at each level of the LH-moments i.e. L_η, ($\eta = 1,2,3,4$), for subzones 2(a)A, 2(a)B, 2(b) and 2(c) are calculated and are given in Table 3.21 to Table 3.24, respectively.

By using these estimated regional parameters, the regional growth factors or quantiles ($\hat{q}_T^\eta$, $\eta = 1,2,3,4$) of three distributions are calculated for all the four LH-moments levels i.e. L_1- to L_2-moments. The regional growth factors or quantiles of GEV, GLO and GPA distributions for all level of LH-moments and for all the four homogeneous subzones are given in Table 3.25 to Table 3.28.

It has been observed from Table 3.25 to Table 3.28 that the growth factors of all the three distributions for all the four homogeneous subzones are decreasing with increase of the level of LH-moments i.e. η.

Table 3.21 Regional parameters of various distributions for subzone 2(a)A

Distributions	L_η-moments	Location	Scale	Shape
GEV	L_1	0.697	0.264	0.083
	L_2	**0.626**	**0.264**	**0.134**
	L_3	0.578	0.266	0.163
	L_4	0.545	0.263	0.174
GLO	L_1	0.800	0.178	-0.068
	L_2	0.728	0.181	0.002
	L_3	0.678	0.188	0.048
	L_4	0.641	0.192	0.074
GPA	L_1	0.464	0.509	0.353
	L_2	0.427	0.454	0.336
	L_3	0.402	0.424	0.326
	L_4	0.389	0.397	0.307

Table 3.22 Regional parameters of various distributions for subzone 2(a)B

Distributions	L_η-moments	Location	Scale	Shape
GEV	L_1	0.545	0.364	0.014
	L_2	0.465	0.343	0.055
	L_3	0.417	0.329	0.077
	L_4	**0.382**	**0.322**	**0.094**
GLO	L_1	0.691	0.253	-0.118
	L_2	0.600	0.245	-0.059
	L_3	0.542	0.242	-0.022
	L_4	0.500	0.242	0.007
GPA	L_1	0.236	0.661	0.250
	L_2	0.216	0.560	0.231
	L_3	0.206	0.500	0.216
	L_4	0.196	0.465	0.210

Table 3.23 Regional parameters of various distributions for subzone 2(b)

Distributions	L_η-moment	Location	Scale	Shape
GEV	L_1	**0.701**	**0.244**	**0.028**
	L_2	0.639	0.227	0.041
	L_3	0.604	0.214	0.044
	L_4	0.576	0.209	0.052
GLO	L_1	0.798	0.169	-0.107
	L_2	0.729	0.163	-0.070
	L_3	0.685	0.160	-0.049
	L_4	0.653	0.159	-0.029
GPA	L_1	0.493	0.448	0.270
	L_2	0.476	0.367	0.212
	L_3	0.468	0.319	0.174
	L_4	0.458	0.295	0.159

Table 3.24 Regional parameters of various distributions for subzone 2(c)

Distributions	L_η-moments	Location	Scale	Shape
GEV	L_1	0.686	0.253	0.019
	L_2	**0.631**	**0.218**	**-0.007**
	L_3	0.586	0.219	0.028
	L_4	0.539	0.240	0.091
GLO	L_1	0.788	0.175	-0.114
	L_2	0.718	0.160	-0.108
	L_3	0.670	0.164	-0.062
	L_4	0.627	0.180	0.004
GPA	L_1	0.471	0.461	0.257
	L_2	0.477	0.341	0.148
	L_3	0.449	0.323	0.154
	L_4	0.401	0.345	0.206

The bold figures (in Table 3.21 to Table 3.24) represent regional parameters of the best fitting distributions

Table 3.25 Regional growth factors of three distributions at different levels of LH-moments for subzone 2(a)A

Dist.	L_η-moments	Return period in T years							
		2	5	10	20	50	100	500	1000
GEV	L_1	0.792	1.069	1.239	1.392	1.577	1.707	1.979	2.085
	L_2	**0.720**	**0.985**	**1.139**	**1.273**	**1.428**	**1.533**	**1.739**	**1.815**
	L_3	0.673	0.932	1.079	1.204	1.346	1.439	1.617	1.681
	L_4	0.638	0.892	1.035	1.155	1.290	1.378	1.544	1.602
GLO	L_1	0.801	1.058	1.221	1.378	1.590	1.756	2.169	2.361
	L_2	0.728	0.979	1.125	1.259	1.430	1.556	1.846	1.970
	L_3	0.678	0.930	1.070	1.194	1.345	1.453	1.688	1.783
	L_4	0.641	0.894	1.030	1.149	1.290	1.389	1.597	1.679
GPA	L_1	0.777	1.089	1.266	1.405	1.544	1.622	1.745	1.780
	L_2	0.708	0.991	1.155	1.284	1.415	1.491	1.611	1.646
	L_3	0.665	0.933	1.089	1.213	1.339	1.413	1.531	1.566
	L_4	0.637	0.893	1.044	1.167	1.293	1.368	1.490	1.527

Table 3.26 Regional growth factors of three distributions at different levels of LH-moments for subzone 2(a)B

Distribution	L_η-moments	Return period in T years							
		2	5	10	20	50	100	500	1000
GEV	L_1	0.678	1.085	1.351	1.604	1.927	2.167	2.711	2.942
	L_2	0.589	0.959	1.191	1.405	1.669	1.859	2.270	2.436
	L_3	0.536	0.883	1.097	1.291	1.526	1.691	2.042	2.179
	L_4	**0.498**	**0.832**	**1.035**	**1.216**	**1.434**	**1.585**	**1.897**	**2.018**
GLO	L_1	0.691	1.072	1.326	1.582	1.941	2.234	3.010	3.391
	L_2	0.600	0.954	1.175	1.388	1.672	1.893	2.438	2.689
	L_3	0.542	0.883	1.087	1.278	1.525	1.712	2.153	2.347
	L_4	0.500	0.834	1.028	1.205	1.429	1.594	1.971	2.132
GPA	L_1	0.657	1.112	1.393	1.630	1.886	2.044	2.321	2.410
	L_2	0.575	0.969	1.216	1.427	1.658	1.804	2.063	2.149
	L_3	0.528	0.886	1.113	1.309	1.526	1.665	1.916	2.000
	L_4	0.496	0.831	1.045	1.230	1.437	1.568	1.810	1.891

Table 3.27 Regional growth factors of three distributions at different levels of LH-moments for subzone 2(b)

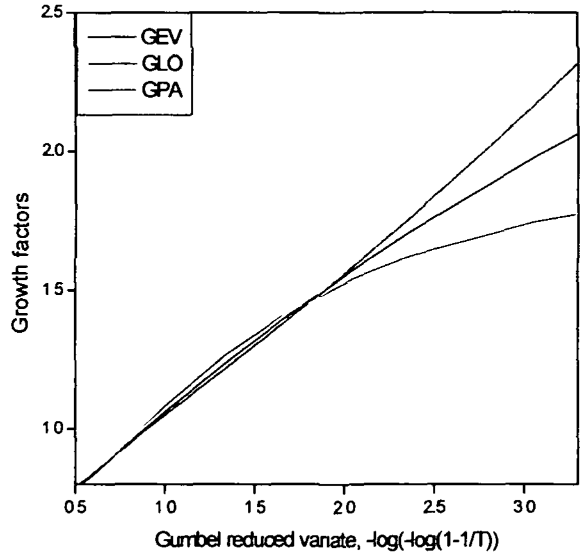
Distribution	L_η-moments	Return period in T years							
		2	5	10	20	50	100	500	1000
GEV	L_1	**0.790**	**1.059**	**1.233**	**1.396**	**1.603**	**1.754**	**2.093**	**2.233**
	L_2	0.722	0.969	1.127	1.274	1.458	1.591	1.884	2.004
	L_3	0.682	0.915	1.063	1.200	1.371	1.495	1.767	1.879
	L_4	0.652	0.878	1.020	1.151	1.314	1.431	1.686	1.789
GLO	L_1	0.798	1.051	1.217	1.383	1.614	1.801	2.289	2.526
	L_2	0.729	0.966	1.116	1.262	1.458	1.612	1.998	2.177
	L_3	0.685	0.915	1.056	1.192	1.371	1.510	1.847	2.000
	L_4	0.653	0.878	1.014	1.142	1.308	1.435	1.735	1.869
GPA	L_1	0.776	1.078	1.261	1.413	1.575	1.674	1.842	1.895
	L_2	0.713	0.976	1.145	1.290	1.452	1.555	1.744	1.807
	L_3	0.676	0.916	1.073	1.213	1.373	1.479	1.680	1.750
	L_4	0.652	0.877	1.027	1.161	1.317	1.421	1.623	1.695

Table 3.28 Regional growth factors of three distributions at different levels of LH-moments for subzone 2(c)

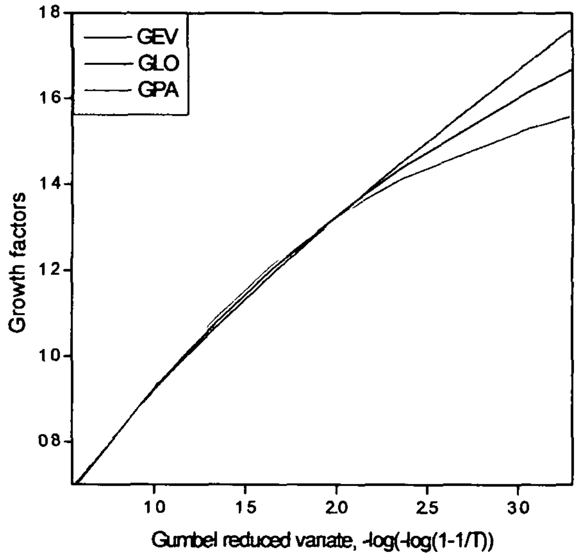
Distribution	L_η-moments	Return period in T years							
		2	5	10	20	50	100	500	1000
GEV	L_1	0.778	1.060	1.243	1.417	1.637	1.800	2.169	2.324
	L_2	**0.711**	**0.960**	**1.125**	**1.285**	**1.493**	**1.650**	**2.015**	**2.174**
	L_3	0.666	0.908	1.064	1.210	1.395	1.531	1.835	1.961
	L_4	0.626	0.875	1.027	1.164	1.327	1.441	1.678	1.770
GLO	L_1	0.788	1.051	1.225	1.400	1.645	1.845	2.370	2.626
	L_2	0.718	0.957	1.115	1.273	1.492	1.670	2.134	2.360
	L_3	0.670	0.907	1.056	1.200	1.392	1.542	1.913	2.084
	L_4	0.627	0.876	1.021	1.154	1.322	1.447	1.731	1.853
GPA	L_1	0.764	1.079	1.272	1.434	1.608	1.716	1.902	1.961
	L_2	0.702	0.965	1.142	1.302	1.490	1.616	1.863	1.952
	L_3	0.661	0.909	1.075	1.224	1.398	1.514	1.741	1.822
	L_4	0.624	0.874	1.034	1.172	1.328	1.427	1.610	1.672

The bold figures (in Table 3.25 to Table 3.28) represent the growth factors of the best fitting distribution

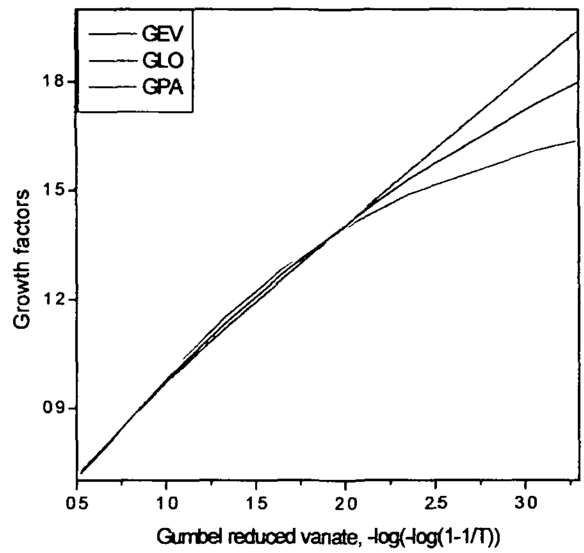
The estimated growth factors of each of the three probability distributions for each level of LH-moments are used for drawing the growth curves of the four homogeneous subzones of North-East India. The growth curves are shown in Figure 3.5 to Figure 3.8. It has been observed from Figure 3.5 that for subzone 2(a)A, the growth curves of GLO distribution is higher and GPA distribution is lower than the GEV distribution at all level of LH-moments i.e. (L_1- to L_4-moments). Furthermore, the growth curves of three distributions are identical for more years of return periods, with increase in the level of LH-moments. Similar case has been observed from Figure 3.6, Figure 3.7 and Figure 3.8 for subzones 2(a)B, 2(b) and 2(c), respectively.



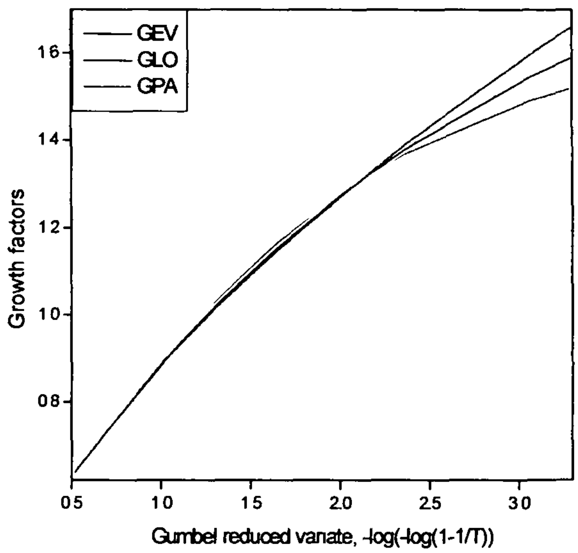
(a) Growth curves for L_1-moments



(c) Growth curves for L_3-moments

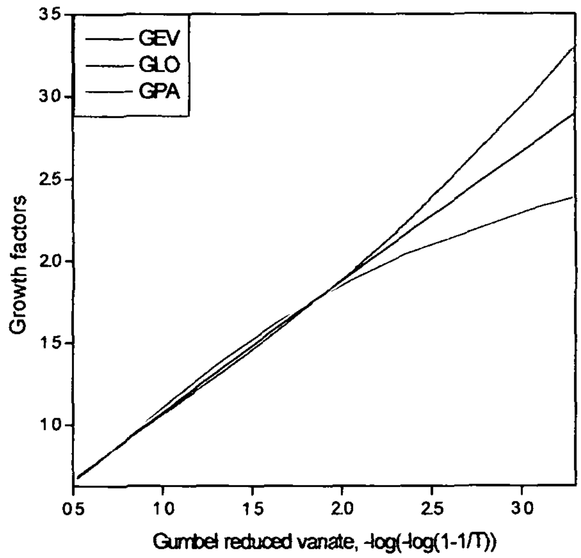


(b) Growth curves for L_2-moments

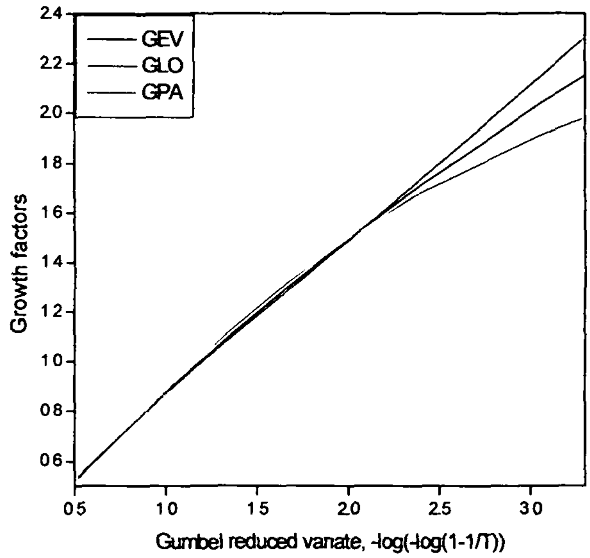


(d) Growth curves for L_4-moments

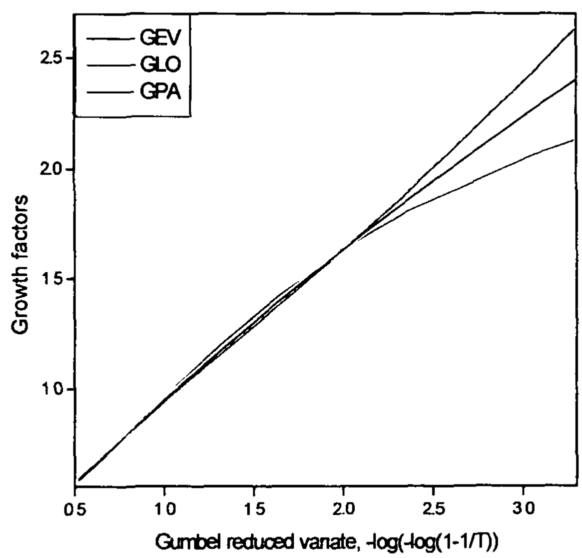
Figure 3.5 Growth curves of three distributions at different level of LH-moments for subzone 2(a)A



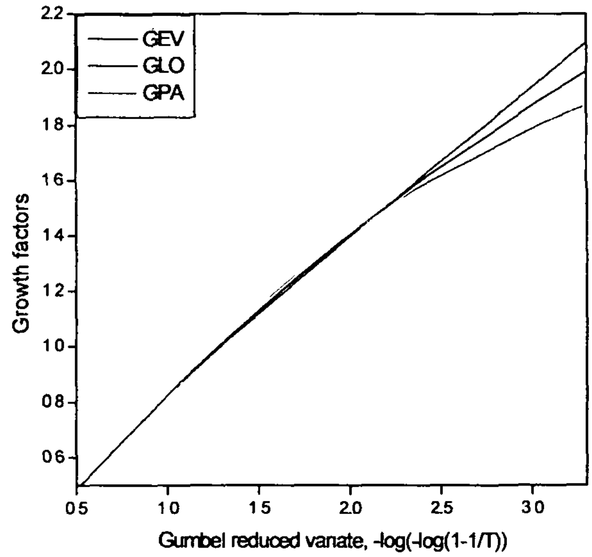
(a) Growth curves for L_1-moments



(c) Growth curves for L_3-moments

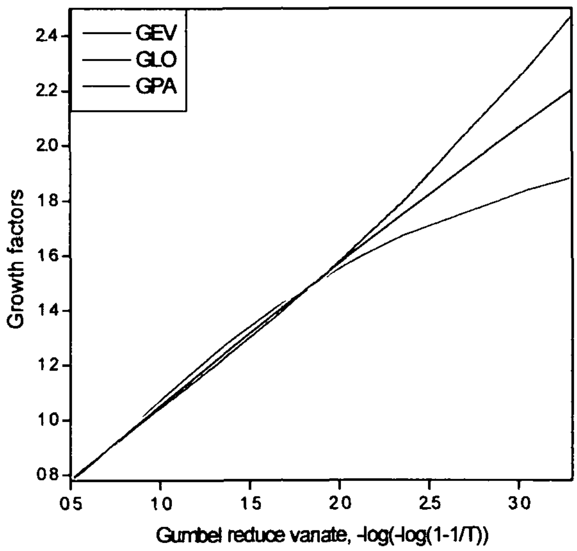


(b) Growth curves for L_2-moments

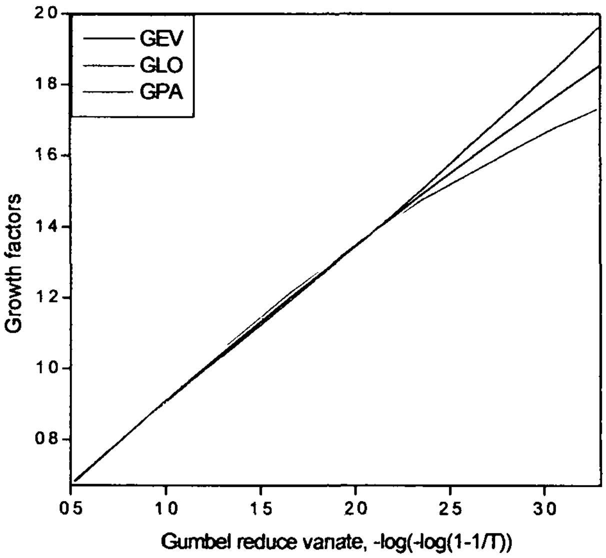


(d) Growth curves for L_4-moments

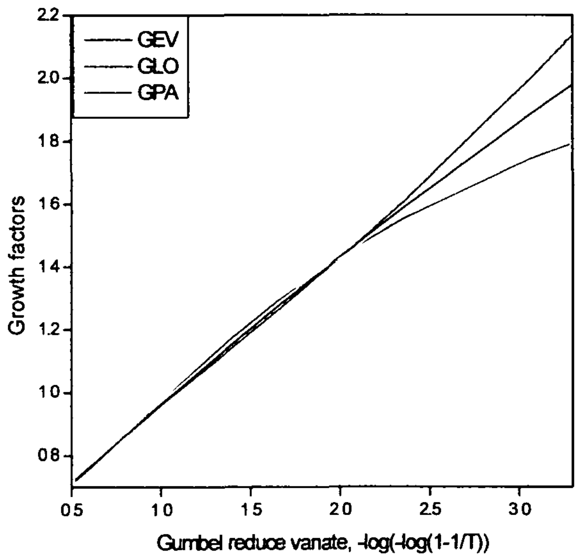
Figure 3.6 Growth curves of three distributions at different level of LH-moments for subzone 2(a)B



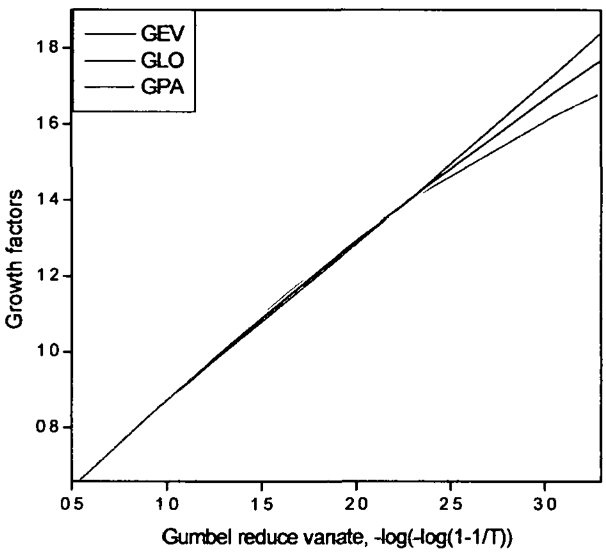
(a) Growth curves for L_1-moments



(c) Growth curves for L_3 -moments

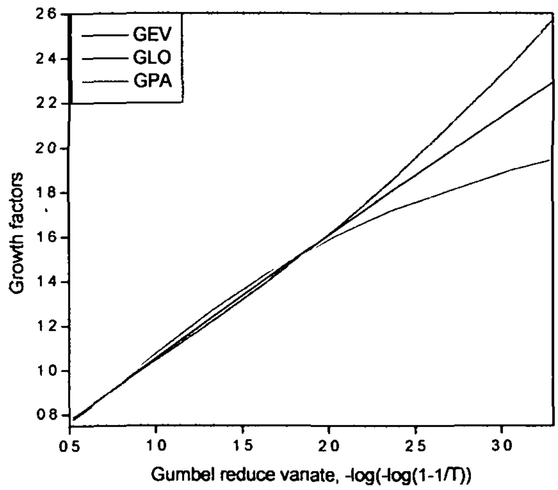


(b) Growth curves for L_2-moments

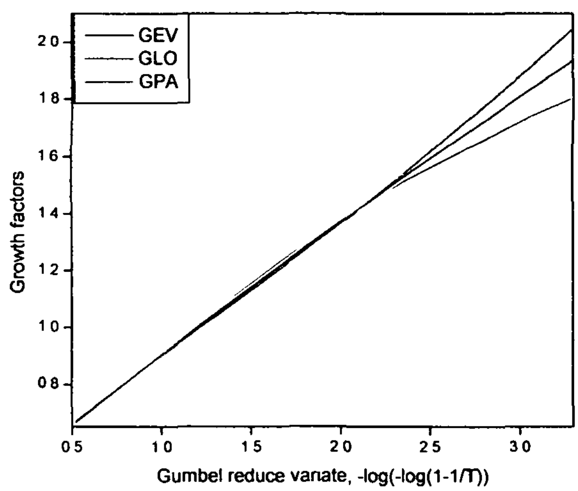


(d) Growth curves for L_4-moments

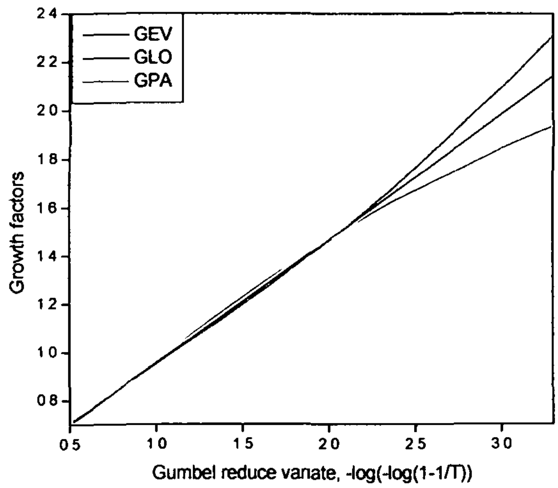
Figure 3.7 Growth curves of three distributions at different level of LH-moments for subzone 2(b)



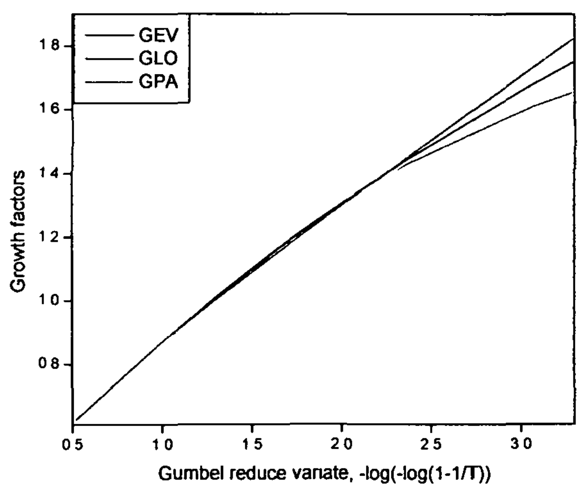
(a) Growth curves for L_1-moments



(c) Growth curves for L_3-moments



(b) Growth curves for L_2-moments



(d) Growth curves for L_4-moments

Figure 3.8 Growth curves of three distributions at different level of LH-moments for subzone 2(c)

3.3.5 Development of regional flood frequency relationships

The index flood procedure is used for development of regional flood frequency relationships, which are given below.

The quantile estimates $\hat{Q}_T^{\eta,i}$ for return period T, at site i, for $\eta = 1,2,3,4,...$ is given as

$$\hat{Q}_T^{\eta,i} = \hat{\lambda}_1^{\eta,i} * \hat{q}_T^{\eta} \tag{3.3.4}$$

where, $\hat{\lambda}_1^{\eta,i}$ is considered as index flood for site i and $\hat{q}_T^{\eta}$ is the regional quantile estimates for return period T at LH-moments level, $\eta = 1,2,3,4,....$

The regional flood frequency relationships for both gauged and ungauged catchments areas of four homogeneous subzones are discussed in the following sub-sections

(a) Gauged catchments

The GEV distributions with L_2-moments, L_4-moments, L_1-moments and L_2-moments has been identified as the best fitting distributions for subzones 2(a)A, 2(a)B, 2(b) and 2(c), respectively, therefore the regional flood frequency relationships have been developed by using this distribution. The regional flood frequency relationships for estimation of floods of various return periods for the gauged catchments of subzones 2(a)A, 2(a)B, 2(b) and 2(c) are given in equations (3.3.5), (3.3.6), (3.3.7) and (3.3.8), respectively and which are given below

$$\hat{Q}_T^{2,i} = \left[2.596 - 1.970 * \left\{ -\ln\left(1 - \frac{1}{T}\right) \right\}^{0\,134} \right] * \hat{\lambda}_1^{2,i} \tag{3.3.5}$$

$$\hat{Q}_T^{4,i} = \left[3.808 - 3.426 * \left\{ -\ln\left(1 - \frac{1}{T}\right) \right\}^{0\,094} \right] * \hat{\lambda}_1^{4,i} \tag{3.3.6}$$

$$\hat{Q}_T^{1,i} = \left[9.415 - 8.714 * \left\{ -\ln\left(1 - \frac{1}{T}\right) \right\}^{0\,028} \right] * \hat{\lambda}_1^{1,i} \tag{3.3.7}$$

$$Q_T^{2,i} = \left[-30.512 + 31.143 * \left\{ -\ln\left(1 - \frac{1}{T}\right) \right\}^{-0\,007} \right] * \hat{\lambda}_1^{2,i} \tag{3.3.8}$$

where $\hat{Q}_T^{2,i}$, $\hat{Q}_T^{4,i}$, $\hat{Q}_T^{1,i}$ and $\hat{Q}_T^{2,i}$ are flood (m^3/s) for T-year return period and $\hat{\lambda}_1^{2,i}, \hat{\lambda}_1^{4,i}, \hat{\lambda}_1^{1,i}$ and $\hat{\lambda}_1^{2,i}$ are the first sample L_2-, L_4- , L_1- and L_2-moments for site i of subzones 2(a)A, 2(a)B, 2(b) and 2(c), respectively.

For estimation of flood of desired return period for gauged catchments of study area above regional flood frequency relationships may be used. Alternatively, floods of various return periods for the homogeneous subzones 2(a)A, 2(a)B, 2(b) and 2(c) may also be obtained by multiplying the first sample LH-moments i.e. $\hat{\lambda}_1^{2,i}$ (Table 3.3 in Appendix-A), $\hat{\lambda}_1^{4,i}$ (Table 3.9 in Appendix-A), $\hat{\lambda}_1^{1,i}$ (Table 3.10 in Appendix-A) and $\hat{\lambda}_1^{2,i}$ (Table 3.15 in Appendix-A) of the catchment by the corresponding values of growth factors given in Table 3.25, Table 3.26, Table 3.27 and Table 3.28, respectively.

(b) Ungauged catchments

For ungauged catchments data are not available to find first sample LH-moments. In such a situation, a regional relationship between $\hat{\lambda}_1^{\eta,i}$ of gauged catchments and catchments areas (A_i) has been developed in log domain by using the least square approaches for all the four homogeneous subzones of North-East region. The developed relationships between first LH-moments and catchments areas of the homogeneous subzones 2(a)A, 2(a)B, 2(b) and 2(c) are given in equations (3.3.9)-(3.3.12), respectively.

$$\hat{\lambda}_1^{2,i} = 2.394 * (A_i)^{0.797} \tag{3.3.9}$$

$$\hat{\lambda}_1^{4,i} = 7.183 * (A_i)^{0.700} \tag{3.3.10}$$

$$\hat{\lambda}_1^{1,i} = 1.951 * (A_i)^{0.705} \tag{3.3.11}$$

$$\hat{\lambda}_1^{2,i} = 3.381 * (A_i)^{0.709} \tag{3.3.12}$$

where, A_i and $\hat{\lambda}_1^{\eta,i}$ ($\eta = 2,4,1,2$), are the catchments areas, in km^2 and the first sample LH-moments in m^3/s for site i of subzone 2(a)A, 2(a)B, 2(b) and 2(c), respectively. For these relationships, the correlation coefficients are, 0.844, 0.549, 0.872 and 0.936, respectively.

Finally, for development of the regional flood frequency relationships for estimation of floods of various return periods for ungauged catchments, the relationships given in equations (3.3.9)-(3.3.12) has been coupled with the regional relationships, given in equations (3.3.5)-(3.3.8), respectively. The following regional frequency relationships has been developed for subzones 2(a)A, 2(a)B, 2(b) and 2(c) and are given as follows

$$\hat{Q}_T^{2,i} = \left[6.215 - 4.716 * \left\{ -\ln\left(1 - \frac{1}{T}\right)\right\}^{0.134} \right] * (A_i)^{0.797} \tag{3.3.13}$$

$$\hat{Q}_T^{4,i} = \left[27.353 - 24.609 * \left\{ -\ln\left(1 - \frac{1}{T}\right)\right\}^{0.094} \right] * (A_i)^{0.700} \tag{3.3.14}$$

$$\hat{Q}_T^{1,i} = \left[18.369 - 17.001 * \left\{ -\ln\left(1 - \frac{1}{T}\right)\right\}^{0.028} \right] * (A_i)^{0.705} \tag{3.3.15}$$

$$\hat{Q}_T^{2,i} = \left[-103.161 + 105.295 * \left\{-\ln\left(1-\frac{1}{T}\right)\right\}^{-0.007}\right] * (A_i)^{0.709} \qquad (3.3.16)$$

3.4 Conclusion

The discordancy measure shows that there is no discordance site found at all the level of LH-moments, $L_\eta, \eta = 1,2,3,4$ for subzones 2(a)A, 2(a)B, 2(b) and 2(c), except the site Nonai of the subzone 2(a)A at L_1-moments. Though, the discordancy measure (D_i^1) is slightly higher than the critical value for this site, we include the site for further steps of regional flood frequency. The heterogeneity measures, $H_i^\eta, \eta = 1,2,3,4$ shows that all the four subzones are found to be homogeneous at all the level of LH-moments, $L_\eta, \eta = 1,2,3,4$. The LH-moment ratio diagram and $\left|Z_\eta^{DIST}\right|$-statistic criteria have been used as goodness of fit criteria for identifying the best fitting distribution for each of the subzone. It has been observed that the LH-moment ratio diagram is not sufficient to identify the best fitting distribution with level of LH-moment. Therefore, only the $\left|Z_\eta^{DIST}\right|$-statistic criteria is used to identify the best fitting probability distribution with level of LH-moment. It has been observed that the GEV distribution with L_2-, L_4-, L_1- and L_2-moments are identified as best fitting probability distribution with level of LH-moments for subzones 2(a)A, 2(a)B, 2(b) and 2(c), respectively. The regional relationships for gauged catchments of all the four homogeneous subzones of North-East India have been developed and are given in equations (3.3.5), (3.3.6), (3.3.7) and (3.3.8). Similarly, for ungauged catchments of these four homogeneous subzones also the regional relationships are developed and which are given in equations (3.3.13), (3.3.14), (3.3.15) and (3.3.16).

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Chapter 4

Regional Flood Frequency Analysis by Using LQ-moments

4.1 Introduction

In this chapter an attempt has been made for regional flood frequency analysis of four subzones, mentioned in previous chapters, by using LQ-moments. The linear quantile estimator as a sample quantile estimator and trimean as 'quick' estimator has been used in this study of regional flood frequency analysis. Five probability distributions viz. generalized extreme value (GEV), generalized logistic (GLO), generalized Pareto (GPA), generalized log-normal (GNO), and Pearson type III (PE3), which are generally used for regional flood frequency analysis by using L-moments, have been used in this study also. The theories of LQ-moments are given below before going to the regional flood frequency analysis of the study area.

4.2 LQ-moments

The r^{th} LQ-moments ζ_r of X proposed by Mudholkar and Hutson (1998) is given by

$$\zeta_r = r^{-1}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\tau_{p,\alpha}(X_{r-k\,r}), \qquad r = 1,2,..., \tag{4.2.1}$$

where $0 \le \alpha \le 1/2, 0 \le p \le 1/2$, and

$$\tau_{p,\alpha}(X_{r-k\,r}) = pQ_{X_{r-k\,r}}(\alpha)+(1-2p)Q_{X_{r-k\,r}}(1/2)+pQ_{X_{r-k\,r}}(1-\alpha) \tag{4.2.2}$$

The linear combination $\tau_{p,\alpha}$ defined above is a 'quick' measure of the location of the sampling distribution of order statistic $X_{r-k\,r}$. With appropriate combinations of

α and p, we can find estimators for $\tau_{p,\alpha}(\cdot)$, which are functions of commonly used estimators such as median, trimean and Gastwirth. In this study, we consider the trimean-based estimator, defined as

$$Q_{X_{r-k\,r}}(1/4)/4 + Q_{X_{r-k\,r}}(1/2)/2 + Q_{X_{r-k\,r}}(3/4)/4 \tag{4.2.3}$$

The first four LQ-moments of the random variable X is given by

$$\zeta_1 = \tau_{p,\alpha}(X) \tag{4.2.4}$$

$$\zeta_2 = \frac{1}{2}[\tau_{p,\alpha}(X_{22}) - \tau_{p,\alpha}(X_{12})] \tag{4.2.5}$$

$$\zeta_3 = \frac{1}{3}[\tau_{p,\alpha}(X_{33}) - 2\tau_{p,\alpha}(X_{23}) + \tau_{p,\alpha}(X_{13})] \tag{4.2.6}$$

$$\zeta_4 = \frac{1}{4}[\tau_{p,\alpha}(X_{44}) - 3\tau_{p,\alpha}(X_{34}) + 3\tau_{p,\alpha}(X_{24}) - \tau_{p,\alpha}(X_{14})] \tag{4.2.7}$$

The LQ-moments ratios (LQMRs) defined by Mudholkar and Hutson (1998) as:

$$\text{LQ-coefficient of variation, } \eta_2 = \zeta_2 / \zeta_1 \tag{4.2.8}$$

$$\text{LQ-skewness, } \eta_3 = \zeta_3 / \zeta_2 \tag{4.2.9}$$

$$\text{LQ-kurtosis, } \eta_4 = \zeta_4 / \zeta_2 \tag{4.2.10}$$

If $Q_X(.) = F_X^{-1}(.)$ is the quantile function of the random variable X then the 'quick' location measure (equation 4,2.2) defined by Mudholkar and Hutson (1998) as

$$\tau_{p,\alpha}(X_{r-k\,r}) = pQ_X[B_{r-k\,r}^{-1}(\alpha)] + (1-2p)Q_X[B_{r-k\,r}^{-1}(1/2)] + pQ_X[B_{r-k\,r}^{-1}(1-\alpha)] \tag{4.2.11}$$

where $B_{r-k\,r}^{-1}(\alpha)$ denotes the corresponding α^{th} quantile of a beta random variable with parameters $r-k$ and $k+1$.

The sample estimates of LQ-moments defined by Mudholkar and Hutson (1998) as follows:

Let $X_{1n} \le X_{2n} \le \ldots \le X_{nn}$ denote the sample order statistics, then the quantile estimator of $Q_X(u)$ is given by

$$\hat{Q}_X(u) = (1-\varepsilon)X_{[n'u]n} + \varepsilon X_{[n'u]+1\,n} \tag{4.2.12}$$

where, $\varepsilon = n'u - [n'u]$ and $n' = n+1$.

Thus for samples of size n, the r^{th} sample LQ-moment is given by

$$\hat{\zeta}_r = r^{-1}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} \hat{\tau}_{p,\alpha}(X_{r-k\,r}) \tag{4.2.13}$$

where $\hat{\tau}_{p,\alpha}(X_{r-k\,r})$, the quick estimator of the location for the distribution of $X_{r-k\,r}$ in a random sample of size r.

The first four sample LQ-moments are given by

$$\hat{\zeta}_1 = \hat{\tau}_{p,\alpha}(X) \tag{4.2.14}$$

$$\hat{\zeta}_2 = \frac{1}{2}[\hat{\tau}_{p,\alpha}(X_{22}) - \hat{\tau}_{p,\alpha}(X_{12})] \tag{4.2.15}$$

$$\hat{\zeta}_3 = \frac{1}{3}[\hat{\tau}_{p,\alpha}(X_{33}) - 2\hat{\tau}_{p,\alpha}(X_{23}) + \hat{\tau}_{p,\alpha}(X_{13})] \tag{4.2.16}$$

$$\hat{\zeta}_4 = \frac{1}{4}[\hat{\tau}_{p,\alpha}(X_{44}) - 3\hat{\tau}_{p,\alpha}(X_{34}) + 3\hat{\tau}_{p,\alpha}(X_{24}) - \hat{\tau}_{p,\alpha}(X_{14})] \tag{4.2.17}$$

where, the quick estimator $\hat{\tau}_{p,\alpha}(X_{r-k\,r})$ of the location of the order statistic $X_{r-k\,r}$ is given by

$$\begin{aligned}\hat{\tau}_{p,\alpha}(X_{r-k\,r}) &= p\hat{Q}_{X_{r-k\,r}}(\alpha) + (1-2p)\hat{Q}_{X_{r-k\,r}}(1/2) + p\hat{Q}_{X_{r-k\,r}}(1-\alpha) \\ &= p\hat{Q}_X[B^{-1}_{r-k\,r}(\alpha)] + (1-2p)\hat{Q}_X[B^{-1}_{r-k\,r}(1/2)] + p\hat{Q}_X[B^{-1}_{r-k\,r}(1-\alpha)]\end{aligned} \tag{4.2.18}$$

$0 \le \alpha \le 1/2, 0 \le p \le 1/2, B^{-1}_{r-k\,r}(\alpha)$ is the α^{th} quantile of beta random variable with parameters $r-k$ and $k+1$, and $\hat{Q}_X(.)$ denotes the linear interpolation estimator given by equation (4.2.12)

The sample LQ-moment ratios can be defined analogous to LQ-moment ratios defined above as

$$\hat{\eta}_2 = \hat{\zeta}_2 / \hat{\zeta}_1 \text{ , } \hat{\eta}_3 = \hat{\zeta}_3 / \hat{\zeta}_2 \text{ and } \hat{\eta}_4 = \hat{\zeta}_4 / \hat{\zeta}_3$$

4.3 Regional flood frequency analysis

The index flood procedure based on L-moments discussed in chapter 2 (Sec. 2.4) has been used here also by replacing index flood as first LQ-moment i.e. $\hat{\zeta}_1$, instead of mean. The steps involved in the regional flood frequency analysis by using L-moments and LH-moments discussed in previous chapters are also used here for LQ-moments based regional flood frequency analysis and the details are given in the following sections.

For all calculations computer programs (in Fortran-77) have been developed and the subroutines used for these programs are given in Appendix-B.

4.3.1 Screening of data

For screening of data, a discordancy measure (D_i^{LQ}) based on LQ-moments is used to recognize those sites that are grossly discordant with the group as a whole and can be defined as follows

$$D_i^{LQ} = \frac{1}{3} N (u_i^{LQ} - \bar{u}^{LQ})^T S_{LQ}^{-1} (u_i^{LQ} - \bar{u}^{LQ}) \qquad (4.3.1)$$

where, $u_i^{LQ} = [\hat{\eta}_2^{(i)} \quad \hat{\eta}_3^{(i)} \quad \hat{\eta}_4^{(i)}]^T$ be a vector containing the sample LQMRs $\hat{\eta}_2^{(i)}$, $\hat{\eta}_3^{(i)}$ and $\hat{\eta}_4^{(i)}$ for the site i; $\bar{u}^{LQ}$ be the group average and S_{LQ} is a matrix of sums and cross product defined similarly as equations (2.5.1) and (2.5.2), respectively.

The same critical values suggested by Hosking and Wallis (1997) for L-moments are used here for LQ-moments.

The sample LQ-moments and discordancy measures (D_i^{LQ}) of all the sites of 2(a)A, 2(a)B, 2(b) and 2(c) subzones are given in Table 4.1 to Table 4.4 respectively. It has been observed from the discordancy measures of each site of the four sub-zones that there is no discordance site found for subzones 2(a)A, 2(a)B, 2(b) and 2(c). Therefore, all sites of each of the four subzones can be considered for further steps of regional frequency analysis procedure based on LQ-moment.

Table 4.1 Name of the sites, sample size, sample LQ-moment statistics and discordancy measures of subzone 2(a)A

Name of sites	Sample size	$\hat{\xi}_1$	$\hat{\eta}_2$	$\hat{\eta}_3$	$\hat{\eta}_4$	D_i^{LQ}
Manas	17	5965.56	0.1739	0.1437	0.1008	0.24
Nanoi	11	91.32	0.2158	0.1580	0.2021	0.07
Borolia	15	194.22	0.2540	-0.0345	0.1656	1.92
Dhansiri	21	1275.50	0.1715	0.2039	0.3430	0.57
Jiabhoroli	36	4015.77	0.2607	0.0856	0.0412	0.52
Subansiri	27	8498.75	0.1777	0.2060	0.1198	0.43
Sankush	12	1865.99	0.1418	0.0703	-0.0942	1.22
Champamati	22	746.67	0.2055	0.3715	0.5916	1.61
NoaNadi	13	40.88	0.1989	-0.1080	-0.2222	1.22
Ranganadi	19	912.49	0.2803	0.1461	-0.1305	2.20

Table 4.2 Name of the sites, sample size, sample LQ-moment statistics and discordancy measures of subzone 2(a)B

Name of sites	Sample size	$\hat{\zeta}_1$	$\hat{\eta}_2$	$\hat{\eta}_3$	$\hat{\eta}_4$	D_i^{LQ}
Puthimari	37	504.78	0.3905	0.2396	-0.0275	0.26
Pagladia	35	580.23	0.3812	0.2150	0.1329	0.12
Pachnoi	22	196.82	0.2930	0.2915	0.1767	0.89
Belsiri	23	269.36	0.3397	0.3096	0.0990	0.48
Gabharu	15	223.91	0.5416	0.2571	-0.3114	1.90
Beki	13	748.60	0.2957	-0.0512	0.1490	1.50
Gaurang	17	921.87	0.3647	0.1937	0.3134	1.18
Ghiladhari	20	62.45	0.3345	0.5547	0.4735	1.69

Table 4.3 Name of the sites, sample size, sample LQ-moment statistics and discordancy measures of subzone 2(b)

Name of sites	Sample size	$\hat{\zeta}_1$	$\hat{\eta}_2$	$\hat{\eta}_3$	$\hat{\eta}_4$	D_i^{LQ}
Dikhow	26	720.34	0.1711	0.0896	0.0038	0.96
Jhanji	13	158.86	0.2833	0.2662	-0.0601	0.57
Bhogdoi	13	196.08	0.2964	0.2793	-0.0550	0.90
Dhansiri	29	1106.94	0.2013	0.1738	0.2057	0.08
Kapili	26	1160.60	0.2223	0.5137	0.1677	1.49
Kulsi	24	105.19	0.1325	0.0870	0.5892	1.74
Krishnai	19	482.03	0.2011	-0.1426	-0.1534	1.25

Table 4.4 Name of the sites, sample size, sample LQ-moment statistics and discordancy measures of subzone 2(c)

Name of sites	Sample size	$\hat{\zeta}_1$	$\hat{\eta}_2$	$\hat{\eta}_3$	$\hat{\eta}_4$	D_i^{LQ}
Barak	11	3560.50	0.1699	0.5653	0.4754	1.38
Dhaleshwari	16	647.82	0.1137	-0.2516	0.6703	1.72
Khowai	19	265.58	0.3457	0.2862	0.1631	1.78
Dhalai	11	179.46	0.1945	0.2795	01408	0.55
Gumti	24	420.77	0.2281	-0.0812	0.2106	0.28
Muhuri	28	343.20	0.2259	0.3035	0.1299	0.32
Manu	12	713.20	0.1942	-0.1946	0.0848	0.97

4.3.2 Identification of homogeneous region

The heterogeneity measure ($H_i^{LQ}, i = 1,2,3$) for LQ-moments has been used for test the regional homogeneity of a region. In similar way as defined for L-moment and LH-moments, the heterogeneity measure (H_i^{LQ}) for LQ-moments can be defined as follows

$$H_i^{LQ} = \frac{V_i^{LQ} - \mu_{V_i^{LQ}}}{\sigma_{V_i^{LQ}}},\ i = 1,2,3 \tag{4.3.2}$$

where, $V_i^{LQ}(i = 1,2,3)$, $\mu_{V_i^{LQ}}$ and $\sigma_{V_i^{LQ}}$ are similar as for L-moments and can be obtained by replacing L-moments with LQ-moments.

For this purpose, the LQ-moments and the parameters of the Kappa distribution based on it have been developed. The values of heterogeneity measures are computed by carrying out 500 simulations using this Kappa distribution. The heterogeneity measures of all the four subzones are given in Table 4.5. It has been observed from heterogeneity measures that all the four subzones i.e. 2(a)A, 2(a)B, 2(b) and 2(c) are found to be homogeneous for LQ-moment based regional frequency analysis.

Table 4.5 Heterogeneity measures based on LQ-moments for all the four subzones

Name of subzones	H_1^{LQ}	H_2^{LQ}	H_3^{LQ}
2(a)A	0.03	-1.00	0.08
2(a)B	-0.10	-1.01	-0.25
2(b)	0.95	0.73	1.65
2(c)	0.77	2.16	1.34

4.3.3 Selection of best fitting probability distribution

(a) Z_{LQ}^{DIST}-Statistic criteria

The Z_{LQ}^{DIST}-statistic for LQ-moments can be defined similarly as defined for L-moments and LH-moments. The formula for calculating Z_{LQ}^{DIST}-statistic can be written as follows

$$Z_{LQ}^{DIST} = (\eta_4^{DIST} - \hat{\eta}_4^R + B_4^{LQ}) / \sigma_4^{LQ} \tag{4.3.3}$$

where, $\hat{\eta}_4^R$ is the regional average value (given in Table 4.6) of $\hat{\eta}_4^{(i)}$ obtained from the data of a given region, the B_4^{LQ}, σ_4^{LQ} are can be defined similarly as equations (2.5.12) and (2.5.13) by replacing L-moment with LQ-moment and η_4^{DIST} are the LQ-kurtosis values for a particular probability distribution for which test statistics applied.

For estimating the η_4^{DIST} values of various distributions, a relationship between η_3 and η_4 have been developed and which are given below.

The regional average LQ-coefficient of variation, LQ-skewness and LQ-kurtosis of all the four subzones are given in Table 4.6

Table 4.6 Regional average LQ-moment ratios of four subzones

Name of subzones	$\hat{\eta}_2^R$	$\hat{\eta}_3^R$	$\hat{\eta}_4^R$
2(a)A	0.2127	0.1398	0.1288
2(a)B	0.3676	0.2610	0.1201
2(b)	0.2040	0.1813	0.1344
2(c)	0.2192	0.1233	0.2505

The relationship between η_3 and η_4 developed for the probability distributions used for our study are given as follows

$$\eta_4^{GEV} = 0.1080 + 0.1131\eta_3 + 0.8178\eta_3^2 - 0.0330\eta_3^3 - 0.0087\eta_3^4 + 0.0064\eta_3^5 - 0.0056\eta_3^6$$
$$\eta_4^{GLO} = 0.1585 + 0.8189\eta_3^2 - 0.0118\eta_3^4 - 0.0037\eta_3^6$$
$$\eta_4^{GPA} = -0.0020 + 0.2229\eta_3 + 0.8626\eta_3^2 - 0.0751\eta_3^3 - 0.0106\eta_3^4 - 0.0013\eta_3^5 - 0.0064\eta_3^6 + 0.0117\eta_3^7$$
$$\eta_4^{GNO} = 0.1202 + 0.7929\eta_3^2 - 0.0044\eta_3^4 - 0.0064\eta_3^6$$
$$\eta_4^{PE3} = 0.1232 - 0.1224\eta_3 + 1.3324\eta_3^2 - 2.3445\eta_3^3 + 2.0100\eta_3^4$$

The $|Z_{LQ}^{DIST}|$-statistic of various three parameter distributions used in our study are calculated for each of the four homogeneous subzones of North-East India and are given in Table 4.7.

Table 4.7 The $|Z_{LQ}^{DIST}|$-statistics of various distributions for four subzones

Distributions	$\|Z_{LQ}^{DIST}\|$			
	2(a)A	2(a)B	2(b)	2(c)
GLO	1.51	2.17	1.35	**1.42**
GEV	0.85	1.79	0.87	1.90
GNO	0.77	1.48	0.73	1.93
PE3	**0.68**	1.05	**0.41**	2.03
GPA	0.91	**0.43**	0.55	3.15

The bold figures represent the lowest $|Z_{LQ}^{DIST}|$-statistic

It has been observed from Table 4.7 that for subzones 2(a)A and 2(b), the $|Z_{LQ}^{DIST}|$-statistic values of all the five distributions are less than the critical value 1.64. However, the $|Z_{LQ}^{DIST}|$-statistic values are found to be the lowest for PE3 distribution than all other distributions for subzones 2(a)A and 2(b), respectively. Therefore, the PE3 distribution can be identified as the best fitting distributions for subzones 2(a)A and 2(b) respectively. For subzone 2(a)B, the $|Z_{LQ}^{DIST}|$-statistic values of GNO, PE3 and GPA have been found to be less than the critical value. But out of these three distributions GPA distribution has the lowest value, therefore GPA distribution can be considered as the best fitting distribution for this sub-zone. Similarly, for subzone 2(c), it has been observed that the $|Z_{LQ}^{DIST}|$-statistic value of only the GLO distribution is less than the critical value out of five distributions used for our study. Therefore, the GLO distribution can be identified as the best fitting distribution for this subzone.

(b) LQ-moment ratio diagram

The LQ-moment ratio diagram is nothing but a plot between LQ-skewness and LQ-kurtosis like L-moment and LH-moments ratio diagram. The relationships given above, between η_3 and η_4 for five probability distributions have been used for drawing the theoretical curves in LQ-moment ratio diagram. The LQ-moment ratio diagrams for all the four subzones of North-East India are given in Figure 4.1 to Figure 4.4. It has been observed from LQ-moment ratio diagrams for subzones 2(a)A (Figure 4.1), 2(a)B (Figure 4.2), 2(b) (Figure 4.3) and 2(c) (Figure 4.4) that the regional average LQ-skewness ($\hat{\eta}_3^R$) and LQ-kurtosis ($\hat{\eta}_4^R$), lies closest to PE3, GPA, PE3 and GLO distributions respectively.

Thus, the $|Z_{LQ}^{DIST}|$-statistic criteria as well as LQ-moment ratio diagram shows that PE3 distribution is the best fitting distribution for subzone 2(a)A. Similarly for subzones 2(a)B, 2(b) and 2(c), it has been observed from the $|Z_{LQ}^{DIST}|$-statistic criteria as well as LQ-moment ratio diagram that the GPA, PE3 and GLO distributions, respectively are the best fitting distributions for these subzones.

the homogeneous regions. The regional quantiles or regional growth factors of each of the five distributions for each of the four homogeneous subzones are given in Table 4.9 to Table 4.12. Again, these regional quantiles given for return periods 2 to 1000 years can be used for drawing the regional growth curves of various distributions for four homogeneous subzones. The regional growth curves of each of the five distributions for each of the four homogeneous subzones are shown in Figure 4.5 to Figure 4.8.

It has been observed from Figure 4.5 and Figure 4.7 that the growth curves of GEV, GLO and GNO distributions are higher and GPA distribution is lower than the growth curve of best fitting PE3 distribution for subzones 2(a)A and 2(b), respectively. Again, from Figure 4.6, it has been observed that the growth curves of GEV, GLO, GNO and PE3 distributions are higher than the growth curve of best fitting GPA distribution for subzone 2(a)B. For subzone 2(c), it has been observed from Figure 4.8 that the growth curves of GEV, GNO, PE3 and GPA distributions are lower than the growth curve of best fitting GLO distribution.

Table 4.8 Regional parameters of various distributions for four subzones

Name of the subzones	Distribution	Regional parameters		
		Location	scale	shape
2(a)A	GEV	0.864	0.333	0.023
	GLO	0.987	0.232	-0.186
	GPA	0.698	0.511	0.573
	GNO	0.986	0.382	-0.326
	PE3	**1.048**	**0.405**	**0.953**
2(a)B	GEV	0.764	0.507	-0.206
	GLO	0.959	0.388	-0.348
	GPA	**0.329**	**0.954**	**0.177**
	GNO	0.955	0.638	-0.610
	PE3	1.158	0.775	1.718
2(b)	GEV	0.869	0.307	-0.057
	GLO	0.984	0.220	-0.242
	GPA	0.588	0.650	0.411
	GNO	0.982	0.363	-0.423
	PE3	**1.060**	**0.400**	**1.224**
2(c)	GEV	0.860	0.349	0.055
	GLO	**0.988**	**0.239**	**-0.164**
	GPA	0.523	0.812	0.590
	GNO	0.987	0.395	-0.287
	PE3	1.044	0.413	0.843

The bold figures represent regional parameters of best fitting distributions

Table 4.9 Regional growth factors of various distributions for subzone 2(a)A

Distributions	Return periods (in years)							
	2	5	10	20	50	100	500	1000
GEV	0.986	1.355	1.594	1.820	2.107	2.318	2.792	2.991
GLO	0.987	1.354	1.617	1.897	2.312	2.672	3.701	4.247
GPA	0.990	1.235	1.351	1.430	1.495	1.526	1.564	1.573
GNO	0.986	1.356	1.594	1.817	2.103	2.316	2.809	3.023
PE3	**0.985**	**1.357**	**1.591**	**1.805**	**2.069**	**2.260**	**2.681**	**2.856**

Table 4.10 Regional growth factors of various distributions for subzone 2(a)B

Distributions	Return periods (in years)							
	2	5	10	20	50	100	500	1000
GEV	0.957	1.655	2.215	2.841	3.801	4.652	7.155	8.514
GLO	0.959	1.650	2.239	2.950	4.164	5.361	9.531	12.178
GPA	**0.951**	**1.665**	**2.133**	**2.547**	**3.022**	**3.333**	**3.925**	**4.132**
GNO	0.955	1.657	2.195	2.762	3.570	4.232	5.962	6.798
PE3	0.948	1.667	2.183	2.688	3.344	3.835	4.963	5.445

Table 4.11 Regional growth factors of various distributions for subzone 2(b)

Distributions	Return periods (in years)							
	2	5	10	20	50	100	500	1000
GEV	0.983	1.350	1.606	1.863	2.211	2.484	3.158	3.468
GLO	0.984	1.346	1.622	1.929	2.406	2.839	4.163	4.911
GPA	0.980	1.353	1.556	1.708	1.853	1.931	2.047	2.077
GNO	0.982	1.349	1.600	1.845	2.170	2.420	3.023	3.295
PE3	**0.980**	**1.352**	**1.596**	**1.825**	**2.114**	**2.326**	**2.800**	**2.999**

Table 4.12 Regional growth factors of various distributions for subzone 2(c)

Distributions	Return periods (in years)							
	2	5	10	20	50	100	500	1000
GEV	0.987	1.362	1.599	1.816	2.086	2.278	2.697	2.866
GLO	**0.988**	**1.360**	**1.620**	**1.893**	**2.290**	**2.627**	**3.568**	**4.054**
GPA	0.985	1.367	1.546	1.664	1.762	1.808	1.864	1.876
GNO	0.987	1.363	1.599	1.817	2.092	2.294	2.755	2.952
PE3	0.987	1.364	1.596	1.807	2.065	2.250	2.655	2.823

The bold figures (in Table 4 9 to Table 4 12) represent growth factors of best fitting distributions

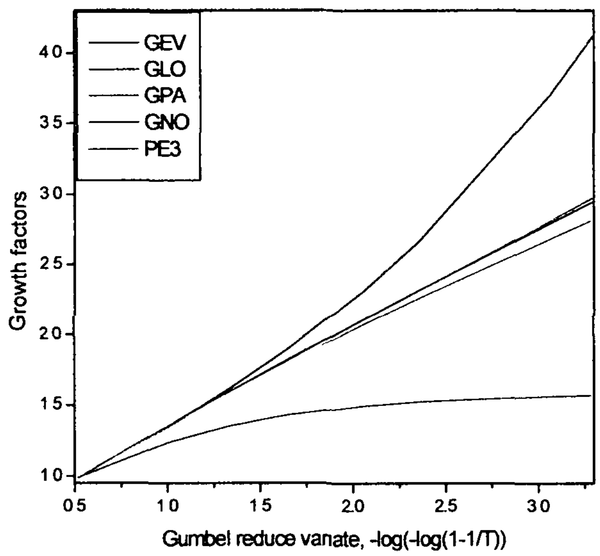

Figure 4.5 Growth curves of various distributions for subzone 2(a)A

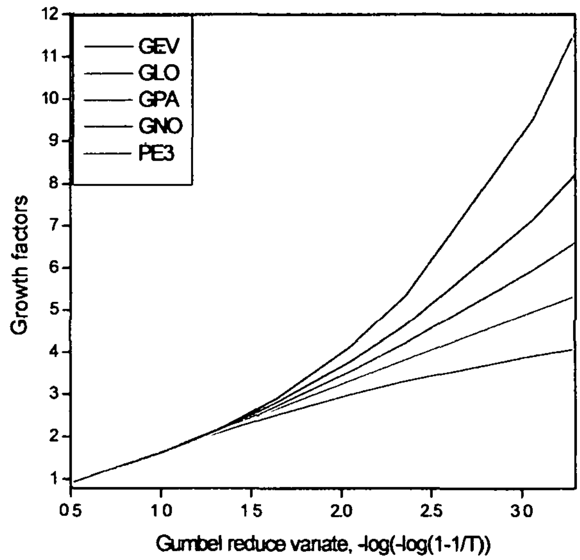

Figure 4.6 Growth curves of various distributions for subzone 2(a)B

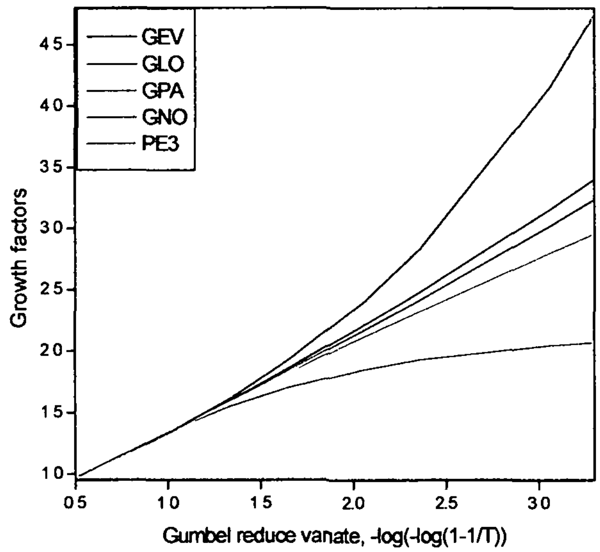

Figure 4.7 Growth curves of various distributions for subzone 2(b)

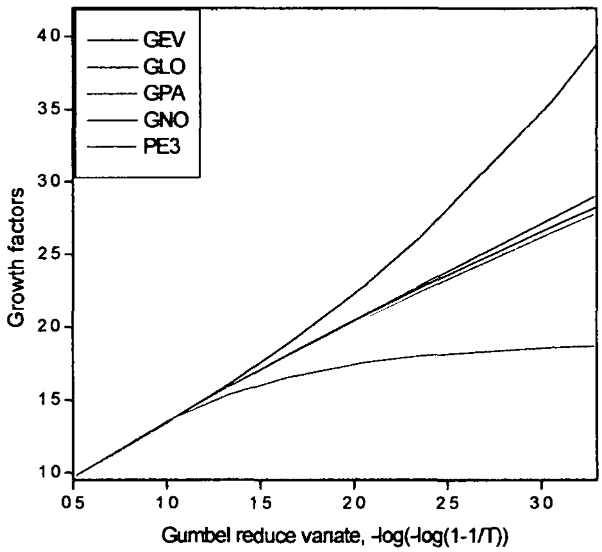

Figure 4.8 Growth curves of various distributions for subzone 2(c)

4.3.5 Development of regional flood frequency relationships

The index flood procedure mentioned in chapter 2 has been used for development of regional flood frequency relationships in which the index flood considered here as the first LQ-moment instead of mean. The quantile of a site i for T years return periods denoted by, $\hat{Q}'_T$, can be estimates by the equation given below

$$\hat{Q}'_T = \hat{\zeta}'_1 * \hat{q}_T \tag{4.3.4}$$

where, $\hat{\zeta}'_1$ is the first LQ-moment of site i and $\hat{q}_T$ is the regional quantile estimates or regional growth factor at return period T.

(a) Gauged catchments

The regional flood frequency relationship for gauged catchments is developed by using the identified best fitting distributions for each of the four homogeneous subzones of the study area. The PE3 distribution has been identified as the best fitting distribution for subzones 2(a)A and 2(b) respectively. The floods of various return periods T for gauged catchments areas of subzones 2(a)A and 2(b) may be computed by multiplying $\hat{\zeta}'_1$ (first LQ-moments) of a catchments by the corresponding values of growth factors of PE3 distributions which are given in Table 4.9 and Table 4.11, respectively.

For subzone 2(a)B, the identified GPA distribution can be used for development of regional flood frequency relationship of gauged catchments, which can be written as

$$\hat{Q}'_T = \left\{5.719 - 5.390 * \left(\frac{1}{T}\right)^{0.177}\right\} * \hat{\zeta}'_1 \tag{4.3.5}$$

Similarly, for subzone 2(c), the identified best fitting GLO distribution can be used for development of regional flood frequency relationships for gauged catchments and which can be written as

$$\hat{Q}'_T = \left\{-0.469 + 1.457 * \left(\frac{1}{T-1}\right)^{-0.164}\right\} * \hat{\zeta}'_1 \tag{4.3.6}$$

where, $\hat{Q}'_T$ and $\hat{\zeta}'_1$ in equations (4.3.5) and (4.3.6) are the floods (in m^3/s) and first sample LQ-moments at site i, for subzones 2(a)B and 2(c), respectively.

Alternatively, for subzones 2(a)B and 2(c) also, the floods of various return periods may also be obtained by multiplying the first LQ-moments of the catchments (given in Table 4.2 and Table 4.4, respectively) by the corresponding values of the regional growth factors given in Table 4.10 and Table 4.12, respectively.

(b) Ungauged catchments

In this case a relationship between the first LQ-moment of gauged catchments in the region and their physiographic catchments characteristic is developed, which is used to estimate the first LQ-moments for an ungauged site. The relationship developed for the subzones in log domain using least squares approach based on the data of the subzones 2(a)A, 2(a)B, 2(b) and 2(c), are given in following equations.

$$\hat{\zeta}_1^i = 1.688 * (A_i)^{0.804} \tag{4.3.7}$$

$$\hat{\zeta}_1^i = 2.626 * (A_i)^{0.752} \tag{4.3.8}$$

$$\hat{\zeta}_1^i = 1.669 * (A_i)^{0.697} \tag{4.3.9}$$

$$\hat{\zeta}_1^i = 3.214 * (A_i)^{0.673} \tag{4.3.10}$$

where, A_i is the catchments area, in sq. km. and $\hat{\zeta}_1^i$ is the first LQ-moments in m^3/s at site i. For the relations given in equations (4.3.7), (4.3.8), (4.3.9) and (4.3.10), the correlation coefficients are, 0.857, 0.561, 0.864 and 0.888, respectively.

Now by coupling the regional flood frequency relationships for gauged catchments and the relationships between first LQ-moments and catchments areas given by equations (4.3.7) to (4.3.10), we obtained the regional flood frequency relationships for ungauged catchments of four subzones. The quantile estimates $\hat{Q}_T^i$ with return period T at ungauged catchments i of the four subzones 2(a)A, 2(a)B, 2(b) and 2(c) are given as follows

$$\hat{Q}_T^i = \hat{C}_T * (A_i)^{0.804} \tag{4.3.11}$$

$$\hat{Q}_T^i = \left\{15.018 - 14.154 * \left(\frac{1}{T}\right)^{0.177}\right\} * (A_i)^{0.752} \tag{4.3.12}$$

$$\hat{Q}_T^i = \hat{C}_T * (A_i)^{0.697} \tag{4.3.13}$$

$$\hat{Q}_T^i = \left\{-1.507 + 4.683 * \left(\frac{1}{T-1}\right)^{-0.164}\right\} * (A_i)^{0.673} \tag{4.3.14}$$

where, $\hat{C}_T$ is the regional coefficient at return period T for subzones 2(a)A and 2(b), and A_i is catchment area in km^2 at ungauged site i, for all the four subzones.

The values of $\hat{C}_T$ for some of the commonly used return periods for subzones 2(a)A and 2(b) are given in Table 4.13.

Table 4.13 Values of regional coefficient, $\hat{C}_T$ for PE3 distribution

Subzones	Return periods (in years)							
	2	5	10	20	50	100	500	1000
	Growth factors							
2(a)A	1.663	2.291	2.686	3.047	3.492	3.815	4.526	4.821
2(b)	1.636	2.256	2.664	3.046	3.528	3.882	4.673	5.005

4.4 Conclusion

The following conclusion can be drawn from our studies which are out lines below.

From discordancy measure, it has been observed that no sites found to be discordant for all the subzones of North-East India. Again, from heterogeneity measure, it has been observed that all subzones are found to be homogeneous. Therefore the regional flood frequency analysis has been carried out for all the four subzones by considering all sites which are considered in previous chapters of our study. The selection of best fitting probability distributions for each of the homogeneous subzones has been made by using two goodness of fit test one is Z_{LQ}^{DIST} -statistic criteria and another one is LQ-moment ratio diagram. It has been observed from goodness of fit tests that the PE3 distribution is the best fitting distribution for two subzones 2(a)A and 2(b). Again the GPA and GLO distributions are identifies as the best fitting distributions for subzones 2(a)B and 2(c), respectively. We have seen from our study that for this method also the same distributions are identified as best fitting distributions for each of the four subzones of North-East India, like the method of L-moments discussed in chapter 2 with having different Z-statistic values. The regional parameters and regional growth factors of each of the five probability distributions are calculated for each of the four homogeneous subzones (given in Table 4.8 to Table 4.12). In last section of this chapter the regional flood frequency relationships for both the gauged as well as ungauged catchments areas are developed and which can be used for estimated the floods of various return periods for both gauged and ungauged catchments areas of these four subzones of North-East India.

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Chapter 5

Comparative Studies between the Estimation Methods

5.1 Introduction

In this chapter, two comparative studies one between L- and LH-moments and another between L- and LQ-moments has been performed for regional flood frequency analysis of our study area. Various measures are available for comparative analysis of parameter estimation methods in statistical literature. In some recent study of hydrological modeling, Monte Carlo simulation method is used for relative bias (RBIAS) and relative root mean square error (RRMSE) calculations. Hosking and Wallis (1997) used RBIAS and RRMSE in presence of Monte Carlo simulation method in assessment of accuracy measure of regional quantile estimates based on L-moments. Again, both these methods are used for comparative study between L-moment and LH-moments by Meshgi and Khalili (2009b). In their study they also used box plots of both RBIAS and RRMSE values as a graphical tool for selection of probability distribution with the level of LH-moments. In both the study they generate some random samples by using random number generator programs available in most of the computer software. The details of the comparative studies are discussed in the following sections.

5.2 L -moments and LH-moments

For comparative studies between the parameter estimation methods of a region the homogeneity of the region is one of the most essential criteria. It has been observed from chapter 2 and chapter 3 that the four subzones i.e. 2(a)A, 2(a)B, 2(b) and 2(c) are found to be homogeneous for L-moments as well as for LH-moments (i.e.

L_1- to L_4-moments), therefore a comparative study can be performed for all the four subzones between the method of L- and LH-moments (i.e. from L_1 to L_4). The $|Z^{DIST}|$- and $|Z_{\eta}^{DIST}|$-statistic values have been used for selecting the best fitting distribution with method of parameter estimation for regional frequency analysis of these four homogeneous subzones. Since, the three probability distributions i.e. GEV, GLO and GPA have been used for LH-moments based regional frequency analysis, therefore comparative study between the methods of L- and LH-moments can be performed only for these three distributions. By comparing, the $|Z^{DIST}|$ and $|Z_{\eta}^{DIST}|,(\eta = 1,2,3,4)$-statistic values (see Table 2.6 and Table 3.20) of three distributions, for four homogeneous subzones of North-East India, it has been observed that the $|Z_2^{GEV}|$, $|Z_4^{GEV}|$ and $|Z_1^{GEV}|$-statistic values are lowest among all other values for subzones 2(a)A, 2(a)B and 2(b), respectively. Therefore, the GEV distribution with L_2-, L_4-, and L_1-moments is the best fitting distribution with levels of LH-moments for subzones 2(a)A, 2(a)B and 2(b), respectively. But in case of subzone 2(c), it has been observed that GLO distribution at L-moments attains the minimum Z-statistic value. Therefore, the GLO distribution with L-moments as parameter estimation method is identified as the best fitting distribution for this subzone.

5.3 L-moments and LQ-moments

The four homogeneous subzones of North-East India for the method of L-moments were again found to be homogeneous for the method of LQ-moments also; therefore a comparative study can be performed between these two methods for regional flood frequency analysis of these four subzones. For this purpose, the $|Z^{DIST}|$ and $|Z_{LQ}^{DIST}|$-statistic values of five probability distributions (see in Table 2.6 and Table 4.7), for four homogeneous subzones have been used. It has been observed from $|Z|$-statistic values for both the methods that the PE3 distribution receives the lowest $|Z|$-statistic value for the method of L-moments for subzones 2(a)A and 2(b). Therefore, if we compare the L- and LQ-moments, the PE3 distribution with L-moments parameter estimation methods is identified as best fitting distributions for these two subzones of North-East India. Similarly, for subzone 2(a)B and 2(c), it has been observed that the $|Z|$-statistic values of GPA and GLO distributions are lowest for the method of L-moments. Therefore, if we compare L-and LQ-moments, the GPA and GLO with L-moments as parameter estimation method are the best suitable distributions for subzones 2(a)B and 2(c), respectively.

5.4 Monte Carlo simulation techniques

A Monte Carlo simulation technique has been used with relative root mean square error (RRMSE) and relative bias (RBIAS) for verification of results obtained from both the comparative study for the four homogeneous subzones based on the Z-statistics values. The equations for RRMSE and RBIAS can be represented by

$$RRMSE = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left\{\frac{Q_l^{[m]} - Q_l}{Q_l}\right\}^2} \tag{5.4.1}$$

$$RBIAS = \frac{1}{M}\sum_{m=1}^{M}\left\{\frac{Q_l^{[m]} - Q_l}{Q_l}\right\} \tag{5.4.2}$$

where, M is the number of simulation, $Q_l^{[m]}$ and Q_l are the simulated and calculated quantiles at return period T, respectively.

The study has been carried out for random sample of sizes 20, 50 and 80 and a 10,000 repetition has been performed in each case.

The RRMSE and RBIAS values of GEV distributions for L- and L_2-moments, L- and L_4-moments and L- and L_1-moments of subzones 2(a)A, 2(a)B and 2(b), respectively have been calculated for sample size 20,50 and 80 and return periods 2 to 1000 years. The RRMSE and RBIAS values for each of these three homogeneous subzones are given in Table 5.1 to Table 5.6 (see in Appendix-A). Again, for subzone 2(c), the RRMSE and RBIAS values of GLO distribution for L- and L_1-moments have been calculated for sample size 20, 50 and 80 and the return periods 2 to 1000 years. The RRMSE and RBIAS values for subzone 2(c) are given in Table 5.7 and Table 5.8 (see in Appendix-A). In subzone 2(c), we consider the L_1-moments for comparative study between L- and LH-moments because for this subzone the identified best fitting GLO distribution attains the next lowest Z-statistic value at L_1-moments.

For subzone 2(a)A, it has been observed from Table 5.1 for sample size 20 to 80 that the RRMSE values of GEV distribution are smaller at L_2-moments than L-moments from 10 year return period onwards. But in 2 and 5 years return period, the RRMSE values of this distribution are smaller at L-moments than L_2-moments. The similar case has been observed for subzones 2(a)B (see Table 5.3) and 2(b) (see Table 5.5). In subzone 2(a)B, for all the sample sizes, the RRMSE values of GEV distribution are smaller at L_4-moments than L-moments from 10 years return period onwards and in 2 and 5 years the RRMSE values of this distribution is bigger at L_4-moments than L-moments. Again in subzone 2(b), the RRMSE values of GEV distribution are smaller at L_1-moments than L-moments from 10 year return period onwards and in 2 and 5 years a reverse trend has been observed.

For subzone 2(c), it has been observed from Table 5.7 for sample size 20 that the RRMSE values of GLO distribution are smaller at L-moments than L_1-moments for return periods 2, 5, 100, 500 and 1000 years but for 10 and 20 years these values are bigger and for 50 years both values are equal. Again, for sample size 50 and 80, it has been observed that the RRMSE values of GLO distribution are smaller at L-moments than L_1-moments for return periods 2, 50, 100, 500 and 1000 years return periods and reverse cases are observed at return periods 10 and 20 years. The RRMSE values of GLO distribution at L- and L_1-moments are equal at 5 years return periods for both 50 and 80 sample sizes.

It has been observed from RRMSE values of each of the probability distributions for all the four subzones that using these values only we are not able to say anything about the parameter estimation method.

In case of RBIAS tests the absolute RBIAS values are considered. It has been observed from RBIAS values of different distributions for all the four homogeneous subzones (see Table 5.2, Table 5.4, Table 5.6 and Table 5.8) that for all the four subzones each distribution carries small RBIAS values for both L- and LH-moments for different return periods. Again in some cases it has been observed that the distributions shared same RBIAS value for both L- and LH-moments for some of the return periods. The increase in sample sizes from 20 to 80 will improved the calculated RBIAS values of the distributions both for L- and LH-moments.

Similarly, the RBIAS test also not able to give a single distribution with method of parameter estimation for a subzone. Therefore a graphical tool known as box plots has been used for this purpose and the details are given in next section of this chapter.

Again, for verification of results obtained from comparative study between L- and LQ-moments, based on Z-statistic values, the Monte Carlo simulation technique with RRMSE and RBIAS values have been used. For this purpose, the RRMSE and RBIAS values of PE3, GPA, PE3 and GLO distributions for the method of L- and LQ-moments for four sub-zones 2(a)A, 2(a)B, 2(b) and 2(c) are calculated for sample sizes 20 to 80 and return periods 2 to 1000 years. The RRMSE and RBIAS values for L- and LQ-moments for four distributions for four subzones are given in Table 5.9 to Table 5.16 (see in Appendix-A).

For subzones 2(a)A and 2(b), it has been observed from Table 5.9 and Table 5.13 for all sample sizes that the RRMSE values of PE3 distribution for the method of L-moments are smaller than the method of LQ-moments for return periods 2 to 1000 years. Similarly, for subzones 2(a)B and 2(c), it has been observed from Table 5.11 and Table 5.15 that for all samples the RRMSE values of GPA and GLO distributions, respectively for the method of L-moments are smaller than the method of LQ-moments for return periods 2 to 1000 years. Therefore, if we compare both L- and LQ-moment methods than the PE3 distribution with L-moments method is suitable best fitting distribution for subzones 2(a)A and 2(b). Similarly the GPA and GLO distribution with L-moments as parameter estimation method are the best fitting regional distributions for subzones 2(a)B and 2(c), respectively.

It has been observed that the RBIAS test also shows the similar results for all the four subzones of North-East India.

The box plots of RRMSE and RBIAS values of L- and LQ-moments are also drawn for each of the four subzones for better representation of our results.

5.5 The Box plots used for selection purpose

The box plots, is widely used graphical tool introduced by Turkey (1977). It is a simple plot of five sample quantities: the minimum value; lower quartile, the median; the upper quartile; and the maximum value. The Box plots can be used to show the location of the median and the associated dispersion of the data at specific probability levels. The criteria for selecting a suitable method is based on the minimum achieved median RRMSE and RBIAS values, as well as the minimum dispersion in the median RRMSE or RBIAS values, indicated by both ends of the box plot. It is noted that a smaller median dispersion in RRMSE or RBIAS values would indicate a better integration of the methods, so it should also be used as a selection criterion.

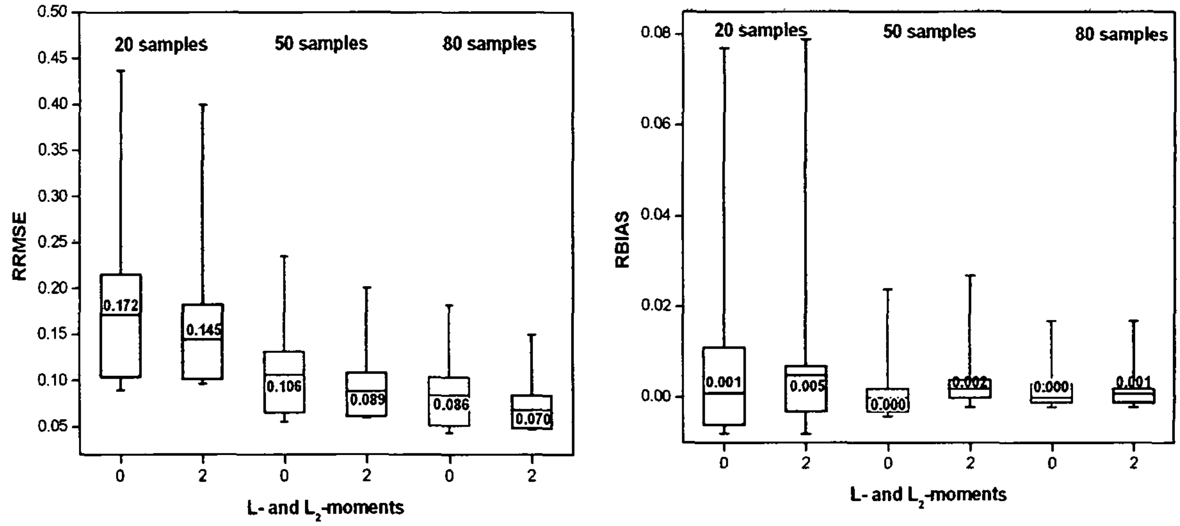
Figure 5.1(a) and Figure 5.1(b), respectively shows the box plots of the relative positions of computed RRMSE and RBIAS values of GEV distribution for L- and L_2-moments for subzone 2(a)A. It has been observed from box plots of RRMSE values for subzone 2(a)A that GEV distribution with level two LH-moments has the smaller median and dispersion in RRMSE for all samples than the GEV distribution with L-moments. Again, from Figure 5.1(b), it has been observed for all the samples that though the median in RBIAS values of GEV distribution at L_2-moments are higher than L-moments but its dispersions from both the ends are lower than L-moments. Therefore, by using the box plots of RBIAS values of GEV distribution for both L- and L_2-moments, it is difficult to say about suitability of the parameter estimation methods. Hence, the box plots of RRMSE values only used for this purpose and it shows that the GEV distribution with level two LH-moments i.e. L_2-moments is suitable for regional flood frequency analysis of subzone 2(a)A. The box plots of RRMSE and RBIAS values of GEV distribution for L- and L_4-moments for subzone 2(a)B are shown in Figure 5.2(a) and Figure 5.2(b), respectively. It has been observed from Figure 5.2(a) that the GEV distribution at L_4-moments has the minimum median and dispersion for all sample sizes. In case of box plots of RBIAS (Figure 5.2(b)), it has been observed for 20 and 80 samples that the GEV distribution at L_4-moments achieved minimum median and dispersion than L-moments. For sample size 50, it has been observed that the median in RBIAS values of GEV distributions are same for both L- and L_4-moments but the dispersion of it from both the ends are higher for L-moments than L_4-moments. Therefore, it can be concluded from both box plots of RRMSE and RBIAS values for subzone 2(a)B that the GEV distribution with L_4-moments is suitable for this subzone. Again, from Figure 5.3(a) for subzone 2(b), it has been observed that the GEV distribution has the minimum median and dispersion in RRMSE for L_1-moments for all sample sizes. But the box plots of RBIAS values (Figure 5.3(b)) for all the sample sizes shows that though the minimum median in RBIAS values are for L-moments but the dispersions are higher in case of L-moments than L_1-moments. In this case also using the box plots of RBIAS values we cannot judge about the suitable parameter estimation method for this subzone. Therefore, the box plots of RRMSE values are used and it shows that the GEV distribution with L_1-moments parameter estimation method is suitable for regional flood frequency analysis of subzone 2(b). For subzone 2(c), the box plots of RRMSE values (Figure 5.4(a)) of GLO distribution for sample sizes 20 to 80 shows that the same median in RRMSE values are achieved by both L- and L_l-moments. But the dispersion of median in RRMSE values from both the ends is achieved by L-moments. In case of box plots of RBIAS values (Figure 5.4(b)) it has been observed that for sample size 20, though the minimum median RBIAS value receives at L-moments but the dispersion of median from both the ends are higher at this method than L_l-moments. For sample sizes 50 and 80, it has been observed that for both the methods median values are same and the minimum dispersion of median from both the ends are achieved by L_l-moments. Therefore, it can be concluded from the box plots of RRMSE of subzone 2(c) for sample sizes 20 to 80 that the GLO distribution with L-moments is best fitting distributions with methods of parameter estimation for regional flood frequency analysis of this subzone.

The contradictory results shows by the box plots of RBIAS values are due to more negative values obtained in RBIAS calculations for both the methods for all the subzones.

The box plots of RRMSE and RBIAS values for the methods of L- and LQ-moments for each of the four subzones are also drawn and the details are given below.

The box plots of RRMSE and RBIAS values of PE3 distribution for L- and LQ-moments for subzones 2(a)A and 2(b) are shown in Figure 5.5 and Figure 5.7, respectively. It has been observed from Figure 5.5 and Figure 5.7 that the minimum medians and dispersions in RRMSE and RBIAS values are obtained for L-moments. Therefore, the PE3 distribution with L-moments has been identified as the suitable distribution for subzones 2(a)A and 2(b). For subzone 2(a)B, the box plots of RRMSE and RBIAS values of GPA distribution are shown in Figure 5.6. It has been observed from both the box plots of RRMSE (Figure 5.6(a)) and RBIAS values (Figure 5.6(b)) that the minimum medians and dispersions in both RRMSE and RBIAS values are produces by the method of L-moments. Therefore, the GPA distribution can be identified as the suitable distribution with L-moments for this subzone. Similarly for subzone 2(c), it has been observed from both the box plots of RRMSE (Figure 5.8(a)) and RBIAS (Figure 5.8(b)) values of GLO distribution for L- and LQ-moments that the minimum medians and dispersions in RRMSE and RBIAS values are produced by the method of L-moments for all sample sizes. Therefore, the GLO distribution with method of L-moments is found to be suitable for this subzone.

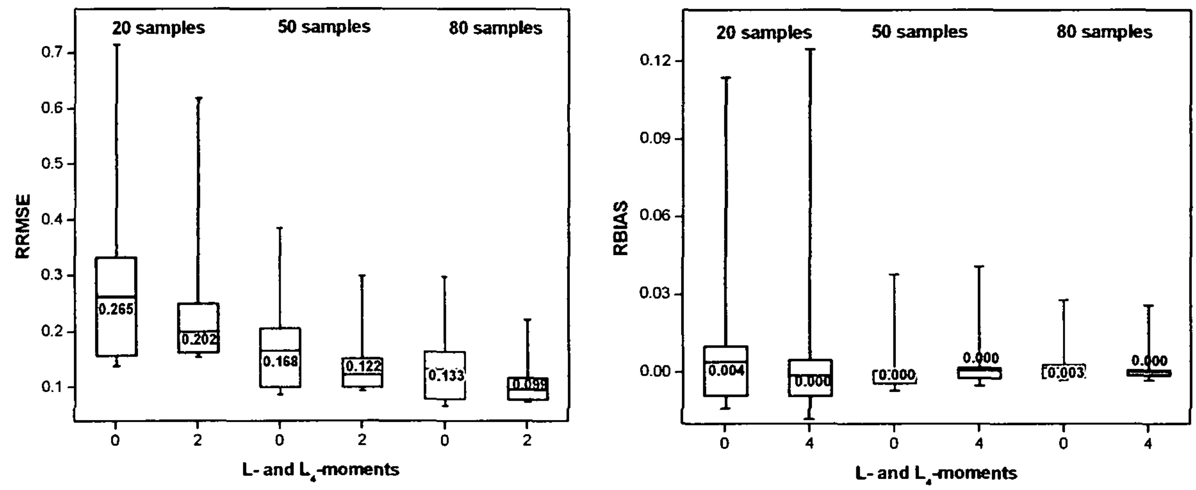
It has been observed that the box plots of RRMSE and RBIAS also shows the similar results as obtained from both the comparative studies based on the Z-statistic values of each of the distributions for three parameter estimation methods.



(a) The relative positions of RRMSE values of GEV distribution

(b) The relative positions of RBIAS values of GEV distribution

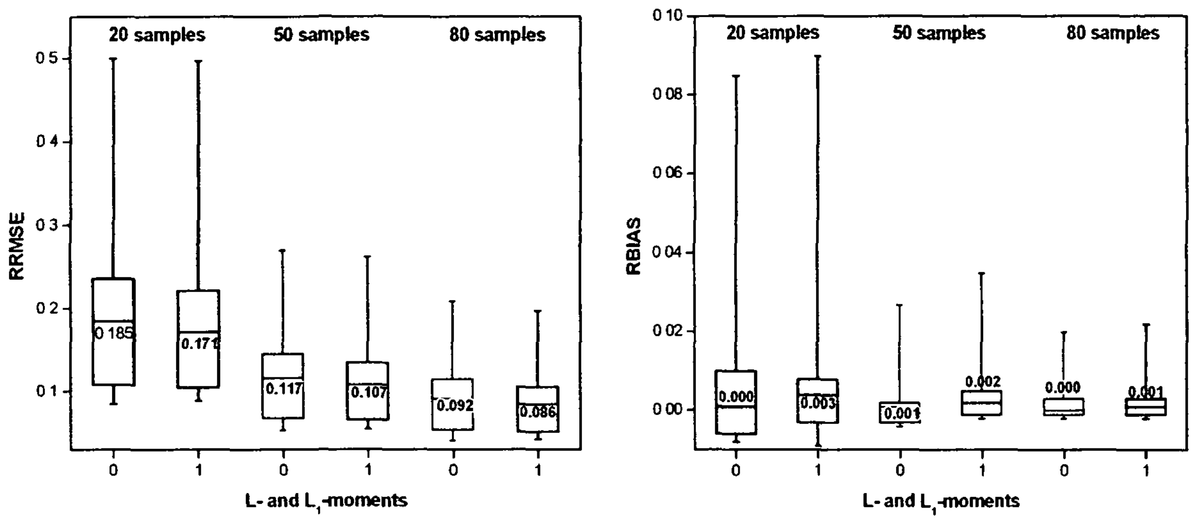
Figure 5.1 Box plots for L- and L_2-moments for subzone 2(a)A



(a) The relative positions of RRMSE values of GEV distribution

(b) The relative positions of RBIAS values of GEV distribution

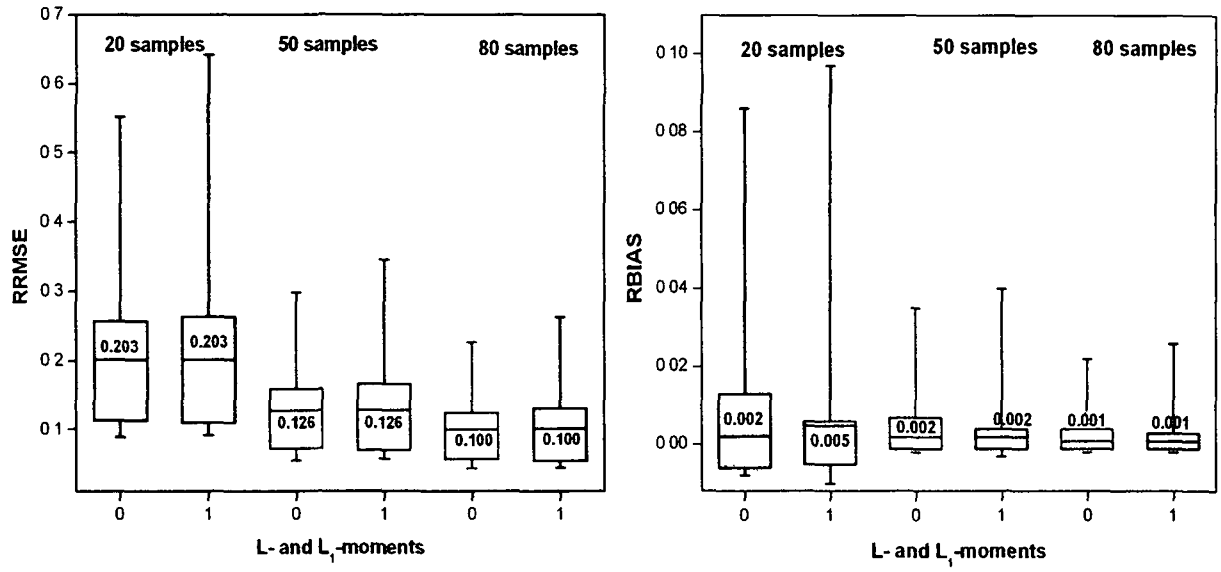
Figure 5.2 Box plots of L- and L_4-moments for subzone 2(a)B



(a) The relative positions of RRMSE values of GEV distribution

(b) The relative positions of RBIAS values of GEV distribution

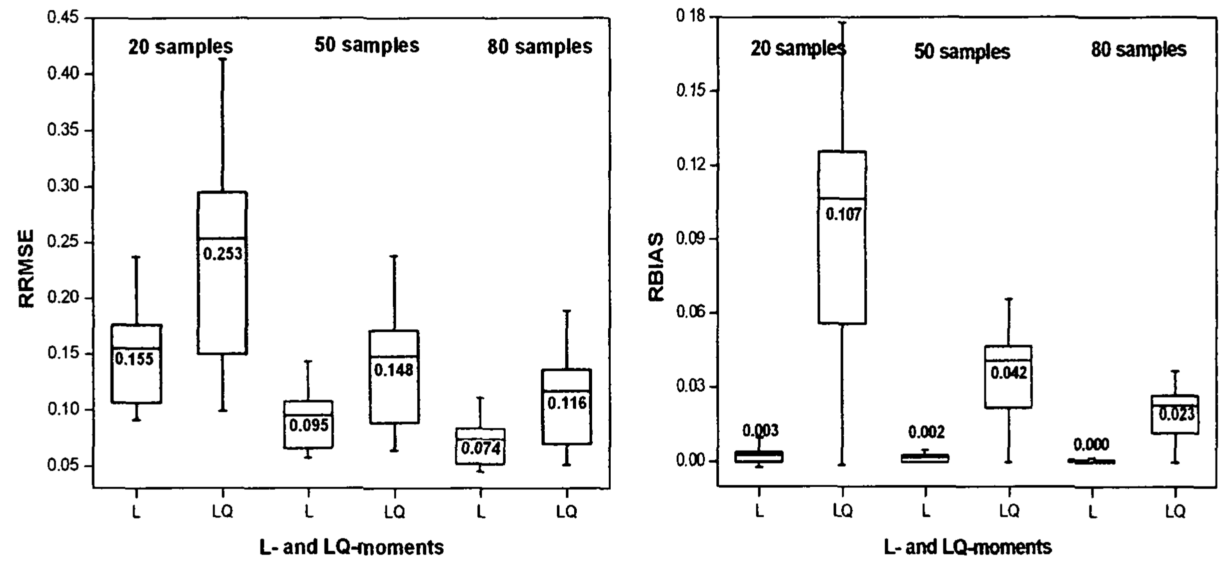
Figure 5.3 Box plots of L- and L_1-moments for subzone 2(b)



(a) The relative positions of RRMSE values of GLO distribution

(b) The relative positions of RBIAS values of GLO distribution

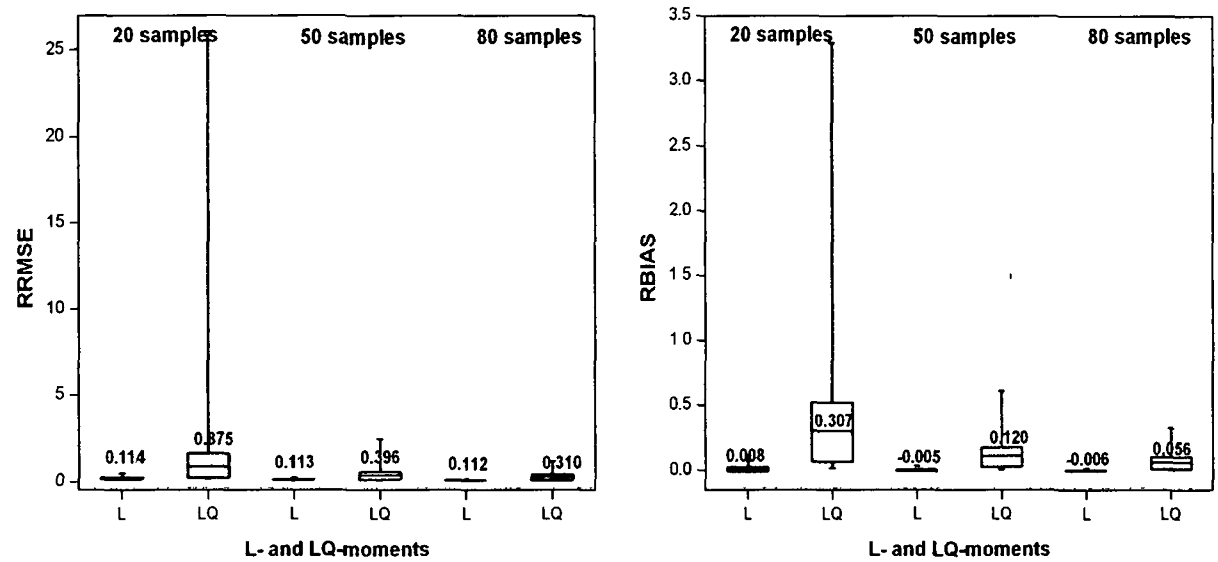
Figure 5.4 Box plots of L- and L_1-moments for subzone 2(c)



(a) The relative positions of RRMSE values of PE3 distribution

(b) The relative positions of RBIAS values of PE3 distribution

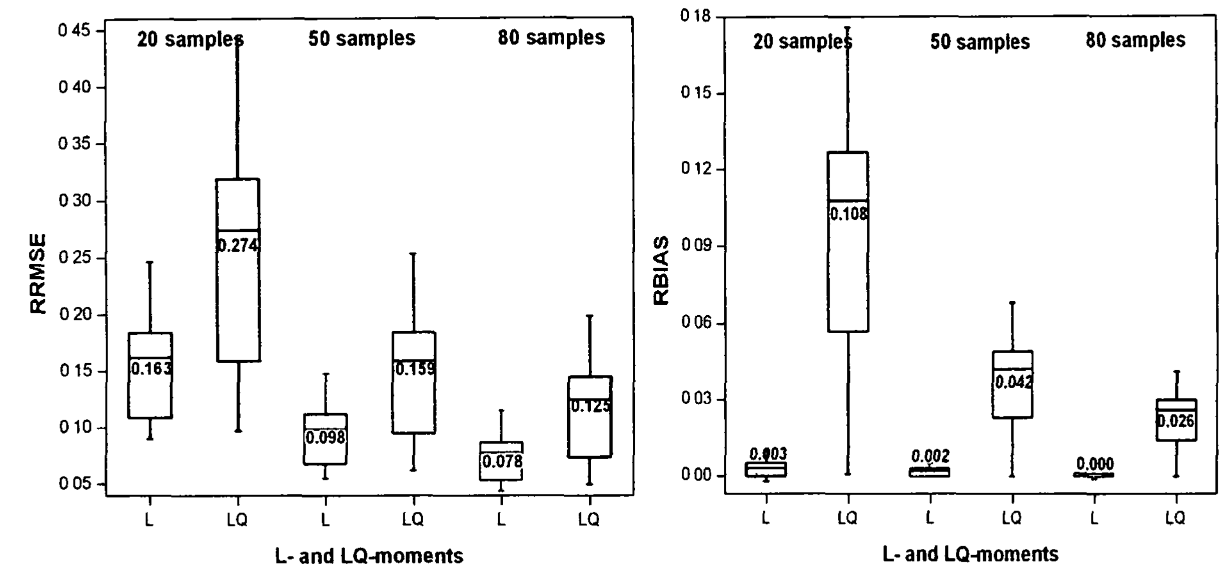
Figure 5.5 Box plots of L- and LQ-moments for subzone 2(a)A



(a) The relative positions of RRMSE values of GPA distribution

(b) The relative positions of RBIAS values of GPA distribution

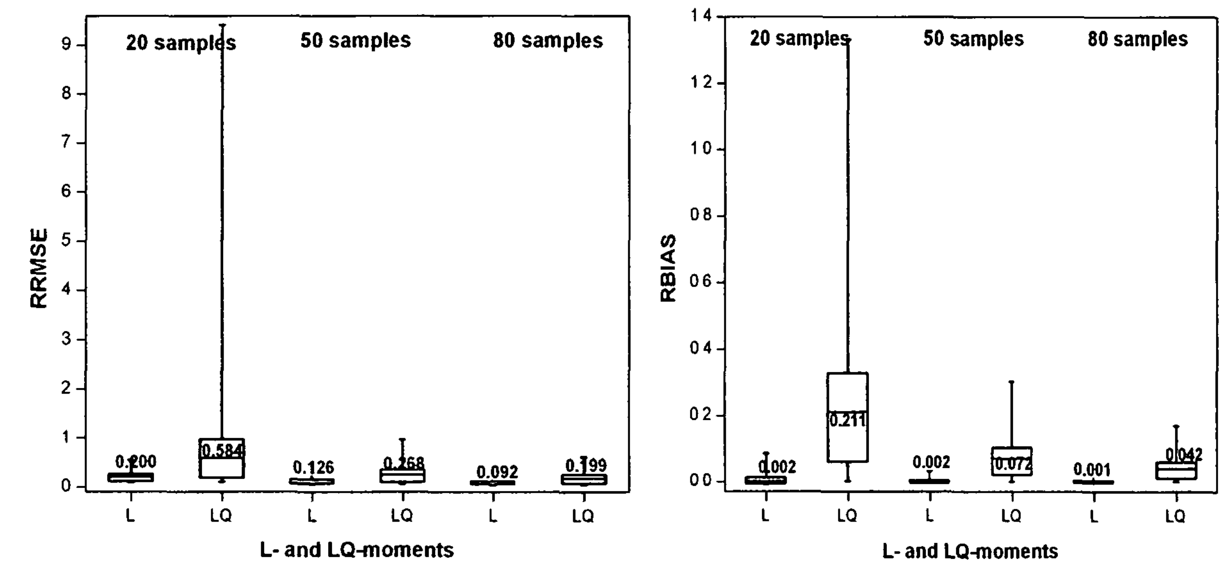
Figure 5.6 Box plots of L- and LQ- moments for subzone 2(a)B



(a) The relative positions of RRMSE values of PE3 distribution

(b) The relative positions of RBIAS values of PE3 distribution

Figure 5.7 Box plots of L- and LQ-moments for subzone 2(b)



(a) The relative positions of RRMSE values of GLO distribution

(b) The relative positions of RBIAS values of GLO distribution

Figure 5.8 Box plots of L- and LQ-moments for sub zone 2(c)

5.6 Conclusion

The following conclusions can be drawn from our study.

The Z-statistic values of each of the probability distributions for each of the three parameter estimation methods are used for the comparative studies of L-moments with LH- and LQ-moments.

The obtained results based on Z-statistic values have been verified by using the Monte Carlo simulation techniques with the help of RRMSE and RBIAS values of each of the probability distributions for all the three parameter estimation methods.

It has been observed from comparative study between L- and LH-moments that the GEV distribution with L_2-moments is found to be suitable than this distribution with L-moments for subzone 2(a)A. Similarly for subzones 2(a)B and 2(b), it has been observed that the GEV with L_4- and L_1-moments, respectively are found to be suitable for these two subzones than this distribution with L-moments. Again for subzone 2(c), it has been observed that the GLO distribution with L-moments is found to be suitable than this distribution with L_1-moments for this subzone.

It has been observed from our comparative study between L- and LQ-moments that the PE3 distribution with L-moments method is found to be suitable for subzones 2(a)A and 2(b) than this distribution with LQ-moments. Similarly, for subzones 2(a)B and 2(c), it has been observed from our comparative study that the GPA and GLO distributions with L-moments parameter estimation methods, respectively are suitable than both these distributions with LQ-moments for these two subzones.

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Chapter 6

Conclusion and Discussion of Results

In this chapter the overall conclusions and results drawn from our study have been discussed and also give some feature scope of our study. In our study we consider whole North-East India as a study region and regional flood frequency analysis has been performed by using three parameter estimation methods i.e. L-, LH- and LQ-moments. The North-East India falls in zone 2 out of 7 hydro-meteorological zones of India. It has three hydro-meteorological subzones namely subzone 2(a), 2(b) and 2(c). For regional frequency analysis of the study area we consider these three hydro-meteorological subzones. The annual maximum discharge data has been collected for some of the gauged sites of these three subzones for which data's are available. Before performing the regional frequency analysis the screening of data and homogeneity of these three subzones have been tested for each of the three parameter estimation methods. It has been observed that the subzone 2(a) consisting of 18 gauged sites is found to be heterogeneous and other two subzones i.e. 2(b) and 2(c) consisting of 7 gauged sites in each subzone found to be homogeneous for L-moment based regional frequency analysis. As the subzone 2(a) is found to be heterogeneous therefore, it has been divided into two homogeneous subzones namely subzone 2(a)A and subzone 2(a)B by using the K-mean cluster analysis techniques. Again from discordancy measures for the method of L-moments, it has been observed that no site found to be discordance for these four homogeneous subzones except the site Beki of subzone 2(a)B. Though the site Beki is found to be discordant we include this site for regional frequency analysis of subzone 2(a)B and the reason of inclusion of this site is mentioned in section 2.5.2. The Z-statistic criteria and L-moment ratio diagram have been used as goodness of fit tests for selection of best fitting regional probability distributions for each of the four homogeneous subzones. It has been observed that the PE3 is identified as best fitting distribution for subzones 2(a)A and 2(b). Similarly, it has been observed that the GPA and GLO distributions are found to be the best fitting regional probability distributions for subzones 2(a)B and 2(c), respectively. The regional flood frequency relationships for both gauged and ungauged catchments areas of these four subzones have been developed by using the identified best fitting regional distributions for each of these subzones. In chapter 3, the regional flood frequency analysis of these four homogeneous subzones identified by the method of L-moments has been carried out by using method of LH-moments. In this method instead of five probability distributions only three probability distributions namely GEV, GLO and GPA have been used for regional frequency analysis. The homogeneity and the discordancy measures of each of the sites of these four subzones have been tested for each level of LH-moments i.e. for L_1 to L_4-moments. It has been observed that these four subzones are found to be homogeneous for all level of LH-moments i.e. L_1- to L_4-moments. The discordancy measures for each level of LH-moments i.e. $L_\eta, (\eta = 1,2,3,4)$ shows that no sites of these four subzones are found to be discordance for each level of LH-moments. But an exceptional case has been observed for the site Beki of subzone 2(a)B, this site is found to be discordance at the L_1-moments level. Though this site is found to be discordant we include this site for regional frequency analysis of subzone 2(a)B by using L_1-moments. The selection of regional probability distributions for each of the four homogeneous subzones for each levels of LH-moments has been performed by using two goodness of fit tests i.e. $|Z_\eta^{DIST}|$-statistic criteria and LH-moments ratio diagram. Though both these are used as goodness of fit test for selection of best fitting distributions but it has been observed that the LH-moments ratio diagram shows different distributions as best fitting distribution at different level of LH-moments. Therefore, only the $|Z_\eta^{DIST}|$-statistic criteria has been used for selection of best fitting distribution with level of LH-moments for each of the four homogeneous subzones considered for our study. It has been observed from the $|Z_\eta^{DIST}|$-statistic criteria that GEV distribution with L_2-moments has been identified as the best fitting regional probability distributions for subzones 2(a)A and 2(c). Similarly, the GEV distribution with L_4-and L_1-moments has been identified as best fitting distribution with method of parameter estimations for subzones 2(a)B and 2(b), respectively. The regional flood frequency relationships for gauged and ungauged catchments areas of these four subzones have been developed by using the identified best fitting distributions with the level of LH-moments. These developed relationships can be used for estimating the flood quantiles of desired return periods for each of the four homogeneous subzones. In chapter 4, the regional frequency analysis of these four subzones has been performed by using another parameter estimation method known as the LQ-moments. Before going to the regional frequency analysis procedure, the homogeneity and discordancy measures have been performed in terms of LQ-moments for each of the four subzones. It has been observed from heterogeneity measure based on LQ-moments that these four subzones are found to be homogeneous for this parameter estimation method. Again from discordancy measures based on LQ-moments shows that there is no sites found to be discordance for all the four subzones. The $|Z_{LQ}^{DIST}|$-statistic criteria and LQ-moment ratio diagram have been used as goodness of fit tests for selection of best fitting distributions for each of the four homogeneous subzones of North-East India. It has been observed from goodness of fit tests that similar to the method of L-moments, the PE3 distribution for subzones 2(a)A and 2(b) and GPA and GLO distributions for subzones 2(a)B and 2(c), respectively are identifies as the best fitting distributions. The regional relationships based on the method of LQ-moments for gauged and ungauged catchments areas of these four subzones have been developed by using the identified best fitting distributions for each of the four homogeneous subzones.

In chapter 5 two comparative studies one between L- and LH-moments and another between L- and LQ-moments has been performed for regional flood frequency analysis of these four subzones. It has been observed from chapter 2, 3 and 4 that all the four sub zones i.e. 2(a)A, 2(a)B, 2(b) and 2(c) are found to be homogeneous for all the three parameter estimation methods. Therefore, a comparative study can be performed between the methods of L- and LH-moments for these four subzones. Similarly a comparative study can also be performed between the methods of L- and LQ-moments for these four subzones. The Z-statistic values are used for selection of probability distributions with the method of parameter estimations in both the comparative studies. For comparative study between L- and

LH-moments, the $|Z^{DIST}|$- and $|Z_{\eta}^{DIST}|$-statistic values of three distributions namely GEV, GLO and GPA have been used. It has been observed from Z-statistic values that the GEV distribution with L_2-moments attains the lowest Z-statistic value among all the distributions for subzone 2(a)A. Therefore, if we compare L- and LH-moments parameter estimation methods for GEV, GLO and GPA distributions, the GEV distribution with L_2-moments can be used as best fitting distributions for this subzone. Similarly, for subzones 2(a)B, 2(b) and 2(c), it has been observed from comparative study between L- and LH-moments that the GEV distribution with L_4- and L_1-moments and GLO distribution with L-moments, respectively, are identified as best fitting distributions with the method of parameters estimation for these subzones. The obtained results from comparative study between L-and LH-moments based on Z-statistic values are again verified by using Monte Carlo simulation techniques in terms of relative root mean square error (RRMSE) and relative bias (RBIAS). The RRMSE and RBIAS values of identified best fitting distribution for LH-moments has been compared with its RRMSE and RBIAS values based on the method of L-moments. The selection criteria used for the method of parameter estimation is based on the minimum RRMSE and RBIAS values. It has been observed that the minimum RRMSE and RBIAS values are produced by different method in different return periods. Therefore, based on RRMSE and RBIAS values only it is not possible to select a proper parameter estimation method out of these L- and LH-moments. Therefore a graphical tool known as box plots for both RRMSE and RBIAS values have been used for proper selection of the superior method of parameter estimation for each of the four subzones. The box plots of RRMSE and RBIAS values for all the four subzones shows similar results as obtained by using the Z-statistic values. Similarly, for comparison of the method of L-moments and LQ-moments also the $|Z^{DIST}|$- and $|Z_{LQ}^{DIST}|$-statistic values of five probability distributions for each of the four subzones have been used. It has been observed that the $|Z^{DIST}|$-statistic values of PE3, GPA, PE3 and GLO distributions for subzones 2(a)A, 2(a)B, 2(b) and 2(c) are less than all other values. Therefore, if we compare both the parameter estimation methods i.e. L-moments and LQ-moments these distributions with L-moments parameter estimation method are suitable best fitting distributions for four subzones of North-East India. The obtained results from the comparative study based on Z-statistic values have been again verified by using Monte Carlo simulation techniques like L- and LH-moments comparison study mentioned above. The box plots are also used in this case for appropriate selection of distribution with the method of estimation of parameters. It has been observed that the box plots also shows similar results for our study area as obtained by using the Z-statistic values for this comparative study.

The regional flood frequency relationships for ungauged sites may be refined for obtaining more accurate flood frequency estimates when the data for more gauging sites become available and catchment and physiographic characteristics other than catchment area are also used for development of the regional relationship. Again, for finding better relationships between mean annual discharge and catchments characteristics the generalized least square approach (GLS) suggested by Stedinger and Tasker (1985, 1986) may be used instead of least square approach.

For regional frequency analysis based on LH-moments the generalized log-normal (GNO) and Pearson type III (PE3) may also be used by developing the LH-moments of these two distributions.

A comparative study may be performed among all the three methods of estimations i.e. L-, LH- and LQ-moments if GNO and PE3 could also be used for regional frequency analysis by using LH-moments.

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Appendix-A

List of Tables

Table 3.2 Name of sites, number of observations, sample L_1-moment statistics and discordancy measures of subzone 2(a)A

Name of sites	Number of observations	$\hat{\lambda}_1^1$	$\hat{\tau}^1$	$\hat{\tau}_3^1$	$\hat{\tau}_4^1$	D_i^1
Manas	17	7080.32	0.1359	0.2782	0.1620	0.25
Nanoi	11	119.88	0.1615	0.4042	0.4557	**2.68**
Borolia	15	233.11	0.1443	0.0844	0.0016	0.36
Dhansiri	21	1580.56	0.1344	0.2309	0.1425	0.08
Jiabhoroli	36	5189.70	0.1636	0.1926	0.0828	0.48
Subansiri	27	10488.15	0.1418	0.3188	0.2062	0.29
Sankush	12	2132.09	0.0884	0.1570	0.0832	1.88
Champamati	22	974.80	0.1678	0.3278	0.0969	1.08
Noanadi	13	45.79	0.0941	-0.1801	-0.0540	2.10
Ranganadi	19	1201.98	0.1498	0.0357	-0.0233	0.79

The bold figures represent discordancy measure greater than critical value

Table 3.3 Name of sites, number of observations, sample L_2-moment statistics and discordancy measures of subzone 2(a)A

Name of sites	Number of observations	$\hat{\lambda}_1^2$	$\hat{\tau}^2$	$\hat{\tau}_3^2$	$\hat{\tau}_4^2$	D_i^2
Manas	17	7721.68	0.1205	0.3012	0.2007	0.16
Nanoi	11	132.79	0.1520	0.5267	0.4921	2.37
Borolia	15	255.54	0.1120	0.0719	-0.0341	0.42
Dhansiri	21	1722.21	0.1158	0.2625	0.0912	0.56
Jiabhoroli	36	5755.81	0.1351	0.2019	0.0979	0.41
Subansiri	27	11479.39	0.1284	0.3486	0.2292	0.22
Sankush	12	2257.68	0.0746	0.1786	0.1966	1.92
Champamati	22	1083.87	0.1505	0.2905	0.0125	1.02
Noanadi	13	48.66	0.0612	-0.2304	0.0589	2.36
Ranganadi	19	1321.99	0.1118	0.0148	-0.0308	0.56

Table 3.4 Name of sites, number of observations, sample L_3-moment statistics and discordancy measures of subzone 2(a)A

Name of sites	Number of observations	$\hat{\lambda}_1^3$	$\hat{\tau}^3$	$\hat{\tau}_3^3$	$\hat{\tau}_4^3$	D_i^3
Manas	17	8186.77	0.1118	0.3316	0.2371	0.19
Nanoi	11	142.88	0.1549	0.5846	0.4928	2.29
Borolia	15	269.85	0.0922	0.0424	-0.0449	0.45
Dhansiri	21	1821.92	0.1056	0.2514	0.0472	0.94
Jiabhoroli	36	6144.56	0.1182	0.2145	0.1154	0.21
Subansiri	27	12216.33	0.1216	0.3732	0.2398	0.23
Sankush	12	2341.92	0.0664	0.2517	0.3463	1.93
Champamati	22	1165.41	0.1369	0.2290	-0.0558	1.09
Noanadi	13	50.16	0.0427	-0.1997	0.1191	2.17
Ranganadi	19	1395.92	0.0891	-0.0052	-0.0001	0.50

Table 3.5 Name of sites, number of observations, sample L_4-moment statistics and discordancy measures of subzone 2(a)A

Name of sites	Number of observations	$\hat{\lambda}_1^4$	$\hat{\tau}^4$	$\hat{\tau}_3^4$	$\hat{\tau}_4^4$	D_i^4
Manas	17	8552.87	0.1069	0.3635	0.2554	0.27
Nanoi	11	151.74	0.1616	0.6082	0.4983	2.46
Borolia	15	279.80	0.0778	0.0149	-0.0455	0.44
Dhansiri	21	1898.86	0.0977	0.2244	0.0178	0.94
Jiabhoroli	36	6435.12	0.1071	0.2298	0.1317	0.07
Subansiri	27	12810:37	0.1179	0.3902	0.2428	0.27
Sankush	12	2404.09	0.0624	0.3624	0.4960	2.03
Champamati	22	1229.24	0.1240	0.1605	-0.1111	1.06
Noanadi	13	51.01	0.0323	-0.1327	0.1667	1.88
Ranganadi	19	1445.65	0.0735	-0.0048	0.0574	0.58

Table 3.6 Name of sites, number of observations, sample L_1-moment statistics and discordancy measures of subzone 2(a)B

Name of sites	Number of observations	$\hat{\lambda}_1^1$	$\hat{\tau}^1$	$\hat{\tau}_3^1$	$\hat{\tau}_4^1$	D_i^1
Puthimari	37	762.83	0.2215	0.2304	0.0882	0.07
Pagladiya	35	887.86	0.2320	0.2363	0.1368	0.32
Pachnoi	22	280.64	0.2082	0.3082	0.1916	0.41
Belsiri	23	387.40	0.1973	0.1943	0.0254	0.96
Gabharu	15	369.89	0.2398	0.0975	-0.0246	1.78
Beki	13	944.26	0.1469	0.0134	0.1543	2.11
Gaurang	17.	1374.95	0.2347	0.3539	0.2846	1.47
Ghiladhari	20	99.59	0.2291	0.3572	0.1055	0.90

Table 3.7 Name of sites, number of observations, sample L_2-moment statistics and discordancy measures of subzone 2(a)B

Name of sites	Number of observations	$\hat{\lambda}_1^2$	$\hat{\tau}^2$	$\hat{\tau}_3^2$	$\hat{\tau}_4^2$	D_i^2
Puthimari	37	875.47	0.1811	0.2291	0.1215	0.06
Pagladiya	35	1025.21	0.1893	0.2622	0.1460	0.16
Pachnoi	22	319.59	0.1801	0.3347	0.2144	0.52
Belsiri	23	438.35	0.1598	0.1673	0.0360	0.54
Gabharu	15	429.01	0.1775	0.0647	0.0422	1.85
Beki	13	1036.72	0.1081	0.1209	0.1189	1.95
Gaurang	17	1590.09	0.2054	0.4103	0.2513	1.13
Ghiladhari	20	114.81	0.2016	0.3109	0.0353	1.78

Table 3.8 Name of sites, number of observations, sample L_3-moment statistics and discordancy measures of subzone 2(a)B

Name of sites	Number of observations	$\hat{\lambda}_1^3$	$\hat{\tau}^3$	$\hat{\tau}_3^3$	$\hat{\tau}_4^3$	D_i^3
Puthimari	37	954.74	0.1574	0.2454	0.1637	0.18
Pagladiya	35	1122.23	0.1667	0.2796	0.1513	0.11
Pachnoi	22	348.36	0.1653	0.3580	0.2306	0.92
Belsiri	23	473.37	0.1357	0.1565	0.0327	0.39
Gabharu	15	467.08	0.1411	0.0805	0.1421	1.69
Beki	13	1092.76	0.0917	0.1680	0.0353	2.03
Gaurang	17	1753.42	0.1935	0.4173	0.1590	0.96
Ghiladhari	20	126.38	0.1811	0.2537	-0.0230	1.71

Table 3.9 Name of sites, number of observations, sample L_4-moment statistics and discordancy measures of subzone 2(a)B

Name of sites	Number of observations	$\hat{\lambda}_1^4$	$\hat{\tau}^{4'}$	$\hat{\tau}_3^4$	$\hat{\tau}_4^4$	D_i^4
Puthimari	37	1014.84	0.1425	0.2739	0.2059	0.26
Pagladiya	35	1197.08	0.1527	0.2915	0.1500	0.11
Pachnoi	22	371.40	0.1567	0.3769	0.2431	0.98
Belsiri	23	499.07	0.1189	0.1474	0.0100	0.45
Gabharu	15	493.45	0.1191	0.1410	0.2441	1.64
Beki	13	1132.82	0.0822	0.1575	0.0092	1.99
Gaurang	17	1889.13	0.1860	0.3815	0.0529	1.07
Ghiladhari	20	135.54	0.1631	0.1937	-0.0788	1.50

Table 3.10 Name of sites, number of observations, sample L_1-moment statistics and discordancy measures of subzone 2(b)

Name of sites	Number of observations	$\hat{\lambda}_1^1$	$\hat{\tau}^1$	$\hat{\tau}_3^1$	$\hat{\tau}_4^1$	D_i^1
Dikhow	26	879.99	0.1334	0.2470	0.1749	0.13
Jhanji	13	202.15	0.1689	0.0851	-0.0469	1.05
Bhogdoi	13	264.77	0.1827	0.1903	0.0662	1.87
Dhansiri	29	1351.14	0.1410	0.2251	0.1348	0.03
Kapili	26	1522.78	0.1583	0.2879	0.0330	1.54
Kulsi	24	124.26	0.1152	0.3291	0.3153	1.17
Krishnai	19	560.33	0.1232	0.1185	0.1630	1.22

Table 3.11 Name of sites, number of observations, sample L_2-moment statistics and discordancy measures of subzone 2(b)

Name of sites	Number of observations	$\hat{\lambda}_1^2$	$\hat{\tau}^2$	$\hat{\tau}_3^2$	$\hat{\tau}_4^2$	D_i^2
Dikhow	26	958.25	0.1161	0.2915	0.2194	0.15
Jhanji	13	224.91	0.1292	0.0399	-0.0982	1.57
Bhogdoi	13	297.02	0.1489	0.1901	0.1309	1.79
Dhansiri	29	1478.19	0.1206	0.2544	0.1353	0.05
Kapili	26	1683.44	0.1393	0.2308	-0.0006	0.79
Kulsi	24	133.80	0.1068	0.4163	0.2709	1.18
Krishnai	19	606.37	0.0992	0.2041	0.2783	1.47

Table 3.12 Name of sites, number of observations, sample L_3-moment statistics and discordancy measures of subzone 2(b)

Name of sites	Number of observations	$\hat{\lambda}_1^3$	$\hat{\tau}^3$	$\hat{\tau}_3^3$	$\hat{\tau}_4^3$	D_i^3
Dikhow	26	1013.91	0.1075	0.3353	0.2442	0.15
Jhanji	13	239.44	0.1035	-0.0225	-0.2124	1.69
Bhogdoi	13	319.13	0.1287	0.2241	0.2307	1.73
Dhansiri	29	1567.29	0.1092	0.2690	0.1275	0.07
Kapili	26	1800.66	0.1235	0.1821	-0.0287	0.71
Kulsi	24	140.95	0.1056	0.4298	0.2548	1.11
Krishnai	19	636.45	0.0884	0.3124	0.3498	1.54

Table 3.13 Name of sites, number of observations, sample L_4-moment statistics and discordancy measures of subzone 2(b)

Name of sites	Number of observations	$\hat{\lambda}_1^4$	$\hat{\tau}^4$	$\hat{\tau}_3^4$	$\hat{\tau}_4^4$	D_i^4
Dikhow	26	1057.50	0.1031	0.3690	0.2418	0.19
Jhanji	13	249.36	0.0843	-0.1320	-0.3356	1.73
Bhogdoi	13	335.56	0.1166	0.2904	0.3358	1.86
Dhansiri	29	1635.75	0.1018	0.2735	0.1221	0.07
Kapili	26	1889.62	0.1101	0.1378	-00578	0.70
Kulsi	24	146.90	0.1055	0.4298	0.2467	0.86
Krishnai	19	658.96	0.0846	0.4021	0.3749	1.58

Table 3.14 Name of sites, number of observations, sample L_1-moment statistics and discordancy measures of subzone 2(c)

Name of sites	Number of observations	$\hat{\lambda}_1^1$	$\hat{\tau}^1{}_i$	$\hat{\tau}_3^1$	$\hat{\tau}_4^1$	D_i^1
Barak	11	4567.19	0.1384	0.3716	0.0938	1.29
Dhaleshwari	16	733.04	0.0874	0.2859	0.4466	1.46
Dhalai	11	218.48	0.1384	0.2705	0.1212	0.23
Khowai	19	367.10	0.2020	0.1863	0.0149	0.99
Gumti	24	511.32	0.1479	0.1319	0.1973	0.87
Muhuri	28	435.15	0.1461	0.1846	0.0562	0.41
Manu	12	950.21	0.1813	0.3862	0.3775	1.76

Table 3.15 Name of sites, number of observations, sample L_2-moment statistics and discordancy measures of subzone 2(c)

Name of sites	Number of observations	$\hat{\lambda}_1^2$	$\hat{\tau}^2$	$\hat{\tau}_3^2$	$\hat{\tau}_4^2$	D_i^2
Barak	11	4988.47	0.1296	0.3119	-0.0474	1.42
Dhaleshwari	16	775.74	0.0802	0.4729	0.3633	1.47
Dhalai	11	238.64	0.1219	0.2727	0.0697	0.12
Khowai	19	416.53	0.1623	0.1552	-0.0066	0.84
Gumti	24	561.73	0.1183	0.2354	0.2186	0.90
Muhuri	28	477.53	0.1212	0.1800	0.0216	0.42
Manu	12	1065.06	0.1669	0.4765	0.3895	1.84

Table 3.16 Name of sites, number of observations, sample L_3-moment statistics and discordancy measures of subzone 2(c)

Name of sites	Number of observations	$\hat{\lambda}_1^3$	$\hat{\tau}^3$	$\hat{\tau}_3^3$	$\hat{\tau}_4^3$	D_i^3
Barak	11	5311.69	0.1204	0.2125	-0.2079	1.69
Dhaleshwari	16	806.85	0.0825	0.5013	0.3334	1.49
Dhalai	11	253.19	0.1115	0.2469	0.0109	0.10
Khowai	19	450.33	0.1367	0.1242	-0.0083	0.92
Gumti	24	594.97	0.1063	0.3006	0.2122	0.45
Muhuri	28	506.48	0.1056	0.1579	-0.0034	0.47
Manu	12	1153.93	0.1651	0.5152	0.3869	1.88

Table 3.17 Name of sites, number of observations, sample L_4-moment statistics and discordancy measures of subzone 2(c)

Name of sites	Number of observations	$\hat{\lambda}_1^4$	$\hat{\tau}^4$	$\hat{\tau}_3^4$	$\hat{\tau}_4^4$	D_i^4
Barak	11	5567.50	0.1089	0.0769	-0.4472	1.86
Dhaleshwari	16	833.47	0.0856	0.5020	0.3180	1.45
Dhalai	11	264.48	0.1028	0.2044	-0.0292	0.06
Khowai	19	474.96	0.1180	0.1021	0.0063	1.02
Gumti	24	620.26	0.1005	0.3328	0.2039	0.24
Muhuri	28	527.87	0.0937	0.1312	-0.0267	0.46
Manu	12	1230.11	0.1669	0.5313	0.3924	1.91

Table 5.1 The RRMSE values of GEV distribution for subzone 2(a)A for L- and L_2-moments

Sample size	Methods	Return period							
		2	5	10	20	50	100	500	1000
20	L	**0.090**	**0.092**	0.104	0.127	0.172	0.216	0.356	0.437
	L_2	0.102	0.097	**0.100**	**0.112**	**0.145**	**0.183**	**0.318**	**0.400**
50	L	**0.056**	**0.058**	0.066	0.080	0.107	0.132	0.201	0.236
	L_2	0.062	0.060	**0.062**	**0.069**	**0.089**	**0.109**	**0.170**	**0.201**
80	L	**0.044**	**0.045**	0.052	0.064	0.085	0.104	0.157	0.183
	L_2	0.049	0.048	**0.049**	**0.055**	**0.069**	**0.085**	**0.129**	**0.151**

Table 5.2 The RBIAS values of GEV distribution for subzone 2(a)A for L- and L_2-moments

Sample size	Methods	Return periods							
		2	5	10	20	50	100	500	1000
20	L	**0.001**	-0.006	-0.008	**-0.006**	**0.001**	0.011	0.052	**0.077**
	L_2	0.005	**-0.002**	**-0.007**	-0.008	-0.003	**0.007**	**0.050**	0.079
50	L	**0.000**	-0.003	-0.004	-0.003	-0.001	**0.002**	**0.016**	**0.024**
	L_2	0.002	**0.000**	**-0.002**	**-0.002**	**0.000**	0.004	0.018	0.027
80	L	**0.000**	-0.001	-0.002	-0.001	0.000	0.003	0.012	0.017
	L_2	0.001	-0.001	-0.002	-0.001	0.000	**0.002**	**0.011**	0.017

Table 5.3 The RRMSE values of GEV distribution for subzone 2(a)B for L- and L_4-moments

Sample size	Methods	Return periods							
		2	5	10	20	50	100	500	1000
20	L	**0.140**	**0.139**	0.158	0.194	0.263	0.333	0.569	0.716
	L_4	0.201	0.158	**0.155**	**0.163**	**0.200**	**0.251**	**0.466**	**0.620**
50	L	**0.088**	**0.088**	0.101	0.124	0.167	0.207	0.324	0.386
	L_4	0.120	0.096	**0.095**	**0.101**	**0.124**	**0.152**	**0.247**	**0.300**
80	L	**0.068**	**0.069**	0.080	0.099	0.134	0.165	0.254	0.299
	L_4	0.095	0.076	**0.075**	**0.079**	**0.097**	**0.118**	**0.187**	**0.223**

Table 5.4 The RBIAS values of GEV distribution for subzone 2(a)B for L- and L_4-moments

Sample size	Methods	Return periods							
		2	5	10	20	50	100	500	1000
20	L	0.004	-0.009	-0.014	**-0.014**	**-0.004**	0.010	0.072	**0.114**
	L_4	**-0.001**	**0.005**	**-0.009**	-0.018	-0.016	**-0.003**	**0.070**	0.125
50	L	0.001	-0.004	-0.007	-0.007	-0.004	0.001	**0.023**	**0.038**
	L_4	**0.000**	**0.002**	**-0.002**	**-0.005**	**-0.003**	0.001	0.025	0.041
80	L	0.001	-0.002	-0.003	-0.003	0.002	0.003	0.018	0.028
	L_4	**0.000**	**0.000**	**-0.002**	-0.003	**-0.001**	**0.001**	**0.017**	**0.026**

Table 5.5 The RRMSE values of GEV distribution for subzone 2(b) for L- and L_1-moments

Sample size	Methods	Return periods							
		2	5	10	20	50	100	500	1000
20	L	**0.085**	**0.092**	0.108	0.134	0.185	0.236	0.401	0.500
	L_1	0.089	0.095	**0.105**	**0.126**	**0.172**	**0.222**	**0.391**	**0.497**
50	L	**0.053**	**0.058**	0.068	0.085	0.116	0.145	0.227	0.270
	L_1	0.055	0.059	**0.066**	**0.079**	**0.108**	**0.135**	**0.218**	**0.262**
80	L	**0.041**	**0.045**	0.054	0.068	0.092	0.115	0.178	0.209
	L_1	0.043	0.047	**0.052**	**0.063**	**0.085**	**0.106**	**0.167**	**0.198**

Table 5.6 The RBIAS values of GEV distribution for subzone 2(b) for L- and L_1-moments

Sample size	Methods	Return periods							
		2	5	10	20	50	100	500	1000
20	L	**0.001**	-0.006	-0.008	**-0.008**	**0.000**	0.010	**0.055**	**0.085**
	L_1	0.004	**-0.003**	-0.008	-0.009	-0.002	**0.008**	0.057	0.090
50	L	**0.001**	-0.003	-0.004	-0.004	-0.002	**0.002**	**0.017**	**0.027**
	L_1	0.002	**-0.001**	**-0.002**	**-0.002**	**0.000**	0.005	0.023	0.035
80	L	**0.000**	-0.001	-0.002	-0.002	0.000	0.003	**0.013**	**0.020**
	L_1	0.001	-0.001	-0.002	-0.002	0.000	0.003	0.015	0.022

Table 5.7 The RRMSE values of GLO distribution for subzone 2(c) for L- and L_1-moments

Sample size	Methods	Return periods							
		2	5	10	20	50	100	500	1000
20	L	**0.089**	**0.092**	0.113	0.144	0.201	**0.257**	**0.441**	**0.552**
	L_1	0.092	0.094	**0.110**	**0.140**	0.201	0.264	0.492	0.642
50	L	**0.055**	0.058	0.072	0.092	**0.127**	**0.158**	**0.250**	**0.298**
	L_1	0.057	0.058	**0.070**	**0.090**	0.128	0.166	0.281	0.346
80	L	**0.044**	0.046	0.057	0.073	**0.100**	**0.124**	**0.192**	**0.227**
	L_1	0.045	0.046	**0.055**	**0.071**	0.102	0.131	0.217	0.264

Table 5.8 The RBIAS values of GLO distribution for subzone 2(c) for L- and L_1-moments

Sample size	Methods	Return periods							
		2	5	10	20	50	100	500	1000
20	L	**0.001**	-0.006	**-0.008**	**-0.006**	**0.002**	0.013	**0.057**	**0.086**
	L_1	0.006	**-0.004**	-0.009	-0.010	-0.005	**0.005**	0.058	0.097
50	L	**0.000**	-0.002	**-0.002**	**-0.001**	0.002	0.007	**0.024**	**0.035**
	L_1	0.002	**-0.001**	-0.003	-0.003	**0.000**	**0.004**	0.026	0.040
80	L	**0.000**	-0.002	-0.002	**-0.001**	0.001	0.004	**0.015**	**0.022**
	L_1	0.001	**-0.001**	-0.002	-0.002	**0.000**	**0.003**	0.017	0.026

Table 5.9 The RRMSE values of PE3 distribution for subzone 2(a)A for L- and LQ-moments

Sample size	Methods	Return periods							
		2	5	10	20	50	100	500	1000
20	L	**0.093**	**0.091**	**0.106**	**0.126**	**0.155**	**0.176**	**0.220**	**0.237**
	LQ	0.099	0.113	0.150	0.194	0.253	0.295	0.381	0.413
50	L	**0.057**	**0.057**	**0.066**	**0.078**	**0.095**	**0.107**	**0.133**	**0.143**
	LQ	0.064	0.069	0.089	0.115	0.148	0.171	0.220	0.238
80	L	**0.045**	**0.045**	**0.052**	**0.062**	**0.074**	**0.084**	**0.103**	**0.111**
	LQ	0.051	0.054	0.070	0.090	0.117	0.136	0.175	0.189

Table 5.10 The RBIAS values of PE3 distribution for subzone 2(a)A for L- and LQ-moments

Sample size	Methods	Return periods							
		2	5	10	20	50	100	500	1000
20	L	**0.000**	**-0.002**	**-0.001**	**0.000**	**0.003**	**0.004**	**0.008**	**0.010**
	LQ	-0.001	0.030	0.056	0.080	0.107	0.126	0.164	0.178
50	L	0.000	**0.000**	**0.000**	**0.001**	**0.002**	**0.003**	**0.004**	**0.005**
	LQ	0.000	0.012	0.022	0.030	0.041	0.047	0.061	0.066
80	L	0.000	**0.000**	**0.000**	**0.000**	**0.001**	**0.001**	**0.002**	**0.002**
	LQ	0.000	0.007	0.012	0.017	0.023	0.027	0.034	0.037

Table 5.11 The RRMSE values of GPA distribution for subzone 2(a)B for L- and LQ-moments

Sample size	Methods	Return periods							
		2	5	10	20	50	100	500	1000
20	L	**0.164**	**0.150**	**0.143**	**0.152**	**0.190**	**0.234**	**0.375**	**0.455**
	LQ	0.181	0.176	0.233	0.391	0.844	1.606	10.188	26.071
50	L	**0.101**	**0.094**	**0.089**	**0.094**	**0.116**	**0.140**	**0.204**	**0.235**
	LQ	0.114	0.110	0.142	0.219	0.382	0.570	1.505	2.450
80	L	**0.080**	**0.075**	**0.070**	**0.074**	**0.091**	**0.108**	**0.155**	**0.176**
	LQ	0.091	0.087	0.112	0.169	0.282	0.399	0.857	1.202

Table 5.12 The RBIAS values of GPA distribution for subzone 2(a)B for L- and LQ-moments

Sample size	Methods	Return periods							
		2	5	10	20	50	100	500	1000
20	L	**0.005**	**-0.007**	**-0.011**	**-0.008**	**0.005**	**0.021**	**0.078**	**0.110**
	LQ	0.021	0.033	0.069	0.139	0.306	0.525	1.825	3.296
50	L	**0.002**	**-0.002**	**-0.003**	**-0.002**	**0.003**	**0.009**	**0.029**	**0.040**
	LQ	0.009	0.013	0.028	0.055	0.114	0.179	0.433	0.613
80	L	**0.001**	**-0.002**	**-0.002**	**-0.002**	**0.001**	**0.005**	**0.017**	**0.023**
	LQ	0.005	0.007	0.016	0.033	0.068	0.106	0.242	0.329

Table 5.13 The RRMSE values of PE3 distribution for subzone 2(b) for L- and LQ-moments

Sample size	Methods	Return periods							
		2	5	10	20	50	100	500	1000
20	L	**0.090**	**0.092**	**0.109**	**0.131**	**0.162**	**0.184**	**0.229**	**0.246**
	LQ	0.097	0.117	0.159	0.209	0.274	0.319	0.410	0.443
50	L	**0.055**	**0.058**	**0.068**	**0.081**	**0.099**	**0.112**	**0.138**	**0.148**
	LQ	0.062	0.071	0.095	0.123	0.159	0.184	0.235	0.253
80	L	**0.044**	**0.046**	**0.054**	**0.064**	**0.078**	**0.087**	**0.107**	**0.115**
	LQ	0.050	0.056	0.074	0.096	0.125	0.145	0.184	0.199

Table 5.14 The RBIAS values of PE3 distribution for subzone 2(b) for L- and LQ-moments

Sample size	Methods	Return periods							
		2	5	10	20	50	100	500	1000
20	L	**0.000**	**-0.002**	**-0.001**	**0.000**	**0.003**	**0.005**	**0.009**	**0.010**
	LQ	0.001	0.031	0.057	0.081	0.108	0.127	0.163	0.176
50	L	0.000	**0.000**	**0.000**	**0.001**	**0.002**	**0.003**	**0.004**	**0.005**
	LQ	0.000	0.013	0.023	0.032	0.042	0.049	0.063	0.068
80	L	0.000	**-0.001**	**0.000**	**0.000**	**0.001**	**0.001**	**0.002**	**0.003**
	LQ	0.000	0.008	0.014	0.019	0.026	0.030	0.038	0.041

Table 5.15 The RRMSE values of GLO distribution for subzone 2(c) for L- and LQ-moments

Sample size	Methods	Return periods							
		2	5	10	20	50	100	500	1000
20	L	**0.089**	**0.092**	**0.113**	**0.144**	**0.201**	**0.257**	**0.441**	**0.552**
	LQ	0.101	0.122	0.189	0.308	0.585	0.969	4.220	9.419
50	L	**0.055**	**0.058**	**0.072**	**0.092**	**0.127**	**0.158**	**0.250**	**0.298**
	LQ	0.063	0.073	0.106	0.158	0.256	0.357	0.722	0.969
80	L	**0.044**	**0.046**	**0.057**	**0.073**	**0.100**	**0.124**	**0.192**	**0.227**
	LQ	0.050	0.057	0.081	0.120	0.190	0.258	0.483	0.621

Table 5.16 The RBIAS values of GLO distribution for subzone 2(c) for L- and LQ-moments

Sample size	Methods	Return periods							
		2	5	10	20	50	100	500	1000
20	L	**0.001**	**-0.006**	**-0.008**	**-0.006**	**0.002**	**0.013**	**0.057**	**0.086**
	LQ	0.003	0.031	0.063	0.112	0.212	0.329	0.868	1.334
50	L	**0.000**	**-0.002**	**-0.002**	**-0.001**	**0.002**	**0.007**	**0.024**	**0.035**
	LQ	0.002	0.011	0.023	0.040	0.072	0.105	0.227	0.304
80	L	**0.000**	**-0.002**	**-0.002**	**-0.001**	**0.001**	**0.004**	**0.015**	**0.022**
	LQ	0.001	0.007	0.013	0.023	0.042	0.061	0.130	0.172

The bold figures (in Table 5.1 to Table 5.16) represent minimum RRMSE and RBIAS (absolute) values

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Appendix-B

List of Subroutines

The original source of subroutines:

- L-moments package developed by Hosking (2005), which can be obtained from http://lib.stat.cmu.edu/general/lmoments.
- Subroutine "DIRECT" developed by Wang (1996)

The subroutines provided in L-moments package and "DIRECT" were not directly usable for LH- and LQ-moment parameter estimation methods. Hence we have modified a few selective subroutines which are required for our computation purpose. The main executable programs, 'EXE Files' have been provided in a CD so that it can be used by others. The modified versions of 32 subroutines have been given below.

1. SUBROUTINE FOR CALCULATING SAMPLE L_l-MOMENT RATIOS OF A DATA SET

```
      SUBROUTINE DIRL1(X, N, XMOM,4)
C     THIS SUBROUTINE HAS BEEN EXTEND TO L1-MOMENTS FROM THE SUBROUTINE DIRECT
C     PROVIDED BY WANG (1996) FOR DIRECT ESTIMATION OF SAMPLE L1-MOMENTS
C     X IS THE INPUT ARRAY OF LENGTH N  CONTAINS THE DATA, IN ASCENDING
C     ORDER
C     N IS THE INPUT NUMBER OF DATA VALUES
C     XMOM IS THE OUTPUT  ARRAY OF LENGTH 4  CONTAINS THE SAMPLE
C     L1- MOMENTS
C
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      DOUBLE PRECISION X(N), XL1(4)
      DATA ZERO/0D0/
      AL1=ZERO
      AL2=ZERO
      AL3=ZERO
      AL4=ZERO
      DO 10 I=1,N
      CL1=I-1
      CL2=CL1*(I-1-1)/2
      CL3=CL2*(I-1-2)/3
      CL4=CL3*(I-1-3)/4
      CR1=N-I
      CR2=CR1*(N-I-1)/2
      CR3=CR2*(N-I-2)/3
      AL1=AL1+CL1*X(I)
      AL2=AL2+(CL2-CL1*CR1)*X(I)
      AL3=AL3+(CL3-2*CL2*CR1+CL1*CR2)*X(I)
      AL4=AL4+(CL4-3*CL3*CR1+3*CL2*CR2-CL1*CR3)*X(I)
10    CONTINUE
      C1=N
      C2=C1*(N-1)/2
      C3=C2*(N-2)/3
      C4=C3*(N-3)/4
      C5=C4*(N-4)/5
       AL1=AL1/C2
       AL2=AL2/C3/2
       AL3=AL3/C4/3
       AL4=AL4/C5/4
       XL1(1)=AL1
       XL1(2)=AL2
       XL1(3)=AL3/AL2
       XL1(4)=AL4/AL2
       RETURN
       END
```

2. SUBROUTINE FOR CALCULATING SAMPLE L_2-MOMENT RATIOS OF A DATA SET

```
      SUBROUTINE DIRL2(X, N, XMOM,4)
C     THIS SUBROUTINE HAS BEEN EXTEND TO L2-MOMENTS FROM THE SUBROUTINE DIRECT
C     PROVIDED BY WANG (1996) FOR DIRECT ESTIMATION OF SAMPLE L2-MOMENTS
C     X IS THE INPUT ARRAY OF LENGTH N  CONTAINS THE DATA, IN ASCENDING
C     ORDER
C     N IS THE INPUT NUMBER OF DATA VALUES
C     XMOM IS THE OUTPUT  ARRAY OF LENGTH 4  CONTAINS THE SAMPLE
C     L2- MOMENTS
C
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      DOUBLE PRECISION X(N), XL2(4)
      DATA ZERO/0D0/
      AL1=ZERO
      AL2=ZERO
      AL3=ZERO
      AL4=ZERO
      DO 10 I=1, N
      CL1=I-1
      CL2=CL1*(I-1-1)/2
      CL3=CL2*(I-1-2)/3
      CL4=CL3*(I-1-3)/4
      CL5=CL4*(I-1-4)/5
      CR1=N-I
      CR2=CR1*(N-I-1)/2
      CR3=CR2*(N-I-2)/3
      CR4=CR3*(N-I-3)/4
      AL1=AL1+CL2*X(I)
      AL2=AL2+(CL3-CL2*CR1)*X(I)
      AL3=AL3+(CL4-2*CL3*CR1+CL2*CR2)*X(I)
      AL4=AL4+(CL5-3*CL4*CR1+3*CL3*CR2-CL2*CR3)*X(I)
10    CONTINUE
      C1=N
      C2=C1*(N-1)/2
      C3=C2*(N-2)/3
      C4=C3*(N-3)/4
      C5=C4*(N-4)/5
      C6=C5*(N-5)/6
      AL1=AL1/C3
      AL2=AL2/C4/2
      AL3=AL3/C5/3
      AL4=AL4/C6/4
      XL2(1)=AL1
      XL2(2)=AL2
      XL2(3)=AL3/AL2
      XL2(4)=AL4/AL2
      RETURN
      END
```

3. SUBROUTINE FOR CALCULATING SAMPLE L_3-MOMENT RATIOS OF A DATA SET

```
      SUBROUTINE DIRL3(X, N, XMOM,4)
C     THIS SUBROUTINE HAS BEEN EXTEND TO L3-MOMENTS FROM THE SUBROUTINE DIRECT
C     PROVIDED BY WANG (1996) FOR DIRECT ESTIMATION OF SAMPLE L3-MOMENTS
C     X IS THE INPUT ARRAY OF LENGTH N  CONTAINS THE DATA, IN ASCENDING
C     ORDER
C     N IS THE INPUT NUMBER OF DATA VALUES
C     XMOM IS THE OUTPUT  ARRAY OF LENGTH 4  CONTAINS THE SAMPLE
C     L3- MOMENTS
C
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      DOUBLE PRECISION X(N), XL3(4)
      DATA ZERO/0D0/
      AL1=ZERO
      AL2=ZERO
      AL3=ZERO
      AL4=ZERO
      DO 10 I=1, N
      CL1=I-1
      CL2=CL1*(I-1-1)/2
      CL3=CL2*(I-1-2)/3
      CL4=CL3*(I-1-3)/4
```

```
      CL5=CL4*(I-1-4)/5
      CL6=CL5*(I-1-5)/6
      CR1=N-I
      CR2=CR1*(N-I-1)/2
      CR3=CR2*(N-I-2)/3
      AL1=AL1+CL3*X(I)
      AL2=AL2+(CL4-CL3*CR1)*X(I)
      AL3=AL3+(CL5-2*CL4*CR1+CL3*CR2)*X(I)
      AL4=AL4+(CL6-3*CL5*CR1+3*CL4*CR2-CL3*CR3)*X(I)
10    CONTINUE
      C1=N
      C2=C1*(N-1)/2
      C3=C2*(N-2)/3
      C4=C3*(N-3)/4
      C5=C4*(N-4)/5
      C6=C5*(N-5)/6
      C7=C6*(N-6)/7
      AL1=AL1/C4
      AL2=AL2/C5/2
      AL3=AL3/C6/3
      AL4=AL4/C7/4
      XL3(1)=AL1
      XL3(2)=AL2
      XL3(3)=AL3/AL2
      XL3(4)=AL4/AL2
      RETURN
      END
```

4. SUBROUTINE FOR CALCULATING SAMPLE L_4-MOMENT RATIOS OF A DATA SET

```
      SUBROUTINE DIRL4(X, N, XMOM,4)
C     THIS SUBROUTINE HAS BEEN EXTEND TO L4-MOMENTS FROM THE SUBROUTINE DIRECT
C     PROVIDED BY  WANG (1996) FOR DIRECT ESTIMATION OF SAMPLE L4-MOMENTS
C     X IS THE INPUT ARRAY OF LENGTH N  CONTAINS THE DATA, IN ASCENDING
C     ORDER
C     N IS THE INPUT NUMBER OF DATA VALUES
C     XMOM IS THE OUTPUT  ARRAY OF LENGTH 4  CONTAINS THE SAMPLE
C     L4- MOMENTS
C
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      DOUBLE PRECISION X(N), XL4(4)
      DATA ZERO/0D0/
      AL1=ZERO
      AL2=ZERO
      AL3=ZERO
      AL4=ZERO
      DO 10 I=1, N
      CL1=I-1
      CL2=CL1*(I-1-1)/2
      CL3=CL2*(I-1-2)/3
      CL4=CL3*(I-1-3)/4
      CL5=CL4*(I-1-4)/5
      CL6=CL5*(I-1-5)/6
      CL7=CL6*(I-1-6)/7
      CR1=N-I
      CR2=CR1*(N-I-1)/2
      CR3=CR2*(N-I-2)/3
      AL1=AL1+CL4*X(I)
      AL2=AL2+(CL5-CL4*CR1)*X(I)
      AL3=AL3+(CL6-2*CL5*CR1+CL4*CR2)*X(I)
      AL4=AL4+(CL7-3*CL6*CR1+3*CL5*CR2-CL4*CR3)*X(I)
10    CONTINUE
      C1=N
      C2=C1*(N-1)/2
      C3=C2*(N-2)/3
      C4=C3*(N-3)/4
      C5=C4*(N-4)/5
      C6=C5*(N-5)/6
      C7=C6*(N-6)/7
      C8=C7*(N-7)/8
      AL1=AL1/C5
      AL2=AL2/C6/2
      AL3=AL3/C7/3
      AL4=AL4/C8/4
      XL4(1)=AL1
```

```
      XL4(2)=AL2
      XL4(3)=AL3/AL2
      XL4(4)=AL4/AL2
      RETURN
      END
```

5. **SUBROUTINE FOR ESTIMATES OF PARAMETERS OF GEV DIST. BY USING L_1-MOMENTS**

```
      SUBROUTINE PELGEVL1(XMOM,PARA)
C     XMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS L1-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C     THE OTHER ROUTINE DLGAMA CAN BE OBTAINED FROM HOSKING'S (2005)
C     L-MOMENTS PACKAGE
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION XL1(3),PARA(3)
      DATA ZERO/0D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/
      DATA A0,A1,A2/ 0 4823D0,-2 1494D0,0 7269D0/
      DATA A3/-0 2103D0/
      T3=XL1(3)
      S=T3*T3
      G=A0+T3*(A1+T3*A2+S*A3)
      GOTO 40
40    PARA(3)=G
      GAM=DEXP(DLGAMA(ONE+G))
      PARA(2)=(TWO*XL1(2)*G)/(THREE*GAM*(TWO**(-G)-THREE**(-G)))
      PARA(1)=XL1(1)-PARA(2)*(ONE-TWO**(-G)*GAM)/G
      RETURN
      END
```

6. **SUBROUTINE FOR ESTIMATES OF PARAMETERS OF GEV DIST. BY USING L_2-MOMENTS**

```
      SUBROUTINE PELGEVL2(XMOM,PARA)
C     XMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS L2-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C     THE OTHER ROUTINE DLGAMA CAN BE OBTAINED FROM HOSKING'S (2005)
C     L-MOMENTS PACKAGE
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION XL2(3),PARA(3)
      DATA ZERO/0D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/,FOUR/4D0/
      DATA A0,A1,A2/ 0 5914D0,-2 3351D0,0 6442D0/
      DATA A3/-0 1616D0/
      T3=XL2(3)
      S=T3*T3
      G=A0+T3*(A1+T3*A2+S*A3)
      PARA(3)=G
      GAM=DEXP(DLGAMA(ONE+G))
      PARA(2)=(XL2(2)*G)/(TWO*GAM*(THREE**(-G)-FOUR**(-G)))
      PARA(1)=XL2(1)-PARA(2)*(ONE-(THREE**(-G))*GAM)/G
      RETURN
      END
```

7. **SUBROUTINE FOR CALCULATING PARAMETERS OF GEV DIST. BY USING L_3-MOMENTS**

```
      SUBROUTINE PELGEVL3(XMOM,PARA)
C     XMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS L3-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C     THE OTHER ROUTINE DLGAMA CAN BE OBTAINED FROM HOSKING'S (2005)
C     L-MOMENTS PACKAGE
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION XL3(3),PARA(3)
      DATA ZERO/0D0/,ONE/1D0/,TWO/2D0/,FOUR/4D0/,FIVE/5D0/
      DATA A0,A1,A2/ 0 6618D0,-2 4548D0,0 5733D0/
      DATA A3/-0 1273D0/
      T3=XL3(3)
      S=T3*T3
      G=A0+T3*(A1+T3*A2+S*A3)
      PARA(3)=G
      GAM=DEXP(DLGAMA(ONE+G))
      PARA(2)=(TWO*XL3(2)*G)/(FIVE*GAM*(FOUR**(-G)-FIVE**(-G)))
      PARA(1)=XL3(1)-PARA(2)*(ONE-(FOUR**(-G))*GAM)/G
      RETURN
      END
```

8. SUBROUTINE FOR CALCULATING PARAMETERS OF GEV DIST. BY USING L_4-MOMENTS

```
      SUBROUTINE PELGEVL4(XMOM,PARA)
C     XMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS L4-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C     THE OTHER ROUTINE DLGAMA CAN BE OBTAINED FROM HOSKING'S (2005)
C     L-MOMENTS PACKAGE.
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION XL4(3),PARA(3)
      DATA ZERO/0D0/,ONE/1D0/,THREE/3D0/,FIVE/5D0/,SIX/6D0/
      DATA A0,A1,A2/ 0.7113D0,-2 5383D0,0 5142D0/
      DATA A3/-0 1027D0/
      T3=XL4(3)
      S=T3*T3
      G=A0+T3*(A1+T3*A2+S*A3)
      PARA(3)=G
      GAM=DEXP(DLGAMA(ONE+G))
      PARA(2)=XL4(2)*G/(THREE*GAM*(FIVE**(-G)-SIX**(-G)))
      PARA(1)=XL4(1)-PARA(2)*(ONE-FIVE**(-G)*GAM)/G
      RETURN
      END
```

9. SUBROUTINE TO ESTIMATES THE PARAMETERS OF GLO DIST. BY USING L_1-MOMENTS

```
      SUBROUTINE PELGLOL1(XMOM,PARA)
C     XMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS L1-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C     THE OTHER ROUTINE DLGAMA CAN BE OBTAINED FROM HOSKING'S (2005)
C     L-MOMENTS PACKAGE
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION XL1(3),PARA(3)
      DATA ZERO/0D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/,FOUR/4D0/
      DATA X20/20D0/,X27/27D0/
      G=-(X27*XL1(3)-FOUR)/X20
      GAM=DEXP(DLGAMA(ONE+G))
      GAMM=DEXP(DLGAMA(TWO-G))
      GAMTH=DEXP(DLGAMA(THREE))
      GAMTW=DEXP(DLGAMA(TWO))
      S=GAM*GAMM
      A=(TWO*XL1(2)*GAMTH)/(THREE*S)
      PARA(1)=XL1(1)-A*(ONE-(S/GAMTW))/G
      PARA(2)=A
      PARA(3)=G
      RETURN
      END
```

10. SUBROUTINE TO ESTIMATES THE PARAMETERS OF GLO DIST. BY USING L_2-MOMENTS

```
      SUBROUTINE PELGLOL2(XMOM,PARA)
C     XMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS L2-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C     THE OTHER ROUTINE DLGAMA CAN BE OBTAINED FROM HOSKING'S (2005)
C     L-MOMENTS PACKAGE
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION XL2(3),PARA(3)
      DATA ZERO/0D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/,FOUR/4D0/
      DATA FIVE/5D0/,FIFTEN/15D0/,X24/24D0/
      G=-(X24*XL2(3)-FIVE)/FIFTEN
      GAM=DEXP(DLGAMA(ONE+G))
      GAMM=DEXP(DLGAMA(THREE-G))
      GAMMM=DEXP(DLGAMA(FOUR))
      GAMMMM=DEXP(DLGAMA(THREE))
      S=GAM*GAMM
      A=(XL2(2)*GAMMM)/(TWO*S)
      PARA(1)=XL2(1)-A*(ONE-(S/GAMMMM))/G
      PARA(2)=A
      PARA(3)=G
      RETURN
      END
```

11. SUBROUTINE TO ESTIMATES THE PARAMETERS OF GLO DIST. BY USING L_3-MOMENTS

```
      SUBROUTINE PELGLOL3(XMOM,PARA)
C     XMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS L3-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C     THE OTHER ROUTINE DLGAMA CAN BE OBTAINED FROM HOSKING'S (2005)
C     L-MOMENTS PACKAGE
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION XL3(3),PARA(3)
      DATA ZERO/0D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/,FOUR/4D0/
      DATA TFIVE/25D0/,FIVE/5D0/,FOURTEN/14D0/,SIX/6D0/
      G=-(TFIVE*XL3(3)-SIX)/FOURTEN
      GAM=DEXP(DLGAMA(ONE+G))
      GAMM=DEXP(DLGAMA(FOUR-G))
      GAMMM=DEXP(DLGAMA(FIVE))
      GAMMMM=DEXP(DLGAMA(FOUR))
      S=GAM*GAMM
      A=(TWO*XL3(2)*GAMMM)/(FIVE*S)
      PARA(1)=XL3(1)-A*(ONE-(S/GAMMMM))/G
      PARA(2)=A
      PARA(3)=G
      RETURN
      END
```

12. SUBROUTINE TO ESTIMATES THE PARAMETERS OF GLO DIST. BY USING L_4-MOMENTS

```
      SUBROUTINE PELGLOL4(XMOM,PARA)
C     XMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS L4-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C     THE OTHER ROUTINE DLGAMA CAN BE OBTAINED FROM HOSKING'S (2005)
C     L-MOMENTS PACKAGE
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION XL4(3),PARA(3)
      DATA ZERO/0D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/,SEVEN/7D0/,FOUR/4D0/
      DATA FIVE/5D0/,X14/14D0/,SIX/6D0/,X27/27D0/
      G=-((X27*XL4(3))-SEVEN)/X14
      GAM=DEXP(DLGAMA(ONE+G))
      GAMM=DEXP(DLGAMA(FIVE-G))
      GAMMM=DEXP(DLGAMA(SIX))
      GAMMMM=DEXP(DLGAMA(FIVE))
      S=GAM*GAMM
      A=(XL4(2)*GAMMM)/(THREE*S)
      PARA(1)=XL4(1)-A*(ONE-(S/GAMMMM))/G
      PARA(2)=A
      PARA(3)=G
      RETURN
      END
```

13. SUBROUTINE TO ESTIMATES THE PARAMETERS OF GPA DIST. BY USING L_1-MOMENTS

```
      SUBROUTINE PELGPAL1(XMOM,PARA)
C     XMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS L1-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C     THE OTHER ROUTINE DLGAMA CAN BE OBTAINED FROM HOSKING'S (2005)
C     L-MOMENTS PACKAGE
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION XL1(3),PARA(3)
      DATA ZERO/0D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/,FOUR/4D0/
      T3=XL1(3)
      G=FOUR*(ONE-THREE*T3)/(THREE*T3+FOUR)
      PARA(3)=G
      GAMTWO=DEXP(DLGAMA(TWO))
      GAM=DEXP(DLGAMA(ONE+G))
      GAMM=DEXP(DLGAMA(THREE+G))
      GAMMM=DEXP(DLGAMA(FOUR+G))
      S=GAMM*GAMMM
      PARA(2)=-(XL1(2)*G*S)/(THREE*GAM*(THREE*GAMM-GAMMM))
      PARA(1)=XL1(1)-PARA(2)*(ONE-(TWO*GAM*GAMTWO/GAMM))/G
      RETURN
      END
```

14. SUBROUTINE TO ESTIMATES THE PARAMETERS OF GPA DIST. BY USING L_2-MOMENTS

```
      SUBROUTINE PELGPAL2(XMOM,PARA)
C     XMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS L2-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C     THE OTHER ROUTINE DLGAMA CAN BE OBTAINED FROM HOSKING'S (2005)
C     L-MOMENTS PACKAGE
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION XL2(3),PARA(3)
      DATA ZERO/0D0/,ONE/1D0/,THREE/3D0/,FOUR/4D0/,FIVE/5D0/
      DATA TWELVE/12D0/
      T3=XL2(3)
      G=FIVE*(ONE-THREE*T3)/(THREE*T3+FIVE)
      PARA(3)=G
      GAMTHREE=DEXP(DLGAMA(THREE))
      GAM=DEXP(DLGAMA(ONE+G))
      GAMM=DEXP(DLGAMA(FOUR+G))
      GAMMM=DEXP(DLGAMA(FIVE+G))
      PARA(2)=-XL2(2)*G*GAMM*GAMMM/(TWELVE*GAM*(FOUR*GAMM-
      GAMMM))
      PARA(1)=XL2(1)-PARA(2)*(ONE-(THREE*GAM*GAMTHREE/GAMM))/G
      RETURN
      END
```

15. SUBROUTINE TO ESTIMATES THE PARAMETERS OF GPA DIST. BY USING L_3-MOMENTS

```
      SUBROUTINE PELGPAL3(XMOM,PARA)
C     XMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS L3-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C     THE OTHER ROUTINE DLGAMA CAN BE OBTAINED FROM HOSKING'S (2005)
C     L-MOMENTS PACKAGE
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION XL3(3),PARA(3)
      DATA ZERO/0D0/,ONE/1D0/,THREE/3D0/,FOUR/4D0/,FIVE/5D0/,SIX/6D0/
      DATA SIXTY/60D0/
      T3=XL3(3)
      G=SIX*(ONE-THREE*T3)/(THREE*T3+SIX)
      PARA(3)=G
      GAMFOUR=DEXP(DLGAMA(FOUR))
      GAM=DEXP(DLGAMA(ONE+G))
      GAMM=DEXP(DLGAMA(FIVE+G))
      GAMMM=DEXP(DLGAMA(SIX+G))
      PARA(2)=-XL3(2)*G*GAMM*GAMMM/(SIXTY*GAM*(FIVE*GAMM-GAMMM))
      PARA(1)=XL3(1)-PARA(2)*(ONE-(FOUR*GAM*GAMFOUR/GAMM))/G
      RETURN
      END
```

16. SUBROUTINE TO ESTIMATES THE PARAMETERS OF GPA DIST. BY USING L_4-MOMENTS

```
      SUBROUTINE PELGPAL4(XMOM,PARA)
C     XMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS L4-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C     THE OTHER ROUTINE DLGAMA CAN BE OBTAINED FROM HOSKING'S (2005)
C     L-MOMENTS PACKAGE
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION XL4(3),PARA(3)
      DATA ZERO/0D0/,ONE/1D0/,THREE/3D0/,FIVE/5D0/,SIX/6D0/,SEVEN/7D0/
      DATA X360/360D0/
      T3=XL4(3)
      G=SEVEN*(ONE-THREE*T3)/(THREE*T3+SEVEN)
      PARA(3)=G
      GAMFIVE=DEXP(DLGAMA(FIVE))
      GAM=DEXP(DLGAMA(ONE+G))
      GAMM=DEXP(DLGAMA(SIX+G))
      GAMMM=DEXP(DLGAMA(SEVEN+G))
      PARA(2)=-XL4(2)*G*GAMM*GAMMM/(X360*GAM*(SIX*GAMM-GAMMM))
      PARA(1)=XL4(1)-PARA(2)*(ONE-(FIVE*GAM*GAMFIVE/GAMM))/G
      RETURN
      END
```

17. SUBROUTINE FOR CALCULATING PARAMETERS OF KAPA DIST. BY USING L_l-MOMENTS

```
      SUBROUTINE PELKAPL1(XMOM,PARA,IFAIL)
C     THIS SUBROUTINE HAS BEEN EXTEND TO L1-MOMENTS FROM SUBROUTINE PELKAP
C     PROVIDED BY HOSKING (2005) IN L-MOMENTS PACKAGE
C     XMOM IS THE INPUT ARRAY OF LENGTH 4  CONTAINS THE L1-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 4  CONTAINS THE PARAMETERS OF
C     THE DISTRIBUTION
C     IFAIL IS THE OUTPUT FAIL FLAG  ON EXIT, IT IS SET AS FOLLOWS
C           0  SUCCESSFUL EXIT
C           1  L1-MOMENTS INVALID
C           2  (TAU-3, TAU-4) LIES ABOVE THE GENERALIZED-LOGISTIC
C              LINE (SUGGESTS THAT L1-MOMENTS ARE NOT CONSISTENT
C              WITH ANY KAPPA DISTRIBUTION WITH H GT -1)
C           3  ITERATION FAILED TO CONVERGE
C           4  UNABLE TO MAKE PROGRESS FROM CURRENT POINT IN
C              ITERATION
C           5  ITERATION ENCOUNTERED NUMERICAL DIFFICULTIES -
C              OVERFLOW WOULD HAVE BEEN LIKELY TO OCCUR
C           6  ITERATION FOR H AND K CONVERGED, BUT OVERFLOW
C              WOULD HAVE OCCURRED WHEN CALCULATING XI AND ALPHA
C      THE OTHER ROUTINES DLGAMA AND DIGAMD CAN BE OBTAINED FROM
C     L-MOMENTS PACKAGE OF HOSKING (2005)
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION XMOM(4),PARA(4)
      DATA ZERO/0D0/,HALF/0 5D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/,FOUR/4D0/
      DATA FIVE/5D0/,SIX/6D0/,SEVEN/7D0/,EIGHT/8D0/,X16/16D0/,X10/10D0/
      DATA X25/25D0/,X75/75D0/,X35/35D0/
      DATA P725/0 725D0/,P8/0 8D0/
      DATA EPS/1D-6/,MAXIT/20/,MAXSR/10/,HSTART/1 001D0/,BIG/10D0/
      DATA OFLEXP/170D0/,OFLGAM/53D0/
      T3=XMOM(3)
      T4=XMOM(4)
      DO 10 I=1,4
   10 PARA(I)=ZERO
      IF(XMOM(2) LE ZERO)GOTO 1000
      IF(DABS(T3) GE ONE OR DABS(T4) GE ONE)GOTO 1000
      IF(T4 LE (FIVE*T3*T3-ONE)/FOUR)GOTO 1000
      IF(T4 GE (FIVE*T3*T3+ONE)/SIX )GOTO 1010
      G=FOUR*(ONE-THREE*T3)/(THREE*T3+FOUR)
      H=HSTART
      Z=G+H*P725
      XDIST=BIG
      DO 100 IT=1,MAXIT
      DO 40 I=1,MAXSR
      IF(G GT OFLGAM)GOTO 1020
      IF(H GT ZERO)GOTO 20
      U1=DEXP(DLGAMA(  -ONE/H-G)-DLGAMA(  -ONE/H+ONE))
      U2=DEXP(DLGAMA(  -TWO/H-G)-DLGAMA(  -TWO/H+ONE))
      U3=DEXP(DLGAMA(-THREE/H-G)-DLGAMA(-THREE/H+ONE))
      U4=DEXP(DLGAMA( -FOUR/H-G)-DLGAMA( -FOUR/H+ONE))
      U5=DEXP(DLGAMA( -FIVE/H-G)-DLGAMA( -FIVE/H+ONE))
      GOTO 30
   20 U1=DEXP(DLGAMA(  ONE/H)-DLGAMA(  ONE/H+ONE+G))
      U2=DEXP(DLGAMA(  TWO/H)-DLGAMA(  TWO/H+ONE+G))
      U3=DEXP(DLGAMA(THREE/H)-DLGAMA(THREE/H+ONE+G))
      U4=DEXP(DLGAMA( FOUR/H)-DLGAMA( FOUR/H+ONE+G))
      U5=DEXP(DLGAMA( FIVE/H)-DLGAMA( FIVE/H+ONE+G))
   30 CONTINUE
      ALAM2=(THREE/TWO)*U2-(THREE/TWO)*U3
      ALAM3=-TWO*U2+(X16/THREE)*U3-(X10/THREE)*U4
      ALAM4=(FIVE/TWO)*U2-(X25/TWO)*U3+(X75/FOUR)*U4
     *         -(X35/FOUR)*U5
      IF(ALAM2 EQ ZERO)GOTO 1020
      TAU3=ALAM3/ALAM2
      TAU4=ALAM4/ALAM2
      E1=TAU3-T3
      E2=TAU4-T4
      DIST=DMAX1(DABS(E1),DABS(E2))
      IF(DIST LT XDIST)GOTO 50
      DEL1=HALF*DEL1
      DEL2=HALF*DEL2
      G=XG-DEL1
```

```
      U6=DEXP(DLGAMA( SIX/H)-DLGAMA( SIX/H+ONE+G))
   30 CONTINUE
      ALAM2=SIX*U3-EIGHT*U4
      ALAM3=-TEN*U3+33 33D0*U4-25D0*U5
      ALAM4=15D0*U3-90D0*U4+157 5D0*U5-84D0*U6
      IF(ALAM2 EQ ZERO)GOTO 1020
      TAU3=ALAM3/ALAM2
      TAU4=ALAM4/ALAM2
      E1=TAU3-T3
      E2=TAU4-T4
      DIST=DMAX1(DABS(E1),DABS(E2))
      IF(DIST LT XDIST)GOTO 50
      DEL1=HALF*DEL1
      DEL2=HALF*DEL2
      G=XG-DEL1
      H=XH-DEL2
   40 CONTINUE
      IFAIL=4
      RETURN
   50 CONTINUE
      IF(DIST LT EPS)GOTO 110
      XG=G
      XH=H
      XZ=Z
      XDIST=DIST
      RHH=ONE/(H*H)
      IF(H GT ZERO)GOTO 60
      U3G=-U3*DIGAMD(-THREE/H-G)
      U4G=-U4*DIGAMD( -FOUR/H-G)
      U5G=-U5*DIGAMD( -FIVE/H-G)
      U6G=-U6*DIGAMD(  -SIX/H-G)
      U3H=THREE*RHH*(-U3G-U3*DIGAMD(-THREE/H+ONE))
      U4H= FOUR*RHH*(-U4G-U4*DIGAMD( -FOUR/H+ONE))
      U5H= FIVE*RHH*(-U5G-U5*DIGAMD( -FIVE/H+ONE))
      U6H= SIX*RHH*( -U6G-U6*DIGAMD(  -SIX/H+ONE))
      GOTO 70
   60 U3G=-U3*DIGAMD(THREE/H+ONE+G)
      U4G=-U4*DIGAMD( FOUR/H+ONE+G)
      U5G=-U5*DIGAMD( FIVE/H+ONE+G)
      U6G=-U6*DIGAMD(  SIX/H+ONE+G)
      U3H=THREE*RHH*(-U3G-U3*DIGAMD(THREE/H))
      U4H= FOUR*RHH*(-U4G-U4*DIGAMD( FOUR/H))
      U5H= FIVE*RHH*(-U5G-U5*DIGAMD( FIVE/H))
      U6H=  SIX*RHH*(-U6G-U6*DIGAMD(  SIX/H))
   70 CONTINUE
      DL2G=SIX*U3G-EIGHT*U4G
      DL2H=SIX*U3H-EIGHT*U4H
      DL3G=-TEN*U3G+33 33D0*U4G-25D0*U5G
      DL3H=-TEN*U3H+33 33D0*U4H-25D0*U5H
      DL4G=15D0*U3G-90D0*U4G+157 5D0*U5G-84D0*U6G
      DL4H=15D0*U3H-90D0*U4H+157 5D0*U5H-84D0*U6H
      D11=(DL3G-TAU3*DL2G)/ALAM2
      D12=(DL3H-TAU3*DL2H)/ALAM2
      D21=(DL4G-TAU4*DL2G)/ALAM2
      D22=(DL4H-TAU4*DL2H)/ALAM2
      DET=D11*D22-D12*D21
      H11= D22/DET
      H12=-D12/DET
      H21=-D21/DET
      H22= D11/DET
      DEL1=E1*H11+E2*H12
      DEL2=E1*H21+E2*H22
      G=XG-DEL1
      H=XH-DEL2
      Z=G+H*P725
      FACTOR=ONE
      IF(G LE -ONE)FACTOR=P8*(XG+ONE)/DEL1
      IF(H LE -ONE)FACTOR=DMIN1(FACTOR,P8*(XH+ONE)/DEL2)
      IF(Z LE -ONE)FACTOR=DMIN1(FACTOR,P8*(XZ+ONE)/(XZ-Z))
      IF(H LE ZERO AND G*H LE -ONE)
     FACTOR=DMIN1(FACTOR,P8*(XG*XH+ONE)/(XG*XH-G*H))
      IF(FACTOR EQ ONE)GOTO 80
      DEL1=DEL1*FACTOR
      DEL2=DEL2*FACTOR
      G=XG-DEL1
```

```
      H=XH-DEL2
      Z=G+H*P725
   80 CONTINUE
  100 CONTINUE
      IFAIL=3
      RETURN
  110 IFAIL=0
      PARA(4)=H
      PARA(3)=G
      TEMP=DLGAMA(ONE+G)
      IF(TEMP GT OFLEXP)GOTO 1030
      GAM=DEXP(TEMP)
      TEMP=(ONE+G)*DLOG(DABS(H))
      IF(TEMP GT OFLEXP)GOTO 1030
      HH=DEXP(TEMP)
      PARA(2)=XMOM(2)*G*HH/(ALAM2*GAM)
      PARA(1)=XMOM(1)-PARA(2)/G*(ONE-THREE*GAM*U3/HH)
      RETURN
 1000 IFAIL=1
      RETURN
 1010 IFAIL=2
      RETURN
 1020 IFAIL=5
      RETURN
 1030 IFAIL=6
      RETURN
      END
```

19. SUBROUTINE FOR CALCULATING PARAMETERS OF KAPA DIST. BY USING L_3-MOMENTS

```
      SUBROUTINE PELKAPL3(XMOM,PARA,IFAIL)
C     THIS SUBROUTINE HAS BEEN EXTEND TO L3-MOMENTS FROM SUBROUTINE PELKAP
C     PROVIDED BY HOSKING (2005) IN L-MOMENTS PACKAGE
C     XMOM IS THE INPUT ARRAY OF LENGTH 4  CONTAINS THE L3-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 4  CONTAINS THE PARAMETERS OF
C     THE DISTRIBUTION
C     IFAIL IS THE OUTPUT FAIL FLAG  ON EXIT, IT IS SET AS FOLLOWS
C            0  SUCCESSFUL EXIT
C            1  L3-MOMENTS INVALID
C            2  (TAU-3, TAU-4) LIES ABOVE THE GENERALIZED-LOGISTIC
C               LINE (SUGGESTS THAT L3-MOMENTS ARE NOT CONSISTENT
C               WITH ANY KAPPA DISTRIBUTION WITH H GT -1)
C            3  ITERATION FAILED TO CONVERGE
C            4  UNABLE TO MAKE PROGRESS FROM CURRENT POINT IN
C               ITERATION
C            5  ITERATION ENCOUNTERED NUMERICAL DIFFICULTIES -
C               OVERFLOW WOULD HAVE BEEN LIKELY TO OCCUR
C            6  ITERATION FOR H AND K CONVERGED, BUT OVERFLOW
C               WOULD HAVE OCCURRED WHEN CALCULATING XI AND ALPHA
C      THE OTHER ROUTINES DLGAMA AND DIGAMD CAN BE OBTAINED FROM THE
C      L-MOMENTS PACKAGE OF HOSKING (2005)
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION XMOM(4),PARA(4)
      DATA ZERO/0D0/,HALF/0 5D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/,FOUR/4D0/
      DATA FIVE/5D0/,SIX/6D0/,SEVEN/7D0/,TEN/10D0/,TWENTY/20D0/
      DATA TWENTY/20D0/,SIXTY/60D0/
      DATA P725/0 725D0/,P8/0 8D0/
      DATA EPS/1D-6/,MAXIT/20/,MAXSR/10/,HSTART/1 001D0/,BIG/10D0/
      DATA OFLEXP/170D0/,OFLGAM/53D0/
      T3=XMOM(3)
      T4=XMOM(4)
      DO 10 I=1,4
   10 PARA(I)=ZERO
      IF(XMOM(2) LE ZERO)GOTO 1000
      IF(DABS(T3) GE ONE OR DABS(T4) GE ONE)GOTO 1000
      IF(T4 LE (FIVE*T3*T3-ONE)/FOUR)GOTO 1000
      IF(T4 GE (FIVE*T3*T3+ONE)/SIX )GOTO 1010
      G=SIX*(ONE-THREE*T3)/(THREE*T3+SIX)
      H=HSTART
      Z=G+H*P725
      XDIST=BIG
      DO 100 IT=1,MAXIT
      DO 40 I=1,MAXSR
```

```
      IF(G GT OFLGAM)GOTO 1020
      IF(H GT ZERO)GOTO 20
      U3=DEXP(DLGAMA(-THREE/H-G)-DLGAMA(-THREE/H+ONE))
      U4=DEXP(DLGAMA( -FOUR/H-G)-DLGAMA( -FOUR/H+ONE))
      U5=DEXP(DLGAMA( -FIVE/H-G)-DLGAMA( -FIVE/H+ONE))
      U6=DEXP(DLGAMA(  -SIX/H-G)-DLGAMA(  -SIX/H+ONE))
      U7=DEXP(DLGAMA(-SEVEN/H-G)-DLGAMA(-SEVEN/H+ONE))
      GOTO 30
   20 U3=DEXP(DLGAMA(THREE/H)-DLGAMA(THREE/H+ONE+G))
      U4=DEXP(DLGAMA( FOUR/H)-DLGAMA( FOUR/H+ONE+G))
      U5=DEXP(DLGAMA( FIVE/H)-DLGAMA( FIVE/H+ONE+G))
      U6=DEXP(DLGAMA(  SIX/H)-DLGAMA(  SIX/H+ONE+G))
      U7=DEXP(DLGAMA(SEVEN/H)-DLGAMA(SEVEN/H+ONE+G))
   30 CONTINUE
      ALAM2=TEN*U4-12 5D0*U5
      ALAM3=-TWENTY*U4+SIXTY*U5-42D0*U6
      ALAM4=35D0*U4-183 75D0*U5+294D0*U6-147D0*U7
      IF(ALAM2 EQ ZERO)GOTO 1020
      TAU3=ALAM3/ALAM2
      TAU4=ALAM4/ALAM2
      E1=TAU3-T3
      E2=TAU4-T4
      DIST=DMAX1(DABS(E1),DABS(E2))
      IF(DIST LT XDIST)GOTO 50
      DEL1=HALF*DEL1
      DEL2=HALF*DEL2
      G=XG-DEL1
      H=XH-DEL2
   40 CONTINUE
      IFAIL=4
      RETURN
   50 CONTINUE
      IF(DIST LT EPS)GOTO 110
      XG=G
      XH=H
      XZ=Z
      XDIST=DIST
      RHH=ONE/(H*H)
      IF(H GT ZERO)GOTO 60
      U3G=-U3*DIGAMD(-THREE/H-G)
      U4G=-U4*DIGAMD( -FOUR/H-G)
      U5G=-U5*DIGAMD( -FIVE/H-G)
      U6G=-U6*DIGAMD(  -SIX/H-G)
      U7G=-U7*DIGAMD(-SEVEN/H-G)
      U3H=THREE*RHH*(-U3G-U3*DIGAMD(-THREE/H+ONE))
      U4H= FOUR*RHH*(-U4G-U4*DIGAMD( -FOUR/H+ONE))
      U5H= FIVE*RHH*(-U5G-U5*DIGAMD( -FIVE/H+ONE))
      U6H=  SIX*RHH*( -U6G-U6*DIGAMD(  -SIX/H+ONE))
      U7H=SEVEN*RHH*( -U7G-U7*DIGAMD(-SEVEN/H+ONE))
      GOTO 70
   60 U3G=-U3*DIGAMD(THREE/H+ONE+G)
      U4G=-U4*DIGAMD( FOUR/H+ONE+G)
      U5G=-U5*DIGAMD( FIVE/H+ONE+G)
      U6G=-U6*DIGAMD(  SIX/H+ONE+G)
      U7G=-U7*DIGAMD(SEVEN/H+ONE+G)
      U3H=THREE*RHH*(-U3G-U3*DIGAMD(THREE/H))
      U4H= FOUR*RHH*(-U4G-U4*DIGAMD( FOUR/H))
      U5H= FIVE*RHH*(-U5G-U5*DIGAMD( FIVE/H))
      U6H=  SIX*RHH*(-U6G-U6*DIGAMD(  SIX/H))
      U7H=SEVEN*RHH*(-U7G-U7*DIGAMD(SEVEN/H))
   70 CONTINUE
      DL2G=TEN*U4G-12 5D0*U5G
      DL2H=TEN*U4H-12 5D0*U5H
      DL3G=-TWENTY*U4G+SIXTY*U5G-42D0*U6G
      DL3H=-TWENTY*U4H+SIXTY*U5H-42D0*U6H
      DL4G=35D0*U4G-183 75D0*U5G+294D0*U6G-147D0*U7G
      DL4H=35D0*U4H-183 75D0*U5H+294D0*U6H-147D0*U7H
      D11=(DL3G-TAU3*DL2G)/ALAM2
      D12=(DL3H-TAU3*DL2H)/ALAM2
      D21=(DL4G-TAU4*DL2G)/ALAM2
      D22=(DL4H-TAU4*DL2H)/ALAM2
      DET=D11*D22-D12*D21
      H11= D22/DET
      H12=-D12/DET
      H21=-D21/DET
```

```
      H22= D11/DET
      DEL1=E1*H11+E2*H12
      DEL2=E1*H21+E2*H22
      G=XG-DEL1
      H=XH-DEL2
      Z=G+H*P725
      FACTOR=ONE
      IF(G LE -ONE)FACTOR=P8*(XG+ONE)/DEL1
      IF(H LE -ONE)FACTOR=DMIN1(FACTOR,P8*(XH+ONE)/DEL2)
      IF(Z LE -ONE)FACTOR=DMIN1(FACTOR,P8*(XZ+ONE)/(XZ-Z))
      IF(H LE ZERO AND G*H LE -ONE)
     * FACTOR=DMIN1(FACTOR,P8*(XG*XH+ONE)/(XG*XH-G*H))
      IF(FACTOR EQ ONE)GOTO 80
      DEL1=DEL1*FACTOR
      DEL2=DEL2*FACTOR
      G=XG-DEL1
      H=XH-DEL2
      Z=G+H*P725
   80 CONTINUE
  100 CONTINUE
      IFAIL=3
      RETURN
  110 IFAIL=0
      PARA(4)=H
      PARA(3)=G
      TEMP=DLGAMA(ONE+G)
      IF(TEMP GT OFLEXP)GOTO 1030
      GAM=DEXP(TEMP)
      TEMP=(ONE+G)*DLOG(DABS(H))
      IF(TEMP GT OFLEXP)GOTO 1030
      HH=DEXP(TEMP)
      PARA(2)=XMOM(2)*G*HH/(ALAM2*GAM)
      PARA(1)=XMOM(1)-PARA(2)/G*(ONE-FOUR*GAM*U4/HH)
      RETURN
 1000 IFAIL=1
      RETURN
 1010 IFAIL=2
      RETURN
 1020 IFAIL=5
      RETURN
 1030 IFAIL=6
      RETURN
      END
```

20. SUBROUTINE FOR CALCULATING PARAMETERS OF KAPA DIST. BY USING L_4-MOMENTS

```
      SUBROUTINE PELKAPL4(XMOM,PARA,IFAIL)
C     THIS SUBROUTINE HAS BEEN EXTEND TO L4-MOMENTS FROM SUBROUTINE PELKAP
C     PROVIDED BY HOSKING (2005) IN L-MOMENTS PACKAGE
C     XMOM IS THE INPUT ARRAY OF LENGTH 4  CONTAINS THE L4-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 4  CONTAINS THE PARAMETERS OF
C     THE DISTRIBUTION
C     IFAIL IS THE OUTPUT FAIL FLAG  ON EXIT, IT IS SET AS FOLLOWS
C            0  SUCCESSFUL EXIT
C            1  L4-MOMENTS INVALID
C            2  (TAU-3, TAU-4) LIES ABOVE THE GENERALIZED-LOGISTIC
C               LINE (SUGGESTS THAT L4-MOMENTS ARE NOT CONSISTENT
C               WITH ANY KAPPA DISTRIBUTION WITH H GT -1)
C            3  ITERATION FAILED TO CONVERGE
C            4  UNABLE TO MAKE PROGRESS FROM CURRENT POINT IN
C               ITERATION
C            5  ITERATION ENCOUNTERED NUMERICAL DIFFICULTIES -
C               OVERFLOW WOULD HAVE BEEN LIKELY TO OCCUR
C            6  ITERATION FOR H AND K CONVERGED, BUT OVERFLOW
C               WOULD HAVE OCCURRED WHEN CALCULATING XI AND ALPHA
C
C      THE OTHER ROUTINES DLGAMA AND DIGAMD CAN BE OBTAINED FROM
C     L-MOMENTS PACKAGE OF HOSKING (2005)
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION XMOM(4),PARA(4)
      DATA ZERO/0D0/,HALF/0 5D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/,FOUR/4D0/
      DATA FIVE/5D0/,SIX/6D0/,SEVEN/7D0/,EIGHT/8D0/,X15/15D0/,X18/18D0/
```

```
      DATA X35/35D0/,X98/98D0/,X70/70D0/,X196/196D0/,X336/336D0/
      DATA X240/240D0/,X504/504D0/
      DATA P725/0 725D0/,P8/0 8D0/
      DATA EPS/1D-6/,MAXIT/20/,MAXSR/10/,HSTART/1 001D0/,BIG/10D0/
      DATA OFLEXP/170D0/,OFLGAM/53D0/
      T3=XMOM(3)
      T4=XMOM(4)
      DO 10 I=1,4
   10 PARA(I)=ZERO
      IF(XMOM(2) LE ZERO)GOTO 1000
      IF(DABS(T3) GE ONE OR DABS(T4) GE ONE)GOTO 1000
      IF(T4 LE (FIVE*T3*T3-ONE)/FOUR)GOTO 1000
      IF(T4 GE (FIVE*T3*T3+ONE)/SIX )GOTO 1010
      G=SEVEN*(ONE-THREE*T3)/(THREE*T3+SEVEN)
      H=HSTART
      Z=G+H*P725
      XDIST=BIG
      DO 100 IT=1,MAXIT
      DO 40 I=1,MAXSR
      IF(G GT OFLGAM)GOTO 1020
      IF(H GT ZERO)GOTO 20
      U5=DEXP(DLGAMA( -FIVE/H-G)-DLGAMA( -FIVE/H+ONE))
      U6=DEXP(DLGAMA( -SIX/H-G)-DLGAMA( -SIX/H+ONE))
      U7=DEXP(DLGAMA(-SEVEN/H-G)-DLGAMA(-SEVEN/H+ONE))
      U8=DEXP(DLGAMA(-EIGHT/H-G)-DLGAMA(-EIGHT/H+ONE))
      GOTO 30
   20 U5=DEXP(DLGAMA( FIVE/H)-DLGAMA( FIVE/H+ONE+G))
      U6=DEXP(DLGAMA( SIX/H)-DLGAMA( SIX/H+ONE+G))
      U7=DEXP(DLGAMA(SEVEN/H)-DLGAMA(SEVEN/H+ONE+G))
      U8=DEXP(DLGAMA(EIGHT/H)-DLGAMA(EIGHT/H+ONE+G))
   30 CONTINUE
      ALAM2=X15*U5-X18*U6
      ALAM3=-X35*U5+X98*U6-((X196*U7)/THREE)
      ALAM4=X70*U5-X336*U6+X504*U7-X240*U8
      IF(ALAM2 EQ ZERO)GOTO 1020
      TAU3=ALAM3/ALAM2
      TAU4=ALAM4/ALAM2
      E1=TAU3-T3
      E2=TAU4-T4
      DIST=DMAX1(DABS(E1),DABS(E2))
      IF(DIST LT XDIST)GOTO 50
      DEL1=HALF*DEL1
      DEL2=HALF*DEL2
      G=XG-DEL1
      H=XH-DEL2
   40 CONTINUE
      IFAIL=4
      RETURN
   50 CONTINUE
      IF(DIST LT EPS)GOTO 110
      XG=G
      XH=H
      XZ=Z
      XDIST=DIST
      RHH=ONE/(H*H)
      IF(H GT ZERO)GOTO 60
      U5G=-U5*DIGAMD( -FIVE/H-G)
      U6G=-U6*DIGAMD( -SIX/H-G)
      U7G=-U7*DIGAMD(-SEVEN/H-G)
      U8G=-U8*DIGAMD(-EIGHT/H-G)
      U5H= FIVE*RHH*(-U5G-U5*DIGAMD( -FIVE/H+ONE))
      U6H= SIX*RHH*( -U6G-U6*DIGAMD( -SIX/H+ONE))
      U7H=SEVEN*RHH*( -U7G-U7*DIGAMD(-SEVEN/H+ONE))
      U8H=EIGHT*RHH*( -U8G-U8*DIGAMD(-EIGHT/H+ONE))
      GOTO 70
   60 U5G=-U5*DIGAMD( FIVE/H+ONE+G)
      U6G=-U6*DIGAMD( SIX/H+ONE+G)
      U7G=-U7*DIGAMD(SEVEN/H+ONE+G)
      U8G=-U8*DIGAMD(EIGHT/H+ONE+G)
      U5H= FIVE*RHH*(-U5G-U5*DIGAMD( FIVE/H))
      U6H= SIX*RHH*(-U6G-U6*DIGAMD( SIX/H))
      U7H=SEVEN*RHH*(-U7G-U7*DIGAMD(SEVEN/H))
      U8H=EIGHT*RHH*(-U8G-U8*DIGAMD(EIGHT/H))
   70 CONTINUE
      DL2G=X15*U5G-X18*U6G
```

```
      DL2H=X15*U5H-X18*U6H
      DL3G=-X35*U5G+X98*U6G-((X196*U7G)/THREE)
      DL3H=-X35*U5H+X98*U6H-((X196*U7H)/THREE)
      DL4G=X70*U5G-X336*U6G+X504*U7G-X240*U8G
      DL4H=X70*U5H-X336*U6H+X504*U7H-X240*U8H
      D11=(DL3G-TAU3*DL2G)/ALAM2
      D12=(DL3H-TAU3*DL2H)/ALAM2
      D21=(DL4G-TAU4*DL2G)/ALAM2
      D22=(DL4H-TAU4*DL2H)/ALAM2
      DET=D11*D22-D12*D21
      H11= D22/DET
      H12=-D12/DET
      H21=-D21/DET
      H22= D11/DET
      DEL1=E1*H11+E2*H12
      DEL2=E1*H21+E2*H22
      G=XG-DEL1
      H=XH-DEL2
      Z=G+H*P725
      FACTOR=ONE
      IF(G LE -ONE)FACTOR=P8*(XG+ONE)/DEL1
      IF(H LE -ONE)FACTOR=DMIN1(FACTOR,P8*(XH+ONE)/DEL2)
      IF(Z LE -ONE)FACTOR=DMIN1(FACTOR,P8*(XZ+ONE)/(XZ-Z))
      IF(H LE ZERO AND G*H LE -ONE)
     * FACTOR=DMIN1(FACTOR,P8*(XG*XH+ONE)/(XG*XH-G*H))
      IF(FACTOR EQ ONE)GOTO 80
      DEL1=DEL1*FACTOR
      DEL2=DEL2*FACTOR
      G=XG-DEL1
      H=XH-DEL2
      Z=G+H*P725
   80 CONTINUE
  100 CONTINUE
      IFAIL=3
      RETURN
  110 IFAIL=0
      PARA(4)=H
      PARA(3)=G
      TEMP=DLGAMA(ONE+G)
      IF(TEMP GT OFLEXP)GOTO 1030
      GAM=DEXP(TEMP)
      TEMP=(ONE+G)*DLOG(DABS(H))
      IF(TEMP GT OFLEXP)GOTO 1030
      HH=DEXP(TEMP)
      PARA(2)=XMOM(2)*G*HH/(ALAM2*GAM)
      PARA(1)=XMOM(1)-PARA(2)/G*(ONE-FIVE*GAM*U5/HH)
      RETURN
 1000 IFAIL=1
      RETURN
 1010 IFAIL=2
      RETURN
 1020 IFAIL=5
      RETURN
 1030 IFAIL=6
      RETURN
      END
```

21. SUBROUTINE FOR CALCULATING THE DISCORDANCY, HETEROGENEITY, Z- STATISTIC VALUES OF THREE DIST. I.E. GEV, GLO AND GPA AND ALSO ESTIMATES THE REGIONAL PARAMETERS AND QUANTILES FOR L_1-MOMENTS

```
      SUBROUTINE REGTSTL1(NSITES,NAMES,LEN,XMOM,A,B,SEED,NSIM,NPROB,
     *  PROB,KPRINT,KOUT,RMOM,D,VOBS,VBAR,VSD,H,Z,PARA)
C     THIS SUBROUTINE HAS BEEN EXTEND TO L1-MOMENTS FROM SUBROUTINE REGTST
C     PROVIDED BY HOSKING (2005) IN L-MOMENTS PACKAGE
C     XMOM IS THE INPUT ARRAY OF DIMENSION (4, NSITES), ARRAY CONTAINING THE
C     FIRST 4 SAMPLE L1-MOMENTS FOR EACH SITE
C     THE OTHER INPUT AND OUTPUT PARAMETERS ARE SAME LIKE SUBROUTINE REGTST
C     THE OTHER ROUTINES I E  SORT, DURAND, QUAGEV, QUAKAP, QUAGLO AND QUAGPA
C     CAN BE OBTAINED FROM L-MOMENTS PACKAGE OF HOSKING (2005)
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER (MAXNS=200,MAXREC=200,MAXQ=30)
      CHARACTER*1 BLANK,STAR,LOOK1,LOOK2
```

```
      CHARACTER*12 NAMES(NSITES)
      CHARACTER*18 DISTRI(3)
      DOUBLE PRECISION D(NSITES),DC1(14),DC2(18),PARA(3,3),H(3),
     * PROB(NPROB),Q(MAXQ),RMOM(4),RPARA(4),SMAT(3,3),TMOM(4),T4FIT(3),
     * VBAR(3),VOBS(3),VSD(3),WORK(MAXNS,3),X(MAXREC),XMOM(4,NSITES),
     * Z(3)
      INTEGER LEN(NSITES)
      DATA BLANK/' '/,STAR/'*'/
      DATA ZERO/0D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/
      DATA DISTRI/
     * 'GEN LOGISTIC    ','GEN EXTREME VALUE',
     * 'GEN PARETO      '/
      DATA GLOC0,GLOC1,GLOC2/0 1167D0, 0 0187D0, 0 8859D0/
      DATA GEVC0,GEVC1,GEVC2,GEVC3,GEVC4/
     * 0 0666D0, 0 1208D0, 0 8711D0,-0 0484D0, 0 0084D0/
      DATA GPAC1,GPAC2,GPAC3,GPAC4/
     * 0 2083D0, 0 9115D0, -0 1134D0, 0 0124D0/
C
C       CRITICAL VALUES FOR D, H AND Z STATISTICS
C
      DATA DC1/4*3D0,1 3330D0,1 6481D0,1 9166D0,2 1401D0,2 3287D0,
     *          2 4906D0,2 6321D0,2 7573D0,2 8694D0,2 9709D0/
      DATA DC2/4*4D0,1 3333D0,1 6648D0,1 9821D0,2 2728D0,2 5337D0,
     *          2 7666D0,2 9748D0,3 1620D0,3 3310D0,3 4844D0,
     *          3 6246D0,3 7532D0,3 8718D0,3 9816D0/
      DATA HCRIT1,HCRIT2/1D0,2D0/
      DATA ZCRIT/1 645D0/
C
C       INITIALIZE ARRAYS
C
      NMAX=0
      SUMLEN=0
      DO 10 I=1,NSITES
      NREC=LEN(I)
      IF(NREC GT NMAX)NMAX=NREC
      SUMLEN=SUMLEN+NREC
   10 D(I)=ZERO
      DO 20 K=1,3
      VOBS(K)=ZERO
      VBAR(K)=ZERO
      VSD(K)=ZERO
      H(K)=ZERO
   20 CONTINUE
      DO 30 IDIST=1,3
   30 Z(IDIST)=ZERO
      DO 40 IPARA=1,3
      DO 40 IDIST=1,3
   40 PARA(IPARA,IDIST)=ZERO
      IF(NSITES GT MAXNS)GOTO 1000
C
C       CALCULATE THE WEIGHTED MEAN OF L1-CV, L1-SKEW, L1-KURTOSIS
C
      DO 60 K=2,4
      RMOM(K)=ZERO
      DO 50 I=1,NSITES
   50 RMOM(K)=RMOM(K)+LEN(I)*XMOM(K,I)
   60 RMOM(K)=RMOM(K)/SUMLEN
      RMOM(1)=ONE
      IF(NSITES LE 3)GOTO 135
      SUM2=ZERO
      SUM3=ZERO
      SUM4=ZERO
      DO 70 I=1,NSITES
      SUM2=SUM2+XMOM(2,I)
      SUM3=SUM3+XMOM(3,I)
      SUM4=SUM4+XMOM(4,I)
   70 CONTINUE
      SUM2=SUM2/NSITES
      SUM3=SUM3/NSITES
      SUM4=SUM4/NSITES
      DO 80 I=1,NSITES
      WORK(I,1)=XMOM(2,I)-SUM2
      WORK(I,2)=XMOM(3,I)-SUM3
      WORK(I,3)=XMOM(4,I)-SUM4
   80 CONTINUE
```

```
      DO 100 J=1,3
      DO 100 K=J,3
      SMAT(J,K)=ZERO
      DO 90 I=1,NSITES
   90 SMAT(J,K)=SMAT(J,K)+WORK(I,J)*WORK(I,K)
  100 CONTINUE
      DO 110 K=1,3
      IF(SMAT(1,1) LE ZERO)GOTO 1030
      TEMP0=ONE/SMAT(1,1)
      TEMP1=-SMAT(1,2)*TEMP0
      TEMP2=-SMAT(1,3)*TEMP0
      IF(K GT 2)TEMP1=-TEMP1
      IF(K GT 1)TEMP2=-TEMP2
      SMAT(1,1)=SMAT(2,2)+TEMP1*SMAT(1,2)
      SMAT(1,2)=SMAT(2,3)+TEMP1*SMAT(1,3)
      SMAT(2,2)=SMAT(3,3)+TEMP2*SMAT(1,3)
      SMAT(1,3)=TEMP1
      SMAT(2,3)=TEMP2
      SMAT(3,3)=TEMP0
  110 CONTINUE
      SMAT(2,1)=SMAT(1,2)
      SMAT(3,1)=SMAT(1,3)
      SMAT(3,2)=SMAT(2,3)
C
C       CALCULATE DISCORDANCY MEASURES (D STATISTICS)
C
      FACTOR=NSITES/THREE
      DO 130 I=1,NSITES
      DO 120 J=1,3
      DO 120 K=1,3
  120 D(I)=D(I)+WORK(I,J)*WORK(I,K)*SMAT(J,K)
      D(I)=D(I)*FACTOR
      WORK(I,1)=D(I)
  130 CONTINUE
      CALL SORT(WORK(1,1),NSITES)
      GOTO 140
  135 DO 138 I=1,NSITES
  138 D(I)=ONE
  140 CONTINUE
      IF(KPRINT LE 0)GOTO 160
      WRITE(KOUT,6000)
      DCRIT1=DC1(1)
      DCRIT2=DC2(1)
      IF(NSITES LE 14)DCRIT1=DC1(NSITES)
      IF(NSITES LE 18)DCRIT2=DC2(NSITES)
      KSTART=1
      DO 150 I=1,NSITES
      LOOK1=BLANK
      LOOK2=BLANK
      IF(D(I) GE DCRIT1)LOOK1=STAR
      IF(D(I) GE DCRIT2)LOOK2=STAR
      IF(D(I) LT DCRIT1)KSTART=KSTART+1
      WRITE(KOUT,6010)I,LEN(I),NAMES(I),(XMOM(K,I),K=2,4),
     *   D(I),LOOK1,LOOK2
  150 CONTINUE
      WRITE(KOUT,6020)(RMOM(K),K=2,4)
      IF(KSTART LE NSITES)WRITE(KOUT,6030)(WORK(K,1),K=KSTART,NSITES)
  160 CONTINUE
      IF(NSIM LE 0)RETURN
      IF(NPROB GT MAXQ)GOTO 1010
      IF(NSIM EQ 1)GOTO 270
      IF(NMAX GT MAXREC)GOTO 1020
      CALL PELKAPL1(RMOM,RPARA,IFAIL)
      IF(IFAIL EQ 0)GOTO 180
      CALL PELGLOL1(RMOM,RPARA)
      RPARA(4)=-ONE
  180 IF(KPRINT GT 0)WRITE(KOUT,6040)(RPARA(K),K=1,4)
      T4BAR=ZERO
      T4SD=ZERO
      DO 220 ISIM=1,NSIM
      SUM2=ZERO
      SUM3=ZERO
      SUM4=ZERO
      DO 200 I=1,NSITES
      NREC=LEN(I)
```

```
      LOOK2=BLANK
      IF(H(J) GE HCRIT1)LOOK1=STAR
      IF(H(J) GE HCRIT2)LOOK2=STAR
      IF(J EQ 1)WRITE(KOUT,6060)VOBS(J),VBAR(J),VSD(J),H(J),LOOK1,LOOK2
      IF(J EQ 2)WRITE(KOUT,6070)VOBS(J),VBAR(J),VSD(J),H(J),LOOK1,LOOK2
      IF(J EQ 3)WRITE(KOUT,6080)VOBS(J),VBAR(J),VSD(J),H(J),LOOK1,LOOK2
  230 CONTINUE
  235 CONTINUE
      S=RMOM(3)
      SS=S*S
      T4FIT(1)=GLOC0+S*(GLOC1+S*GLOC2)
      T4FIT(2)=GEVC0+S*(GEVC1+S*(GEVC2+S*(GEVC3+S*GEVC4)))
      T4FIT(3)=S*(GPAC1+S*(GPAC2+S*(GPAC3+S*GPAC4)))
C
C      CALCULATE GOODNESS-OF-FIT MEASURES (Z STATISTICS)
C
      T4BAR=T4BAR/NSIM
      T4SD=DSQRT((T4SD-NSIM*T4BAR**2)/(NSIM-ONE))
      DO 240 IDIST=1,3
      Z(IDIST)=(T4FIT(IDIST)+T4BAR-TWO*RMOM(4))/T4SD
  240 CONTINUE
      IF(KPRINT LE 0)GOTO 260
      WRITE(KOUT,6090)NSIM
      DO 250 IDIST=1,3
      LOOK1=BLANK
      IF(DABS(Z(IDIST)) LT ZCRIT)LOOK1=STAR
  250 WRITE(KOUT,6100)DISTRI(IDIST),T4FIT(IDIST),Z(IDIST),LOOK1
  260 CONTINUE
  270 CONTINUE
      CALL PELGLOL1(RMOM,PARA(1,1))
      CALL PELGEVL1(RMOM,PARA(1,2))
      CALL PELGPAL1(RMOM,PARA(1,3))
      IF(KPRINT LE 0)GOTO 320
      IF(NSIM EQ 1)WRITE(KOUT,6110)
      IF(NSIM GT 1)WRITE(KOUT,6120)
      DO 280 IDIST=1,3
      IF(DABS(Z(IDIST)) LE ZCRIT)
    * WRITE(KOUT,6130)DISTRI(IDIST),(PARA(IPARA,IDIST),IPARA=1,3)
  280 CONTINUE
      WRITE(KOUT,6130)DISTRI(3),(PARA(IPARA,3),IPARA=1,3)
      IF(NPROB EQ 0)GOTO 320
      WRITE(KOUT,6140)PROB
      DO 300 IDIST=1,3
      IF(DABS(Z(IDIST)) GT ZCRIT)GOTO 300
      DO 290 IQ=1,NPROB
      IF(IDIST EQ 1)Q(IQ)=QUAGLO(PROB(IQ),PARA(1,1))
      IF(IDIST EQ 2)Q(IQ)=QUAGEV(PROB(IQ),PARA(1,2))
      IF(IDIST EQ 3)Q(IQ)=QUAGPA(PROB(IQ),PARA(1,3))
  290 CONTINUE
      WRITE(KOUT,6150)DISTRI(IDIST),(Q(IQ),IQ=1,NPROB)
  300 CONTINUE
  320 CONTINUE
      RETURN
 1000 WRITE(KOUT,7000)'MAXNS'
      RETURN
 1010 WRITE(KOUT,7000)'MAXQ'
      RETURN
 1020 WRITE(KOUT,7000)'MAXREC'
      RETURN
 1030 WRITE(KOUT,7010)
      GOTO 140
 6000 FORMAT(/' SITE   N     NAME      L1-CV  L1-SKEW  L1-KURT   D(I)')
 6010 FORMAT(2I5,2X,A12,3F8 4,F7 2,2X,2A1)
 6020 FORMAT(/5X,'WEIGHTED MEANS',5X,6F8 4)
 6030 FORMAT(/' FLAGGED TEST VALUES'/(15F5 1))
 6040 FORMAT(/' PARAMETERS OF REGIONAL KAPPA DISTRIBUTION ',4F8 4)
 6050 FORMAT(//' ***** HETEROGENEITY MEASURES *****'/
    * '(NUMBER OF SIMULATIONS  =',I6,')')
 6060 FORMAT(/' OBSERVED     S D  OF GROUP L1-CV        =',F8 4/
    *    ' SIM  MEAN OF S D  OF GROUP L1-CV        =',F8 4/
    *    ' SIM  S D  OF S D  OF GROUP L1-CV        =',F8 4/
    *    ' STANDARDIZED TEST VALUE H(1)           =',F6 2,2X,2A1)
 6070 FORMAT(/' OBSERVED AVE   OF L1-CV / L1-SKEW DISTANCE =',F8 4/
    *    ' SIM  MEAN OF AVE  L1-CV / L1-SKEW DISTANCE =',F8 4/
    *    ' SIM  S D  OF AVE  L1-CV / L1-SKEW DISTANCE =',F8 4/
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     *     ' STANDARDIZED TEST VALUE H(2)          =',F6 2,2X,2A1)
 6080 FORMAT(/' OBSERVED AVE  OF L1-SKEW/L1-KURT DISTANCE =',F8 4/
     *     ' SIM  MEAN OF AVE  L1-SKEW/L1-KURT DISTANCE =',F8 4/
     *     ' SIM  S D  OF AVE  L1-SKEW/L1-KURT DISTANCE =',F8 4/
     *     ' STANDARDIZED TEST VALUE H(3)          =',F6 2,2X,2A1)

 6090 FORMAT(//' ***** GOODNESS-OF-FIT MEASURES *****'/
     * ' (NUMBER OF SIMULATIONS  =',I6,')'/)
 6100 FORMAT(1X,A18,2X,' L-KURTOSIS=',F6 3,2X,' Z VALUE=',F6 2,1X,A1)
 6110 FORMAT(//' PARAMETER ESTIMATES'/)
 6120 FORMAT(//' PARAMETER ESTIMATES FOR DISTRIBUTIONS ACCEPTED AT THE',
     * ' 90% LEVEL'/)
 6130 FORMAT(1X,A18,1X,5F7 3)
 6140 FORMAT(/' QUANTILE ESTIMATES'/19X,(1X,14F7 3))
 6150 FORMAT(1X,A18,(1X,14F7 3))
 7000 FORMAT(' *** ERROR *** ROUTINE REGTSTL1  ',
     * ' INSUFFICIENT WORKSPACE - RECOMPILE WITH LARGER VALUE OF ',A6)
 7010 FORMAT(' *** ERROR *** ROUTINE REGTSTL1  UNABLE TO INVERT',
     * ' SUM-OF-SQUARES MATRIX '/31X,'D STATISTICS NOT CALCULATED ')
      END
```

22. SUBROUTINE FOR CALCULATING THE DISCORDANCY, HETEROGENEITY, Z-STATISTICS VALUE OF THREE DIST. I.E. GEV, GLO AND GPA AND ALSO ESTIMATES THE REGIONAL PARAMETERS AND QUANTILES FOR L_2-MOMENTS

```
      SUBROUTINE REGTSTL2(NSITES,NAMES,LEN,XMOM,A,B,SEED,NSIM,NPROB,
     * PROB,KPRINT,KOUT,RMOM,D,VOBS,VBAR,VSD,H,Z,PARA)
C     THIS SUBROUTINE HAS BEEN EXTEND TO L2-MOMENTS FROM SUBROUTINE REGTST
C     PROVIDED BY HOSKING (2005) IN L-MOMENTS PACKAGE
C     XMOM IS THE INPUT ARRAY OF DIMENSION (4, NSITES), ARRAY CONTAINING THE
C     FIRST 4 SAMPLE L2-MOMENTS FOR EACH SITE
C     THE OTHER INPUT AND OUTPUT PARAMETERS ARE SAME LIKE SUBROUTINE REGTST
C     THE OTHER ROUTINES I E  SORT, DURAND, QUAGEV, QUAKAP, QUAGLO AND QUAGPA
C     CAN BE OBTAINED FROM L-MOMENTS PACKAGE OF HOSKING (2005)
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER (MAXNS=200,MAXREC=200,MAXQ=30)
      CHARACTER*1 BLANK,STAR,LOOK1,LOOK2
      CHARACTER*12 NAMES(NSITES)
      CHARACTER*18 DISTRI(3)
      DOUBLE PRECISION D(NSITES),DC1(14),DC2(18),PARA(3,3),H(3),
     * PROB(NPROB),Q(MAXQ),RMOM(4),RPARA(4),SMAT(3,3),TMOM(4),T4FIT(3),
     * VBAR(3),VOBS(3),VSD(3),WORK(MAXNS,3),X(MAXREC),XMOM(4,NSITES),
     * Z(3)
      INTEGER LEN(NSITES)
      DATA BLANK/' '/,STAR/'*'/
      DATA ZERO/0D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/
      DATA DISTRI/
     * 'GEN  LOGISTIC    ','GEN  EXTREME VALUE',
     * 'GEN  PARETO      '/
      DATA GLOC0,GLOC1,GLOC2/0 0889D0, 0 0467D0, 0 8960D0/
      DATA GEVC0,GEVC1,GEVC2,GEVC3,GEVC4/
     * 0 0483D0, 0 1357D0, 0 8710D0,-0 0317D0,
     * 0 0045D0/
      DATA GPAC1,GPAC2,GPAC3,GPAC4/
     * 0 2143D0, 0 8816D0, -0 0754D0, 0 0059D0/
      DATA DC1/4*3D0,1 3330D0,1 6481D0,1 9166D0,2 1401D0,2 3287D0,
     *          2 4906D0,2 6321D0,2 7573D0,2 8694D0,2 9709D0/
      DATA DC2/4*4D0,1 3333D0,1 6648D0,1 9821D0,2 2728D0,2 5337D0,
     *          2 7666D0,2 9748D0,3 1620D0,3 3310D0,3 4844D0,
     *          3 6246D0,3 7532D0,3 8718D0,3 9816D0/
      DATA HCRIT1,HCRIT2/1D0,2D0/
      DATA ZCRIT/1 645D0/
      NMAX=0
      SUMLEN=0
      DO 10 I=1,NSITES
      NREC=LEN(I)
      IF(NREC GT NMAX)NMAX=NREC
      SUMLEN=SUMLEN+NREC
   10 D(I)=ZERO
      DO 20 K=1,3
      VOBS(K)=ZERO
      VBAR(K)=ZERO
```

```
      VSD(K)=ZERO
      H(K)=ZERO
   20 CONTINUE
      DO 30 IDIST=1,3
   30 Z(IDIST)=ZERO
      DO 40 IPARA=1,3
      DO 40 IDIST=1,3
   40 PARA(IPARA,IDIST)=ZERO
      IF(NSITES GT MAXNS)GOTO 1000
C
C      CALCULATE THE WEIGHTED MEAN OF L2-CV, L2-SKEW, L2-KURTOSIS
C
      DO 60 K=2,4
      RMOM(K)=ZERO
      DO 50 I=1,NSITES
   50 RMOM(K)=RMOM(K)+LEN(I)*XMOM(K,I)
   60 RMOM(K)=RMOM(K)/SUMLEN
      RMOM(1)=ONE
      IF(NSITES LE 3)GOTO 135
      SUM2=ZERO
      SUM3=ZERO
      SUM4=ZERO
      DO 70 I=1,NSITES
      SUM2=SUM2+XMOM(2,I)
      SUM3=SUM3+XMOM(3,I)
      SUM4=SUM4+XMOM(4,I)
   70 CONTINUE
      SUM2=SUM2/NSITES
      SUM3=SUM3/NSITES
      SUM4=SUM4/NSITES
      DO 80 I=1,NSITES
      WORK(I,1)=XMOM(2,I)-SUM2
      WORK(I,2)=XMOM(3,I)-SUM3
      WORK(I,3)=XMOM(4,I)-SUM4
   80 CONTINUE
      DO 100 J=1,3
      DO 100 K=J,3
      SMAT(J,K)=ZERO
      DO 90 I=1,NSITES
   90 SMAT(J,K)=SMAT(J,K)+WORK(I,J)*WORK(I,K)
  100 CONTINUE
      DO 110 K=1,3
      IF(SMAT(1,1) LE ZERO)GOTO 1030
      TEMP0=ONE/SMAT(1,1)
      TEMP1=-SMAT(1,2)*TEMP0
      TEMP2=-SMAT(1,3)*TEMP0
      IF(K GT 2)TEMP1=-TEMP1
      IF(K GT 1)TEMP2=-TEMP2
      SMAT(1,1)=SMAT(2,2)+TEMP1*SMAT(1,2)
      SMAT(1,2)=SMAT(2,3)+TEMP1*SMAT(1,3)
      SMAT(2,2)=SMAT(3,3)+TEMP2*SMAT(1,3)
      SMAT(1,3)=TEMP1
      SMAT(2,3)=TEMP2
      SMAT(3,3)=TEMP0
  110 CONTINUE
      SMAT(2,1)=SMAT(1,2)
      SMAT(3,1)=SMAT(1,3)
      SMAT(3,2)=SMAT(2,3)
C
C      CALCULATE DISCORDANCY MEASURES (D STATISTICS)
C
      FACTOR=NSITES/THREE
      DO 130 I=1,NSITES
      DO 120 J=1,3
      DO 120 K=1,3
  120 D(I)=D(I)+WORK(I,J)*WORK(I,K)*SMAT(J,K)
      D(I)=D(I)*FACTOR
      WORK(I,1)=D(I)
  130 CONTINUE
      CALL SORT(WORK(1,1),NSITES)
      GOTO 140
  135 DO 138 I=1,NSITES
  138 D(I)=ONE
  140 CONTINUE
      IF(KPRINT LE 0)GOTO 160
```

```
      VBAR(3)=VBAR(3)+V3
      VSD(1)=VSD(1)+V1**2
      VSD(2)=VSD(2)+V2**2
      VSD(3)=VSD(3)+V3**2
  215 CONTINUE
  220 CONTINUE
C
C      CALCULATE HETEROGENEITY V-STATISTICS FOR OBSERVED DATA
C
      IF(NSITES EQ 1)GOTO 235
      V1=ZERO
      V2=ZERO
      V3=ZERO
      DO 225 I=1,NSITES
      NREC=LEN(I)
      TEMP2=(XMOM(2,I)-RMOM(2))**2
      TEMP3=(XMOM(3,I)-RMOM(3))**2
      TEMP4=(XMOM(4,I)-RMOM(4))**2
      V1=V1+NREC*TEMP2
      V2=V2+NREC*DSQRT(TEMP2+TEMP3)
      V3=V3+NREC*DSQRT(TEMP3+TEMP4)
  225 CONTINUE
      VOBS(1)=DSQRT(V1/SUMLEN)
      VOBS(2)=V2/SUMLEN
      VOBS(3)=V3/SUMLEN
      IF(KPRINT GT 0)WRITE(KOUT,6050)NSIM
      DO 230 J=1,3
      VBAR(J)=VBAR(J)/NSIM
      VSD(J)=DSQRT((VSD(J)-NSIM*VBAR(J)**2)/(NSIM-ONE))
      H(J)=(VOBS(J)-VBAR(J))/VSD(J)
      IF(KPRINT LE 0)GOTO 230
      LOOK1=BLANK
      LOOK2=BLANK
      IF(H(J) GE HCRIT1)LOOK1=STAR
      IF(H(J) GE HCRIT2)LOOK2=STAR
      IF(J EQ 1)WRITE(KOUT,6060)VOBS(J),VBAR(J),VSD(J),H(J),LOOK1,LOOK2
      IF(J EQ 2)WRITE(KOUT,6070)VOBS(J),VBAR(J),VSD(J),H(J),LOOK1,LOOK2
      IF(J EQ 3)WRITE(KOUT,6080)VOBS(J),VBAR(J),VSD(J),H(J),LOOK1,LOOK2
  230 CONTINUE
  235 CONTINUE
      S=RMOM(3)
      SS=S*S
      T4FIT(1)=GLOC0+S*(GLOC1+S*GLOC2)
      T4FIT(2)=GEVC0+S*(GEVC1+S*(GEVC2+S*(GEVC3+S*GEVC4)))
      T4FIT(3)=S*(GPAC1+S*(GPAC2+S*(GPAC3+S*GPAC4)))
      T4BAR=T4BAR/NSIM
      T4SD=DSQRT((T4SD-NSIM*T4BAR**2)/(NSIM-ONE))
      DO 240 IDIST=1,3
      Z(IDIST)=(T4FIT(IDIST)+T4BAR-TWO*RMOM(4))/T4SD
  240 CONTINUE
      IF(KPRINT LE 0)GOTO 260
      WRITE(KOUT,6090)NSIM
      DO 250 IDIST=1,3
      LOOK1=BLANK
      IF(DABS(Z(IDIST)) LT ZCRIT)LOOK1=STAR
  250 WRITE(KOUT,6100)DISTRI(IDIST),T4FIT(IDIST),Z(IDIST),LOOK1
  260 CONTINUE
  270 CONTINUE
      CALL PELGLOL2(RMOM,PARA(1,1))
      CALL PELGEVL2(RMOM,PARA(1,2))
      CALL PELGPAL2(RMOM,PARA(1,3))
      IF(KPRINT LE 0)GOTO 320
      IF(NSIM EQ 1)WRITE(KOUT,6110)
      IF(NSIM GT 1)WRITE(KOUT,6120)
      DO 280 IDIST=1,3
      IF(DABS(Z(IDIST)) LE ZCRIT)
     * WRITE(KOUT,6130)DISTRI(IDIST),(PARA(IPARA,IDIST),IPARA=1,3)
  280 CONTINUE
      WRITE(KOUT,6130)DISTRI(3),(PARA(IPARA,3),IPARA=1,3)
      IF(NPROB EQ 0)GOTO 320
      WRITE(KOUT,6140)PROB
      DO 300 IDIST=1,3
      IF(DABS(Z(IDIST)) GT ZCRIT)GOTO 300
      DO 290 IQ=1,NPROB
      IF(IDIST EQ 1)Q(IQ)=QUAGLO(PROB(IQ),PARA(1,1))
```

```
      DATA BLANK/' '/,STAR/'*'/
      DATA ZERO/0D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/
      DATA DISTRI/
     * 'GEN  LOGISTIC    ','GEN  EXTREME VALUE',
     * 'GEN  PARETO      '/
      DATA GLOC0,GLOC1,GLOC2/0 0714D0, 0 0714D0, 0 8929D0/
      DATA GEVC0,GEVC1,GEVC2,GEVC3,GEVC4/
     * 0 0378D0, 0 1491D0, 0 8644D0,-0 0222D0,
     * 0 0026D0/
      DATA GPAC1,GPAC2,GPAC3,GPAC4/
     * 0 2187D0, 0 8813D0, -0 0538D0, 0 0031D0/
      DATA DC1/4*3D0,1 3330D0,1 6481D0,1 9166D0,2 1401D0,2 3287D0,
     *         2 4906D0,2 6321D0,2 7573D0,2 8694D0,2 9709D0/
      DATA DC2/4*4D0,1 3333D0,1 6648D0,1 9821D0,2 2728D0,2 5337D0,
     *         2 7666D0,2 9748D0,3 1620D0,3 3310D0,3 4844D0,
     *         3 6246D0,3 7532D0,3 8718D0,3 9816D0/
      DATA HCRIT1,HCRIT2/1D0,2D0/
      DATA ZCRIT/1 645D0/
      NMAX=0
      SUMLEN=0
      DO 10 I=1,NSITES
      NREC=LEN(I)
      IF(NREC GT NMAX)NMAX=NREC
      SUMLEN=SUMLEN+NREC
   10 D(I)=ZERO
      DO 20 K=1,3
      VOBS(K)=ZERO
      VBAR(K)=ZERO
      VSD(K)=ZERO
      H(K)=ZERO
   20 CONTINUE
      DO 30 IDIST=1,3
   30 Z(IDIST)=ZERO
      DO 40 IPARA=1,3
      DO 40 IDIST=1,3
   40 PARA(IPARA,IDIST)=ZERO
      IF(NSITES GT MAXNS)GOTO 1000
C
C      CALCULATE THE WEIGHTED MEAN OF L3-CV, L3-SKEW, L3-KURTOSIS
C
      DO 60 K=2,4
      RMOM(K)=ZERO
      DO 50 I=1,NSITES
   50 RMOM(K)=RMOM(K)+LEN(I)*XMOM(K,I)
   60 RMOM(K)=RMOM(K)/SUMLEN
      RMOM(1)=ONE
      IF(NSITES LE 3)GOTO 135
      SUM2=ZERO
      SUM3=ZERO
      SUM4=ZERO
      DO 70 I=1,NSITES
      SUM2=SUM2+XMOM(2,I)
      SUM3=SUM3+XMOM(3,I)
      SUM4=SUM4+XMOM(4,I)
   70 CONTINUE
      SUM2=SUM2/NSITES
      SUM3=SUM3/NSITES
      SUM4=SUM4/NSITES
      DO 80 I=1,NSITES
      WORK(I,1)=XMOM(2,I)-SUM2
      WORK(I,2)=XMOM(3,I)-SUM3
      WORK(I,3)=XMOM(4,I)-SUM4
   80 CONTINUE
      DO 100 J=1,3
      DO 100 K=J,3
      SMAT(J,K)=ZERO
      DO 90 I=1,NSITES
   90 SMAT(J,K)=SMAT(J,K)+WORK(I,J)*WORK(I,K)
  100 CONTINUE
      DO 110 K=1,3
      IF(SMAT(1,1) LE ZERO)GOTO 1030
      TEMP0=ONE/SMAT(1,1)
      TEMP1=-SMAT(1,2)*TEMP0
      TEMP2=-SMAT(1,3)*TEMP0
      IF(K GT 2)TEMP1=-TEMP1
```

```
      IF(K GT 1)TEMP2=-TEMP2
      SMAT(1,1)=SMAT(2,2)+TEMP1*SMAT(1,2)
      SMAT(1,2)=SMAT(2,3)+TEMP1*SMAT(1,3)
      SMAT(2,2)=SMAT(3,3)+TEMP2*SMAT(1,3)
      SMAT(1,3)=TEMP1
      SMAT(2,3)=TEMP2
      SMAT(3,3)=TEMP0
  110 CONTINUE
      SMAT(2,1)=SMAT(1,2)
      SMAT(3,1)=SMAT(1,3)
      SMAT(3,2)=SMAT(2,3)
C
C      CALCULATE DISCORDANCY MEASURES (D STATISTICS)
C
      FACTOR=NSITES/THREE
      DO 130 I=1,NSITES
      DO 120 J=1,3
      DO 120 K=1,3
  120 D(I)=D(I)+WORK(I,J)*WORK(I,K)*SMAT(J,K)
      D(I)=D(I)*FACTOR
      WORK(I,1)=D(I)
  130 CONTINUE
      CALL SORT(WORK(1,1),NSITES)
      GOTO 140
  135 DO 138 I=1,NSITES
  138 D(I)=ONE
  140 CONTINUE
      IF(KPRINT LE 0)GOTO 160
      WRITE(KOUT,6000)
      DCRIT1=DC1(1)
      DCRIT2=DC2(1)
      IF(NSITES LE 14)DCRIT1=DC1(NSITES)
      IF(NSITES LE 18)DCRIT2=DC2(NSITES)
      KSTART=1
      DO 150 I=1,NSITES
      LOOK1=BLANK
      LOOK2=BLANK
      IF(D(I) GE DCRIT1)LOOK1=STAR
      IF(D(I) GE DCRIT2)LOOK2=STAR
      IF(D(I) LT DCRIT1)KSTART=KSTART+1
      WRITE(KOUT,6010)I,LEN(I),NAMES(I),(XMOM(K,I),K=2,4),
     * D(I),LOOK1,LOOK2
  150 CONTINUE
      WRITE(KOUT,6020)(RMOM(K),K=2,4)
      IF(KSTART LE NSITES)WRITE(KOUT,6030)(WORK(K,1),K=KSTART,NSITES)
  160 CONTINUE
      IF(NSIM LE 0)RETURN
      IF(NPROB GT MAXQ)GOTO 1010
      IF(NSIM EQ 1)GOTO 270
      IF(NMAX GT MAXREC)GOTO 1020
      CALL PELKAPL3(RMOM,RPARA,IFAIL)
      IF(IFAIL EQ 0)GOTO 180
      CALL PELGLOL3(RMOM,RPARA)
      RPARA(4)=-ONE
  180 IF(KPRINT GT 0)WRITE(KOUT,6040)(RPARA(K),K=1,4)
      T4BAR=ZERO
      T4SD=ZERO
      DO 220 ISIM=1,NSIM
      SUM2=ZERO
      SUM3=ZERO
      SUM4=ZERO
      DO 200 I=1,NSITES
      NREC=LEN(I)
      CALL DURAND(SEED,NREC,X)
      DO 190 J=1,NREC
      X(J)=QUAKAP(X(J),RPARA)
  190 CONTINUE
      CALL SORT(X,NREC)
      CALL DIRL3(X,NREC,TMOM,4)
      CV=TMOM(2)/TMOM(1)
      WORK(I,1)=CV
      WORK(I,2)=TMOM(3)
      WORK(I,3)=TMOM(4)
      SUM2=SUM2+NREC*CV
      SUM3=SUM3+NREC*TMOM(3)
```

```
      SUM4=SUM4+NREC*TMOM(4)
  200 CONTINUE
      SUM2=SUM2/SUMLEN
      SUM3=SUM3/SUMLEN
      SUM4=SUM4/SUMLEN
      T4BAR=T4BAR+SUM4
      T4SD=T4SD+SUM4**2
C
C      CALCULATE HETEROGENEITY V-STATISTICS FOR SIMULATED DATA
C
      IF(NSITES EQ 1)GOTO 215
      V1=ZERO
      V2=ZERO
      V3=ZERO
      DO 210 I=1,NSITES
      NREC=LEN(I)
      TEMP2=(WORK(I,1)-SUM2)**2
      TEMP3=(WORK(I,2)-SUM3)**2
      TEMP4=(WORK(I,3)-SUM4)**2
      V1=V1+NREC*TEMP2
      V2=V2+NREC*DSQRT(TEMP2+TEMP3)
      V3=V3+NREC*DSQRT(TEMP3+TEMP4)
  210 CONTINUE
      V1=DSQRT(V1/SUMLEN)
      V2=V2/SUMLEN
      V3=V3/SUMLEN
      VBAR(1)=VBAR(1)+V1
      VBAR(2)=VBAR(2)+V2
      VBAR(3)=VBAR(3)+V3
      VSD(1)=VSD(1)+V1**2
      VSD(2)=VSD(2)+V2**2
      VSD(3)=VSD(3)+V3**2
  215 CONTINUE
  220 CONTINUE
C
C      CALCULATE HETEROGENEITY V-STATISTICS FOR OBSERVED DATA
C
      IF(NSITES EQ 1)GOTO 235
      V1=ZERO
      V2=ZERO
      V3=ZERO
      DO 225 I=1,NSITES
      NREC=LEN(I)
      TEMP2=(XMOM(2,I)-RMOM(2))**2
      TEMP3=(XMOM(3,I)-RMOM(3))**2
      TEMP4=(XMOM(4,I)-RMOM(4))**2
      V1=V1+NREC*TEMP2
      V2=V2+NREC*DSQRT(TEMP2+TEMP3)
      V3=V3+NREC*DSQRT(TEMP3+TEMP4)
  225 CONTINUE
      VOBS(1)=DSQRT(V1/SUMLEN)
      VOBS(2)=V2/SUMLEN
      VOBS(3)=V3/SUMLEN
C
C      CALCULATE AND PRINT HETEROGENEITY MEASURES (H STATISTICS)
C
      IF(KPRINT GT 0)WRITE(KOUT,6050)NSIM
      DO 230 J=1,3
      VBAR(J)=VBAR(J)/NSIM
      VSD(J)=DSQRT((VSD(J)-NSIM*VBAR(J)**2)/(NSIM-ONE))
      H(J)=(VOBS(J)-VBAR(J))/VSD(J)
      IF(KPRINT LE 0)GOTO 230
      LOOK1=BLANK
      LOOK2=BLANK
      IF(H(J) GE HCRIT1)LOOK1=STAR
      IF(H(J) GE HCRIT2)LOOK2=STAR
      IF(J EQ 1)WRITE(KOUT,6060)VOBS(J),VBAR(J),VSD(J),H(J),LOOK1,LOOK2
      IF(J EQ 2)WRITE(KOUT,6070)VOBS(J),VBAR(J),VSD(J),H(J),LOOK1,LOOK2
      IF(J EQ 3)WRITE(KOUT,6080)VOBS(J),VBAR(J),VSD(J),H(J),LOOK1,LOOK2
  230 CONTINUE
  235 CONTINUE
      S=RMOM(3)
      SS=S*S
      T4FIT(1)=GLOC0+S*(GLOC1+S*GLOC2)
      T4FIT(2)=
```

```
  * ' 90% LEVEL'/)
6130 FORMAT(1X,A18,1X,5F7 3)
6140 FORMAT(/' QUANTILE ESTIMATES'/19X,(1X,14F7 3))
6150 FORMAT(1X,A18,(1X,14F7 3))
7000 FORMAT(' *** ERROR *** ROUTINE REGTSTL3 ',
  * ' INSUFFICIENT WORKSPACE - RECOMPILE WITH LARGER VALUE OF ',A6)
7010 FORMAT(' *** ERROR *** ROUTINE REGTSTL3  UNABLE TO INVERT',
  * ' SUM-OF-SQUARES MATRIX '/31X,'D STATISTICS NOT CALCULATED ')

     END
```

24. SUBROUTINE FOR CALCULATING THE DISCORDANCY, HETEROGENEITY, Z-STATISTICS VALUE OF THREE DIST. I.E. GEV, GLO AND GPA AND ALSO ESTIMATES THE REGIONAL PARAMETERS AND QUANTILES FOR L4-MOMENTS

```
     SUBROUTINE REGTSTL4(NSITES,NAMES,LEN,XMOM,SEED,NSIM,NPROB,
     *    PROB,KPRINT,KOUT,RMOM,D,VOBS,VBAR,VSD,H,Z,PARA)
C    THIS SUBROUTINE HAS BEEN EXTEND TO L4-MOMENTS FROM SUBROUTINE REGTST
C    PROVIDED BY HOSKING (2005) IN L-MOMENTS PACKAGE
C    XMOM IS THE INPUT ARRAY OF DIMENSION (4, NSITES), ARRAY CONTAINING THE
C    FIRST 4 SAMPLE L4-MOMENTS FOR EACH SITE
C    THE OTHER INPUT AND OUTPUT PARAMETERS ARE SAME LIKE SUBROUTINE REGTST
C    THE OTHER ROUTINES I E  SORT, DURAND, QUAGEV, QUAKAP, QUAGLO AND QUAGPA
C    CAN BE OBTAINED FROM L-MOMENTS PACKAGE OF HOSKING (2005)
C
     IMPLICIT DOUBLE PRECISION (A-H,O-Z)
     PARAMETER (MAXNS=200,MAXREC=200,MAXQ=30)
     CHARACTER*1 BLANK,STAR,LOOK1,LOOK2
     CHARACTER*12 NAMES(NSITES)
     CHARACTER*18 DISTRI(3)
     DOUBLE PRECISION D(NSITES),DC1(14),DC2(18),PARA(3,3),H(3),
    * PROB(NPROB),Q(MAXQ),RMOM(4),RPARA(4),SMAT(3,3),TMOM(4),T4FIT(3),
    * VBAR(3),VOBS(3),VSD(3),WORK(MAXNS,3),X(MAXREC),XMOM(4,NSITES),
    * Z(3)
     INTEGER LEN(NSITES)
     DATA BLANK/' '/,STAR/'*'/
     DATA ZERO/0D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/
     DATA DISTRI/
    * 'GEN  LOGISTIC    ','GEN  EXTREME VALUE',
    * 'GEN  PARETO      '/
     DATA GLOC0,GLOC1,GLOC2/0 0595D0,0 0918D0,0 8856D0/
     DATA GEVC0,GEVC1,GEVC2,GEVC3,GEVC4/
    * 0 0310D0,0 1602D0,0 8564D0,-0 0163D0,
    * 0 0017D0/
     DATA GPAC1,GPAC2,GPAC3,GPAC4/
    * 0 2212D0,0 8374D0,-0 0665D0,-0 0112D0/
     DATA DC1/4*3D0,1 3330D0,1 6481D0,1 9166D0,2 1401D0,2 3287D0,
    *         2 4906D0,2 6321D0,2 7573D0,2 8694D0,2 9709D0/
     DATA DC2/4*4D0,1 3333D0,1 6648D0,1 9821D0,2 2728D0,2 5337D0,
    *         2 7666D0,2 9748D0,3 1620D0,3 3310D0,3 4844D0,
    *         3 6246D0,3 7532D0,3 8718D0,3 9816D0/
     DATA HCRIT1,HCRIT2/1D0,2D0/
     DATA ZCRIT/1 645D0/
     NMAX=0
     SUMLEN=0
     DO 10 I=1,NSITES
     NREC=LEN(I)
     IF(NREC GT NMAX)NMAX=NREC
     SUMLEN=SUMLEN+NREC
  10 D(I)=ZERO
     DO 20 K=1,3
     VOBS(K)=ZERO
     VBAR(K)=ZERO
     VSD(K)=ZERO
     H(K)=ZERO
  20 CONTINUE
     DO 30 IDIST=1,3
  30 Z(IDIST)=ZERO
     DO 40 IPARA=1,3
     DO 40 IDIST=1,3
  40 PARA(IPARA,IDIST)=ZERO
     IF(NSITES GT MAXNS)GOTO 1000
C
C     CALCULATE THE WEIGHTED MEAN OF L4-CV, L4-SKEW, L4-KURTOSIS
C
```

```
      DO 60 K=2,4
      RMOM(K)=ZERO
      DO 50 I=1,NSITES
   50 RMOM(K)=RMOM(K)+LEN(I)*XMOM(K,I)
   60 RMOM(K)=RMOM(K)/SUMLEN
      RMOM(1)=ONE
      IF(NSITES LE 3)GOTO 135
      SUM2=ZERO
      SUM3=ZERO
      SUM4=ZERO
      DO 70 I=1,NSITES
      SUM2=SUM2+XMOM(2,I)
      SUM3=SUM3+XMOM(3,I)
      SUM4=SUM4+XMOM(4,I)
   70 CONTINUE
      SUM2=SUM2/NSITES
      SUM3=SUM3/NSITES
      SUM4=SUM4/NSITES
      DO 80 I=1,NSITES
      WORK(I,1)=XMOM(2,I)-SUM2
      WORK(I,2)=XMOM(3,I)-SUM3
      WORK(I,3)=XMOM(4,I)-SUM4
   80 CONTINUE
      DO 100 J=1,3
      DO 100 K=J,3
      SMAT(J,K)=ZERO
      DO 90 I=1,NSITES
   90 SMAT(J,K)=SMAT(J,K)+WORK(I,J)*WORK(I,K)
  100 CONTINUE
      DO 110 K=1,3
      IF(SMAT(1,1) LE ZERO)GOTO 1030
      TEMP0=ONE/SMAT(1,1)
      TEMP1=-SMAT(1,2)*TEMP0
      TEMP2=-SMAT(1,3)*TEMP0
      IF(K GT 2)TEMP1=-TEMP1
      IF(K GT 1)TEMP2=-TEMP2
      SMAT(1,1)=SMAT(2,2)+TEMP1*SMAT(1,2)
      SMAT(1,2)=SMAT(2,3)+TEMP1*SMAT(1,3)
      SMAT(2,2)=SMAT(3,3)+TEMP2*SMAT(1,3)
      SMAT(1,3)=TEMP1
      SMAT(2,3)=TEMP2
      SMAT(3,3)=TEMP0
  110 CONTINUE
      SMAT(2,1)=SMAT(1,2)
      SMAT(3,1)=SMAT(1,3)
      SMAT(3,2)=SMAT(2,3)
C
C       CALCULATE DISCORDANCY MEASURES (D STATISTICS)
C
      FACTOR=NSITES/THREE
      DO 130 I=1,NSITES
      DO 120 J=1,3
      DO 120 K=1,3
  120 D(I)=D(I)+WORK(I,J)*WORK(I,K)*SMAT(J,K)
      D(I)=D(I)*FACTOR
      WORK(I,1)=D(I)
  130 CONTINUE
      CALL SORT(WORK(1,1),NSITES)
      GOTO 140
  135 DO 138 I=1,NSITES
  138 D(I)=ONE
  140 CONTINUE
      IF(KPRINT LE 0)GOTO 160
      WRITE(KOUT,6000)
      DCRIT1=DC1(1)
      DCRIT2=DC2(1)
      IF(NSITES LE 14)DCRIT1=DC1(NSITES)
      IF(NSITES LE 18)DCRIT2=DC2(NSITES)
      KSTART=1
      DO 150 I=1,NSITES
      LOOK1=BLANK
      LOOK2=BLANK
      IF(D(I) GE DCRIT1)LOOK1=STAR
      IF(D(I) GE DCRIT2)LOOK2=STAR
      IF(D(I) LT DCRIT1)KSTART=KSTART+1
```

```
      WRITE(KOUT,6010)I,LEN(I),NAMES(I),(XMOM(K,I),K=2,4),
     * D(I),LOOK1,LOOK2
  150 CONTINUE
      WRITE(KOUT,6020)(RMOM(K),K=2,4)
      IF(KSTART LE NSITES)WRITE(KOUT,6030)(WORK(K,1),K=KSTART,NSITES)
  160 CONTINUE
      IF(NSIM LE 0)RETURN
      IF(NPROB GT MAXQ)GOTO 1010
      IF(NSIM EQ 1)GOTO 270
      IF(NMAX GT MAXREC)GOTO 1020
      CALL PELKAPL4(RMOM,RPARA,IFAIL)
      IF(IFAIL EQ 0)GOTO 180
      CALL PELGLOL4(RMOM,RPARA)
      RPARA(4)=-ONE
  180 IF(KPRINT GT 0)WRITE(KOUT,6040)(RPARA(K),K=1,4)
      T4BAR=ZERO
      T4SD=ZERO
      DO 220 ISIM=1,NSIM
      SUM2=ZERO
      SUM3=ZERO
      SUM4=ZERO
      DO 200 I=1,NSITES
      NREC=LEN(I)
      CALL DURAND(SEED,NREC,X)
      DO 190 J=1,NREC
      X(J)=QUAKAP(X(J),RPARA)
  190 CONTINUE
      CALL SORT(X,NREC)
      CALL DIRL4(X,NREC,TMOM,4)
      CV=TMOM(2)/TMOM(1)
      WORK(I,1)=CV
      WORK(I,2)=TMOM(3)
      WORK(I,3)=TMOM(4)
      SUM2=SUM2+NREC*CV
      SUM3=SUM3+NREC*TMOM(3)
      SUM4=SUM4+NREC*TMOM(4)
  200 CONTINUE
      SUM2=SUM2/SUMLEN
      SUM3=SUM3/SUMLEN
      SUM4=SUM4/SUMLEN
      T4BAR=T4BAR+SUM4
      T4SD=T4SD+SUM4**2
C
C      CALCULATE HETEROGENEITY V-STATISTICS FOR SIMULATED DATA
C
      IF(NSITES EQ 1)GOTO 215
      V1=ZERO
      V2=ZERO
      V3=ZERO
      DO 210 I=1,NSITES
      NREC=LEN(I)
      TEMP2=(WORK(I,1)-SUM2)**2
      TEMP3=(WORK(I,2)-SUM3)**2
      TEMP4=(WORK(I,3)-SUM4)**2
      V1=V1+NREC*TEMP2
      V2=V2+NREC*DSQRT(TEMP2+TEMP3)
      V3=V3+NREC*DSQRT(TEMP3+TEMP4)
  210 CONTINUE
      V1=DSQRT(V1/SUMLEN)
      V2=V2/SUMLEN
      V3=V3/SUMLEN
      VBAR(1)=VBAR(1)+V1
      VBAR(2)=VBAR(2)+V2
      VBAR(3)=VBAR(3)+V3
      VSD(1)=VSD(1)+V1**2
      VSD(2)=VSD(2)+V2**2
      VSD(3)=VSD(3)+V3**2
  215 CONTINUE
  220 CONTINUE
C
C      CALCULATE HETEROGENEITY V-STATISTICS FOR OBSERVED DATA
C
      IF(NSITES EQ 1)GOTO 235
      V1=ZERO
      V2=ZERO
```

```
      V3=ZERO
      DO 225 I=1,NSITES
      NREC=LEN(I)
      TEMP2=(XMOM(2,I)-RMOM(2))**2
      TEMP3=(XMOM(3,I)-RMOM(3))**2
      TEMP4=(XMOM(4,I)-RMOM(4))**2
      V1=V1+NREC*TEMP2
      V2=V2+NREC*DSQRT(TEMP2+TEMP3)
      V3=V3+NREC*DSQRT(TEMP3+TEMP4)
  225 CONTINUE
      VOBS(1)=DSQRT(V1/SUMLEN)
      VOBS(2)=V2/SUMLEN
      VOBS(3)=V3/SUMLEN
C
C      CALCULATE AND PRINT HETEROGENEITY MEASURES (H STATISTICS)
C
      IF(KPRINT GT 0)WRITE(KOUT,6050)NSIM
      DO 230 J=1,3
      VBAR(J)=VBAR(J)/NSIM
      VSD(J)=DSQRT((VSD(J)-NSIM*VBAR(J)**2)/(NSIM-ONE))
      H(J)=(VOBS(J)-VBAR(J))/VSD(J)
      IF(KPRINT LE 0)GOTO 230
      LOOK1=BLANK
      LOOK2=BLANK
      IF(H(J) GE HCRIT1)LOOK1=STAR
      IF(H(J) GE HCRIT2)LOOK2=STAR
      IF(J EQ 1)WRITE(KOUT,6060)VOBS(J),VBAR(J),VSD(J),H(J),LOOK1,LOOK2
      IF(J EQ 2)WRITE(KOUT,6070)VOBS(J),VBAR(J),VSD(J),H(J),LOOK1,LOOK2
      IF(J EQ 3)WRITE(KOUT,6080)VOBS(J),VBAR(J),VSD(J),H(J),LOOK1,LOOK2
  230 CONTINUE
  235 CONTINUE
      S=RMOM(3)
      SS=S*S
      T4FIT(1)=GLOC0+S*(GLOC1+S*GLOC2)
      T4FIT(2)=
     * GEVC0+S*(GEVC1+S*(GEVC2+S*(GEVC3+S*GEVC4)))
      T4FIT(3)=S*(GPAC1+S*(GPAC2+S*(GPAC3+S*GPAC4)))
C
C      CALCULATE GOODNESS-OF-FIT MEASURES (Z STATISTICS)
C
      T4BAR=T4BAR/NSIM
      T4SD=DSQRT((T4SD-NSIM*T4BAR**2)/(NSIM-ONE))
      DO 240 IDIST=1,3
      Z(IDIST)=(T4FIT(IDIST)+T4BAR-TWO*RMOM(4))/T4SD
  240 CONTINUE
      IF(KPRINT LE 0)GOTO 260
      WRITE(KOUT,6090)NSIM
      DO 250 IDIST=1,3
      LOOK1=BLANK
      IF(DABS(Z(IDIST)) LT ZCRIT)LOOK1=STAR
  250 WRITE(KOUT,6100)DISTRI(IDIST),T4FIT(IDIST),Z(IDIST),LOOK1
  260 CONTINUE
  270 CONTINUE
      CALL PELGLOL4(RMOM,PARA(1,1))
      CALL PELGEVL4(RMOM,PARA(1,2))
      CALL PELGPAL4(RMOM,PARA(1,3))
      IF(KPRINT LE 0)GOTO 320
      IF(NSIM EQ 1)WRITE(KOUT,6110)
      IF(NSIM GT 1)WRITE(KOUT,6120)
      DO 280 IDIST=1,3
      IF(DABS(Z(IDIST)) LE ZCRIT)
     * WRITE(KOUT,6130)DISTRI(IDIST),(PARA(IPARA,IDIST),IPARA=1,3)
  280 CONTINUE
      WRITE(KOUT,6130)DISTRI(3),(PARA(IPARA,3),IPARA=1,3)
      IF(NPROB EQ 0)GOTO 320
      WRITE(KOUT,6140)PROB
      DO 300 IDIST=1,3
      IF(DABS(Z(IDIST)) GT ZCRIT)GOTO 300
      DO 290 IQ=1,NPROB
      IF(IDIST EQ 1)Q(IQ)=QUAGLO(PROB(IQ),PARA(1,1))
      IF(IDIST EQ 2)Q(IQ)=QUAGEV(PROB(IQ),PARA(1,2))
      IF(IDIST EQ 3)Q(IQ)=QUAGPA(PROB(IQ),PARA(1,3))
  290 CONTINUE
      WRITE(KOUT,6150)DISTRI(IDIST),(Q(IQ),IQ=1,NPROB)
  300 CONTINUE
```

```
 320 CONTINUE
     RETURN
1000 WRITE(KOUT,7000)'MAXNS'
     RETURN
1010 WRITE(KOUT,7000)'MAXQ'
     RETURN
1020 WRITE(KOUT,7000)'MAXREC'
     RETURN
1030 WRITE(KOUT,7010)
     GOTO 140
6000 FORMAT(/' SITE   N     NAME     L4-CV  L4-SKEW  L4-KURT   D(I)')
6010 FORMAT(2I5,2X,A12,3F8 4,F7 2,2X,2A1)
6020 FORMAT(/5X,'WEIGHTED MEANS',5X,6F8 4)
6030 FORMAT(/' FLAGGED TEST VALUES'/(15F5 1))
6040 FORMAT(/' PARAMETERS OF REGIONAL KAPPA DISTRIBUTION ',4F8 4)
6050 FORMAT(//' ***** HETEROGENEITY MEASURES *****'/
   * ' (NUMBER OF SIMULATIONS  =',I6,')')
6060 FORMAT(/' OBSERVED     S D  OF GROUP L4-CV        =',F8 4/
   *      ' SIM  MEAN OF S D  OF GROUP L4-CV        =',F8 4/
   *      ' SIM  S D  OF S D  OF GROUP L4-CV        =',F8 4/
   *      ' STANDARDIZED TEST VALUE H(1)            =',F6 2,2X,2A1)
6070 FORMAT(/' OBSERVED AVE   OF L4-CV / L4-SKEW DISTANCE =',F8 4/
   *      ' SIM  MEAN OF AVE  L4-CV / L4-SKEW DISTANCE =',F8 4/
   *      ' SIM  S D  OF AVE  L4-CV / L4-SKEW DISTANCE =',F8 4/
   *      ' STANDARDIZED TEST VALUE H(2)            =',F6 2,2X,2A1)
6080 FORMAT(/' OBSERVED AVE   OF L4-SKEW/L4-KURT DISTANCE =',F8 4/
   *      ' SIM  MEAN OF AVE  L4-SKEW/L4-KURT DISTANCE =',F8 4/
   *      ' SIM  S D  OF AVE  L4-SKEW/L4-KURT DISTANCE =',F8 4/
   *      ' STANDARDIZED TEST VALUE H(3)            =',F6 2,2X,2A1)
6090 FORMAT(//' ***** GOODNESS-OF-FIT MEASURES *****'/
   * ' (NUMBER OF SIMULATIONS  =',I6,')'/)
6100 FORMAT(1X,A18,2X,' L4-KURTOSIS=',F6 3,2X,' Z VALUE=',F6 2,1X,A1)
6110 FORMAT(//' PARAMETER ESTIMATES'/)
6120 FORMAT(//' PARAMETER ESTIMATES FOR DISTRIBUTIONS ACCEPTED AT THE',
   * ' 90% LEVEL'/)
6130 FORMAT(1X,A18,1X,5F7 3)
6140 FORMAT(/' QUANTILE ESTIMATES'/19X,(1X,14F7 3))
6150 FORMAT(1X,A18,(1X,14F7 3))
7000 FORMAT(' *** ERROR *** ROUTINE REGTSTL4  ',
   * ' INSUFFICIENT WORKSPACE - RECOMPILE WITH LARGER VALUE OF ',A6)
7010 FORMAT(' *** ERROR *** ROUTINE REGTSTL4  UNABLE TO INVERT',
   * ' SUM-OF-SQUARES MATRIX '/31X,'D STATISTICS NOT CALCULATED ')

     END
```

25. SUBROUTINE FOR CALCULATING SAMPLE LQ-MOMENTS OF A DATA SET BY USING TRIMEAN ESTIMATOR

```
      SUBROUTINE SAMLQM(X, N, QMOM, 4)
C     X IS THE INPUT ARRAY OF LENGTH N  CONTAINS THE DATA, IN ASCENDING
C     ORDER
C     N IS THE INPUT NUMBER OF DATA VALUES
C     QMOM IS THE OUTPUT  ARRAY OF LENGTH 4  CONTAINS THE SAMPLE
C     LQ- MOMENTS
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION X(N),U1(3),U2(4),U3(9),U4(12),C1(3),C2(4),
      * C3(9),C4(12),DESUN1(3),DESUN2(4),DESUN3(9),DESUN4(12),E1(3),
      * E2(4),E3(9),E4(12),QXU1(3),QXU2(4),QXU3(9),QXU4(12),S1(3),
      * S2(4),S3(9),S4(12),QMOM(NMOM)
      INTEGER NDESU1(3),NDESU2(4),NDESU3(9),NDESU4(12)
      DATA ZERO/0D0/,ONE/1D0/,THREE/3D0/,SIX/6D0/,EIGHT/8D0/,
      * SIXTEEN/16D0/,TWELVE/12D0/
      U1(1)=0 750D0
      U1(2)=0 500D0
      U1(3)=0 250D0
      C1(1)=0 250D0
      C1(2)=0 500D0
      C1(3)=0 250D0
      U2(1)=0 866D0
      U2(2)=0 707D0
      U2(3)=0 293D0
      U2(4)=0 134D0
      C2(1)=0 125D0
```

```
      C2(2)=0 250D0
      C2(3)=-0 250D0
      C2(4)=-0 125D0
      U3(1)=0 909D0
      U3(2)=0 794D0
      U3(3)=0 674D0
      U3(4)=0 630D0
      U3(5)=0 500D0
      U3(6)=0 370D0
      U3(7)=0 326D0
      U3(8)=0 206D0
      U3(9)=0 091D0
      C3(1)=ONE/TWELVE
      C3(2)=ONE/SIX
      C3(3)=-ONE/SIX
      C3(4)=ONE/TWELVE
      C3(5)=-ONE/THREE
      C3(6)=ONE/TWELVE
      C3(7)=-ONE/SIX
      C3(8)=ONE/SIX
      C3(9)=ONE/TWELVE
      U4(1)=0 931D0
      U4(2)=0 841D0
      U4(3)=0 757D0
      U4(4)=0 707D0
      U4(5)=0 614D0
      U4(6)=0 544D0
      U4(7)=0 456D0
      U4(8)=0 386D0
      U4(9)=0 293D0
      U4(10)=0 243D0
      U4(11)=0 159D0
      U4(12)=0 069D0
      C4(1)=ONE/SIXTEEN
      C4(2)=ONE/EIGHT
      C4(3)=-THREE/SIXTEEN
      C4(4)=ONE/SIXTEEN
      C4(5)=-THREE/EIGHT
      C4(6)=THREE/SIXTEEN
      C4(7)=-THREE/SIXTEEN
      C4(8)=THREE/EIGHT
      C4(9)=-ONE/SIXTEEN
      C4(10)=THREE/SIXTEEN
      C4(11)=-ONE/EIGHT
      C4(12)=-ONE/SIXTEEN
      NDES=N+1
      SUM=ZERO
      SUM1=ZERO
      SUM2=ZERO
      SUM3=ZERO
      DO 10 I=1,3
      DESUN1(I)=NDES*U1(I)
      NDESU1(I)=DESUN1(I)
      E1(I)=DESUN1(I)-NDESU1(I)
      S1(I)=1-E1(I)
      QXU1(I)=S1(I)*(X(NDESU1(I)))+E1(I)*(X(NDESU1(I)+1))
      SUM=SUM+C1(I)*QXU1(I)
10    CONTINUE
      DO 15 I=1,4
      DESUN2(I)=NDES*U2(I)
      NDESU2(I)=DESUN2(I)
      E2(I)=DESUN2(I)-NDESU2(I)
      S2(I)=1-E2(I)
      QXU2(I)=S2(I)*(X(NDESU2(I)))+E2(I)*(X(NDESU2(I)+1))
      SUM1=SUM1+C2(I)*QXU2(I)
15    CONTINUE
      DO 20 I=1,9
      DESUN3(I)=NDES*U3(I)
      NDESU3(I)=DESUN3(I)
      E3(I)=DESUN3(I)-NDESU3(I)
      S3(I)=1-E3(I)
      QXU3(I)=S3(I)*(X(NDESU3(I)))+E3(I)*(X(NDESU3(I)+1))
      SUM2=SUM2+C3(I)*QXU3(I)
20    CONTINUE
      DO 25 I=1,12
```

```
      DESUN4(I)=NDES*U4(I)
      NDESU4(I)=DESUN4(I)
      E4(I)=DESUN4(I)-NDESU4(I)
      S4(I)=1-E4(I)
      QXU4(I)=S4(I)*(X(NDESU4(I)))+E4(I)*(X(NDESU4(I)+1))
      SUM3=SUM3+C4(I)*QXU4(I)
25    CONTINUE
      QMOM(1)=SUM
      QMOM(2)=SUM1
      QMOM(3)=SUM2/SUM1
      QMOM(4)=SUM3/SUM1
      RETURN
      END
```

26. SUBROUTINE FOR CALCULATING PARAMETERS OF GLO DISTRIBUTION BY USING LQ-MOMENTS BASED ON TRIMEAN ESTIMATOR

```
      SUBROUTINE PELGLOQ(QMOM,PARA)
C     QMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS LQ-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION QMOM(3),PARA(3)
      DATA ONE/1D0/,TWO/2D0/,EIGHT/8D0/
      DATA A1,A3,A5/-1 3330D0,-0 0277D0,0 0153D0/
      T3=QMOM(3)
      S=T3*T3
      G=T3*(A1+S*(A3+S*A5))
      PARA(3)=G
      U1=((ONE-0 707D0)/0 707D0)
      U2=((ONE-0 293D0)/0 293D0)
      U3=((ONE-0 866D0)/0 866D0)
      U4=((ONE-0 134D0)/0 134D0)
      U5=((ONE-0 250D0)/0 250D0)
      U6=((ONE-0 500D0)/0 500D0)
      U7=((ONE-0 750D0)/0 750D0)
      V1=U1**G
      V2=U2**G
      V3=U3**G
      V4=U4**G
      V5=U5**G
      V6=U6**G
      V7=U7**G
      PARA(2)=(QMOM(2)*EIGHT*G)/(-TWO*V1-V3+V4+TWO*V2)
      PARA(1)=QMOM(1)-(PARA(2)/G)*(0 250D0*(ONE-V5)+0 500D0*(ONE-V6)
     *   +0 250D0*(ONE-V7))
      RETURN
      END
```

27. SUBROUTINE FOR CALCULATING PARAMETERS OF GPA DISTRIBUTION BY USING LQ-MOMENTS BASED ON TRIMEAN ESTIMATOR

```
      SUBROUTINE PELGPAQ(QMOM,PARA)
C     QMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS LQ-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION QMOM(3),PARA(3)
      DATA ONE/1D0/,TWO/2D0/,EIGHT/8D0/
      DATA A0,A1,A2/1 0003D0,-3 5015D0,1 5146D0/
      DATA A3,A4,A5,A6/-0 8005D0,0 3870D0,-0 0764D0,-0 0253D0/
      T3=QMOM(3)
      G=A0+T3*(A1+T3*(A2+T3*(A3+T3*(A4+T3*(A5+T3*A6)))))
      PARA(3)=G
      U1=ONE-0 750D0
      U2=ONE-0 500D0
      U3=ONE-0 250D0
      U4=ONE-0 707D0
      U5=ONE-0 293D0
      U6=ONE-0 866D0
      U7=ONE-0 134D0
      V1=(ONE-DEXP(G*DLOG(U1)))/G
      V2=(ONE-DEXP(G*DLOG(U2)))/G
```

```
      V3=(ONE-DEXP(G*DLOG(U3)))/G
      V4=(ONE-DEXP(G*DLOG(U4)))/G
      V5=(ONE-DEXP(G*DLOG(U5)))/G
      V6=(ONE-DEXP(G*DLOG(U6)))/G
      V7=(ONE-DEXP(G*DLOG(U7)))/G
      PARA(2)=(QMOM(2)*EIGHT)/(TWO*V4-TWO*V5+V6-V7)
      PARA(1)=QMOM(1)-PARA(2)*(0 250D0*V3+0 500D0*V2+0 250D0*V1)
      RETURN
      END
```

28. SUBROUTINE FOR CALCULATING PARAMETERS OF PE3 DISTRIBUTION BY USING LQ-MOMENTS BASED ON TRIMEAN ESTIMATOR

```
      SUBROUTINE PELPE3Q(QMOM,PARA)
C     QMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS LQ-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C     THE OTHER ROUTINE QUASTN CAN BE OBTAINED FROM L-MOMENTS PACKAGE OF
C     HOSKING (2005)
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION QMOM(3),PARA(3)
      DATA ONE/1D0/,TWO/2D0/,EIGHT/8D0/,SIX/6D0/,THIRTYSIX/36D0/
      DATA A0,A1,A2/0 0117D0,6 6784D0,1 7839D0/
      DATA A3,A4/-11 0797D0,8 3365D0/
      T3=QMOM(3)
      G=A0+T3*(A1+T3*(A2+T3*(A3+T3*A4)))
      PARA(3)=G
      U1=0 750D0
      U2=0 500D0
      U3=0 250D0
      U4=0 707D0
      U5=0 293D0
      U6=0 866D0
      U7=0 134D0
      S=G*G/THIRTYSIX
      V1=G*QUASTN(U1)/SIX
      V2=G*QUASTN(U2)/SIX
      V3=G*QUASTN(U3)/SIX
      V4=G*QUASTN(U4)/SIX
      V5=G*QUASTN(U5)/SIX
      V6=G*QUASTN(U6)/SIX
      V7=G*QUASTN(U7)/SIX
      X1=ONE+V1-S
      X2=ONE+V2-S
      X3=ONE+V3-S
      X4=ONE+V4-S
      X5=ONE+V5-S
      X6=ONE+V6-S
      X7=ONE+V7-S
      Y1=X1*X1*X1
      Y2=X2*X2*X2
      Y3=X3*X3*X3
      Y4=X4*X4*X4
      Y5=X5*X5*X5
      Y6=X6*X6*X6
      Y7=X7*X7*X7
      W1=TWO*(Y1-ONE)/G
      W2=TWO*(Y2-ONE)/G
      W3=TWO*(Y3-ONE)/G
      W4=TWO*(Y4-ONE)/G
      W5=TWO*(Y5-ONE)/G
      W6=TWO*(Y6-ONE)/G
      W7=TWO*(Y7-ONE)/G
      PARA(2)=(QMOM(2)*EIGHT)/(TWO*W4-TWO*W5+W6-W7)
      PARA(1)=QMOM(1)-PARA(2)*(0 250D0*W3+0 500D0*W2+0 250D0*W1)
      RETURN
      END
```

29. SUBROUTINE FOR CALCULATING PARAMETERS OF GEV DISTRIBUTION BY USING LQ-MOMENTS BASED ON TRIMEAN ESTIMATOR

```
      SUBROUTINE PELGEVQ(QMOM,PARA)
C     QMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS LQ-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION QMOM(3),PARA(3)
      DATA ONE/1D0/,TWO/2D0/,EIGHT/8D0/
      DATA A0,A1,A2/0 2985D0,-2 0229D0,0 3738D0/
      DATA A3,A4,A5,A6/-0 1488D0,0 0400D0,0 0270D0,-0 0171D0/
      T3=QMOM(3)
      G=A0+T3*(A1+T3*(A2+T3*(A3+T3*(A4+T3*(A5+T3*A6)))))
      PARA(3)=G
      U1=0 750D0
      U2=0 500D0
      U3=0 250D0
      U4=0 707D0
      U5=0 293D0
      U6=0 866D0
      U7=0 134D0
      V1=-DLOG(U1)
      V2=-DLOG(U2)
      V3=-DLOG(U3)
      V4=-DLOG(U4)
      V5=-DLOG(U5)
      V6=-DLOG(U6)
      V7=-DLOG(U7)
      X1=V1**G
      X2=V2**G
      X3=V3**G
      X4=V4**G
      X5=V5**G
      X6=V6**G
      X7=V7**G
      W1=(ONE-X1)/G
      W2=(ONE-X2)/G
      W3=(ONE-X3)/G
      W4=(ONE-X4)/G
      W5=(ONE-X5)/G
      W6=(ONE-X6)/G
      W7=(ONE-X7)/G
      PARA(2)=(QMOM(2)*EIGHT)/(TWO*W4-TWO*W5+W6-W7)
      PARA(1)=QMOM(1)-PARA(2)*(0 250D0*W3+0 500D0*W2+0 250D0*W1)
      RETURN
      END
```

30. SUBROUTINE FOR CALCULATING PARAMETERS OF GNO DISTRIBUTION BY USING LQ-MOMENTS BASED ON TRIMEAN ESTIMATOR

```
      SUBROUTINE PELGNOQ(QMOM,PARA)
C     QMOM IS THE INPUT ARRAY OF LENGTH 3  CONTAINS LQ-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 3
C     THE OTHER ROUTINE QUASTN CAN BE OBTAINED FROM L-MOMENTS PACKAGE OF
C     HOSKING (2005)
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION QMOM(3),PARA(3)
      DATA ONE/1D0/,TWO/2D0/,EIGHT/8D0/
      DATA A1,A3,A5/-2 3283D0,-0 1232D0,0 0322D0/
      T3=QMOM(3)
      S=T3*T3
      G=T3*(A1+S*(A3+S*A5))
      PARA(3)=G
      U1=0 750D0
      U2=0 500D0
      U3=0 250D0
      U4=0 707D0
      U5=0 293D0
      U6=0 866D0
      U7=0 134D0
      V1=QUASTN(U1)
      V2=QUASTN(U2)
```

```
      V3=QUASTN(U3)
      V4=QUASTN(U4)
      V5=QUASTN(U5)
      V6=QUASTN(U6)
      V7=QUASTN(U7)
      X1=DEXP(-G*V1)
      X2=DEXP(-G*V2)
      X3=DEXP(-G*V3)
      X4=DEXP(-G*V4)
      X5=DEXP(-G*V5)
      X6=DEXP(-G*V6)
      X7=DEXP(-G*V7)
      W1=(ONE-X1)/G
      W2=(ONE-X2)/G
      W3=(ONE-X3)/G
      W4=(ONE-X4)/G
      W5=(ONE-X5)/G
      W6=(ONE-X6)/G
      W7=(ONE-X7)/G
      PARA(2)=(QMOM(2)*EIGHT)/(TWO*W4-TWO*W5+W6-W7)
      PARA(1)=QMOM(1)-PARA(2)*(0 250D0*W3+0 500D0*W2+0 250D0*W1)
      RETURN
      END
```

31. SUBROUTINE FOR CALCULATING PARAMETERS OF KAPA DISTRIBUTION BY USING LQ-MOMENTS BASED ON TRIMEAN ESTIMATOR

```
      SUBROUTINE PELKAPQ(QMOM,PARA,IFAIL)
C     THIS SUBROUTINE HAS BEEN EXTEND TO LQ-MOMENTS FROM SUBROUTINE PELKAP
C     PROVIDED BY HOSKING (2005) IN L-MOMENTS PACKAGE
C     QMOM IS THE INPUT ARRAY OF LENGTH 4  CONTAINS THE LQ-MOMENTS
C     PARA IS THE OUTPUT ARRAY OF LENGTH 4  CONTAINS THE PARAMETERS OF
C     THE DISTRIBUTION
C     IFAIL IS THE OUTPUT FAIL FLAG  ON EXIT, IT IS SET AS FOLLOWS
C           0  SUCCESSFUL EXIT
C           1  LQ-MOMENTS INVALID
C           2  (TAU-3, TAU-4) LIES ABOVE THE GENERALIZED-LOGISTIC
C             LINE (SUGGESTS THAT LQ-MOMENTS ARE NOT CONSISTENT
C             WITH ANY KAPPA DISTRIBUTION WITH H GT -1)
C           3  ITERATION FAILED TO CONVERGE
C           4  UNABLE TO MAKE PROGRESS FROM CURRENT POINT IN
C             ITERATION
C           5  ITERATION ENCOUNTERED NUMERICAL DIFFICULTIES -
C             OVERFLOW WOULD HAVE BEEN LIKELY TO OCCUR
C           6  ITERATION FOR H AND K CONVERGED, BUT OVERFLOW
C             WOULD HAVE OCCURRED WHEN CALCULATING XI AND ALPHA
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION QMOM(4),PARA(4)
      DATA ZERO/0D0/,HALF/0 5D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/,FOUR/4D0/
      DATA FIVE/5D0/,SIX/6D0/,TWELVE/12D0/,SIXTEEN/16D0/,EIGHT/8D0/
      DATA P725/0 725D0/,P8/0 8D0/
      DATA EPS/1D-6/,MAXIT/20/,MAXSR/10/,HSTART/1 001D0/,BIG/10D0/
      DATA C0,C1,C2/1 0003D0,-3 5015D0,1 5146D0/
      DATA C3,C4,C5,C6/-0 8005D0,0 3870D0,-0 0764D0,-0 0253D0/
      T3=QMOM(3)
      T4=QMOM(4)
      DO 10 I=1,4
   10 PARA(I)=ZERO
      IF(QMOM(2) LE ZERO)GOTO 1000
      IF(DABS(T3) GE ONE OR DABS(T4) GE ONE)GOTO 1000
      IF(T4 LE (FIVE*T3*T3-ONE)/FOUR)GOTO 1000
      IF(T4 GE (FIVE*T3*T3+ONE)/SIX )GOTO 1010
      G=C0+T3*(C1+T3*(C2+T3*(C3+T3*(C4+T3*(C5+T3*C6)))))
      H=HSTART
      Z=G+H*P725
      XDIST=BIG
      DO 100 IT=1,MAXIT
      DO 40 I=1,MAXSR
      A1=ONE/TWELVE
      A2=ONE/SIX
      A3=ONE/THREE
```

```
B1=ONE/SIXTEEN
B2=ONE/EIGHT
B3=THREE/SIXTEEN
B4=THREE/EIGHT
U1=DEXP(H*DLOG(0 707D0))
U2=DEXP(H*DLOG(0 866D0))
U3=DEXP(H*DLOG(0 134D0))
U4=DEXP(H*DLOG(0 293D0))
U5=DEXP(H*DLOG(0 091D0))
U6=DEXP(H*DLOG(0 206D0))
U7=DEXP(H*DLOG(0 326D0))
U8=DEXP(H*DLOG(0 370D0))
U9=DEXP(H*DLOG(0 500D0))
U10=DEXP(H*DLOG(0 630D0))
U11=DEXP(H*DLOG(0 674D0))
U12=DEXP(H*DLOG(0 794D0))
U13=DEXP(H*DLOG(0 909D0))
U14=DEXP(H*DLOG(0 841D0))
U15=DEXP(H*DLOG(0 931D0))
U16=DEXP(H*DLOG(0 456D0))
U17=DEXP(H*DLOG(0 614D0))
U18=DEXP(H*DLOG(0 757D0))
U19=DEXP(H*DLOG(0 243D0))
U20=DEXP(H*DLOG(0 386D0))
U21=DEXP(H*DLOG(0 544D0))
U22=DEXP(H*DLOG(0 069D0))
U23=DEXP(H*DLOG(0 159D0))
U24=DEXP(H*DLOG(0 250D0))
U25=DEXP(H*DLOG(0 750D0))
V1=DEXP(G*(DLOG(ONE-U1)-DLOG(H)))
V2=DEXP(G*(DLOG(ONE-U2)-DLOG(H)))
V3=DEXP(G*(DLOG(ONE-U3)-DLOG(H)))
V4=DEXP(G*(DLOG(ONE-U4)-DLOG(H)))
V5=DEXP(G*(DLOG(ONE-U5)-DLOG(H)))
V6=DEXP(G*(DLOG(ONE-U6)-DLOG(H)))
V7=DEXP(G*(DLOG(ONE-U7)-DLOG(H)))
V8=DEXP(G*(DLOG(ONE-U8)-DLOG(H)))
V9=DEXP(G*(DLOG(ONE-U9)-DLOG(H)))
V10=DEXP(G*(DLOG(ONE-U10)-DLOG(H)))
V11=DEXP(G*(DLOG(ONE-U11)-DLOG(H)))
V12=DEXP(G*(DLOG(ONE-U12)-DLOG(H)))
V13=DEXP(G*(DLOG(ONE-U13)-DLOG(H)))
V14=DEXP(G*(DLOG(ONE-U14)-DLOG(H)))
V15=DEXP(G*(DLOG(ONE-U15)-DLOG(H)))
V16=DEXP(G*(DLOG(ONE-U16)-DLOG(H)))
V17=DEXP(G*(DLOG(ONE-U17)-DLOG(H)))
V18=DEXP(G*(DLOG(ONE-U18)-DLOG(H)))
V19=DEXP(G*(DLOG(ONE-U19)-DLOG(H)))
V20=DEXP(G*(DLOG(ONE-U20)-DLOG(H)))
V21=DEXP(G*(DLOG(ONE-U21)-DLOG(H)))
V22=DEXP(G*(DLOG(ONE-U22)-DLOG(H)))
V23=DEXP(G*(DLOG(ONE-U23)-DLOG(H)))
V24=DEXP(G*(DLOG(ONE-U24)-DLOG(H)))
V25=DEXP(G*(DLOG(ONE-U25)-DLOG(H)))
XI1=0 25D0*(ONE-V24)+0 5D0*(ONE-V9)+0 25D0*(ONE-V25)
XI2=0 25D0*(ONE-V1)+0 125D0*(ONE-V2)-0 125D0*(ONE-V3)
#-0 25D0*(ONE-V4)
XI3=A1*(ONE-V10)+A2*(ONE-V12)+A1*(ONE-V13)
#-A2*(ONE-V7)-A3*(ONE-V9)-A2*(ONE-V11)
#+A1*(ONE-V5)+A2*(ONE-V6)+A1*(ONE-V8)
XI4=B1*(ONE-V1)+B2*(ONE-V14)+B1*(ONE-V15)
#-B3*(ONE-V16)-B4*(ONE-V17)-B3*(ONE-V18)
#+B3*(ONE-V19)+B4*(ONE-V20)+B3*(ONE-V21)
#-B1*(ONE-V22)-B2*(ONE-V23)-B1*(ONE-V4)
IF(XI2 EQ ZERO)GOTO 1020
TAU3=XI3/XI2
TAU4=XI4/XI2
E1=TAU3-T3
E2=TAU4-T4
DIST=DMAX1(DABS(E1),DABS(E2))
IF(DIST LT XDIST)GOTO 50
DEL1=HALF*DEL1
DEL2=HALF*DEL2
G=XG-DEL1
H=XH-DEL2
```

```
40 CONTINUE
   IFAIL=4
   RETURN
50 CONTINUE
   IF(DIST LT EPS)GOTO 110
   XG=G
   XH=H
   XZ=Z
   XDIST=DIST
   V1G=V1*(DLOG(ONE-U1)-DLOG(H))
   V2G=V2*(DLOG(ONE-U2)-DLOG(H))
   V3G=V3*(DLOG(ONE-U3)-DLOG(H))
   V4G=V4*(DLOG(ONE-U4)-DLOG(H))
   V5G=V5*(DLOG(ONE-U5)-DLOG(H))
   V6G=V6*(DLOG(ONE-U6)-DLOG(H))
   V7G=V7*(DLOG(ONE-U7)-DLOG(H))
   V8G=V8*(DLOG(ONE-U8)-DLOG(H))
   V9G=V9*(DLOG(ONE-U9)-DLOG(H))
   V10G=V10*(DLOG(ONE-U10)-DLOG(H))
   V11G=V11*(DLOG(ONE-U11)-DLOG(H))
   V12G=V12*(DLOG(ONE-U12)-DLOG(H))
   V13G=V13*(DLOG(ONE-U13)-DLOG(H))
   V14G=V14*(DLOG(ONE-U14)-DLOG(H))
   V15G=V15*(DLOG(ONE-U15)-DLOG(H))
   V16G=V16*(DLOG(ONE-U16)-DLOG(H))
   V17G=V17*(DLOG(ONE-U17)-DLOG(H))
   V18G=V18*(DLOG(ONE-U18)-DLOG(H))
   V19G=V19*(DLOG(ONE-U19)-DLOG(H))
   V20G=V20*(DLOG(ONE-U20)-DLOG(H))
   V21G=V21*(DLOG(ONE-U21)-DLOG(H))
   V22G=V22*(DLOG(ONE-U22)-DLOG(H))
   V23G=V23*(DLOG(ONE-U23)-DLOG(H))
   V24G=V24*(DLOG(ONE-U24)-DLOG(H))
   V25G=V25*(DLOG(ONE-U25)-DLOG(H))
   F1=ONE/(ONE-U1)
   F2=ONE/(ONE-U2)
   F3=ONE/(ONE-U3)
   F4=ONE/(ONE-U4)
   F5=ONE/(ONE-U5)
   F6=ONE/(ONE-U6)
   F7=ONE/(ONE-U7)
   F8=ONE/(ONE-U8)
   F9=ONE/(ONE-U9)
   F10=ONE/(ONE-U10)
   F11=ONE/(ONE-U11)
   F12=ONE/(ONE-U12)
   F13=ONE/(ONE-U13)
   F14=ONE/(ONE-U14)
   F15=ONE/(ONE-U15)
   F16=ONE/(ONE-U16)
   F17=ONE/(ONE-U17)
   F18=ONE/(ONE-U18)
   F19=ONE/(ONE-U19)
   F20=ONE/(ONE-U20)
   F21=ONE/(ONE-U21)
   F22=ONE/(ONE-U22)
   F23=ONE/(ONE-U23)
   F24=ONE/(ONE-U24)
   F25=ONE/(ONE-U25)
   R=ONE/H
   V1H=V1*G*(-F1*U1*DLOG(0.707D0)-R)
   V2H=V2*G*(-F2*U2*DLOG(0 866D0)-R)
   V3H=V3*G*(-F3*U3*DLOG(0 134D0)-R)
   V4H=V4*G*(-F4*U4*DLOG(0 293D0)-R)
   V5H=V5*G*(-F5*U5*DLOG(0 091D0)-R)
   V6H=V6*G*(-F6*U6*DLOG(0 206D0)-R)
   V7H=V7*G*(-F7*U7*DLOG(0 326D0)-R)
   V8H=V8*G*(-F8*U8*DLOG(0.370D0)-R)
   V9H=V9*G*(-F9*U9*DLOG(0 500D0)-R)
   V10H=V10*G*(-F10*U10*DLOG(0 630D0)-R)
   V11H=V11*G*(-F11*U11*DLOG(0 674D0)-R)
   V12H=V12*G*(-F12*U12*DLOG(0 794D0)-R)
   V13H=V13*G*(-F13*U13*DLOG(0.909D0)-R)
   V14H=V14*G*(-F14*U14*DLOG(0 841D0)-R)
   V15H=V15*G*(-F15*U15*DLOG(0 931D0)-R)
```

```
      V16H=V16*G*(-F16*U16*DLOG(0 456D0)-R)
      V17H=V17*G*(-F17*U17*DLOG(0 614D0)-R)
      V18H=V18*G*(-F18*U18*DLOG(0 757D0)-R)
      V19H=V19*G*(-F19*U19*DLOG(0 243D0)-R)
      V20H=V20*G*(-F20*U20*DLOG(0 386D0)-R)
      V21H=V21*G*(-F21*U21*DLOG(0 544D0)-R)
      V22H=V22*G*(-F22*U22*DLOG(0 069D0)-R)
      V23H=V23*G*(-F23*U23*DLOG(0 159D0)-R)
      V24H=V24*G*(-F24*U24*DLOG(0 250D0)-R)
      V25H=V25*G*(-F25*U25*DLOG(0 750D0)-R)
      DL2G=-0 25D0*V1G-0 125D0*V2G+0 125D0*V3G+0 25D0*V4G
      DL2H=-0 25D0*V1H-0 125D0*V2H+0 125D0*V3H+0 25D0*V4H
      DL3G=-A1*V10G-A2*V12G-A1*V13G
     #+A2*V7G+A3*V9G+A2*V11G-A1*V5G-A2*V6G-A1*V8G
      DL3H=-A1*V10H-A2*V12H-A1*V13H
     #+A2*V7H+A3*V9H+A2*V11H-A1*V5H-A2*V6H-A1*V8H
      DL4G=-B1*V1G-B2*V14G-B1*V15G
     #+B3*V16G+B4*V17G+B3*V18G-B3*V19G-B4*V20G-B3*V21G
     #+B1*V22G+B2*V23G+B1*V4G
      DL4H=-B1*V1H-B2*V14H-B1*V15H
     #+B3*V16H+B4*V17H+B3*V18H-B3*V19H-B4*V20H-B3*V21H
     #+B1*V22H+B2*V23H+B1*V4H
      D11=(DL3G-TAU3*DL2G)/XI2
      D12=(DL3H-TAU3*DL2H)/XI2
      D21=(DL4G-TAU4*DL2G)/XI2
      D22=(DL4H-TAU4*DL2H)/XI2
      DET=D11*D22-D12*D21
      H11= D22/DET
      H12=-D12/DET
      H21=-D21/DET
      H22= D11/DET
      DEL1=E1*H11+E2*H12
      DEL2=E1*H21+E2*H22
      G=XG-DEL1
      H=XH-DEL2
      Z=G+H*P725
      FACTOR=ONE
      IF(G LE -ONE)FACTOR=P8*(XG+ONE)/DEL1
      IF(H LE -ONE)FACTOR=DMIN1(FACTOR,P8*(XH+ONE)/DEL2)
      IF(Z LE -ONE)FACTOR=DMIN1(FACTOR,P8*(XZ+ONE)/(XZ-Z))
      IF(H LE ZERO AND G*H LE -ONE)
     * FACTOR=DMIN1(FACTOR,P8*(XG*XH+ONE)/(XG*XH-G*H))
      IF(FACTOR EQ ONE)GOTO 80
      DEL1=DEL1*FACTOR
      DEL2=DEL2*FACTOR
      G=XG-DEL1
      H=XH-DEL2
      Z=G+H*P725
   80 CONTINUE
  100 CONTINUE
      IFAIL=3
      RETURN
  110 IFAIL=0
      PARA(4)=H
      PARA(3)=G
      PARA(2)=QMOM(2)*G/XI2
      PARA(1)=QMOM(1)-(PARA(2)*XI1/G)
      RETURN
 1000 IFAIL=1
      RETURN
 1010 IFAIL=2
      RETURN
 1020 IFAIL=5
      RETURN
      END
```

32. SUBROUTINE FOR CALCULATING THE DISCORDANCY, HETEROGENEITY, Z-STATISTICS VALUE OF THREE DIST. I.E. GEV, GLO AND GPA AND ALSO ESTIMATES THE REGIONAL PARAMETERS AND QUANTILES FOR LQ-MOMENTS

```
      SUBROUTINE REGTSTLQ(NSITES,NAMES,LEN,XMOM,SEED,NSIM,NPROB,PROB,
     *     KPRINT,KOUT,RMOM,D,VOBS,VBAR,VSD,H,Z,PARA)
C     THIS SUBROUTINE HAS BEEN EXTEND TO LQ-MOMENTS FROM SUBROUTINE REGTST
C     PROVIDED BY HOSKING (2005) IN L-MOMENTS PACKAGE
C     XMOM IS THE INPUT ARRAY OF DIMENSION (4, NSITES), ARRAY CONTAINING THE
C     FIRST 4 SAMPLE LQ-MOMENTS FOR EACH SITE
C     THE OTHER INPUT AND OUTPUT PARAMETERS ARE SAME LIKE SUBROUTINE REGTST
C     THE OTHER ROUTINES I E  SORT, DURAND, QUAGEV, QUAKAP, QUAGLO, QUAGPA
C     QUAPE3 AND QUAGNO CAN BE OBTAINED FROM L-MOMENTS PACKAGE OF HOSKING (2005)
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER (MAXNS=200,MAXQ=30,MAXREC=200)
      CHARACTER*1 BLANK,STAR,LOOK1,LOOK2
      CHARACTER*12 NAMES(NSITES)
      CHARACTER*18 DISTRI(5)
      DOUBLE PRECISION D(NSITES),DC1(14),DC2(18),H(3),PARA(5,5),
     * PROB(NPROB),Q(MAXQ),RMOM(5),RPARA(4),SMAT(3,3),TMOM(4),T4FIT(5),
     * VBAR(3),VOBS(3),VSD(3),WORK(MAXNS,3),X(MAXREC),XMOM(4,NSITES),
     * Z(5)
      INTEGER LEN(NSITES)
      DATA BLANK/' '/,STAR/'*'/
      DATA ZERO/0D0/,ONE/1D0/,TWO/2D0/,THREE/3D0/
      DATA DISTRI/
     * 'GEN  LOGISTIC    ','GEN  EXTREME VALUE','GEN  NORMAL
     * 'PEARSON TYPE III ','GEN  PARETO
      DATA GLOC0,GLOC2,GLOC4,GLOC6
     * /0 1585D0,0 8189D0,-0 0118D0,-0 0037D0/
      DATA GEVC0,GEVC1,GEVC2,GEVC3,GEVC4,GEVC5,GEVC6/
     * 0 1080D0, 0 1131D0, 0 8178D0,-0 0330D0,-0 0087D0,
     * 0 0064D0,-0 0056D0/
      DATA GNOC0,GNOC2,GNOC4,GNOC6/
     * 0 1202D0,0 7929D0,-0 0044D0,-0 0064D0/
      DATA PE3C0,PE3C1,PE3C2,PE3C3,PE3C4/
     * 0 1232D0,-0 1224D0,1 3324D0,-2 3445D0,2 0100D0/
      DATA GPAC0,GPAC1,GPAC2,GPAC3,GPAC4,GPAC5,GPAC6,GPAC7/
     * -0 0020D0,0 2229D0,0 8626D0,-0 0751D0,-0 0106D0,
     * -0 0013D0,-0 0064D0,0 0117D0/
      DATA DC1/4*3D0,1 3330D0,1 6481D0,1 9166D0,2 1401D0,2 3287D0,
     *         2 4906D0,2 6321D0,2 7573D0,2 8694D0,2 9709D0/
      DATA DC2/4*4D0,1 3333D0,1 6648D0,1 9821D0,2 2728D0,2 5337D0,
     *         2 7666D0,2 9748D0,3 1620D0,3 3310D0,3 4844D0,
     *         3 6246D0,3 7532D0,3 8718D0,3 9816D0/
      DATA HCRIT1,HCRIT2/1D0,2D0/
      DATA ZCRIT/1 645D0/
      NMAX=0
      SUMLEN=0
      DO 10 I=1,NSITES
      NREC=LEN(I)
      IF(NREC GT NMAX)NMAX=NREC
      SUMLEN=SUMLEN+NREC
   10 D(I)=ZERO
      DO 20 K=1,3
      VOBS(K)=ZERO
      VBAR(K)=ZERO
      VSD(K)=ZERO
      H(K)=ZERO
   20 CONTINUE
      DO 30 IDIST=1,5
   30 Z(IDIST)=ZERO
      DO 40 IPARA=1,5
      DO 40 IDIST=1,5
   40 PARA(IPARA,IDIST)=ZERO
      IF(NSITES GT MAXNS)GOTO 1000
C
C      CALCULATE THE WEIGHTED MEAN OF LQ-CV, LQ-SKEW, LQ-KURTOSIS
C
      DO 60 K=2,4
      RMOM(K)=ZERO
      DO 50 I=1,NSITES
   50 RMOM(K)=RMOM(K)+LEN(I)*XMOM(K,I)
   60 RMOM(K)=RMOM(K)/SUMLEN
```

```
      RMOM(1)=ONE
      IF(NSITES LE 3)GOTO 135
      SUM2=ZERO
      SUM3=ZERO
      SUM4=ZERO
      DO 70 I=1,NSITES
      SUM2=SUM2+XMOM(2,I)
      SUM3=SUM3+XMOM(3,I)
      SUM4=SUM4+XMOM(4,I)
   70 CONTINUE
      SUM2=SUM2/NSITES
      SUM3=SUM3/NSITES
      SUM4=SUM4/NSITES
      DO 80 I=1,NSITES
      WORK(I,1)=XMOM(2,I)-SUM2
      WORK(I,2)=XMOM(3,I)-SUM3
      WORK(I,3)=XMOM(4,I)-SUM4
   80 CONTINUE
      DO 100 J=1,3
      DO 100 K=J,3
      SMAT(J,K)=ZERO
      DO 90 I=1,NSITES
   90 SMAT(J,K)=SMAT(J,K)+WORK(I,J)*WORK(I,K)
  100 CONTINUE
      DO 110 K=1,3
      IF(SMAT(1,1) LE ZERO)GOTO 1030
      TEMP0=ONE/SMAT(1,1)
      TEMP1=-SMAT(1,2)*TEMP0
      TEMP2=-SMAT(1,3)*TEMP0
      IF(K GT 2)TEMP1=-TEMP1
      IF(K GT 1)TEMP2=-TEMP2
      SMAT(1,1)=SMAT(2,2)+TEMP1*SMAT(1,2)
      SMAT(1,2)=SMAT(2,3)+TEMP1*SMAT(1,3)
      SMAT(2,2)=SMAT(3,3)+TEMP2*SMAT(1,3)
      SMAT(1,3)=TEMP1
      SMAT(2,3)=TEMP2
      SMAT(3,3)=TEMP0
  110 CONTINUE
      SMAT(2,1)=SMAT(1,2)
      SMAT(3,1)=SMAT(1,3)
      SMAT(3,2)=SMAT(2,3)
C
C      CALCULATE DISCORDANCY MEASURES (D STATISTICS)
C
      FACTOR=NSITES/THREE
      DO 130 I=1,NSITES
      DO 120 J=1,3
      DO 120 K=1,3
  120 D(I)=D(I)+WORK(I,J)*WORK(I,K)*SMAT(J,K)
      D(I)=D(I)*FACTOR
      WORK(I,1)=D(I)
  130 CONTINUE
      CALL SORT(WORK(1,1),NSITES)
      GOTO 140
  135 DO 138 I=1,NSITES
  138 D(I)=ONE
  140 CONTINUE
      IF(KPRINT LE 0)GOTO 160
      WRITE(KOUT,6000)
      DCRIT1=DC1(1)
      DCRIT2=DC2(1)
      IF(NSITES LE 14)DCRIT1=DC1(NSITES)
      IF(NSITES LE 18)DCRIT2=DC2(NSITES)
      KSTART=1
      DO 150 I=1,NSITES
      LOOK1=BLANK
      LOOK2=BLANK
      IF(D(I) GE DCRIT1)LOOK1=STAR
      IF(D(I) GE DCRIT2)LOOK2=STAR
      IF(D(I) LT DCRIT1)KSTART=KSTART+1
      WRITE(KOUT,6010)I,LEN(I),NAMES(I),(XMOM(K,I),K=2,4),
     * D(I),LOOK1,LOOK2
  150 CONTINUE
      WRITE(KOUT,6020)(RMOM(K),K=2,4)
      IF(KSTART LE NSITES)WRITE(KOUT,6030)(WORK(K,1),K=KSTART,NSITES)
```

```
  160 CONTINUE
      IF(NSIM LE 0)RETURN
      IF(NPROB GT MAXQ)GOTO 1010
      IF(NSIM EQ 1)GOTO 270
      IF(NMAX GT MAXREC)GOTO 1020
      CALL PELKAPQ(RMOM,RPARA,IFAIL)
      IF(IFAIL EQ 0)GOTO 180
      CALL PELGLOQ(RMOM,RPARA)
      RPARA(4)=-ONE
  180 IF(KPRINT GT 0)WRITE(KOUT,6040)(RPARA(K),K=1,4)
      T4BAR=ZERO
      T4SD=ZERO
      DO 220 ISIM=1,NSIM
      SUM2=ZERO
      SUM3=ZERO
      SUM4=ZERO
      DO 200 I=1,NSITES
      NREC=LEN(I)
      CALL DURAND(SEED,NREC,X)
      DO 190 J=1,NREC
      X(J)=QUAKAP(X(J),RPARA)
  190 CONTINUE
      CALL SORT(X,NREC)
      CALL SAMLQM(X,NREC,TMOM,4)
      CV=TMOM(2)/TMOM(1)
      WORK(I,1)=CV
      WORK(I,2)=TMOM(3)
      WORK(I,3)=TMOM(4)
      SUM2=SUM2+NREC*CV
      SUM3=SUM3+NREC*TMOM(3)
      SUM4=SUM4+NREC*TMOM(4)
  200 CONTINUE
      SUM2=SUM2/SUMLEN
      SUM3=SUM3/SUMLEN
      SUM4=SUM4/SUMLEN
      T4BAR=T4BAR+SUM4
      T4SD=T4SD+SUM4**2
C
C      CALCULATE HETEROGENEITY V-STATISTICS FOR SIMULATED DATA
C
      IF(NSITES EQ 1)GOTO 215
      V1=ZERO
      V2=ZERO
      V3=ZERO
      DO 210 I=1,NSITES
      NREC=LEN(I)
      TEMP2=(WORK(I,1)-SUM2)**2
      TEMP3=(WORK(I,2)-SUM3)**2
      TEMP4=(WORK(I,3)-SUM4)**2
      V1=V1+NREC*TEMP2
      V2=V2+NREC*DSQRT(TEMP2+TEMP3)
      V3=V3+NREC*DSQRT(TEMP3+TEMP4)
  210 CONTINUE
      V1=DSQRT(V1/SUMLEN)
      V2=V2/SUMLEN
      V3=V3/SUMLEN
      VBAR(1)=VBAR(1)+V1
      VBAR(2)=VBAR(2)+V2
      VBAR(3)=VBAR(3)+V3
      VSD(1)=VSD(1)+V1**2
      VSD(2)=VSD(2)+V2**2
      VSD(3)=VSD(3)+V3**2
  215 CONTINUE
  220 CONTINUE
C
C      CALCULATE HETEROGENEITY V-STATISTICS FOR OBSERVED DATA
C
      IF(NSITES EQ 1)GOTO 235
      V1=ZERO
      V2=ZERO
      V3=ZERO
      DO 225 I=1,NSITES
      NREC=LEN(I)
      TEMP2=(XMOM(2,I)-RMOM(2))**2
      TEMP3=(XMOM(3,I)-RMOM(3))**2
```

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      TEMP4=(XMOM(4,I)-RMOM(4))**2
      V1=V1+NREC*TEMP2
      V2=V2+NREC*DSQRT(TEMP2+TEMP3)
      V3=V3+NREC*DSQRT(TEMP3+TEMP4)
  225 CONTINUE
      VOBS(1)=DSQRT(V1/SUMLEN)
      VOBS(2)=V2/SUMLEN
      VOBS(3)=V3/SUMLEN
C
C       CALCULATE AND PRINT HETEROGENEITY MEASURES (H STATISTICS)
C
      IF(KPRINT GT 0)WRITE(KOUT,6050)NSIM
      DO 230 J=1,3
      VBAR(J)=VBAR(J)/NSIM
      VSD(J)=DSQRT((VSD(J)-NSIM*VBAR(J)**2)/(NSIM-ONE))
      H(J)=(VOBS(J)-VBAR(J))/VSD(J)
      IF(KPRINT LE 0)GOTO 230
      LOOK1=BLANK
      LOOK2=BLANK
      IF(H(J) GE HCRIT1)LOOK1=STAR
      IF(H(J) GE HCRIT2)LOOK2=STAR
      IF(J EQ 1)WRITE(KOUT,6060)VOBS(J),VBAR(J),VSD(J),H(J),LOOK1,LOOK2
      IF(J EQ 2)WRITE(KOUT,6070)VOBS(J),VBAR(J),VSD(J),H(J),LOOK1,LOOK2
      IF(J EQ 3)WRITE(KOUT,6080)VOBS(J),VBAR(J),VSD(J),H(J),LOOK1,LOOK2
  230 CONTINUE
  235 CONTINUE
      S=RMOM(3)
      SS=S*S
      T4FIT(1)=
     * GLOC0+SS*(GLOC2+SS*(GLOC4+SS*GLOC6))
      T4FIT(2)=
     * GEVC0+S*(GEVC1+S*(GEVC2+S*(GEVC3+S*(GEVC4+S*(GEVC5+S*GEVC6)))))
      T4FIT(3)=GNOC0+SS*(GNOC2+SS*(GNOC4+SS*GNOC6))
      T4FIT(4)=
     * PE3C0+S*(PE3C1+S*(PE3C2+S*(PE3C3+S*PE3C4)))
      T4FIT(5)=GPAC0+S*(GPAC1+S*(GPAC2+S*(GPAC3+S*(GPAC4+S*(GPAC5+
     * S*(GPAC6+S*GPAC7))))))
C
C       CALCULATE GOODNESS-OF-FIT MEASURES (Z STATISTICS)
C
      T4BAR=T4BAR/NSIM
      T4SD=DSQRT((T4SD-NSIM*T4BAR**2)/(NSIM-ONE))
      DO 240 IDIST=1,5
      Z(IDIST)=(T4FIT(IDIST)+T4BAR-TWO*RMOM(4))/T4SD
  240 CONTINUE
      IF(KPRINT LE 0)GOTO 260
      WRITE(KOUT,6090)NSIM
      DO 250 IDIST=1,5
      LOOK1=BLANK
      IF(DABS(Z(IDIST)) LT ZCRIT)LOOK1=STAR
  250 WRITE(KOUT,6100)DISTRI(IDIST),T4FIT(IDIST),Z(IDIST),LOOK1
  260 CONTINUE
  270 CONTINUE
      CALL PELGLOQ(RMOM,PARA(1,1))
      CALL PELGEVQ(RMOM,PARA(1,2))
      CALL PELGNOQ(RMOM,PARA(1,3))
      CALL PELPE3Q(RMOM,PARA(1,4))
      CALL PELGPAQ(RMOM,PARA(1,5))
      IF(KPRINT LE 0)GOTO 320
      IF(NSIM EQ 1)WRITE(KOUT,6110)
      IF(NSIM GT 1)WRITE(KOUT,6120)
      DO 280 IDIST=1,5
      IF(DABS(Z(IDIST)) LE ZCRIT)
     *   WRITE(KOUT,6130)DISTRI(IDIST),(PARA(IPARA,IDIST),IPARA=1,3)
  280 CONTINUE
      IF(NPROB EQ 0)GOTO 320
      WRITE(KOUT,6140)PROB
      DO 300 IDIST=1,5
      IF(DABS(Z(IDIST)) GT ZCRIT)GOTO 300
      DO 290 IQ=1,NPROB
      IF(IDIST EQ 1)Q(IQ)=QUAGLO(PROB(IQ),PARA(1,1))
      IF(IDIST EQ 2)Q(IQ)=QUAGEV(PROB(IQ),PARA(1,2))
      IF(IDIST EQ 3)Q(IQ)=QUAGNO(PROB(IQ),PARA(1,3))
      IF(IDIST EQ 4)Q(IQ)=QUAPE3(PROB(IQ),PARA(1,4))
      IF(IDIST EQ 5)Q(IQ)=QUAGPA(PROB(IQ),PARA(1,5))
```

```
 290 CONTINUE
     WRITE(KOUT,6150)DISTRI(IDIST),(Q(IQ),IQ=1,NPROB)
 300 CONTINUE
 320 CONTINUE
      RETURN
1000 WRITE(KOUT,7000)'MAXNS'
     RETURN
1010 WRITE(KOUT,7000)'MAXQ'
     RETURN
1020 WRITE(KOUT,7000)'MAXREC'
     RETURN
1030 WRITE(KOUT,7010)
     GOTO 140
6000 FORMAT(/' SITE   N     NAME     LQ-CV  LQ-SKEW LQ-KURT  D(I)')
6010 FORMAT(2I5,2X,A12,3F8 4,F7 2,2X,2A1)
6020 FORMAT(/5X,'WEIGHTED MEANS',5X,6F8 4)
6030 FORMAT(/' FLAGGED TEST VALUES'/(15F5 1))
6040 FORMAT(/' PARAMETERS OF REGIONAL KAPPA DISTRIBUTION ',4F8 4)
6050 FORMAT(//' ***** HETEROGENEITY MEASURES *****'/
  * ' (NUMBER OF SIMULATIONS  =',I6,')')
6060 FORMAT(/' OBSERVED     S D  OF GROUP LQ-CV          =',F8 4/
  *     ' SIM  MEAN OF S D  OF GROUP LQ-CV         =',F8 4/
  *     ' SIM  S D  OF S D  OF GROUP LQ-CV         =',F8 4/
  *     ' STANDARDIZED TEST VALUE H(1)             =',F6 2,2X,2A1)
6070 FORMAT(/' OBSERVED AVE   OF LQ-CV / LQ-SKEW DISTANCE =',F8 4/
  *     ' SIM  MEAN OF AVE  LQ-CV / LQ-SKEW DISTANCE =',F8 4/
  *     ' SIM  S D  OF AVE  LQ-CV / LQ-SKEW DISTANCE =',F8 4/
  *     ' STANDARDIZED TEST VALUE H(2)             =',F6 2,2X,2A1)
6080 FORMAT(/' OBSERVED AVE   OF LQ-SKEW/LQ-KURT DISTANCE =',F8 4/
  *     ' SIM  MEAN OF AVE  LQ-SKEW/LQ-KURT DISTANCE =',F8 4/
  *     ' SIM  S D  OF AVE  LQ-SKEW/LQ-KURT DISTANCE =',F8 4/
  *     ' STANDARDIZED TEST VALUE H(3)             =',F6 2,2X,2A1)
6090 FORMAT(//' ***** GOODNESS-OF-FIT MEASURES *****'/
  * ' (NUMBER OF SIMULATIONS  =',I6,')'/)
6100 FORMAT(1X,A18,2X,' L-KURTOSIS=',F6 3,2X,' Z VALUE=',F6 2,1X,A1)
6110 FORMAT(//' PARAMETER ESTIMATES'/)
6120 FORMAT(//' PARAMETER ESTIMATES FOR DISTRIBUTIONS ACCEPTED A'
  * ' 90% LEVEL'/)
6130 FORMAT(1X,A18,1X,5F7 3)
6140 FORMAT(/' QUANTILE ESTIMATES'/19X,(1X,14F7 3))
6150 FORMAT(1X,A18,(1X,14F7 3))
7000 FORMAT(' *** ERROR *** ROUTINE REGTSTLQ  ',
  * ' INSUFFICIENT WORKSPACE - RECOMPILE WITH LARGER VALUE OF ',A6)
7010 FORMAT(' *** ERROR *** ROUTINE REGTST   UNABLE TO INVERT',
  * ' SUM-OF-SQUARES MATRIX '/31X,'D STATISTICS NOT CALCULATED ')
     END
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List of Publications

List of papers published in Journals/ Proceedings

1. Bhuyan, A. and Borah, M. Best fitting probability distributions for annual maximum discharge data of the river Kopili, Assam, *Journal of Applied and Natural Science* **1(1)**, 50- 52 (2009)

2. Bhuyan, A., Borah, M. and Kumar, R. Regional flood frequency analysis of north-bank of the river Brahmaputra by using LH-moments. *Water Res. Manage.* **24(9),** 1779-1790 (2010)

3. Bhuyan, A. and Borah, M. Flood frequency analysis of Tripura region of North East India by using L-moments. *Proceedings Seminar on Integrated Water Resources development and Management, **May 30,*** 45- 51 (2008)

4. Bhuyan, A. and Borah, M. Modeling of annual maximum water level of Buri Dihing sub- basin by using extreme value distribution. *Proceedings of National Seminar on Mathematical Modeling, **March 3-5,*** 152-161 (2008)

5. Bhuyan, A. and Borah, M. Flood frequency analysis of some selected rivers of south bank of the river Brahmaputra, Assam. *Proceedings Seminar on Shared Water-Shared Opportunities, **May, 29,*** 62- 70 (2009)

List of papers presented in National Seminar

1. Bhuyan, A. and Borah, M. Comparison of various probability distributions for flood frequency analysis of the river Kopili of Assam. *National Research Scholar Meet for Mathematics and Statistics,* 6-10, December, IIT, Kanpur, India (2008)

2. Bhuyan, A. and Borah, M. (2008): Assessment of developed regional flood frequency of Tripura region of North East India, *96th Science Congress,* 3-7 January, (2009)

List of papers communicate for publication

1. Bhuyan, A. and Borah, M. Assessment of regional flood frequency of Tripura region of North East India. Communicated to *Journal of Applied Hydrology, Andhra University, Dept. of Geophysics, Andhra* (2008)

2. Bhuyan, A. and Borah, M. Regional flood frequency analysis of Tripura based on L-moment. Communicated to *Hydrology Journal, Indian Association of Hydrologist* (2008)