SOME CASCADE MODELS IN THE THEORY OF RELIABILITY

A Thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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Abstract

Our effort in this research work is to develop different cascade reliability models. The evaluation of reliability is considered mainly for cascade systems. For this purpose several distributions are considered namely, exponential, Weibull, gamma, Rayleigh, Lindley, uniform and two-point distribution.

First, the attenuation factor K is taken to be a uniform random variable in $(0,1)$. Then unconditional reliability of the system is given by the expected value of $R_n(K)$, where expectation is taken w.r.t. the distribution of K. Then some well known distributions namely, exponential, gamma and Weibull distribution are used to evaluate the reliability of n-cascade system. Again stress-strength is considered as a mixture of either two exponentials or two Rayleighs or two Weibulls, reliability of n-cascade system is obtained.

More precisely an n-cascade system is also used where components may fail in different ways. For the estimation of cascade system reliability three failure models have been considered. Further, two distributions viz. exponential and Rayleigh are used to find out the expressions of reliability for all the failure models. Again stress-strength distributions are considered as one parameter exponential and the parameters involved are assumed to be random with known prior distribution. Two cases are considered i.e. (i) strength parameter is random but stress parameter is a constant and (ii) stress parameter is random but strength parameter is a constant. In each case, two prior distributions viz. uniform and two-point distributions are considered for the parameter(s) involved. The system reliability of a 2-cascade system is obtained for all the cases.

An n-cascade system with $P(X<Y<Z)$ is considered where Y is the stress on the component subjected to two strengths X and Z. Reliability expressions of an n-cascade system is obtained when stress-strength both are either exponential or Rayleigh or Lindley distribution.

Often the expressions of reliability are not simple enough to give an idea of their change with different stress-strength parameters. Hence, a few numerical values of the

reliabilities are tabulated against the parameter(s) involved, in each chapter, to show the effect of various parameters on the system reliability. The graphical representations are described for a particular set up.

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DECLARATION

I, **Chumchum Doloi**, hereby declare that the subject matter in this thesis entitled "**Some Cascade Models in the theory of Reliability**" is the record of work done by me, that the contents of this thesis did not form basis of the award of any previous degree to me or to the best of my knowledge to anybody else and that the thesis has not been submitted by me for any research degree in any other university/institute.

This thesis is being submitted to the Tezpur University for the degree of Doctor of Philosophy in Mathematical Sciences.

Place: Napaam, Tezpur

Date: 01/05/2012

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CERTIFICATE

This is to certify that the thesis entitled "**Some Cascade Models in the theory of Reliability**" submitted to the School of Science and Technology, Tezpur University in partial fulfillment for the award of the degree of Doctor of Philosophy in Mathematical Sciences is a record of research work carried out by **Ms. Chumchum Doloi** under my supervision and guidance.

All help received by her from various sources have been duly acknowledged.

No part of this thesis has been submitted elsewhere for award of any other degree.

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- Doloi. C, Borah. M. and Sriwastav G. L. A Cascade Model with Random Parameters, Communicated for publication in ***IAPQR Transactions***.

Chapter 1

Introduction

Chapter 1

1 Introduction

Reliability theory is a well established scientific discipline with its own principles and methods for solving its problems. Probability theory and mathematical statistics play a major role in most of the problems in reliability theory. In fact reliability is often defined in terms of probability. Standby redundancy is a well known technique to increase the reliability of a system. In standby redundancy, it is assumed that a component, taking the place of a failed component works exactly in the same environment, i.e. it faces the same stress. But it may not be necessarily so. An n-cascade system is a special type of n-standby system in which a new component faces k (called an attenuation factor) times the stress on the preceding component. The purpose of this thesis is to estimate the reliability of various cascade models using different stress-strength (S-S) distributions.

1.1 Background of the study

Reliability theory uses existing mathematical methods as well as develops new once. There are a large number of definitions of reliability scattered in the literature. They defined reliability in different ways and in different contexts. Some of them are:

1. 'Reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered' (Radio- Electronics-Television Manufactures Association, 1955, cf. Barlow and Proschan, 1965).

   Symbolically, $R(t) = \int_t^{\infty} d\,F(x)$

where $F(t)$ represents the failure time distribution of the system.

2. 'Reliability is the integral of the distribution of probabilities of failure-free operation from the instant of switch-on to the first failure (cf. Polvoko, 1968).

3. 'The reliability $R(t)$ of a component (or a system) is the probability that the component (or system) will not fail for a time t (cf. Polvoko, 1968).

Reliability models, in general, can be broadly divided into two groups: (i) Time dependent models and (ii) Stress-strength models or interference models.

In the time dependent models time is the important random variable and different measures of reliability theory such as Reliability or Survival function, Availability, Maintainability, Failure rate etc. are obtained from Time-to-Failure (TTF) distributions of the unit (system or component) under study. In such models the underlying idea is that the characteristics of the unit gradually changes and failure occurs when it goes beyond the specified limits. Ordinarily, here failure probability is an increasing function of time and similarly other measures are also functions of time (except failure rate for exponential TTF distribution). Majority of studies in reliability theory are based on the time dependent models. A case in favor of such models is presented by Disney and Seth (1968), Yadav (1973), Kapur and Lamberson (1977), Dhillon (1980) and many references cited by them. Some such models are considered in the present study. In time dependent models the time is the dominating factor and in interference models stress is the dominating factor. Throughout this thesis strength and stress are considered to be continuous random variable, though they may be discrete also (cf. Charalambides, 1974, Winterbottom, 1974).

The word 'stress' and 'strength' used in the reliability theory in a broader sense, applicable in many situations well beyond the traditional, mechanical or structural systems. In reliability theory by 'stress' we mean any agency which tends to produce failure of a component, a device or a material. The term agency may be a mechanical load, environmental hazard, electric voltage etc. The 'strength' represents an agency resisting failure of the system and it is measured by the mean stress required to cause the failure of the system. The Interference theory is based upon the fact that when the strength of a component or a device or a material is less than the stress imposed on it, the failure occurs (Dhillon 1980). Here the reliability R, of a component (or system) is defined as the probability that the strength of the component, say X (a r.v), is not less than the stress, say Y (another r.v), on it. Symbolically,

$$R = P(X \geq Y) \tag{1.1.1}$$

The S-S models are also called interference models because here the reliability can be represented in terms of interference area between stress and strength densities. Once the respective distributions of stress and strength are known (or estimated), one can obtain reliability of a system by employing equation (1.1.1).

If $f(x)$ and $g(y)$ are the densities of X and Y respectively then from (1.1.1)

$$\begin{aligned} R &= \int_{-\infty}^{\infty}\left[\int_{y}^{\infty} f(x)dx\right] g(y)dy \\ &= \int_{-\infty}^{\infty} \bar{F}(y) g(y)dy \end{aligned} \tag{1.1.2}$$

$$\begin{aligned} \text{or } R &= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{x} g(y)dy\right] f(x)dx \\ &= \int_{-\infty}^{\infty} G(x) f(x)dx \end{aligned} \tag{1.1.3}$$

where $\bar{F}(x) = 1 - F(x)$, $F(X)$ and $G(Y)$ are the cumulative distribution functions of X and Y, respectively.

The distribution of stress and strength which are commonly used in reliability theory are exponential, gamma, normal, Weibull, log-normal, Rayleigh and Lindley distributions. Different distributions are used in different situations.

Mostly in S-S models each stress-strength is represented by a single distribution, but in some cases this may not be suitable and a mixture of distributions may represent them better. Sometimes, the distributions with fixed parameters may not represent the stress and/or strength distributions adequately. For example, if a particular component, having certain strength distribution is manufactured in different lots, then for a particular lot the parameters of the strength distribution may remain fixed but may vary from lot to lot. In such situations the parameters of the strength distributions may themselves be taken as random variables. Similar reasoning can be given for the distribution of stress also. So stress and strength may be represented by compound distributions.

In the studies of S-S models, generally stress-strength of a component is supposed to be independent random variables. But in many situations they may be correlated also. The stresses and strengths together and even stress and strength separately may be correlated.

Increase in the complexity of jobs to be performed increases the complexity of the device, increasing the number of components in it and the possibility of inter-actions. In general, as the number of components increases the reliability of the system decreases. But at the same time the reliability must be kept high in order to meet the increasing importance of the task. Better maintenance results in higher reliability, but some times it is not possible to achieve high reliability goals with any amount of maintenance. In such cases the only alternative way to achieve high reliability is to incorporate redundancy.

Redundancy is the technique in which more components (or units) than the minimum required for normal operations of the system are attached to it in such a way that even if only a few units are working the system works. Pieruschka (1963) has described the following forms of redundancy:

(i) Parallel redundancy

(ii) Standby redundancy

In a parallel redundant system n components are connected in a parallel arrangement, and to start with all n components are operating. The system continues to operate till at least k of the components are operating. The system is also referred to as k-out-of-n system. When $k = n$, it is the series system, when $k = 1$ it is called completely parallel system (Lloyed and Lipow, 1962).

Standby redundancy is a well known technique to increase the reliability of a system. Here a number of redundant components are attached to one or more essential components of the system. If a particular component along with its redundant units is called one set, the standby system works till at least one component in each set is in working order. In general, in standby redundancy, it is assumed that a component, taking the place of a failed component works exactly in the same environment, i.e. it faces the same stress. But it may not be necessarily so.

In addition to the redundancy discussed above there is another type of redundancy i.e., 'Cascade Redundancy' (cf. Pandit and Sriwastav, 1975). Cascade reliability is a special kind of standby system for stress-strength models. An n-cascade system may be described as follows:

The system consists of n components, arranged in order of activation. The component strengths are identically distributed independent random variables following a specified distribution. The system is working under impacts of random stresses. Initially the first component only is active and other $(n-1)$ remain as standbys, i.e. when a stress comes on the system, it is only this first component that initially is subjected to it. If this random stress exceeds the strength of this component, it fails and the second component in order gets activated. However, the stress to which this new component is subjected to k times the stress on the previous component. If, under the circumstances, the second component fails, the third component in order succeeds it, to face again a changed stress. The chain continues until, at some stage the residual stress becomes less than the strength of the currently activated component or all the components in the system have failed. In the first case the system itself survives, though possibly with the less of a few components; in the later case, the system fails. The attenuation factor k (an integer constant) by which the stress keeps on changing in the successive steps may be constant or random variable. Such a system is termed as an n-cascade system.

Let $X_1, X_2, \ldots X_n$ be the strengths of the n components in the order of activation and Y_1 be the stress applied to the first component. If $Y_1 \leq X_1$, the component works; otherwise it fails and the second component, with strength X_2, takes its place. However the magnitude of the stress coming on the second component will be $Y_2 = k\,Y_1$, where k is an integer constant. If $Y_2 \leq X_2$, the second component works; otherwise it also fails and the third component will now be subjected to a stress $Y_3 = k^2\,Y_2$. In general, if the ith component fails, the $(i+1)^{th}$ component, with strength X_{i+1}, takes its place and is subjected to a stress $Y_{i+1} = k^i\,Y_i$, i=1,2,..., $(n-1)$. The system fails only if all the n components in cascade fail.

In the above discussions, only the time dependent failure models are considered. But as remarked earlier a realistic model should consider both stress and time. For example, though the system's failure depends upon the magnitude of stress but it may also depend on the number of stresses impinging to the system in a particular interval of time period. Thus considering the factors- stress and time, certain type of models can be considered which are known as Stress-Strength-Time (SST) models (cf. Kapur and Lamberson, 1977).

The S-S models discussed so far, assume that the stress and strength are random variables. However, more generally, they may be stochastic processes. Taking the system strength and stress on it as two stochastic processes $X(t)$ and $Y(t)$ respectively, the reliability of the system can be obtained from the 'Difference-process', viz.

$$Z(t)=\{\, X(t) - Y(t) \,\}$$

The system fails when, for the 'first-time', the stochastic process Z(t) crosses zero from the above (Sriwastav and Pandit, 1978).

1.2 Objectives

This research aims to develop some cascade models to estimate the system reliability. For this purpose several distributions are considered viz. exponential, gamma, Weibull, Rayleigh, Lindley, uniform and two-point distribution. The main objectives of this thesis are outlined as follows:

- **Cascade system with random attenuation factor**

  Here we have made an n-cascade system for which the attenuation factor k is a uniform random variable in $(0,1)$. For this purpose, several stress-strength distributions have been considered to obtain the expressions of the unconditional reliability of the system.

- **Mixture of distributions in cascade system**

  In this study, n-cascade system has been taken when stress-strength of each component are assumed to be mixture of two distributions with different mix-parameters. Various distributions have been considered to describe the expression for reliability of n-cascade system.

- **Cascade reliability in different types of failure models**

  For the estimation of cascade system reliability three failure models have been considered here. In particular when stress-strength of the components follows different distributions, expressions of reliability have been worked out for all the models.

- **Cascade model with random parameters**

  Here we have made a 2-cascade system by considering stress-strength are exponential variates and one of the parameters (stress or strength) be a random with a known prior distribution, other parameter remaining constant. Some well known probability distributions viz. uniform and two-point distribution has been taken as the prior distributions; reliability of the 2-cascade system is obtained.

- **An n-cascade system with P(X <Y <Z)**

  Here a general model is developed for n-cascade system with $P(X < Y < Z)$, where Y is the stress on the component subjected to two strengths X and Z. The reliability expressions of an n-cascade system is obtained when the stress-strength of the components follow particular distributions.

1.3 A Brief Survey on Reliability Models

A review of some of the works of other authors in the interference theory, which are relevant to the work done in the present thesis, follows:

A system may be a single component system or a multicomponent system. In a multicomponent system one may be interested in finding reliability of each component, each of which may be treated as a system itself and then knowing the structure of the system and using probability logic the reliability of the complete system may be evaluated.

Pandit and Sriwastav (1975, 1978) have considered n-cascade system and obtained the expressions for reliability where stress and strength distributions are exponential, gamma and normal, assuming the attenuation factor, k to be a constant and also when it is random.

Pandit and Sriwastav (1976) have obtained the distribution of the number of attacks to failure for a cascade system and called it generalized geometric distribution. They have also considered the cascade system subjected to stress arriving at a random process, viz. Poisson process and obtained with reliability expressions for 2- and 3-cascade system. Gogoi et al. (2010) has considered n-standby S-S system where the number of impacts of stresses faced by the system is a Poisson process. They obtained the system reliability when both stress and strength follow either exponential or gamma or normal or Weibull distribution.

Kapur and Lamberson (1977) studied the time dependent S-S model by considering repeated application of stress and also the deterioration of strength with time. They have obtained the expression for reliability of the system for a single component by considering the deterministic and random cycle times. Gopalan and Vankateswarlu (1982) have considered the reliability of time dependent 2-cascade and 3-cascade system using S-S models by considering each of the stress and strength variables as deterministic or random fixed or random independent. They assume the number of cycles in any period of time 't' to be deterministic. Assuming attenuation factor k_i's to be constants they have obtained the expressions for system reliability where stress and strength distributions are exponential.

Beg (1980) has considered the two-parameter exponential distribution for stress and strength random variables to derive the MVUE of R. Beg (1980) obtained MVU and Bayes estimators for a n-cascade system where X and Y are exponential. A cascade reliability model for n-warm standby system is considered by Dutta and Bhowal (1999) and cascade model with imperfect switching is considered by Sriwastav (1992).

Kakaty (1983) considered the mixture of distributions in stress-strength model for standby system and obtained the system reliability when the stress and strength are the mixture of two exponential, two gammas, two Weibull distributions and an exponential and a gamma distribution. Cohen (1965), Harris and Singpurwalla (1968), Mann et al. (1974) have considered time to failure distributions as mixtures of distributions.

Shooman (1968) has assumed that the parameter of strength distribution is a deterministic function of time. Tarman and Kapur (1975) have assumed that the parameters of the stress-strength distributions are variables but not random variables. Sriwastav and Kakaty (1980) have considered that the parameters (stress or strength) of the stress-strength distributions are random variable. Although all the parameters involved may be taken as random variables, they have considered only one parameter random with a known prior distribution, at a time, and the others remain constants. Then, from compound distribution of stress-strength they have obtained the reliability of the system.

Singh (1980) has considered the estimation of $R = P(X_1 < Y < X_2)$ where X_1 and X_2 are independent random stress variables and Y is independent of X_1 and X_2 is a random strength variables. Chandra and Owen (1975) obtained the estimation of reliability of a component subjected to several different stresses. They obtained the estimate $R = P[Max(Y_1, Y_2, \ldots, Y_k) < X]$ when $(Y_1, Y_2, \ldots, Y_k)$ are i.i.d. normal distributions and X as an independent normal distribution. Hanagal (1997) has estimated the reliability of a component subjected to two different stresses which are independent of the strength of a component. In another paper (2003) he estimated the system reliability in multicomponent series stress-strength models.

Rekha et al. (1988) obtained the reliability of n-cascade system where stress and strength are log-normal and Weibull. In another investigation Rekha et al. (1992) have derived an expression for the reliability of a single component system where the strength of the component and the imminent stress on the system are random and follow non-identical probability distribution. They assumed that after successive arrivals, the strengths on the successive components are attenuated by specified deterministic factors. They have considered survival function for the stress and strength following exponential distribution.

Raghavachar et al. (1983) have considered survival functions under stress attenuation in cascade reliability. Rekha and Shyam Sunder (1997) have derived an expression for survival function for the strength attenuation system with stress-strength following exponential distribution. They have obtained the lower and upper bounds when the strength attenuation factor $k_i^* = k_i$.

1.4 Organization of the thesis

This thesis consists of seven chapters followed by references. The organization of the thesis is as follows:

Chapter 1 is the introductory one, provides background of the present study and objective of the thesis. It also provides a brief survey on the reliability models.

Chapter 2 deals with an n-cascade system for which the attenuation factor K is a random variable distributed uniformly over $(0,1)$. Here the unconditional reliability of the n-cascade system is given by the expected value of $R_n(K)$, where expectation is taken w.r.t. the distribution of K. Various stress-strength distributions like exponential and Weibull have been considered to evaluate the unconditional reliability of the system. We also consider the case when stress follows gamma and strength follows exponential distribution. For some particular values of the parameters involved numerical values of the reliability are tabulated for each case and graphical representations are described for a particular set up.

Chapter 3 demonstrate the estimation for reliability of an n-cascade system where stress-strength of each component can be represented by a mixture of distributions. Considering that each stress and strength is a mixture of either two exponentials or two Rayleighs or two Weibulls, reliability of n-cascade system, $n \leq 3$ is obtained. A few numerical values of reliabilities R_1, R_2, R_3 are tabulated and reliability graphs are also drawn.

In Chapter 4, an attempt has been made to estimate an n-cascade system where components may fail in different ways. For the estimation of cascade system reliability three failure models have been considered i.e. (i) an active component faces m different stresses and it fails if the strength of the active component is less than any one of the stresses and after the failure of the first component, the second component faces m stresses which are k times the corresponding previous stresses and so on, (ii) for the working of an active component the m stresses on the component lie in an interval (a_j, b_j). The component fails even if one of stresses on the component falls outside the specified limits and (iii) this model is similar to the model II except that the components are not identical. In particular when stress-strength of the components are either exponential or Rayleigh distributions, expressions of reliability have been worked out for all the models. Some numerical values of reliability have also been presented in tabular form for some selected values of the parameters. Some graphs are plotted for selected values of the parameters to facilitate the direct reading of reliability.

Chapter 5 provides the reliability estimation of 2-cascade system where stress strength are exponential variates and one of the parameters (stress or strength) is assumed to be a random with a known prior distribution, other parameter remaining constant. Using the derived compound distribution, reliability of the 2-cascade system is obtained. Uniform and two-point distributions are taken as the prior distributions for the parameters concerned. For all the cases some numerical values of reliability are tabulated. To make the things clear, a few graphs are also drawn.

In Chapter 6, an n-cascade system with $P(X < Y < Z)$ where Y is the stress on the component subjected to two strengths X and Z has been demonstrated. Reliability expressions of an n-cascade system is obtained when stress-strength both are either exponential or Rayleigh or Lindley distribution. Another two cases are considered; first, when both strengths are one parameter exponential and stress follows Lindley distributions and second, when both strengths are one parameter exponential and stress follows two parameters gamma distributions. Various reliability parameters have been computed and analyzed by tabular illustrations and some graphs are also drawn for selected values of the parameters.

The last Chapter 7 includes the summary of the thesis and future works. The references and the tables are cited in the text of the thesis are appended at the end.

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Chapter 2

Cascade System with Random Attenuation Factor

Chapter 2

Cascade System with Random Attenuation Factor

2.1 Introduction

Cascade system were first developed and studied by Pandit and Sriwastav (1975). Cascade reliability is a special kind of standby system for stress-strength models. In a standby system, i.e. a system with standby redundancy, there are a number of components only one of which works at a time and the others remain as standbys. When an impact of stress exceeds the strength of the active component, for the first time, it fails and another from standbys, if there is any, is activated and faces the impact of stresses. The system fails when all the components have failed. In cascade system, the strength of the components are independent and stress on a component is k times the stress on its preceding component, called attenuation factor.

Attenuation factor is generally assumed to be a constant for all the components or to be a parameter having different fixed values for different components (Pandit and Sriwastav, 1975). But an attenuation factor may be a random variable also (Pandit and Sriwastav, 1978). Here we have considered an n-cascade system for which the attenuation factor 'K' is a random variable. Then, instead of talking about the reliability of the system we talk about the expected reliability of the system. The expectation is taken w.r.t. the distribution of K. Much of the materials of this chapter are based on Doloi and Borah (2010).

The organization of this chapter is as follows: In Section-2.2, the problem is formulated in mathematical terms. In Section-2.3, a simple case is considered when K is a uniform r.v. in (0,1). In Sub-Sections 2.3.1, 2.3.2 and 2.3.3, all the distributions are assumed to be exponential, Weibull and exponential-gamma respectively to evaluate the expressions of the unconditional reliability of the system. Some graphs are plotted for each case in Section-2.4. To observe the change in the values of reliabilities with parameters involved, some numerical values of reliabilities are tabulated against the parameters involved in the **Table** 2.1, **Table** 2.2 and **Table** 2.3 (cf. Appendix) and results and discussions are given in Section 2.5.

2.2 Mathematical Formulation

Let us consider an n-cascade system and suppose that n components are numbered from 1 to n in their order of activation. Let X_i be the strength of the i[th] component, in the order of activation, and when activated faces the stress Y_i, $i=1,2,\ldots,n$. For a cascade system with attenuation factor 'K' (considered to be an integer random variable)

$$Y_i = K^{i-1}Y_1, i = 1,2,\ldots,n \tag{2.2.1}$$

The reliability of the system is given by

$$R_n(K) = R(1,K) + R(2,K) + \ldots + R(n,K) \tag{2.2.2}$$

where $R(i,K) = P[X_1 < Y_1, X_2 < K\,Y_1, \ldots, X_{i-1} < K^{i-1}Y_1, X_i > K^i Y_1], i = 2,3,\ldots,n$ (2.2.3)

Now, if the attenuation factor, say 'K', is a r.v. then $R_n(K)$ is the conditional reliability of the system on the condition that $K=k$. Let $h(k)$ be the density function of K. Then, unconditional reliability of the system is given by the expected value of $R_n(K)$, where expectation is taken w.r.t $h(k)$,

i.e. Unconditional reliability of the n-cascade system $= E[R_n(K)]$

$$= \sum_{i=1}^{n} E[R(i,K)], \tag{2.2.4}$$

where $$E[R(i,K)] = \int_{-\infty}^{\infty} R(i,k)h(k)dk \tag{2.2.5}$$

Thus, when the attenuation factor is a r.v. we talk of expected reliability of the system instead of the reliability of the system.

We may note that $R(1,K)$ is independent of 'K' and hence

$$E[R(1,K)] = R(1,K) = R(1), \text{ say} \tag{2.2.6}$$

2.3 The Distribution of K is Uniform

We consider here the case when 'K' follows a uniform distribution in (0,1),

i.e. $$h(k) = 1,\ 0 \le k \le 1 \tag{2.3.1}$$

Then from (2.2.5)

$$E[R(i,K)] = \int_0^1 R(i,k)dk,\ i = 1,2,..n \tag{2.3.2}$$

The strength (X) and stress (Y) may follow any distribution. We consider here only three cases viz., (1) when X_i and Y_i both are exponential variates, (2) when X_i and Y_i both are Weibull variates and (3) when X_i's are exponential variates and Y_i's are gamma variates.

2.3.1 Exponential Stress-Strength

Suppose X_i and Y_i are i.i.d. as exponential variates with mean $(1/\lambda)$ and $(1/\mu)$, respectively. Then for $\rho = \frac{\lambda}{\mu}$

$$R(1) = \frac{1}{(1+\rho)}, \quad \text{(Independent of } K\text{)}, \tag{2.3.3}$$

$$R(2,K)=\left(\frac{1}{(1+\rho K)}\right)-\left(\frac{1}{(1+\rho+\rho K)}\right) \tag{2.3.4}$$

$$R(3,K)=\left(\frac{1}{(1+\rho K^2)}\right)-\left(\frac{1}{(1+\rho+\rho K^2)}\right)-\left(\frac{1}{(1+\rho K+\rho K^2)}\right)+\left(\frac{1}{(1+\rho+\rho K+\rho K^2)}\right) \tag{2.3.5}$$

So, from (2.2.5), (2.3.4), and (2.3.5) after some simple calculations we get

$$E[R(2,K)]=\frac{1}{\rho}\log\left\{\frac{(1+\rho)^2}{(1+2\rho)}\right\} \tag{2.3.6}$$

and
$$E[R(3,K)]=\rho^{-1/2}\begin{bmatrix}\tan^{-1}(\rho)^{1/2}-\frac{1}{\sqrt{1+\rho}}\tan^{-1}\left(\frac{\rho}{1+\rho}\right)^{1/2}\\ -\frac{2}{\sqrt{4-\rho}}\tan^{-1}3\left(\frac{\rho}{4-\rho}\right)^{1/2}+\frac{2}{\sqrt{4-\rho}}\tan^{-1}\left(\frac{\rho}{4-\rho}\right)^{1/2}\\ +\frac{2}{\sqrt{4+3\rho}}\tan^{-1}3\left(\frac{\rho}{4+3\rho}\right)^{1/2}-\tan^{-1}\left(\frac{\rho}{4+3\rho}\right)^{1/2}\end{bmatrix} \tag{2.3.7}$$

For $i\geq 4$, closed form expression could not be obtained. Of course, one may use numerical integrations.

Then, the unconditional reliability (or expected reliability), R_2 for a 2- cascade system, from (2.2.4), is given by

$$R_2 = R(1)+E[R(2,K)] \tag{2.3.8}$$

where, $R(1)$ and $E[R(2,K)]$ are given by (2.3.3) and (2.3.6), respectively.

Similarly, the unconditional reliability R_3 for a 3- cascade system, is given by

$$R_3 = R_2 + E[R(3,K)] \tag{2.3.9}$$

A few values of R_1, R_2 and R_3 are tabulated in **Table** 2.1 (cf. Appendix) for different values of ρ .

2.3.2 Weibull Stress-Strength

Suppose X_t and Y_t are Weibull distribution with p.d.f.'s

$$f(x) = cx^{c-1}\exp\{-(x/\theta)^c\}/\theta^c \ , \ x > 0$$

$$g(y) = ay^{a-1}\exp\{-(y/\lambda)^a\}/\lambda^a \ , \ y > 0$$

Then

$$R(1) = \int_0^\infty \exp[-\{t + (\lambda/\theta)^c t^{c/a}\}]dt \ , \tag{2.3.10}$$

$$R(2) = \int_0^\infty \exp[-\{t + (\lambda k/\theta)^c t^{c/a}\}]dt - \int_0^\infty \exp[-\{t + \{(\lambda/\theta)^c + (\lambda k/\theta)^c\}t^{c/a}\}]dt \tag{2.3.11}$$

So from (2.2.5) and (2.3.11) we get

$$E[R(2,k)] = \int_0^1 \left[\int_0^\infty \exp[-\{t + (\lambda k/\theta)^c t^{c/a}\}]dt - \int_0^\infty \exp[-\{t + \{(\lambda/\theta)^c + (\lambda k/\theta)^c\}t^{c/a}\}]dt \right] dk \tag{2.3.12}$$

For $i \geq 3$ closed form expression could not be obtained. Using the Gauss Laguerre Integration method and Trapezoidal rule we have evaluated $R(1)$ and $E[R(2,K)]$ for different values of c, θ, a and λ .

Then the unconditional reliability, R_2 is given by (2.3.8).

A few values of R_1 and R_2 are tabulated in **Table** 2.2 (cf. Appendix) for different values of c, θ, a, λ .

2.3.3 Exponential Strength and Gamma Stress

Suppose, X_i for all i, are i.i.d. exponential variates with mean $(\frac{1}{\lambda})$ and Y_i for all i, are i.i.d. gamma variates with scale parameter unity and degrees of freedom l, respectively. Then,

$$R(1) = \frac{1}{(1+\lambda)^l} \tag{2.3.13}$$

$$R(2,K) = \frac{1}{(1+\lambda K)^l} - \frac{1}{(1+\lambda+\lambda K)^l} \tag{2.3.14}$$

So, from (2.2.5) and (2.3.14), after some simple calculations we get,

$$E[R(2,k)] = \frac{1}{\lambda(-l+1)}\left[2(1+\lambda)^{-l+1} - 1 - (1+2\lambda)^{-l+1}\right] \tag{2.3.15}$$

Then the unconditional reliability R_2 can be easily obtained by the expression (2.3.8).

For some particular values of l and λ we have tabulated the values of R_1 and R_2 in **Table** 2.3 (cf. Appendix).

2.4 Graphical Representations

Some graphs are plotted in **Fig.** 2.1 to 2.3 by taking different parameters along the horizontal axis and the corresponding reliability along the vertical axis for different parametric values. **Fig.** 2.1 signifies that R_2 decreases steadily with increasing ρ. These graphs may be used to read the intermediate values directly. For example, for ρ=0.25, we get from the graph, R_2=0.9643 whereas by actual calculation we get R_2=0.9633. The difference is only 0.10%. **Fig.** 2.2 represents the curves of R_1 which were drawn against c for different parameter values of θ, a and λ. From these graphs we get R_1=0.8542. For

c=1.5, θ=10, a=6, λ=2 while the computed value is R_1=0.8536. The difference is only 0.06%. Taking l along the horizontal axis and the corresponding R_2 along the vertical axis graphs are plotted for different values of λ in **Fig.** 2.3. One can read the values of R_2 for intermediate values of l, from these graphs. Thus, for λ=0.2 we get R_2=0.7731 for l=5 from graphical extrapolation, while the computed value is R_2=0.7716. The difference is only 0.15%.

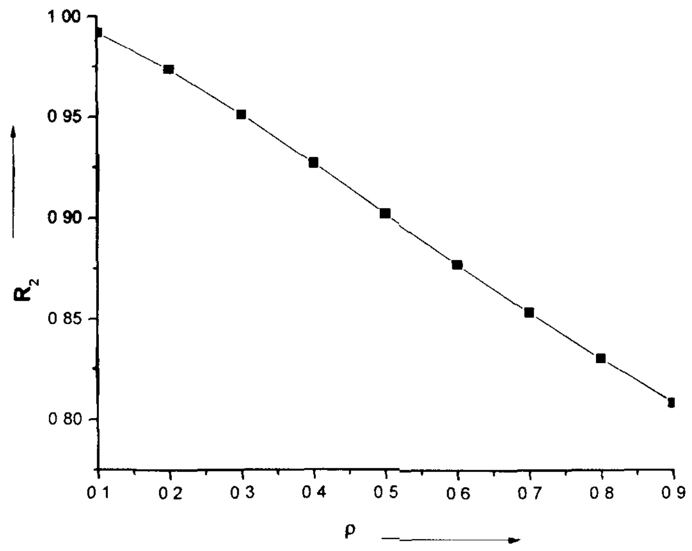

Fig. 2.1 Exponential Stress-Strength: Graph for R_2

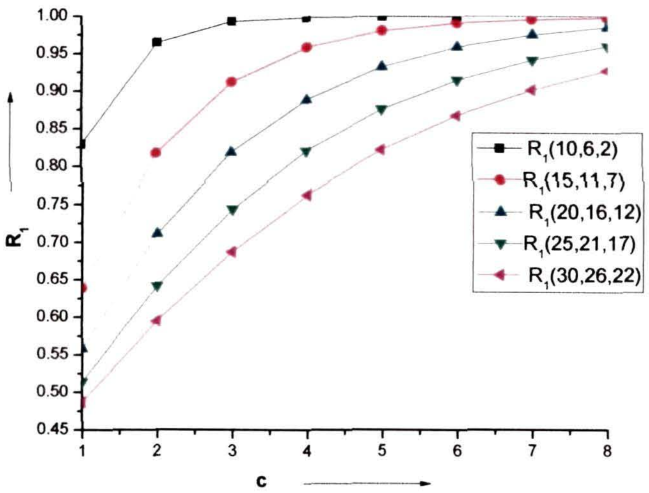

Fig. 2.2 Weibull Stress-Strength: Graph for R_1 for different fixed values of θ, a and λ i.e. $R_1(\theta, a, \lambda)$.

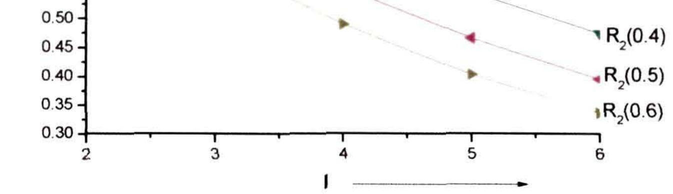

Fig. 2.3 Exponential Strength and Gamma Stress: Graph for R_2 for different fixed values of λ i.e $R_2(\lambda)$.

2.5 Results and Discussions

For some specific values of the parameters involved in the expressions of R_i, $i=1,2,3$ we evaluate R_1, R_2, R_3 for exponential distribution for different values of ρ and R_1, R_2 for Weibull and exponential-gamma distributions from their expressions obtained in Sub-Section 2.3.1-2.3.3. The computation is carried out using the software Matlab 6.

Table 2.1 (cf. Appendix) presents a few values of R_1, R_2, R_3 for different values of the parameter ρ for exponential distribution. From the table it is clear that reliabilities decreases with increasing values of ρ.

A few values of R_1, R_2 are tabulated for Weibull distribution, in **Table** 2.2 (cf. Appendix) for different values of the parameter c, θ, a, λ. Here the change in the values of reliability is as expected. The increase in the values of shape parameter increases the reliability. But increase in the values of scale parameter decreases the reliability.

Table 2.3 (cf. Appendix) presents some values of R_1 and R_2 for exponential-gamma distributions. Here the values of R_1 and R_2 decreases with increasing values of l and λ which is expected as well.

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Chapter 3

Mixture of Distributions in Cascade System

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Mixture of Distributions in Cascade System

3.1 Introduction

Standby redundancy is one of the means to achieve highly reliable system with less dependable units. In an n-standby system, initially there are n components, one of which is working and the remaining (n-1) are standbys. Whenever the working component fails one from standbys takes its place and the system works. This goes on. The system fails when all the components have failed. Let $X_1, X_2, \ldots, X_n$ be the strengths of n-components in the order of activation and let $Y_1, Y_2, \ldots, Y_n$ are the stresses working on them. In cascade system after every failure the stress is modified by a factor k which is given by the equation (2.2.1).

Often, it is assumed that stress and strength can be represented by a single distribution. But in general, stress and strength may be the effects of many random causes, each having its own distribution. Hence, the distributions of stress-strength may be better represented by mixtures of distributions rather than single distributions.

If there are 'l' different stresses working on the system simultaneously, and the system's failure may be due to either of the 'l' stresses, then the stress distribution $G(y)$ can be represented by a mixture of 'l' distributions $G_j(y)$, j=1,2,..., l as

$$G(y) = \sum_{j=1}^{l} q_j G_j(y), \quad 0 \le q_j \le 1 \quad \sum_{j=1}^{l} q_j = 1$$

where $G_j(y)$ is the c.d.f. of the j[th] stress and q_j, $j=1,2,\ldots,\ l$ is the corresponding mix parameter (Mann et al., 1974).

Similar reasoning can be given for the distribution of strength also. Various factors such as raw materials, shape, manufacturing process etc., which determines the strength (X) of a system, may be random in nature and as such can be represented by random variables following different distributions. The resultant of these distributions may not be adequately represented by a single distribution. On the other hand it may also happen that though the system is taken as one unit, actually it consists of several components having their own strength distributions which cannot be converted into a single distribution. Then like a stress distribution, it may be possible to represent a strength distribution, $F(x)$, by mixture of distributions, as

$$F(x)=\sum_{i=1}^{m}p_iF_i(x),\quad 0\le p_i\le 1\quad \sum_{i=1}^{m}p_i=1$$

where $F_i(x)$ is the c.d.f. of the i[th] factor of the strength and p_i, $i=1,2,\ldots,m$ is the corresponding mix parameter.

Ahmad and Ali (2009) have derived a method of combining two Weibull distributions. It showed how to produce a mixture distribution by including a mixing parameter, which represents the proportions of mixing of the two component Weibull distributions. Nassar (1988) has obtained two properties characterizing mixtures of exponential distributions. Abraham and Nair (1997) have obtained two characterizations of the mixtures of exponential, Lomax and beta densities through relationship between (i) failure rate MRL and (ii) second moment of residual life and failure data.

Some authors (Cohen, 1965, Harris and Singpurwalla, 1968, Kao, 1959 and Mann et al., 1974) have considered time-to-failure distributions as mixture of distributions. Mann et al. (1974) pointed out; in support of using a mixture of distribution that a single time distribution represents either a random (time independent) or a wear out (time dependent) failure but in practice a unit can suffer either of these failures.

Some of the results of this chapter are based on Doloi and Borah (2012). Here we have considered a 2-cascade and 3-cascade systems only. It is assumed that the strength distribution of each component can be represented by a mixture of two distributions. Similarly, the distribution of stress on each component can be represented by a mixture of two distributions. The reliability of the system, 'R_n', for $n < 4$ is obtained in terms of two distributions and density functions, in the mixtures.

The organization of this chapter is as follows: In Section-3.2 mixture of distributions is applied to n-cascade system. In Section-3.3 different specific distributions are considered for stress-strength. Generally it is assumed that the distributions that are mixed belong to same family. In Sub-Sections 3.3.1 to 3.3.3, the reliabilities of a cascade system are obtained by taking it into consideration that each stress and strength is a mixture either of two exponential or of two Rayleigh or of two Weibull distributions. Reliabilities of a cascade system for particular cases of exponential, Rayleigh and Weibull distributions have been obtained in Sub-Sections 3.3.1(a), 3.3.2(a) and 3.3.3(a). Some graphs are plotted for each case in Section-3.4. Some numerical values of reliabilities for particular cases given above are tabulated against the parameters involved in the **Table** 3.1, **Table** 3.2, **Table** 3.3 and **Table** 3.4 (cf. Appendix) and results and discussions are given in Section-3.5.

3.2 Mixture of Distribution: An n-Cascade System

Here we have considered an n-cascade system in which the strength $X_1, X_2, \ldots, X_n$ are i.i.d. with p.d.f. $f(x)$ which is a mixture of two densities $f'(x)$ and $f''(x)$ with mix parameters p' and p'', i.e.

$$f(x) = p'f'(x) + p''f''(x), \quad 0 < p', p'' < 1, \quad p' + p'' = 1 \tag{3.2.1}$$

By the definition of a cascade system the stress on the ith component, $Y_i = K^{i-1}Y_1$, where Y_1 is the stress on the first component. Here we assume that the density of Y_1 is a mixture of two densities $g'(y_1)$ and $g''(y_1)$ with mix parameters q' and q'', i.e.

$$g(y_1) = q'g'(y_1) + q''g''(y_1) \quad 0 < q', q'' < 1, \quad q' + q'' = 1 \tag{3.2.2}$$

Now the marginal reliability $R(1), R(2), R(3), \ldots, R(n)$ may be obtained as

$$R(1) = \int_{-\infty}^{\infty} \overline{F}(y_1) g(y_1) dy_1 \tag{3.2.3}$$

$$R(2) = \int_{-\infty}^{\infty} F(y_1) \overline{F}(ky_1) g(y_1) dy_1 \tag{3.2.4}$$

$$R(3) = \int_{-\infty}^{\infty} F(y_1) F(ky_1) \overline{F}(k^2 y_1) g(y_1) dy_1 \tag{3.2.5}$$

$$R(4) = \int_{-\infty}^{\infty} F(y_1) F(ky_1) F(k^2 y_1) \overline{F}(k^3 y_1) g(y_1) dy_1 \tag{3.2.6}$$

Similarly

$$R(n) = \int_{-\infty}^{\infty} F(y_1) F(ky_1) F(k^2 y_1) \ldots \overline{F}(k^{n-1} y_1)\ g(y_1) dy_1 \tag{3.2.7}$$

The reliability of an n-cascade system is given as

$$R_n = R(1) + R(2) + \ldots + R(n), \tag{3.2.8}$$

where r^{th} component marginal reliability may be given as

$$R(r) = P[X_1 < Y_1, X_2 < k\,Y_1, \ldots, X_{r-1} < k^{r-2} Y_1, X_r \geq k^{r-1} Y_1] \tag{3.2.9}$$

3.3 Stress- Strength follows Mixture of Distributions

In this section we have considered stress-strength both are mixtures of either two exponential or two Rayleigh or two Weibull distributions. In the following Sub-Sections we have obtained the reliability expression for 2-cascade and 3-cascade systems and some particular cases in each case, for $n > 3$, the expressions become too complex.

3.3.1 Mixture of two Exponentials: Cascade System

Let the strength of the n components be i.i.d. with p.d.f. $f(x)$ which is a mixture of two exponential distributions with parameters λ and θ, respectively, i.e.

$$f(x) = p'\lambda e^{-\lambda x} + p''\theta e^{-\theta x}, \quad x \ge 0, \ \lambda, \theta > 0 \tag{3.3.1}$$

where p' and p'' are mix parameters.

Let the p.d.f. of Y_1 be also mixture of two exponential densities with parameters μ and β, i.e.

$$g(y_1) = q'\mu e^{-\mu y_1} + q''\beta e^{-\beta y_1}, \quad y_1 \ge 0, \ \mu, \beta > 0 \tag{3.3.2}$$

where q' and q'' are mix parameters.

Then marginal reliability $R(1), R(2), R(3)$ of mixture of two exponential distributions may be obtained from (3.2.3), (3.2.4) and (3.2.5) respectively.

$$R(1) = q'\mu\left[\frac{p'}{\lambda+\mu} + \frac{p''}{\theta+\mu}\right] + q''\beta\left[\frac{p'}{\lambda+\beta} + \frac{p''}{\theta+\beta}\right]$$

$$R(2) = q'\mu\left[\frac{p'}{\lambda k+\mu} + \frac{p''}{\theta k+\mu} - \frac{p'^2}{\lambda k+\lambda+\mu} - \frac{p''^2}{\theta k+\mu+\theta}\right] + q''\beta\left[\begin{array}{l}\dfrac{p'}{\lambda k+\beta} + \dfrac{p''}{\theta k+\beta} - \dfrac{p'^2}{\lambda k+\lambda+\beta} - \\ \dfrac{p''^2}{\theta k+\theta+\beta}\end{array}\right]$$

$$- p'p''q'\mu\left[\frac{1}{\lambda k+\theta+\mu} + \frac{1}{\theta k+\lambda+\mu}\right] - p'p''q''\beta\left[\frac{1}{\lambda k+\theta+\beta} + \frac{1}{\theta k+\lambda+\beta}\right]$$

$$R(3) = q'\mu\left[\begin{array}{l}\dfrac{p'}{\lambda k^2+\mu} + \dfrac{p''}{\theta k^2+\mu} - \dfrac{p'^2}{\lambda k^2+\lambda+\mu} - \dfrac{p''^2}{\theta k^2+\theta+\mu} - \dfrac{p'^2}{\lambda k^2+\lambda k+\mu} - \dfrac{p''^2}{\theta k^2+\theta k+\mu} + \\ \dfrac{p'^3}{\lambda k^2+\lambda k+\mu+\lambda} + \dfrac{p''^3}{\theta k^2+\theta k+\theta+\mu}\end{array}\right]$$

$$+ q''\beta\left[\begin{array}{l}\dfrac{p'}{\lambda k^2+\beta} + \dfrac{p''}{\theta k^2+\beta} - \dfrac{p'^2}{\lambda k^2+\lambda+\beta} - \dfrac{p''^2}{\theta k^2+\theta+\beta} - \dfrac{p'^2}{\lambda k^2+\lambda k+\beta} - \dfrac{p''^2}{\theta k^2+\theta k+\beta} + \\ \dfrac{p'^3}{\lambda k^2+\lambda k+\lambda+\beta} + \dfrac{p''^3}{\theta k^2+\theta+\theta k+\beta}\end{array}\right]$$

$$-p'p''q'\mu\left[\begin{array}{l}\frac{1}{\lambda k^2+\theta+\mu}+\frac{1}{\theta k^2+\lambda+\mu}+\frac{1}{\lambda k^2+\theta k+\mu}+\frac{1}{\theta k^2+\lambda k+\mu}-\frac{p'}{\lambda k^2+\lambda k+\theta+\mu}-\\ \frac{p''}{\theta k^2+\theta k+\lambda+\mu}-\frac{p'}{\lambda k^2+\theta k+\lambda+\mu}-\frac{p''}{\theta k^2+\lambda k+\theta+\mu}-\frac{p'}{\theta k^2+\lambda k+\lambda+\mu}\\ -\frac{p''}{\lambda k^2+\theta k+\theta+\mu}\end{array}\right]$$

$$-p'p''q''\beta\left[\begin{array}{l}\frac{1}{\lambda k^2+\theta+\beta}+\frac{1}{\theta k^2+\lambda+\beta}+\frac{1}{\lambda k^2+\theta k+\beta}+\frac{1}{\theta k^2+\lambda k+\beta}-\frac{p'}{\lambda k^2+\lambda k+\theta+\beta}-\\ \frac{p''}{\theta k^2+\theta k+\lambda+\beta}-\frac{p'}{\lambda k^2+\theta k+\lambda+\beta}-\frac{p''}{\theta k^2+\lambda k+\theta+\beta}-\frac{p'}{\theta k^2+\lambda k+\lambda+\beta}-\\ \frac{p''}{\lambda k^2+\theta k+\theta+\beta}\end{array}\right]$$

Then from the relation (3.2.8) the reliabilities R_2 and R_3 for the mixture of two exponentials of a 2-Cascade and 3-cascade system, may be obtained as

$$R_2 = R(1) + R(2) \tag{3.3.3}$$

$$R_3 = R(1) + R(2) + R(3) \tag{3.3.4}$$

(a) Particular case of mixture of two Exponentials: Cascade System

When $\lambda = \mu$ and $\theta = \beta$ then

$$R(1) = \frac{1}{2}\left[p'q' + p''q''\right] + \frac{1}{\beta+\mu}\left[p''q'\mu + p'q''\beta\right]$$

$$R(2) = \frac{1}{k+1}\left[p'q' + p''q''\right] - \frac{1}{k+2}\left[p'^2q' + p''^2q''\right] - \frac{p'}{\mu k+\mu+\beta}\left[p''q'\mu + p'q''\beta\right] - \frac{p''}{\beta k+\beta+\mu}\left[p''q'\mu + p'q''\beta\right]$$
$$+ p''q'\mu\left[\frac{1}{\beta k+\mu} - \frac{p'}{\beta k+2\mu}\right] + p'q''\beta\left[\frac{1}{\mu k+\beta} - \frac{p''}{\mu k+2\beta}\right]$$

$$R(3)=\frac{1}{k^2+1}\left[p'q'+p''q''\right]-\frac{1}{k^2+2}\left[p'^2q'+p''^2q''\right]-\frac{1}{k^2+k+1}\left[p'^2q'+p''^2q''\right]+\frac{1}{k^2+k+2}\left[p'^3q'+p''^3q''\right]$$
$$-\frac{p'}{\mu(k^2+1)+\beta}\left[p''q'\mu+p'q''\beta\right]-\frac{p''}{\beta(k^2+1)+\mu}\left[p''q'\mu+p'q''\beta\right]+\frac{p'^2}{\mu(k^2+k+1)+\beta}\left[p''q'\mu+p'q''\beta\right]+$$
$$\frac{p''^2}{\beta(k^2+k+1)+\mu}\left[p''q'\mu+p'q''\beta\right]+\frac{p'p''}{\mu(k^2+1)+\beta(k+1)}\left[p''q'\mu+p'q''\beta\right]+\frac{p'p''}{\beta(k^2+1)+\mu(k+1)}\left[p''q'\mu+p'q''\beta\right]+$$
$$p''q'\mu\left[\frac{1}{\beta k^2+\mu}-\frac{p''}{\beta k^2+\beta k+\mu}\right]+p'q''\beta\left[\frac{1}{\mu k^2+\beta}-\frac{p'}{\mu k^2+\mu k+\beta}\right]-p'p''q'\mu$$
$$\left[\frac{1}{\beta k^2+\mu k+\mu}-\frac{p'}{\beta k^2+\mu k+2\mu}+\frac{1}{\beta k^2+2\mu}-\frac{p''}{\beta k^2+\beta k+2\mu}+\frac{1}{\mu k^2+\beta k+\mu}-\frac{p'}{\mu k^2+\beta k+2\mu}\right]-p'p''q''\beta$$
$$\left[\frac{1}{\mu k^2+\beta k+\beta}-\frac{p''}{\mu k^2+\beta k+2\beta}+\frac{1}{\mu k^2+2\beta}-\frac{p'}{\mu k^2+\mu k+2\beta}+\frac{1}{\beta k^2+\mu k+\beta}-\frac{p''}{\beta k^2+\mu k+2\beta}\right]$$

Then the reliabilities R_2 and R_3 for the mixture of two exponentials of 2-cascade and 3-Cascade system for the particular case $\lambda=\mu$ and $\theta=\beta$ may be obtained from (3.3.3) and (3.3.4) respectively.

A few values of R_1, R_2 and R_3 are tabulated in **Table** 3.1 (cf. Appendix) for different values of $k, \mu, \lambda, \theta, \beta, p'$ and q'.

3.3.2 Mixture of two Rayleighs: Cascade System

Let the strength of the n components be i.i.d. with p.d.f. $f(x)$ which is a mixture of two Rayleigh distributions with parameters σ_1 and σ_2, respectively, i.e.

$$f(x)=\frac{p'x}{\sigma_1^2}e^{-x^2/2\sigma_1^2}+\frac{p''x}{\sigma_2^2}e^{-x^2/2\sigma_2^2},\quad x\geq 0,\ \sigma_1,\sigma_2>0 \tag{3.3.5}$$

Similarly, let the p.d.f. of Y_1 be also mixture of two Rayleigh densities with parameters σ_3 and σ_4, i.e.

$$g(y_1) = \frac{q' y_1}{\sigma_3^2} e^{-y_1^2/2\sigma_3^2} + \frac{q'' y_1}{\sigma_4^2} e^{-y_1^2/2\sigma_4^2}, \; y_1 \geq 0, \; \sigma_3, \sigma_4 > 0 \tag{3.3.6}$$

Then marginal reliability $R(1)$, $R(2)$, $R(3)$ for mixture of two Rayleigh distributions may be obtained from (3.2.3), (3.2.4) and (3.2.5) respectively.

$$R(1) = \frac{q'}{\sigma_3^2}\left[\frac{p'}{(1/\sigma_1^2 + 1/\sigma_3^2)} + \frac{p''}{(1/\sigma_2^2 + 1/\sigma_3^2)}\right] + \frac{q''}{\sigma_4^2}\left[\frac{p'}{(1/\sigma_1^2 + 1/\sigma_4^2)} + \frac{p''}{(1/\sigma_2^2 + 1/\sigma_4^2)}\right]$$

$$R(2) = \frac{q'}{\sigma_3^2}\left[\frac{p'}{(k^2/\sigma_1^2 + 1/\sigma_3^2)} + \frac{p''}{(k^2/\sigma_2^2 + 1/\sigma_3^2)} - \frac{p'^2}{(1/\sigma_1^2 + k^2/\sigma_1^2 + 1/\sigma_3^2)} - \frac{p''^2}{(1/\sigma_2^2 + k^2/\sigma_2^2 + 1/\sigma_3^2)}\right] + \frac{q''}{\sigma_4^2}$$
$$\left[\frac{p'}{(k^2/\sigma_1^2 + 1/\sigma_4^2)} + \frac{p''}{(k^2/\sigma_2^2 + 1/\sigma_4^2)} - \frac{p'^2}{(1/\sigma_1^2 + k^2/\sigma_1^2 + 1/\sigma_4^2)} - \frac{p''^2}{(1/\sigma_2^2 + k^2/\sigma_2^2 + 1/\sigma_4^2)}\right]$$
$$- \frac{p'p''q'}{\sigma_3^2}\left[\frac{1}{(1/\sigma_2^2 + k^2/\sigma_1^2 + 1/\sigma_3^2)} + \frac{1}{(1/\sigma_1^2 + k^2/\sigma_2^2 + 1/\sigma_3^2)}\right] - \frac{p'p''q''}{\sigma_4^2}$$
$$\left[\frac{1}{(1/\sigma_1^2 + k^2/\sigma_2^2 + 1/\sigma_4^2)} + \frac{1}{(1/\sigma_2^2 + k^2/\sigma_1^2 + 1/\sigma_4^2)}\right]$$

$$R(3) = \frac{q'}{\sigma_3^2}\left[\begin{array}{l} \frac{p'}{(k^4/\sigma_1^2 + 1/\sigma_3^2)} + \frac{p''}{(k^4/\sigma_2^2 + 1/\sigma_3^2)} - \frac{p'^2}{(k^4/\sigma_1^2 + 1/\sigma_1^2 + 1/\sigma_3^2)} - \\ \frac{p''^2}{(k^4/\sigma_2^2 + 1/\sigma_2^2 + 1/\sigma_3^2)} - \frac{p'^2}{(k^4/\sigma_1^2 + k^2/\sigma_1^2 + 1/\sigma_3^2)} - \frac{p''^2}{(k^4/\sigma_2^2 + k^2/\sigma_2^2 + 1/\sigma_3^2)} + \\ \frac{p'^3}{(k^4/\sigma_1^2 + k^2/\sigma_1^2 + 1/\sigma_1^2 + 1/\sigma_3^2)} + \frac{p''^3}{(k^4/\sigma_2^2 + k^2/\sigma_2^2 + 1/\sigma_2^2 + 1/\sigma_3^2)} \end{array}\right]$$

$$+\frac{q''}{\sigma_4^2}\left[\begin{array}{l}\frac{p'}{(k^4/\sigma_1^2+1/\sigma_4^2)}+\frac{p''}{(k^4/\sigma_2^2+1/\sigma_4^2)}-\frac{p'^2}{(k^4/\sigma_1^2+1/\sigma_1^2+1/\sigma_4^2)}-\\ \frac{p''^2}{(k^4/\sigma_2^2+1/\sigma_2^2+1/\sigma_4^2)}-\frac{p'^2}{(k^4/\sigma_1^2+k^2/\sigma_1^2+1/\sigma_4^2)}-\frac{p''^2}{(k^4/\sigma_2^2+k^2/\sigma_2^2+1/\sigma_4^2)}+\\ \frac{p'^3}{(k^4/\sigma_1^2+k^2/\sigma_1^2+1/\sigma_1^2+1/\sigma_4^2)}+\frac{p''^3}{(k^4/\sigma_2^2+k^2/\sigma_2^2+1/\sigma_2^2+1/\sigma_4^2)}\end{array}\right]$$

$$-\frac{p'p''q'}{\sigma_3^2}\left[\begin{array}{l}\frac{1}{(k^4/\sigma_1^2+1/\sigma_2^2+1/\sigma_3^2)}+\frac{1}{(k^4/\sigma_2^2+1/\sigma_1^2+1/\sigma_3^2)}+\frac{1}{(k^4/\sigma_1^2+k^2/\sigma_2^2+1/\sigma_3^2)}+\\ \frac{1}{(k^4/\sigma_2^2+k^2/\sigma_1^2+1/\sigma_3^2)}-\frac{p'}{(k^4/\sigma_1^2+k^2/\sigma_2^2+1/\sigma_1^2+1/\sigma_3^2)}-\\ \frac{p''}{(k^4/\sigma_2^2+k^2/\sigma_1^2+1/\sigma_2^2+1/\sigma_3^2)}-\frac{p'}{(k^4/\sigma_2^2+k^2/\sigma_1^2+1/\sigma_1^2+1/\sigma_3^2)}-\\ \frac{p''}{(k^4/\sigma_1^2+k^2/\sigma_2^2+1/\sigma_2^2+1/\sigma_3^2)}-\frac{p'}{(k^4/\sigma_1^2+k^2/\sigma_1^2+1/\sigma_2^2+1/\sigma_3^2)}-\\ \frac{p''}{(k^4/\sigma_2^2+k^2/\sigma_2^2+1/\sigma_1^2+1/\sigma_3^2)}\end{array}\right]$$

$$-\frac{p'p''q''}{\sigma_4^2}\left[\begin{array}{l}\frac{1}{(k^4/\sigma_1^2+1/\sigma_2^2+1/\sigma_4^2)}+\frac{1}{(k^4/\sigma_2^2+1/\sigma_1^2+1/\sigma_4^2)}+\frac{1}{(k^4/\sigma_2^2+k^2/\sigma_1^2+1/\sigma_4^2)}\\ +\frac{1}{(k^4/\sigma_1^2+k^2/\sigma_2^2+1/\sigma_4^2)}-\frac{p'}{(k^4/\sigma_1^2+k^2/\sigma_2^2+1/\sigma_1^2+1/\sigma_4^2)}-\\ \frac{p''}{(k^4/\sigma_2^2+k^2/\sigma_1^2+1/\sigma_2^2+1/\sigma_4^2)}-\frac{p'}{(k^4/\sigma_1^2+k^2/\sigma_1^2+1/\sigma_2^2+1/\sigma_4^2)}-\\ \frac{p''}{(k^4/\sigma_2^2+k^2/\sigma_2^2+1/\sigma_1^2+1/\sigma_4^2)}-\frac{p'}{(k^4/\sigma_2^2+k^2/\sigma_1^2+1/\sigma_1^2+1/\sigma_4^2)}-\\ \frac{p''}{(k^4/\sigma_1^2+k^2/\sigma_2^2+1/\sigma_2^2+1/\sigma_4^2)}\end{array}\right]$$

Then the reliabilities R_2 and R_3 for the mixture of two Rayleighs of 2-cascade and 3-Cascade system may be obtained from (3.3.3) and (3.3.4) respectively.

(a) Particular case of mixture of two Rayleighs: Cascade System

When $\sigma_1 = \sigma_3$ and $\sigma_2 = \sigma_4$ then

$$R(1)=\frac{1}{2}\left[p'q'+p''q''\right]+\frac{1}{\sigma_3^{\,2}+\sigma_4^{\,2}}\left[p'q''\sigma_3^{\,2}+p''q'\sigma_4^{\,2}\right]$$

$$R(2)=\frac{1}{k^2+1}\left[p'q'+p''q''\right]-\frac{1}{k^2+2}\left[p'^2q'+p''^2q''\right]-\frac{p'}{\sigma_4^{\,2}(k^2+1)+\sigma_3^{\,2}}\left[p''q'\sigma_4^{\,2}+p'q''\sigma_3^{\,2}\right]-$$
$$\frac{p''}{\sigma_3^{\,2}(k^2+1)+\sigma_4^{\,2}}\left[p''q'\sigma_4^{\,2}+p'q''\sigma_3^{\,2}\right]+p'q''\sigma_3^{\,2}\left[\frac{1}{\sigma_4^{\,2}k^2+\sigma_3^{\,2}}-\frac{p''}{\sigma_4^{\,2}k^2+2\sigma_3^{\,2}}\right]+p''q'\sigma_4^{\,2}$$
$$\left[\frac{1}{\sigma_3^{\,2}k^2+\sigma_4^{\,2}}-\frac{p'}{\sigma_3^{\,2}k^2+2\sigma_4^{\,2}}\right]$$

$$R(3)=\frac{1}{k^4+1}\left[p'q'+p''q''\right]-\frac{1}{k^4+2}\left[p'^2q'+p''^2q''\right]-\frac{1}{k^4+k^2+1}\left[p'^2q'+p''^2q''\right]+\frac{1}{k^4+k^2+2}$$
$$\left[p'^3q'+p''^3q''\right]-\frac{p'}{\sigma_4^{\,2}(k^4+1)+\sigma_3^{\,2}}\left[p''q'\sigma_4^{\,2}+p'q''\sigma_3^{\,2}\right]-\frac{p''}{\sigma_3^{\,2}(k^4+1)+\sigma_4^{\,2}}\left[p''q'\sigma_4^{\,2}+p'q''\sigma_3^{\,2}\right]$$
$$+\frac{p'^2}{\sigma_4^{\,2}(k^4+k^2+1)+\sigma_3^{\,2}}\left[p''q'\sigma_4^{\,2}+p'q''\sigma_3^{\,2}\right]+\frac{p''^2}{\sigma_3^{\,2}(k^4+k^2+1)+\sigma_4^{\,2}}\left[p''q'\sigma_4^{\,2}+p'q''\sigma_3^{\,2}\right]+$$
$$\frac{p'p''}{\sigma_4^{\,2}(k^4+1)+\sigma_3^{\,2}(k^2+1)}\left[p''q'\sigma_4^{\,2}+p'q''\sigma_3^{\,2}\right]+\frac{p'p''}{\sigma_3^{\,2}(k^4+1)+\sigma_4^{\,2}(k^2+1)}\left[p''q'\sigma_4^{\,2}+p'q''\sigma_3^{\,2}\right]+$$
$$p''q'\sigma_4^{\,2}\left[\frac{1}{\sigma_3^{\,2}k^4+\sigma_4^{\,2}}-\frac{p''}{\sigma_3^{\,2}k^4+\sigma_3^{\,2}k^2+\sigma_4^{\,2}}\right]+p'q''\sigma_3^{\,2}\left[\frac{1}{\sigma_4^{\,2}k^4+\sigma_3^{\,2}}-\frac{p'}{\sigma_4^{\,2}k^4+\sigma_4^{\,2}k^2+\sigma_3^{\,2}}\right]-$$
$$p'p''q'\sigma_4^{\,2}\left[\begin{array}{l}\frac{1}{\sigma_3^{\,2}k^4+\sigma_4^{\,2}k^2+\sigma_4^{\,2}}-\frac{1}{\sigma_3^{\,2}k^4+\sigma_4^{\,2}k^2+2\sigma_4^{\,2}}+\frac{1}{\sigma_3^{\,2}k^4+2\sigma_4^{\,2}}-\\ \frac{p'}{\sigma_3^{\,2}k^4+\sigma_3^{\,2}k^2+2\sigma_4^{\,2}}+\frac{1}{\sigma_4^{\,2}k^4+\sigma_3^{\,2}k^2+\sigma_4^{\,2}}-\frac{p'}{\sigma_4^{\,2}k^4+\sigma_3^{\,2}k^2+2\sigma_4^{\,2}}\end{array}\right]-$$
$$p'p''q'\sigma_3^{\,2}\left[\begin{array}{l}\frac{1}{\sigma_4^{\,2}k^4+\sigma_3^{\,2}k^2+\sigma_3^{\,2}}-\frac{1}{\sigma_4^{\,2}k^4+\sigma_3^{\,2}k^2+2\sigma_3^{\,2}}+\frac{1}{\sigma_4^{\,2}k^4+2\sigma_3^{\,2}}-\\ \frac{p'}{\sigma_4^{\,2}k^4+\sigma_4^{\,2}k^2+2\sigma_3^{\,2}}+\frac{1}{\sigma_3^{\,2}k^4+\sigma_4^{\,2}k^2+\sigma_3^{\,2}}-\frac{p''}{\sigma_3^{\,2}k^4+\sigma_4^{\,2}k^2+2\sigma_3^{\,2}}\end{array}\right]$$

So, as usual the reliabilities R_2 and R_3 for the mixture of two Rayleigh distributions of 2-cascade and 3-Cascade system for the particular case $\sigma_1=\sigma_3$ and $\sigma_2=\sigma_4$ may be obtained from (3.3.3) and (3.3.4) respectively.

For some particular values of $k, \sigma_1, \sigma_2, \sigma_3, \sigma_4, p'$ and q' we have tabulated the values of R_1, R_2 and R_3 in **Table** 3.2 (cf. Appendix)

3.3.3 Mixture of two Weibulls: Cascade System

Let the strength of the n components be i.i.d. with density $f(x)$. Now $f(x)$ is assumed to be a mixture of two Weibull p.d.fs, as

$$f(x) = \frac{p'A}{\lambda^A} x^{A-1} e^{-(x/\lambda)^A} + \frac{p''B}{\theta^B} x^{B-1} e^{-(x/\theta)^B}, \quad x \geq 0, \ A, B, \lambda, \theta > 0 \tag{3.3.7}$$

Similarly, let the p.d.f. of Y_1 be also a mixture of two Weibull densities, given by

$$g(y_1) = \frac{q'C}{\mu^C} y_1^{C-1} e^{-(y_1/\mu)^C} + \frac{q''D}{\beta^D} y_1^{D-1} e^{-(y_1/\beta)^D}, \quad y_1 \geq 0, \ C, D, \mu, \beta > 0 \tag{3.3.8}$$

Then marginal reliability $R(1)$, $R(2)$, $R(3)$ of mixture of two Weibull distributions may be obtained from (3.2.3), (3.2.4) and (3.2.5) respectively.

$$R(1) = \int_0^\infty \left[p' e^{-(y_1/\lambda)^A} + p'' e^{-(y_1/\theta)^B}\right] \left[\frac{q'C}{\mu^C} y_1^{C-1} e^{-(y_1/\mu)^C} + \frac{q''D}{\beta^D} y_1^{D-1} e^{-(y_1/\beta)^D}\right] dy_1$$

$$R(2) = \int_0^\infty \left[1 - p' e^{-(y_1/\lambda)^A} - p'' e^{-(y_1/\theta)^B}\right] \left[p' e^{-(k y_1/\lambda)^A} + p'' e^{-(k y_1/\theta)^B}\right]$$
$$\left[\frac{q'C}{\mu^C} y_1^{C-1} e^{-(y_1/\mu)^C} + \frac{q''D}{\beta^D} y_1^{D-1} e^{-(y_1/\beta)^D}\right] dy_1$$

$$R(3) = \int_0^\infty \left[1 - p' e^{-(y_1/\lambda)^A} - p'' e^{-(y_1/\theta)^B}\right] \left[1 - p' e^{-(k y_1/\lambda)^A} - p'' e^{-(k y_1/\theta)^B}\right]$$
$$\left[p' e^{-(k^2 y_1/\lambda)^A} + p'' e^{-(k^2 y_1/\theta)^B}\right] \left[\frac{q'C}{\mu^C} y_1^{C-1} e^{-(y_1/\mu)^C} + \frac{q''D}{\beta^D} y_1^{D-1} e^{-(y_1/\beta)^D}\right] dy_1 .$$

Hence the expressions for $R(1)$, $R(2)$ and $R(3)$ can be evaluated numerically. The reliabilities R_2 and R_3 for the mixture of two Weibulls of 2-cascade and 3-Cascade system may be obtained from (3.3.3) and (3.3.4) respectively.

(a) Particular cases of mixture of two Weibulls: Cascade System

(1) When shape parameters are equal i.e. $A = C$ and $B = D$ then

$$R(1) = \frac{p'q'\lambda^C}{\mu^C + \lambda^C} + \frac{p''q''\theta^D}{\beta^D + \theta^D} + \frac{p''q'C}{\mu^C}\int_0^\infty y_1^{C-1} e^{-\left[\left(\frac{y_1}{\theta}\right)^D + \left(\frac{y_1}{\mu}\right)^C\right]} dy_1 + \frac{p'q''D}{\beta^D}\int_0^\infty y_1^{D-1} e^{-\left[\left(\frac{y_1}{\lambda}\right)^C + \left(\frac{y_1}{\beta}\right)^D\right]} dy_1$$

$$= \frac{p'q'\lambda^C}{\mu^C + \lambda^C} + \frac{p''q''\theta^D}{\beta^D + \theta^D} + \alpha_1 + \alpha_2$$

where α_1 and α_2 can be evaluated numerically.

$$R(2) = \frac{p'q'\lambda^C}{(k\mu)^C + \lambda^C} + \frac{p''q''\theta^D}{(k\beta)^D + \theta^D} - \frac{p'^2q'\lambda^C}{(k\mu)^C + \lambda^C + \mu^C} - \frac{p''^2q''\theta^D}{(k\beta)^D + \theta^D + \beta^D} + \frac{p''q'C}{\mu^C}$$

$$\int_0^\infty y_1^{C-1} e^{-\left[\left(\frac{k y_1}{\theta}\right)^B + \left(\frac{y_1}{\mu}\right)^C\right]} dy_1 + \frac{p'q''D}{\beta^D}\int_0^\infty y_1^{D-1} e^{-\left[\left(\frac{k y_1}{\lambda}\right)^C + \left(\frac{y_1}{\beta}\right)^D\right]} dy_1 - \frac{p'p''q'C}{\mu^C}\int_0^\infty y_1^{C-1} e^{-\left[\left(\frac{k y_1}{\theta}\right)^D + \left(\frac{y_1}{\mu}\right)^C + \left(\frac{y_1}{\lambda}\right)^C\right]} dy_1 -$$

$$\frac{p'^2q''D}{\beta^D}\int_0^\infty y_1^{D-1} e^{-\left[\left(\frac{k y_1}{\lambda}\right)^C + \left(\frac{y_1}{\beta}\right)^D + \left(\frac{y_1}{\lambda}\right)^C\right]} dy_1 - \frac{p'p''q''D}{\beta^D}\int_0^\infty y_1^{D-1} e^{-\left[\left(\frac{k y_1}{\theta}\right)^D + \left(\frac{y_1}{\beta}\right)^D + \left(\frac{y_1}{\lambda}\right)^C\right]} dy_1 -$$

$$\frac{p'p''q'C}{\mu^C}\int_0^\infty y_1^{C-1} e^{-\left[\left(\frac{k y_1}{\lambda}\right)^C + \left(\frac{y_1}{\mu}\right)^C + \left(\frac{y_1}{\theta}\right)^D\right]} dy_1 - \frac{p''^2q'C}{\mu^C}\int_0^\infty y_1^{C-1} e^{-\left[\left(\frac{k y_1}{\theta}\right)^D + \left(\frac{y_1}{\mu}\right)^C + \left(\frac{y_1}{\theta}\right)^D\right]} dy_1 -$$

$$\frac{p'p''q''D}{\beta^D}\int_0^\infty y_1^{D-1} e^{-\left[\left(\frac{k y_1}{\lambda}\right)^C + \left(\frac{y_1}{\beta}\right)^D + \left(\frac{y_1}{\theta}\right)^D\right]} dy_1$$

$$= \frac{p'q'\lambda^C}{(k\mu)^C + \lambda^C} + \frac{p''q''\theta^D}{(k\beta)^D + \theta^D} - \frac{p'^2q'\lambda^C}{(k\mu)^C + \lambda^C + \mu^C} - \frac{p''^2q''\theta^D}{(k\beta)^D + \theta^D + \beta^D} + \eta_1 + \eta_2 - \eta_3 - \eta_4 - \eta_5 - \eta_6 - \eta_7 - \eta_8$$

where $\eta_i, i = 1,2,\ldots,8$ can be evaluated numerically.

$$R(3)=\frac{p'q'\lambda^C}{(k^2\mu)^C+\lambda^C}-\frac{p'^2q'\lambda^C}{(k^2\mu)^C+\lambda^C+\mu^C}-\frac{p'^2q'\lambda^C}{(k^2\mu)^C+(k\mu)^C+\lambda^C}+\frac{p'^3q'\lambda^C}{(k^2\mu)^C+(k\mu)^C+\mu^C+\lambda^C}+$$

$$\frac{p''q''\theta^D}{(k^2\beta)^D+\theta^D}-\frac{p''^2q''\theta^D}{(k^2\beta)^D+\beta^D+\theta^D}-\frac{p''^2q''\theta^D}{(k^2\beta)^D+(k\beta)^D+\theta^D}+\frac{p''^3q''\theta^D}{(k^2\beta)^D+(k\beta)^D+\beta^D+\theta^D}-\frac{p'p''q'C}{\mu^C}$$

$$\int_0^\infty y_1^{C-1}e^{-\left[\left(\frac{k^2y_1}{\lambda}\right)^C+\left(\frac{y_1}{\mu}\right)^C+\left(\frac{y_1}{\theta}\right)^D\right]}dy_1+\frac{p'^2p''q'C}{\mu^C}\int_0^\infty y_1^{C-1}e^{-\left[\left(\frac{k^2y_1}{\lambda}\right)^C+\left(\frac{ky_1}{\lambda}\right)^A+\left(\frac{y_1}{\mu}\right)^C+\left(\frac{y_1}{\theta}\right)^D\right]}dy_1-\frac{p'p''q'C}{\mu^C}$$

$$\int_0^\infty y_1^{C-1}e^{-\left[\left(\frac{k^2y_1}{\lambda}\right)^C+\left(\frac{ky_1}{\theta}\right)^D+\left(\frac{y_1}{\mu}\right)^C\right]}dy_1+\frac{p'^2p''q'C}{\mu^C}\int_0^\infty y_1^{C-1}e^{-\left[\left(\frac{k^2y_1}{\lambda}\right)^C+\left(\frac{ky_1}{\theta}\right)^D+\left(\frac{y_1}{\mu}\right)^C+\left(\frac{y_1}{\theta}\right)^C\right]}dy_1-\frac{p'p''^2q'C}{\mu^C}$$

$$\int_0^\infty y_1^{C-1}e^{-\left[\left(\frac{k^2y_1}{\lambda}\right)^C+\left(\frac{ky_1}{\theta}\right)^D+\left(\frac{y_1}{\mu}\right)^C+\left(\frac{y_1}{\theta}\right)^D\right]}dy_1+\frac{p'q''D}{\beta^D}\int_0^\infty y_1^{D-1}e^{-\left[\left(\frac{k^2y_1}{\lambda}\right)^C+\left(\frac{y_1}{\beta}\right)^D\right]}dy_1-\frac{p'^2q''D}{\beta^D}$$

$$\int_0^\infty y_1^{D-1}e^{-\left[\left(\frac{k^2y_1}{\lambda}\right)^C+\left(\frac{y_1}{\beta}\right)^D+\left(\frac{y_1}{\lambda}\right)^C\right]}dy_1-\frac{p'p''q''D}{\beta^D}\int_0^\infty y_1^{D-1}e^{-\left[\left(\frac{k^2y_1}{\lambda}\right)^C+\left(\frac{y_1}{\theta}\right)^D+\left(\frac{y_1}{\beta}\right)^D\right]}dy_1-\frac{p'^2q''D}{\beta^D}$$

$$\int_0^\infty y_1^{D-1}e^{-\left[\left(\frac{k^2y_1}{\lambda}\right)^C+\left(\frac{ky_1}{\lambda}\right)^C+\left(\frac{y_1}{\beta}\right)^D\right]}dy_1+\frac{p'^3q''D}{\beta^D}\int_0^\infty y_1^{D-1}e^{-\left[\left(\frac{k^2y_1}{\lambda}\right)^C+\left(\frac{ky_1}{\lambda}\right)^C+\left(\frac{y_1}{\lambda}\right)^C+\left(\frac{y_1}{\beta}\right)^D\right]}dy_1+\frac{p'^2p''q''D}{\beta^D}$$

$$\int_0^\infty y_1^{D-1}e^{-\left[\left(\frac{k^2y_1}{\lambda}\right)^C+\left(\frac{ky_1}{\lambda}\right)^C+\left(\frac{y_1}{\theta}\right)^D+\left(\frac{y_1}{\beta}\right)^D\right]}dy_1-\frac{p'p''q''D}{\beta^D}\int_0^\infty y_1^{D-1}e^{-\left[\left(\frac{k^2y_1}{\lambda}\right)^C+\left(\frac{ky_1}{\theta}\right)^D+\left(\frac{y_1}{\beta}\right)^D\right]}dy_1+\frac{p'^2p''q''D}{\beta^D}$$

$$\int_0^\infty y_1^{D-1}e^{-\left[\left(\frac{k^2y_1}{\lambda}\right)^C+\left(\frac{ky_1}{\theta}\right)^D+\left(\frac{y_1}{\lambda}\right)^C+\left(\frac{y_1}{\beta}\right)^D\right]}dy_1+\frac{p'p''^2q''D}{\beta^D}\int_0^\infty y_1^{D-1}e^{-\left[\left(\frac{k^2y_1}{\lambda}\right)^C+\left(\frac{ky_1}{\theta}\right)^D+\left(\frac{y_1}{\theta}\right)^D+\left(\frac{y_1}{\beta}\right)^D\right]}dy_1+$$

$$\frac{p''q'C}{\mu^C}\int_0^\infty y_1^{C-1}e^{-\left[\left(\frac{k^2y_1}{\theta}\right)^D+\left(\frac{y_1}{\mu}\right)^C\right]}dy_1-\frac{p'p''q'C}{\mu^C}\int_0^\infty y_1^{C-1}e^{-\left[\left(\frac{k^2y_1}{\theta}\right)^D+\left(\frac{y_1}{\lambda}\right)^C+\left(\frac{y_1}{\mu}\right)^C\right]}dy_1-\frac{p''^2q'C}{\mu^C}$$

$$\int_0^\infty y_1^{C-1}e^{-\left[\left(\frac{k^2y_1}{\theta}\right)^D+\left(\frac{y_1}{\theta}\right)^D+\left(\frac{y_1}{\mu}\right)^C\right]}dy_1-\frac{p'p''q'C}{\mu^C}\int_0^\infty y_1^{C-1}e^{-\left[\left(\frac{k^2y_1}{\theta}\right)^D+\left(\frac{ky_1}{\lambda}\right)^C+\left(\frac{y_1}{\mu}\right)^C\right]}dy_1+\frac{p'^2p''q'C}{\mu^C}$$

$$\int_0^\infty y_1^{C-1}e^{-\left[\left(\frac{k^2y_1}{\theta}\right)^D+\left(\frac{ky_1}{\lambda}\right)^C+\left(\frac{y_1}{\lambda}\right)^C+\left(\frac{y_1}{\mu}\right)^C\right]}dy_1+\frac{p'p''^2q'C}{\mu^C}\int_0^\infty y_1^{C-1}e^{-\left[\left(\frac{k^2y_1}{\theta}\right)^D+\left(\frac{ky_1}{\lambda}\right)^C+\left(\frac{y_1}{\theta}\right)^D+\left(\frac{y_1}{\mu}\right)^C\right]}dy_1-$$

$$\frac{p''^2q'C}{\mu^C}\int_0^\infty y_1^{C-1}e^{-\left[\left(\frac{k^2y_1}{\theta}\right)^D+\left(\frac{ky_1}{\theta}\right)^D+\left(\frac{y_1}{\mu}\right)^C\right]}dy_1+\frac{p'p''^2q'C}{\mu^C}\int_0^\infty y_1^{C-1}e^{-\left[\left(\frac{k^2y_1}{\theta}\right)^D+\left(\frac{ky_1}{\theta}\right)^D+\left(\frac{y_1}{\lambda}\right)^C+\left(\frac{y_1}{\mu}\right)^C\right]}dy_1+$$

$$\frac{p''^3q'C}{\mu^C}\int_0^\infty y_1^{C-1}e^{-\left[\left(\frac{k^2y_1}{\theta}\right)^D+\left(\frac{ky_1}{\theta}\right)^D+\left(\frac{y_1}{\theta}\right)^D+\left(\frac{y_1}{\mu}\right)^C\right]}dy_1-\frac{p'p''q''D}{\beta^D}\int_0^\infty y_1^{D-1}e^{-\left[\left(\frac{k^2y_1}{\theta}\right)^D+\left(\frac{y_1}{\lambda}\right)^C+\left(\frac{y_1}{\beta}\right)^D\right]}dy_1-$$

$$\frac{p'p''q''D}{\beta^D}\int_0^\infty y_1^{D-1}e^{-\left[\left(\frac{k^2y_1}{\theta}\right)^D+\left(\frac{ky_1}{\lambda}\right)^C+\left(\frac{y_1}{\beta}\right)^D\right]}dy_1+\frac{p'^2p''q''D}{\beta^D}\int_0^\infty y_1^{D-1}e^{-\left[\left(\frac{k^2y_1}{\theta}\right)^D+\left(\frac{ky_1}{\lambda}\right)^C+\left(\frac{y_1}{\lambda}\right)^C+\left(\frac{y_1}{\beta}\right)^D\right]}dy_1+$$

$$\frac{p'p''^2q''D}{\beta^D}\int_0^\infty y_1^{D-1}e^{-\left[\left(\frac{k^2y_1}{\theta}\right)^D+\left(\frac{ky_1}{\lambda}\right)^C+\left(\frac{y_1}{\theta}\right)^D+\left(\frac{y_1}{\beta}\right)^D\right]}dy_1+\frac{p'p''^2q''D}{\beta^D}\int_0^\infty y_1^{D-1}e^{-\left[\left(\frac{k^2y_1}{\theta}\right)^D+\left(\frac{ky_1}{\lambda}\right)^C+\left(\frac{y_1}{\lambda}\right)^C+\left(\frac{y_1}{\beta}\right)^D\right]}dy_1$$

$$=\frac{p'q'\lambda^C}{(k^2\mu)^C+\lambda^C}-\frac{p'^2q'\lambda^C}{(k^2\mu)^C+\lambda^C+\mu^C}-\frac{p'^2q'\lambda^C}{(k^2\mu)^C+(k\mu)^C+\lambda^C}+\frac{p'^3q'\lambda^C}{(k^2\mu)^C+(k\mu)^C+\mu^C+\lambda^C}+$$

$$\frac{p''q''\theta^D}{(k^2\beta)^D+\theta^D}-\frac{p''^2q''\theta^D}{(k^2\beta)^D+\beta^D+\theta^D}-\frac{p''^2q''\theta^D}{(k^2\beta)^D+(k\beta)^D+\theta^D}+\frac{p''^3q''\theta^D}{(k^2\beta)^D+(k\beta)^D+\beta^D+\theta^D}-\delta_1+$$

$$\delta_2-\delta_3+\delta_4-\delta_5+\delta_6-\delta_7-\delta_8-\delta_9+\delta_{10}+\delta_{11}-\delta_{12}+\delta_{13}+\delta_{14}+\delta_{15}-\delta_{16}-\delta_{17}-\delta_{18}+\delta_{19}+\delta_{20}-$$

$$\delta_{21}+\delta_{22}+\delta_{23}-\delta_{24}-\delta_{25}+\delta_{26}+\delta_{27}+\delta_{28}$$

where δ_i, i =1,2...,28 can be evaluated numerically.

The reliabilities R_2 and R_3 for the mixture of two Weibulls of 2-cascade and 3-cascade system when shape parameters are equal i.e. $A=C$ and $B=D$, may be obtained from (3.3.3) and (3.3.4) respectively.

A few values of R_1, R_2 and R_3 are tabulated in **Table** 3.3 (cf. Appendix) for different values of $k, A, B, C, D, \mu, \lambda, \theta, \beta, p'$ *and* q'.

(2) When scale parameters are equal i.e. $\lambda=\mu$ and $\theta=\beta$ then

$$R(1)=\int_0^\infty\left[p'e^{-(y_1/\mu)^A}+p''e^{-(y_1/\beta)^B}\right]\left[\frac{q'C}{\mu^C}y_1^{C-1}e^{-(y_1/\mu)^C}+\frac{q''D}{\beta^D}y_1^{D-1}e^{-(y_1/\beta)^D}\right]dy_1$$

$$R(2)=\int_0^\infty\left[1-p'e^{-(y_1/\mu)^A}-p''e^{-(y_1/\beta)^B}\right]\left[p'e^{-(ky_1/\mu)^A}+p''e^{-(ky_1/\beta)^B}\right]$$

$$\left[\frac{q'C}{\mu^C}y_1^{C-1}e^{-(y_1/\mu)^C}+\frac{q''D}{\beta^D}y_1^{D-1}e^{-(y_1/\beta)^D}\right]dy_1$$

$$R(3)=\int_0^\infty \left[1-p' e^{-(y_1/\mu)^A}-p'' e^{-(y_1/\beta)^B}\right]\left[1-p' e^{-(k y_1/\mu)^A}-p'' e^{-(k y_1/\beta)^B}\right]$$
$$\left[p' e^{-(k^2 y_1/\mu)^A}+p'' e^{-(k^2 y_1/\beta)^B}\right]\left[\frac{q'C}{\mu^C} y_1^{C-1} e^{-(y_1/\mu)^C}+\frac{q''D}{\beta^D} y_1^{D-1} e^{-(y_1/\beta)^D}\right]dy_1$$

The expressions for $R(1)$, $R(2)$ and $R(3)$ can be evaluated numerically. The reliabilities R_2 and R_3 for the mixture of two Weibulls of 2-cascade and 3-cascade system when scale parameters are equal i.e. $\lambda=\mu$ and $\theta=\beta$, may be obtained from (3.3.3) and (3.3.4) respectively.

A few values of R_1, R_2 and R_3 are tabulated in **Table** 3.4 (cf. Appendix) for different values of $k, A, B, C, D, \mu, \lambda, \theta, \beta, p'\ and\ q'$.

3.4 Graphical Representations

A few graphs of R_1, R_2 and R_3 against p' for different values of q' are drawn in **Fig.** 3.1(a)-3.1(f), **Fig.** 3.2(a)-3.2(f), **Fig.** 3.3(a)-3.3(f) and **Fig.** 3.4(a)-3.4(f) for corresponding parametric values involved. In **Fig.** 3.1(a)-3.1(f), reliabilities are steadily increasing with p' whereas in **Fig.** 3.2(a)-3.2(f), **Fig.** 3.3(a)-3.3(f) and **Fig.** 3.4(a)-3.4(f) it is decreasing with increasing p'.

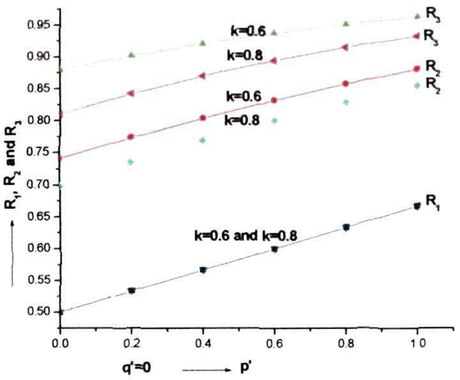

Fig. 3.1(a) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of exponential distribution

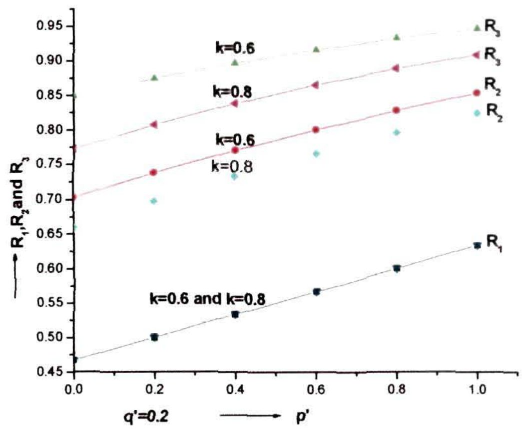

Fig. 3.1(b) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of exponential distribution

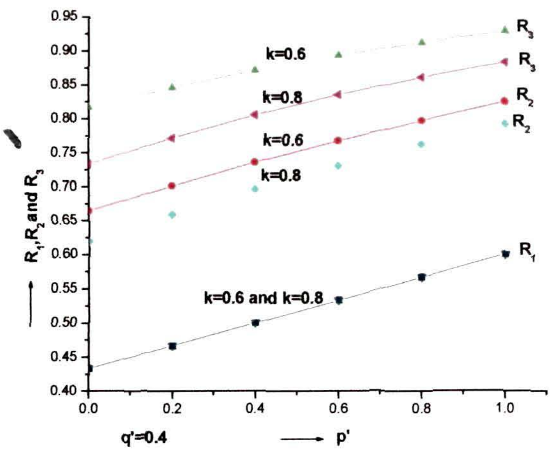

Fig. 3.1(c) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of exponential distribution

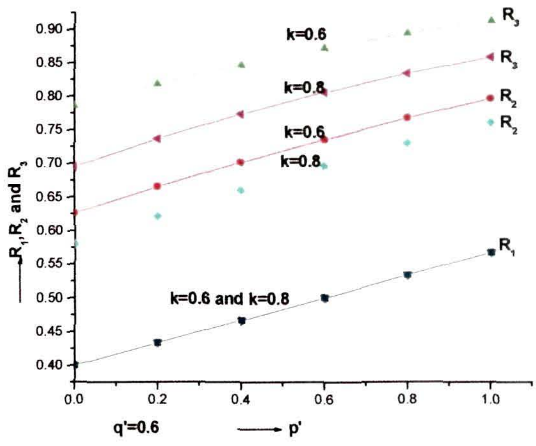

Fig. 3.1(d) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of exponential distribution

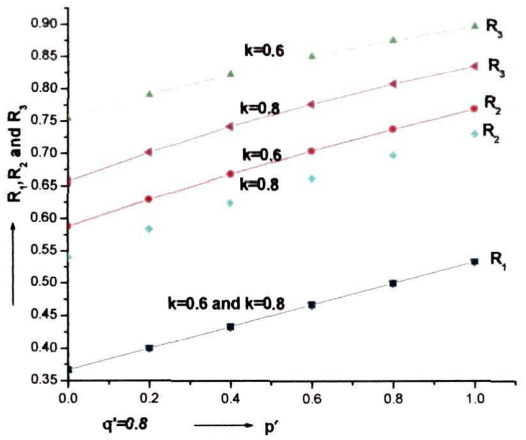

Fig. 3.1(e) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of exponential distribution

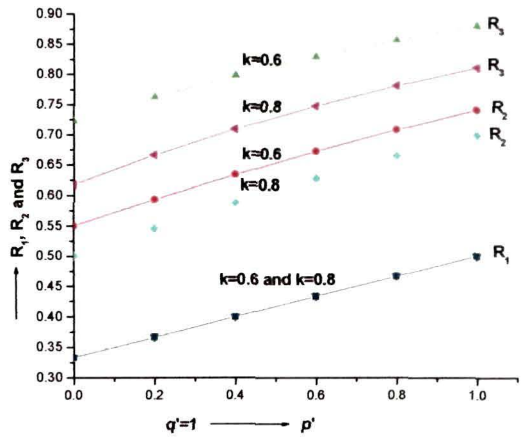

Fig. 3.1(f) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of exponential distribution

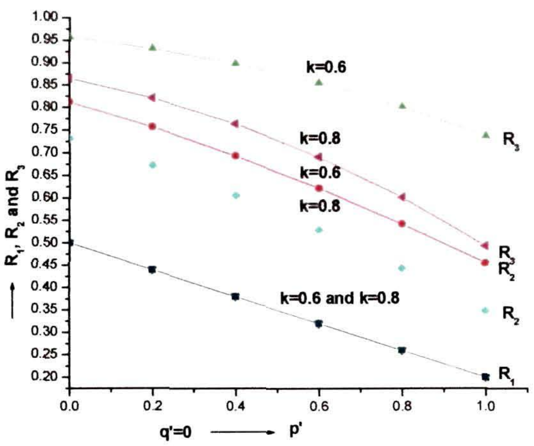

Fig. 3.2(a) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Rayleigh distribution

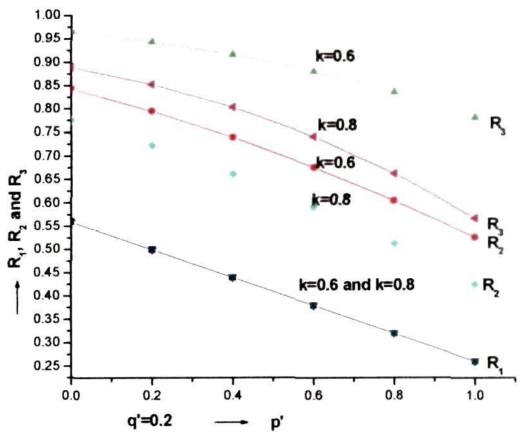

Fig. 3.2(b) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Rayleigh distribution

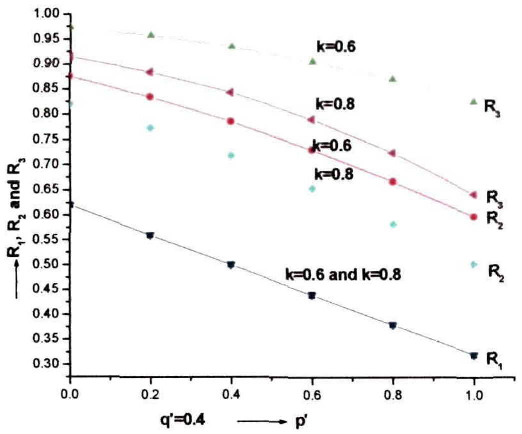

Fig.3.2(c) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Rayleigh distribution

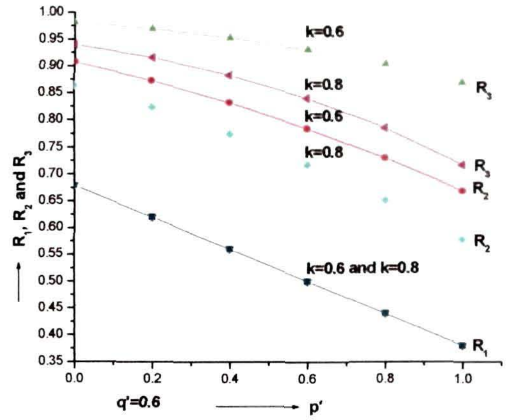

Fig. 3.2(d) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Rayleigh distribution

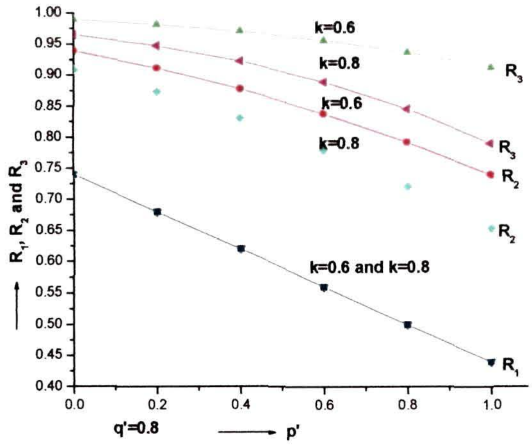

Fig. 3.2(e) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Rayleigh distribution

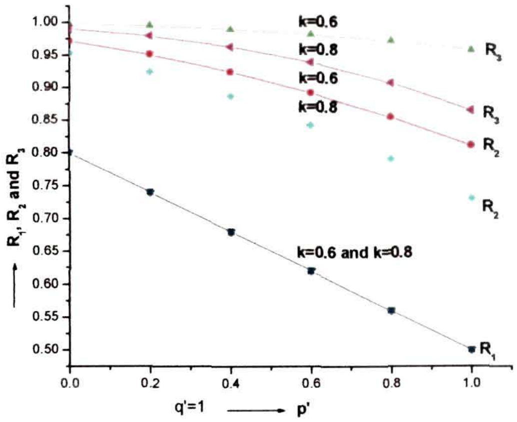

Fig. 3.2(f) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Rayleigh distribution

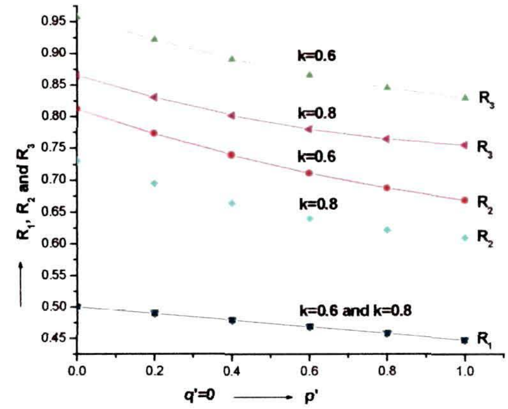

Fig. 3.3(a) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Weibull distribution

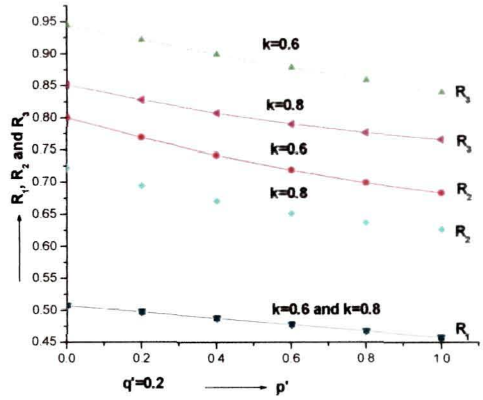

Fig. 3.3(b) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Weibull distribution

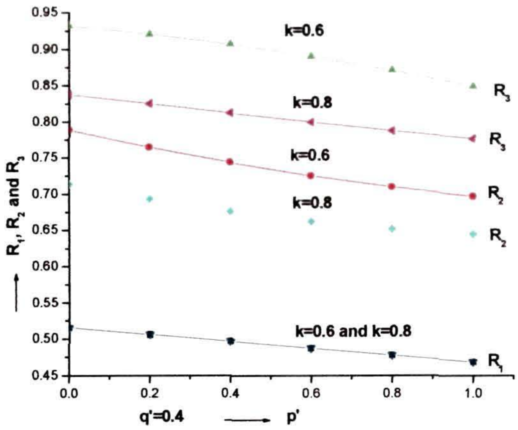

Fig. 3.3(c) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Weibull distribution

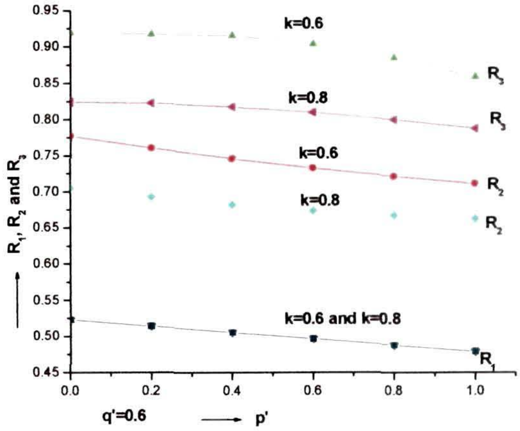

Fig. 3.3(d) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Weibull distribution

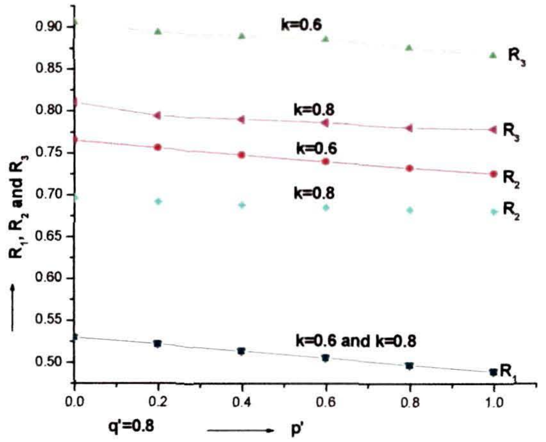

Fig. 3.3(e) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Weibull distribution

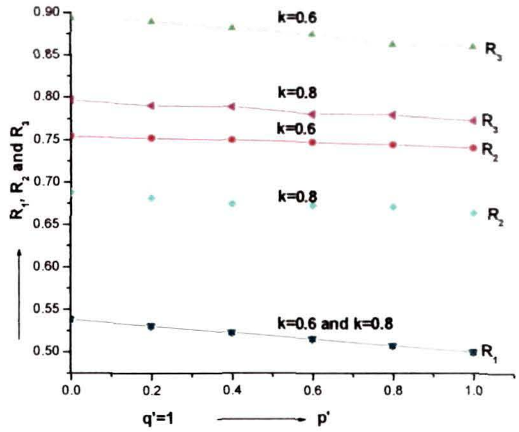

Fig. 3.3(f) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Weibull distribution

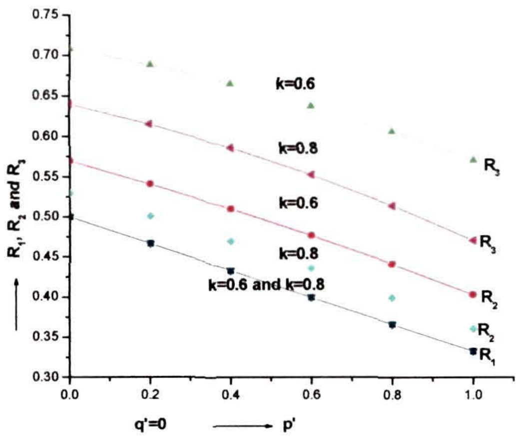

Fig. 3.4(a) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Weibull distribution

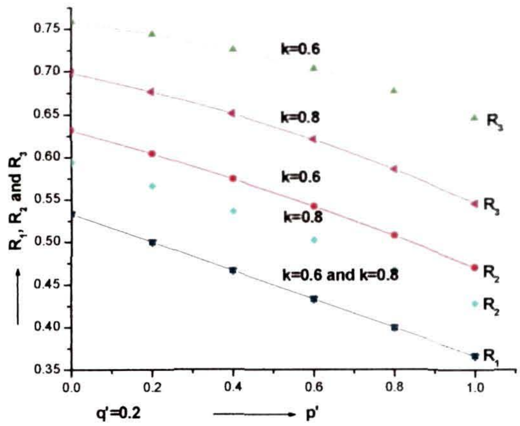

Fig. 3.4(b) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Weibull distribution

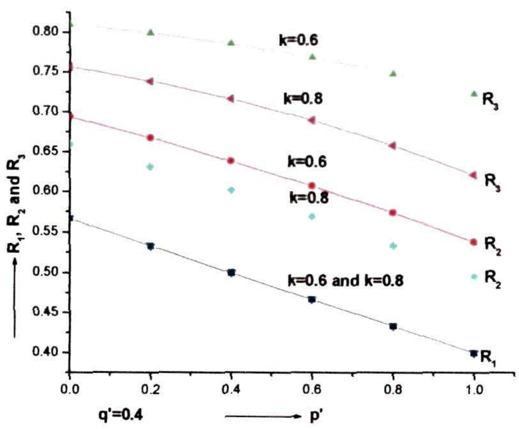

Fig. 3.4(c) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Weibull distribution

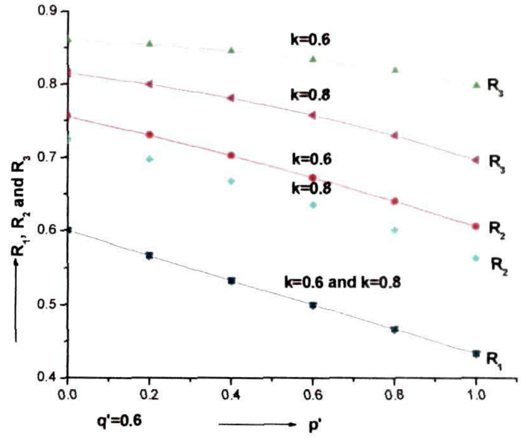

Fig. 3.4(d) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Weibull distribution

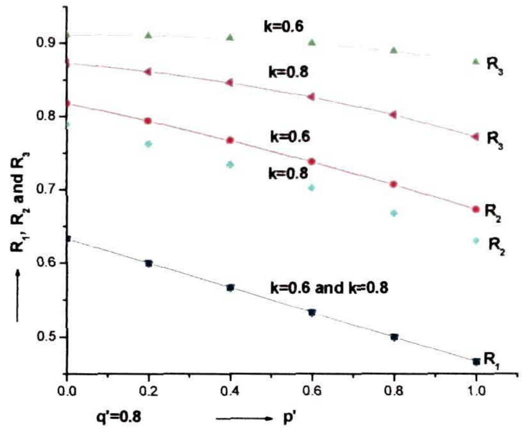

Fig. 3.4(e) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Weibull distribution

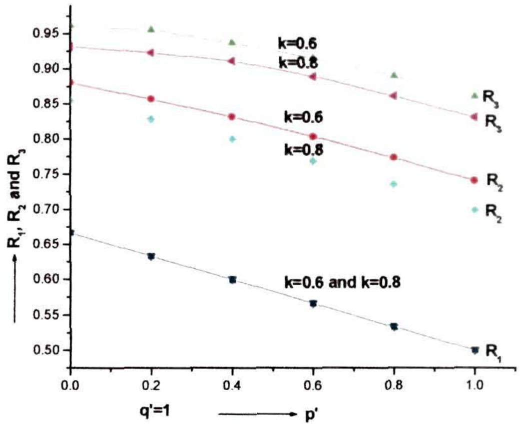

Fig. 3.4(f) Graph of R_1, R_2 and R_3 when Stress and Strength are mixture of Weibull distribution

3.5 Results and Discussions

A few selected values of the parameters involved in the expressions of R_i, i=1,2,3 we evaluate R_1, R_2, R_3 for particular cases of exponential, Rayleigh and Weibull distributions and presented in **Table** 3.1, **Table** 3.2, **Table** 3.3 and **Table** 3.4. These numerical results for exponential distributions are obtained for specific values of μ,λ,θ,β when $\lambda = \mu$ *and* $\theta = \beta$. Similar results have been obtained for Rayleigh and Weibull distributions, when $\sigma_1 = \sigma_3$, $\sigma_2 = \sigma_4$ and shape parameters say $A = C, B = D$ and scale parameters say $\lambda = \mu$, $\theta = \beta$. In all the cases the attenuation factor k takes values 0.6 and 0.8.

From the **Table** 3.1 we see that, in case of mixture of exponential distribution, maximum value R_1= 0.6667, R_2= 0.8803 and R_3= 0.9619 have been obtained when p'=1 and q'=0 for $\mu = \lambda = 1, \theta = \beta$, k=0.6 and $\beta = 2$. Minimum value R_1=0.3333, R_2= 0.5006 and R_3= 0.6178 have been obtained when p'=0 and q'=1 for $\mu = \lambda = 1, \theta = \beta$, k=0.8 and $\beta = 2$. It is noted that as q' increases, values of R_1, R_2 and R_3 decreases.

For different values of k, $\sigma_1, \sigma_2, \sigma_3, \sigma_4, p'$ *and* q' we have tabulated the values of R_1, R_2, R_3 in case of mixture of Rayleigh distribution in **Table** 3.2. From that table we see that, maximum value R_1= 0.8000, R_2= 0.9712 and R_3= 0.9977 have been obtained when p'=0 and q'=1 for $\sigma_1 = \sigma_3 = 1$, $\sigma_2 = \sigma_4$, k=0.6 and $\sigma_4 = 2$. Minimum value R_1=0.2000, R_2=0. 3486 and R_3= 0.4934 have been obtained when p'=1 and q'=0 for $\sigma_1 = \sigma_3 = 1$, $\sigma_2 = \sigma_4$, k=0.8 and $\sigma_4 = 2$. It is noted that as q' increases, values of R_1, R_2 and R_3 increases.

Table 3.3 presents a few values of R_1, R_2, R_3 for different values of k, A, B, C, D, $\mu, \lambda, \theta, \beta, p'$ *and* q' in case of mixture of Weibull distribution. From the **Table** we see that, maximum and minimum value of R_1= 0.5382 (when p'=0,q'=1) and 0.4475 (when p'=1,q'=0) for $\mu = \lambda = \theta = \beta = 1$, $A = C = 1, B = D$, $k = 0.6$ and D=2. Similarly maximum value R_2= 0.8116 and R_3= 0.9576 have been obtained when p'=0 and q'=0 for $\mu = \lambda = \theta = \beta = 1$, $A = C = 1$, $B = D$, $k = 0.6$ and D=2. Minimum value R_2= 0.6094 and R_3=0.7546 have been obtained when p'=1 and q'=0 for $\mu = \lambda = \theta = \beta = 1$, $A = C = 1, B = D$, $k = 0.8$ and D=2. It is noted that as q' increases, values of R_2 and R_3 decreases. Only R_1 increases with increasing value of q'.

From the **Table** 3.4 we see that, in case of mixture of Weibull distribution, maximum value R_1= 0.6667, R_2= 0.8803 and R_3= 0.9619 have been obtained when p'=0 and q'=1 for $\lambda = \mu = 1, \theta = \beta$, $A = B = C = D = 1$, k=0.6 and β=2. Minimum value R_1=0.3329, R_2=0.3605 and R_3=0.4704 have been obtained when p'=1 and q'=0 for $\lambda = \mu = 1, \theta = \beta$, $A = B = C = D = 1$, k=0.8 and β=2. It is noted that values of R_1, R_2 and R_3 increases with increasing value of q'.

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Chapter 4

Cascade Reliability in Different Types of Failure Models

Chapter 4

Cascade Reliability in Different Types of Failure Models

4.1 Introduction

In many situations a component may fail in several ways as electrical failure, mechanical failure and the like. Each type of failure may be attributed to different stresses which in turn can be represented by different random variables. Let us assume that a component may fail in m different ways for which m different stresses are responsible. Now if X is a random variable representing the strength of a component then the reliability of the component is given by

$$R = P[X \geq Y_1, X \geq Y_2, \ldots, X \geq Y_m] \tag{4.1.1}$$

An n-cascade system where components may fail in different ways is considered in this chapter. The following three models are considered:

Model- I: Here we have assumed that in an n-cascade system an active component faces m different stresses (which are responsible for different types of failures) and it fails if the strength of the active component is less than any one of the stresses on it. After the failure of the first component, the second component faces m stresses which are k times the corresponding previous stresses and so on.

Model-II: For an n-cascade system we assume that for the working of an active component it is essential that the m stresses on the component lie in an interval (a_j, b_j), $j=1,2,\ldots, m$. The component fails even if one of stresses on the component falls outside the specified limits. The two limits a_j and b_j are assumed to be constants. All the components are assumed to be identical.

Model- III: Here also an n-cascade system is considered. This model is similar to the model II except that the components are not identical. Here the limits of the different stresses, though constants are assumed to be different for different components, say (a_{ij}, b_{ij}), for the ith component, $j=1,2,\ldots, m$.

For model II and model III strength of the components do not come into the picture directly. But the constants (a_j, b_j) and (a_{ij}, b_{ij}) must have been fixed on the basis of the strength of the components.

Sriwastav and Dutta (1986) considered the case of an n-standby system with different types of failure in S-S model. For multicomponent systems Hilton and Fergen (1960) have considered structural reliability problem where failure modes of the components are independent. A similar problem is considered by Moser and Kinser (1967). Heller and Donat (1967) have evaluated the reliability of 'multiple-load-path' structure, in which the system with m components initially, may work even with $(m-1)$ failed components. The applied stress to the system is redistributed among the surviving components, at every failure. The system fails when all the components fail. They have assumed a statistical dependence among different types of failure modes.

Here we have considered an n-cascade system. We have not come across any study where cascade model is considered for such a models. The main aim of this chapter is to obtain the system reliability R_n under this three failure models.

This chapter is organized as follows: In Section-4.2 the general expressions for all the three models are developed. In Section-4.3 the reliability expressions of an n-cascade system is obtained for all the models when the stress-strength of the components follow particular distributions. In Sub-Section 4.3.1 to 4.3.2 the expressions of R_n, $n < 4$ is obtained under the three models when stress-strength distributions are either exponential or Rayleigh. Some graphs are also plotted for some values in Section 4.4. In all the cases, numerical results for particular values of relevant parameters are tabulated in **Table** 4.1, **Table** 4.2, **Table** 4.3, **Table** 4.4, **Table** 4.5 and **Table** 4.6 (cf. Appendix) and some results and discussions are given in Section-4.5.

4.2 Development of the Mathematical Models

Let us consider an n-cascade system working under the impact of m different stresses. Here we shall obtain the reliability of this system under the three different models, one-by-one.

Model I: Let $X_1, X_2,..., X_n$ be the strengths of n-components and each component faces m stresses simultaneously. The stresses on the first component are $Y_1, Y_2,..., Y_m$. After the failure of the first component the second component faces m stresses $kY_1, kY_2,..., kY_m$, the third component faces m stresses $k^2Y_1, k^2Y_2,..., k^2Y_m$ and so on. The stresses on the th component are $k^{i-1}Y_1, k^{i-1}Y_2,..k^{i-1}Y_m$. Let the attenuation factor k be a constant quantity. It is assumed that X_i, $i=1,2,...,n$ and Y_j, $j=1,2,...,m$ are independent random variables. Then the probability that the ith component works is given by

$$R'(i) = P\left[X_i \geq k^{i-1}Y_1, X_i \geq k^{i-1}Y_2,..., X_i \geq k^{i-1}Y_m\right], i = 1,2,...,n \tag{4.2.1}$$

Then, the reliability of the system is given by

$$R_n = R'(1) + [1 - R'(1)]R'(2) + [1 - R'(1)][1 - R'(2)]R'(3) + ... + [1 - R'(1)] \\ [1 - R'(2)]..[1 - R'(n-1)]R'(n) \tag{4.2.2}$$

$$= R(1) + R(2) + ... + R(n), \; say \tag{4.2.3}$$

where $R(i)$, $i=1,2,\ldots,n$ is the marginal reliability due to the ith component, given by

$$R(i)=[1-R'(1)][1-R'(2)]..[1-R'(i-1)]R'(i) \tag{4.2.4}$$

Let $f_i(x)$ and $g_j(y)$ be the p.d.f. of X_i and Y_j, $i=1,2,\ldots,n$, $j=1,2,\ldots,m$ respectively. Since, the stress and strength are independent, we have

$$R'(i)=\prod_{j=1}^{m}\int_{-\infty}^{\infty}\overline{F_i}(k^{i-1}y_j)g_j(y)dy_j \quad i=1,2,\ldots,n \tag{4.2.5}$$

where $F_i(x)=\int_{-\infty}^{x}f_i(x)dx$ and $\overline{F_i}(x)=1-F_i(x)$

Substituting i =1,2,3 in (4.2.5) and using (4.2.4) we get,

$$\begin{aligned}R(1)&=\prod_{j=1}^{m}\int_{-\infty}^{\infty}\overline{F_1}(y_j)g_j(y)dy_j\\&=R'(1)\end{aligned} \tag{4.2.6}$$

$$\begin{aligned}R(2)&=[1-R'(1)]R'(2)\\&=R'(2)-R'(1)R'(2)\\&=\prod_{j=1}^{m}\int_{-\infty}^{\infty}\overline{F_2}(k\,y_j)g_j(y)dy_j-\prod_{j=1}^{m}\int_{-\infty}^{\infty}\overline{F_1}(y_j)\overline{F_2}(k\,y_j)g_j(y)dy_j\end{aligned} \tag{4.2.7}$$

$$\begin{aligned}R(3)&=[1-R'(1)][1-R'(2)]R'(3)\\&=R'(3)-R'(2)R'(3)-R'(1)R'(3)+R'(1)R'(2)R'(3)\\&=\prod_{j=1}^{m}\int_{-\infty}^{\infty}\overline{F_3}(k^2y_j)g_j(y)dy_j-\prod_{j=1}^{m}\int_{-\infty}^{\infty}\overline{F_1}(y_j)\overline{F_3}(k^2y_j)g_j(y)dy_j\\&\quad-\prod_{j=1}^{m}\int_{-\infty}^{\infty}\overline{F_2}(k\,y_j)\overline{F_3}(k^2y_j)g_j(y)dy_j+\prod_{j=1}^{m}\int_{-\infty}^{\infty}\overline{F_1}(y_j)\overline{F_2}(k\,y_j)\overline{F_3}(k^2y_j)g_j(y)dy_j\end{aligned} \tag{4.2.8}$$

Substituting $R(1),R(2),R(3),\ldots,R(n)$ in (4.2.3) we get R_n.

Model II: We have an n-cascade system. As in model I let $Y_1, Y_2, ..., Y_m$ be the stresses on the first component. We assume that for the working of a component the jth stress must lie in a specified interval, say (a_j, b_j), $j = 1,2,\ldots,m$. After the failure of the first component the second component faces m stresses $kY_1, kY_2, ..., kY_m$, the 3rd component faces m stresses $k^2Y_1, k^2Y_2, ..., k^2Y_m$, and so on. Here we assume that the components are identical i.e., the limits (a_j, b_j) is same for all the components. Here also the reliability R_n of the system is given by (4.2.2) and (4.2.3) but now,

$$\begin{aligned} R'(i) &= P\left[(a_1 < k^{i-1}Y_1 < b_1),(a_2 < k^{i-1}Y_2 < b_2),...,(a_m < k^{i-1}Y_m < b_m)\right] \\ &= P\left(a_1 < k^{i-1}Y_1 < b_1\right)P\left(a_2 < k^{i-1}Y_2 < b_2\right)..P\left(a_m < k^{i-1}Y_m < b_m\right) \end{aligned} \quad (4.2.9)$$

$i = 1,2,\ldots,n$, since the stresses on the components are independent random variables.

Let $g_j(y)$ be the p.d.f. of Y_j, then

$$R'(i) = \prod_{j=1}^{m} \int_{\frac{a_j}{k^{i-1}}}^{\frac{b_j}{k^{i-1}}} g_j(y)dy_j \quad i = 1,2,...,n \quad (4.2.10)$$

Now from (4.2.4) and (4.2.10), we get

$$\begin{aligned} R(1) &= R'(1) \\ &= \prod_{j=1}^{m} \int_{a_j}^{b_j} g_j(y)dy_j \end{aligned} \quad (4.2.11)$$

$$\begin{aligned} R(2) &= R'(2) - R'(1)R'(2) \\ &= \prod_{j=1}^{m} \int_{a_j/k}^{b_j/k} g_j(y)dy_j - \left(\prod_{j=1}^{m} \int_{a_j}^{b_j} g_j(y)dy_j\right)\left(\prod_{j=1}^{m} \int_{a_j/k}^{b_j/k} g_j(y)dy_j\right) \end{aligned} \quad (4.2.12)$$

$$
\begin{aligned}
R(3) &= R'(3) - R'(2)R'(3) - R'(1)R'(3) + R'(1)R'(2)R'(3) \\
&= \prod_{j=1}^{m} \int_{a_j/k^2}^{b_j/k^2} g_j(y)dy_j - \left(\prod_{j=1}^{m} \int_{a_j/k}^{b_j/k} g_j(y)dy_j\right)\left(\prod_{j=1}^{m} \int_{a_j/k^2}^{b_j/k^2} g_j(y)dy_j\right) - \left(\prod_{j=1}^{m} \int_{a_j}^{b_j} g_j(y)dy_j\right) \\
&\left(\prod_{j=1}^{m} \int_{a_j/k^2}^{b_j/k^2} g_j(y)dy_j\right) + \left(\prod_{j=1}^{m} \int_{a_j}^{b_j} g_j(y)dy_j\right)\left(\prod_{j=1}^{m} \int_{a_j/k}^{b_j/k} g_j(y)dy_j\right)\left(\prod_{j=1}^{m} \int_{a_j/k^2}^{b_j/k^2} g_j(y)dy_j\right)
\end{aligned}
\tag{4.2.13}
$$

Substituting $R(1), R(2), R(3), \ldots, R(n)$ in (4.2.3) we can obtain R_n.

Model III: This model is similar to model II except the components are not identical. Let $k^{i-1}Y_1, k^{i-1}Y_2, \ldots, k^{i-1}Y_m$ be the stresses on the ith active component and let (a_{ij}, b_{ij}); ($i=1,2,\ldots,n$; $j=1,2,\ldots,m$) be the required limits of the m stresses for the working of the ith active component where (a_{ij}, b_{ij}) corresponds to the jth stress on the ith component. Reliability of the system under this model is given by (4.2.2) and (4.2.3)

$$
\begin{aligned}
R'(i) &= P\left[(a_{i1} < k^{i-1}Y_1 < b_{i1}), (a_{i2} < k^{i-1}Y_2 < b_{i2}), \ldots, (a_{im} < k^{i-1}Y_m < b_{im})\right] \\
&= P\left(\frac{a_{i1}}{k^{i-1}} < Y_1 < \frac{b_{i1}}{k^{i-1}}\right) P\left(\frac{a_{i2}}{k^{i-1}} < Y_2 < \frac{b_{i2}}{k^{i-1}}\right) \ldots P\left(\frac{a_{im}}{k^{i-1}} < Y_m < \frac{b_{im}}{k^{i-1}}\right)
\end{aligned}
\tag{4.2.14}
$$

since $Y_1, Y_2, \ldots, Y_m$ are independent.

Let $g_j(y)$ be the p.d.f. of Y_j, then

$$
R'(i) = \prod_{j=1}^{m} \int_{\frac{a_{ij}}{k^{i-1}}}^{\frac{b_{ij}}{k^{i-1}}} g_j(y)dy_j, \quad i = 1,2,\ldots,n \tag{4.2.15}
$$

Substituting the values of $R'(i)$, $i=1,2,\ldots,n$ from (4.2.15) in (4.2.2) we get the system reliability R_n.

4.3 Stress-Strength follows Specific Distributions

In this section we have considered stress-strength, which are either exponential or Rayleigh distributions and they may be with different parameters. In the following Sub-Sections we have obtained the reliability of a 3-cascade system under the three models as discussed in section 4.2.

4.3.1 Exponential Stress-Strength Distribution

Model I: Let $f_i(x)$ and $g_j(y)$ be the exponential densities with means $1/\theta_i$ and $1/\alpha_j$, $i=1,2,\ldots,n$; $j=1,2,\ldots,m$ respectively. Then from (4.2.5) we get,

$$\begin{aligned} R'(i) &= \prod_{j=1}^{m}\int_{0}^{\infty} e^{-k^{i-1}\theta_i y_j}\alpha_j e^{-\alpha_j y_j}\,dy_j \\ &= \prod_{j=1}^{m}\frac{\alpha_j}{\alpha_j+k^{i-1}\theta_i};\quad i=1,2,\ldots,n \end{aligned} \tag{4.3.1}$$

Now from (4.3.1) using (4.2.4) we get the marginal reliabilities $R(1), R(2), R(3)$ as follows.

$$\begin{aligned} R(1) &= R'(1) \\ &= \prod_{j=1}^{m}\frac{\alpha_j}{\alpha_j+\theta_1} \end{aligned} \tag{4.3.2}$$

$$\begin{aligned} R(2) &= R'(2)-R'(1)R'(2) \\ &= \prod_{j=1}^{m}\frac{\alpha_j}{\alpha_j+k\theta_2}-\prod_{j=1}^{m}\frac{\alpha_j}{\alpha_j+\theta_1+k\theta_2} \end{aligned} \tag{4.3.3}$$

$$\begin{aligned} R(3) &= R'(3)-R'(2)R'(3)-R'(1)R'(3)+R'(1)R'(2)R'(3) \\ &= \prod_{j=1}^{m}\frac{\alpha_j}{\alpha_j+k^2\theta_3}-\prod_{j=1}^{m}\frac{\alpha_j}{\alpha_j+\theta_1+k^2\theta_3}-\prod_{j=1}^{m}\frac{\alpha_j}{\alpha_j+k\theta_2+k^2\theta_3}+\prod_{j=1}^{m}\frac{\alpha_j}{\alpha_j+\theta_1+k\theta_2+k^2\theta_3} \end{aligned} \tag{4.3.4}$$

Substituting the values of $R(1), R(2), R(3)$ in (4.2.3) we can obtain R_3, the reliability of a 3 cascade system.

Model II: Let $g_j(y)$ be the exponential densities with mean $1/\alpha_j$, $j = 1,2,\ldots,m$ respectively.

Then from (4.2.10) we get,

$$R'(i) = \prod_{j=1}^{m}\left(e^{-\frac{\alpha_j a_j}{k^{i-1}}} - e^{-\frac{\alpha_j b_j}{k^{i-1}}}\right); \quad i = 1,2,\ldots,n \tag{4.3.5}$$

Now from (4.3.5) using (4.2.4) we get the marginal reliabilities $R(1), R(2), R(3)$ as follows:

$$\begin{aligned} R(1) &= R'(1) \\ &= \prod_{j=1}^{m}\left(e^{-\alpha_j a_j} - e^{-\alpha_j b_j}\right) \end{aligned} \tag{4.3.6}$$

$$R(2) = \prod_{j=1}^{m}\left(e^{-\frac{\alpha_j a_j}{k}} - e^{-\frac{\alpha_j b_j}{k}}\right) - \prod_{j=1}^{m}\left(e^{-\alpha_j a_j} - e^{-\alpha_j b_j}\right)\prod_{j=1}^{m}\left(e^{-\frac{\alpha_j a_j}{k}} - e^{-\frac{\alpha_j b_j}{k}}\right) \tag{4.3.7}$$

$$\begin{aligned} R(3) = {} & \prod_{j=1}^{m}\left(e^{\frac{\alpha_j a_j}{k^2}} - e^{-\frac{\alpha_j b_j}{k^2}}\right) - \prod_{j=1}^{m}\left(e^{-\alpha_j a_j} - e^{-\alpha_j b_j}\right)\prod_{j=1}^{m}\left(e^{\frac{\alpha_j a_j}{k^2}} - e^{\frac{\alpha_j b_j}{k^2}}\right) - \prod_{j=1}^{m}\left(e^{-\frac{\alpha_j a_j}{k}} - e^{-\frac{\alpha_j b_j}{k}}\right) \\ & \prod_{j=1}^{m}\left(e^{-\frac{\alpha_j a_j}{k^2}} - e^{-\frac{\alpha_j b_j}{k^2}}\right) + \prod_{j=1}^{m}\left(e^{-\alpha_j a_j} - e^{-\alpha_j b_j}\right)\prod_{j=1}^{m}\left(e^{\frac{\alpha_j a_j}{k}} - e^{\frac{\alpha_j b_j}{k}}\right)\prod_{j=1}^{m}\left(e^{\frac{\alpha_j a_j}{k^2}} - e^{\frac{\alpha_j b_j}{k^2}}\right) \end{aligned} \tag{4.3.8}$$

Substituting the values of $R(1), R(2), R(3)$ from (4.3.6) to (4.3.8) in (4.2.3) we get R_3.

Model III: Let $g_j(y)$ be the exponential densities with means $1/\alpha_j$, $j = 1,2,\ldots,m$ respectively.

Then from (4.2.14) we get,

$$R'(i) = \prod_{j=1}^{m}\left(e^{-\frac{\alpha_j a_{ij}}{k^{i-1}}} - e^{-\frac{\alpha_j b_{ij}}{k^{i-1}}}\right); \quad i = 1,2,\ldots,n \tag{4.3.9}$$

Now from (4.3.9) and (4.2.4) we get the different marginal reliabilities as follows:

$$R(1)=\prod_{j=1}^{m}\left(e^{-\alpha_j a_{1j}}-e^{-\alpha_j b_{1j}}\right) \tag{4.3.10}$$

$$R(2)=\prod_{j=1}^{m}\left(e^{-\frac{\alpha_j a_{2j}}{k}}-e^{-\frac{\alpha_j b_{2j}}{k}}\right)-\prod_{j=1}^{m}\left(e^{-\alpha_j a_{1j}}-e^{-\alpha_j b_{1j}}\right)\prod_{j=1}^{m}\left(e^{-\frac{\alpha_j a_{2j}}{k}}-e^{-\frac{\alpha_j b_{2j}}{k}}\right) \tag{4.3.11}$$

$$R(3)=\prod_{j=1}^{m}\left(e^{-\frac{\alpha_j a_{3j}}{k^2}}-e^{-\frac{\alpha_j b_{3j}}{k^2}}\right)-\prod_{j=1}^{m}\left(e^{-\alpha_j a_{1j}}-e^{-\alpha_j b_{1j}}\right)\prod_{j=1}^{m}\left(e^{-\frac{\alpha_j a_{3j}}{k^2}}-e^{-\frac{\alpha_j b_{3j}}{k^2}}\right)-\prod_{j=1}^{m}\left(e^{-\frac{\alpha_j a_{2j}}{k}}-e^{-\frac{\alpha_j b_{2j}}{k}}\right)$$
$$\prod_{j=1}^{m}\left(e^{-\frac{\alpha_j a_{3j}}{k^2}}-e^{-\frac{\alpha_j b_{3j}}{k^2}}\right)+\prod_{j=1}^{m}\left(e^{-\alpha_j a_{1j}}-e^{-\alpha_j b_{1j}}\right)\prod_{j=1}^{m}\left(e^{-\frac{\alpha_j a_{2j}}{k}}-e^{-\frac{\alpha_j b_{2j}}{k}}\right)\prod_{j=1}^{m}\left(e^{-\frac{\alpha_j a_{3j}}{k^2}}-e^{-\frac{\alpha_j b_{3j}}{k^2}}\right)$$

(4.3.12)

Substituting the values of $R(1), R(2), R(3)$ in (4.2.3) we get R_3.

4.3.2 Rayleigh Stress-Strength Distribution

Model I: Let $f_i(x)$ and $g_j(y)$ be the Rayleigh densities with parameters σ_i^2 and β_j^2, $i=1,2,\ldots,n$; $j=1,2,\ldots,m$ respectively. Then from (4.2.5) we get,

$$R'(i)=\prod_{j=1}^{m}\frac{\sigma_i^2}{\sigma_i^2+k^{2i-2}\beta_j^2};\quad i=1,2,\ldots,n \tag{4.3.13}$$

Now from (4.3.13) using (4.2.4) we get the marginal reliabilities $R(1), R(2), R(3)$ as follows.

$$R(1)=R'(1)$$
$$=\prod_{j=1}^{m}\frac{\sigma_1^2}{\sigma_1^2+\beta_j^2} \tag{4.3.14}$$

$$R(2) = R'(2) - R'(1)R'(2)$$

$$= \prod_{j=1}^{m} \frac{\sigma_2^{\,2}}{\sigma_2^{\,2} + k^2\beta_j^{\,2}} - \prod_{j=1}^{m} \frac{\sigma_1^{\,2}\sigma_2^{\,2}}{\sigma_1^{\,2}\sigma_2^{\,2} + \sigma_2^{\,2}\beta_j^{\,2} + k^2\sigma_1^{\,2}\beta_j^{\,2}} \tag{4.3.15}$$

$$R(3) = R'(3) - R'(2)R'(3) - R'(1)R'(3) + R'(1)R'(2)R'(3)$$

$$= \prod_{j=1}^{m} \frac{\sigma_3^{\,2}}{\sigma_3^{\,2} + k^4\beta_j^{\,2}} - \prod_{j=1}^{m} \frac{\sigma_2^{\,2}\sigma_3^{\,2}}{\sigma_2^{\,2}\sigma_3^{\,2} + k^2\sigma_3^{\,2}\beta_j^{\,2} + k^4\sigma_2^{\,2}\beta_j^{\,2}} - \prod_{j=1}^{m} \frac{\sigma_1^{\,2}\sigma_3^{\,2}}{\sigma_1^{\,2}\sigma_3^{\,2} + \sigma_3^{\,2}\beta_j^{\,2} + k^4\sigma_1^{\,2}\beta_j^{\,2}}$$

$$+ \prod_{j=1}^{m} \frac{\sigma_1^{\,2}\sigma_2^{\,2}\sigma_3^{\,2}}{\sigma_1^{\,2}\sigma_2^{\,2}\sigma_3^{\,2} + \sigma_2^{\,2}\sigma_3^{\,2}\beta_j^{\,2} + k^2\sigma_1^{\,2}\sigma_3^{\,2}\beta_j^{\,2} + k^4\sigma_1^{\,2}\sigma_2^{\,2}\beta_j^{\,2}} \tag{4.3.16}$$

Substituting the values of $R(1), R(2), R(3)$ in (4.2.3) we can obtain R_3, the reliability of a 3 cascade system.

Model II: Let $g_j(y)$ be the Rayleigh densities with parameters $\beta_j^{\,2}$, $j = 1,2,\ldots,m$ respectively.

Then from (4.2.10) we get,

$$R'(i) = \prod_{j=1}^{m} \left(e^{-\frac{a_j}{k^{i-1}}} - e^{-\frac{b_j}{k^{i-1}}} \right); \quad i = 1,2,\ldots,n \tag{4.3.17}$$

Now from (4.3.17) using (4.2.4) we get the marginal reliabilities $R(1), R(2), R(3)$ as follows:

$$R(1) = R'(1)$$

$$= \prod_{j=1}^{m} \left(e^{-a_j} - e^{-b_j} \right) \tag{4.3.18}$$

$$R(2) = \prod_{j=1}^{m} \left(e^{-\frac{a_j}{k}} - e^{-\frac{b_j}{k}} \right) - \prod_{j=1}^{m} \left(e^{-a_j} - e^{-b_j} \right) \prod_{j=1}^{m} \left(e^{-\frac{a_j}{k}} - e^{-\frac{b_j}{k}} \right) \tag{4.3.19}$$

$$R(3)=\prod_{j=1}^{m}\left(e^{-\frac{a_j}{k^2}}-e^{-\frac{b_j}{k^2}}\right)-\prod_{j=1}^{m}\left(e^{-a_j}-e^{-b_j}\right)\prod_{j=1}^{m}\left(e^{-\frac{a_j}{k^2}}-e^{-\frac{b_j}{k^2}}\right)-\prod_{j=1}^{m}\left(e^{-\frac{a_j}{k}}-e^{-\frac{b_j}{k}}\right)$$
$$\prod_{j=1}^{m}\left(e^{-\frac{a_j}{k^2}}-e^{-\frac{b_j}{k^2}}\right)+\prod_{j=1}^{m}\left(e^{-a_j}-e^{-b_j}\right)\prod_{j=1}^{m}\left(e^{-\frac{a_j}{k}}-e^{-\frac{b_j}{k}}\right)\prod_{j=1}^{m}\left(e^{-\frac{a_j}{k^2}}-e^{-\frac{b_j}{k^2}}\right) \tag{4.3.20}$$

Substituting the values of $R(1), R(2), R(3)$ from (4.3.18) to (4.3.20) in (4.2.3) we get R_3.

Model III: Let $g_j(y)$ be the Rayleigh densities with parameters $\beta_j^{\,2}$, $j=1,2,\ldots,m$ respectively.

Then from (4.2.14) we get,

$$R'(i)=\prod_{j=1}^{m}\left(e^{-\frac{a_{ij}}{k^{i-1}}}-e^{-\frac{b_{ij}}{k^{i-1}}}\right);\quad i=1,2,\ldots,n \tag{4.3.21}$$

Now from (4.3.21) and (4.2.4) we get the different marginal reliabilities as follows:

$$R(1)=\prod_{j=1}^{m}\left(e^{-a_{1j}}-e^{-b_{1j}}\right) \tag{4.3.22}$$

$$R(2)=\prod_{j=1}^{m}\left(e^{-\frac{a_{2j}}{k}}-e^{-\frac{b_{2j}}{k}}\right)-\prod_{j=1}^{m}\left(e^{-a_{1j}}-e^{-b_{1j}}\right)\prod_{j=1}^{m}\left(e^{-\frac{a_{2j}}{k}}-e^{-\frac{b_{2j}}{k}}\right) \tag{4.3.23}$$

$$R(3)=\prod_{j=1}^{m}\left(e^{-\frac{a_{3j}}{k^2}}-e^{-\frac{b_{3j}}{k^2}}\right)-\prod_{j=1}^{m}\left(e^{-a_{1j}}-e^{-b_{1j}}\right)\prod_{j=1}^{m}\left(e^{-\frac{a_{3j}}{k^2}}-e^{-\frac{b_{3j}}{k^2}}\right)-\prod_{j=1}^{m}\left(e^{-\frac{a_{2j}}{k}}-e^{-\frac{b_{2j}}{k}}\right)$$
$$\prod_{j=1}^{m}\left(e^{-\frac{a_{3j}}{k^2}}-e^{-\frac{b_{3j}}{k^2}}\right)+\prod_{j=1}^{m}\left(e^{-a_{1j}}-e^{-b_{1j}}\right)\prod_{j=1}^{m}\left(e^{-\frac{a_{2j}}{k}}-e^{-\frac{b_{2j}}{k}}\right)\prod_{j=1}^{m}\left(e^{-\frac{a_{3j}}{k^2}}-e^{-\frac{b_{3j}}{k^2}}\right) \tag{4.3.24}$$

Substituting the values of $R(1), R(2), R(3)$ in (4.2.3) we get R_3.

4.4 Graphical Representations

Some graphs are plotted in **Fig.** 4.1(a) - 4.1(b), **Fig.** 4.2(a) - 4.2(b), **Fig.** 4.3(a) - 4.3(b), **Fig.** 4.4(a) - 4.4(b), **Fig.** 4.5(a) - 4.5(b) and **Fig.** 4.6(a) - 4.6(b) by taking different parameters along the horizontal axis and the corresponding reliabilities along the vertical axis. **Fig.** 4.1(a) - 4.1(b) represents the curves for $\theta_2 = 1, \theta_3 = 2$ and it is seen that reliabilities decrease steadily with increasing k. **Fig.** 4.2(a)-4.2(b) represents the values for $a_1 = 0.1, a_2 = 0.2, a_3 = 0.3 = 1, b_1 = b_2 = b_3 = 4, \alpha_1 = \alpha_2 = \alpha_3 = 0.2$ respectively. One can read the values of $R(1), R(2), R(3)$ and R_3 for intermediate values of k, from these graphs of exponential distribution. In **Fig.** 4.3(a) - 4.3(b), some graphs of reliabilities against k are plotted for some fix values of $a_{11} = a_{12} = a_{13} = a_{21} = a_{22} = a_{23} = 0.1$, $b_{11}, b_{12}, b_{13}, b_{21}, b_{22}, b_{23}, b_{31}, b_{32}, b_{33}$ and $\alpha_1 = \alpha_2 = \alpha_3 = 0.2$. It is to be observed that, reliability steadily decreases with increasing k. Again **Fig.** 4.4(a) - 4.4(b) represents the curves for $\sigma_2 = 1, \sigma_3 = 2$ and it is seen that reliability decreases steadily with increasing k. **Fig.** 4.5(a) - 4.5(b) represents the different values of a_1, a_2, b_1, b_2 respectively. One can read the values of $R(1), R(2), R(3)$ and R_3 for intermediate values of k, from these graphs of Rayleigh distribution. Similarly **Fig.** 4.6(a) - 4.6(b) it is seen that some graphs of reliabilities against k are plotted for some fix values of $a_{11}, a_{12}, a_{21}, a_{22}, a_{31}, a_{32}$ and $b_{11}, b_{12},, b_{21}, b_{22}, b_{31}, b_{32}$. It is to be observed that, $R(2), R(3)$ and R_3 are steadily increases with increasing k but the graph of $R(1)$ seems to be a straight line since it is independent of k.

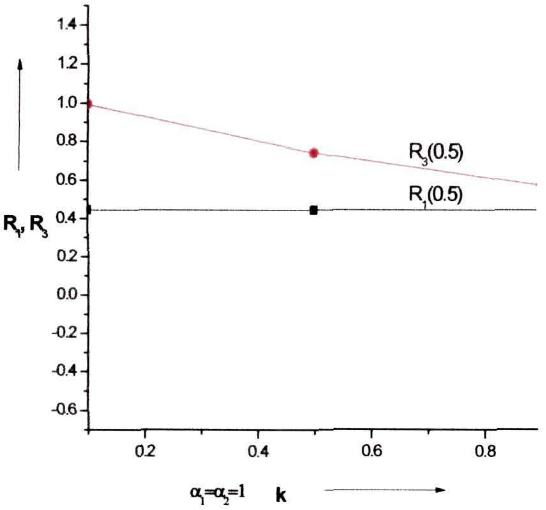

Fig. 4.1(a) Exponential Stress-Strength for model I:
Graph for R_1, R_3 for fixed values of θ_1

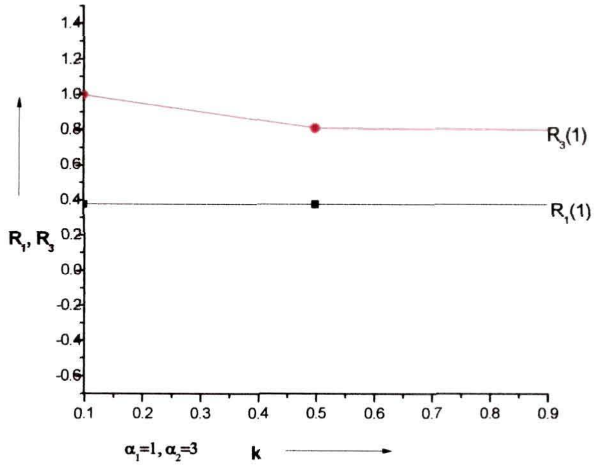

Fig. 4.1(b) Exponential Stress-Strength for model I:
Graph for R_1, R_3 for fixed values of θ_1

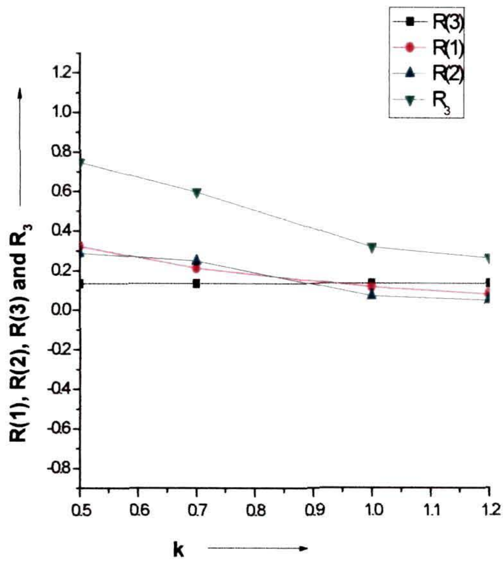

Fig. 4.2(a) Exponential Stress-Strength for model II:
Graph for $R(1)$, $R(2)$, $R(3)$, R_3

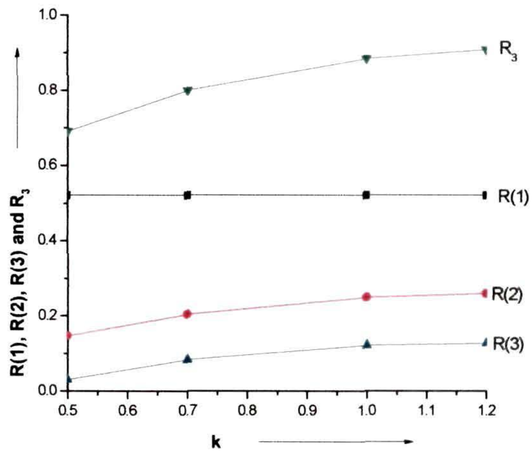

Fig. 4.2(b) Exponential Stress-Strength for model II:
Graph for $R(1)$, $R(2)$, $R(3)$, R_3

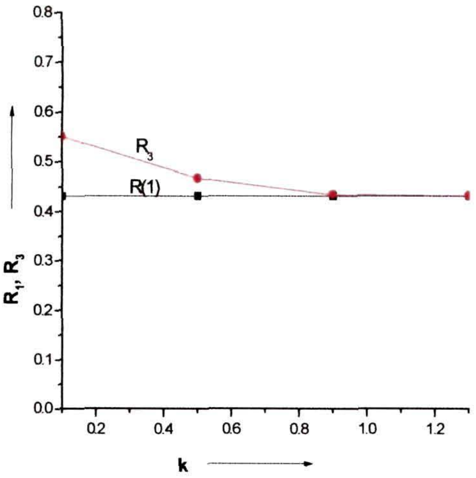

Fig. 4.3(a) Exponential Stress-Strength for model III: Graph for R_1, R_3.

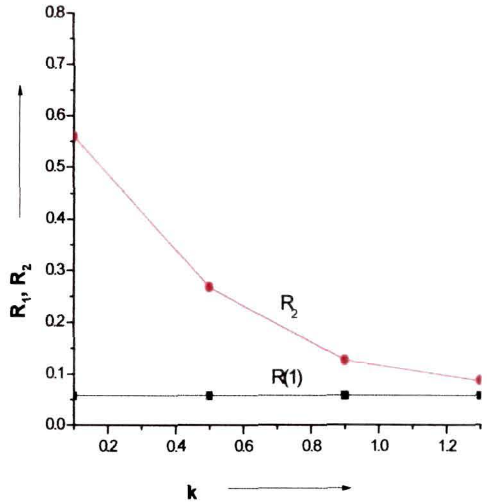

Fig. 4.3(b) Exponential Stress-Strength for model III: Graph for R_1, R_2

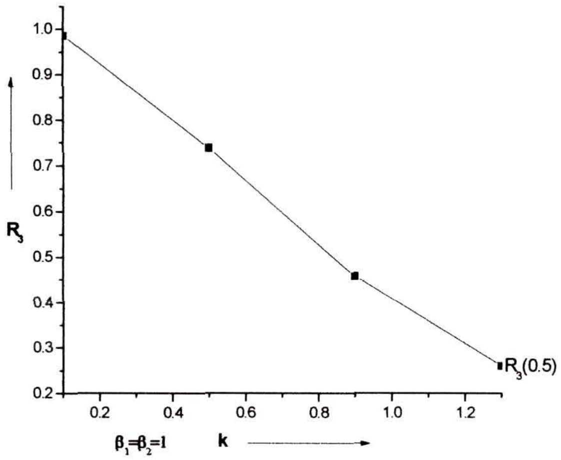

Fig. 4.4(a) Rayleigh Stress-Strength for model I: Graph for R_3 for fixed values of σ_1

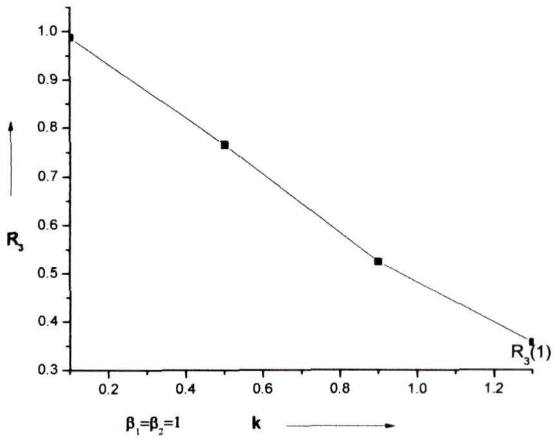

Fig. 4.4(b) Rayleigh Stress-Strength for model I: Graph for R_3 for fixed values of σ_1

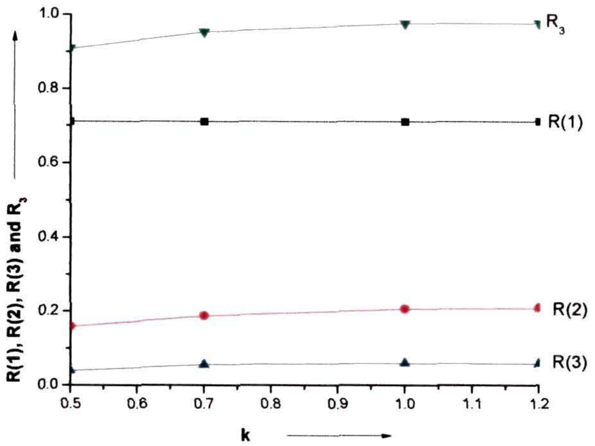

Fig. 4.5(a) Rayleigh Stress-Strength for model II:
Graph for $R(1)$, $R(2)$, $R(3)$, R_3

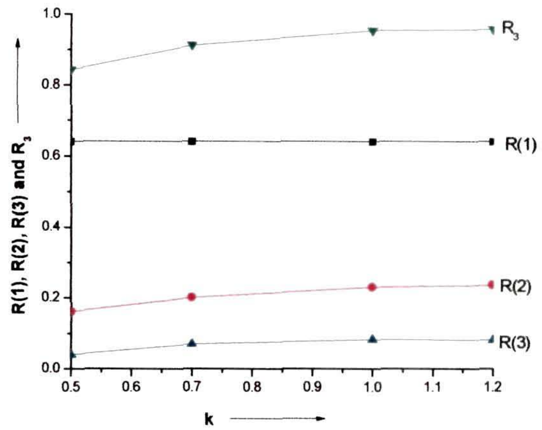

Fig. 4.5(b) Rayleigh Stress-Strength for model II:
Graph for $R(1)$, $R(2)$, $R(3)$, R_3

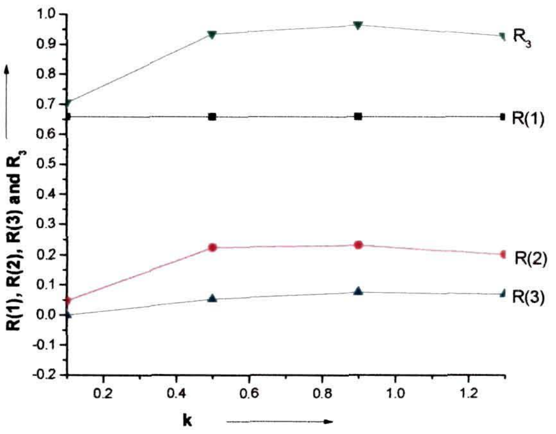

Fig. 4.6(a) Rayleigh Stress-Strength for model III:
Graph for $R(1)$, $R(2)$, $R(3)$, R_3

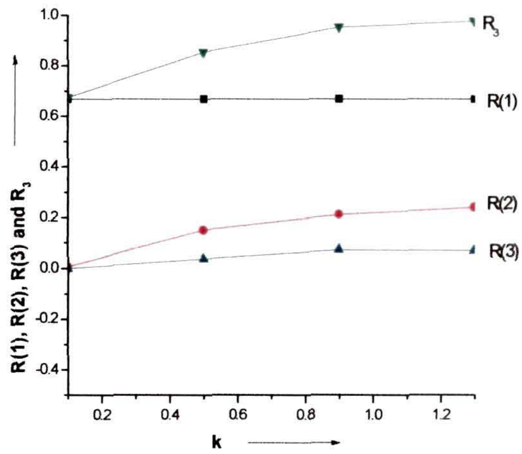

Fig. 4.6 (b) Rayleigh Stress-Strength for model III:
Graph for $R(1)$, $R(2)$, $R(3)$, R_3

4.5 Results and Discussions

From the **Table** 4.1 we observe that when k increases, α and θ remaining the same, $R(2)$ decreases rapidly whereas there is slight decreases in $R(3)$ which results in decrease in R_3. For example, when k=0.1, α_1=1, $\alpha_2 = 1.0$ and $\theta_1 = 0.5, \theta_2 = 1.0, \theta_3 = 2.0$ we find $R(2)$=0.4358, $R(3)$=0.1122 and system reliability R_3=0.9925 but k=0.5, for same set of parameter values $R(2)$=0.1944, $R(3)$=0.1044 and R_3=0.7433. When θ_1 increase $R(1)$ increases. Similar conclusion may be drawn for increase in θ_2 and θ_3. But when θ_1 and θ_2 increases we find increase in the values of $R(i)$; i=1,2,3 and consequently R_3.

From the **Table** 4.2, we see that $R(i)$, i=2,3 increase when k increases. Also change in values of α_i, i=1,2,3 effects all $R(i)$. When any α_i increases (i.e., mean stress decreases), keeping the limits fixed $R(i)$ increases but there is decrease in $R(2)$ and $R(3)$. For instance when $a_1 = 0.1, b_1 = 4.0, a_2 = 0.2, b_2 = 4.0, a_3 = 0.3, b_3 = 4.0, \alpha_1 = 0.2$ $\alpha_2 = 0.2, \alpha_3 = 0.2$, then $R(1)$=0.1337, $R(2)$=0.3248, $R(3)$=0.2891 and keeping all other parameters fixed if $\alpha_1 = 0.1, \alpha_2 = 1.0, \alpha_3 = 1.0$, we get $R(1)$=0.5217, $R(2)$=0.1466, $R(3)$=0.0309. When any of the upper limits decreases, $R(i)$, i=2,3 will decrease.

From the **Table** 4.3, we notice that all the limits (a_{ij}, b_{ij}), i,j=1,2,3 and stress parameter α_i, i=1,2,3 remaining the same R_3 increases when $k \leq 0.5$ and R_3 decrease when k>0.5. In general we see that when α_i (i=1, 2, 3) increases, all $R(i)$ will increase. For instance when $\alpha_1 = \alpha_2 = \alpha_3 = 0.2$, a_{ij}=0.1, i,j=1,2,3; $b_{11} = b_{21} = b_{31}$=2.0 and $b_{12} = b_{13} = b_{22} = b_{23} = b_{32} = b_{33}$ =3, $R(1)$=0.5767, $R(2)$=0.5026, $R(3)$=0.0011. Whereas when $\alpha_1 = \alpha_2 = \alpha_3 = 0.5$, other values remaining constant, $R(1)$=0.3093, $R(2)$=0.1541, $R(3)$=0.0001. Also it is observed that keeping all other parameters fixed $R(i)$ can be increased, by increasing b_{ij} values.

From the **Table** 4.4, we observe that when k increases, σ and β remaining the same, $R(2)$ decreases rapidly whereas there is slight increases in $R(3)$ which results in decrease in R_3. For example, when k=0.1, β_1=1, $\beta_2 = 1.0$ and $\sigma_1 = 0.5, \sigma_2 = 1.0, \sigma_3 = 2.0$ we find $R(2)$=0.8290, $R(3)$=0.0048 and system reliability R_3=0.9857 but k=0.5, for same set of parameter values $R(2)$=0.4949, $R(3)$=0.0826 and R_3=0.7375. When σ_1 increase $R(1)$ increases. Similar conclusion may be drawn for increase in σ_2 and σ_3. But when σ_1 and σ_2 increase we find increase in the values of $R(i)$; i=1,2,3 and consequently R_3.

From the **Table** 4.5, we see that $R(i)$, i=2,3 increase when k increases. For instance when $a_1 = 0.1, b_1 = 4.0, a_2 = 0.2, b_2 = 4.0$, then $R(1)$=0.7096, $R(2)$=0.1592, $R(3)$=0.0395. When any of the upper limits increases, $R(i)$ (i=2,3) will decrease some times and similarly when any of the lower limits increases, $R(i)$, (i=2,3) will decrease.

From the **Table** 4.6, we notice that all the limits (a_{ij}, b_{ij}), i=1,2,3 and j=1,2 remaining the same R_3 increases when $k \leq 0.9$ and R_3 decreases when $k > 0.9$. Also it is observed that keeping all other parameters fixed, $R(i)$, i=2,3 can be decreased, by increasing b_{ij} values and also $R(i)$, i=2,3 and R_3 can be decreased, by increasing a_{ij} values.

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Chapter 5

A Cascade Model with Random Parameters

Chapter 5

A Cascade Model with Random Parameters

5.1 Introduction

Most of the discussions of interference models assume that the parameters of stress and strength distributions are constants (Beg, 1980, Enis and Geisser, 1971, Harris and Singpurwalla, 1968, Kelley et al., 1976 etc.). But in many cases this assumption may not be true and the parameters may be assumed themselves (parameters) to be random variables. In other words, the distributions with fixed parameters may not represent the stress and or strength distributions adequately; distributions with random parameters may model the situations better. For example, if a particular component, having certain strength distribution is manufactured in different lots, for a particular lot the parameters of the strength distribution may remain fixed but may vary randomly from lot to lot. In such situations the parameters of the strength distribution may themselves be taken as random variables. Similarly, stress applied on a component (or system) is due to different factors such as vibration, pressure, temperature, humidity etc. Generally one of these factors will be dominant and will be the main cause for the stress on the component and stress distribution will be the distribution of this factor. But the other factors may vary at different time or at different places in such a way that, though they may not alter the form of the distribution, they may bring random fluctuations in the values of the parameters of the stress distribution. For example, solutions corrosive action may be highly influenced by variation in its temperature (Kakati, 1983) and hence the distribution of stress (corrosive action) may have different parametric values which vary randomly with temperature or in other words, the stress parameter may be taken as a random variable.

Further, if a prior knowledge exists about the parameters involved, it will be a waste of available data if we do not use a random parameter model i.e. a Bayesian model. In order to use the Bayesian approach the subjective information must be quantified and represented in the form of a prior distribution of the parameter concerned (Kapur and Lamberson, 1977).

Harris and Singpurwalla (1968) have derived an unconditional time to failure distribution by assuming that a parameter of the failure distribution (viz. exponential or Weibull) is a random variable. Using the derived compound distribution and Bayesian techniques they have estimated the system's reliability. They considered two-point, uniform and gamma distributions as prior distributions. Krishnamoorthy et al. (2007) proposed in estimating the parameter R which is referred to as the reliability parameter. i.e. $R = (X > Y)$, where its strength X and stress Y are independent random variables.

Shooman (1968) has assumed that the parameter of strength distribution is a deterministic function of time. Tarman and Kapur (1975) have assumed that the parameters of the stress-strength distributions are variables but not random variables. They have found optimal values of the parameters involved subject to resource and design constraints.

In this chapter we have calculated the system reliability of a 2-cascade system when the parameters of the stress-strength distributions are random variable. Here stress and strength are considered to be exponential random variable with parameters, say λ and μ, respectively. We further assume that either λ or μ is a random variable with a known prior. The following two cases are considered:

- When strength parameter is random but stress parameter is a constant.
- When stress parameter is random but strength parameter is a constant.

The prior distributions considered for stress-strength parameters, in all the above two cases, are either uniform or two-point distributions.

In Section-5.2 the general model is formulated. In Section- 5.3 to 5.4 the above two cases are considered. The reliabilities of a cascade system are evaluated in each case. For all the above cases, some numerical values of these reliabilities are tabulated in the **Table** 5.1, **Table** 5.2, **Table** 5.3 and **Table** 5.4 (cf. Appendix) for different set of values of the parameters. To make the things clear, a few graphs are drawn in Section 5.5 for selected values of the parameters. Results and discussions are devoted to Section 5.6.

5.2 Notations and Formulation of the Model

Here we have assumed that strength 'X' and stress 'Y' are exponential variates with means $1/\lambda$ and $1/\mu$, respectively. The parameters λ and μ may be random. Let X_r's be i.i.d. with distribution function $F(x)$ and let p.d.f. of Y_1 be $g(y_1)$

$$R(r) = \int_{-\infty}^{\infty} \left[F(y_1)F(ky_1)F(k^2 y_1)...F(k^{r-1} y_1)\right] g(y_1)\,dy_1 \tag{5.2.1}$$

Let

$P(\lambda), p(\lambda)$ = The prior distribution function and p.d.f. (or p.m.f.) of the random strength parameter λ.

$Q(\mu), q(\mu)$ = The prior distribution function and p.d.f. (or p.m.f.) of the random stress parameter μ.

$f(x/\lambda)$= The conditional p.d.f. of the r.v. X for a given value of λ.

$g(y/\mu)$=The conditional p.d.f. of the r.v. Y for a given value of μ.

$f_x(x)$= The unconditional p.d.f. of the r.v. X.

$g_y(y)$= The unconditional p.d.f. of the r.v. Y.

Here $f(x/\lambda) = f_x(x)$ if λ is constant and $g(y/\mu) = g_y(y)$ if μ is constant.

Now

$$f_X(x) = \int_{-\infty}^{\infty} f(x/\lambda)\, dP(\lambda) \tag{5.2.2}$$

$$g_Y(y) = \int_{-\infty}^{\infty} g(y/\mu)\, dQ(\mu) \tag{5.2.3}$$

Then from (5.2.1)

$$R(1) = \int_{-\infty}^{\infty} \overline{F_X}(y_1) g_Y(y_1)\, dy_1 \tag{5.2.4}$$

$$R(2) = \int_{-\infty}^{\infty} F_X(y_1)\overline{F_X}(ky_1) g_Y(y_1)\, dy_1 \tag{5.2.5}$$

5.3 Random Strength Parameter

Suppose the components under study are manufactured in different lots so that its strength distribution may be represented by distribution with random parameters. But are used in the same environment exerting similar stresses, of course with random fluctuations, i.e. stress distribution is having constant parameter. As already assumed stress-strength are exponential with parameter μ and λ, so in this case the strength parameter λ is a r.v. whereas the stress parameter μ remains constant.

i.e.

$$f(x/\lambda) = \lambda e^{-\lambda x} \quad and \quad g_Y(y) = \mu e^{-\mu y}$$

Two types of prior distributions are considered for λ :

(a) uniform and (b) two-point

(a) Uniform Prior for λ

In a situation where each lot is homogeneous within itself but different lots may have different values of λ and taking all the possible sources of the lots together, each value of λ

appears equally frequently, a uniform prior distribution will be suitable for λ (Harris and Singpurwalla, 1968).

Let λ be uniformly distributed in the range (a,b)

$$i.e.,\ p(\lambda) = \frac{1}{(b-a)},\ a < \lambda < b$$

Then the unconditional p.d.f of X is given by

$$f_x(x) = \frac{1}{(b-a)} \int_a^b \lambda e^{-\lambda x} d\lambda$$

Hence,

$$R(1) = \int_0^\infty \left[\int_y^\infty \frac{1}{b-a} \int_a^b \lambda e^{-\lambda x} d\lambda\, dx \right] \mu e^{-\mu y} dy$$

$$= \frac{\mu}{b-a} \log_e \frac{b+\mu}{a+\mu}$$

$$R(2) = \int_0^\infty F_X(y_1) \overline{F_X}(ky_1) g_Y(y_1) dy_1$$

$$= \int_0^\infty \left[1 - \frac{1}{b-a} \frac{e^{-ay_1} - e^{-by_1}}{y_1} \right] \left[\frac{1}{b-a} \frac{e^{-aky_1} - e^{-bky_1}}{ky_1} \right] \mu e^{-\mu y_1} dy_1$$

$$= \frac{\mu}{k(b-a)} \log_e \frac{bk+\mu}{ak+\mu} - \frac{\mu}{k(b-a)^2} \int_0^\infty \frac{1}{y^2} \left[e^{-y(a+ak+\mu)} - e^{-y(b+ak+\mu)} - e^{-y(a+bk+\mu)} + e^{-y(b+bk+\mu)} \right] dy$$

$$= \frac{\mu}{k(b-a)} \log_e \frac{bk+\mu}{ak+\mu} - \frac{\mu}{k(b-a)^2} \eta_1$$

where $\eta_1 = \int_0^\infty \frac{1}{y^2} \left[e^{-y(a+ak+\mu)} - e^{-y(b+ak+\mu)} - e^{-y(a+bk+\mu)} + e^{-y(b+bk+\mu)} \right] dy$

The expression η_1 can be evaluated numerically.

Then the reliability R_2 for a 2-cascade system, from the equation (3.2.8), is given by

$R_2 = R(1) + R(2)$

Table 5.1 (cf. Appendix) gives a few values of R_1 and R_2 for different values of parameters μ, a, b and attenuation factor k.

(b) Two- Point prior for λ

In a situation where it is known that λ can take two only values λ_1 and λ_2 (say) with probabilities p and $(1-p)$, respectively, a two point prior distribution for λ is appropriate (Harris and Singpurwalla,1968).

Let λ have a two-point distribution, given by,

$$\Pr(\lambda = \lambda_1) = p(\lambda_1) \; and \; \Pr(\lambda = \lambda_2) = p(\lambda_2)$$

Then

$$f_x(x) = \sum_{i=1}^{2} f(x/\lambda_i)p(\lambda_i) = p\lambda_1 e^{-\lambda_1 x} + (1-p)\lambda_2 e^{-\lambda_2 x} \tag{5.3.1}$$

Hence

$$R(1) = \left(\frac{p}{\lambda_1 + \mu} + \frac{1-p}{\lambda_2 + \mu}\right)\mu$$

$$R(2) = \frac{p\mu}{\mu + \lambda_1 k} - \frac{p^2\mu}{\mu + \lambda_1 + \lambda_1 k} - \frac{\mu p(1-p)}{\mu + \lambda_1 k + \lambda_2} + \frac{\mu(1-p)}{\mu + \lambda_2 k} - \frac{\mu p(1-p)}{\mu + \lambda_1 + \lambda_2 k} - \frac{\mu(1-p)^2}{\mu + \lambda_2 + \lambda_2 k}$$

Then $R_2 = R(1) + R(2)$

A few values of R_1 and R_2 are tabulated in **Table** 5.2 (cf. Appendix) for different values of parameters $\mu, p, \lambda_1, \lambda_2$ and attenuation factor k.

5.4 Random Stress Parameter

The situation may be opposite of that considered in Section 5.3. That is, the components might have come from the same lot or otherwise also the strength parameters may not vary from one lot to another. In other words, the parameter of component's strength distribution remains constant. But as discussed earlier the stress on the component may not suitably be represented by a constant parameter distribution. So we shall consider a stress distribution with random parameters. Let, as in Section 5.3, X and Y be both exponential with parameter λ and μ i.e. stress and strength are exponential with mean $1/\mu$ and $1/\lambda$ respectively. But now μ is a random variable and λ remains constant.

i.e. $g(y/\mu) = \mu e^{-\mu y}$ and $f_X(x) = \lambda e^{-\lambda x}$

For μ also the prior distributions considered are uniform and two-point distributions, respectively, in the following sub-sections.

(a) Uniform Prior for μ

Similar situations, as that in case of λ, may be envisaged for the use of this distribution as the prior distribution for μ also. For example, the components may be working in such an environment where the values taken by μ in a given range are equally likely, and then we can assume a uniform distribution for μ.

Let μ be distributed uniformly in the range (c, d), then

$$q(\mu) = \frac{1}{d-c}, \; c \le \mu \le d$$

Then, $g_Y(y) = \frac{1}{d-c}\int_c^d \mu e^{-\mu y} d\mu$

Hence,

$$R(1) = 1 - \frac{\lambda}{d-c}\log_e \frac{d+\lambda}{c+\lambda}$$

$$R(2) = \frac{\lambda}{d-c}\left[k \log\frac{d+\lambda k+\lambda}{c+\lambda k+\lambda} + \log\frac{d+\lambda k+\lambda}{c+\lambda k+\lambda} - k\log\frac{d+\lambda k}{c+\lambda k}\right]$$

Then $R_2 = R(1) + R(2)$

For different values of the parameters the reliabilities of R_1 and R_2 are tabulated in **Table** 5.3 (cf. Appendix).

(b) Two- Point prior for μ

Similar to Section 5.3(b), if it is known that μ can take only two values μ_1 and μ_2 with probabilities q and $(1-q)$, respectively, then we have two-point prior distributions for μ given as,

$$\Pr(\mu = \mu_1) = q(\mu_1) \ \textit{and} \ \Pr(\mu = \mu_2) = q(\mu_2)$$

Then,

$$g_Y(y) = \sum_{j=1}^{2} g(y/\mu_j)q(\mu_j) = q\mu_1 e^{-\mu_1 y} + (1-q)\mu_2 e^{-\mu_2 y} \tag{5.4.1}$$

Hence,

$$R(1) = \frac{q\mu_1}{\mu_1 + \lambda} + \frac{(1-q)\mu_2}{\mu_2 + \lambda}$$

$$R(2) = q\mu_1\left(\frac{1}{\mu_1 + \lambda k} - \frac{1}{\mu_1 + \lambda + \lambda k}\right) + (1-q)\mu_2\left(\frac{1}{\mu_2 + \lambda k} - \frac{1}{\mu_2 + \lambda + \lambda k}\right)$$

and as usual $R_2 = R(1) + R(2)$

Table 5.4 (cf. Appendix) gives a few values of R_1 and R_2 for different values of parameters μ_1, μ_2, λ, q and attenuation factor k.

5.5 Graphical Representations

A few graphs of R_1, R_2 are drawn in **Fig.** 5.1(a), **Fig.** 5.1(b), **Fig.** 5.2(a), **Fig.** 5.2(b), **Fig.** 5.3(a), **Fig.** 5.3(b), **Fig.** 5.4(a) and **Fig.** 5.4(b) for different parametric values involved. From these graphs one can read directly the values of reliabilities R_1, R_2 for intermediate values of μ, λ and k. In **Fig.** 5.1(a), **Fig.** 5.1(b), **Fig.** 5.2(a) and **Fig.** 5.2(b) reliabilities are steadily increasing with μ and k whereas in **Fig.** 5.3(a), **Fig.** 5.3(b), **Fig.** 5.4(a) and **Fig.** 5.4(b) it is decreasing with increasing λ and k.

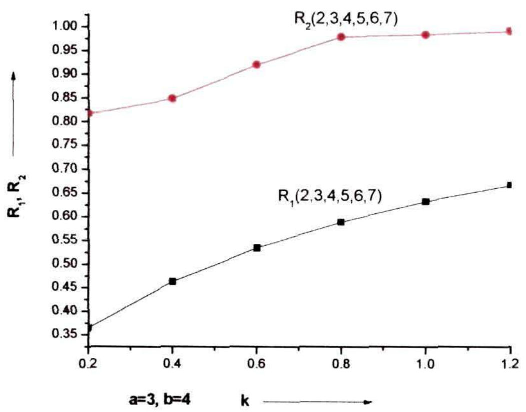

Fig. 5.1(a) Graph of R_1, R_2 for exponential Stress-Strength: Strength parameter λ is random and uniformly distributed in the range (a,b)

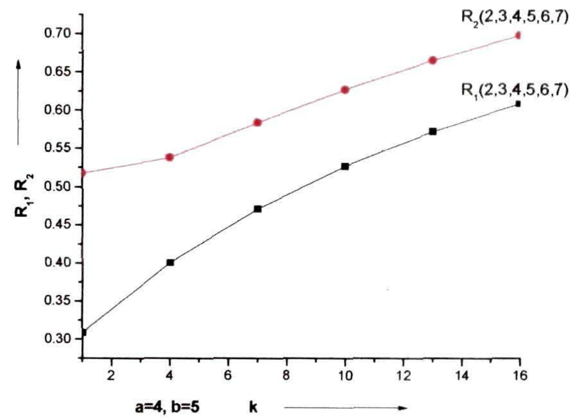

Fig. 5.1(b) Graph of R_1, R_2 for exponential Stress-Strength: Strength parameter λ is random and uniformly distributed in the range (a,b)

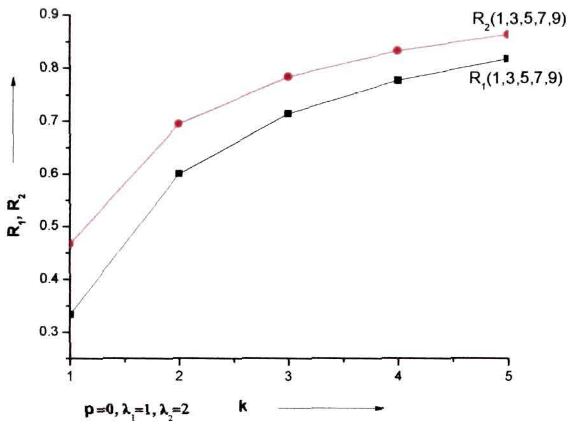

Fig. 5.2(a) Graph of R_1, R_2 for exponential Stress-Strength: Strength parameter λ is random and has a two-point distribution

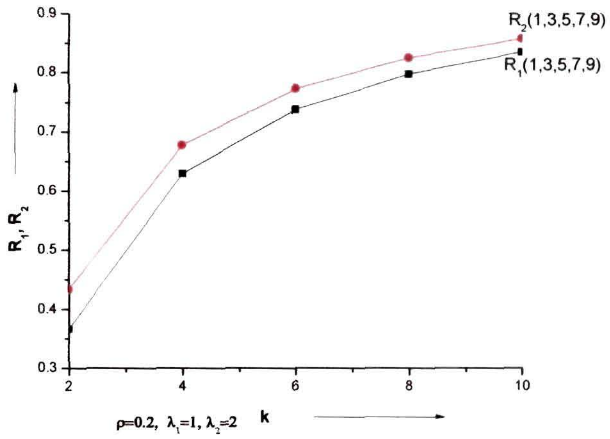

Fig. 5.2(b) Graph of R_1, R_2 for exponential Stress-Strength: Strength parameter λ is random and has a two-point distribution

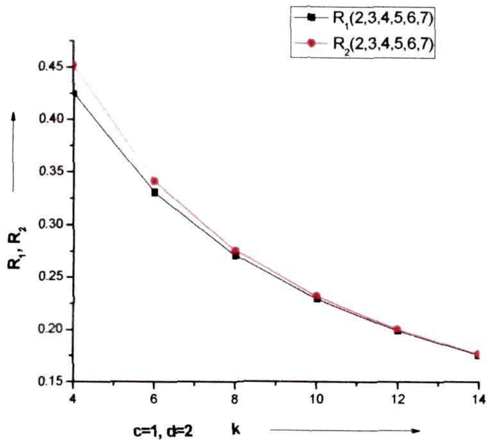

Fig. 5.3(a) Graph of R_1, R_2 for exponential Stress-Strength: Stress parameter μ is random and uniformly distributed in the range (c, d)

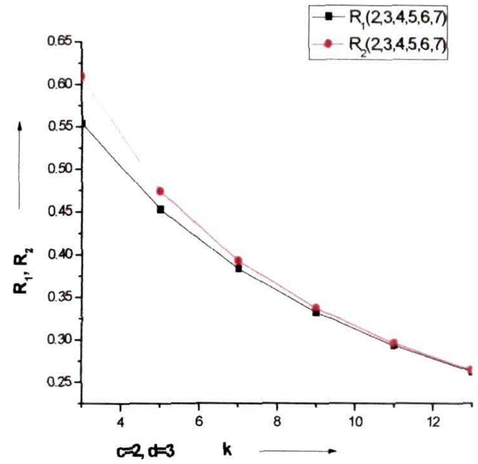

Fig. 5.3(b) Graph of R_1, R_2 for exponential Stress-Strength: Stress parameter μ is random and uniformly distributed in the range (c, d)

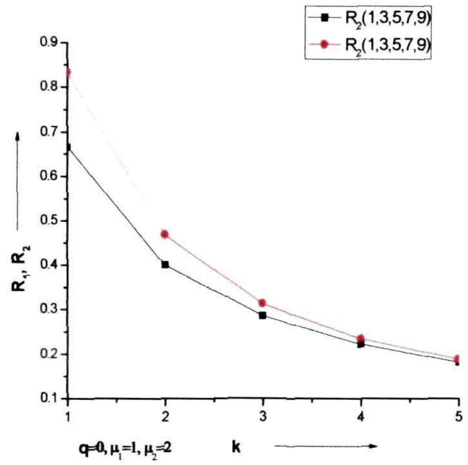

Fig. 5.4(a) Graph of R_1, R_2 for exponential Stress-Strength: Stress parameter μ is random and has a two-point distribution

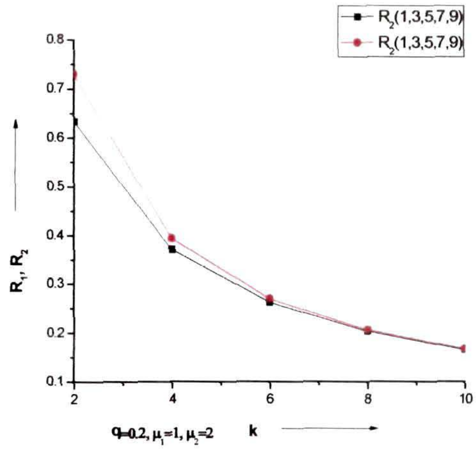

Fig. 5.4(b) Graph of R_1, R_2 for exponential Stress-Strength: Stress parameter μ is random and has a two-point distribution

5.6 Results and Discussions

A few values of R_1 and R_2 are tabulated for the 1st case when strength parameter is random but stress parameter is a constant. Uniform distributions are considered as a prior distribution for λ in **Table** 5.1 (cf. Appendix) for different values of μ, a, b and attenuation factor k. From the table we have seen that reliabilities are steadily increasing with μ and k but decrease with increasing value of a and b. Similarly in **Table** 5.2 (cf. Appendix), two-point distributions are considered as the prior distribution for λ for the 1st case we have tabulated some values of R_1 and R_2 for different values of $\mu, p, \lambda_1, \lambda_2$ and attenuation factor k. Here also we have seen that the reliabilities are steadily increasing with μ and k.

For different values of c, d, k and λ we have tabulated the values of R_1 and R_2 in **Table** 5.3 (cf. Appendix) in the 2nd case when stress parameter is random but strength parameter is a constant and uniform distributions are considered as a prior distribution for μ. The values of the reliability are on expected line. From the table we see that, increase in the values of λ and k decrease the reliability but reliabilities are steadily increasing with c and d. Similarly, when two-point distributions are considered as the prior distribution for μ in the 2nd case we have tabulated some values of R_1 and R_2 for different values of μ_1, μ_2, λ, q and attenuation factor k in **Table** 5.4 (cf. Appendix). Here also we have seen that the reliabilities are decreasing with increasing values of q, λ and k.

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Chapter 6

Cascade System with $P(X<Y<Z)$

Chapter 6

Cascade System with P(X<Y<Z)

6.1 Introduction

In Stress-Strength models, generally, a component fails when stress (Y) on it exceeds its strength (X) and the reliability, R, of the component is given by the equation (1.1.1). In this chapter we have considered an n-cascade system. In literature this cascade system has been considered by many authors viz. Pandit and Sriwastav (1978), Sriwastav and Kakaty (1981), Bhowal (1999), Rekha et.al (1988), Beg (1980), Gopalan and Venkateswarlu (1982), Pandit and Sriwastav (1975), Sriwastav and Kakati (1980) etc.

But sometimes a component (or system) can work only when the stress Y on it is not only less than certain values, say Z, but also must be greater than some other value, say X, i.e. stress is within certain limits. These limits may or may not be constants. For example, many electronic components cannot work at very high voltage or at a very low voltage. Here the limits are generally constant for particular equipment. Similarly a person's blood pressure has two limits-systolic and diastolic pressures. For a healthy person his blood pressure must lie within these two limits. Both these pressure may vary within certain ranges beyond which a person cannot survive.

The reliability of a component (or system) under such a situation may be defined as

$$R = P(X < Y < Z) \tag{6.1.1}$$

where Y is the stress on the component and X and Z may termed as strengths. We shall call them lower and upper strengths. These X, Y and Z are random variables.

Singh (1980) has considered the estimation in stress-strength model under the assumption that strength of a component lies in an interval and estimates the probability

$$R = P(X_1 < Y < X_2)$$

where X_1 and X_2 are independent random stress variables and Y independent of X_1 and X_2 is random strength variable. Chandra and Owen (1975) obtained the estimation of reliability of a component subjected to several different stresses. They obtained the estimate $R = P[Max(Y_1, Y_2, ..., Y_k) < X]$ when $(Y_1, Y_2, ..., Y_k)$ are i.i.d. normal distributions and X as an independent normal distribution. Hanagal (1997) has estimated the reliability of a component subjected to two different stresses which are independent of the strength of a component.

Some of the results from this chapter have been accepted for publication in 'Journal of Informatics and Mathematical Sciences'. Here we have considered an n-cascade system with this model. We have not come across any study where cascade model is considered for such a model. The main aim of this chapter is to obtain the system reliability R_n for this model where stress on the component is subjected to two strengths.

This chapter is organized as follows: In Section-6.2 the general model is developed for an n-cascade system. In Section-6.3 the reliability expressions of an n-cascade system is obtained when the stress-strength of the components follow particular distributions. In Sub-Section 6.3.1 to 6.3.5 the expressions of R_n, is obtained when stress-strength are either exponential or Rayleigh or Lindley and when both strengths are one-parameter exponential and stress follows Lindley and when both strengths are one-parameter exponential and stress follows two-parameters gamma distributions. In Section 6.4 some graphs are plotted for selected values of the parameters to facilitate the direct reading of reliability. Some numerical values of reliabilities $R(1), R(2), R(3)$ and R_3 are tabulated in **Table** 6.1, **Table** 6.2, **Table** 6.3, **Table** 6.4 and **Table** 6.5 (cf. Appendix) for each cases and some results and discussions are given in Section-6.5.

6.2 Mathematical Formulation

Let us consider a system with n components working under the impact of stresses. Let X_i and Z_i be the lower and upper strengths, respectively, on the i[th] component and Y_i be the stress on it, $i=1,2,\ldots,n$. For a cascade system after every failure the stress is modified by a factor k which is given by the equation (2.2.1)

It is obvious that once the distribution of Y_1 is specified the distribution of Y_2, $Y_3,\ldots,Y_n$ are automatically specified. The i[th] component works if the stress $k^{i-1}Y_1$ lie in the interval (X_i , Z_i). Whenever a stress falls outside these two limits, the component fails and another from standby takes the place of the failed component and the system continues to work. The system fails only if all the n components in cascade fail. It is further assumed that all the components work independently. Then the reliability, R_n, of the system is given by the equation (3.2.8). Here $R(r)$ is different from the preceding chapters.

Let $f_i(x), h_i(z)$ be the probability density function of X_i, Z_i , $i=1,2,\ldots,n$ and $g_i(y_1)$ be the p.d.f. of Y_1

Now we have,

$$R(1) = P(X_1 < Y_1 < Z_1)$$
$$= P(Y_1 > X_1) - P(Y_1 > X_1, Y_1 > Z_1)$$
$$= \int_{-\infty}^{\infty} F_1(y_1) g_1(y_1) dy_1 - \int_{-\infty}^{\infty} F_1(y_1) H_1(y_1) g_1(y_1) dy_1 \tag{6.2.1}$$

where $F_i(x)$ and $H_i(z)$ are c.d.f.'s of X_i and Z_i respectively.

$$R(2) = P(X_1 < Y_1 < Z_1)^c \, P(X_2 < kY_1 < Z_2)$$

$$= [1 - R(1)][P(kY_1 > X_1) - P(kY_1 > X_2, kY_1 > Z_2)]$$
$$= [1 - R(1)]\left[\int_{-\infty}^{\infty} F_2(ky_1) g_2(y_1) dy_1 - \int_{-\infty}^{\infty} F_2(k y_1) H_2(k y_1) g_2(y_1) dy_1\right] \tag{6.2.2}$$

Similarly

$$R(3) = [1 - R(1)][1 - R(2)]\left[\int_{-\infty}^{\infty} F_3(k^2 y_1) g_3(y_1) dy_1 - \int_{-\infty}^{\infty} F_3(k^2 y_1) H_3(k^2 y_1) g_3(y_1) dy_1\right] \quad (6.2.3)$$

In general, we get

$$R(r) = [1 - R(1)][1 - R(2)]..[1 - R(r-1)]\left[\int_{-\infty}^{\infty} F_r(k^{r-1} y_1) g_r(y_1) dy_1 - \int_{-\infty}^{\infty} F_r(k^{r-1} y_1) H_r(k^{r-1} y_1) g_r(y_1) dy_1\right]$$

(6.2.4)

6.3 Stress-Strength follows Specific Distributions

When Stress-Strength follows particular distributions we can evaluate the expression (6.2.4) to get $R(r)$ and thereby obtain the system reliability. In the following five Sub-Sections we assume different particular distributions of all the Stress-Strength involved and obtain expressions of system reliability.

6.3.1 Stress-Strength follows Exponential Distributions

Let $f_i(x)$, $g_i(y_1)$ and $h_i(z)$ be all exponential densities with means $1/\lambda_i, 1/\mu_i, 1/\gamma_i$ respectively, $i=1,2,\ldots,n$ i.e.

$$f_i(x,\lambda) = \lambda_i e^{-\lambda_i x_i} \qquad x_i \geq 0,\ \lambda_i \geq 0$$
$$h_i(z,\gamma) = \gamma_i e^{-\gamma_i z_i} \qquad z_i \geq 0,\ \gamma_i \geq 0$$
$$g_i(y_1,\mu) = \mu_i e^{-\mu_i y_1} \qquad y_1 \geq 0,\ \mu_i \geq 0$$

then from (6.2.1) to (6.2.4) we have

$$R(1) = \frac{\mu_1}{\mu_1 + \gamma_1} - \frac{\mu_1}{\mu_1 + \gamma_1 + \lambda_1} \quad (6.3.1)$$

$$R(2) = [1 - R(1)]\left[\frac{\mu_2}{\mu_2 + \gamma_2 k} - \frac{\mu_2}{\mu_2 + \gamma_2 k + \lambda_2 k}\right] \quad (6.3.2)$$

$$R(3) = [1 - R(1)][1 - R(2)]\left[\frac{\mu_3}{\mu_3 + \gamma_3 k^2} - \frac{\mu_3}{\mu_3 + \gamma_3 k^2 + \lambda_3 k^2}\right] \quad (6.3.3)$$

In general,

$$R(r) = [1 - R(1)][1 - R(2)]...[1 - R(r-1)]\left[\frac{\mu_r}{\mu_r + \gamma_r k^{r-1}} - \frac{\mu_r}{\mu_r + \gamma_r k^{r-1} + \lambda_r k^{r-1}}\right] \quad (6.3.4)$$

Substituting the values of $R(r)$, $r = 1,2,... n$ in (3.2.8) we can obtain R_n, the reliability of the system.

Particular case

Let the strengths of the n components be i.i.d. with p.d.f. $f(x)$ and $h(z)$ which follows exponential distributions with means $1/\lambda$ and $1/\gamma$ and the p.d.f. of Y_1 be exponential density with parameter μ i.e.

$$f(x,\lambda) = \lambda e^{-\lambda x} \qquad x \geq 0,\ \lambda \geq 0$$
$$h(z,\gamma) = \gamma e^{-\gamma z} \qquad z \geq 0,\ \gamma \geq 0$$
$$g(y_1,\mu) = \mu e^{-\mu y_1} \qquad y_1 \geq 0,\ \mu \geq 0$$

then from (6.2.1) to (6.2.4) we have

$$R(1) = \frac{\mu}{\mu + \gamma} - \frac{\mu}{\mu + \gamma + \lambda} \quad (6.3.5)$$

$$R(2) = [1 - R(1)]\left[\frac{\mu}{\mu + \gamma k} - \frac{\mu}{\mu + \gamma k + \lambda k}\right] \quad (6.3.6)$$

$$R(3) = [1 - R(1)][1 - R(2)]\left[\frac{\mu}{\mu + \gamma k^2} - \frac{\mu}{\mu + \gamma k^2 + \lambda k^2}\right] \quad (6.3.7)$$

In general,

$$R(r)=[1-R(1)][1-R(2)]..[1-R(r-1)]\left[\frac{\mu}{\mu+\gamma k^{r-1}}-\frac{\mu}{\mu+\gamma k^{r-1}+\lambda k^{r-1}}\right] \qquad (6.3.8)$$

A few numerical values of $R(1), R(2), R(3)$ and R_3 are tabulated in **Table** 6.1 (cf. Appendix) for different values of the parameters.

6.3.2 Stress-Strength follows Rayleigh Distributions

Let $f_i(x)$, $g_i(y_1)$ and $h_i(z)$ be all Rayleigh densities with parameters σ_i, b_i and s_i respectively, $i=1,2,\dots,n$ i.e.

$$f_i(x,\sigma)=\frac{x_i}{\sigma_i^2}e^{-x_i^2/2\sigma_i^2} \qquad x_i\ge 0, \sigma_i>0$$

$$h_i(z,s)=\frac{z_i}{s_i^2}e^{-z_i^2/2s_i^2} \qquad z_i\ge 0, s_i>0$$

$$g_i(y_1,b)=\frac{y_1}{b_i^2}e^{-y_1^2/2b_i^2} \qquad y_1\ge 0, b_i>0$$

then from (6.2.1) to (6.2.4) we have

$$R(1)=\frac{s_1^2}{b_1^2+s_1^2}-\frac{\sigma_1^2 s_1^2}{\sigma_1^2 s_1^2+b_1^2 s_1^2+\sigma_1^2 b_1^2} \qquad (6.3.9)$$

$$R(2)=[1-R(1)]\left[\frac{s_2^2}{k^2b_2^2+s_2^2}-\frac{\sigma_2^2 s_2^2}{\sigma_2^2 s_2^2+k^2b_2^2 s_2^2+k^2\sigma_2^2 b_2^2}\right] \qquad (6.3.10)$$

$$R(3)=[1-R(1)][1-R(2)]\left[\frac{s_3^2}{k^4b_3^2+s_3^2}-\frac{\sigma_3^2 s_3^2}{\sigma_3^2 s_3^2+k^4b_3^2 s_3^2+k^4\sigma_3^2 b_3^2}\right] \qquad (6.3.11)$$

In general,

$$R(r) = [1 - R(1)][1 - R(2)]...[1 - R(r-1)]\left[\frac{s_r^{\ 2}}{k^{2r-2}b_r^{\ 2} + s_r^{\ 2}} - \frac{\sigma_r^{\ 2}s_r^{\ 2}}{\sigma_r^{\ 2}s_r^{\ 2} + k^{2r-2}b_r^{\ 2}s_r^{\ 2} + k^{2r-2}\sigma_r^{\ 2}b_r^{\ 2}}\right]$$

(6.3.12)

Substituting the values of $R(r)$, $r = 1,2,... n$ in (3.2.8) we can obtain R_n, the reliability of the system.

Particular case

Let the strengths of the n components be i.i.d. with p.d.f. $f(x)$ and $h(z)$ which follows Rayleigh distributions with parameters σ and s, and the p.d.f. of Y_1 be Rayleigh density with parameter b i.e.

$$f(x,\sigma) = \frac{x}{\sigma^2}e^{-x^2/2\sigma^2} \qquad x \geq 0, \sigma > 0$$

$$h(z,s) = \frac{z}{s^2}e^{-z^2/2s^2} \qquad x \geq 0, s > 0$$

$$g(y_1,b) = \frac{y_1}{b^2}e^{-y_1^2/2b^2} \qquad y_1 \geq 0, b > 0$$

then from (6.2.1) to (6.2.4) we have

$$R(1) = \frac{s^2}{b^2 + s^2} - \frac{\sigma^2 s^2}{\sigma^2 s^2 + b^2 s^2 + \sigma^2 b^2} \tag{6.3.13}$$

$$R(2) = [1 - R(1)]\left[\frac{s^2}{k^2b^2 + s^2} - \frac{\sigma^2 s^2}{\sigma^2 s^2 + k^2b^2s^2 + k^2\sigma^2 b^2}\right] \tag{6.3.14}$$

$$R(3) = [1 - R(1)][1 - R(2)]\left[\frac{s^2}{k^4b^2 + s^2} - \frac{\sigma^2 s^2}{\sigma^2 s^2 + k^4b^2s^2 + k^4\sigma^2 b^2}\right] \tag{6.3.15}$$

In general,

$$R(r)=[1-R(1)][1-R(2)]...[1-R(r-1)]\left[\frac{s^2}{k^{2r-2}b^2+s^2}-\frac{\sigma^2 s^2}{\sigma^2 s^2+k^{2r-2}b^2 s^2+k^{2r-2}\sigma^2 b^2}\right] \quad (6.3.16)$$

A few numerical values of $R(1), R(2), R(3)$ and R_3 are tabulated in **Table** 6.2 (cf. Appendix) for different values of the parameters.

6.3.3 Stress-Strength follows Lindley Distributions

Let $f_i(x)$, $g_i(y_1)$ and $h_i(z)$ be all Lindley densities with parameters θ_i, μ_i and γ_i respectively, $i=1,2,\ldots,n$ i.e.

$$f_i(x,\theta)=\frac{\theta_i^2}{(1+\theta_i)}(1+x_i)e^{-\theta_i x_i} \qquad x_i>0, \theta_i>0$$

$$h_i(z,\gamma)=\frac{\gamma_i^2}{(1+\gamma_i)}(1+z_i)e^{-\gamma_i z} \qquad z_i>0, \gamma_i>0$$

$$g_i(y_1,\mu)=\frac{\mu_i^2}{(1+\mu_i)}(1+y_1)e^{-\mu_i y_1} \qquad y_1>0, \mu_i>0$$

then from (6.2.1) to (6.2.4) we have

$$R(1)=\frac{\mu_1^2}{1+\mu_1}\left[\begin{array}{l}\frac{1}{\mu_1+\gamma_1}+\frac{1}{(\mu_1+\gamma_1)^2}+\frac{\gamma_1}{(1+\gamma_1)(\mu_1+\gamma_1)^2}+\frac{2\gamma_1}{(1+\gamma_1)(\mu_1+\gamma_1)^3}-\frac{1}{\theta_1+\gamma_1+\mu_1}\\ -\frac{1}{(\theta_1+\gamma_1+\mu_1)^2}-\frac{\theta_1}{(1+\theta_1)(\theta_1+\gamma_1+\mu_1)^2}-\frac{\gamma_1}{(1+\gamma_1)(\theta_1+\gamma_1+\mu_1)^2}-\\ \frac{2\theta_1}{(1+\theta_1)(\theta_1+\gamma_1+\mu_1)^3}-\frac{2\gamma_1}{(1+\gamma_1)(\theta_1+\gamma_1+\mu_1)^3}-\frac{2\theta_1\gamma_1}{(1+\theta_1)(1+\gamma_1)(\theta_1+\gamma_1+\mu_1)^3}\\ -\frac{6\theta_1\gamma_1}{(1+\theta_1)(1+\gamma_1)(\theta_1+\gamma_1+\mu_1)^4}\end{array}\right]$$

(6.3.17)

$$R(2) = [1 - R(1)]\left[\frac{\mu_2^{\,2}}{1+\mu_2} \left\{ \begin{array}{l} \dfrac{1}{\mu_2 + \gamma_2 k} + \dfrac{1}{(\mu_2 + \gamma_2 k)^2} + \dfrac{\gamma_2 k}{(1+\gamma_2)(\mu_2 + \gamma_2 k)^2} + \dfrac{2\gamma_2 k}{(1+\gamma_2)(\mu_2 + \gamma_2 k)^3} \\ - \dfrac{1}{\theta_2 k + \gamma_2 k + \mu_2} - \dfrac{1}{(\theta_2 k + \gamma_2 k + \mu_2)^2} - \dfrac{\theta_2 k}{(1+\theta_2)(\theta_2 k + \gamma_2 k + \mu_2)^2} - \\ \dfrac{\gamma_2 k}{(1+\gamma_2)(\theta_2 k + \gamma_2 k + \mu_2)^2} - \dfrac{2\theta_2 k}{(1+\theta_2)\,(\theta_2 k + \gamma_2 k + \mu_2)^3} - \\ \dfrac{2\gamma_2 k}{(1+\gamma_2)\,(\theta_2 k + \gamma_2 k + \mu_2)^3} \dfrac{2\theta_2 \gamma_2 k^2}{(1+\theta_2)(1+\gamma_2)(\theta_2 k + \gamma_2 k + \mu_2)^3} \\ - \dfrac{6\theta_2 \gamma_2 k^2}{(1+\theta_2)(1+\gamma_2)(\theta_2 k + \gamma_2 k + \mu_2)^4} \end{array} \right\} \right]$$

(6.3.18)

$$R(3) = [1 - R(1)][1 - R(2)]$$

$$\left[\frac{\mu_3^{\,2}}{1+\mu_3} \left\{ \begin{array}{l} \dfrac{1}{\mu_3 + \gamma_3 k^2} + \dfrac{1}{(\mu_3 + \gamma_3 k^2)^2} + \dfrac{\gamma_3 k^2}{(1+\gamma_3)(\mu_3 + \gamma_3 k^2)^2} + \dfrac{2\gamma_3 k^2}{(1+\gamma_3)(\mu_3 + \gamma_3 k^2)^3} - \\ \dfrac{1}{\theta_3 k^2 + \gamma_3 k^2 + \mu_3} - \dfrac{1}{(\theta_3 k^2 + \gamma_3 k^2 + \mu_3)^2} - \dfrac{\theta k^2}{(1+\theta_3)(\theta_3 k^2 + \gamma_3 k^2 + \mu_3)^2} - \\ \dfrac{\gamma_3 k^2}{(1+\gamma_3)(\theta_3 k^2 + \gamma_3 k^2 + \mu_3)^2} - \dfrac{2\theta_3 k^2}{(1+\theta_3)\,(\theta_3 k^2 + \gamma_3 k^2 + \mu_3)^3} - \\ \dfrac{2\gamma_3 k^2}{(1+\gamma_3)\,(\theta_3 k^2 + \gamma_3 k^2 + \mu_3)^3} - \dfrac{2\theta_3 \gamma_3 k^4}{(1+\theta_3)(1+\gamma_3)(\theta_3 k^2 + \gamma_3 k^2 + \mu_3)^3} - \\ \dfrac{6\theta_3 \gamma_3 k^4}{(1+\theta_3)(1+\gamma_3)(\theta_3 k^2 + \gamma_3 k^2 + \mu_3)^4} \end{array} \right\} \right]$$

(6.3.19)

In general,

$R(r) = [1 - R(1)][1 - R(2)]...[1 - R(r-1)]$

$$\left[\frac{\mu_r^{\,2}}{1+\mu_r}\left\{\begin{array}{l}\dfrac{1}{\mu_r+\gamma_r k^{r-1}}+\dfrac{1}{(\mu_r+\gamma_r k^{r-1})^2}+\dfrac{\gamma_r k^{r-1}}{(1+\gamma_r)(\mu_r+\gamma_r k^{r-1})^2}+\dfrac{2\gamma_r k^{r-1}}{(1+\gamma_r)(\mu_r+\gamma_r k^{r-1})^3}-\\ \dfrac{1}{\theta_r k^{r-1}+\gamma_r k^{r-1}+\mu_r}-\dfrac{1}{(\theta_r k^{r-1}+\gamma_r k^{r-1}+\mu_r)^2}-\dfrac{\theta_r k^{r-1}}{(1+\theta_r)(\theta_r k^{r-1}+\gamma_r k^{r-1}+\mu_r)^2}\\ -\dfrac{\gamma_r k^{r-1}}{(1+\gamma_r)(\theta_r k^{r-1}+\gamma_r k^{r-1}+\mu_r)^2}-\dfrac{2\theta_r k^{r-1}}{(1+\theta_r)(\theta_r k^{r-1}+\gamma_r k^{r-1}+\mu_r)^3}-\\ \dfrac{2\gamma_r k^{r-1}}{(1+\gamma_r)(\theta_r k^{r-1}+\gamma_r k^{r-1}+\mu_r)^3}-\dfrac{2\theta_r\gamma_r k^{2(r-1)}}{(1+\theta_r)(1+\gamma_r)(\theta_r k^{r-1}+\gamma_r k^{r-1}+\mu_r)^3}-\\ \dfrac{6\theta_r\gamma_r k^{2(r-1)}}{(1+\theta_r)(1+\gamma_r)(\theta_r k^{r-1}+\gamma_r k^{r-1}+\mu_r)^4}\end{array}\right\}\right]$$

(6.3.20)

Substituting the values of $R(r)$, $r=1,2,\ldots n$ in (3.2.8) we can obtain R_n, the reliability of the system.

Particular case

Let the strengths of the n components be i.i.d. with p.d.f. $f(x)$ and $h(z)$ which follows Lindley distributions with parameters θ *and* γ and the p.d.f. of Y_1 be Lindley density with parameter μ i.e.

$$f(x,\theta) = \frac{\theta^2}{(1+\theta)}(1+x)e^{-\theta x} \qquad x>0,\ \theta>0$$

$$h(z,\gamma) = \frac{\gamma^2}{(1+\gamma)}(1+z)e^{-\gamma z} \qquad z>0,\ \gamma>0$$

$$g(y_1,\mu) = \frac{\mu^2}{(1+\mu)}(1+y_1)e^{-\mu y_1} \qquad y_1>0,\ \mu>0$$

then from (6.2.1) to (6.2.4) we have

$$R(1)=\frac{\mu^2}{1+\mu}\left[\begin{array}{l}\frac{1}{\mu+\gamma}+\frac{1}{(\mu+\gamma)^2}+\frac{\gamma}{(1+\gamma)(\mu+\gamma)^2}+\frac{2\gamma}{(1+\gamma)(\mu+\gamma)^3}-\frac{1}{\theta+\gamma+\mu}-\frac{1}{(\theta+\gamma+\mu)^2}\\ -\frac{\theta}{(1+\theta)(\theta+\gamma+\mu)^2}-\frac{\gamma}{(1+\gamma)(\theta+\gamma+\mu)^2}-\frac{2\theta}{(1+\theta)\,(\theta+\gamma+\mu)^3}-\frac{2\gamma}{(1+\gamma)\,(\theta+\gamma+\mu)^3}\\ -\frac{2\theta\gamma}{(1+\theta)(1+\gamma)(\theta+\gamma+\mu)^3}-\frac{6\theta\gamma}{(1+\theta)(1+\gamma)(\theta+\gamma+\mu)^4}\end{array}\right]$$

(6.3.21)

$$R(2)=[1-R(1)]\left[\frac{\mu^2}{1+\mu}\left\{\begin{array}{l}\frac{1}{\mu+\gamma k}+\frac{1}{(\mu+\gamma k)^2}+\frac{\gamma k}{(1+\gamma)(\mu+\gamma k)^2}+\frac{2\gamma k}{(1+\gamma)(\mu+\gamma k)^3}-\\ \frac{1}{\theta k+\gamma k+\mu}-\frac{1}{(\theta k+\gamma k+\mu)^2}-\frac{\theta k}{(1+\theta)(\theta k+\gamma k+\mu)^2}-\\ \frac{\gamma k}{(1+\gamma)(\theta k+\gamma k+\mu)^2}-\frac{2\theta k}{(1+\theta)\,(\theta k+\gamma k+\mu)^3}-\frac{2\gamma k}{(1+\gamma)\,(\theta k+\gamma k+\mu)^3}\\ -\frac{2\theta\gamma k^2}{(1+\theta)(1+\gamma)(\theta k+\gamma k+\mu)^3}-\frac{6\theta\gamma k^2}{(1+\theta)(1+\gamma)(\theta k+\gamma k+\mu)^4}\end{array}\right\}\right]$$

(6.3.22)

$$R(3)=[1-R(1)][1-R(2)]$$

$$\left[\frac{\mu^2}{1+\mu}\left\{\begin{array}{l}\frac{1}{\mu+\gamma k^2}+\frac{1}{(\mu+\gamma k^2)^2}+\frac{\gamma k^2}{(1+\gamma)(\mu+\gamma k^2)^2}+\frac{2\gamma k^2}{(1+\gamma)(\mu+\gamma k^2)^3}-\frac{1}{\theta k^2+\gamma k^2+\mu}\\ -\frac{1}{(\theta k^2+\gamma k^2+\mu)^2}-\frac{\theta k^2}{(1+\theta)(\theta k^2+\gamma k^2+\mu)^2}-\frac{\gamma k^2}{(1+\gamma)(\theta k^2+\gamma k^2+\mu)^2}\\ -\frac{2\theta k^2}{(1+\theta)\,(\theta k^2+\gamma k^2+\mu)^3}-\frac{2\gamma k^2}{(1+\gamma)\,(\theta k^2+\gamma k^2+\mu)^3}-\frac{2\theta\gamma k^4}{(1+\theta)(1+\gamma)(\theta k^2+\gamma k^2+\mu)^3}\\ -\frac{6\theta\gamma k^4}{(1+\theta)(1+\gamma)(\theta k^2+\gamma k^2+\mu)^4}\end{array}\right\}\right]$$

6.3.23)

In general,

$$R(r) = [1 - R(1)][1 - R(2)]...[1 - R(r-1)]$$

$$\left[\frac{\mu^2}{1+\mu}\left\{\begin{array}{l}\frac{1}{\mu+\gamma k^{r-1}}+\frac{1}{(\mu+\gamma k^{r-1})^2}+\frac{\gamma k^{r-1}}{(1+\gamma)(\mu+\gamma k^{r-1})^2}+\frac{2\gamma k^{r-1}}{(1+\gamma)(\mu+\gamma k^{r-1})^3}-\\ \frac{1}{\theta k^{r-1}+\gamma k^{r-1}+\mu}-\frac{1}{\left(\theta k^{r-1}+\gamma k^{r-1}+\mu\right)^2}-\frac{\theta k^{r-1}}{(1+\theta)(\theta k^{r-1}+\gamma k^{r-1}+\mu)^2}-\\ \frac{\gamma k^{r-1}}{(1+\gamma)(\theta k^{r-1}+\gamma k^{r-1}+\mu)^2}-\frac{2\theta k^{r-1}}{(1+\theta)\,(\theta k^{r-1}+\gamma k^{r-1}+\mu)^3}-\frac{2\gamma k^{r-1}}{(1+\gamma)\,(\theta k^{r-1}+\gamma k^{r-1}+\mu)^3}\\ -\frac{2\theta\gamma k^{2(r-1)}}{(1+\theta)(1+\gamma)(\theta k^{r-1}+\gamma k^{r-1}+\mu)^3}-\frac{6\theta\gamma k^{2(r-1)}}{(1+\theta)(1+\gamma)(\theta k^{r-1}+\gamma k^{r-1}+\mu)^4}\end{array}\right\}\right]$$

(6.3.24)

A few numerical values of $R(1), R(2), R(3)$ and R_3 are tabulated in **Table** 6.3 (cf. Appendix) for different values of the parameters.

6.3.4 Both Strength follows One-Parameter Exponential and Stress follows Lindley Distributions

Let $f_i(x)$ and $h_i(z)$ be one-parameter exponential densities with parameters λ_i, θ_i and let $g_i(y_1)$ be Lindley densities with parameters μ_i respectively, $i=1,2,\ldots,n$ i.e.

$$f_i(x,\lambda) = \lambda_i e^{-\lambda_i x_i} \qquad x \ge 0,\ \lambda \ge 0$$

$$h_i(z,\theta) = \theta_i e^{-\theta_i z_i} \qquad z \ge 0,\ \theta \ge 0$$

$$g_i(y_1,\mu) = \frac{\mu_i^2}{(1+\mu_i)}(1+y_1)e^{-\mu_i y_1} \qquad y_1 > 0,\ \mu > 0$$

then from (6.2.1) to (6.2.4) we have

$$R(1) = \frac{\mu_1^2}{1+\mu_1}\left[\frac{1}{\theta_1+\mu_1}+\frac{1}{(\theta_1+\mu_1)^2}-\frac{1}{\theta_1+\mu_1+\lambda_1}-\frac{1}{(\theta_1+\mu_1+\lambda_1)^2}\right] \qquad (6.3.25)$$

$$R(2)=[1-R(1)]\left[\frac{\mu_2^{\,2}}{1+\mu_2}\left\{\frac{1}{\theta_2 k+\mu_2}+\frac{1}{(\theta_2 k+\mu_2)^2}-\frac{1}{\theta_2 k+\lambda_2 k+\mu_2}-\frac{1}{(\theta_2 k+\lambda_2 k+\mu_2)^2}\right\}\right]$$

(6.3.26)

$$R(3)=[1-R(1)][1-R(2)]\left[\frac{\mu_3^{\,2}}{1+\mu_3}\left\{\begin{array}{l}\frac{1}{\theta_3 k^2+\mu_3}+\frac{1}{(\theta_3 k^2+\mu_3)^2}-\frac{1}{\theta_3 k^2+\lambda_3 k^2+\mu_3}-\\ \frac{1}{(\theta_3 k^2+\lambda_3 k^2+\mu_3)^2}\end{array}\right\}\right] \quad (6.3.27)$$

In general,

$$R(r)=[1-R(1)][1-R(2)]...[1-R(r-1)]$$

$$\left[\frac{\mu_r^{\,2}}{1+\mu_r}\left\{\frac{1}{\theta_r k^{r-1}+\mu_r}+\frac{1}{(\theta_r k^{r-1}+\mu_r)^2}-\frac{1}{\theta_r k^{r-1}+\lambda_r k^{r-1}+\mu_r}-\frac{1}{(\theta_r k^{r-1}+\lambda_r k^{r-1}+\mu_r)^2}\right\}\right]$$

(6.3.28)

Substituting the values of $R(r)$, $r=1,2,\ldots,n$ in (3.2.8) we can obtain R_n, the reliability of the system.

<u>Particular case</u>

Let the strengths of the n components be i.i.d. with p.d.f. $f(x)$ and $h(z)$ which follows one-parameter exponential with means $1/\lambda$ and $1/\theta$ and the p.d.f. of Y_1 be Lindley density with parameter μ i.e.

$$f(x,\lambda)=\lambda e^{-\lambda x} \qquad x\geq 0,\ \lambda\geq 0$$

$$h(z,\theta)=\theta e^{-\theta z} \qquad z\geq 0,\ \theta\geq 0$$

$$g(y_1,\mu)=\frac{\mu^2}{(1+\mu)}(1+y_1)e^{-\mu y_1} \qquad y_1>0,\ \mu>0$$

then from (6.2.1) to (6.2.4) we have

$$R(1)=\frac{\mu^2}{1+\mu}\left[\frac{1}{\theta+\mu}+\frac{1}{(\theta+\mu)^2}-\frac{1}{\theta+\mu+\lambda}-\frac{1}{(\theta+\mu+\lambda)^2}\right] \tag{6.3.29}$$

$$R(2)=[1-R(1)]\left[\frac{\mu^2}{1+\mu}\left\{\frac{1}{\theta k+\mu}+\frac{1}{(\theta k+\mu)^2}-\frac{1}{\theta k+\lambda k+\mu}-\frac{1}{(\theta k+\lambda k+\mu)^2}\right\}\right] \tag{6.3.30}$$

$$R(3)=[1-R(1)][1-R(2)]\left[\frac{\mu^2}{1+\mu}\left\{\frac{1}{\theta k^2+\mu}+\frac{1}{(\theta k^2+\mu)^2}-\frac{1}{\theta k^2+\lambda k^2+\mu}-\frac{1}{(\theta k^2+\lambda k^2+\mu)^2}\right\}\right] \tag{6.3.31}$$

In general,

$$R(r)=[1-R(1)][1-R(2)]...[1-R(r-1)]$$
$$\left[\frac{\mu^2}{1+\mu}\left\{\frac{1}{\theta k^{r-1}+\mu}+\frac{1}{(\theta k^{r-1}+\mu)^2}-\frac{1}{\theta k^{r-1}+\lambda k^{r-1}+\mu}-\frac{1}{(\theta k^{r-1}+\lambda k^{r-1}+\mu)^2}\right\}\right] \tag{6.3.32}$$

A few numerical values of $R(1), R(2), R(3)$ and R_3 are tabulated in **Table** 6.4 (cf. Appendix) for different values of the parameters.

6.3.5 Both Strength follows One-Parameter Exponential and Stress follows Two-Parameter Gamma Distributions

Let $f_i(x)$ and $h_i(z)$ be one-parameter exponential densities with parameters λ_i, θ_i and let $g_i(y_1)$ be two-parameter gamma densities with parameters γ_i and μ_i respectively, $i=1,2,\ldots,n$ i.e.

$$f_i(x,\lambda)=\lambda_i e^{-\lambda_i x_i} \qquad x_i \geq 0,\ \lambda_i \geq 0$$
$$h_i(z,\theta)=\theta_i e^{-\theta_i z_i} \qquad z_i \geq 0,\ \theta_i \geq 0$$
$$g_i(y_1,\gamma,\mu)=\frac{1}{\gamma_i^{\mu_i}\Gamma\mu_i} y_1^{\mu_i-1} e^{-y_1/\gamma_i} \qquad y_1>0,\ \gamma_i,\mu_i>0$$

then from (6.2.1) to (6.2.4) we have

$$R(1)=\frac{1}{(1+\theta_1\gamma_1)^{\mu_1}}-\frac{1}{(1+\theta_1\gamma_1+\lambda_1\gamma_1)^{\mu_1}} \tag{6.3.33}$$

$$R(2)=[1-R(1)]\left[\frac{1}{(1+\theta_2\gamma_2 k)^{\mu_2}}-\frac{1}{(1+\theta_2\gamma_2 k+\lambda_2\gamma_2 k)^{\mu_2}}\right] \tag{6.3.34}$$

$$R(3)=[1-R(1)][1-R(2)]\left[\frac{1}{(1+\theta_3\gamma_3 k^2)^{\mu_3}}-\frac{1}{(1+\theta_3\gamma_3 k^2+\lambda_3\gamma_3 k^2)^{\mu_3}}\right] \tag{6.3.35}$$

In general,

$$R(r)=[1-R(1)][1-R(2)]\ldots[1-R(r-1)]\left[\frac{1}{(1+\theta_r\gamma_r k^{r-1})^{\mu_r}}-\frac{1}{(1+\theta_r\gamma_r k^{r-1}+\lambda_r\gamma_r k^{r-1})^{\mu_r}}\right] \tag{6.3.36}$$

Substituting the values of $R(r)$, $r=1,2,\ldots,n$ in (3.2.8) we can obtain R_n, the reliability of the system.

Particular case

Let the strengths of the n components be i.i.d. with p.d.f. $f(x)$ and $h(z)$ which follows one-parameter exponential with means $1/\lambda$ and $1/\theta$ and $g(y_1)$ be two-parameters gamma densities with degrees of freedom γ and μ respectively and unit scale parameters i.e.

$$f(x,\lambda)=\lambda e^{-\lambda x} \qquad x\ge 0,\ \lambda\ge 0$$
$$h(z,\theta)=\theta e^{-\theta z} \qquad z\ge 0,\ \theta\ge 0$$
$$g(y_1,\gamma,\mu)=\frac{1}{\gamma^{\mu}\Gamma\mu}y_1^{\mu-1}e^{-y_1/\gamma} \qquad y_1>0,\ \gamma,\mu>0$$

then from (6.2.1) to (6.2.4) we have

$$R(1) = \frac{1}{(1+\theta\gamma)^{\mu}} - \frac{1}{(1+\theta\gamma+\lambda\gamma)^{\mu}} \tag{6.3.37}$$

$$R(2) = [1-R(1)]\left[\frac{1}{(1+\theta\gamma k)^{\mu}} - \frac{1}{(1+\theta\gamma k+\lambda\gamma k)^{\mu}}\right] \tag{6.3.38}$$

$$R(3) = [1-R(1)][1-R(2)]\left[\frac{1}{(1+\theta\gamma k^{2})^{\mu}} - \frac{1}{(1+\theta\gamma k^{2}+\lambda\gamma k^{2})^{\mu}}\right] \tag{6.3.39}$$

In general,

$$R(r) = [1-R(1)][1-R(2)]...[1-R(r-1)]\left[\frac{1}{(1+\theta\gamma k^{r-1})^{\mu}} - \frac{1}{(1+\theta\gamma k^{r-1}+\lambda\gamma k^{r-1})^{\mu}}\right] \tag{6.3.40}$$

A few numerical values of $R(1), R(2), R(3)$ and R_3 are tabulated in Table 6.5 (cf. Appendix) for different values of the parameters.

6.4 Graphical Representations

Some graphs are plotted in **Fig.** 6.1(a), **Fig.** 6.1(b), **Fig.** 6.2(a), **Fig.** 6.2(b), **Fig.** 6.3(a), **Fig.** 6.3(b), **Fig.** 6.4(a), **Fig.** 6.4(b), **Fig.** 6.5(a) and **Fig.** 6.5(b) taking different parameters along the horizontal axis and the corresponding reliabilities along the vertical axis for different parametric values. **Fig.** 6.1(a) - **Fig.** 6.1(b) signifies that reliabilities increase steadily with increasing γ. These graphs may be used to read the intermediate values directly. In **Fig.** 6.2(a) - **Fig.** 6.2(b) it is seen that graphs of $R(1), R(2), R(3)$ and R_3 against σ_2 for fix values of σ_1 and k are plotted for different values of σ_3. From these graphs it is observed that if the stress parameter σ_2 and strength parameter σ_3 increase, $R(1), R(2), R(3)$ and R_3 also increases. Similarly it is also seen that in **Fig.** 6.3(a) - **Fig.** 6.3(b) and **Fig.** 6.4(a) - **Fig.** 6.4(b), reliabilities are decreases with increasing values of their parameters. But in **Fig.** 6.5(a) - **Fig.** 6.5(b) taking the attenuation factor k along the horizontal axis and the corresponding reliability along the vertical axis for different values of $\mu, \gamma, \lambda, \theta$, it is to be observed that reliability is increasing with k.

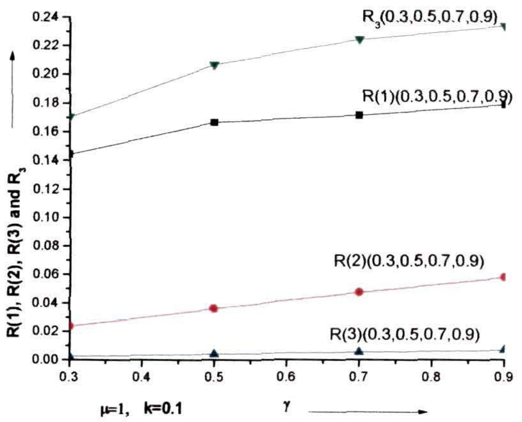

Fig. 6.1(a) Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of λ for sub-section 6.3.1.

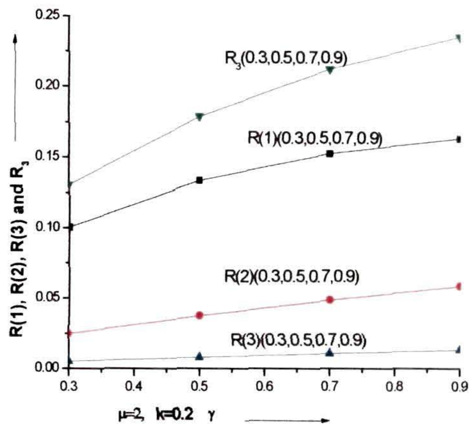

Fig. 6.1(b) Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of λ for sub-section 6.3.1.

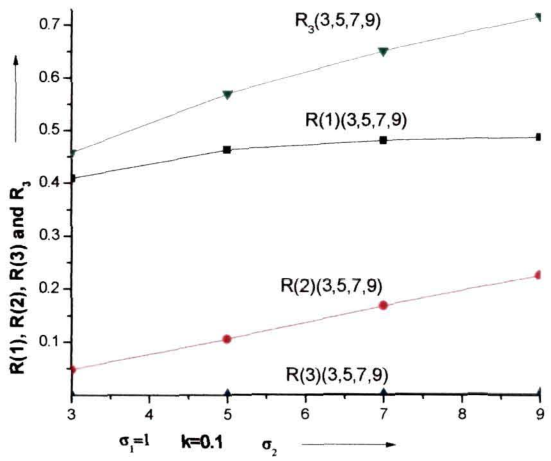

Fig. 6.2(a) Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of σ_3 for sub-section 6.3.2.

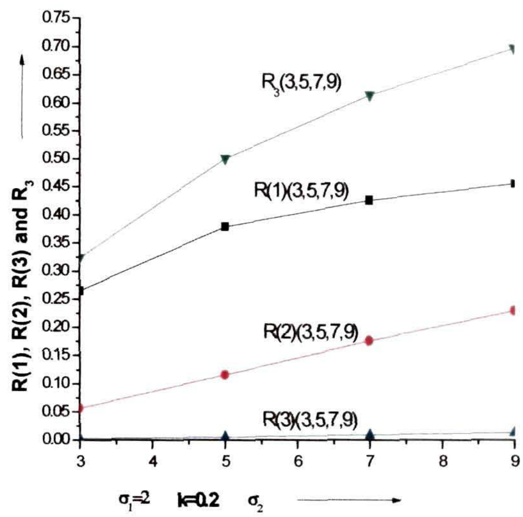

Fig. 6.2(b) Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of σ_3 for sub-section 6.3.2.

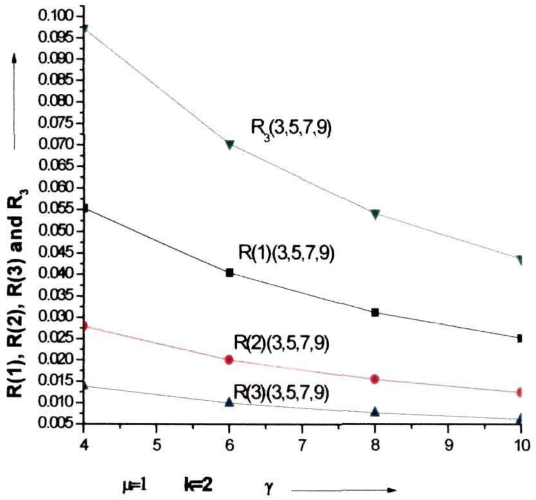

Fig. 6.3(a) Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of θ for sub-section 6.3.3.

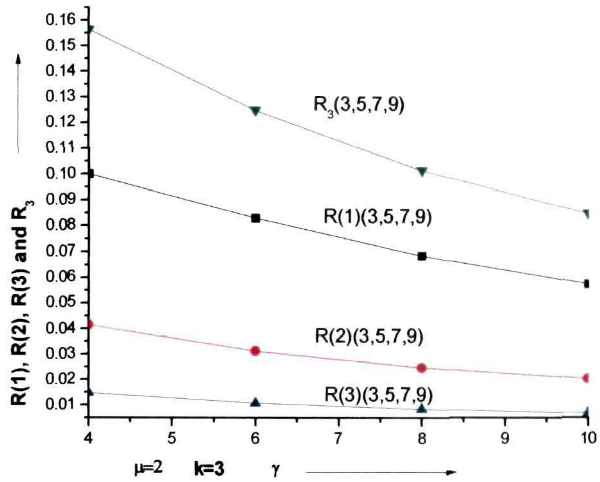

Fig. 6.3(b) Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of θ for sub-section 6.3.3.

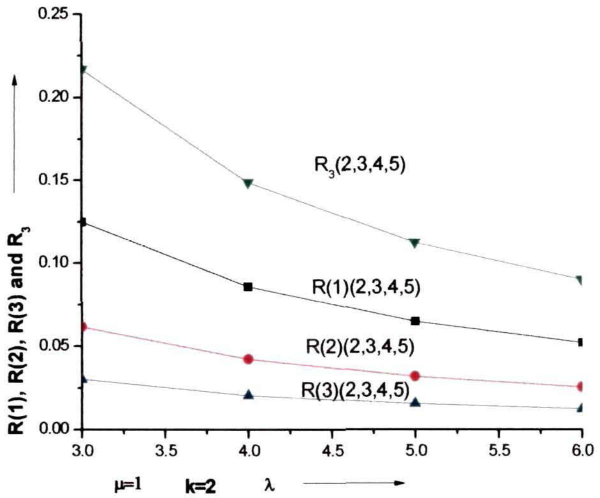

Fig. 6.4(a) Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of θ for sub-section 6.3.4.

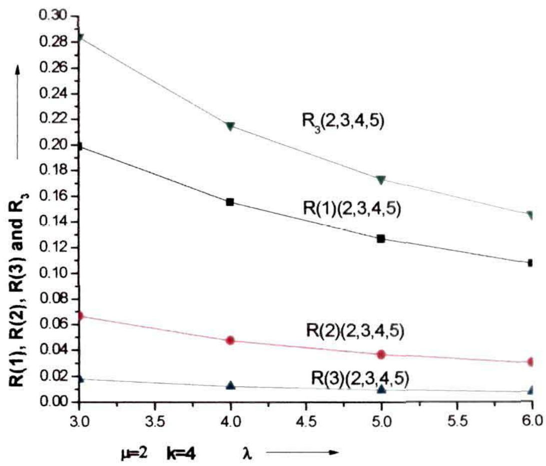

Fig. 6.4(b) Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of θ for sub-section 6.3.4.

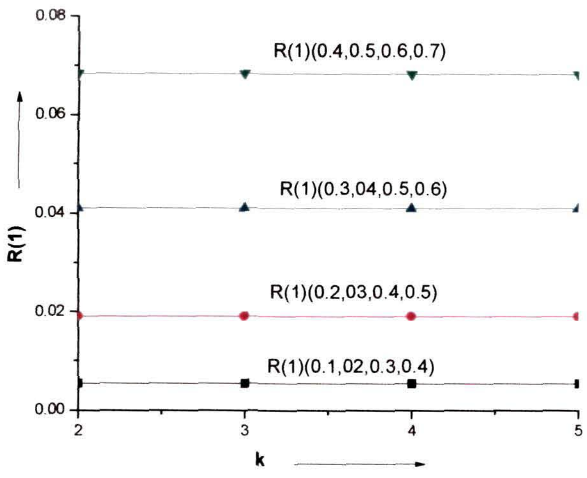

Fig. 6.5(a) Graph of $R(1)$ for different fixed values of $\mu, \gamma, \lambda, \theta$ for sub-section 6.3.5.

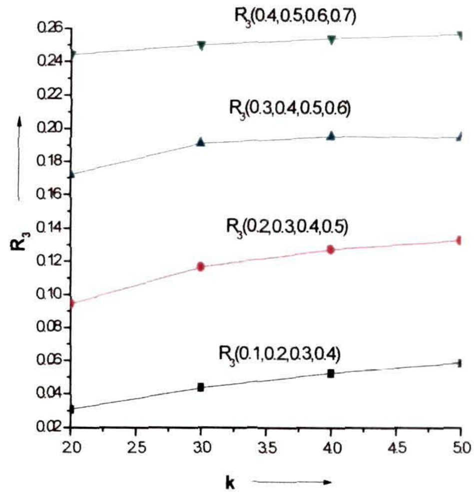

Fig. 6.5(b) Graph for R_3 for different fixed values of $\mu, \gamma, \lambda, \theta$ for sub-section 6.3.5.

6.5 Results and Discussions

For some specific values of the parameters involved in the expressions of $R(r)$, $r = 1,2,3$ we evaluate the marginal reliabilities $R(1), R(2), R(3)$ and system reliability R_3 for the above five cases from their expressions obtained in Sub-Section 6.3.1-6.3.5.

From the **Table** 6.1, we notice that if the strength parameter λ and γ increases then the system reliability R_3 increase. When the stress parameter μ increases $R(1)$ decreases but $R(2)$ and $R(3)$ increases. For instance, if $\mu = 1$, $R(1) = 0.1442$ and if $\mu = 2$, $R(1) = 0.1003$. In general we see that when γ, λ increases then $R(2)$ and $R(3)$ will also increases. When the attenuation factor k increases then the marginal reliabilities $R(1), R(2), R(3)$ and system reliability R_3 decreases.

Table 6.2, shows that with some set of values of the parameters, if σ_1 increases, the system reliability decrease. i.e., if $\sigma_1 = 1$, $R_3 = 0.4575$ and if $\sigma_1 = 2$, $R_3 = 0.3235$. But if the stress parameter σ_2 and strength parameter σ_3 increase, $R(1), R(2)$ and $R(3)$ also increase. Here it is seen that when the attenuation factor k increases the marginal reliabilities $R(1), R(2), R(3)$ and the system reliability R_3 decreases.

From the **Table** 6.3, it is clear that when the strength parameters θ *and* γ increases the system reliability R_3 and marginal reliabilities $R(1), R(2), R(3)$ decreases. But if the stress parameter μ and attenuation factor k increases, reliability also increases.

In **Table** 6.4, it is seen that with some set of values of the parameters if θ *and* λ increases then the system reliability decreases and $R(1), R(2), R(3)$ also decreases. Here also it is apparent that when the attenuation factor k and stress parameter μ increases, the marginal reliabilities $R(1), R(2), R(3)$ and the system reliability R_3 increases.

From the tabulated value of **Table** 6.5, we observe that when the strength parameters λ *and* θ and stress parameters μ *and* γ increases, marginal reliabilities $R(1), R(2), R(3)$ and system reliability R_3 increases with constant value of k. When the attenuation factor k increases there are significant increase in the values of $R(2), R(3)$ and R_3 but no significant difference in the values of $R(1)$.

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Chapter 7

Conclusion

Chapter 7

Conclusion

7.1 Summary of the thesis

In our study, different cascade models are considered to estimate the system reliability. For this estimation, several distributions viz. exponential, gamma, Weibull, Rayleigh, Lindley, two-point and uniform distribution are considered. Generally in cascade system the attenuation factor is assumed to be a constant for all the components. But in the 2nd chapter the attenuation factor K is taken to be uniform random variable to evaluate the expressions of the unconditional reliability of a system and exponential, gamma and Weibull distribution have been used to obtain the cascade reliability. It has been observed that when stress-strength follows exponential distribution, reliability decreases with increasing values of the parameters. But for Weibull distribution, increase in the values of shape parameters increases the reliability and increase in the values of scale parameters decreases the reliability.

Cascade reliability can also be obtained if stress and strength are represented by a mixture of two distributions, which is discussed in chapter 3. It has been observed from the numerical values of reliabilities that for fix values of one mix parameter and attenuation factor, reliabilities are decreasing for increased values of another mix parameter and is mentioned in **Table** 3.1 (cf. Appendix). It is also seen that the increase of one mix parameter increases the reliability.

In chapter 4, an n-cascade system with three failure models have been discussed. Using these models marginal reliability and system reliability of n-cascade system has been developed and their values are presented graphically. Two distributions viz. exponential and Rayleigh has been used to find out the reliabilities. When exponential distribution is used for the first failure model then we observed that increase in the parametric values result a corresponding increase of the reliabilities. Similarly in model II we considered n- cascade system with identical components where m stresses on the component lie in an interval (a_j, b_j). The component fails even if one of stresses on the component falls outside the specified limits. Here model II and model III are almost similar except that the components are not identical. It is seen from the **Table** 4.2 (cf. Appendix) and **Table** 4.3 (cf. Appendix) that keeping all other parameters fixed, reliability can be increased by increasing the upper limits. But sometimes it is seen that reliabilities decrease when any one of the lower limits increases.

In chapter 5, the system reliability of 2-cascade system has been formulated, when the parameters of the stress-strength distributions are considered as random variable. For this purpose, stress and strength are assumed as exponential random variables with certain parameters. Further it is assumed that either stress or strength parameter is a random variable with a known prior distribution. The obtained expressions of the system reliabilities are verified using some numerical values of the parameters. It has been found from **Table** 5.1(cf. Appendix) and **Table** 5.2 (cf. Appendix) that when the prior distributions are uniform and two-point type, the reliabilities are steadily increasing with increasing value of the stress parameter and the attenuation factor. But when stress parameter is random then reliabilities are decreasing with increasing value of the strength parameter. In stress-strength model the reliability of a component is defined as the probability that its strength is not less than the stress working on it. But sometimes a component can work only when the stress Y on it is not only less than certain values, say Z, but must be greater than some other value, say X, i.e. stress is within certain limits, where X and Z are identified as lower and upper strengths. This assumptions has been followed in chapter 6, where reliability of n-cascade system under such process has been obtained using different stress-strength distribution. It has been observed from the numerical values of the reliability that when the attenuation factor increases, marginal reliabilities and system reliability decreases. The results may vary from distribution to distribution.

7.2 Future Works

In this research, the reliability of the cascade system is obtained from the distribution of its strength and that of stress working on it. Cascade reliability can also be evaluated for some truncated stress-strength. Such type of problem is considered as a future research prospect. Again for finding the system reliability of n-cascade system, we considered one case where stress on the component is subjected to two strengths. But it is also possible to obtain the system reliability where the strength of the components lying between two stresses, which is not included in this research due to mathematical complication and the limited time. Therefore, it may be a new research interest.

In most of the studies of S-S models, stress-strength are taken into consideration in evaluating the reliability of the system, passage of time has no effect on it. So by taking time as an important factor, one time dependent cascade model with different stress-strength distribution will be considered in our next study. We have not come across any study of time dependent cascade model where parameters of the distributions are function of time. So our future research work will be to study such type of models.

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Appendix

APPENDIX

List of Tables

Table 2.1 Values of R_1, R_2, R_3 when Stress-Strength follows Exponential Distribution

ρ	R_1	R_2	R_3
0.1	0.9091	0.9921	0.9990
0.2	0.8333	0.9742	0.9947
0.3	0.7692	0.9516	0.9872
0.4	0.7143	0.9272	0.9773
0.5	0.6667	0.9022	0.9659
0.6	0.6250	0.8776	0.9535
0.7	0.5882	0.8536	0.9405
0.8	0.5556	0.8306	0.9274
0.9	0.5263	0.8086	0.9142
1.1	0.4762	0.7678	0.8883
1.2	0.4545	0.7488	0.8758
1.3	0.4348	0.7308	0.8636
1.4	0.4167	0.7138	0.8517
1.5	0.4000	0.6975	0.8401
1.6	0.3846	0.6821	0.8289
1.7	0.3704	0.6674	0.8180
1.8	0.3571	0.6534	0.8075
1.9	0.3448	0.6400	0.7974
.01	0.9901	0.9999	1.0000
.02	0.9804	0.9996	1.0000
.03	0.9709	0.9992	1.0000
.04	0.9615	0.9985	0.9999
.05	0.9524	0.9978	0.9999
.06	0.9434	0.9969	0.9998
.07	0.9346	0.9959	0.9996
.08	0.9259	0.9947	0.9995
.09	0.9174	0.9934	0.9993
10	0.0909	0.2660	0.8482
20	0.0476	0.1664	0.5870
30	0.0323	0.1242	0.4685
40	0.0244	0.1002	0.3980
50	0.0196	0.0846	0.3502
60	0.0164	0.0735	0.3153
70	0.0141	0.0652	0.2884
80	0.0123	0.0587	0.2669
90	0.0110	0.0535	0.2492

Table 2.2 Values of R_1, R_2 when Stress-Strength follows Weibull Distribution

c	θ	a	λ	R_1	R_2
1	10	6	2	0.8295	0.9846
1	15	11	7	0.6389	0.9297
1	20	16	12	0.5582	0.8930
1	25	21	17	0.5142	0.8689
1	30	26	22	0.4866	0.8525
2	10	6	2	0.9646	0.9996
2	15	11	7	0.8167	0.9877
2	20	16	12	0.7110	0.9693
2	25	21	17	0.6420	0.9525
2	30	26	22	0.5949	0.9390
3	10	6	2	0.9929	1.000
3	15	11	7	0.9118	0.9978
3	20	16	12	0.8184	0.9908
3	25	21	17	0.7438	0.9821
3	30	26	22	0.6873	0.9730
4	10	6	2	0.9985	1.000
4	15	11	7	0.9584	0.9996
4	20	16	12	0.8884	0.9971
4	25	21	17	0.8202	0.9929
4	30	26	22	0.7625	0.9876
5	10	6	2	0.9997	1.0000
5	15	11	7	0.9804	0.9998
5	20	16	12	0.9322	0.9991
5	25	21	17	0.8754	0.9972
5	30	26	22	0.8216	0.9942
6	10	6	2	0.9999	1.0000
6	15	11	7	0.9908	0.9999
6	20	16	12	0.9591	0.9998
6	25	21	17	0.9143	0.9989
6	30	26	22	0.8671	0.9972
7	10	6	2	1.0000	1.0000
7	15	11	7	0.9956	0.9999
7	20	16	12	0.9753	0.9998
7	25	21	17	0.9413	0.9995
7	30	26	22	0.9015	0.9986
8	10	6	2	1.0000	1.0000
8	15	11	7	0.9979	0.9980
8	20	16	12	0.9851	0.9999
8	25	21	17	0.9599	0.9997
8	30	26	22	0.9272	0.9993

Table 2.3 Values of R_1, R_2 when Strength follows Exponential and Stress follows Gamma Distribution

l	λ	R_1	R_2
2	0.1	0.8264	0.9780
2	0.2	0.6944	0.9325
2	0.3	0.5917	0.8802
2	0.4	0.5102	0.8277
2	0.5	0.4444	0.7778
2	0.6	0.3906	0.7315
3	0.1	0.7513	0.9591
3	0.2	0.5787	0.8820
3	0.3	0.4552	0.8005
3	0.4	0.3644	0.7247
3	0.5	0.2963	0.6574
3	0.6	0.2441	0.5986
4	0.1	0.6830	0.9366
4	0.2	0.4823	0.8273
4	0.3	0.3501	0.7210
4	0.4	0.2603	0.6291
4	0.5	0.1975	0.5525
4	0.6	0.1526	0.4891
5	0.1	0.6209	0.9115
5	0.2	0.4019	0.7716
5	0.3	0.2693	0.6463
5	0.4	0.1859	0.5451
5	0.5	0.1317	0.4654
5	0.6	0.0954	0.4027
6	0.1	0.5645	0.8845
6	0.2	0.3349	0.7171
6	0.3	0.2072	0.5783
6	0.4	0.1328	0.4733
6	0.5	0.0878	0.3949
6	0.6	0.0596	0.3358
7	0.1	0.5132	0.8564
7	0.2	0.2791	0.6649
7	0.3	0.1594	0.5178
7	0.4	0.0949	0.4131
7	0.5	0.0585	0.3385
7	0.6	0.0373	0.2844

Table 3.1 Reliabilities R_1, R_2 and R_3 when Stress and Strength are mixture of Exponential Distribution for $\mu = \lambda = 1, \theta = \beta$, $\beta = 2$ where $p' + p'' = 1$ and $q' + q'' = 1$

k =0.6 | k =0.8

p'	q'	R_1	R_2	R_3	R_1	R_2	R_3
0	0	0.5	0.7404	0.8796	0.5	0.6984	0.8102
0.2	0	0.5333	0.773	0.9015	0.5333	0.7348	0.8422
0.4	0	0.5667	0.8034	0.9205	0.5667	0.7686	0.87
0.6	0	0.6	0.8314	0.9367	0.6	0.7998	0.894
0.8	0	0.6333	0.857	0.9505	0.6333	0.8285	0.9145
1	0	0.6667	0.8803	0.9619	0.6667	0.8546	0.9318
0	0.2	0.4667	0.7023	0.8483	0.4667	0.6588	0.7718
0.2	0.2	0.5	0.7371	0.8737	0.5	0.6969	0.807
0.4	0.2	0.5333	0.7695	0.8959	0.5333	0.7324	0.8378
0.6	0.2	0.5667	0.7995	0.9151	0.5667	0.7653	0.8647
0.8	0.2	0.6	0.8271	0.9316	0.6	0.7957	0.8878
1	0.2	0.6333	0.8524	0.9455	0.6333	0.8234	0.9075
0	0.4	0.4333	0.6641	0.817	0.4333	0.6193	0.7333
0.2	0.4	0.4667	0.7011	0.8459	0.4667	0.6591	0.7718
0.4	0.4	0.5	0.7355	0.8714	0.5	0.6963	0.8057
0.6	0.4	0.5333	0.7676	0.8935	0.5333	0.7308	0.8353
0.8	0.4	0.5667	0.7972	0.9126	0.5667	0.7628	0.8611
1	0.4	0.6	0.8244	0.929	0.6	0.7921	0.8832
0	0.6	0.4	0.626	0.7857	0.4	0.5797	0.6948
0.2	0.6	0.4333	0.6651	0.8182	0.4333	0.6212	0.7365
0.4	0.6	0.4667	0.7016	0.8468	0.4667	0.6601	0.7735
0.6	0.6	0.5	0.7357	0.8719	0.5	0.6963	0.806
0.8	0.6	0.5333	0.7673	0.8937	0.5333	0.7299	0.8343
1	0.6	0.5667	0.7964	0.9125	0.5667	0.7609	0.8589
0	0.8	0.3667	0.5879	0.7544	0.3667	0.5401	0.6563
0.2	0.8	0.4	0.6291	0.7904	0.4	0.5834	0.7013
0.4	0.8	0.4333	0.6677	0.8223	0.4333	0.6239	0.7413
0.6	0.8	0.4667	0.7038	0.8503	0.4667	0.6618	0.7766
0.8	0.8	0.5	0.7374	0.8748	0.5	0.6971	0.8076
1	0.8	0.5333	0.7684	0.8961	0.5333	0.7297	0.8346
0	1	0.3333	0.5498	0.7231	0.3333	0.5006	0.6178
0.2	1	0.3667	0.5931	0.7626	0.3667	0.5455	0.6661
0.4	1	0.4	0.6338	0.7977	0.4	0.5878	0.7091
0.6	1	0.4333	0.6719	0.8287	0.4333	0.6273	0.7473
0.8	1	0.4667	0.7075	0.8559	0.4667	0.6642	0.7809
1	1	0.5	0.7404	0.8796	0.5	0.6984	0.8102

Table 3.2 Reliabilities R_1, R_2 and R_3 when Stress and Strength are mixture of Rayleigh Distribution for $\sigma_1 = \sigma_3 = 1$, $\sigma_2 = \sigma_4$, $\sigma_4 = 2$ where $p' + p'' = 1$ and $q' + q'' = 1$

		k=0.6			k=0.8		
p'	q'	R_1	R_2	R_3	R_1	R_2	R_3
0	0	0.5	0.8116	0.9576	0.5	0.731	0.8654
0.2	0	0.44	0.7564	0.9321	0.44	0.6728	0.8211
0.4	0	0.38	0.6932	0.8985	0.38	0.6055	0.7638
0.6	0	0.32	0.6218	0.8556	0.32	0.529	0.6914
0.8	0	0.26	0.5422	0.8023	0.26	0.4434	0.602
1	0	0.2	0.4546	0.7376	0.2	0.3486	0.4934
0	0.2	0.56	0.8435	0.9656	0.56	0.7753	0.8902
0.2	0.2	0.5	0.7953	0.9446	0.5	0.723	0.8527
0.4	0.2	0.44	0.7394	0.9167	0.44	0.6618	0.8036
0.6	0.2	0.38	0.6759	0.8809	0.38	0.5918	0.741
0.8	0.2	0.32	0.6047	0.8362	0.32	0.5129	0.6631
1	0.2	0.26	0.526	0.7816	0.26	0.4251	0.5678
0	0.4	0.62	0.8754	0.9736	0.62	0.8197	0.9151
0.2	0.4	0.56	0.8341	0.9571	0.56	0.7732	0.8843
0.4	0.4	0.5	0.7856	0.9349	0.5	0.7181	0.8434
0.6	0.4	0.44	0.73	0.9061	0.44	0.6545	0.7906
0.8	0.4	0.38	0.6672	0.87	0.38	0.5823	0.7241
1	0.4	0.32	0.5974	0.8256	0.32	0.5016	0.6422
0	0.6	0.68	0.9073	0.9817	0.68	0.8641	0.9399
0.2	0.6	0.62	0.8729	0.9696	0.62	0.8234	0.9159
0.4	0.6	0.56	0.8318	0.9531	0.56	0.7745	0.8833
0.6	0.6	0.5	0.7841	0.9314	0.5	0.7173	0.8402
0.8	0.6	0.44	0.7297	0.9038	0.44	0.6518	0.7852
1	0.6	0.38	0.6688	0.8696	0.38	0.578	0.7166
0	0.8	0.74	0.9392	0.9897	0.74	0.9085	0.9648
0.2	0.8	0.68	0.9117	0.9821	0.68	0.8736	0.9475
0.4	0.8	0.62	0.878	0.9713	0.62	0.8308	0.9231
0.6	0.8	0.56	0.8382	0.9567	0.56	0.78	0.8898
0.8	0.8	0.5	0.7922	0.9376	0.5	0.7212	0.8463
1	0.8	0.44	0.7402	0.9136	0.44	0.6545	0.791
0	1	0.8	0.9712	0.9977	0.8	0.9528	0.9896
0.2	1	0.74	0.9505	0.9946	0.74	0.9238	0.9792
0.4	1	0.68	0.9242	0.9895	0.68	0.8871	0.9629
0.6	1	0.62	0.8923	0.9819	0.62	0.8427	0.9394
0.8	1	0.56	0.8548	0.9715	0.56	0.9707	0.9074
1	1	0.5	0.8116	0.9576	0.5	0.731	0.8654

Table 3.3 Reliabilities R_1, R_2 and R_3 when Stress and Strength are mixture of Weibull Distribution for $\mu = \lambda = \theta = \beta = 1$, $A = C = 1$, $B = D$, $D = 2$ where $p' + p'' = 1$ and $q' + q'' = 1$

		k =0.6			k =0.8		
p'	q'	R_1	R_2	R_3	R_1	R_2	R_3
0	0	0.5	0.8116	0.9576	0.5	0.731	0.8654
0.2	0	0.4895	0.7731	0.9217	0.4895	0.6945	0.8303
0.4	0	0.4790	0.7396	0.891	0.479	0.6641	0.8018
0.6	0	0.4685	0.7109	0.8654	0.4685	0.6398	0.7798
0.8	0	0.458	0.6871	0.8449	0.458	0.6216	0.7641
1	0	0.4475	0.6682	0.8293	0.4475	0.6094	0.7546
0	0.2	0.5076	0.8001	0.9448	0.5076	0.7223	0.8516
0.2	0.2	0.4977	0.769	0.9213	0.4977	0.6939	0.8278
0.4	0.2	0.4878	0.7417	0.8991	0.4878	0.6701	0.8071
0.6	0.2	0.4778	0.7182	0.8782	0.4778	0.6511	0.7898
0.8	0.2	0.4679	0.6985	0.8583	0.4679	0.6368	0.776
1	0.2	0.458	0.6826	0.8394	0.458	0.6272	0.7658
0	0.4	0.5153	0.7886	0.932	0.5153	0.7137	0.8379
0.2	0.4	0.5059	0.7648	0.9208	0.5059	0.6933	0.8252
0.4	0.4	0.4966	0.7438	0.9072	0.4966	0.6762	0.8124
0.6	0.4	0.4872	0.7254	0.8909	0.4872	0.6624	0.7999
0.8	0.4	0.4778	0.7099	0.8717	0.4778	0.6521	0.7879
1	0.4	0.4685	0.697	0.8494	0.4685	0.645	0.7769
0	0.6	0.5229	0.7772	0.9192	0.5229	0.7051	0.8241
0.2	0.6	0.5141	0.7607	0.917	0.5141	0.6927	0.8227
0.4	0.6	0.5053	0.7458	0.9153	0.5053	0.6822	0.8177
0.6	0.6	0.4966	0.7327	0.9037	0.4966	0.6738	0.8099
0.8	0.6	0.4878	0.7213	0.8852	0.4878	0.6673	0.7998
1	0.6	0.479	0.7115	0.8595	0.479	0.6628	0.788
0	0.8	0.5306	0.7657	0.9064	0.5306	0.6964	0.8104
0.2	0.8	0.5223	0.7565	0.8939	0.5223	0.692	0.7941
0.4	0.8	0.5141	0.7479	0.8893	0.5141	0.6883	0.7901
0.6	0.8	0.5059	0.74	0.8856	0.5059	0.6851	0.7865
0.8	0.8	0.4977	0.7326	0.8761	0.4977	0.6826	0.7811
1	0.8	0.4895	0.7259	0.867	0.4895	0.6806	0.7791
0	1	0.5382	0.7542	0.8937	0.5382	0.6878	0.7966
0.2	1	0.5306	0.7523	0.8895	0.5306	0.6814	0.7903
0.4	1	0.5229	0.75	0.8815	0.5229	0.6743	0.7883
0.6	1	0.5153	0.7472	0.8742	0.5153	0.6722	0.78
0.8	1	0.5076	0.744	0.8621	0.5076	0.6707	0.7791
1	1	0.5	0.7404	0.8596	0.5	0.6636	0.7728

Table 3.4 Reliabilities R_1, R_2 and R_3 when Stress and Strength are mixture of Weibull Distribution for $\lambda = \mu = 1, \theta = \beta$, $A = B = C = D = 1$, β=2 where $p' + p'' = 1$ and $q' + q'' = 1$

		k=0.6			k=0.8		
p'	q'	R_1	R_2	R_3	R_1	R_2	R_3
0	0	0.5	0.5694	0.7083	0.5	0.5292	0.64
0.2	0	0.4666	0.5408	0.6882	0.4666	0.5006	0.6151
0.4	0	0.4331	0.5098	0.6648	0.4331	0.4694	0.586
0.6	0	0.3997	0.4765	0.6376	0.3997	0.4356	0.5524
0.8	0	0.3663	0.4409	0.6064	0.3663	0.3993	0.5139
1	0	0.3329	0.4029	0.571	0.3329	0.3605	0.4704
0	0.2	0.5333	0.6316	0.759	0.5333	0.5943	0.6984
0.2	0.2	0.4999	0.604	0.7437	0.4999	0.5662	0.6768
0.4	0.2	0.4665	0.5741	0.7252	0.4665	0.5355	0.6511
0.6	0.2	0.4331	0.5419	0.7031	0.4331	0.5022	0.621
0.8	0.2	0.3997	0.5073	0.6771	0.3997	0.4664	0.586
1	0.2	0.3663	0.4704	0.6468	0.3663	0.4281	0.5459
0	0.4	0.5667	0.6938	0.8097	0.5667	0.6594	0.7567
0.2	0.4	0.5333	0.6673	0.7992	0.5333	0.6318	0.7384
0.4	0.4	0.4999	0.6384	0.7857	0.4999	0.6016	0.7162
0.6	0.4	0.4665	0.6073	0.7687	0.4665	0.5689	0.6896
0.8	0.4	0.4331	0.5738	0.7479	0.4331	0.5336	0.6581
1	0.4	0.3997	0.538	0.7227	0.3997	0.4958	0.6214
0	0.6	0.6	0.756	0.8605	0.6	0.7245	0.8151
0.2	0.6	0.5666	0.7305	0.8547	0.5666	0.6974	0.8001
0.4	0.6	0.5333	0.7028	0.8462	0.5333	0.6677	0.7813
0.6	0.6	0.4999	0.6727	0.8343	0.4999	0.6355	0.7582
0.8	0.6	0.4665	0.6402	0.8186	0.4665	0.6007	0.7302
1	0.6	0.4331	0.6055	0.7985	0.4331	0.5634	0.6969
0	0.8	0.6333	0.8182	0.9112	0.6333	0.7896	0.8734
0.2	0.8	0.6	0.7938	0.9102	0.6	0.7629	0.8617
0.4	0.8	0.5666	0.7671	0.9067	0.5666	0.7338	0.8464
0.6	0.8	0.5333	0.738	0.8999	0.5333	0.7021	0.8268
0.8	0.8	0.4999	0.7067	0.8893	0.4999	0.6678	0.8023
1	0.8	0.4666	0.673	0.8744	0.4666	0.631	0.7724
0	1	0.6667	0.8803	0.9619	0.6667	0.8546	0.9318
0.2	1	0.6333	0.857	0.9557	0.6333	0.8285	0.9234
0.4	1	0.6	0.8314	0.9371	0.6	0.7999	0.9115
0.6	1	0.5667	0.8034	0.9155	0.5667	0.7687	0.8897
0.8	1	0.5333	0.7731	0.89	0.5333	0.735	0.8612
1	1	0.5	0.7405	0.8606	0.5	0.6987	0.8309

Table 4.1 Values of $R(1)$, $R(2)$, $R(3)$ and R_3 for Failure model I where Stress-Strength Distributions are Exponential

θ_1	θ_2	θ_3	α_1	α_2	k	$R(1)$	$R(2)$	$R(3)$	R_3
0.5	1	2	1	1	0.1	0.4444	0.4358	0.1122	0.9925
1	1	2	1	1	0.1	0.2500	0.5997	0.1414	0.9911
0.5	1	2	1	3	0.1	0.5714	0.3589	0.0662	0.9966
1	1	2	1	3	0.1	0.3750	0.5313	0.0894	0.9957
0.5	1	2	3	1	0.1	0.5714	0.3589	0.0662	0.9966
1	1	2	3	1	0.1	0.3750	0.5313	0.0894	0.9957
0.5	1	2	3	3	0.1	0.7347	0.2421	0.0227	0.9994
1	1	2	3	3	0.1	0.5625	0.4011	0.0355	0.9992
0.5	1	2	1	2	0.1	0.5333	0.3850	0.0774	0.9958
1	1	2	1	2	0.1	0.3333	0.5586	0.1029	0.9948
0.5	1	2	2	1	0.1	0.5333	0.3850	0.0774	0.9958
1	1	2	2	1	0.1	0.3333	0.5586	0.1029	0.9948
0.5	1	2	1	1	0.5	0.4440	0.1944	0.1044	0.7433
1	1	2	1	1	0.5	0.2500	0.2844	0.1456	0.6800
0.5	1	2	1	3	0.5	0.5714	0.1964	0.0881	0.8560
1	1	2	1	3	0.5	0.3750	0.3048	0.1298	0.8095
0.5	1	2	3	1	0.5	0.5714	0.1964	0.0881	0.8560
1	1	2	3	1	0.5	0.3750	0.3048	0.1298	0.8095
0.5	1	2	3	3	0.5	0.7347	0.1722	0.0541	0.9610
1	1	2	3	3	0.5	0.5625	0.2902	0.0877	0.9405
0.5	1	2	1	2	0.5	0.5333	0.2000	0.0952	0.8286
1	1	2	1	2	0.5	0.3333	0.3048	0.1381	0.7762
0.5	1	2	2	1	0.5	0.5333	0.2000	0.0952	0.8286
1	1	2	2	1	0.5	0.3333	0.3048	0.1381	0.7762
0.5	1	2	1	1	0.9	0.4444	0.1034	0.0241	0.5720
1	1	2	1	1	0.9	0.2500	0.1581	0.0376	0.4457
0.5	1	2	1	3	0.9	0.5714	0.1208	0.0296	0.7218
1	1	2	1	3	0.9	0.3750	0.1937	0.0474	0.6165
0.5	1	2	3	1	0.9	0.5714	0.1208	0.0296	0.7218
1	1	2	3	1	0.9	0.3750	0.1937	0.0478	0.6165
0.5	1	2	3	3	0.9	0.7347	0.1268	0.0313	0.8928
1	1	2	3	3	0.9	0.5625	0.2169	0.0530	0.8324
0.5	1	2	1	2	0.9	0.5333	0.1179	0.0287	0.6799
1	1	2	1	2	0.9	0.3333	0.1861	0.0457	0.5652
0.5	1	2	2	1	0.9	0.5333	0.1179	0.0287	0.6799
1	1	2	2	1	0.9	0.3333	0.1861	0.0457	0.5652

Table 4.2 Values of $R(1)$, $R(2)$, $R(3)$ and R_3 for Failure model II where Stress-Strength Distributions are Exponential

a_1	a_2	a_3	b_1	b_2	b_3	α_1	α_2	α_3	k	$R(1)$	$R(2)$	$R(3)$	R_3
0.1	0.2	0.3	4	4	4	0.2	0.2	0.2	0.5	0.1337	0.3248	0.2891	0.7477
0.1	0.2	0.3	4	4	4	0.2	0.2	0.2	0.7	0.1337	0.2119	0.2505	0.5961
0.1	0.2	0.3	4	4	4	0.2	0.2	0.2	1	0.1337	0.1158	0.0703	0.3198
0.1	0.2	0.3	4	4	4	0.2	0.2	0.2	1.2	0.1337	0.0809	0.0498	0.2644
0.1	0.2	0.3	4	4	4	1	1	1	0.5	0.5217	0.1466	0.0309	0.6901
0.1	0.2	0.3	4	4	4	1	1	1	0.7	0.5217	0.2041	0.0831	0.7999
0.1	0.2	0.3	4	4	4	1	1	1	1	0.5217	0.2498	0.1218	0.8843
0.1	0.2	0.3	4	4	4	1	1	1	1.2	0.5217	0.2597	0.1270	0.9084
1	0.2	0.3	4	4	4	0.6	0.6	0.6	0.5	0.5045	0.2337	0.0620	0.8002
1	0.2	0.3	4	4	4	0.6	0.6	0.6	0.7	0.5045	0.2504	0.0884	0.8433
1	0.2	0.3	4	4	4	0.6	0.6	0.6	1	0.5045	0.2519	0.1239	0.8803
1	0.2	0.3	4	4	4	0.6	0.6	0.6	1.2	0.5045	0.2657	0.1354	0.9056
0.3	0.3	0.3	5	5	5	0.5	0.5	0.5	0.5	0.4720	0.2088	0.0527	0.7336
0.3	0.3	0.3	5	5	5	0.5	0.5	0.5	0.7	0.4720	0.2496	0.1084	0.8300
0.3	0.3	0.3	5	5	5	0.5	0.5	0.5	1	0.4720	0.2592	0.1316	0.8628
0.3	0.3	0.3	5	5	5	0.5	0.5	0.5	1.2	0.4720	0.2699	0.1435	0.8854
0.1	0.4	0.5	5	4	3	0.8	0.8	0.8	0.5	0.3594	0.1265	0.0209	0.5069
0.1	0.4	0.5	5	4	3	0.8	0.8	0.8	0.7	0.3594	0.1887	0.0865	0.6347
0.1	0.4	0.5	5	4	3	0.8	0.8	0.8	1	0.3594	0.2302	0.1475	0.7372
0.1	0.4	0.5	5	4	3	0.8	0.8	0.8	1.2	0.3594	0.2333	0.1487	0.7414
0.2	0.4	0.6	4	4	4	0.9	0.9	0.9	0.5	0.3008	0.0802	0.0082	0.3893
0.2	0.4	0.6	4	4	4	0.9	0.9	0.9	0.7	0.3008	0.1450	0.0609	0.5067
0.2	0.4	0.6	4	4	4	0.9	0.9	0.9	1	0.3008	0.2103	0.1471	0.6582
0.2	0.4	0.6	4	4	4	0.9	0.9	0.9	1.2	0.3008	0.2303	0.1582	0.6894
0.3	0.5	0.7	3	4	5	0.1	0.1	0.1	0.5	0.0230	0.0977	0.2289	0.3495
0.3	0.5	0.7	3	4	5	0.1	0.1	0.1	0.7	0.0230	0.0502	0.0961	0.1693
0.3	0.5	0.7	3	4	5	0.1	0.1	0.1	1	0.0230	0.0225	0.0219	0.0674
0.3	0.5	0.7	3	4	5	0.1	0.1	0.1	1.2	0.0230	0.0144	0.0090	0.0464
0.2	0.4	0.6	4	4	4	0.1	0.1	0.1	0.5	0.0244	0.1034	0.2392	0.3670
0.2	0.4	0.6	4	4	4	0.1	0.1	0.1	0.7	0.0244	0.0533	0.1014	0.1791
0.2	0.4	0.6	4	4	4	0.1	0.1	0.1	1	0.0244	0.0238	0.0233	0.0715
0.2	0.4	0.6	4	4	4	0.1	0.1	0.1	1.2	0.0244	0.0153	0.0095	0.0493
0.3	0.5	0.7	3	4	5	0.9	0.9	0.9	0.5	0.2924	0.0808	0.0083	0.3815
0.3	0.5	0.7	3	4	5	0.9	0.9	0.9	0.7	0.2924	0.1447	0.0616	0.4987
0.3	0.5	0.7	3	4	5	0.9	0.9	0.9	1	0.2924	0.2069	0.1464	0.6457
0.3	0.5	0.7	3	4	5	0.9	0.9	0.9	1.2	0.2924	0.2250	0.1562	0.6736

Table 4.3 Values of $R(1)$, $R(2)$, $R(3)$ and R_3 for Failure model III where Stress-Strength Distributions are Exponential for $a_{11} = a_{12} = a_{13} = a_{21} = a_{22} = 0.1$, $a_{23} = a_{31} = a_{32} = a_{33} = 0.1$

b_{11}	b_{12}	b_{13}	b_{21}	b_{22}	b_{23}	b_{31}	b_{32}	b_{33}	α_1	α_2	α_3	k	$R(1)$	$R(2)$	$R(3)$	R_3
2	3	3	2	3	3	2	3	3	0.2	0.2	0.2	0.1	0.0577	0.5025	0.0011	0.5613
2	3	3	2	3	3	2	3	3	0.2	0.2	0.2	0.5	0.0577	0.2097	0.3661	0.6335
2	3	3	2	3	3	2	3	3	0.2	0.2	0.2	0.9	0.0577	0.0685	0.0794	0.2056
2	3	3	2	3	3	2	3	3	0.2	0.2	0.2	1.3	0.0577	0.0294	0.0149	0.1022
2	3	3	2	3	3	2	3	3	0.5	0.5	0.5	0.1	0.3093	0.1541	0.0001	0.4634
2	3	3	2	3	3	2	3	3	0.5	0.5	0.5	0.5	0.3093	0.3886	0.1611	0.8589
2	3	3	2	3	3	2	3	3	0.5	0.5	0.5	0.9	0.3093	0.2442	0.1778	0.7313
2	3	3	2	3	3	2	3	3	0.5	0.5	0.5	1.3	0.3093	0.1442	0.1714	0.5248
2	3	3	2	3	3	2	3	3	1.0	1.0	1.0	0.1	0.5626	0.0218	0.0001	0.5844
2	3	3	2	3	3	2	3	3	1.0	1.0	1.0	0.5	0.5626	0.2337	0.0615	0.8573
2	3	3	2	3	3	2	3	3	1.0	1.0	1.0	0.9	0.5626	0.2539	0.1083	0.9248
2	3	3	2	3	3	2	3	3	1.0	1.0	1.0	1.3	0.5626	0.2125	0.0855	0.0606
6	8	9	6	8	9	6	8	9	0.2	0.2	0.2	0.1	0.4307	0.1188	0.0011	0.5505
6	8	9	6	8	9	6	8	9	0.2	0.2	0.2	0.5	0.4307	0.0061	0.0296	0.4664
6	8	9	6	8	9	6	8	9	0.2	0.2	0.2	0.9	0.4307	0.0013	0.0017	0.4337
6	8	9	6	8	9	6	8	9	0.2	0.2	0.2	1.3	0.4307	0.0005	0.0002	0.4314
4	5	6	4	5	6	4	5	6	0.3	0.3	0.3	0.1	0.4027	0.2428	0.0001	0.6456
4	5	6	4	5	6	4	5	6	0.3	0.3	0.3	0.5	0.4027	0.4146	0.1258	0.9431
4	5	6	4	5	6	4	5	6	0.3	0.3	0.3	0.9	0.4027	0.2721	0.1652	0.8400
4	5	6	4	5	6	4	5	6	0.3	0.3	0.3	1.3	0.4027	0.1666	0.0769	0.6462
1	2	3	1	2	3	1	2	3	0.4	0.4	0.4	0.1	0.0980	0.2641	0.0001	0.3621
1	2	3	1	2	3	1	2	3	0.4	0.4	0.4	0.5	0.0980	0.2566	0.2874	0.6419
1	2	3	1	2	3	1	2	3	0.4	0.4	0.4	0.9	0.0980	0.1075	0.1139	0.3194
1	2	3	1	2	3	1	2	3	0.4	0.4	0.4	1.3	0.0980	0.0520	0.0273	0.1773

Table 4.4 Values of $R(1)$, $R(2)$, $R(3)$ and R_3 for Failure model I where Stress-Strength Distributions are Rayleigh

σ_1	σ_2	σ_3	β_1	β_2	k	$R(1)$	$R(2)$	$R(3)$	R_3
0.5	1	2	1	1	0.1	0.1600	0.8209	0.0048	0.9857
1	1	2	1	1	0.1	0.2500	0.7328	0.0043	0.9871
0.5	1	2	1	3	0.1	0.0216	0.8868	0.0229	0.9313
1	1	2	1	3	0.1	0.0500	0.8590	0.0227	0.9318
0.5	1	2	3	1	0.1	0.0216	0.8868	0.0229	0.9313
1	1	2	3	1	0.1	0.0500	0.8590	0.0227	0.9318
0.5	1	2	1	2	0.1	0.0471	0.9052	0.0119	0.9642
1	1	2	1	2	0.1	0.1000	0.8533	0.0117	0.9650
0.5	1	2	2	1	0.1	0.0471	0.9052	0.0119	0.9642
1	1	2	2	1	0.1	0.1000	0.8533	0.0117	0.9650
0.5	1	2	1	1	0.5	0.1600	0.4949	0.0826	0.7375
1	1	2	1	1	0.5	0.2500	0.4425	0.0735	0.7659
0.5	1	2	1	3	0.5	0.0216	0.2267	0.1570	0.4054
1	1	2	1	3	0.5	0.0500	0.2099	0.1542	0.4141
0.5	1	2	3	1	0.5	0.0216	0.2267	0.1570	0.4054
1	1	2	3	1	0.5	0.0500	0.2099	0.1542	0.4141
0.5	1	2	1	2	0.5	0.0471	0.3577	0.1347	0.5395
1	1	2	1	2	0.5	0.1000	0.3259	0.1296	0.5555
0.5	1	2	2	1	0.5	0.0471	0.3577	0.1347	0.5395
1	1	2	2	1	0.5	0.1000	0.3259	0.1296	0.5555
0.5	1	2	1	1	0.9	0.1600	0.1867	0.1109	0.4576
1	1	2	1	1	0.9	0.2500	0.1786	0.0952	0.5238
0.5	1	2	1	3	0.9	0.0216	0.0511	0.0724	0.1451
1	1	2	1	3	0.9	0.0500	0.0461	0.0682	0.1642
0.5	1	2	3	1	0.9	0.0216	0.0511	0.0724	0.1451
1	1	2	3	1	0.9	0.0500	0.0461	0.0682	0.1642
0.5	1	2	1	2	0.9	0.0471	0.0963	0.1009	0.2442
1	1	2	1	2	0.9	0.1000	0.0871	0.0928	0.2800
0.5	1	2	2	1	0.9	0.0471	0.0963	0.1009	0.2442
1	1	2	2	1	0.9	0.1000	0.0871	0.0928	0.2800
0.5	1	2	1	1	1.3	0.1600	0.0488	0.0511	0.2600
1	1	2	1	1	1.3	0.2500	0.0648	0.0425	0.3572
0.5	1	2	1	3	1.3	0.0216	0.0115	0.0147	0.0478
1	1	2	1	3	1.3	0.0500	0.0122	0.0126	0.0748
0.5	1	2	3	1	1.3	0.0216	0.0115	0.0147	0.0478
1	1	2	3	1	1.3	0.0500	0.0122	0.0126	0.0748
0.5	1	2	1	2	1.3	0.0471	0.0227	0.0272	0.0970
1	1	2	1	2	1.3	0.1000	0.0249	0.0231	0.1479
0.5	1	2	2	1	1.3	0.0471	0.0227	0.0272	0.0970
1	1	2	2	1	1.3	0.1000	0.0249	0.0231	0.1479

Table 4.5 Values of $R(1)$, $R(2)$, $R(3)$ and R_3 for Failure model II where Stress-Strength Distributions are Rayleigh

a_1	a_2	b_1	b_2	k	$R(1)$	$R(2)$	$R(3)$	R_3
0.1	0.2	4	4	0.5	0.7096	0.1592	0.0395	0.9083
0.1	0.2	4	4	0.7	0.7096	0.1876	0.0557	0.9529
0.1	0.2	4	4	1	0.7096	0.2061	0.0598	0.9755
0.1	0.2	4	4	1.2	0.7096	0.2082	0.0578	0.9757
0.1	0.3	4	4	0.5	0.6405	0.1614	0.0400	0.8419
0.1	0.3	4	4	0.7	0.6405	0.2012	0.0699	0.9116
0.1	0.3	4	4	1	0.6405	0.2303	0.0828	0.9535
0.1	0.3	4	4	1.2	0.6405	0.2363	0.0805	0.9572
1	0.4	4	4	0.5	0.2279	0.0468	0.0027	0.2774
1	0.4	4	4	0.7	0.2279	0.1024	0.0383	0.3687
1	0.4	4	4	1	0.2279	0.1760	0.1359	0.5398
1	0.4	4	4	1.2	0.2279	0.2097	0.1709	0.6086
0.3	0.5	5	5	0.5	0.4403	0.1130	0.0182	0.5715
0.3	0.5	5	5	0.7	0.4403	0.1780	0.0746	0.6929
0.3	0.5	5	5	1	0.4403	0.2464	0.1379	0.8247
0.3	0.5	5	5	1.2	0.4403	0.2750	0.1502	0.8655
0.1	0.6	5	4	0.5	0.4764	0.1290	0.0240	0.6294
0.1	0.6	5	4	0.7	0.4764	0.1909	0.0796	0.7470
0.1	0.6	5	4	1	0.4764	0.2494	0.1306	0.8565
0.1	0.6	5	4	1.2	0.4764	0.2703	0.1363	0.8831
0.2	0.7	5	4	0.5	0.3884	0.1010	0.0140	0.5033
0.2	0.7	5	4	0.7	0.3884	0.1674	0.0707	0.6264
0.2	0.7	5	4	1	0.3884	0.2375	0.1453	0.7712
0.2	0.7	5	4	1.2	0.3884	0.2655	0.1606	0.8145
0.3	0.4	6	7	0.5	0.4943	0.1247	0.0232	0.6421
0.3	0.4	6	7	0.7	0.4943	0.1860	0.0766	0.7569
0.3	0.4	6	7	1	0.4943	0.2500	0.1264	0.8706
0.3	0.4	6	7	1.2	0.4943	0.2786	0.1356	0.9085
0.2	0.6	2	2	0.5	0.2826	0.1323	0.0237	0.4386
0.2	0.6	2	2	0.7	0.2826	0.1827	0.0960	0.5613
0.2	0.6	2	2	1	0.2826	0.2027	0.1454	0.6307
0.2	0.6	2	2	1.2	0.2826	0.1970	0.1325	0.6121
0.5	0.8	6	6	0.5	0.2699	0.0542	0.0037	0.3279
0.5	0.8	6	6	0.7	0.2699	0.1139	0.0434	0.4272
0.5	0.8	6	6	1	0.2699	0.1971	0.1439	0.6109
0.5	0.8	6	6	1.2	0.2699	0.2414	0.1886	0.6999
0.3	0.4	5	5	0.5	0.4871	0.1265	0.0235	0.6371
0.3	0.4	5	5	0.7	0.4871	0.1882	0.0778	0.7531
0.3	0.4	5	5	1	0.4871	0.2498	0.1281	0.8651
0.3	0.4	5	5	1.2	0.4871	0.2744	0.1353	0.8968

Table 4.6 Values of $R(1)$, $R(2)$, $R(3)$ and R_3 for Failure model III where Stress-Strength Distributions are Rayleigh

a_{11}	a_{12}	a_{21}	a_{22}	a_{31}	a_{32}	b_{11}	b_{12}	b_{21}	b_{22}	b_{31}	b_{32}	k	$R(1)$	$R(2)$	$R(3)$	R_3
0.1	0.1	0.1	0.1	0.1	0.1	2	3	2	3	2	3	0.1	0.6580	0.0463	0.0001	0.7043
0.1	0.1	0.1	0.1	0.1	0.1	2	3	2	3	2	3	0.5	0.6580	0.2235	0.0532	0.9343
0.1	0.1	0.1	0.1	0.1	0.1	2	3	2	3	2	3	0.9	0.6580	0.2311	0.0762	0.9652
0.1	0.1	0.1	0.1	0.1	0.1	2	3	2	3	2	3	1.3	0.6580	0.2011	0.0694	0.9284
0.2	0.2	0.2	0.2	0.2	0.2	6	8	6	8	6	8	0.1	0.6680	0.0061	0.0001	0.6741
0.2	0.2	0.2	0.2	0.2	0.2	6	8	6	8	6	8	0.5	0.6680	0.1492	0.0369	0.8541
0.2	0.2	0.2	0.2	0.2	0.2	6	8	6	8	6	8	0.9	0.6680	0.2125	0.0729	0.9534
0.2	0.2	0.2	0.2	0.2	0.2	6	8	6	8	6	8	1.3	0.6680	0.2406	0.0691	0.9777
2	4	2	4	2	4	0.3	0.3	0.3	0.3	0.3	0.3	0.1	0.4375	0.0014	0.0001	0.4389
2	4	2	4	2	4	0.3	0.3	0.3	0.3	0.3	0.3	0.5	0.4375	0.1637	0.0361	0.6373
2	4	2	4	2	4	0.3	0.3	0.3	0.3	0.3	0.3	0.9	0.4375	0.2411	0.1331	0.8116
2	4	2	4	2	4	0.3	0.3	0.3	0.3	0.3	0.3	1.3	0.4375	0.2437	0.1259	0.8071
6	8	6	8	6	8	0.4	0.4	0.4	0.4	0.4	0.4	0.1	0.4474	0.0002	0.0001	0.4476
6	8	6	8	6	8	0.4	0.4	0.4	0.4	0.4	0.4	0.5	0.4474	0.1116	0.0180	0.5770
6	8	6	8	6	8	0.4	0.4	0.4	0.4	0.4	0.4	0.9	0.4474	0.2267	0.1212	0.7954
6	8	6	8	6	8	0.4	0.4	0.4	0.4	0.4	0.4	1.3	0.4474	0.2937	0.1536	0.8948
7	9	7	9	7	9	0.5	0.5	0.5	0.5	0.5	0.5	0.1	0.3673	0.0002	0.0001	0.3673
7	9	7	9	7	9	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.3673	0.0856	0.0100	0.4629
7	9	7	9	7	9	0.5	0.5	0.5	0.5	0.5	0.5	0.9	0.3673	0.2081	0.1235	0.6989
7	9	7	9	7	9	0.5	0.5	0.5	0.5	0.5	0.5	1.3	0.3673	0.2908	0.1840	0.8420
5	6	5	6	5	6	0.6	0.6	0.6	0.6	0.6	0.6	0.1	0.2962	0.0001	0.0002	0.2962
5	6	5	6	5	6	0.6	0.6	0.6	0.6	0.6	0.6	0.5	0.2962	0.0638	0.0053	0.3653
5	6	5	6	5	6	0.6	0.6	0.6	0.6	0.6	0.6	0.9	0.2962	0.1837	0.1176	0.5974
5	6	5	6	5	6	0.6	0.6	0.6	0.6	0.6	0.6	1.3	0.2962	0.2659	0.1912	0.7533

Table 5.1 Values of R_1, R_2 for Exponential Stress-Strength when Strength parameter λ is random and Uniformly distributed in the range (a,b)

a	b	μ	k	R_1	R_2
1	2	2	4	0.5754	0.6877
1	2	3	6	0.6694	0.8332
1	2	4	8	0.7293	0.9317
1	2	5	10	0.7708	0.9671
1	2	6	12	0.8012	0.9810
1	2	7	14	0.8245	0.9917
2	3	2	3	0.4463	0.5778
2	3	3	5	0.5470	0.7094
2	3	4	7	0.6166	0.7922
2	3	5	9	0.6677	0.8469
2	3	6	11	0.7067	0.8860
2	3	7	13	0.7375	0.9159
3	4	2	0.2	0.3646	0.8152
3	4	3	0.4	0.4625	0.8478
3	4	4	0.6	0.5341	0.9195
3	4	5	0.8	0.5889	0.9786
3	4	6	1.0	0.6322	0.9836
3	4	7	1.2	0.6672	0.9903
4	5	2	1	0.3083	0.5173
4	5	3	4	0.4006	0.5378
4	5	4	7	0.4711	0.5838
4	5	5	10	0.5268	0.6271
4	5	6	13	0.5719	0.6652
4	5	7	16	0.6091	0.6980

Table 5.2 Values of R_1, R_2 for Exponential Stress-Strength when Strength parameter λ is random having Two-Point Distribution

p	λ_1	λ_2	μ	k	R_1	R_2
0	1	2	1	1	0.3333	0.4667
0	1	2	3	2	0.6000	0.6952
0	1	2	5	3	0.7143	0.7842
0	1	2	7	4	0.7778	0.8327
0	1	2	9	5	0.8182	0.8633
0.2	1	2	1	2	0.3667	0.4332
0.2	1	2	3	4	0.6300	0.6779
0.2	1	2	5	6	0.7381	0.7732
0.2	1	2	7	8	0.7972	0.8248
0.2	1	2	9	10	0.8345	0.8572
0.4	1	2	1	0.1	0.4000	0.8884
0.4	1	2	3	0.2	0.6600	0.9480
0.4	1	2	5	0.3	0.7619	0.9649
0.4	1	2	7	0.4	0.8167	0.9734
0.4	1	2	9	0.5	0.8509	0.9785
0.6	1	2	1	0.2	0.4333	0.8334
0.6	1	2	3	0.4	0.6900	0.9237
0.6	1	2	5	0.6	0.7857	0.9492
0.6	1	2	7	0.8	0.8361	0.9617
0.6	1	2	9	1	0.8673	0.9692
0.7	1	2	1	1	0.4500	0.6137
0.7	1	2	3	3	0.7050	0.7787
0.7	1	2	5	5	0.7976	0.8453
0.7	1	2	7	7	0.8458	0.8811
0.7	1	2	9	9	0.8755	0.9034
0.8	1	2	1	1	0.4667	0.6320
0.8	1	2	3	3	0.7200	0.7935
0.8	1	2	5	5	0.8095	0.8569
0.8	1	2	7	7	0.8556	0.8905
0.8	1	2	9	9	0.8836	0.9113
0.9	1	2	1	3	0.4833	0.5333
0.9	1	2	3	5	0.7350	0.7773
0.9	1	2	5	7	0.8214	0.8542
0.9	1	2	7	9	0.8653	0.8917
0.9	1	2	9	11	0.8918	0.9138
1	1	2	1	10	0.5000	0.5076
1	1	2	3	12	0.7500	0.7625
1	1	2	5	14	0.8333	0.8465
1	1	2	7	16	0.8750	0.8877
1	1	2	9	18	0.9000	0.9119

Table 5.3 Values of R_1, R_2 for Exponential Stress-Strength when Stress parameter μ is random and Uniformly distributed in the range (c,d)

c	d	λ	k	R_1	R_2
1	2	2	4	0.4246	0.4518
1	2	3	6	0.3306	0.3408
1	2	4	8	0.2707	0.2755
1	2	5	10	0.2292	0.2318
1	2	6	12	0.1988	0.2003
1	2	7	14	0.1755	0.1765
2	3	2	3	0.5537	0.6095
2	3	3	5	0.4530	0.4739
2	3	4	7	0.3834	0.3929
2	3	5	9	0.3323	0.3373
2	3	6	11	0.2933	0.2962
2	3	7	13	0.2625	0.2643
3	4	2	0.2	0.6354	0.9400
3	4	3	0.4	0.5375	0.8275
3	4	4	0.6	0.4659	0.7051
3	4	5	0.8	0.4111	0.5973
3	4	6	1.0	0.3678	0.5101
3	4	7	1.2	0.3328	0.4415
4	5	2	1	0.6917	0.8545
4	5	3	4	0.5994	0.6413
4	5	4	7	0.5289	0.5440
4	5	5	10	0.4732	0.4801
4	5	6	13	0.4281	0.4318
4	5	7	16	0.3909	0.3931

Table 5.4 Values of R_1, R_2 for Exponential Stress-Strength when Stress parameter μ is random having Two-Point Distribution

q	μ_1	μ_2	λ	k	R_1	R_2
0	1	2	1	1	0.6667	0.8333
0	1	2	3	2	0.4000	0.4682
0	1	2	5	3	0.2857	0.3125
0	1	2	7	4	0.2222	0.2348
0	1	2	9	5	0.1818	0.1887
0.2	1	2	1	2	0.6333	0.7300
0.2	1	2	3	4	0.3700	0.3931
0.2	1	2	5	6	0.2619	0.2696
0.2	1	2	7	8	0.2028	0.2061
0.2	1	2	9	10	0.1655	0.1672
0.4	1	2	1	0.1	0.6000	0.9575
0.4	1	2	3	0.2	0.3400	0.7503
0.4	1	2	5	0.3	0.2381	0.5464
0.4	1	2	7	0.4	0.1833	0.3999
0.4	1	2	9	0.5	0.1491	0.3014
0.6	1	2	1	0.2	0.5667	0.9076
0.6	1	2	3	0.4	0.3100	0.5883
0.6	1	2	5	0.6	0.2143	0.3776
0.6	1	2	7	0.8	0.1639	0.2611
0.6	1	2	9	1	0.1327	0.1939
0.7	1	2	1	3	0.5500	0.6050
0.7	1	2	3	5	0.2950	0.3072
0.7	1	2	5	7	0.2024	0.2067
0.7	1	2	7	9	0.1542	0.1561
0.7	1	2	9	11	0.1245	0.1256
0.8	1	2	1	1	0.5333	0.7000
0.8	1	2	3	3	0.2800	0.3063
0.8	1	2	5	5	0.1905	0.1978
0.8	1	2	7	7	0.1444	0.1474
0.8	1	2	9	9	0.1164	0.1178
0.9	1	2	1	1	0.5167	0.6833
0.9	1	2	3	4	0.2650	0.2805
0.9	1	2	5	7	0.1786	0.1823
0.9	1	2	7	10	0.1347	0.1361
0.9	1	2	9	13	0.1082	0.1088
1	1	2	1	10	0.5000	0.5076
1	1	2	3	12	0.2500	0.2520
1	1	2	5	14	0.1667	0.1676
1	1	2	7	16	0.1250	0.1255
1	1	2	9	18	0.1000	0.1003

Table 6.1 Values of $R(1)$, $R(2)$, $R(3)$ and R_3 when Stress-Strength follows Exponential Distribution

μ	γ	λ	k	$R(1)$	$R(2)$	$R(3)$	R_3
1	0.3	0.3	0.1	0.1442	0.0235	0.0025	0.1702
1	0.5	0.5	0.1	0.1667	0.0361	0.0040	0.2067
1	0.7	0.7	0.1	0.1716	0.0475	0.0054	0.2245
1	0.9	0.9	0.1	0.1792	0.0581	0.0069	0.2342
2	0.3	0.3	0.2	0.1003	0.0247	0.0052	0.1302
2	0.5	0.5	0.2	0.1333	0.0375	0.0081	0.1789
2	0.7	0.7	0.2	0.1525	0.0486	0.0108	0.2120
2	0.9	0.9	0.2	0.1633	0.0585	0.0134	0.2353
3	0.3	0.3	0.3	0.0758	0.0254	0.0079	0.1090
3	0.5	0.5	0.3	0.1071	0.0387	0.0123	0.1581
3	0.7	0.7	0.3	0.1290	0.0500	0.0163	0.1953
3	0.9	0.9	0.3	0.1442	0.0599	0.0201	0.2242
4	0.3	0.3	0.4	0.0607	0.0258	0.0106	0.0971
4	0.5	0.5	0.4	0.0889	0.0394	0.0165	0.1448
4	0.7	0.7	0.4	0.1103	0.0511	0.0218	0.1832
4	0.9	0.9	0.4	0.1267	0.0611	0.0266	0.2144
5	0.3	0.3	0.5	0.0505	0.0261	0.0133	0.0899
5	0.5	0.5	0.5	0.0758	0.0400	0.0206	0.1364
5	0.7	0.7	0.5	0.0959	0.0519	0.0271	0.1749
5	0.9	0.9	0.5	0.1122	0.0621	0.0329	0.2072
6	0.3	0.3	0.6	0.0433	0.0263	0.0159	0.0855
6	0.5	0.5	0.6	0.0659	0.0404	0.0246	0.1310
6	0.7	0.7	0.6	0.0847	0.0525	0.0322	0.1695
6	0.9	0.9	0.6	0.1003	0.0630	0.0390	0.2023
7	0.3	0.3	0.7	0.0379	0.0264	0.0185	0.0828
7	0.5	0.5	0.7	0.0583	0.0408	0.0285	0.1276
7	0.7	0.7	0.7	0.0758	0.0530	0.0372	0.1660
7	0.9	0.9	0.7	0.0906	0.0636	0.0448	0.1991
8	0.3	0.3	0.8	0.0336	0.0266	0.0210	0.0812
8	0.5	0.5	0.8	0.0523	0.0410	0.0324	0.1257
8	0.7	0.7	0.8	0.0685	0.0535	0.0420	0.1640
8	0.9	0.9	0.8	0.0825	0.0642	0.0504	0.1972
9	0.3	0.3	0.9	0.0302	0.0266	0.0235	0.0804
9	0.5	0.5	0.9	0.0474	0.0412	0.0361	0.1247
9	0.7	0.7	0.9	0.0625	0.0538	0.0467	0.1629
9	0.9	0.9	0.9	0.0758	0.0647	0.0557	0.1962
10	0.3	0.3	1	0.0275	0.0267	0.0260	0.0802
10	0.5	0.5	1	0.0433	0.0414	0.0397	0.1244
10	0.7	0.7	1	0.0574	0.0541	0.0512	0.1626
10	0.9	0.9	1	0.0700	0.0651	0.0608	0.1959

Table 6.2 Values of $R(1)$, $R(2)$, $R(3)$ and R_3 when Stress-Strength follows Rayleigh Distribution

σ_1	σ_2	σ_3	k	$R(1)$	$R(2)$	$R(3)$	R_3
1	3	3	0.1	0.4091	0.0479	0.0005	0.4575
1	5	5	0.1	0.4630	0.1055	0.0012	0.5697
1	7	7	0.1	0.4804	0.1681	0.0021	0.6506
1	9	9	0.1	0.4880	0.2256	0.0032	0.7168
2	3	3	0.2	0.2647	0.0563	0.0025	0.3235
2	5	5	0.2	0.3788	0.1158	0.0054	0.5000
2	7	7	0.2	0.4258	0.1756	0.0090	0.6144
2	9	9	0.2	0.4551	0.2294	0.0131	0.6976
3	3	3	0.3	0.1667	0.0583	0.0062	0.2312
3	5	5	0.3	0.2907	0.1241	0.0135	0.4256
3	7	7	0.3	0.3657	0.1805	0.0216	0.5678
3	9	9	0.3	0.4091	0.2311	0.0304	0.6706
4	3	3	0.4	0.1098	0.0553	0.0114	0.1764
4	5	5	0.4	0.2193	0.1193	0.0252	0.3638
4	7	7	0.4	0.3025	0.1786	0.0397	0.5207
4	9	9	0.4	0.3584	0.2274	0.0542	0.6400
5	3	3	0.5	0.0763	0.0496	0.0171	0.1430
5	5	5	0.5	0.1667	0.1111	0.0387	0.3165
5	7	7	0.5	0.2475	0.1695	0.0608	0.4778
5	9	9	0.5	0.3092	0.2173	0.0815	0.6079
6	3	3	0.6	0.0556	0.0431	0.0223	0.1210
6	5	5	0.6	0.1289	0.0995	0.0512	0.2796
6	7	7	0.6	0.2025	0.1553	0.0805	0.4383
6	9	9	0.6	0.2647	0.2018	0.1066	0.5731
7	3	3	0.7	0.0421	0.0366	0.0256	0.1042
7	5	5	0.7	0.1016	0.0866	0.0595	0.2477
7	7	7	0.7	0.1667	0.1384	0.0939	0.3990
7	9	9	0.7	0.2263	0.1829	0.1236	0.5327
8	3	3	0.8	0.0328	0.0307	0.0261	0.0896
8	5	5	0.8	0.0817	0.0741	0.0615	0.2173
8	7	7	0.8	0.1384	0.1209	0.0978	0.3571
8	9	9	0.8	0.1938	0.1625	0.1288	0.4851
9	3	3	0.9	0.0263	0.0255	0.0242	0.0760
9	5	5	0.9	0.0668	0.0626	0.0576	0.1870
9	7	7	0.9	0.1161	0.1040	0.0924	0.3126
9	9	9	0.9	0.1667	0.1423	0.1225	0.4315
10	3	3	1	0.0215	0.0211	0.0206	0.0632
10	5	5	1	0.0556	0.0525	0.0497	0.1577
10	7	7	1	0.0984	0.0887	0.0808	0.2679
10	9	9	1	0.1441	0.1234	0.1081	0.3756

Table 6.3 Values of $R(1)$, $R(2)$, $R(3)$ and R_3 when Stress-Strength follows Lindley Distribution

μ	θ	γ	k	$R(1)$	$R(2)$	$R(3)$	R_3
1	3	4	2	0.0553	0.0279	0.0139	0.0972
1	5	6	2	0.0404	0.0200	0.0099	0.0703
1	7	8	2	0.0311	0.0154	0.0076	0.0541
1	9	10	2	0.0252	0.0124	0.0062	0.0437
2	3	4	3	0.1000	0.0415	0.0148	0.1564
2	5	6	3	0.0829	0.0312	0.0108	0.1248
2	7	8	3	0.0683	0.0246	0.0084	0.1013
2	9	10	3	0.0574	0.0203	0.0069	0.0846
3	3	4	4	0.1217	0.0481	0.0135	0.1833
3	5	6	4	0.1111	0.0366	0.0098	0.1575
3	7	8	4	0.0964	0.0293	0.0077	0.1334
3	9	10	4	0.0839	0.0243	0.0063	0.1145
4	3	4	5	0.1305	0.0521	0.0122	0.1948
4	5	6	5	0.1285	0.0398	0.0087	0.1770
4	7	8	5	0.1165	0.0319	0.0068	0.1553
4	9	10	5	0.1043	0.0266	0.0056	0.1365
5	3	4	6	0.1326	0.0550	0.0110	0.1986
5	5	6	6	0.1317	0.0420	0.0078	0.1885
5	7	8	6	0.1306	0.0337	0.0061	0.1703
5	9	10	6	0.1198	0.0281	0.0050	0.1529
6	3	4	7	0.1312	0.0572	0.0100	0.1984
6	5	6	7	0.1443	0.0436	0.0070	0.1949
6	7	8	7	0.1402	0.0349	0.0054	0.1805
6	9	10	7	0.1314	0.0291	0.0045	0.1650
7	3	4	8	0.1279	0.0590	0.0092	0.1961
7	5	6	8	0.1468	0.0449	0.0064	0.1931
7	7	8	8	0.1465	0.0359	0.0049	0.1873
7	9	10	8	0.1401	0.0299	0.0040	0.1740
8	3	4	9	0.1238	0.0606	0.0085	0.1929
8	5	6	9	0.1473	0.0459	0.0059	0.1921
8	7	8	9	0.1506	0.0366	0.0045	0.1917
8	9	10	9	0.1464	0.0305	0.0037	0.1806
9	3	4	10	0.1194	0.0619	0.0079	0.1891
9	5	6	10	0.1464	0.0469	0.0054	0.1987
9	7	8	10	0.1529	0.0373	0.0042	0.1943
9	9	10	10	0.1510	0.0310	0.0034	0.1853
10	3	4	11	0.1148	0.0630	0.0073	0.1852
10	5	6	11	0.1447	0.0477	0.0051	0.1974
10	7	8	11	0.1539	0.0379	0.0038	0.1956
10	9	10	11	0.1541	0.0314	0.0031	0.1887

Table 6.4 Values of $R(1)$, $R(2)$, $R(3)$ and R_3 when both Strength follows One-Parameter Exponential and Stress follows Lindley Distributions

μ	θ	λ	k	$R(1)$	$R(2)$	$R(3)$	R_3
1	2	3	2	0.1250	0.0616	0.0302	0.2168
1	3	4	2	0.0859	0.0421	0.0206	0.1487
1	4	5	2	0.0650	0.0318	0.0156	0.1124
1	5	6	2	0.0521	0.0255	0.0125	0.0901
2	2	3	4	0.1990	0.0667	0.0179	0.2836
2	3	4	4	0.1554	0.0474	0.0124	0.2152
2	4	5	4	0.1270	0.0368	0.0095	0.1734
2	5	6	4	0.1072	0.0301	0.0077	0.1451
3	2	3	6	0.2236	0.0697	0.0130	0.3063
3	3	4	6	0.1900	0.0495	0.0089	0.2484
3	4	5	6	0.1642	0.0387	0.0068	0.2097
3	5	6	6	0.1442	0.0318	0.0056	0.1816
4	2	3	8	0.2272	0.0724	0.0104	0.3099
4	3	4	8	0.2051	0.0510	0.0070	0.2631
4	4	5	8	0.1849	0.0397	0.0053	0.2299
4	5	6	8	0.1675	0.0326	0.0043	0.2045
5	2	3	10	0.2219	0.0748	0.0088	0.3055
5	3	4	10	0.2098	0.0523	0.0058	0.2679
5	4	5	10	0.1955	0.0405	0.0044	0.2404
5	5	6	10	0.1816	0.0332	0.0036	0.2184
6	2	3	12	0.2132	0.0771	0.0076	0.2979
6	3	4	12	0.2089	0.0535	0.0050	0.2674
6	4	5	12	0.2000	0.0412	0.0038	0.2450
6	5	6	12	0.1897	0.0337	0.0030	0.2264
7	2	3	14	0.2032	0.0792	0.0067	0.2892
7	3	4	14	0.2050	0.0546	0.0044	0.2640
7	4	5	14	0.2007	0.0419	0.0033	0.2458
7	5	6	14	0.1938	0.0341	0.0026	0.2305
8	2	3	16	0.1931	0.0811	0.0061	0.2803
8	3	4	16	0.1996	0.0556	0.0039	0.2591
8	4	5	16	0.1991	0.0425	0.0029	0.2445
8	5	6	16	0.1951	0.0345	0.0023	0.2320
9	2	3	18	0.1834	0.0828	0.0055	0.2717
9	3	4	18	0.1934	0.0566	0.0036	0.2535
9	4	5	18	0.1960	0.0431	0.0026	0.2417
9	5	6	18	0.1946	0.0349	0.0021	0.2316
10	2	3	20	0.1742	0.0843	0.0051	0.2636
10	3	4	20	0.1869	0.0575	0.0033	0.2477
10	4	5	20	0.1921	0.0437	0.0024	0.2382
10	5	6	20	0.1929	0.0353	0.0019	0.2301

Table 6.5 Values of $R(1)$, $R(2)$, $R(3)$ and R_3 when both Strength follows One-Parameter Exponential and Stress follows Two-Parameter Gamma Distributions

μ	γ	λ	θ	k	$R(1)$	$R(2)$	$R(3)$	R_3
0.1	0.2	0.3	0.4	2	0.0054	0.0096	0.0159	0.0308
0.2	0.3	0.4	0.5	2	0.0191	0.0310	0.0042	0.0944
0.3	0.4	0.5	0.6	2	0.0411	0.0590	0.0720	0.1721
0.4	0.5	0.6	0.7	2	0.0684	0.0858	0.0898	0.2441
0.1	0.2	0.3	0.4	3	0.0054	0.0131	0.0250	0.0435
0.2	0.3	0.4	0.5	3	0.0191	0.0395	0.0578	0.1165
0.3	0.4	0.5	0.6	3	0.0411	0.0700	0.0799	0.1910
0.4	0.5	0.6	0.7	3	0.0684	0.0947	0.0867	0.2498
0.1	0.2	0.3	0.4	4	0.0054	0.0160	0.0311	0.0525
0.2	0.3	0.4	0.5	4	0.0191	0.0456	0.0628	0.1275
0.3	0.4	0.5	0.6	4	0.0411	0.0765	0.0799	0.1955
0.4	0.5	0.6	0.7	4	0.0684	0.0983	0.0775	0.2542
0.1	0.2	0.3	0.4	5	0.0054	0.0185	0.0348	0.0586
0.2	0.3	0.4	0.5	5	0.0191	0.0502	0.0636	0.1330
0.3	0.4	0.5	0.6	5	0.0411	0.0805	0.0734	0.1950
0.4	0.5	0.6	0.7	5	0.0684	0.0993	0.0689	0.2566
0.1	0.2	0.3	0.4	6	0.0054	0.0206	0.0368	0.0628
0.2	0.3	0.4	0.5	6	0.0191	0.0537	0.0628	0.1356
0.3	0.4	0.5	0.6	6	0.0411	0.0830	0.0687	0.1928
0.4	0.5	0.6	0.7	6	0.0684	0.0991	0.0616	0.2291
0.1	0.2	0.3	0.4	7	0.0054	0.0224	0.0379	0.0657
0.2	0.3	0.4	0.5	7	0.0191	0.0564	0.0612	0.1367
0.3	0.4	0.5	0.6	7	0.0411	0.0845	0.0643	0.1899
0.4	0.5	0.6	0.7	7	0.0684	0.0983	0.0557	0.2224
0.1	0.2	0.3	0.4	8	0.0054	0.0240	0.0385	0.0678
0.2	0.3	0.4	0.5	8	0.0191	0.0585	0.0595	0.1371
0.3	0.4	0.5	0.6	8	0.0411	0.0854	0.0604	0.1869
0.4	0.5	0.6	0.7	8	0.0684	0.0971	0.0508	0.2163
0.1	0.2	0.3	0.4	9	0.0054	0.0254	0.0387	0.0694
0.2	0.3	0.4	0.5	9	0.0191	0.0602	0.0577	0.1370
0.3	0.4	0.5	0.6	9	0.0411	0.0859	0.0569	0.1840
0.4	0.5	0.6	0.7	9	0.0684	0.0958	0.0467	0.2109
0.1	0.2	0.3	0.4	10	0.0054	0.0266	0.0387	0.0707
0.2	0.3	0.4	0.5	10	0.0191	0.0616	0.0560	0.1367
0.3	0.4	0.5	0.6	10	0.0411	0.0861	0.0539	0.1811
0.4	0.5	0.6	0.7	10	0.0684	0.0943	0.0433	0.2061
0.1	0.2	0.3	0.4	11	0.0054	0.0277	0.0386	0.0716
0.2	0.3	0.4	0.5	11	0.0191	0.0627	0.0544	0.1362
0.3	0.4	0.5	0.6	11	0.0411	0.0860	0.0513	0.1785
0.4	0.5	0.6	0.7	11	0.0684	0.0929	0.0404	0.2017