STUDIES OF SOME STRESS-STRENGTH MODELS IN THE INTERFERENCE THEORY OF RELIABILITY

A Thesis submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy

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Dedicated to my beloved Parents...

Abstract

An attempt has been made in this thesis to study some stress-strength (S-S) models in the interference theory of reliability. In interference theory, reliability of a system (or component) is studied from interference of strength of the system and stress working on it. To evaluation of reliability mainly standby and cascade systems have been considered. Several continuous distributions viz. exponential, gamma, normal, Weibull, uniform and Lindley distribution are considered and also two-point distribution is used among the discrete distributions.

An n-standby system is considered where the number of stresses impinging on the system in time $(0,t)$ follows a Poisson distribution. The system reliability at time t is the probability that the system stands 'r' impacts that is at least one component is working at time t. Again n-standby system with imperfect switching for a single repair facility has been considered. The stresses are impinging on the system in cycles and the life-time (discrete) of a system is measured by number of cycles it can withstand. Switch and repair time is also measured in cycles. For a 3-standby system, the reliability of the system at the N^{th} cycle is calculated by using different stress-strength distributions.

In general, parameters of stress-strength distributions are assumed to be constant. But in many situations this assumption may not be true and the parameters themselves may be random variables. In this case stress-strength distributions are assumed as exponential variates and one of the parameters involved may be random and other parameter remaining constant with a known prior distribution. The prior distributions are considered as uniform and two-point distributions for the parameters concerned.

Again from comparative study between warm and cold standby system with imperfect switching for identical strength and stress, it has been observed that in case of warm standby system, values of the system reliability becomes smaller than that of cold standby system. An n-standby and n-cascade systems have been considered to evaluate the reliability expressions, where all the stresses and strengths are independent random variables. Also stress-strength distributions are assumed to be dissimilar. Again cascade reliability for warm standby system with imperfect switching has been considered for our study. For this

purpose, exponential and gamma distributions are used to obtain the reliability expressions up to 4-cascade system.

Finally, it is observed that, values of reliability are on expected line. In each Chapter, the reliability of the system with the model under consideration is obtained. Often the expressions of reliability are not simple enough to give an idea of their change with relevant stress-strength parameters, so numerical results for reliability are tabulated against the parameter(s) involved, in each case, to show the effect of various parameters on the system reliability. Some graphs are plotted to illustrate our theoretical findings.

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DECLARATION

I, **Jonali Gogoi**, hereby declare that the present thesis entitled "**Studies of Some Stress-Strength Models in the Interference Theory of Reliability**" is the record of work done by me under the supervision of Prof. Munindra Borah, Professor, Department of Mathematical Sciences, Tezpur University, Tezpur. The contents of this thesis represent my original work that has not been previously submitted for a degree or diploma in any other university or institution of higher education.

This thesis is being submitted to the Tezpur University for the degree of Doctor of Philosophy in Mathematical Sciences.

Place: Napaam, Tezpur

Date: 09/04/2012

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CERTIFICATE

This is to certify that the thesis entitled "**Studies of Some Stress-Strength Models in the Interference Theory of Reliability**" which is being submitted by **Miss Jonali Gogoi,** for the award of the Degree of Doctor of Philosophy, to the School of Science and Technology, Tezpur University, Tezpur, Assam (India), under my supervision and guidance.

All help received by her from various sources have been duly acknowledged. This thesis as a whole or any part thereof has not been submitted elsewhere for award of any other degree.

9/4/2012

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Place: Napaam, Tezpur

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With Regards

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- Gogoi, J. and M. Borah. Identical Stress-Strength Model with Random Parameters in Reliability Theory, *Journal of Statistics and Applications*, **6(1-2)**, 15-24, 2011.
- Gogoi, J. and M. Borah. Warm Standby with Imperfect Switching in Cascade Model, *International Journal of Statistics and Analysis*, **1(4)**, 359-366, 2011.
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- Gogoi, J. and M. Borah. Identical Strength for Warm and Cold Standby System with Imperfect Switch: A Comparative Study, (Accepted in *Journal of Indian Statistical Association*).
- Gogoi, J. and M. Borah. Stress-Strength Model for Exponential and Lindley Distributions with Redundancy, Presented in the ***National Conference on Statistical Theory and Its Practice***, at Gauhati University, Guwahati, on December 23-24, 2011.

Chapter 1

Introduction

Chapter 1

1 Introduction

The Interference theory concept is based upon the fact that when the strength of a component or a device or a material is less than the stress imposed on it, the failure occurs. The stress-strength (S-S) models are also called interference models, because here the reliability can be represented in terms of interference area between stress and strength densities. The main emphasis of this thesis is to obtain the reliabilities of different stress-strength distributions.

1.1 Background of the Study

The word 'reliability' is used often in very different contexts, covering different areas and disciplines like Educational Testing, Material Testing, Engineering, to name a few. Depending on the context, the word takes on different shades of meaning. For example, a psychological test for measuring the skills is considered reliable, if the scores obtained through the test for the same individual at different times or individuals known to be of the same ability will be nearly equal.

Technological developments lead to an increase in the number of complicated systems as well as an increase in the complexity of the systems themselves. With remarkable advancements made in electronics and communications, systems became more and more sophisticated. Because of their varied natures, these problems have attracted the attention of scientists from various disciplines especially the system engineers, software engineers and applied probabilistic. An overall scientific discipline called reliability theory that deals with the methods and techniques to ensure the maximum effectiveness of systems has developed. Reliability theory introduces quantitative indices of the quality production (Gnedenko et al.,

1969) and there is carried through from the design and subsequent manufacturing process to the use and storage of technical devices. Due to the nature of the subjects, the methods of Probability theory and Mathematical statistics play an important role in the problem solving of reliability theory. In fact reliability is often defined in terms of probability.

The present study is an attempt to study of appropriate interference models to describe the performance of a system with components of random quality operating in a random environment. Different researchers have defined 'reliability' in different ways and in its own context. Some of these definitions are listed below:

1. 'Reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered' (Radio-Electronics-Television Manufactures Association, 1995, cf. Barlow and Proschan, 1965).

   Symbolically, $R(t) = \int_t^{\infty} dF(x)$

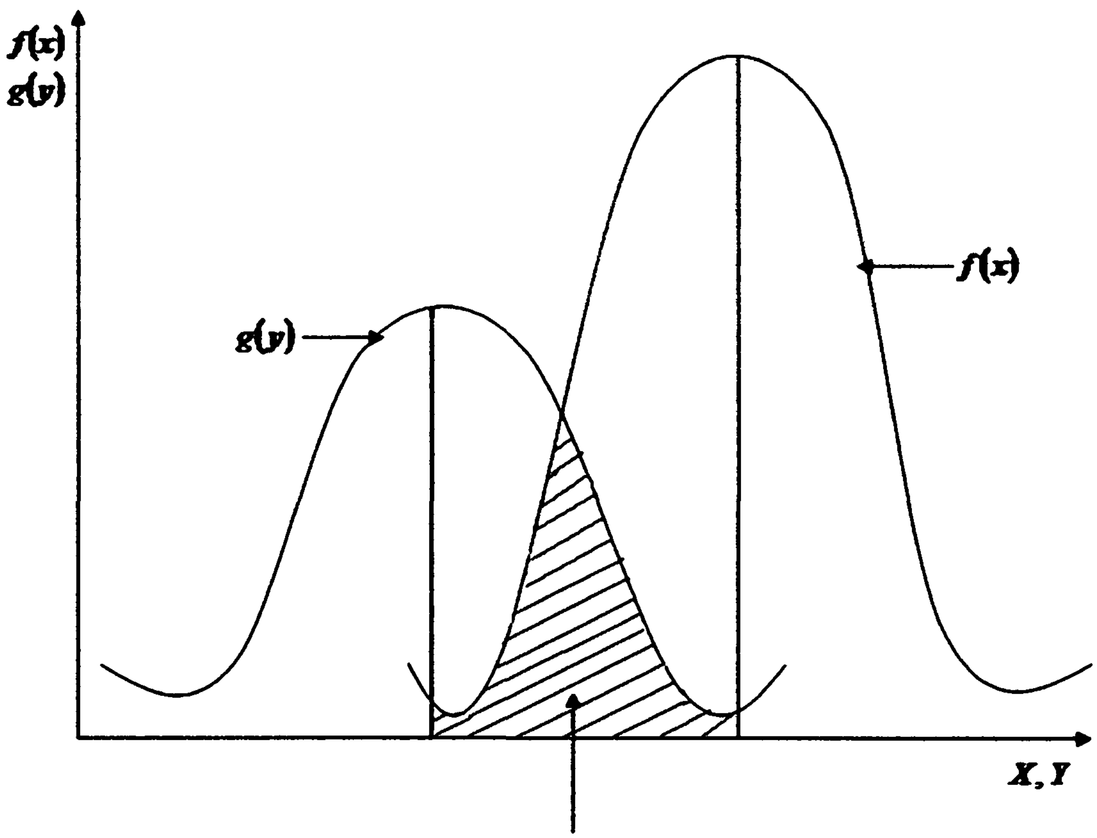
   where $F(t)$ represents the failure time distribution of the system.
2. 'Reliability is the integral of the distribution of probabilities of failure- free operation from the instant of switch on to the first failure. (cf. Polovko, 1968)
3. The reliability $R(t)$ of a component (or a system) is the probability that the component (or system) will not fail for a time t. (cf. Polvoko, 1968)
4. The reliability of a unit (or a system) is defined as the probability that it will perform satisfactorily at least for a specified period of time without a major breakdown. (cf. Sinha and Kale, 1980)

Reliability models, can be broadly classified into the following two groups---
(i) Time-Dependent Models and (ii) Stress-Strength Models or Interference Models

In the time-dependent models time is the important random variable and different measures of reliability theory such as Reliability or Survival function, Availability, Maintainability, Failure rate etc. are obtained from Time-to-Failure (TTF) distributions of the unit (system or component) under study. In such models the underlying idea is that the characteristics of the unit gradually changes and failure occurs when it goes beyond the specified limits. Ordinarily, here failure probability is an increasing function of time and similarly other measures are also functions of time (except failure rate for exponential TTF distribution). Majority of studies in reliability theory are based on the time dependent models. A case in favour of such models is presented by Disney and Seth (1968), Yadav (1973), Kapur and Lamberson (1977), Dhillon (1980) and many references cited by them. Some such models are considered in the present study. In time-dependent models the time is the dominating factor and in interference models stress is the dominating factor.

In S-S models, strength of the system and the stress working on it are the quantities of interest. The words 'stress' and 'strength' used in the reliability theory are not restricted to mechanical loadings. It is used in a broader sense, applicable in many situations well beyond the traditional, mechanical or structural systems. In reliability theory by 'stress' we mean any agency which tends to produce failure of a component, a device or a material. The term agency may be a mechanical load, environmental hazard, electric voltage etc. and the 'strength' represents an agency resisting failure of the system and it is measured by the minimum stress required to cause the failure of the system (cf. Kapur and Lamberson, 1977).

Let us suppose that X and Y are continuous random variables with densities $f(x)$ and $g(y)$ respectively and these are represented graphically as in the following figure. Obviously, the probability of failure $(=1-R)$ is represented by the shaded area in the following figure. In other words, it is represented by the area of interference of stress and strength densities. Hence the term "Interference models" is used when studying reliability taking stress and strength into consideration and this is called interference theory of reliability (cf. Kapur and Lamberson, 1977). Here it is understood that the system works under impact of stresses i.e., the stress is not working continuously on the system but is working as discrete impact (or impacts). From **Fig.** 1.1, it is intuitively clear that the greater the amount of overlap between the curves $f(x)$ and $g(y)$, the larger will be the probability of failure. Under this assumption, if the two curves coincide, the probability of success will be 50 percent (cf. Roberts, 1964 and Commente by Pandit and Sriwastav, 1975).



Interference Area

Fig. 1.1 Stress-Strength Interference Model

Since imperfection and non-uniformity occur during the manufacture of system, the system strength 'X' can be assumed to follow a probability distribution with density $f(x)$. Similarly the stresses 'Y' that impinge on the system are also independently and identically distributed (i.i.d.) random variables with density function $g(y)$. The reliability 'R' of the system may be defined as the probability that 'X' is greater than 'Y'. Symbolically,

$$R = P(X \geq Y) = P(X - Y \geq 0) = P(Z \geq 0), \text{ where } X - Y = Z \tag{1.1.1}$$

Once the respective distributions of stress and strength are known (or estimated), one can obtain reliability of a system by employing equation (1.1.1). If $f(x)$ and $g(y)$ are the densities of X and Y respectively then from (1.1.1)

$$R = \int_{-\infty}^{\infty}\left[\int_{y}^{\infty} f(x)\,dx\right] g(y)\,dy$$
$$= \int_{-\infty}^{\infty} \overline{F}(y)\,g(y)\,dy \tag{1.1.2}$$

or $$R = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{x} g(y)\,dy\right] f(x)\,dx$$
$$= \int_{-\infty}^{\infty} G(x)\,f(x)\,dx \tag{1.1.3}$$

where $\overline{F}(x) = 1 - F(x)$, $F(x)$ and $G(y)$ are the distribution function of strength and stress, respectively.

In our study, stress and strength are considered to be continuous random variables, though they may be discrete also (cf. Charalambides, 1974, Winterbottom, 1974). The stress-strength distributions may be of any type, but most commonly used distributions in reliability theory are exponential, gamma, normal, Weibull, lognormal, Lindley and extreme value distributions. Different types of distributions are used to represent the stress-strength in different situations.

Sometimes, the distributions with fixed parameters may not represent the stress and/or strength distributions adequately. For instance, if a particular component, having certain strength distribution is manufactured in different lots, then for a particular lot the parameters of the strength distribution may remain fixed but may vary from lot to lot. In such situations the parameters of the strength distributions may themselves be taken as random variables. Similar reasoning can be given for the distribution of stress also. So stress and strength may be represented by compound distributions.

In the studies of S-S models, generally stress-strength of a component is supposed to be independent random variables. But in many situations they may be correlated also. The stresses and strengths together and even stress and strength separately may be correlated.

A number of techniques are available to enhance the system reliability. For example,

1. Use of overrated components
2. Effective or creative design
3. System simplification
4. Maintenance
5. Redundancy

The effect of the first four is limited. We cannot increase the system reliability beyond a certain limit by their use. But redundancy may held in achieving any reliability goal. Redundancy is the most effective way to increase the system reliability. Theoretically a reliability are arbitrarily close to one (unity) can be achieved by incorporating redundancy into the system in a suitable manner.

Redundancy is the technique in which more components than the minimum required for normal operation (i.e., essential components) of the systems are attached to it. In such a way that even if only a few components are working, the system works. The essential components which are working initially may be termed as primary components and additional components are termed as redundant components. The redundant components may also remain active through out or may be activated after the failure of the active component. Pieruschka (1963) has described the following forms of redundancy

(i) Active or Parallel redundancy

(ii) Standby redundancy

(i) **Parallel redundancy**: In a parallel redundant system n-components are connected in a parallel arrangement, and to start with all n-components are operating. The system continues to operate till at least k of the components are operating. The system is also referred to as k-out-of-n system. When $k = n$, it is the series system, when $k=1$ it is called completely parallel system (Lloyed and Lipow, 1962).

(ii) **Standby redundancy**: In standby redundancy the redundant units (components) do not work simultaneously with the primary unit, they remain in active. In an n-standby system initially there is one primary unit which is working and there are $(n-1)$ standby units. When the primary units fail one from the standbys is activated (manually or automatically, generally automatically) in its place and the system continues to work. Now this unit becomes primary (or active) unit and the number of standbys reduces to $(n-2)$. When this unit also fails another from standbys takes its place and the system remains working and so on. The system fails when all the n units have failed until and unless stated. Otherwise all the units work (and fail) independently.

Standby redundancy depending upon the nature of failures of standbys is further divided into following three groups (Gnedenko et al., 1969)

(i) Hot or active standby redundancy

(ii) Cold or completely inactive standby redundancy and

(iii) Warm or tepid standby redundancy

(i) **Hot or active standby redundancy**: Here each component has the same failure rate regardless of whether it is in standby or in operation. The situation is equivalent to the case of pure parallel redundancy when $k=1$. Many systems with electronic components are of this type.

(ii) **Cold or completely inactive standby redundancy**: In this type of systems, only one component will be working at any given time, the others being standbys and not working. One of the standby components starts working only when the currently working component fails. The system works until all of its components fail. Mostly mechanical systems are of this type.

(iii) **Warm or tepid standby redundancy**: In practice the assumptions of hot and cold standbys are often not true. Generally, the standbys may fail but fail with lesser probability then they are activated. Such standbys are called warm standbys and the system is called a warm standby system. Here the redundant units are in a partially energized state up to the instant they take the place of active unit. Obviously hot and cold standbys are two extreme cases of warm standbys.

In addition to the redundancy discussed above there is another type of redundancy i.e., 'Cascade redundancy' (cf. Pandit and Sriwastav, 1975). Cascade redundancy is a special kind of standby redundant system in which a new component faces a stress k times the stress on the preceding component, k being a constant or a random variable, called an attenuation factor.

To take out a unit that has failed and insert in its place by a standby unit, we need a device which is termed as switch. In general, it is assumed that the switching mechanism is perfect i.e., it never fails to activate a redundant unit (if one is available). Also it is assume that activation of a redundant unit is instantaneous i.e., as soon as the active unit fails at once one unit from standbys start working in its place (i.e., there is no time-lag). Of course in practice both the assumptions may not be true but unless otherwise stated we shall assume that both the assumptions are varied. However, in reality the switch(es) are subject to failure i.e., the switch is imperfect. The failure of the switch(es) may be either of the following types (Gnedenko et al., 1969):

(i) The switch does not work when it is needed, i.e., the operating unit fails, but the switch does not connect the standbys and so the system fails.

(ii) The switch removes an operating unit from the operation when it should not and does not replace it with a standby. So, there is no operating unit even if the units are good, and hence the system fails.

Type (ii) switch failure is called false switching (Nakagawa, 1977).

The S-S models discussed so far, assume that the stress and strength are random variables. However it is more general, they may be stochastic processes. Taking the system strength and stress on it as two stochastic processes $X(t)$ and $Y(t)$ respectively, the concept of reliability of the system can be obtained from the 'Difference-process', viz. $Z(t) = \{X(t) - Y(t)\}$. The system fails when, for the 'first-time', the stochastic process $Z(t)$ crosses zero from the above (Sriwastav and Pandit, 1978).

1.2 Review of Literature

In S-S models two types of studies are carried out (i) evaluations of system reliability (or structural reliability) making certain assumptions about the strength distribution of the system and the distribution of stresses applied to it and (ii) reliability inferences. As in the present thesis we have studied only the structural reliability. Hence, reviews of some of the works of other authors' articles on structural reliability which are relevant to the present work are discussed here.

Brinbaum (1955) is one of the pioneers in the field of reliability estimation in S-S models. He considered a distribution free method for estimation of the probability viz.

$$p = P(X \leq Y) = 1 - R$$

where X and Y are random variables representing strength and stress of a system and R is the reliability, under the sole weak condition that the distribution functions of X and Y are continuous.

Kapur and Lamberson (1977) have presented some S-S models taking the cases when stress is constant as well as random variable. For different distribution of X and Y they have evaluated reliability for a single component systems.

Pandit and Sriwastav (1976) have obtained the distribution of the number of attacks to failure for a cascade system and called it generalized geometric distribution. They have also considered the cascade system subjected to stress arriving at a random process, viz. Poisson process and obtained with reliability expressions for 2- and 3-cascade system (Pandit and Sriwastav, 1978).

Maiti (1995) has obtained reliability under S-S model in the geometric case. Sriwastav and Kakati (1980) have considered that the parameters of the stress-strength distributions are random variables. Although all the parameters involved may be taken as random variables, they have considered only one parameter random and the others remain constants. Then from compound distribution of stress-strength, they have obtained the reliability of the system.

Uma Maheswari et al. (1992) have considered the reliability of a system where n-stresses act on a single component. The probability distributions considered are exponential, gamma and normal. They have reported that when n-stresses act on a single component with an exponential distribution, the component has the same reliability as single stress and strength components which are connected in a series. They also observed that normal and gamma distributions do not follow this rule.

Sriwastav and Dutta (1986) considered the standby redundant system with different types of failure in S-S model. Sriwastav and Kakati (1981) have considered a n-standby redundant system and obtained the system reliability when the stress and strength of the system follows exponential, gamma or normal distributions. They have also evaluated the system reliability for exponential strength and gamma stress, gamma strength and exponential stress, normal strength and exponential stress, exponential strength and normal stress for a n-cascade system. Pandit and Sriwastav (1975, 1978) have considered an n-cascade system and obtained the expressions for reliability where stress and strength distributions are exponential, gamma and normal, assuming the attenuation factor, k to be a constant and also when it is random.

Warm standby in stress-strength model is studied by Sriwastav and Dutta (1989). They have considered an n-unit warm standby redundant system for stress-strength model to obtain the reliability expressions. Cascade model for warm standby is studied by Bhowal (1999).

Rekha et al. (1988) obtained the reliability of n-cascade system where stress and strength are Log-normal and Weibull. Again Rekha et al. (1992) have derived an expression for the reliability of a single component system where the strength of the component and the imminent stress on the system are random and follow non-identical probability distribution. They assumed that after successive arrivals, the strengths on the successive components are attenuated by specified deterministic factors. They have considered survival function for the stress and strength following exponential distribution. Rekha and Shyam Sunder (1997) have considered an n-cascade system when the strengths of the components follow an exponential distribution and the imminent stress is impinged on the first component with a gamma distribution to obtain the reliability.

Hanagal (1997) has estimated the reliability of a component subjected to two different stresses which are independent of the strength of a component. In another paper (2003) he estimated the system reliability in multicomponent series stress-strength models.

Raghavachar et al. (1983) have considered survival functions under stress attenuation in cascade reliability. Rekha and Shyamsunder (1997) have derived an expression for survival function for the strength attenuation system with stress-strength following exponential distribution. They have obtained the lower and upper bounds when the strength attenuation factor $k_i^* = k_i$.

Apart from the above time independent S-S models we have come across some studies in time dependent S-S models.

Kapur and Lamberson (1977) studied the time dependent S-S model by considering repeated application of stress and also the deterioration of strength with time. They have obtained the expression for reliability of the system for a single component by considering the deterministic and random cycle times.

Gopalan and Venkateswarlu (1982) have considered the reliability analysis of time-dependent cascade system in stress-strength models by considering each of the stress and strength variables as deterministic or random fixed or random independent. They have considered the number of cycles in any period of time 't' to be deterministic. They (1983) further extended this problem to the random cycle times, i.e., the number of cycles in any period of time 't' is assumed to be random. They obtained the reliability expressions for 2- and 3-cascade systems for any time period 't' considering the attenuation factor to be constant and the number of cycles per unit time follows a Poisson distribution. Assuming components to be identical and attenuation factor k_i's to be constants, they have also obtained the expression for system reliability when stress and strength follow exponential distributions.

Gopalan and Venkateswarlu (1985) further considered the repairman problem in stress-strength model. They have carried out the reliability and availability analysis of a repairable dissimilar two-unit standby system in S-S model with a single repair facility. The time taken to repair a unit is either deterministic or random. They carried out the analysis for arbitrary stress-strength and repair time distribution.

Shooman (1968) has obtained the reliability of a system from stress-strength time model assuming that the stresses are coming to the system in a Poisson process with parameter λ as

$$R(f) = e^{-p} f^{\lambda}$$

where p_f is the probability of failure of the system with a single impact. He has introduced the time factor in another way also, viz. by considering the deterioration of strength parameter with time.

Shaw (1973) has obtained the reliability expression for components operating in environments with repeated stress. They have considered time variations of stress and strength. Raghavachar et al. (1984) have studied a system exposed to shocks at time points from a discrete set and random time points. Tumolillo (1974) has considered the situation where stresses change the failure rate of components stochastically.

Sriwastav (2005) have considered a stress-strength standby system with a single repair facility. For a 3-standby system they calculated the reliability of the system at the N^{th} cycle when stress-strength is either exponential or gamma or normal assuming that the number of cycles in time $(0,t)$ follows a Poisson distribution.

Xu, Guo, Yu, Zhu (2005) have studied the asymptotic stability of a repairable system with repair time of failed system that follows arbitrary distribution. Srinivasan and Gopalan

(1973) have considered a two-unit standby system with a single repair facility. They have considered the cases where stresses are constants as well as random variables to obtain the system reliability.

Subramanian and Anantharaman (1995) have considered probabilistic analysis of a three unit cold standby system where the lifetime of the unit and the repair time are random variables with arbitrary distributions. Srinivasan and Subramanian (2006) have considered reliability analysis of a three unit warm standby redundant system with repair. Here they were using imbedded renewal points to obtain the reliability and availability functions.

There are very few works available for unreliable switches in interference models. Of course for time-to-failure models there are a few studies on imperfect switches. For example, Osaki (1972) found the Laplace-Stieltjes transform of the distribution function of the time up to the first system failure, for a two dissimilar unit standby system with repair and imperfect switchover, using the exponential distribution for the unit. Srinivasan (1968) has considered a non-instantaneous switchover where switchover time is a random variable. Nakagawa and Osaki (1975) have obtained the stochastic behavior of a two-unit standby system with repair and imperfect switchover. Gopalan (1975a) has considered the availability and reliability of 1-server-2-unit system with imperfect switch. Alidrisi (1992) has obtained the reliability considering imperfect switching for dynamic warm standby system in TTF (time-to-failure) model.

Sriwastav (2004) has obtained the reliability expressions for the n-standby system where the standbys are warm standbys with an imperfect switch when all the stress-strength are random variables with given density. Gajjar and Patel (2010) have considered an n-cold standby system with imperfect switches in stress-strength model. They assume estimation of standby reliability with imperfect switching under (a) geometric stress and geometric strength (b) geometric stress and exponential strength.

A problem of system reliability of a standby system, when switches and the components follow dissimilar continuous distributions is considered by Dutta and Bhowal

(2000). A cascade reliability model for n-warm standby system is considered by Dutta and Bhowal (1998) and cascade model with imperfect switching is considered by Sriwastav (1992).

1.3 Objectives

In this investigation, our objectives were to study the stress-strength of the components in the interference systems. To achieve the main goal of the study to be presented in the thesis the following objectives have been undertaken.

- To study an interference model with number of stresses which follows a Poisson process. Here we assumed that the number of stresses impinging on the system during time $(0,t)$ is a stochastic process and follows a Poisson distribution.
- To study an n-standby repairable system with imperfect switching. For this purpose, we have considered the repair of switch as well as the components and evaluate reliability and other characteristics of reliability.
- To study the identical stress-strength model with random parameters. In this study, we have considered interference model where stress-strength are exponential variates and one of the parameter (stress or strength) be a random with a known prior distribution, other parameter remaining constant.
- To study identical stress-strength model for warm and cold standby system with imperfect switching. Here we have considered warm and cold standby system where switching mechanism is not perfect.
- To study the stress-strength models for standby redundancy and cascade redundancy.
- To study cascade model for warm standby system with imperfect switching.

1.4 Outline and Organization of the Thesis

With relevant to the objectives mentioned above, the present thesis is organized in nine chapters under broad headings.

- Chapter 1 Introduction
- Chapter 2 An Interference Model with Number of Stresses a Poisson Process

- Chapter 3 An n-Standby Repairable System with Imperfect Switching
- Chapter 4 Identical Stress-Strength Model with Random Parameters in Reliability Theory
- Chapter 5 Identical Strength for Warm and Cold Standby System with Imperfect Switch: A Comparative Study
- Chapter 6 Identical Stress for Warm and Cold Standby System with Imperfect Switch: A Comparative Study
- Chapter 7 Stress-Strength Model with Standby Redundancy and Cascade Redundancy
- Chapter 8 Cascade Model for Warm Standby System with Imperfect Switching
- Chapter 9 Summary and Future Works

A brief summary of each chapter of the thesis mentioned above is highlighted below.

Chapter 2 is concerned an n-standby stress-strength system where the number of impacts of stresses faced by the system is a Poisson process. i.e., the number of stresses impinging on the system in time $(0,t)$ follows a Poisson distribution. The general expression of the reliability of the system R_n is obtained for $n = 1, 2, 3, 4$ when both stress-strength of the components follow exponential, gamma, normal and Weibull distributions. Numerical integration method is used to obtain the system reliability in some cases. Some numerical values of reliability $R_1(t), R_2(t), R_{31}(t), R_4(t)$ have also been tabulated, for different set of values of the parameters of the stress-strength distributions in all the cases. To make the things clear, a few graphs are drawn for selected values of the parameters. These graphs are smooth enough to facilitate direct reading of reliability for intermediate values of the parameters.

An n-standby stress-strength system with a single repair facility has been made in Chapter 3. Here assumed that the stresses are impinging on the system in cycles and the life time (discrete) of the system is measured by number of cycles it can withstand and also the switch works under the impact of stresses. When stress-strength of the components and the switch follow exponential, gamma and normal distributions and repair time distribution is geometric variate, reliability of a 3-component standby system at the N^{th} cycle is obtained. Some numerical values of the reliability corresponding to different values of the parameters involved are also tabulated.

Chapter 4 deals with the identical stress-strength model where they are exponential variates and one of the parameters (stress or strength) is assumed to be a random with a known prior distribution, other parameters remaining constant. Here we have considered only two cases. First, strength parameter is random but stress parameter is a constant. Secondly, stress parameter is random but strength parameter is a constant. Uniform and two-point distributions are taken as the prior distributions for the parameters concerned. Using the derived compound distribution, general expressions of reliability of the system is obtained. In order to see how system reliabilities change with parameters involved, we have tabulated some values of R_1, R_2, R_3, R_4 for the distributions from their expressions. To make the things clear, a few graphs are also drawn.

A comparative study between warm and cold standby system with imperfect switching for identical strength has been discussed in Chapter 5. Similarly, a comparative study for identical stress has been discussed in Chapter 6. The strengths of different components and the stresses on them during functioning and when they are standbys, are all independent random variables. When stress-strength of the components and the switch follow particular distributions viz. exponential, gamma and normal then the general expressions of system reliability R_n is obtained for $n \leq 3$. Various reliability parameters have been computed and analyzed by tabular illustrations. Some graphs are drawn for selected values of the parameters in Chapter 6.

Chapter 7 is concerned with the determination of system reliability when stress-strength distributions are assumed to be dissimilar for n-standby and n-cascade systems. Here general n-standby and n-cascade systems have been considered where all stresses and strengths are independent random variables. Depending upon the nature of $f_i(x)$'s and

$g_i(y)$'s different cases may be considered. Explicit expressions of reliability $R(r)$, $r = 1,2,...,n$ and hence R_n is obtained for all the cases in an n-standby system. For n-cascade system, the reliability expressions considering the different cases are obtained up to 3-cascade system and a recursive rule is indicated, except in one case, for obtaining the expression $R(r)$. Some numerical values of reliabilities $R(1)$, $R(2)$, $R(3)$ and R_3 are tabulated against the different values of the parameters and graphical technique is described for a particular set up.

In Chapter 8, the general cascade reliability model for n-warm standby system with imperfect switching has been developed and the reliability expressions are obtained when the stress-strength of the components and the switch follow particular distributions. Following two cases have been considered for our study.

- Stress-strength for the active component, standby component and the switch follow exponential distribution
- Stress-strength for the active component and the switch follow Exponential distribution and standby component follows gamma distribution.

For both the cases the marginal reliability $R(1)$, $R(2)$, $R(3)$, $R(4)$ and the system reliability R_4 for a 4-cascade system are obtained. Also some numerical values of the reliability, in each case, are estimated and presented in tabular form against the parameters involved.

Lastly, overall summary and future works have been given in Chapter 9 and some chapter-wise information/results in the form of tables, references and literatures cited in the thesis of the present work are listed at the end.

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Chapter 2

An Interference Model with Number of Stresses a Poisson Process

Chapter 2

An Interference Model with Number of Stresses a Poisson Process

2.1 Introduction

In interference models in reliability theory it is assumed that a system is working under impacts of stresses. The system has some property which withstands the impacts of stresses; that property is called its strength and is measured by the minimum stress required for the failure of the system. They are called the stress-strength of the system and are assumed to be random variables. The system works if an impact of stress is smaller than or equal to its strength. The reliability of the system is the probability that it works. Such types of models are studied by many, e.g. [Dhillon (1980), Frudenthal (1996), Kapur and Lamberson (1977), Pandit and Sriwastav (1978), Sriwastav (1994, 2003), Sriwastav and Kakati (1981)].

In most of the studies of interference models, in evaluating the reliability of the system, only its stress-strength are taken into consideration as if the passage of time has no effect on it. But to assume that passage of time has no effect on the reliability of a system seems to be somewhat unrealistic. A more realistic situation will be one in which both time and stress directly affect the system reliability. In some studies e.g. Sriwastav (1994) time has come into the picture in an indirect way. To bring time into the model, directly, it may be assumed that the number of stresses impinging on the system in the time $(0,t)$ is a stochastic process. Here we have considered an n-standby system. We have assumed that the number of stresses faced by the system is a Poisson process i.e., the number of stresses impinging on the system in time $(0,t)$ follows a Poisson distribution. Here the system reliability at time 't'

is defined as the probability that 'r' impacts of stresses impinges on the system in time $(0,t)$ and the system stands 'r' impacts i.e., at least one component is working at time 't'. Pandit and Sriwastav (1978) have mentioned this problem for cascade reliability.

The organization of this chapter is as follows: In Section 2.2, the general model is presented mathematically. The reliability of the system can be obtained if the forms of the stress and strength are specified. In Section 2.3, assuming that stress-strength both is either exponential or gamma or normal or Weibull, the general expressions of the reliability of the system are obtained in each case. For some particular values of the parameters involved numerical values of the reliability are tabulated in the **Table** 2.1, **Table** 2.2, **Table** 2.3 and **Table** 2.4 (cf. Appendix) and some graphs are plotted for all the cases in Section 2.4. Results and discussions are given in Section 2.5.

2.2 Mathematical Description of the Model

Here we have an n-standby system. Let $X_1, X_2, \ldots, X_n$, be the strengths of the n-components in the system arranged in the order of activation. Let $Y_1, Y_2, \ldots, Y_n$, be the stresses faced, respectively, by 1st, 2nd,..., n^{th} component when they are activated. X_i's and Y_i's are all independent. For detail description of such a system one may refer Sriwastav and Kakati (1981).

The reliability R_n of an n-standby system for a single impact of stress is given by, [Sriwastav and Kakati (1981)]

$$R_n = R(1) + R(2) + \ldots + R(n) \tag{2.2.1}$$

where, $R(i)$ is the increment in the system reliability due to the i^{th} component, defined as

$$R(i) = P\left[X_1 < Y_1, X_2 < Y_2, \ldots, X_{i-1} < Y_{i-1}, X_i \geq Y_i\right] \tag{2.2.2}$$

Here we are to find the reliability of the system at time 't' when the number of impacts of stresses on the system during the time $(0,t)$ is 'r' (a random variable), following a Poisson distribution given by,

$$p(r) = \frac{e^{-\alpha t}(\alpha t)^r}{r!}, \quad r = 0,1,2,... \tag{2.2.3}$$

In an n-standby system, in the starting, there are n good components, of which one is active and faces the impact of stress; the remaining $(n-1)$ are cold standbys. Here we would like to note that at least one impact is required for the failure of a component and more than one component may fail in a single impact. Further if $m(<n)$ components fail in a single impact then at the next impact system is an $(n-m)$ standby system and the $(m+1)^{th}$ component in the original order of activation behaves as the 1^{st} component in the order of activation, $(m+2)^{th}$ as the 2^{nd} component, etc.

We have to obtain first the probability that the system survives 'r' attacks. Here we have considered the case when $n \le 4$, though the expressions for any finite n can be obtained but the complexity of the expressions increases rapidly with increasing n.

Now, if $R_n(r)$, $r = 1,2,3,4,$ is the reliability of the n-standby system at the r^{th} attack then Sriwastav (2003)

$$R_1(r) = R_1^r \tag{2.2.4}$$

$$R_2(r) = R_1^r + R(2)\binom{r}{1}R_1^{r-1} \tag{2.2.5}$$

$$R_3(r) = R_2(r) + R(3)\binom{r}{1}R_1^{r-1} + \{R(2)\}^2\binom{r}{2}R_1^{r-2} \tag{2.2.6}$$

and

$$R_4(r) = R_3(r) + R(4)\binom{r}{1}R_1^{r-1} + 2R(2)R(3)\binom{r}{2}R_1^{r-2}R_1^{r-2} + \{R(2)\}^3\binom{r}{3}R_1^{r-3} \tag{2.2.7}$$

where $R(1) = R_1$.

In this way we can find $R_n(r)$ for any finite n.

Since, as per our assumptions here r is a random variable follows the Poisson distribution employing the equation (2.2.3), then the reliability of the system, $R_n(t)$, at time t is given by

$$R_n(t) = \sum_{r=0}^{\infty} p(r) R_n(r) \tag{2.2.8}$$

Then, substituting the values of $p(r)$ from (2.2.3) and $R_n(r)$ from (2.2.4) to (2.2.7), we get

$$R_1(t) = \sum_{r=0}^{\infty} p(r) R_1(r) = \sum_{r=0}^{\infty} \frac{e^{-\alpha t}(\alpha t)^r}{r!} R_1^r = e^{-\alpha t(1-R_1)} \tag{2.2.9}$$

$$\begin{aligned} R_2(t) &= \sum_{r=0}^{\infty} p(r) R_2(r) \\ &= \sum_{r=0}^{\infty} \frac{e^{-\alpha t}(\alpha t)^r}{r!} \left[R_1^r + R(2)\binom{r}{1} R_1^{r-1} \right] = e^{-\alpha t(1-R_1)}\left[1 + \alpha t R(2)\right] \end{aligned}$$

(2.2.10)

$$\begin{aligned} R_3(t) &= \sum_{r=0}^{\infty} p(r) R_3(r) \\ &= \sum_{r=0}^{\infty} \frac{e^{-\alpha t}(\alpha t)^r}{r!} \left[R_2(r) + R(3)\binom{r}{1} R_1^{r-1} + \{R(2)\}^2 \binom{r}{2} R_1^{r-2} \right] \\ &= e^{-\alpha t(1-R_1)}\left[1 + \alpha t\{R(2) + R(3)\} + \{\alpha t R(2)\}^2/(2!)\right] \end{aligned}$$

(2.2.11)

$$\begin{aligned} R_4(t) &= \sum_{r=0}^{\infty} p(r) R_4(r) \\ &= \sum_{r=0}^{\infty} \frac{e^{-\alpha t}(\alpha t)^r}{r!} \left[R_3(r) + R(4)\binom{r}{1} R_1^{r-1} + 2R(2)R(3)\binom{r}{2} R_1^{r-2} + \{R(2)\}^3 \binom{r}{3} R_1^{r-3} \right] \\ &= e^{-\alpha t(1-R_1)}\left[\begin{matrix} 1 + \alpha t\{R(2) + R(3) + R(4)\} + \\ (\alpha t)^2 R(2)\{R(2) + 2R(3)\}/(2!) + \{\alpha t R(2)\}^3/(3!) \end{matrix} \right] \end{aligned}$$

(2.2.12)

If the stress-strength distributions are specified then from equation (2.2.2) we can obtain $R(i)$, from (2.2.4) to (2.2.7) we can obtain $R_n(r)$ and from (2.2.9) to (2.2.12) we can obtain $R_n(t)$, for $n \leq 4$.

2.3 Stress-Strength follows Specific Distributions

Exponential, gamma, normal and Weibull these are the most common distributions used in reliability theory. What follows, we have assumed that the distributions of stress and strength both for every component are either exponential or gamma or normal or Weibull variates.

2.3.1 Stress and Strength Exponentially Distributed

Let us assume that both strength (X_i) and Stress (Y_i) of the i^{th} component, $i = 1,2,\ldots,n$ are exponentially distributed with densities

$$\left.\begin{array}{l} f(x_i) = \lambda_i e^{-\lambda_i x_i},\ i = 1,2,\ldots,n;\ 0 \leq x_i \leq \infty,\ \lambda_i > 0, \\ \textit{and} \\ g(y_i) = \mu_i e^{-\mu_i y_i},\ i = 1,2,\ldots,n;\ 0 \leq y_i \leq \infty,\ \mu_i > 0 \end{array}\right\}, \text{ respectively} \tag{2.3.1}$$

Then we can easily see [Sriwastav and Kakati (1981)] that

$$R(i) = \overline{R}(\rho_1)\overline{R}(\rho_2)\ldots\overline{R}(\rho_{n-1})R(\rho_n) \tag{2.3.2}$$

$$\text{where } \rho_i = \frac{\lambda_i}{\mu_i},\quad R(\rho_i) = \frac{1}{1+\rho_i} \quad \text{and} \quad \overline{R}(\rho_i) = 1 - R(\rho_i) \tag{2.3.3}$$

Usually, the components used in a standby system are identical and the working conditions also do not change drastically. So strengths of all the components may be assumed to be identically distributed, similarly all the stresses may also be identical i.e.,

$$\rho_1 = \rho_2 = \ldots = \rho_n = \rho \text{ (say)}$$

Then, (2.3.2) becomes

$$R(i) = \left[\overline{R}(\rho)\right]^{i-1} R(\rho) \qquad (2.3.4)$$

Substituting these values in (2.2.9) to (2.2.12) we get

$$R_1(t) = \exp\left[-\alpha t\overline{R}(\rho)\right], \qquad (2.3.5)$$

$$R_2(t) = \exp\left[-\alpha t\overline{R}(\rho)\right]\left\{1 + \alpha t\overline{R}(\rho)R(\rho)\right\}, \qquad (2.3.6)$$

$$R_3(t) = \exp\left[-\alpha t\overline{R}(\rho)\right]\left[1 + \alpha t\overline{R}(\rho)R(\rho)\left\{1 + \overline{R}(\rho)\right\} + \left\{\alpha t\overline{R}(\rho)R(\rho)\right\}^2/2\right], \qquad (2.3.7)$$

$$R_4(t) = \exp\left[-\alpha t\overline{R}(\rho)\right]\begin{bmatrix} 1 + \alpha t\overline{R}(\rho)R(\rho)\left\{1 + \overline{R}(\rho) + \left(\overline{R}(\rho)\right)^2\right\} + \\ \left\{\alpha t\overline{R}(\rho)R(\rho)\right\}^2\left\{0.5 + \overline{R}(\rho)\right\} + \left\{\alpha t\overline{R}(\rho)R(\rho)\right\}^3/(3!) \end{bmatrix} \qquad (2.3.8)$$

2.3.2 Stress and Strength Distributed as Gamma Variates

Let us assume that both stress (Y_i) and strength (X_i) of the i^{th} component, $i = 1,2,\ldots,n$ are distributed as gamma variates with densities,

$$\left.\begin{aligned} f(x_i) &= \frac{1}{\Gamma m_i} e^{-x_i} x_i^{m_i - 1};\ 0 \le x_i \le \infty,\ m_i \ge 1, \\ &\textit{and} \\ g(y_i) &= \frac{1}{\Gamma l_i} e^{-y_i} y_i^{l_i - 1};\ 0 \le y_i \le \infty,\ l_i \ge 1,\ i = 1,2,\ldots,n, \end{aligned}\right\}\text{, respectively} \qquad (2.3.9)$$

Then from Sriwastav and Kakati (1981)

$$R(i) = \overline{R}(m_1, l_1)\overline{R}(m_2, l_2)\ldots\overline{R}(m_{i-1}, l_{i-1})R(m_i, l_i), \qquad (2.3.10)$$

where $$R(m_i, l_i) = \sum_{j=0}^{m_i - 1} \frac{\Gamma(m_i + l_i - j - 1)}{\Gamma l(m_i - j - 1)!2^{m_i + l_i - j - 1}},\ i = 1,2,\ldots,n. \qquad (2.3.11)$$

$\overline{R}(m_i, l_i) = 1 - R(m_i, l_i)$ and m_i's are integers.

As commented in Sub section 2.3.1 taking stresses and strengths identical, so that $m_i = m,\ l_i = l\ \forall\ i$, we have

$$R(i) = \left[\bar{R}(m,l)\right]^{i-1} R(m,l) \tag{2.3.12}$$

Then, from (2.2.9) to (2.2.12) we have,

$$R_1(t) = e^{-\alpha t \bar{R}(m,l)} \tag{2.3.13}$$

$$R_2(t) = e^{-\alpha t \bar{R}(m,l)}\left[1 + \alpha t \bar{R}(m,l)R(m,l)\right] \tag{2.3.14}$$

$$R_3(t) = e^{-\alpha t \bar{R}(m,l)}\left[1 + \alpha t \bar{R}(m,l)R(m,l)\{1 + \bar{R}(m,l)\} + \{\alpha t \bar{R}(m,l)R(m,l)\}^2/(2!)\right] \tag{2.3.15}$$

$$R_4(t) = e^{-\alpha t \bar{R}(m,l)}\begin{bmatrix} 1 + \alpha t \bar{R}(m,l)R(m,l)\left\{1 + \bar{R}(m,l) + \left(\bar{R}(m,l)\right)^2\right\} \\ + \alpha^2 t^2 \{\bar{R}(m,l)R(m,l)\}^2\left\{\frac{1}{2} + \bar{R}(m,l)\right\} + \frac{\alpha^3 t^3}{3!}\{\bar{R}(m,l)R(m,l)\}^3 \end{bmatrix} \tag{2.3.16}$$

2.3.3 Stress and Strength Normally Distributed

Let X_i be a $N(\mu_i, \sigma_i)$ variate and Y_i be a $N(\lambda_i, \theta_i)$ variate. Then from Sriwastav and Kakati (1981)

$$R(i) = \bar{\phi}(A_1)\bar{\phi}(A_2)\ldots\bar{\phi}(A_{i-1})\phi(A_i), \tag{2.3.17}$$

where $A_i = \dfrac{\mu_i - \lambda_i}{\sqrt{\sigma_i^2 + \theta_i^2}}$ and $\bar{\phi}(A_i) = 1 - \phi(A_i),\ i = 1,2,\ldots,n.$ (2.3.18)

Further, let us assume that X_i's are identical and Y_i's are also identically distributed, then

$$A_i = A = \frac{\mu - \lambda}{\sqrt{\sigma^2 + \theta^2}},\ \forall i = 1,2,\ldots,n. \tag{2.3.19}$$

and so, $R(i) = \left[\bar{\phi}(A)\right]^{i-1}\phi(A),\ i = 1,2,\ldots,n$ (2.3.20)

Then, substituting these values in (2.2.9) to (2.2.12) we get

$$R_1(t) = e^{-\alpha t\bar{\phi}(A)}, \tag{2.3.21}$$

$$R_2(t) = e^{-\alpha t\bar{\phi}(A)}\left[1 + \alpha t\bar{\phi}(A)\phi(A)\right], \tag{2.3.22}$$

$$R_3(t) = e^{-\alpha t\bar{\phi}(A)}\left[1 + \alpha t\bar{\phi}(A)\phi(A)\{1 + \bar{\phi}(A)\} + \frac{1}{2}\{\alpha t\bar{\phi}(A)\phi(A)\}^2\right], \tag{2.3.23}$$

$$R_4(t) = e^{-\alpha t\bar{\phi}(A)}\begin{bmatrix} 1 + \alpha t\bar{\phi}(A)\phi(A)\{1 + \bar{\phi}(A) + (\bar{\phi}(A))^2\} + \frac{1}{2}\{\alpha t\bar{\phi}(A)\phi(A)\}^2\{1 + 2\bar{\phi}(A)\} \\ + \frac{1}{3!}\{\alpha t\bar{\phi}(A)\phi(A)\}^3 \end{bmatrix} \tag{2.3.24}$$

2.3.4 Stress and Strength Distributed as Weibull Variates

Let us assume that both stress (Y_i) and strength (X_i) of the i^{th} component, $i = 1,2,\ldots,n$ are distributed as Weibull variates with densities,

$$\left.\begin{array}{l} f(x_i) = bx_i^{b-1}e^{-\left(\frac{x_i}{\theta_i}\right)^b} / \theta_i^b,\ 0 \le x_i \le \infty,\ \theta\rangle 0 \\ and \\ g(x_i) = cy_i^{c-1}e^{-\left(\frac{y_i}{\lambda_i}\right)^c} / \lambda_i^c,\ 0 \le y_i \le \infty,\ \lambda\rangle 0 \end{array}\right\}, \text{ respectively} \tag{2.3.25}$$

Then we have,

$$R(i) = \bar{R}(\theta_1,\lambda_1)\bar{R}(\theta_2,\lambda_2)\ldots\bar{R}(\theta_{i-1},\lambda_{i-1})R(\theta_i,\lambda_i) \tag{2.3.26}$$

where, $R(\theta_i,\lambda_i) = 1 - \frac{c}{\lambda_i^c}\int_0^\infty e^{-\left[\left(\frac{y_i}{\theta_i}\right)^b + \left(\frac{y_i}{\lambda_i}\right)^c\right]} y_i^{c-1} dy_i$

$\bar{R}(\theta_i,\lambda_i) = 1 - R(\theta_i,\lambda_i)$

As commented in Sub-Section 2.3.1 taking stresses and strengths identical, so that

$\theta_i = \theta,\ \lambda_i = \lambda\ \forall i$ we have,

$$R(i) = \left[\bar{R}(\theta,\lambda)\right]^{-1} R(\theta,\lambda) \tag{2.3.27}$$

Then from (2.2.9) to (2.2.12) we have,

$$R_1(t) = e^{-\alpha t\bar{R}(\theta,\lambda)} \tag{2.3.28}$$

$$R_2(t) = e^{-\alpha t\bar{R}(\theta,\lambda)}\left[1 + \alpha t\left\{\bar{R}(\theta,\lambda)R(\theta,\lambda)\right\}\right] \tag{2.3.29}$$

$$R_3(t) = e^{-\alpha t\bar{R}(\theta,\lambda)}\left[1 + \alpha t\bar{R}(\theta,\lambda)R(\theta,\lambda)\left\{1 + \bar{R}(\theta,\lambda)\right\} + \left\{\alpha t\bar{R}(\theta,\lambda)R(\theta,\lambda)\right\}^2 /(2!)\right] \tag{2.3.30}$$

$$R_4(t) = e^{-\alpha t\bar{R}(\theta,\lambda)}\begin{bmatrix} 1 + \alpha t\bar{R}(\theta,\lambda)R(\theta,\lambda)\left\{1 + \bar{R}(\theta,\lambda) + \bar{R}(\theta,\lambda)^2\right\} + \\ (\alpha t)^2\left\{\bar{R}(\theta,\lambda)R(\theta,\lambda)\right\}^2\left\{\frac{1}{2} + \bar{R}(\theta,\lambda)\right\} + \frac{\alpha^3 t^3}{3!}\left\{\bar{R}(\theta,\lambda)R(\theta,\lambda)\right\}^3 \end{bmatrix} \tag{2.3.31}$$

2.4 Graphical Representations

Some graphs are plotted in **Fig.** 2.1 to **Fig.** 2.4 by taking different parameters along the horizontal axis and the corresponding reliability along the vertical axis for different parametric values. In **Fig.** 2.1 taking αt along the horizontal axis and the corresponding $R_1(t)$ along the vertical axis graphs are plotted for different values of ρ. One can read the values of $R_1(t)$ for intermediate values of αt, from this graph. Thus, for ρ=0.5 we get $R_1(t)$= 0.4337. For αt=2.52 from graphical extrapolation, while the computed value is $R_1(t)$=0.4317. The difference is only 0.20%. From **Fig.** 2.2 Graphs of $R_3(t)$ against αt are plotted for different pairs of the values of m and l. These graphs may be used for reading the values of $R_3(t)$, corresponding to intermediate values of αt. For instance, for m=1, l=1 and αt=3.5 from the graph we get $R_3(t)$=0.4707, whereas actual calculation yields the value as $R_3(t)$=0.4684. The difference is only 0.23%. **Fig.** 2.3, from the graph, as expected, it is seen that $R_2(t)$ decreases steadily with increasing αt. These graphs may be used to read the intermediate values directly. For example, for αt =4.5, μ=1 and σ=1 we get from the graph $R_2(t)$=0.6192 whereas by actual calculation we get $R_2(t)$=0.6188. The difference is only 0.16%. **Fig.** 2.4, based on the values of $R_4(t)$, extensively tabulated in just above these graphs, curves of $R_4(t)$ were drawn against αt for different parameter values of b,θ,c and λ. From these graphs we get $R_4(t)$=0.5799. For αt=0.357, b=0.8, θ=0.6, c=0.7, λ=0.2 while the computed value is $R_4(t)$=0.5768. The difference is only 0.31%.

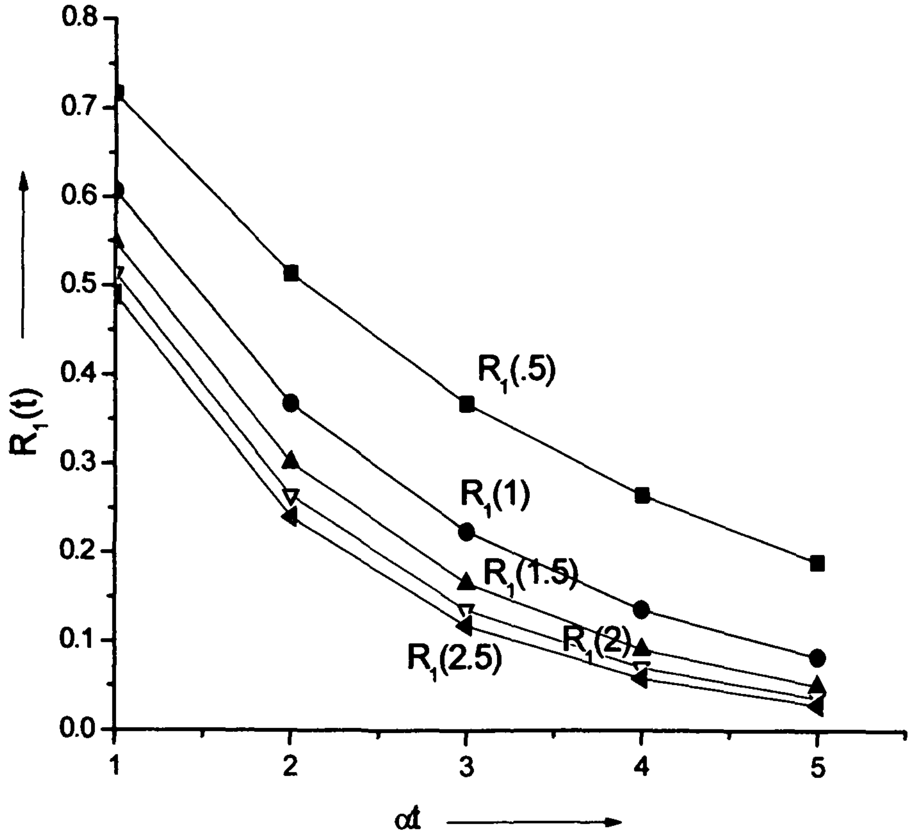

Fig. 2.1 **Exponential Stress-Strength**: Graph for $R_1(t)$ for different fixed values of ρ i.e., $R_1(\rho)$.

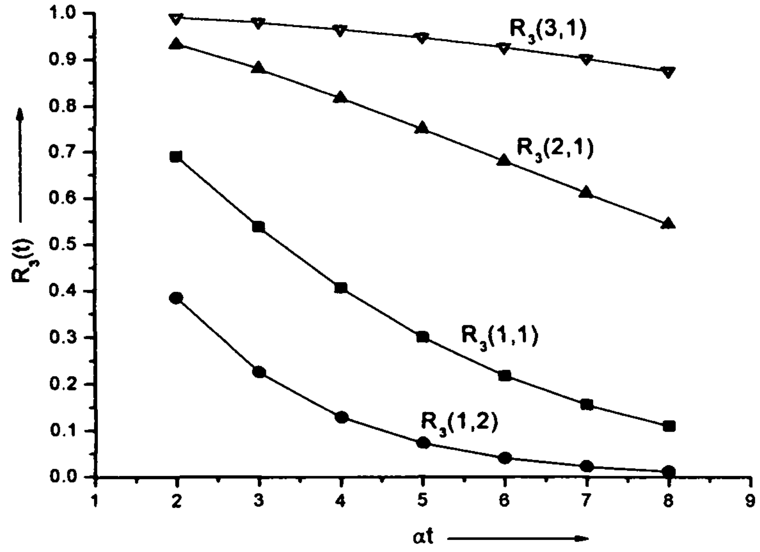

Fig. 2.2 Gamma Stress-Strength: Graph for $R_3(t)$ for different fixed values of m and l i.e., $R_3(m,l)$.

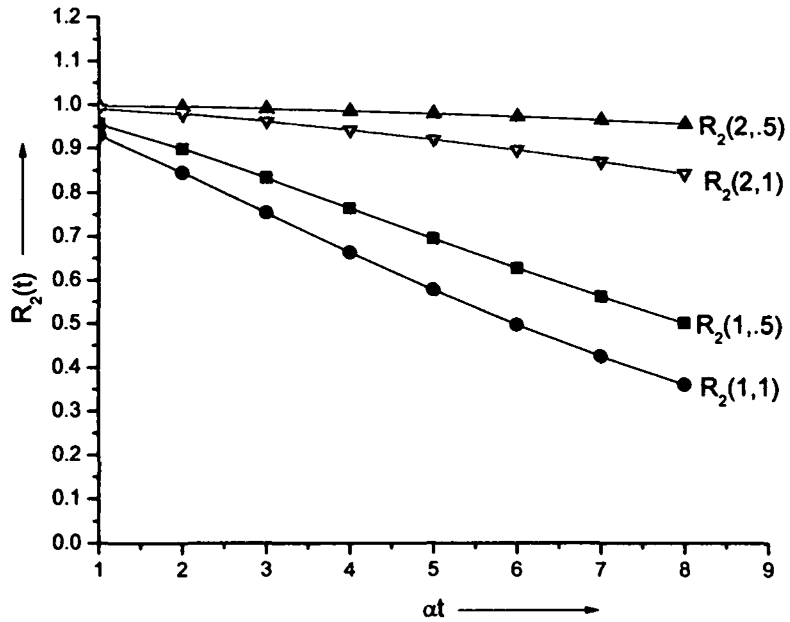

Fig. 2.3 Normal Stress-Strength: Graph for $R_2(t)$ for different fixed values of μ and σ i.e., $R_2(\mu,\sigma)$.

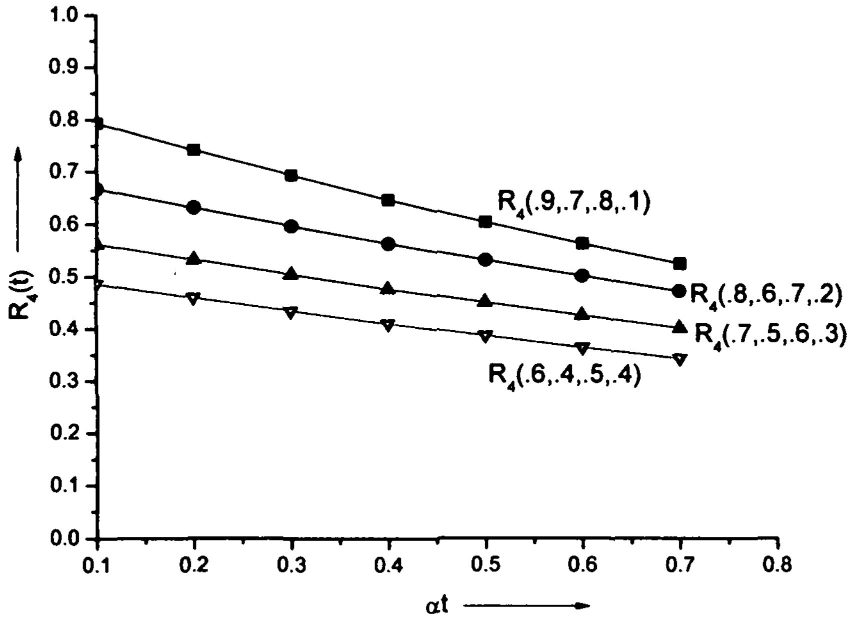

Fig. 2.4 **Weibull Stress-Strength**: Graph for $R_4(t)$ for different fixed values of b,θ,c and λ i.e., $R_4(b,\theta,c,\lambda)$.

2.5 Results and Discussions

For some specific values of the parameters involved in the expressions of $R_i(t)$, $i=1,2,3,4$ and for given values of 't' we evaluate $R_1(t)$ to $R_4(t)$, for different distributions, from their expressions obtained in the last section. From the expressions of $R_i(t)$, it is clear that their values depend upon the values of 'αt' (= mean no. of stresses in time 0 to t) rather than the individual values of α and t.

Table 2.1 (cf. Appendix) presents the values of $R_1(t)$, $R_2(t)$, $R_3(t)$ and $R_4(t)$ for different values of the parameter ρ for exponential distribution. From this table, it is clear that the values of the reliability are on expected line. Increase in the values of αt decreases the reliability. Increase in the values of ρ also decreases the reliability. For instance, when $\alpha t=1$, $\rho=.5$ then $R_1(t)=0.7165$, $R_2(t)=0.8758$, $R_3(t)=0.9465$, and $R_4(t)=0.9973$.

Again when $\alpha t = 2,\ \rho = 1$ then $R_1(t) = 0.3679,\ R_2(t) = 0.5518,\ R_3(t) = 0.6898$ and $R_4(t) = 0.7894$.

Table 2.2 (cf. Appendix) presents the values of $R_1(t),\ R_2(t),\ R_3(t)$ and $R_4(t)$ for different values of the parameter l and m in case of gamma distribution. From this table, we have seen that here also the change in the values of reliability is as expected. The increase in the values of αt decreases the reliability. From the table we have seen that when $\alpha t = 2$ then the reliabilities are $R_1(t) = 0.2231,\ R_2(t) = 0.3068,\ R_3(t) = 0.3852$ and $R_4(t) = 0.4578$. Again when $\alpha t = 3$ then $R_1(t) = 0.1054,\ R_2(t) = 0.1647,\ R_3(t) = 0.2258$ and $R_4(t) = 0.2873$. When m increases, reliability increases. For example, if $m = 1$ then $R_1(t) = 0.3679,\ R_2(t) = 0.5518,\ R_3(t) = 0.6898$ and $R_4(t) = 0.7894$ and if $m = 2$ then $R_1(t) = 0.6065,\ R_2(t) = 0.8340,\ R_3(t) = 0.9335$ and $R_4(t) = 0.9744$ i.e., reliability increases. But when l increases, reliability decreases. For instance, when $l = 1$ then $R_1(t) = 0.3679,\ R_2(t) = 0.5518,\ R_3(t) = 0.6898$ and $R_4(t) = 0.7894$ and when $l = 2$ then $R_1(t) = 0.2231,\ R_2(t) = 0.3068,\ R_3(t) = 0.3852$ and $R_4(t) = 0.4578$.

For normal distribution, the reliabilities $R_i(t),\ i = 1,2,3,4$ are tabulated in **Table** 2.3 (cf. Appendix) for different values of the parameter μ and σ. This table is also self explanatory. With increasing αt reliability decreases where as with increasing μ increases but decreases with increasing σ. Of course, the effect of μ is more than that of σ.

Similarly the reliability values of $R_1(t),\ R_2(t),\ R_3(t)\ and\ R_4(t)$ for different values of the parameter b, θ, c and λ are presented in **Table** 2.4 (cf. Appendix). Here also the change in the values of reliability is as expected. The increase in the values of αt decreases the reliability.

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Chapter 3

An n-Standby Repairable System with Imperfect Switching

Chapter 3

An n-Standby Repairable System with Imperfect Switching

3.1 Introduction

As mentioned in chapter 1, switching mechanisms are required in standby redundant systems. Here we have considered a standby repairable system with imperfect switching.

In a standby system when the active component fails to remove it and to insert its place a component from standbys a switching mechanism is required. In an stress-strength (S-S) standby system a component with strength (X) works under certain stress (Y), both X and Y are random variables. The component fails when the stress on it exceeds its strength; the reliability of the component is defined by the equation (1.1.1) as given in Chapter 1.

In an S-S standby system with repair when the active component fails, it is sent for repair and the next component (if there remains any) is instantly activated by some device which is termed as switch. In general it is assumed that the switching mechanism is perfect i.e., absolutely reliable and switching is instantaneous. Hence, in the evaluation of system reliability, reliability of switch is not taken into account. But in practice the switch may also fail and this will change the reliability structure of the system. It is assumed that the impacts of stresses are coming in cycles and the life-time of a system is measured in number of cycles, (Pandit and Sriwastav, 1976). Here, it is assumed that in an impact (or cycle) only one component can fail. The repair time is also measured in cycles and at least one cycle is required to repair a failed component. The system will work if the first component works or at least one of the standby component along with switch works.

In this chapter we have considered a 3-component standby system with a single repair facility with imperfect switch and obtain the reliability of the system at the N^{th} cycle of the stress.

Studies of standby system with imperfect switch for S-S models are considered by many persons, for example, Kapur and Lamberson (1977), Sriwastav and Dutta (1984), Dutta and Bhowal (1997, 1998, 1999) etc. However, standby system with repair are considered by several authors including Gopalan and Venkateswarlu (1985), Subramanian and Anantharaman (1995), Xu, Guo, Yu and Zhu (2005), Sriwastav (2005), Srinivasan and Subramanian (2006) etc.

Organization of this chapter is as follows: Section 3.2 deals with the description of the system. In Section 3.3, the general model for a 3-component standby system with imperfect switching is developed. In Section 3.4, numerical evaluation is presented. In Section 3.5, three specific distributions are considered to obtain the reliability. In Sub-Sections 3.5.1, 3.5.2 and 3.5.3, all the distributions are assumed to be exponential, gamma and normal respectively for the components and the switch. To see the effect of changes in different parameters on reliabilities, some numerical values of $R_3(N)$, $N = 3,4,5$ are obtained in each case and tabulated against the parameters in the **Table** 3.1, **Table** 3.2 and **Table** 3.3 (cf. Appendix). Results and discussions are reported in Section 3.6.

3.2 Description of the System

Let us consider a 3-standby system where the components are arranged in the order of activation and numbered accordingly. To start with the first component is working; the second and third components are standbys. The components are dissimilar. When the first component fails, the second component starts working in its place by a switch and the system continues to work and the first component goes for repair and if its repair is completed before the second and third component fails, it will become the standby. There is a single repair facility, which takes up the components for repair on first-come-first serve basis. When the second component fails, the third components starts working by a switch in its place and the system continues to work. The second component goes for repair. The system will work till the switch works and either the first component works or there is a standby to work. The system fails when all the three components have failed or the active component fails and switch fails i.e., one is under repair and the two are in the queue for repair. The switch is also repaired by the same repair facility.

As assumed in Section 3.1, the components and the switch are working under impact of stresses and stresses are impinging on the system in cycles. If the working component fails in a particular cycle the next component will face a stress in the next cycle. i.e., only one component can fail in one impact (or cycle) of the stress. Similarly components repair time is also measured in cycles and at least one cycle is required for the repair of a component. If a component is repaired in i^{th} cycle, it will be available for use from $(i+1)^{th}$ cycle. Though the active component faces in cycles but the switch faces a stress only when it is to activate a standby unit. The switching is instantaneous i.e., if the switch works the next component is immediately activated and is ready to face the next cycle of stress on the system.

3.3 General Model for a 3-Component Standby System

As we have assumed that the switch also works under the impact of stresses. Let U be the strength of the switch and V be the stress on it. The switch fails when $U < V$, where U and V are assumed to be independent random variables. Let X_i be the strength of the i^{th} component and Y_i be the impact of stress on it, X_i and Y_i are both random variables. The switch and the components are assumed to work independently. Then a_i the reliability of the i^{th} component $i = 1,2,3$ is given by

$$a_i = P[X_i \geq Y_i] \tag{3.3.1}$$

and the reliability of the switch is

$$R^s = P[U \geq V] \tag{3.3.2}$$

Let $p_i(j)$, $i = 1,2,3;\ \ j = 0,1,2,\ldots$ be the probability that the repair of the i^{th} component is completed exactly at the j^{th} cycles. Then the probability that the repair of the i^{th} component is completed on or before the k^{th} cycle is given by

$$P_i(k) = \sum_{j=0}^{k} p_i(j) \tag{3.3.3}$$

The reliability of the 3-standby system at the N^{th} cycle, $N = 0,1,2,\ldots$ with imperfect switching is given by,

$$R_3(N) = a_1^N + (1-a_1)\sum_{k=1}^{N} a_1^{k-1} P(U \geq V)\left[a_2^{N-k} + (1-a_2)\sum_{j=1}^{N-k} a_2^{j-1} P(U \geq V) F_{3,j}(N-k-j)\right] \tag{3.3.4}$$

where $F_{3,j}(N-k-j)$ is the survival probability of the system for the remaining $(N-k-j)$ cycles starting with the 3rd component. (1st and 2nd component have already failed; 2nd component worked for $(j-1)$ cycles and failed at the j^{th} cycle. i.e.,

$$F_{3,j}(N-k-j) = a_3^{N-k-j} + (1-a_3)\sum_{i=1}^{N-k-j} a_3^{i-1} P(U \geq V) P_1(j+1) F_{1,i}(N-k-j-1) \tag{3.3.5}$$

By the same argument as in (3.3.5)

$$F_{1,j}(N-k-j-i) = a_1^{N-k-j-i} + (1-a_1)\sum_{l=1}^{N-k-j-i} a_1^{l-1} P(U \geq V) P_2(i+l) F_{2,l}(N-k-j-i-l) \tag{3.3.6}$$

and

$$F_{2,l}(N-k-j-i-l) = a_2^{N-k-j-i-l} + (1-a_2)\sum_{n=1}^{N-k-j-i-l} a_2^{n-1} P(U \geq V) P_3(l+n) F_{3,n}(N-k-j-i-l-n) \tag{3.3.7}$$

In general, we can write,

$$F_{r,s}(m) = a_r^m + (1-a_r)\sum_{i=1}^{m} a_r^{i-1} P(U \geq V) P_{r+1}(s+i) F_{r+1,i}(m-i),\ \ r = 1,2,3;\ \ s = 1,2,\ldots \leq N-2 \tag{3.3.8}$$

Since there are only three components hence for $r = 3$, $r+1 = 1$, because of the components are numbered in the order of activation and are repaired in the order they fail. Since, by assumption, at least one cycle is required for the repair of a component, hence

$$P_i(0) = 0,\ i = 1,2,3 \tag{3.3.9}$$

Further, with the convention that

$$\sum_{i}^{j}(.) = 0,\ \ for\ j\langle i, \tag{3.3.10}$$

we can easily see from (3.3.5), (3.3.6) and (3.3.7) that

$$F_{i,d}(0) = 1,\ i = 1,2,3;\ d = 1,2,... \tag{3.3.11}$$

Since only one component can fail in an impact (or cycle), so obviously

$$R_3(0) = 1,\ R_3(1) = 1,\ R_3(2) = 1 \tag{3.3.12}$$

we can see this from (3.3.4) under the convention (3.3.10) and using (3.3.11).

3.4 Numerical Evaluation

Here the expression (3.3.4) could not be obtained in closed form but if stress-strength distributions are known we can find the values of a_i's, $i = 1,2,3$ and R^s's if repair time distribution is known, then $P_i(.)$'s, $i = 1,2,3$ are also known. Substituting these values in (3.3.5), (3.3.6), (3.3.7) and from (3.3.4) we can obtain the reliability $R_3(N)$ for any finite N.

Here reliability $R_3(N)$ is obtained for $N = 3,4,5$ when both stress-strength and the switch follow either exponential or gamma or normal variates and repair time distribution is geometric variate.

Now,

$$R_3(3) = a_1^3 + (1-a_1)P(U \geq V)\sum_{i=1}^{3} A_i a_i^{i-1} \tag{3.4.1}$$

where, $A_i = a_2^{3-i} + (1-a_2)\sum_{j=1}^{3-i} a_2^{j-1} P(U \geq V) F_{3,j}(3-i-j\};\ i = 1,2,3$

$$R_3(4) = a_1^4 + (1-a_1)P(U \geq V)\sum_{i=1}^{4} B_i a_i^{i-1} \tag{3.4.2}$$

where, $B_i = a_2^{4-i} + (1-a_2)\sum_{j=1}^{4-i} a_2^{j-1} P(U \geq V) F_{3,j}(4-i-j);\ i = 1,2,3,4$

$$R_3(5) = a_1^5 + (1-a_1)P(U \geq V)\sum_{i=1}^{5} C_i a_i^{i-1} \tag{3.4.3}$$

where, $C_i = a_2^{5-i} + (1-a_2)\sum_{j=1}^{5-i} a_2^{j-1} P(U \geq V) F_{3,j}(5-i-j);\ i = 1,2,3,4,5$

From (3.3.8) we have,

$$F_{r,s}(1) = a_r^1 + (1-a_r)P(U \geq V)P_{r+1}(s+1);\ r = 1,2,3;\ s = 1,2,3,4 \tag{3.4.4}$$

$$F_{r,s}(2) = a_r^2 + (1-a_r)P(U \geq V)\left[P_{r+1}(s+1)F_{r+1,1}(1) + a_r^1 P_{r+1}(s+2)\right] \tag{3.4.5}$$

$$F_{r,s}(3) = a_r^3 + (1-a_r)P(U \geq V)\left[P_{r+1}(s+1)F_{r+1,1}(2) + a_r^1 P_{r+1}(s+2)F_{r+1,2}(1) + a_r^2 P_{r+1}(s+3)\right] \tag{3.4.6}$$

$$F_{r,s}(4) = a_r^4 + (1-a_r)P(U \geq V)\begin{bmatrix} P_{r+1}(s+1)F_{r+1,1}(3) + a_r^1 P_{r+1}(s+2)F_{r+1,2}(2) + \\ a_r^2 P_{r+1}(s+3)F_{r+1,3}(1) + a_r^3 P_{r+1}(s+4) \end{bmatrix},$$

$$r = 1,2,3;\ s = 1,2,3,4 \text{ and for } r = 3,\ r+1 = 1 \tag{3.4.7}$$

3.5 Reliability for Specific Distributions

In this section we consider stress-strength of the components and the switch follow particular distributions, viz. exponential, gamma and normal.

3.5.1 Exponential Stress-Strength for the Switch and the Components

Let us assume that both strength (X_i) and stress (Y_i) for the i^{th}, $i = 1,2,3$ component are exponential with mean $1/\mu_i$ and $1/\lambda_i$ respectively and let U and V be exponential with mean $1/\alpha$ and $1/\beta$ for the switch. Then, Kapur and Lamberson (1977)

$$a_i = P(X_i \geq Y_i) = \frac{\mu_i}{(\lambda_i + \mu_i)} = \frac{1}{1+\rho}, \text{ where } \rho_i = \frac{\mu_i}{\lambda_i} \qquad (3.5.1)$$

and

$$R^s = b = P(U \geq V) = \frac{\alpha}{\beta + \alpha} = \frac{1}{1+\upsilon}, \text{ where } \upsilon = \frac{\alpha}{\beta} \qquad (3.5.2)$$

Further let us assume that, repair time distribution of the i^{th} component and the switch is a geometric variate with probability of repair being completed in a cycle p_i then,

$$P_k(m) = 1 - q_i^m,\ m = 0,1,2,\ldots;\ \ q_i = 1 - p_i \qquad (3.5.3)$$

The reliabilities $R_3(N)$, for $N = 3,4,5$ are computed and presented in **Table** 3.1 (cf. Appendix) for ρ=.1, .5, 1.0, 1.5, 2; υ=.2, .3, .5, .7, .9 and p_i=.9, .8, .6, .4, .2. In general, the components used in standby systems are identical, so reliability are calculated assuming $\rho_1 = \rho_2 = \rho_3 = \rho$ (say) i.e., $a_1 = a_2 = a_3 = a$ (say) and $p_1 = p_2 = p_3 = p$ (say) i.e., $q_1 = q_2 = q_3 = q$ (say).

3.5.2 Gamma Stress-Strength for the Switch and the Components

Let us assume that both stress and strength of the i^{th} components are gamma variates with scale parameters unity and shape parameters (or degrees of freedom) l_i and m_i respectively. Further let U and V be gamma variates for the switch with degrees of freedom c and d respectively. If either l_i or m_i is an integer then the reliability a_i and R^s of the i^{th} component and the switch is given by Kapur and Lamberson (1977),

$$a_i = \sum_{j=0}^{m_i-1} \frac{\Gamma(m_i + l_i - j - 1)}{\Gamma l_i (m_i - j - 1)! 2^{m_i + l_i j - 1}},\ i = 1,2,3 \qquad (3.5.4)$$

and

$$R^s = b = \sum_{j=0}^{d-1} \frac{\Gamma(d + c - j - 1)}{\Gamma c (d - j - 1)! 2^{d+c-j-1}} \qquad (3.5.5)$$

As assumed in Sub-Section 3.5.1, generally, the components are identical. i.e., $m_1 = m_2 = m_3 = m$ (say) and $l_1 = l_2 = l_3 = l$ (say) or in other words $a_1 = a_2 = a_3 = a$ (say). The reliabilities $R_3(N)$, for $N = 3,4,5$ are computed and presented in **Table** 3.2 (cf. Appendix) for some selected values of the parameters.

3.5.3 Normal Stress-Strength for the Switch and the Components

Let the stress-strength of the i^{th} component are $N(0,1)$ and $N(\mu_i, \sigma_i^2)$ variates and let U and V be normal with $N(0,1)$ and $N(\alpha, \tau^2)$ variates, respectively for the switch. Then its reliability is given by Kapur and Lamberson (1977), as

$$a_i = \phi\left(\frac{\mu_i}{\sqrt{1+\sigma_i^2}}\right), \quad i = 1,2,3 \tag{3.5.6}$$

and

$$R^s = b = \psi\left(\frac{\alpha}{\sqrt{1+\tau^2}}\right) \tag{3.5.7}$$

For illustration purpose reliabilities are computed for the components a_i and the switch R^s assuming the repair time distribution is geometric. The reliability of the 3-standby system $R_3(N)$, at the N^{th} cycle $N = 3,4,5$ is presented in the **Table** 3.3 (cf. Appendix).

3.6 Results and Discussions

From the **Table** 3.1 (cf. Appendix), it is observed that the values of the reliability are on expected line in case of exponential distribution. For example, when the number of cycles N increases $R_3(N)$, $N = 3,4,5$ decreases. i.e., $R_3(3) > R_3(4) > R_3(5)$. This should be the case, because we use the components more than one times. If we use one component more than one or two times than its life goes to decrease (i.e., they have no capacity to work like as first time because of it's used before). But the rate of decrease is very slow for large values of a's , b's and p's i.e., small q's. For example, when $a = 0.9091$, $b = 0.8333$, $q = .1$ i.e., $p = .9$ then the values of reliability $R_3(N)$, $N = 3,4,5$ becomes 0.9552, 0.9407 and 0.9265 respectively. Again the rate of decrease is faster for small a's, b's and p's i.e., large q's. For example, when $a = 0.3333$, $b = 0.5263$, $q = .8$ i.e., $p = .2$ then the reliabilities $R_3(N)$ become 0.2927, 0.1728 and 0.0990 respectively. The effect of changes in the values of a's is more than that of q's. Even for comparatively unreliable components with highly reliable repair facility, at least for first few cycles high reliability can be achieved. The effect of decreasing values of a's and b's is more for smaller p's (i.e., larger q's) than for large p's. Similar conclusion can be made for the **Table** 3.2 and **Table** 3.3 (cf. Appendix) in case of gamma and normal stress-strength.

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Chapter 4

Identical Stress-Strength Model with Random Parameters in Reliability Theory

Chapter 4

Identical Stress-Strength Model with Random Parameters in Reliability Theory

4.1 Introduction

Mostly discussions of interference models assume that the parameters of S-S distributions are constants Beg (1980), Enis and Geisser (1971), Harris and Singpurwalla (1968), Kapur and Lamberson (1977), Kelly, Kelly, J.A. and Schucany (1976), Sriwastav (1976). But in many cases this assumption may not be true and the parameters may be assumed themselves to be random variables. In other words, the distributions with fixed parameters may not represent the stress and strength distributions adequately and a distribution with random parameters may represent the situations better. For example, if a particular component, having certain strength distribution is manufactured in different lots, then for a particular lot the parameters of the strength distribution may remain fixed but may vary randomly from lot to lot. In such situations the parameters of the strength distribution may themselves be taken as random variables Harris and Singpurwalla (1968). Similarly, the stress applied on a component (or system) is due to different factors such as temperature, pressure, vibration, humidity etc. Generally, one of these factors will be dominant and will be the main cause for the stress on the component and the stress distribution will be the distribution of this factor. But the other factors may vary at different times or at different places in such a way that, though they do not alter the nature of the distribution, they bring random changes in the parameters of the stress distribution. For example, the vibration at high temperature will be more severe for a joint than at low temperature Robert (1964) and hence the distribution of stress (due to vibration) may have different parameter value at different temperature or in other words we can say that the stress parameters are random variables.

Further, if a prior knowledge exists about the parameters involved, it will be a waste of available data if we do not use a random parameter model i.e., a Bayesian model. In order to use the Bayesian approach, it is generally assumed that the subjective knowledge can be quantified somehow and represented in the form of a prior distribution of the parameter involved Kapur and Lamberson (1977). When prior distribution is known, the unconditional distribution of the random variable (stress and/or strength) can be recovered Harris and Singpurwalla (1968), Kapur and Lamberson (1977).

Harris and Singpurwalla (1968) have considered life-time distributions with random parameters. They have taken uniform, two-point and gamma distributions as prior distributions for the parameters. They have considered estimation problem for this model. Here we have considered only uniform and two-point distributions as prior distributions to estimate the system reliability but not considered any estimation problem. Shooman (1968) has considered the parameter of strength distribution as a deterministic function of time. Tarman and Kapur (1975) have assumed that the parameters of the stress-strength distributions are variables but not random variables.

In this chapter some of the results were presented in Gogoi and Borah (2011). Here stress-strength model is considered where they are exponential variates. Also assumed that the parameters of the stress-strength distributions are random variables. Though all the parameters involved may be taken as random variables for simplicity, only one parameter, at a time, is taken to be random with a known prior distribution, and other parameters remaining constant. The main aim of this chapter is to obtain the system reliability R_n for identical stress-strength model. The following two cases have been considered for this investigation.

Case I: When strength parameter is random but stress parameter is a constant.

Case II: When stress parameter is random but strength parameter is a constant.

For the above two cases, the prior distributions considered for stress-strength parameters are either uniform or two-point distributions.

Section 4.2 deals with the general model for identical stress-strength. In Section 4.3, we consider that the strength parameter λ is a random variable whereas the stress parameter μ remains constant. Prior distribution of λ is assumed to be uniform and two-point distributions respectively in Sub-Sections 4.3.1 and 4.3.2. Section 4.4 presents the opposite of that considered in Section 4.3. i.e., we consider here the stress parameter μ is a random variable and λ remains constant. Sub-Section 4.4.1 and 4.4.2 deals with the uniform and two-point prior distributions for μ. The **Table** 4.1 (cf. Appendix) to **Table** 4.4 (cf. Appendix) can be used for making a system reliability analysis. To make the things clear, a few graphs are drawn in Section 4.5 for selected values of the parameters. These graphs are smooth enough to facilitate direct reading of reliability for intermediate values of the parameters. Section 4.6 is devoted to a discussion on the results obtained in Section 4.3 and 4.4 respectively.

4.2 Notations, Definitions and Formulation of the Model

Here we have assumed that strength 'X' and stress 'Y' are exponential variates with means $\frac{1}{\lambda}$ and $\frac{1}{\mu}$, respectively. The parameters λ and μ may be random.

Let $P(\lambda)$, $p(\lambda)$= The prior distribution function and p.d.f. (or p.m.f.) of the random strength parameter λ

$Q(\mu)$, $q(\mu)$= The prior distribution function and p.d.f. (or p.m.f.) of the random stress parameter μ

$f(x/\lambda)$= The conditional p.d.f. of the random variable X for a given value of λ

$g(y/\mu)$= The conditional p.d.f. of the random variable Y for a given value of μ

$f_x(x)$= The unconditional p.d.f. of the random variable X

$g_y(y)$= The unconditional p.d.f. of the random variable Y

where $$f_x(x) = \int_{-\infty}^{\infty} f(x/\lambda)dP(\lambda) \tag{4.2.1}$$

and $$g_y(y) = \int_{-\infty}^{\infty} g(y/\mu)dQ(\mu) \tag{4.2.2}$$

Here $f(x/\lambda) = f_x(x)$ if λ is constant and $g(y/\mu) = g_y(y)$ if μ is constant.

Let $X_1, X_2, ..., X_n$ be the strengths of the n components in order of activation, and $Y_1, Y_2, ..., Y_n$ be the respective stresses on them. Here X_i's and Y_i's, $i = 1,2,...,n$, are all independent random variables reliability R_n, of the system is given by the equation (2.2.1). where, $R(r)$ denotes the marginal reliability due to the r^{th} component.

$$\text{i.e. } R(r) = P(X_1 < Y_1, X_2 < Y_2, ..., X_{r-1} < Y_{r-1}, X_r \geq Y_r) \tag{4.2.3}$$

Let $f_i(x)$ be the p.d.f. of X_i and $g_i(y)$ be that of Y_i, $i = 1,2,...,n$ then from (4.2.3)

$$R(r) = \left[\int_{-\infty}^{\infty} F_1(y) g_1(y) dy\right]\left[\int_{-\infty}^{\infty} F_2(y) g_2(y) dy\right] \cdots \left[\int_{-\infty}^{\infty} F_{r-1}(y) g_{r-1}(y) dy\right]\left[\int_{-\infty}^{\infty} \overline{F}_r(y) g_r(y) dy\right] \tag{4.2.4}$$

where $F_i(x) = \int_{-\infty}^{x} f_i(x) dx$ and $\overline{F}_i(x) = F_i(x)$

If all the components having some strength distributions are working under the same environment (stress) then we can assume that stress-strength distributions of all the components are identical, i.e., all the X_i's and Y_i's are i.i.d with p.d.f.s $f(x)$ and $g(y)$, respectively, $i = 1,2,...,n$ then (4.2.4) reduces to,

$$R(r) = \left[\int_{-\infty}^{\infty} F(y) g(y) dy\right]^{r-1}\left[\int_{-\infty}^{\infty} \overline{F}(y) g(y) dy\right] \tag{4.2.5}$$

4.3 Random Strength Parameter

Here we assume that the strength parameter λ is a random variable whereas the stress parameter μ remains constant, i.e., $f(x/\lambda) = \lambda e^{-\lambda x}$ and $g_y(y) = \mu e^{-\mu y}$

Two types of prior distributions are considered for λ

4.3.1 Uniform and **4.3.2** Two-point

4.3.1 Uniform Prior for λ

In a situation where the components are homogeneous within each lot but different lots have different values of λ and taking all the possible sources of the lots together, each value of λ appears equally frequently, a uniform prior distribution will be suitable for λ Harris and Singpurwalla (1968).

Let λ be uniformly distributed in the range (a,b), i.e., $p(\lambda)=\frac{1}{b-a}$, $0 \le a < \lambda < b$

Then the unconditional p.d.f. of X is given by

$$f_x(x)=\frac{1}{b-a}\int_a^b \lambda e^{-\lambda x}dx$$

Hence from (4.2.5) we have,

$$R(1)=\int_0^\infty\left[\int_y^\infty f_x(x)dx\right]g_y(y)dy$$

$$=\int_0^\infty\left[\int_y^\infty\frac{1}{b-a}\int_a^b \lambda e^{-\lambda x}d\lambda dx\right]\mu e^{-\mu y}dy=\frac{\mu}{b-a}\log_e^{\frac{b+\mu}{a+\mu}}$$

$$R(2)=\left[\int_{-\infty}^{\infty}F_y(y)g_y(y)\right]^{2-1}\left[\int_{-\infty}^{\infty}\overline{F}_y(y)g_y(y)dy\right]$$

$$=\left[\int_0^\infty\left(1-\frac{1}{b-a}.\frac{e^{-ay}-e^{-by}}{y}\right)\mu e^{-\mu y}dy\right]R(1)$$

$$=[1-R(1)]R(1)$$

$$R(3)=\left[\int_{-\infty}^{\infty}F_y(y)g_y(y)\right]^{3-1}\left[\int_{-\infty}^{\infty}\overline{F}_y(y)g_y(y)dy\right]=[1-R(1)]^2R(1)$$

Then in general we can write,

$$R(r)=[1-R(1)]^{r-1}R(1)$$

4.3.2 Two-Point Prior for λ

In a situation where it is known that λ can take two only values λ_1 and λ_2 (say), with probabilities p and $(1-p)$, respectively, a two-point distribution for λ is appropriate Harris and Singpurwalla (1968).

Let λ have a two-point distribution, given by,

$$\Pr(\lambda = \lambda_1) = p(\lambda_1) \; and \; \Pr(\lambda = \lambda_2) = p(\lambda_2)$$

Then,

$$f_x(x) = \sum_{i=1}^{2} \lambda_i p(\lambda_i) = p\lambda_1 e^{-\lambda_1 x} + (1-p)\lambda_2 e^{-\lambda_2 x} \tag{4.3.1}$$

Hence from (4.2.5) we have,

$$R(1) = \int_0^\infty \left[\int_y^\infty f_x(x)dx \right] g_y(y)dy$$

$$= \left(\frac{p}{\lambda_1 + \mu} + \frac{1-p}{\lambda_2 + \mu} \right) \mu$$

$$R(2) = \left[\int_{-\infty}^{\infty} F_y(y) g_y(y) \right]^{2-1} \left[\int_{-\infty}^{\infty} \overline{F}_y(y) g_y(y) dy \right]$$

$$= \left[1 - \left(\frac{p}{\lambda_1 + \mu} + \frac{1-p}{\lambda_2 + \mu} \right) \mu \right] \left[\left(\frac{p}{\lambda_1 + \mu} + \frac{1-p}{\lambda_2 + \mu} \right) \mu \right]$$

$$= [1 - R(1)]R(1)$$

$$R(3) = \left[\int_{-\infty}^{\infty} F_y(y) g_y(y) \right]^{3-1} \left[\int_{-\infty}^{\infty} \overline{F}_y(y) g_y(y) dy \right] = [1 - R(1)]^2 R(1)$$

Then in general we can write,

$$R(r) = [1 - R(1)]^{r-1} R(1)$$

System reliability R_n may be computed by substituting the values of $R(r)$, $r = 1,2,\ldots,n$ in the equation (2.2.1). For different values of the parameter a, b, μ and $p, \lambda_1, \lambda_2, \mu$ the values of R_1, R_2, R_3, R_4 from (4.2.3) are tabulated in **Table** 4.1 (cf. Appendix) and **Table** 4.2 (cf. Appendix).

4.4 Random Stress Parameter

Here also the stress and strength are exponential with mean $\frac{1}{\mu}$ and $\frac{1}{\lambda}$ respectively. But now μ is a random variable and λ remains constant, i.e. $g(y/\mu) = \mu e^{-\mu y}$ and $f_x(x) = \lambda e^{-\lambda x}$.

For μ also the prior distributions considered are uniform and two-point distributions respectively, in the following

4.4.1 Uniform Prior for μ

Let μ be distributed uniformly in the range (c, d), then

$$q(\mu) = \frac{1}{d-c}, \quad 0 \le c < \mu < d$$

Then,

$$g_y(y) = \frac{1}{d-c} \int_c^d \mu e^{-\mu y} d\mu$$

Hence from (4.2.5) we have,

$$R(1) = \int_0^\infty \left[\int_y^\infty f_x(x) dx \right] g_y(y) dy$$

$$= 1 - \frac{\lambda}{d-c} \log \frac{d+\lambda}{c+\lambda}$$

$$R(2)=\left[\int_{-\infty}^{\infty}F_y(y)g_y(y)\right]^{2-1}\left[\int_{-\infty}^{\infty}\overline{F}_y(y)g_y(y)dy\right]$$

$$=\left[\int_0^{\infty}\left(1-e^{-\lambda y}\right)\int_c^d\frac{1}{d-c}\mu e^{-\mu y}d\mu dy\right]\left[1-\frac{\lambda}{d-c}\log\frac{d+\lambda}{c+\lambda}\right]=\left[1-R(1)\right]R(1)$$

$$R(3)=\left[\int_{-\infty}^{\infty}F_y(y)g_y(y)\right]^{3-1}\left[\int_{-\infty}^{\infty}\overline{F}_y(y)g_y(y)dy\right]=\left[1-R(1)\right]^2R(1)$$

Then in general we can write,

$$R(r)=\left[1-R(1)\right]^{r-1}R(1)$$

4.4.2 Two-Point Prior for μ

Here if it is assumed that μ can take only two values μ_1 and μ_2 with probabilities q and $(1-q)$, respectively. We have two-point prior distribution for μ given as,

$$\Pr(\mu=\mu_1)=q(\mu_1)\ \textit{and}\ \Pr(\mu=\mu_2)=q(\mu_2)$$

Then,

$$g_y(y)=\sum_{j=1}^{2}\mu_j q(\mu_j)=q\mu_1e^{-\mu_1 y}+(1-q)\mu_2e^{-\mu_2 y} \tag{4.4.1}$$

Hence from (4.2.5) we have,

$$R(1)=\int_0^{\infty}\left[\int_y^{\infty}f_x(x)dx\right]g_y(y)dy$$

$$=\int_0^{\infty}e^{-\lambda y}\left[\mu_1qe^{-\mu_1 y}+(1-q)\mu_2e^{-\mu_2 y}\right]dy$$

$$=\frac{q\mu_1}{\lambda+\mu_1}+\frac{(1-q)\mu_2}{\lambda+\mu_2}$$

$$R(2)=\left[\int_{-\infty}^{\infty}F(y)g(y)dy\right]^{2-1}\left[\int_{-\infty}^{\infty}\overline{F}(y)g(y)dy\right]$$

$$=\left[1-\left(\frac{q\mu_1}{\mu_1+\lambda}+\frac{(1-q)\mu_2}{\mu_2+\lambda}\right)\right]\left[\frac{q\mu_1}{\mu_1+\lambda}+\frac{(1-q)\mu_2}{\mu_2+\lambda}\right]$$

$$=[1-R(1)]R(1)$$

$$R(3)=[1-R(1)]^2R(1)$$

Then in general we can write,

$$R(r)=[1-R(1)]^{r-1}R(1)$$

Hence we can obtain the system reliability R_n by substituting the values of $R(r)$, $r=1,2,\dots,n$ in (2.2.1). The values of R_1, R_2, R_3, R_4 are tabulated in **Table** 4.3 (cf. Appendix) and **Table** 4.4 (cf. Appendix) for parametric values c,d,λ and q,μ_1,μ_2,λ.

4.5 Graphical Representations

A few graphs of R_1, R_2, R_3, R_4 are drawn in **Fig.** 4.1(a)-4.1(d) and **Fig.** 4.2(a)-4.2(d) for different parametric values involved. In **Fig.** 4.1(a) to **Fig.** 4.1(d) show the graphs of R_1, R_2, R_3, R_4 respectively, taking μ along the horizontal axis and the corresponding reliabilities along the vertical axis graphs are plotted for different pairs of a,b. From these graphs one can read directly the values of reliabilities R_1, R_2, R_3, R_4 for intermediate values of μ. It is observed from the graphs that reliability is steadily increasing with μ increases whereas in **Fig.** 4.2(a) to **Fig.** 4.2(d) it is decreasing with increasing λ.

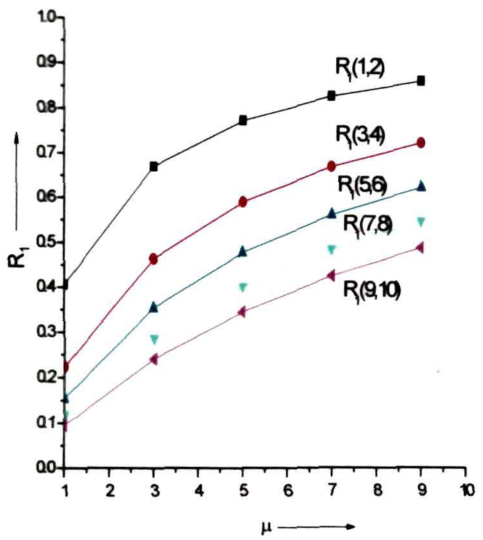

Fig. 4.1(a) Exponential Stress-Strength: Strength parameter λ is random and uniformly distributed in the range (a,b): Graph of R_1 vs μ

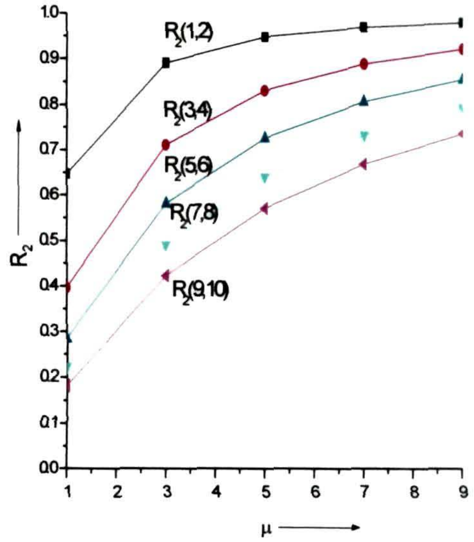

Fig. 4.1(b) Exponential Stress-Strength: Strength parameter λ is random and uniformly distributed in the range (a,b): Graph of R_2 vs μ

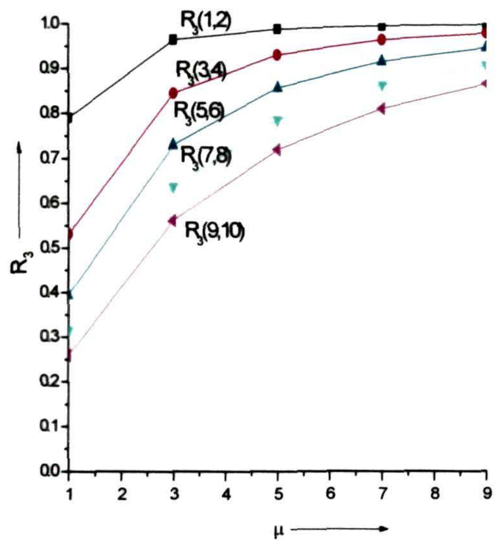

Fig. 4.1(c) Exponential Stress-Strength: Strength parameter λ is random and uniformly distributed in the range (a,b): Graph of R_3 vs μ

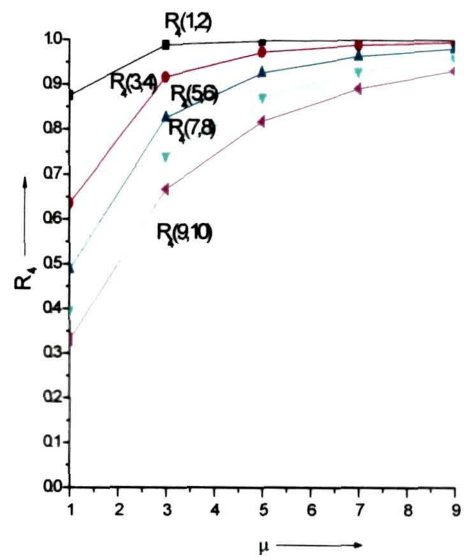

Fig. 4.1(d) Exponential Stress-Strength: Strength parameter λ is random and uniformly distributed in the range (a,b): Graph of R_4 vs μ

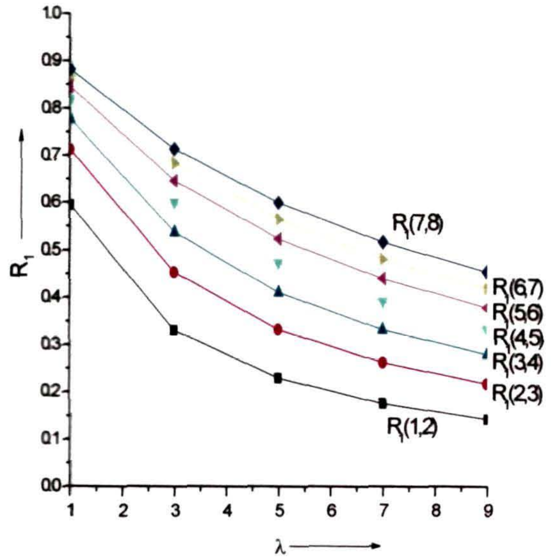

Fig. 4.2(a) Exponential Stress-Strength: Stress parameter μ is random and uniformly distributed in the range (c,d): Graph of R_1 vs λ

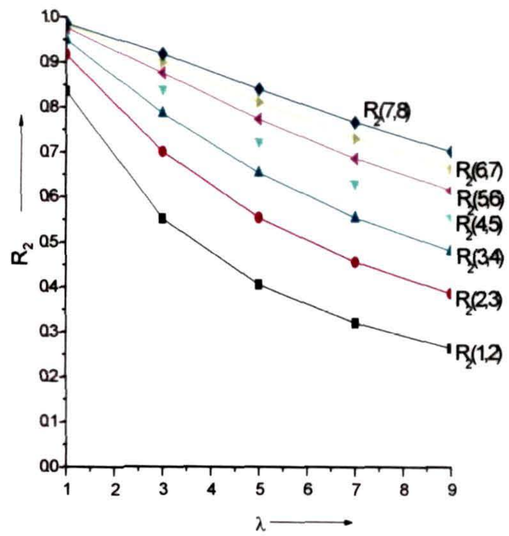

Fig. 4.2(b) Exponential Stress-Strength: Stress parameter μ is random and uniformly distributed in the range (c,d): Graph of R_2 vs λ

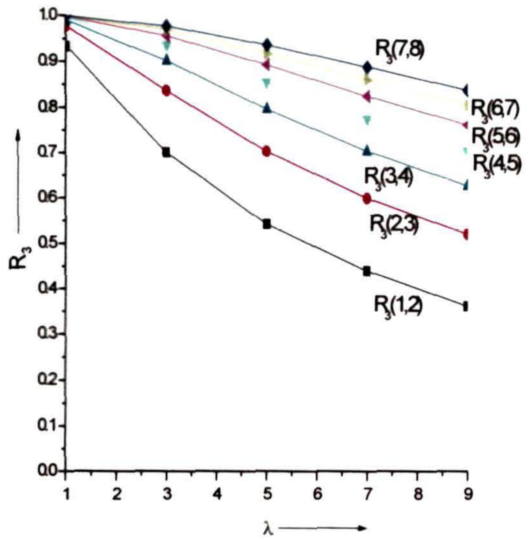

Fig. 4.2(c) Exponential Stress-Strength: Stress parameter μ is random and uniformly distributed in the range (c,d): Graph of R_3 vs λ

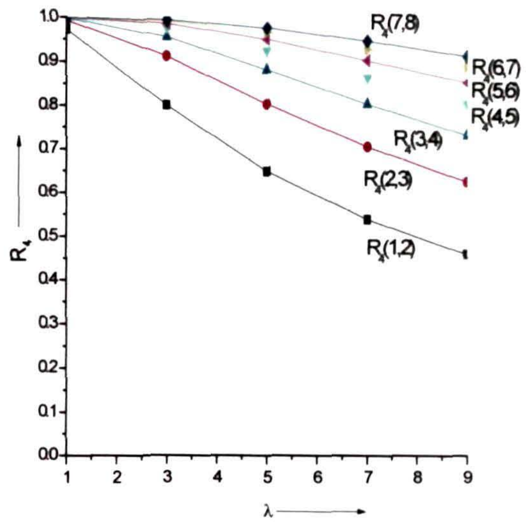

Fig. 4.2(d) Exponential Stress-Strength: Stress parameter μ is random and uniformly distributed in the range (c,d): Graph of R_4 vs λ

4.6 Results and Discussions

In order to see how system reliabilities change with the parameters involved, we have tabulated some values of R_1, R_2, R_3, R_4 for both the distributions from their expressions obtained in the last section. **Table** 4.1 (cf. Appendix) presents the values of R_1, R_2, R_3, R_4 when strength parameter is random but stress parameter is a constant and uniform distributions are considered as a prior distribution for λ for different values of a,b and μ. Here we have seen that the reliabilities are steadily increasing with μ increases but decreases with increasing values of a and b. Similarly, when two-point distributions are considered as the prior distribution for λ for the Case I, we have tabulated some values of R_1, R_2, R_3, R_4 for different values of $p,\lambda_1,\lambda_2,\mu$ in **Table** 4.2 (cf. Appendix). From this table we have also seen that the reliabilities are steadily increasing with μ increases and reliabilities are decreases (increases) with increasing p for $\lambda_1 > \lambda_2(\lambda_1 < \lambda_2)$.

We have tabulated the reliabilities for different values of c,d,λ in **Table** 4.3 (cf. Appendix) when the stress parameter is random but strength parameter is a constant and uniform distributions are considered as a prior distribution for μ. From the table, it is clear that the reliabilities are decreases with increasing λ, intuitively also this should be the case. But reliabilities are steadily increasing with increasing c and d. Similarly, when the prior distributions are considered as the two-point distribution for μ for the Case II, some of the values are presented in respective case in **Table** 4.4 (cf. Appendix) for different values of q, μ_1, μ_2 and λ. From the table we observe that reliabilities are decreases with increasing values of λ and reliabilities are increases (decreases) with increasing q, if $\mu_1 > \mu_2(\mu_1 < \mu_2)$.

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Chapter 5

Identical Strength for Warm and Cold Standby System with Imperfect Switch: A Comparative Study

Chapter 5

Identical Strength for Warm and Cold Standby System with Imperfect Switch: A Comparative Study

5.1 Introduction

As discussed in Chapter 1 switching mechanisms are quite prevalent in standby redundant systems. In a standby redundant system when the active component fails the next component (if there remains any) is instantly activated by some device which is called a switch. In general, it is assumed that the switch is absolutely reliable i.e., perfect. However in the real situation the switch may also fail. i.e., the switch is imperfect and has its own failure pattern. Hence when evaluating the reliability of a standby redundant system not only the failure mechanisms of the different components are to be considered but also that of the switch is to be taken into account.

In a standby system the standby components may be in any one of the three different states viz. hot, cold and warm. In hot standbys the standby units are subjected to the same law of failure as the active unit. i.e., the probability of failure of a standby component in the same as that of an active component. In a cold standby system the standbys, by hypothesis, cannot fail unless they take the place of active units and in case of warm standby system the redundant units are in a partially energized state up to the instant they are put in place of the primary units. During the period they are as standby, they can fail but the probability of failure is less than the probability of failure of the active unit. In this chapter we have considered an n-standby system with cold and warm standbys with imperfect switching for a stress-strength model. Here we assumed that strengths are identical for exponential, gamma and normal distributions for both cold and warm standbys.

Stress-Strength reliability has been discussed in Kapur and Lamberson (1977). Studies on imperfect switching for cold standby systems in S-S model have been considered by Sriwastav and Dutta (1984). They have considered both the switches and the components following similar distributions. Studies on imperfect switching for dynamic warm standby system in TTF (time-to-failure) model have been considered by Alidrisi (1992). Imperfect switching with identical strength for a cold standby redundant system have been considered by Dutta and Bhowal (1997). The system reliability of a standby system, when switches and the components follow dissimilar continuous distributions, is considered by Dutta and Bhowal (2000). To obtain the system reliability for identical stress-strength model when the parameters of the distributions are random variable have been considered by Gogoi and Borah (2011). Studies on warm standby system with imperfect switching in S-S model have been considered by Sriwastav (2004). He considered switches and the components following dissimilar continuous distributions. Warm standby with imperfect switching in cascade model is considered by Gogoi and Borah (2011). Also the problem of system reliability of a cold standby system with imperfect switching in discrete S-S model is considered by Gajjar and Patel (2010). But we have not come across any comparative study on identical strength for a cold and warm standby system with imperfect switching for similar continuous distributions.

The main aim of this chapter is to obtain the system reliability R_3 for cold and warm standby system with imperfect switching for identical strength and comparing the results for both the systems.

In Section 5.2, mathematical formulations of the models are presented. In Sub-Sections 5.2.1 and 5.2.2, the reliability of an n-cold and n-warm standby system with imperfect switch for identical strength is obtained. In Section 5.3, we have assumed some specific distributions to find out the reliability for the stress and strength involved. viz. exponential, gamma and normal. To observe the change in the values of reliabilities with parameters involved, some numerical values of reliabilities are tabulated in **Table** 5.1, **Table** 5.2 and **Table** 5.3 (cf. Appendix). Results and discussions are given in Section 5.4.

5.2 Mathematical Formulation of the Model

5.2.1 Mathematical Formulation of n-Cold Standby Model with Imperfect Switching for Identical Strength

Let us consider an n-standby system working under the impact of stresses. Here we assume that standbys are cold standbys i.e., they cannot fail till put into operation. Let the strength of the n-components are the same say, X. Let $Y_1, Y_2, \ldots, Y_n$ be the set of independent random variables representing the stresses on the n components, when they are activated. It is further assumed that the switch also works under the impact of stresses. Let U be the strength of the switch and V be the stress on it. The switch fails whenever $U < V$, U and V are assumed to be independent random variables. The switch and the components are assumed to work independently. Thus X, Y_i, $i = 1,2,\ldots,n$, U and V are all independent random variables. Then the reliability, R_n of the system is given by (2.2.1) where $R(r)$, $r = 1,2,\ldots,n$ is the marginal reliability due to the r^{th} component. But now $R(r)$, $r = 1,2,\ldots,n$ is given as follows

$$R(1) = P[X \geq Y_1] \tag{5.2.1}$$

$$R(2) = P[X < Y_1, U \geq V \;\; and \;\; X \geq Y_2] \tag{5.2.2}$$

$$R(3) = P[X < Y_1, U \geq V \;\; and \;\; X < Y_2, U \geq V \;\; and \;\; X \geq Y_3] \tag{5.2.3}$$

Then in general, we have

$$R(r) = P[X < Y_1, U \geq V \;\; and \;\; X < Y_2, \ldots U \geq V \;\; and \;\; X < Y_{r-1}, U \geq V \;\; and \;\; X \geq Y_r] \tag{5.2.4}$$

Let $f(x)$, $g_i(y)$, $h(u)$ and $k(v)$, $i = 1,2,\ldots,n$ be the p.d.f.'s of X, Y_i, U and V respectively. Since all the components and the switch are working independently, we have

$$R(1)=\int_{-\infty}^{\infty}\overline{F}(y)g_1(y)dy \tag{5.2.5}$$

$$R(2)=\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\overline{F}(y)g_2(y)dy\right\} \tag{5.2.6}$$

$$\begin{aligned}R(3)=&\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}F(y)g_2(y)dy\right\}\\&\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\overline{F}(y)g_3(y)dy\right\}\end{aligned} \tag{5.2.7}$$

Then,

$$\begin{aligned}R(r)=&\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}F(y)g_2(y)dy\right\}\ldots\\&\left\{\int_{-\infty}^{\infty}F(y)g_{r-1}(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\overline{F}(y)g_r(y)dy\right\}\end{aligned} \tag{5.2.8}$$

where $F(x)$ and $H(u)$ are the cumulative distribution functions (c.d.f.) of X and U respectively. i.e., $F(x)=\int_{-\infty}^{x}f(x)dx$ & $\overline{F}(x)=1-F(x)$ & $\overline{H}(u)=\int_{u}^{\infty}h(u)du$

5.2.2 Mathematical Formulation of n-Warm Standby Model with Imperfect Switching for Identical Strength

Let the strength of the n-components are the same say, X. Let $Y_1, Y_2, \ldots, Y_n$ and $Z_2, Z_3, \ldots, Z_n$ be the stresses on the n^{th} component when it is active and it is standby respectively. Let U be the strength and V be the stress on the switch. The switch fails when $U<V$. We assume that X, Y_i, Z_j, U and V are all independent random variables. The

reliability, R_n of the system is given by the equation (2.2.1). But now $R(r)$, $r = 1,2,\ldots,n$ is given as follows

$$R(1) = P[X \geq Y_1] \tag{5.2.9}$$

$$R(2) = P[X < Y_1, \{X \geq Z_2, (U \geq V \, and \, X \geq Y_2)\}] \tag{5.2.10}$$

$$R(3) = P\begin{bmatrix} X < Y_1, \{X \geq Z_2, (U \geq V \, and \, X < Y_2) \, or \, X < Z_2\} \\ \{X \geq Z_3, (U \geq V \, and \, X \geq Y_3)\} \end{bmatrix} \tag{5.2.11}$$

Then in general, we have

$$R(r) = P\begin{bmatrix} X < Y_1, \{X \geq Z_2, (U \geq V \text{ and } X < Y_2) or \, X < Z_2\} \\ \{X \geq Z_3, (U \geq V \text{ and } X < Y_3) \text{ or } X < Z_3\}, \\ \ldots, \{X \geq Z_{r-1}, (U \geq V \text{ and } X < Y_{r-1}) \text{ or } X < Z_{r-1}\} \\ \{X \geq Z_r, (U \geq V \text{ and } X \geq Y_r)\} \end{bmatrix} \tag{5.2.12}$$

Let $f(x)$, $g_i(y)$, $w_j(z)$, $h(u)$ and $k(v)$, $i = 1,2,\ldots,n$, $j = 2,3,\ldots,n$ be the p.d.f.'s of X, Y_i, Z_j, U and V respectively. Since all the components and the switch are working independently, we have

$$R(1) = \int_{-\infty}^{\infty} \overline{F}(y) g_1(y) dy \tag{5.2.13}$$

$$R(2) = \int_{-\infty}^{\infty} F(y) g_1(y) dy \int_{-\infty}^{\infty} \overline{F}_2(z) w_2(z) dz \int_{-\infty}^{\infty} \overline{H}(v) k(v) dv \int_{-\infty}^{\infty} \overline{F}(y) g_2(y) dy \tag{5.2.14}$$

$$R(3) = \left\{\int_{-\infty}^{\infty} F(y) g_1(y) dy\right\} \left\{\int_{-\infty}^{\infty} \overline{F}_2(z) w_2(z) dz \int_{-\infty}^{\infty} \overline{H}(v) k(v) dv \int_{-\infty}^{\infty} F(y) g_2(y) dy + \int_{-\infty}^{\infty} F_2(z) w_2(z) dz\right\}$$
$$\left\{\int_{-\infty}^{\infty} \overline{F}_3(z) w_3(z) dz \int_{-\infty}^{\infty} \overline{H}(v) k(v) dv \int_{-\infty}^{\infty} \overline{F}(y) g_3(y) dy\right\}$$

(5.2.15)

Then,

$$R(r)=\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{F}_2(z)w_2(z)dz\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\int_{-\infty}^{\infty}F(y)g_2(y)dy+\int_{-\infty}^{\infty}F_2(z)w_2(z)dz\right\}$$
$$\left\{\int_{-\infty}^{\infty}\overline{F}_3(z)w_3(z)dz\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\int_{-\infty}^{\infty}\overline{F}(y)g_3(y)dy+\int_{-\infty}^{\infty}F_3(z)w_3(z)dz\right\}\cdots$$
$$\left\{\int_{-\infty}^{\infty}\overline{F}_r(z)w_r(z)dz\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\int_{-\infty}^{\infty}\overline{F}(y)g_r(y)dy\right\}\quad where,\ r=1,2,\ldots,n \tag{5.2.16}$$

Here $F(x)$ and $H(u)$ are the c.d.f.'s of X and U respectively.

5.3 Reliability for Specific Distributions

5.3.1 Exponential Stress-Strength: Cold Standby for Identical Strength

Let $f(x)$, $g_i(y)$, $h(u)$ and $k(v)$, $i=1,2,\ldots,n$ be all exponential densities with means $\frac{1}{\theta}, \frac{1}{\alpha_i}, \frac{1}{\lambda}$ and $\frac{1}{\mu}$ respectively, $i=1,2,\ldots,n$ i.e.,

$$f(x)=\begin{cases}\theta e^{-\theta x}, & x\ge 0,\ \theta\ge 0\\ 0, & otherwise\end{cases}$$

$$g_i(y)=\begin{cases}\alpha_i e^{-\alpha_i y_i}, & y_i\ge 0,\ \alpha_i\ge 0\\ 0, & otherwise\end{cases}$$

$$h(u)=\begin{cases}\lambda e^{-\lambda u}, & u\ge 0,\ \lambda\ge 0\\ 0, & otherwise\end{cases}$$

$$k(v)=\begin{cases}\mu e^{-\mu v}, & v\ge 0,\ \mu\ge 0\\ 0, & otherwise\end{cases}$$

If λ is a positive integer then,

$$\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv=\int_{0}^{\infty}e^{-\lambda v}\mu e^{-\mu v}dv=\frac{\mu}{\mu+\lambda}=\frac{1}{1+\rho}\quad \text{where } \rho=\frac{\lambda}{\mu}$$

From, (5.2.5) we get,

$$R(1)=\int_{-\infty}^{\infty}\overline{F}(y)g_1(y)dy=\int_{0}^{\infty}e^{\theta y_1}\alpha_1 e^{-\alpha_1 y_1}dy_1=\frac{\alpha_1}{\theta+\alpha_1}$$

From, (5.2.6) we get,

$$\begin{aligned}R(2)&=\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\overline{F}(y)g_2(y)dy\right\}\\&=\left[\int_{0}^{\infty}\left(1-e^{-\theta y_1}\right)\alpha_1 e^{-\alpha_1 y_1}dy_1\right]\left[\int_{0}^{\infty}e^{-\lambda v}\mu e^{-\mu v}dv\right]\left[\int_{0}^{\infty}e^{-\theta y_2}\alpha_2 e^{-\alpha_2 y_2}dy_2\right]\\&=\frac{\theta}{\alpha_1+\theta}\frac{1}{1+\rho}\frac{\alpha_2}{\alpha_2+\theta}\end{aligned}$$

From, (5.2.7) we get,

$$\begin{aligned}R(3)&=\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}F(y)g_2(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\overline{F}(y)g_3(y)dy\right\}\\&=\frac{\theta}{\alpha_1+\theta}\frac{\mu}{\mu+\lambda}\frac{\theta}{\alpha_2+\theta}\frac{\mu}{\mu+\lambda}\frac{\alpha_3}{\alpha_3+\theta}\\&=\left(\frac{1}{1+\rho}\right)^2\frac{\theta}{\alpha_1+\theta}\frac{\theta}{\alpha_2+\theta}\frac{\alpha_3}{\alpha_3+\theta}\end{aligned}$$

Then the system reliability R_3 is given by the equation (2.2.1).

5.3.2 Exponential Stress-Strength: Warm Standby for Identical Strength

Let $f(x)$, $g_i(y)$, $w_j(z)$, $h(u)$ and $k(v)$, $i=1,2,\ldots,n$, $j=2,3,\ldots,n$ be all exponential densities with means $\frac{1}{\theta},\frac{1}{\alpha_i},\frac{1}{\beta_j},\frac{1}{\lambda}$ and $\frac{1}{\mu}$ respectively.

Now,

$$f(x)=\begin{cases}\theta e^{-\theta x}, & x\ge 0,\ \theta\ge 0\\ 0, & otherwise\end{cases}$$

$$g_i(y)=\begin{cases}\alpha_i e^{-\alpha_i y_i}, & y_i\ge 0,\ \alpha_i\ge 0\\ 0, & otherwise\end{cases}$$

$$w_j(z)=\begin{cases}\beta_j e^{-\beta_j z_j}, & z_i\ge 0,\ \beta_j\ge 0\\ 0, & otherwise\end{cases}$$

$$h(u)=\begin{cases}\lambda e^{-\lambda u}, & u\ge 0,\ \lambda\ge 0\\ 0, & otherwise\end{cases}$$

$$k(v)=\begin{cases}\mu e^{-\mu v}, & v\ge 0,\ \mu\ge 0\\ 0, & otherwise\end{cases}$$

From (5.2.13), (5.2.14) and (5.2.15) we get

$$R(1)=\int_{-\infty}^{\infty}\overline{F}(y)g_1(y)dy=\int_{0}^{\infty}e^{-\theta y_1}\alpha_1 e^{-\alpha_1 y_1}dy_1=\frac{\alpha_1}{\alpha_1+\theta}$$

$$\begin{aligned}R(2)&=\int_{-\infty}^{\infty}F(y)g_1(y)dy\int_{-\infty}^{\infty}\overline{F}_2(z)w_2(z)dz\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\int_{-\infty}^{\infty}\overline{F}(y)g_2(y)dy\\&=\left[\int_{0}^{\infty}\left(1-e^{-\theta y_1}\right)\alpha_1 e^{-\alpha_1 y_1}dy_1\right]\left[\int_{0}^{\infty}e^{-\theta z_2}\beta_2 e^{-\beta_2 z_2}dz_2\right]\left[\int_{0}^{\infty}e^{-\lambda v}\mu e^{-\mu v}dv\right]\left[\int_{0}^{\infty}e^{-\theta y_2}\alpha_2 e^{-\alpha_2 y_2}dy_2\right]\\&=\frac{\theta}{\alpha_1+\theta}\frac{\beta_2}{\beta_2+\theta}\frac{1}{1+\rho}\frac{\alpha_2}{\alpha_2+\theta}\end{aligned}$$

Similarly,

$$R(3)=\frac{\theta}{\alpha_1+\theta}\left[\left\{\frac{\beta_2}{\beta_2+\theta}\frac{1}{1+\rho}\frac{\theta}{\alpha_2+\theta}\right\}+\frac{\theta}{\beta_2+\theta}\right]\left[\frac{\beta_3}{\beta_3+\theta}\frac{1}{1+\rho}\frac{\alpha_3}{\alpha_3+\theta}\right]$$

Then the system reliability R_3 is given by the equation (2.2.1).

5.3.3 Gamma Stress-Strength: Cold Standby for Identical Strength

Let $f(x)$, $g_i(y)$, $h(u)$ and $k(v)$, $i=1,2,\ldots,n$ be all gamma densities with shape parameters θ,α_i,λ and μ respectively and scale parameters equal to unity.

Then,

$$f(x)=\begin{cases}\frac{1}{\Gamma\theta}e^{-x}x^{\theta-1}, & x\geq 0,\ \theta\geq 1\\ 0, & otherwise\end{cases}$$

$$g_i(y)=\begin{cases}\frac{1}{\Gamma\alpha_i}e^{-y_i}y_i^{\alpha_i-1}, & y_i\geq 0,\ \alpha_i\geq 1\\ 0, & otherwise\end{cases}$$

$$h(u)=\begin{cases}\frac{1}{\Gamma\lambda}e^{-u}u^{\lambda-1}, & u\geq 0,\ \lambda\geq 1\\ 0, & otherwise\end{cases}$$

$$k(v)=\begin{cases}\frac{1}{\Gamma\mu}e^{-v}v^{\mu-1}, & v\geq 0,\ \mu\geq 1\\ 0, & otherwise\end{cases}$$

From, (5.2.5) we get,

$$
\begin{aligned}
R(1) = \int_{-\infty}^{\infty} \bar{F}(y) g_1(y) dy &= \int_0^{\infty} \left[\int_y^{\infty} f(x) dx \right] g_1(y) dy \\
&= \int_0^{\infty} \left[\int_y^{\infty} \frac{1}{\Gamma\theta} e^{-x} x^{\theta-1} dx \right] \frac{1}{\Gamma\alpha_1} e^{-y_1} y_1^{\alpha_1 - 1} \\
&= \sum_{i=0}^{\theta-1} \frac{\Gamma(\theta + \alpha_1 - i - 1)}{\Gamma\alpha_1 (\theta - i - 1)! 2^{\theta+\alpha_1 - i - 1}} \\
&= R(\theta, \alpha_1), \ say
\end{aligned}
$$

From, (5.2.6) we get,

$$
\begin{aligned}
R(2) &= \left\{ \int_{-\infty}^{\infty} F(y) g_1(y) dy \right\} \left\{ \int_{-\infty}^{\infty} \bar{H}(v) k(v) dv \right\} \left\{ \int_{-\infty}^{\infty} \bar{F}(y) g_2(y) dy \right\} \\
&= \left(1 - \sum_{i=0}^{\theta-1} \frac{\Gamma(\theta + \alpha_1 - i - 1)}{\Gamma\alpha_1 (\theta - i - 1)! 2^{\theta+\alpha_1 - i - 1}} \right) \left(\sum_{i=0}^{\theta-1} \frac{\Gamma(\theta + \alpha_2 - i - 1)}{\Gamma\alpha_2 (\theta - i - 1)! 2^{\theta+\alpha_2 - i - 1}} \right) \left(\sum_{i=0}^{\lambda-1} \frac{\Gamma(\lambda + \mu - i - 1)}{\Gamma\mu (\lambda - i - 1)! 2^{\lambda+\mu - i - 1}} \right) \\
&= \bar{R}(\theta, \alpha_1) R(\theta, \alpha_2) R(\lambda, \mu)
\end{aligned}
$$

Similarly from, (5.2.7) we get,

$$
\begin{aligned}
R(3) &= \left\{ \int_{-\infty}^{\infty} F(y) g_1(y) dy \right\} \left\{ \int_{-\infty}^{\infty} \bar{H}(v) k(v) dv \right\} \left\{ \int_{-\infty}^{\infty} F(y) g_2(y) dy \right\} \left\{ \int_{-\infty}^{\infty} \bar{H}(v) k(v) dv \right\} \left\{ \int_{-\infty}^{\infty} \bar{F}(y) g_3(y) dy \right\} \\
&= \bar{R}(\theta, \alpha_1) \bar{R}(\theta, \alpha_2) R(\theta, \alpha_3) [R(\lambda, \mu)]^2
\end{aligned}
$$

Then the system reliability R_3 is given by the equation (2.2.1).

5.3.4 Gamma Stress-Strength: Warm Standby for Identical Strength

Let $f(x)$, $g_i(y)$, $w_j(z)$, $h(u)$ and $k(v)$, $i = 1,2,\ldots,n$, $j = 2,3,\ldots,n$ be all gamma densities with shape parameters $\theta, \alpha_i, \beta_j, \lambda$ and μ respectively and scale parameters equal to unity.

Then,

$$f(x)=\begin{cases}\dfrac{1}{\Gamma\theta}e^{-x}x^{\theta-1}, & x\geq 0,\ \theta\geq 1\\ 0, & otherwise\end{cases}$$

$$g_i(y)=\begin{cases}\dfrac{1}{\Gamma\alpha_i}e^{-y_i}y_i^{\alpha_i-1}, & y_i\geq 0,\ \alpha_i\geq 1\\ 0, & otherwise\end{cases}$$

$$w_j(z)=\begin{cases}\dfrac{1}{\Gamma\beta_j}e^{-z_j}z_j^{\beta_j-1}, & z_j\geq 0,\ \beta_j\geq 1\\ 0, & otherwise\end{cases}$$

$$h(u)=\begin{cases}\dfrac{1}{\Gamma\lambda}e^{-u}\,u^{\lambda-1}, & u\geq 0,\ \lambda\geq 1\\ 0, & otherwise\end{cases}$$

$$k(v)=\begin{cases}\dfrac{1}{\Gamma\mu}e^{-v}v^{\mu-1}, & v\geq 0,\ \mu\geq 1\\ 0, & otherwise\end{cases}$$

From (5.2.13), (5.2.14) and (5.2.15) we get

$$R(1)=\int_{-\infty}^{\infty}\overline{F}(y)g_1(y)dy=\sum_{i=0}^{\theta-1}\frac{\Gamma(\theta+\alpha_1-i-1)}{\Gamma\alpha_1(\theta-i-1)!2^{\theta+\alpha_1-i-1}}=R(\theta,\alpha_1),\ say$$

$$\begin{aligned}R(2)&=\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{F}_2(z)w_2(z)dz\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\overline{F}(y)g_2(y)dy\right\}\\&=\left(1-\sum_{i=0}^{\theta-1}\frac{\Gamma(\theta+\alpha_1-i-1)}{\Gamma\alpha_1(\theta-i-1)!2^{\theta+\alpha_1-i-1}}\right)\left(\sum_{i=0}^{\theta-1}\frac{\Gamma(\theta+\beta_2-i-1)}{\Gamma\beta_2(\theta-i-1)!2^{\theta+\beta_2-i-1}}\right)\\&\quad\left(\sum_{i=0}^{\lambda-1}\frac{\Gamma(\lambda+\mu-i-1)}{\Gamma\mu(\lambda-i-1)!2^{\lambda+\mu-i-1}}\right)\left(\sum_{i=0}^{\theta-1}\frac{\Gamma(\theta+\alpha_2-i-1)}{\Gamma\alpha_2(\theta-i-1)!2^{\theta+\alpha_2-i-1}}\right)\\&=\overline{R}(\theta,\alpha_1)R(\theta,\beta_2)R(\lambda,\mu)R(\theta,\alpha_2)\end{aligned}$$

Similarly,

$$R(3)=\overline{R}(\theta,\alpha_1)\{R(\theta,\beta_2)R(\lambda,\mu)\overline{R}(\theta,\alpha_2)+\overline{R}(\theta,\beta_2)\}\{R(\theta,\beta_3)R(\lambda,\mu)R(\theta,\alpha_3)\}$$

Then the system reliability R_3 is given by the equation (2.2.1).

5.3.5 Normal Stress-Strength: Cold Standby for Identical Strength

Let $f(x)$, $g_i(y)$, $h(u)$ and $k(v)$ be $N(\theta,\sigma)$, $N(\alpha_i,\tau_i)$, $N(\lambda,\upsilon)$ and $N(\mu,\rho)$ respectively, $i=1,2,\ldots,n$. Let us define $Z=U-V>0$ and $Z_i=X-Y_i>0$. Then the random variables Z and Z_i are normally distributed with mean $(\lambda-\mu)$ and $(\theta-\alpha_i)$ and standard deviations $\sqrt{\upsilon^2+\rho^2}$ and $\sqrt{\sigma^2+\tau_i^2}$ respectively, $i=1,2,\ldots,n$.

Then from (5.2.5) we get,

$$\begin{aligned}R(1)=\int_{-\infty}^{\infty}\overline{F}(y)g_1(y)dy=P(Z_1\geq 0)&=\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{\sigma^2+\tau_1^2}}\int_0^{\infty}e^{-\frac{1}{2}\left[\frac{z_1-(\theta-\alpha_1)}{\sqrt{\sigma^2+\tau_1^2}}\right]^2}dz_1\\&=\frac{1}{\sqrt{2\pi}}\int_{-\frac{\theta-\alpha_1}{\sqrt{\sigma^2+\tau_1^2}}}^{\infty}e^{-\frac{1}{2}t_1^2}dt_1 \quad where\ t_1=\frac{Z_1-(\theta-\alpha_1)}{\sqrt{\sigma^2+\tau_1^2}}\\&=1-\Phi\left(-\frac{\theta-\alpha_1}{\sqrt{\sigma^2+\tau_1^2}}\right)\\&=\Phi\left(\frac{\theta-\alpha_1}{\sqrt{\sigma^2+\tau_1^2}}\right)\\&=\Phi(A_1),\ say\end{aligned}$$

Then from (5.2.6) we get,

$$\begin{aligned}R(2)&=\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\overline{F}(y)g_2(y)dy\right\}\\&=P(Z_1<0)P(Z\geq 0)P(Z_2\geq 0)\end{aligned}$$

Now as above, $P(Z \geq 0) = \Phi\left(\frac{\lambda - \mu}{\sqrt{\upsilon^2 + \rho^2}}\right) = \Phi(S_w)$, *say*

Then,

$$R(2) = \left[1 - \Phi\left(\frac{\theta - \alpha_1}{\sqrt{\sigma^2 + \tau_1^2}}\right)\right]\left[\Phi\left(\frac{\theta - \alpha_2}{\sqrt{\sigma^2 + \tau_2^2}}\right)\right]\left[\Phi\left(\frac{\lambda - \mu}{\sqrt{\upsilon^2 + \rho^2}}\right)\right]$$
$$= \Phi(\overline{A_1})\Phi(A_2)\Phi(S_w)$$

Similarly from (5.2.7) we get,

$$R(3) = \left\{\int_{-\infty}^{\infty} F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty} \overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty} F(y)g_2(y)dy\right\}\left\{\int_{-\infty}^{\infty} \overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty} \overline{F}(y)g_3(y)dy\right\}$$
$$= \Phi(\overline{A_1})\Phi(\overline{A_2})\Phi(A_3)[\Phi(S_w)]^2$$

Then the system reliability R_3 is given by the equation (2.2.1).

5.3.6 Normal Stress-Strength: Warm Standby for Identical Strength

Let $f(x)$, $g_i(y)$, $w_j(z)$ be $N(\theta,\sigma)$, $N(\alpha_i,\tau_i)$ and $N(\beta_j,\gamma_j)$ respectively, $i = 1,2,\ldots,n$; $j = 2,3,\ldots,n$ and $h(u)$ and $k(v)$ be $N(\lambda,\upsilon)$ and $N(\mu,\rho)$ respectively. Let us define $Z = U - V > 0$ and $Z_i = X - Y_i > 0$; $i = 1,2,\ldots,n$, $T_j = X - Z_j$; $j = 2,3,\ldots,n$ are normally distributed with mean $(\lambda - \mu)$, $(\theta - \alpha_i)$, $(\theta - \beta_j)$ and standard deviations $\sqrt{\upsilon^2 + \rho^2}$, $\sqrt{\sigma^2 + \tau_i^2}$ and $\sqrt{\sigma^2 + \gamma_j^2}$ respectively.

Then from (5.2.13), (5.2.14) and (5.2.15) we get

$$R(1) = \int_{-\infty}^{\infty} \overline{F}(y)g_1(y)dy = P(Z_1 \geq 0) = \Phi\left(\frac{\theta - \alpha_1}{\sqrt{\sigma^2 + \tau_1^2}}\right)$$
$$= \Phi(A_1), \ say$$

$$R(2)=\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{F}_2(z)w_2(z)dz\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\overline{F}(y)g_2(y)dy\right\}$$
$$=P(Z_1<0)P(T_2\geq 0)P(Z\geq 0)P(Z_2\geq 0)$$

Now, $P(Z\geq 0)=\Phi\left(\dfrac{\lambda-\mu}{\sqrt{\upsilon^2+\rho^2}}\right)=\Phi(S_w)$, *say*

$$P(T_2\geq 0)=\Phi\left(\frac{\theta-\beta_2}{\sqrt{\sigma^2+\gamma_2^2}}\right)=\Phi(B_2),\ say$$

Then,

$$R(2)=\left[1-\Phi\left(\frac{\theta-\alpha_1}{\sqrt{\sigma^2+\tau_1^2}}\right)\right]\left[\Phi\left(\frac{\theta-\beta_2}{\sqrt{\sigma^2+\gamma_2^2}}\right)\right]\left[\Phi\left(\frac{\lambda-\mu}{\sqrt{\upsilon^2+\rho^2}}\right)\right]\left[\Phi\left(\frac{\theta-\alpha_2}{\sqrt{\sigma^2+\tau_2^2}}\right)\right]$$

$$=\Phi\left(\overline{A_1}\right)\Phi(B_2)\Phi(S_w)\Phi(A_2)$$

Similarly,

$$R(3)=\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{F}_2(z)w_2(z)dz\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\int_{-\infty}^{\infty}F(y)g_2(y)dy+\int_{-\infty}^{\infty}F_2(z)w_2(z)dz\right\}$$
$$\left\{\int_{-\infty}^{\infty}\overline{F}_3(z)w_3(z)dz\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\int_{-\infty}^{\infty}\overline{F}(y)g_3(y)dy\right\}$$
$$=P(Z_1<0)[P(T_2\geq 0)P(Z\geq 0)P(Z_2<0)+P(T_2<0)][P(T_3\geq 0)P(Z\geq 0)P(Z_3\geq 0)]$$

$$=\Phi\left(\overline{A}_1\right)\left[\Phi(B_2)\Phi(S_w)\Phi\left(\overline{A}_2\right)+\Phi\left(\overline{B}_2\right)\right]\left[\Phi(B_3)\Phi(S_w)\Phi(A_3)\right]$$

Then the system reliability R_3 is given by the equation (2.2.1).

5.4 Results and Discussions

For a few values of the parameters involved in the expressions of $R(r)$, $r = 1,2,3$ we evaluate $R(1)$, $R(2)$, $R(3)$ and R_3 for different distributions from their expressions obtained in the last section.

From the **Table** 5.1 (cf. Appendix), it is observed that if the strength parameter θ increases then reliability R_3 decreases. When the stress parameter α_1 increases $R(1)$ also increases. The value of marginal reliability $R(1)$ becomes 0.7500, 0.6000, 0.5000, 0.4286 and 0.3750 respectively for θ=.1,.2,.3,.4.5. It can be noted from the **Table** 5.1 (cf. Appendix) that values of $R(1)$ remains same for both cold and warm standby systems. In case of cold standby system, $R(2)$ and $R(3)$ become 0.0694 and 0.0053 respectively for $\alpha_2 = \alpha_3 = 1.1$. Similarly, when $\alpha_2 = \alpha_3 = 1.3$ then $R(2)$ and $R(3)$ become 0.0603 and 0.0039 respectively. But in case of warm standby system, $R(2)$ and $R(3)$ become 0.0347 and 0.0187 respectively when $\alpha_2 = \alpha_3 = 1.1$. Similarly for $\alpha_2 = \alpha_3 = 1.3$, $R(2)$ and $R(3)$ become 0.0301 and 0.0161 respectively. It is also observed that the parameters β_2, β_3 in case of exponential distribution, are seems to be very sensitive for warm standby system. Hence, in case of warm standby system, the system reliability becomes smaller than that of cold standby system.

From the **Table** 5.2 (cf. Appendix), it is clear that the system reliability increases as the values of corresponding θ increases. In case of cold standby system, the system reliability R_3 becomes 0.6563, 0.8555, 0.9331, 0.9677, 0.9841 and 0.9921 respectively for θ=1,2,3,4,5,6. But in case of warm standby system, the system reliability R_3 becomes 0.6016, 0.8445, 0.9314, 0.9675, 0.9841 and 0.9921 respectively for θ=1,2,3,4,5,6. The marginal reliability $R(1)$ remains same for both cold and warm standby systems. In case of gamma distribution, the parameters β_2, β_3 are seems to be very sensitive for warm standby system. Hence, the values of the system reliability in case of warm standby system become smaller than that of cold standby system.

From the **Table** 5.3 (cf. Appendix), it is observed that when the strength parameter θ increases system reliability R_3 also increases but when σ increases the system reliability R_3 decreases. For example, in case of cold standby system the system reliability R_3 becomes 0.8998, 0.9561, 0.9816, 0.9929, 0.9976 and 0.9993 respectively for θ=1,2,3,4,5,6. But R_3 becomes 0.8998, 0.7965 and 0.7528 respectively for σ=1,2,3. Again in case of warm standby system, the system reliability R_3 becomes 0.7925, 0.8682, 0.9256, 0.9661, 0.9877 and 0.9964 respectively for θ=1,2,3,4,5,6. But R_3 becomes 0.7925, 0.7106, 0.6767 for σ=1,2,3. When the stress parameters τ_1, τ_2 and τ_3 increase there are significant decreases in the values of $R(1)$ with increasing υ and ρ. Similarly, in case of normal distribution the stress parameters γ_2 and γ_3 are seemed to be very sensitive due to which the values of the system reliabilities of warm standby system becomes smaller than that of cold standby system.

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Chapter 6

Identical Stress for Warm and Cold Standby System with Imperfect Switch: A Comparative Study

Chapter 6

Identical Stress for Warm and Cold Standby System with Imperfect Switch: A Comparative Study

6.1 Introduction

As mentioned in chapter 1 and Chapter 5 switching mechanisms are required in standby redundant systems. Also warm and cold standby systems are discussed earlier. In the previous chapter, we have discussed a comparative study between cold and warm standby system with imperfect switching for identical strength. But in this chapter we have discussed a comparative study between cold and warm standby system with imperfect switching for identical stress considering exponential, gamma and normal distributions. Some of the results of this chapter have been accepted for publication in IAPQR journal.

This chapter is organized as follows: In Section 6.2, the general mathematical models are developed. In Sub-Section 6.2.1 reliability of n-cold standby system with imperfect switch for identical stress is obtained and in Sub-Section 6.2.2 reliability of n-warm standby system with imperfect switch for identical stress is obtained. In Section 6.3, marginal reliability expressions, $R(1)$, $R(2)$ and $R(3)$ are obtained when stress-strength of the components and that of the switch follow particular distributions. In Sub-Sections 6.3.1 and 6.3.2, stress-strength distribution for the components and the switch are taken as exponential, in Sub-Sections 6.3.3 and 6.3.4, stress-strength distribution for the components and the switch involved are gamma and in Sub-Sections 6.3.5 and 6.3.6, stress-strength and the switch involved are normal with identical stress for cold and warm standbys and the marginal reliability expressions $R(1)$, $R(2)$ and $R(3)$ are obtained. Also the system reliability R_3 is obtained for all the cases. To testing the validity of the derived model system reliabilities has been estimated with various parameters involved in the system and is also presented in tabular forms in the **Table** 6.1, **Table** 6.2 and **Table** 6.3 (cf. Appendix). To make the things clear, a few graphs are plotted for each case in Section 6.4 for selected values of the parameters. Results and discussions are devoted to Section 6.5.

6.2 Mathematical Formulation of the Model

6.2.1 Mathematical Formulation of n-Cold Standby Model with Imperfect Switching for Identical Stresses

Let $X_1, X_2, ..., X_n$ be the strengths of the n-components in order of activation and let $Y_1 = Y_2 = ... = Y_n = Y$ be the stress on them. Now to activate the standby components there is a switch when strength and stress U and V respectively. The switch fails when $U < V$. All the components and the switch are working independently i.e., X_i, Y, U and V are all independent random variables $(i = 1,2,...,n)$. The system reliability R_n is given by (2.2.1) where $R(r)$, $r = 1,2,...,n$ is the marginal reliability due to the r^{th} component. But now, $R(r)$, $r = 1,2,...,n$ is given as follows

$$R(1) = P[X_1 \geq Y] \tag{6.2.1}$$

$$R(2) = P[X_1 < Y, U \geq V \; and \; X_2 \geq Y] \tag{6.2.2}$$

$$R(3) = P[X_1 < Y, U \geq V \; and \; X_2 < Y, U \geq V \; and \; X_3 \geq Y] \tag{6.2.3}$$

Then in general, we have

$$R(r) = P[X_1 < Y, U \geq V \; and \; X_2 < Y, ..U \geq V \; and \; X_{r-1} < Y, U \geq V \; and \; X_r \geq Y] \tag{6.2.4}$$

Let $f_i(x)$, $g(y)$, $h(u)$ and $k(v)$ denote the p.d.f.'s of X_i, Y, U and V respectively. Then we have,

$$R(1) = \int_{-\infty}^{\infty} \bar{F}_1(y) g(y) dy \tag{6.2.5}$$

$$R(2)=\left\{\int_{-\infty}^{\infty}F_1(y)g(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\overline{F}_2(y)g(y)dy\right\} \tag{6.2.6}$$

$$\begin{aligned}R(3)=&\left\{\int_{-\infty}^{\infty}F_1(y)g(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}F_2(y)g(y)dy\right\}\\&\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\overline{F}_3(y)g(y)dy\right\}\end{aligned} \tag{6.2.7}$$

Then,

$$\begin{aligned}R(r)=&\left\{\int_{-\infty}^{\infty}F_1(y)g(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}F_2(y)g(y)dy\right\}\ldots\\&\left\{\int_{-\infty}^{\infty}F_{r-1}(y)g(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\overline{F}_r(y)g(y)dy\right\}\end{aligned} \tag{6.2.8}$$

Substituting the values of $R(r)$, $r=1,2,\ldots,n$ in the expression (2.2.1) we can get the system reliability R_n.

6.2.2 Mathematical Formulation of n-Warm Standby Model with Imperfect Switching for Identical Stresses

Let us consider that all the components are working under the same stresses, under the same environment. Then we can take all the Y_i's to be i.i.d. Let us consider an n-standby system working under the impact of stresses. Initially there is one active component and $(n-1)$ warm standby components. Let $X_1, X_2, \ldots, X_n$ be the strengths of the n-components in order of activation. Let $Y_1 = Y_2 = \ldots = Y_n = Y$ be the stresses on these n-components, respectively, when in operation. The $(n-1)$ components as warm standbys face $(n-1)$ stresses viz. $Z_2, Z_3, \ldots, Z_n$ respectively. Let U and V be the strength and stress of the switch respectively. The switch fails when $U < V$.

All the components and the switch are working independently i.e., X_i, Y, Z_j, U and V are all independent random variables $(i = 1,2,...,n;\ j = 2,3,...,n)$. Now the i^{th} active component fails if $X_i < Y_i$ and the j^{th} standby component fails if $X_j < Z_j$. The system fails when all the components have failed, either in operation or as standbys. The reliability of the system is given by (2.2.1). But now, $R(r)$, $r = 1,2,...,n$ is given as follows

$$R(1) = P[X_1 \geq Y] \tag{6.2.9}$$

$$R(2) = P[X_1 \langle Y, \{X_2 \geq Z_2, (U \geq V \text{ and } X_2 \geq Y)\}] \tag{6.2.10}$$

$$R(3) = P\begin{bmatrix} X_1 \langle Y, \{X_2 \geq Z_2, (U \geq V \text{ and } X_2 \langle Y) \text{ or } X_2 \langle Z_2\} \\ \{X_3 \geq Z_3, (U \geq V \text{ and } X_3 \geq Y)\} \end{bmatrix} \tag{6.2.11}$$

Then in general, we have

$$R(r) = P\begin{bmatrix} X_1 \langle Y, \{X_2 \geq Z_2, (U \geq V \text{ and } X_2 \langle Y) \text{or } X_2 \langle Z_2\} \\ \{X_3 \geq Z_3, (U \geq V \text{ and } X_3 \langle Y) \text{ or } X_3 \langle Z_3\}, \\ ..., \{X_{r-1} \geq Z_{r-1}, (U \geq V \text{ and } X_{r-1} \langle Y) \text{ or } X_{r-1} \langle Z_{r-1}\} \\ \{X_r \geq Z_r, (U \geq V \text{ and } X_r \geq Y)\} \end{bmatrix} \tag{6.2.12}$$

Let $f_i(x)$, $g(y)$, $w_j(z)$, $h(u)$ and $k(v)$, $i = 1,2,...,n;\ j = 2,3,...,n$ be the p.d.f.'s of X_i, Y, Z_j, U and V respectively. Since all the components and the switch work independently, we have

$$R(1) = \int_{-\infty}^{\infty} \overline{F_1}(y) g(y) dy \tag{6.2.13}$$

$$R(2) = \int_{-\infty}^{\infty} F_1(y) g(y) dy \int_{-\infty}^{\infty} \overline{F_2}(z) w_2(z) dz \int_{-\infty}^{\infty} \overline{H}(v) k(v) dv \int_{-\infty}^{\infty} \overline{F_2}(y) g(y) dy \tag{6.2.14}$$

$$R(3)=\left\{\int_{-\infty}^{\infty}F_1(y)g(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{F_2}(z)w_2(z)dz\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\int_{-\infty}^{\infty}F_2(y)g(y)dy+\int_{-\infty}^{\infty}F_2(z)w_2(z)dz\right\}$$
$$\left\{\int_{-\infty}^{\infty}\overline{F_3}(z)w_3(z)dz\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\int_{-\infty}^{\infty}\overline{F_3}(y)g(y)dy\right\}$$

(6.2.15)

Then,

$$R(r)=\left\{\int_{-\infty}^{\infty}F_1(y)g(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{F_2}(z)w_2(z)dz\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\int_{-\infty}^{\infty}F_2(y)g(y)dy+\int_{-\infty}^{\infty}F_2(z)w_2(z)dz\right\}$$
$$\left\{\int_{-\infty}^{\infty}\overline{F_3}(z)w_3(z)dz\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\int_{-\infty}^{\infty}\overline{F_3}(y)g(y)dy+\int_{-\infty}^{\infty}F_3(z)w_3(z)dz\right\}\ldots$$
$$\left\{\int_{-\infty}^{\infty}\overline{F_r}(z)w_r(z)dz\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\int_{-\infty}^{\infty}\overline{F_r}(y)g(y)dy\right\}\quad where,\ r=1,2,\ldots,n$$

(6.2.16)

Substituting the values of $R(r)$, $r=1,2,\ldots,n$ in the expression (2.2.1) we can get the system reliability R_n.

6.3 Reliability for Specific Distributions

6.3.1 Exponential Stress-Strength: Cold Standby for Identical Stress

Let $f_i(x)$, $g(y)$, $h(u)$ and $k(v)$, $i=1,2,\ldots,n$ be all exponential densities with means $\frac{1}{\theta_i}, \frac{1}{\alpha}, \frac{1}{\lambda}$ and $\frac{1}{\mu}$ respectively, i.e.,

$$f_i(x)=\begin{cases}\theta_i e^{-\theta_i x_i}, & x_i\geq 0,\ \theta_i\geq 0\\ 0, & otherwise\end{cases}$$

$$g(y)=\begin{cases}\alpha e^{-\alpha y}, & y\geq 0,\ \alpha\geq 0\\ 0, & otherwise\end{cases}$$

$$h(u) = \begin{cases} \lambda e^{-\lambda u}, & u \geq 0,\ \lambda \geq 0 \\ 0, & otherwise \end{cases}$$

$$k(v) = \begin{cases} \mu e^{-\mu v}, & v \geq 0,\ \mu \geq 0 \\ 0, & otherwise \end{cases}$$

If λ is a positive integer then,

$$\int_{-\infty}^{\infty} \overline{H}(v) k(v) dv = \int_{0}^{\infty} e^{-\lambda v} \mu e^{-\mu v} dv = \frac{\mu}{\mu + \lambda} = \frac{1}{1+\rho} \quad \text{where } \rho = \frac{\lambda}{\mu}$$

From, (6.2.5), (6.2.6) and (6.2.7) marginal reliabilities $R(1)$, $R(2)$ and $R(3)$ may be obtained as

$$R(1) = \int_{-\infty}^{\infty} \overline{F_1}(y) g(y) dy = \int_{0}^{\infty} e^{\theta_1 y_1} \alpha e^{-\alpha y_1} dy_1 = \frac{\alpha}{\alpha + \theta_1} \tag{6.3.1}$$

$$\begin{aligned} R(2) &= \left\{ \int_{-\infty}^{\infty} F_1(y) g(y) dy \right\} \left\{ \int_{-\infty}^{\infty} \overline{H}(v) k(v) dv \right\} \left\{ \int_{-\infty}^{\infty} \overline{F_2}(y) g(y) dy \right\} \\ &= \left[\int_{0}^{\infty} \left(1 - e^{-\theta_1 y_1}\right) \alpha_1 e^{-\alpha_1 y_1} dy_1 \right] \left[\int_{0}^{\infty} e^{-\lambda v} \mu e^{-\mu v} dv \right] \left[\int_{0}^{\infty} e^{-\theta_2 y_2} \alpha e^{-\alpha y_2} dy_2 \right] \\ &= \frac{\theta_1}{\alpha + \theta_1} \frac{1}{1+\rho} \frac{\alpha}{\alpha + \theta_2} \end{aligned} \tag{6.3.2}$$

$$\begin{aligned} R(3) &= \left\{ \int_{-\infty}^{\infty} F_1(y) g(y) dy \right\} \left\{ \int_{-\infty}^{\infty} \overline{H}(v) k(v) dv \right\} \left\{ \int_{-\infty}^{\infty} F_2(y) g(y) dy \right\} \left\{ \int_{-\infty}^{\infty} \overline{H}(v) k(v) dv \right\} \left\{ \int_{-\infty}^{\infty} \overline{F}_3(y) g(y) dy \right\} \\ &= \left(\frac{1}{1+\rho} \right)^2 \frac{\theta_1}{\alpha + \theta_1} \frac{\theta_2}{\alpha + \theta_2} \frac{\alpha}{\alpha + \theta_3} \end{aligned} \tag{6.3.3}$$

Then the system reliability R_3 may be obtained from the equation (2.2.1) in terms of the above marginal reliabilities $R(1)$, $R(2)$ and $R(3)$ as given in (6.3.1), (6.3.2) and (6.3.3).

6.3.2 Exponential Stress-Strength: Warm Standby for Identical Stress

Let $f_i(x)$, $g(y)$, $w_j(z)$, $h(u)$ and $k(v)$, $i = 1,2,\ldots,n$; $j = 2,3,\ldots,n$ be all exponential densities with means $\frac{1}{\theta_i}, \frac{1}{\alpha}, \frac{1}{\beta_j}, \frac{1}{\lambda}$ and $\frac{1}{\mu}$ respectively.

Now,

$$f_i(x) = \begin{cases} \theta_i e^{-\theta_i x_i}, & x_i \geq 0,\, \theta_i \geq 0 \\ 0, & otherwise \end{cases}$$

$$g(y) = \begin{cases} \alpha e^{-\alpha y}, & y \geq 0,\, \alpha \geq 0 \\ 0, & otherwise \end{cases}$$

$$w_j(z) = \begin{cases} \beta_j e^{-\beta_j z_j}, & z_i \geq 0,\, \beta_j \geq 0 \\ 0, & otherwise \end{cases}$$

$$h(u) = \begin{cases} \lambda e^{-\lambda u}, & u \geq 0,\, \lambda \geq 0 \\ 0, & otherwise \end{cases}$$

$$k(v) = \begin{cases} \mu e^{-\mu v}, & v \geq 0,\, \mu \geq 0 \\ 0, & otherwise \end{cases}$$

From (6.2.13), (6.2.14) and (6.2.15), marginal reliabilities $R(1)$, $R(2)$ and $R(3)$ may be obtained as

$$R(1) = \frac{\alpha}{\alpha + \theta_1} \tag{6.3.4}$$

$$R(2)=\frac{\theta_1}{\alpha+\theta_1}\frac{\beta_2}{\beta_2+\theta_2}\frac{1}{1+\rho}\frac{\alpha}{\alpha+\theta_2} \tag{6.3.5}$$

$$R(3)=\frac{\theta_1}{\alpha+\theta_1}\left[\left\{\frac{\beta_2}{\beta_2+\theta_2}\frac{1}{1+\rho}\frac{\theta_2}{\alpha+\theta_2}\right\}+\frac{\theta_2}{\beta_2+\theta_2}\right]\left[\frac{\beta_3}{\beta_3+\theta_3}\frac{1}{1+\rho}\frac{\alpha}{\alpha+\theta_3}\right] \tag{6.3.6}$$

Then the system reliability R_3 may be obtained from the equation (2.2.1) in terms of the above marginal reliabilities $R(1)$, $R(2)$ and $R(3)$ as given in (6.3.4), (6.3.5) and (6.3.6).

6.3.3 Gamma Stress-Strength: Cold Standby for Identical Stress

Let $f_i(x)$, $g(y)$, $h(u)$ and $k(v)$, be all gamma densities with scale parameters equal to unity and degrees of freedom $\theta_i, \alpha, \lambda$ and μ respectively, $i = 1,2,...,n$ i.e.,

Then,

$$f_i(x)=\begin{cases}\frac{1}{\Gamma\theta_i}e^{-x_i}x_i^{\theta_i-1}, & x_i\ge 0,\ \theta_i\ge 1\\ 0, & otherwise\end{cases}$$

$$g_i(y)=\begin{cases}\frac{1}{\Gamma\alpha}e^{-y}y^{\alpha-1}, & y\ge 0,\ \alpha\ge 1\\ 0, & otherwise\end{cases}$$

$$h(u)=\begin{cases}\frac{1}{\Gamma\lambda}e^{-u}u^{\lambda-1}, & u\ge 0,\ \lambda\ge 1\\ 0, & otherwise\end{cases}$$

$$k(v)=\begin{cases}\frac{1}{\Gamma\mu}e^{-v}v^{\mu-1}, & v\ge 0,\ \mu\ge 1\\ 0, & otherwise\end{cases}$$

From, (6.2.5), (6.2.6) and (6.2.7) marginal reliabilities $R(1)$, $R(2)$ and $R(3)$ may be obtained as

$$R(1)=\int_{-\infty}^{\infty}\overline{F_1}(y)g(y)dy = R(\theta_1,\alpha) \tag{6.3.7}$$

$$\begin{aligned}R(2)&=\left\{\int_{-\infty}^{\infty}F_1(y)g(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\overline{F_2}(y)g(y)dy\right\}\\&=\left(1-\sum_{i=0}^{\theta_1-1}\frac{\Gamma(\theta_1+\alpha-i-1)}{\Gamma\alpha(\theta_1-i-1)!2^{\theta_1+\alpha-i-1}}\right)\left(\sum_{i=0}^{\theta_2-1}\frac{\Gamma(\theta_2+\alpha-i-1)}{\Gamma\alpha(\theta_2-i-1)!2^{\theta_2+\alpha-i-1}}\right)\left(\sum_{i=0}^{\lambda-1}\frac{\Gamma(\lambda+\mu-i-1)}{\Gamma\mu(\lambda-i-1)!2^{\lambda+\mu-i-1}}\right)\\&=\overline{R}(\theta_1,\alpha)R(\theta_2,\alpha)R(\lambda,\mu)\end{aligned} \tag{6.3.8}$$

$$R(3)=\overline{R}(\theta_1,\alpha)\overline{R}(\theta_2,\alpha)R(\theta_3,\alpha)[R(\lambda,\mu)]^2 \tag{6.3.9}$$

Then the system reliability R_3 may be obtained from the equation (2.2.1) in terms of the above marginal reliabilities $R(1)$, $R(2)$ and $R(3)$ as given in (6.3.7), (6.3.8) and (6.3.9).

6.3.4 Gamma Stress-Strength: Warm Standby for Identical Stress

Let $f_i(x)$, $g(y)$, $w_j(z)$, $h(u)$ and $k(v)$, $i=1,2,\ldots,n$; $j=2,3,\ldots,n$ be all gamma densities with shape parameters $\theta_i,\alpha,\beta_j,\lambda$ and μ respectively and scale parameters equal to unity. Then,

$$f_i(x)=\begin{cases}\dfrac{1}{\Gamma\theta_i}e^{-x_i}x_i^{\theta_i-1}, & x_i\geq 0,\ \theta_i\geq 1\\ 0, & otherwise\end{cases}$$

$$g(y)=\begin{cases}\dfrac{1}{\Gamma\alpha}e^{-y}y^{\alpha-1}, & y\geq 0,\ \alpha\geq 1\\ 0, & otherwise\end{cases}$$

$$w_j(z) = \begin{cases} \dfrac{1}{\Gamma\beta_j} e^{-z_j} z_j^{\beta_j - 1}, & z_j \geq 0,\ \beta_j \geq 1 \\ 0, & otherwise \end{cases}$$

$$h(u) = \begin{cases} \dfrac{1}{\Gamma\lambda} e^{-u} u^{\lambda - 1}, & u \geq 0,\ \lambda \geq 1 \\ 0, & otherwise \end{cases}$$

$$k(v) = \begin{cases} \dfrac{1}{\Gamma\mu} e^{-v} v^{\mu - 1}, & v \geq 0,\ \mu \geq 1 \\ 0, & otherwise \end{cases}$$

From (6.2.13), (6.2.14) and (6.2.15), marginal reliabilities $R(1)$, $R(2)$ and $R(3)$ may be obtained as

$$R(1) = R(\theta_1, \alpha) \tag{6.3.10}$$

$$R(2) = \overline{R}(\theta_1, \alpha) R(\theta_2, \beta_2) R(\lambda, \mu) R(\theta_2, \alpha) \tag{6.3.11}$$

$$R(3) = \overline{R}(\theta_1, \alpha)\{R(\theta_2, \beta_2) R(\lambda, \mu) \overline{R}(\theta_2, \alpha) + \overline{R}(\theta_2, \beta_2)\}\{R(\theta_3, \beta_3) R(\lambda, \mu) R(\theta_3, \alpha)\} \tag{6.3.12}$$

where, $R(\theta_1, \alpha) = \sum_{i=0}^{\theta_1 - \alpha} \dfrac{\Gamma(\theta_1 + \alpha - i - 1)}{\Gamma\alpha(\theta_1 - i - 1)! 2^{\theta_1 + \alpha - i - 1}}$, $\quad \overline{R}(\theta_1, \alpha) = 1 - R(\theta_1, \alpha)$

$$R(\theta_2, \alpha) = \sum_{i=0}^{\theta_2 - \alpha} \frac{\Gamma(\theta_2 + \alpha - i - 1)}{\Gamma\alpha(\theta_2 - i - 1)! 2^{\theta_2 + \alpha - i - 1}}, \qquad \overline{R}(\theta_2, \alpha) = 1 - R(\theta_2, \alpha)$$

$$R(\theta_3,\alpha)=\sum_{i=0}^{\theta_3-\alpha}\frac{\Gamma(\theta_3+\alpha-i-1)}{\Gamma\alpha(\theta_3-i-1)!2^{\theta_3+\alpha-i-1}}$$

$$R(\theta_2,\beta_2)=\sum_{i=0}^{\theta_2-1}\frac{\Gamma(\theta_2+\beta_2-i-1)}{\Gamma\alpha(\theta_2-i-1)!2^{\theta_2+\alpha-i-1}},\qquad \overline{R}(\theta_2,\beta_2)=1-R(\theta_2,\beta_2)$$

$$R(\theta_3,\beta_3)=\sum_{i=0}^{\theta_3-1}\frac{\Gamma(\theta_3+\beta_3-i-1)}{\Gamma\alpha(\theta_3-i-1)!2^{\theta_3+\alpha-i-1}}$$

$$R(\lambda,\mu)=\sum_{i=0}^{\lambda-1}\frac{\Gamma(\lambda+\mu-i-1)}{\Gamma\mu(\lambda-i-1)!2^{\lambda+\mu-i-1}}$$

Then the system reliability R_3 may be obtained from the equation (2.2.1) in terms of the above marginal reliabilities $R(1)$, $R(2)$ and $R(3)$ as given in (6.3.10), (6.3.11) and (6.3.12).

6.3.5 Normal Stress-Strength: Cold Standby for Identical Stress

Let $f_i(x)$, $g(y)$, $h(u)$ and $k(v)$ be $N(\theta_i,\sigma_i)$, $N(\alpha,\tau)$, $N(\lambda,\upsilon)$ and $N(\mu,\rho)$ respectively, $i=1,2,\ldots,n$. Let us define $Z=U-V>0$ and $Z_i=X_i-Y>0$. Then the random variables Z and Z_i are normally distributed with mean $(\lambda-\mu)$ and $(\theta_i-\alpha)$ and standard deviations $\sqrt{\upsilon^2+\rho^2}$ and $\sqrt{\sigma_i^2+\tau^2}$ respectively, $i=1,2,\ldots,n$.

From, (6.2.5), (6.2.6) and (6.2.7) marginal reliabilities $R(1)$, $R(2)$ and $R(3)$ may be obtained as

$$R(1)=\int_{-\infty}^{\infty}\overline{F}_1(y)g(y)dy=P(Z_1\geq 0)=\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{\sigma_1^2+\tau^2}}\int_0^{\infty}e^{-\frac{1}{2}\left[\frac{Z_1-(\theta_1-\alpha)}{\sqrt{\sigma_1^2+\tau^2}}\right]^2}dz_1$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\frac{\theta_1-\alpha}{\sqrt{\sigma_1^2+\tau^2}}}^{\infty}e^{-\frac{1}{2}t_1^2}dt_1 \quad where\ t_1=\frac{Z_1-(\theta_1-\alpha)}{\sqrt{\sigma_1^2+\tau^2}}$$

$$=1-\Phi\left(-\frac{\theta_1-\alpha}{\sqrt{\sigma_1^2+\tau^2}}\right)$$

$$=\Phi\left(\frac{\theta_1-\alpha}{\sqrt{\sigma_1^2+\tau^2}}\right)$$

$$=\Phi(A_1),\ say$$

(6.3.13)

$$R(2)=\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\overline{F}(y)g_2(y)dy\right\}$$
$$=P(Z_1<0)P(Z\geq 0)P(Z_2\geq 0)$$

Now as above, $P(Z\geq 0)=\Phi\left(\frac{\lambda-\mu}{\sqrt{\upsilon^2+\rho^2}}\right)=\Phi(S_w),\ say$

Then,

$$R(2)=\left[1-\Phi\left(\frac{\theta_1-\alpha}{\sqrt{\sigma_1^2+\tau^2}}\right)\right]\left[\Phi\left(\frac{\theta_2-\alpha}{\sqrt{\sigma_2^2+\tau^2}}\right)\right]\left[\Phi\left(\frac{\lambda-\mu}{\sqrt{\upsilon^2+\rho^2}}\right)\right]$$
$$=\Phi(\overline{A_1})\Phi(A_2)\Phi(S_w)$$

(6.3.14)

$$R(3)=\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}F(y)g_2(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\overline{F}(y)g_3(y)dy\right\}$$
$$=\Phi(\overline{A}_1)\Phi(\overline{A}_2)\Phi(A_3)[\Phi(S_w)]^2$$

(6.3.15)

Then the system reliability R_3 may be obtained from the equation (2.2.1) in terms of the above marginal reliabilities $R(1)$, $R(2)$ and $R(3)$ as given in (6.3.13), (6.3.14) and (6.3.15).

6.3.6 Normal Stress-Strength: Warm Standby for Identical Stress

Let $f_i(x)$, $g(y)$, $w_j(z)$ be $N(\theta_i,\sigma_i)$, $N(\alpha,\tau)$ and $N(\beta_j,\gamma_j)$ respectively, $i=1,2,\ldots,n$; $j=2,3,\ldots,n$ and $h(u)$ and $k(v)$ be $N(\lambda,\upsilon)$ and $N(\mu,\rho)$ respectively. Let us define $Z=U-V>0$ and $Z_i=X_i-Y>0$; $i=1,2,\ldots,n$, $T_j=X_i-Z_j$; $j=2,3,\ldots,n$ are normally distributed with mean $(\lambda-\mu)$, $(\theta_i-\alpha)$ and $(\theta_i-\beta_j)$ and standard deviations $\sqrt{\upsilon^2+\rho^2}$, $\sqrt{\sigma_i^2+\tau^2}$ and $\sqrt{\sigma_i^2+\gamma_j^2}$ respectively.

From (6.2.13), (6.2.14) and (6.2.15), marginal reliabilities $R(1)$, $R(2)$ and $R(3)$ may be obtained as

$$\begin{aligned} R(1) = \int_{-\infty}^{\infty} \overline{F_1}(y)g(y)dy = P(Z_1 \geq 0) &= \Phi\left(\frac{\theta_1-\alpha}{\sqrt{\sigma_1^2+\tau^2}}\right) \\ &= \Phi(A_1),\ say \end{aligned} \tag{6.3.16}$$

$$\begin{aligned} R(2) &= \left\{\int_{-\infty}^{\infty} F_1(y)g(y)dy\right\}\left\{\int_{-\infty}^{\infty} \overline{F}_2(z)w_2(z)dz\right\}\left\{\int_{-\infty}^{\infty} \overline{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty} \overline{F_2}(y)g(y)dy\right\} \\ &= P(Z_1<0)P(T_2 \geq 0)P(Z \geq 0)P(Z_2 \geq 0) \end{aligned}$$

Now, $P(Z \geq 0) = \Phi\left(\dfrac{\lambda-\mu}{\sqrt{\upsilon^2+\rho^2}}\right) = \Phi(S_w),\ say$

$$P(T_2 \geq 0) = \Phi\left(\frac{\theta_2-\beta_2}{\sqrt{\sigma_2^2+\gamma_2^2}}\right) = \Phi(B_2),\ say$$

Then,

$$R(2)=\left[1-\Phi\left(\frac{\theta_1-\alpha}{\sqrt{\sigma_1^2+\tau^2}}\right)\right]\left[\Phi\left(\frac{\theta_2-\beta_2}{\sqrt{\sigma_2^2+\gamma_2^2}}\right)\right]\left[\Phi\left(\frac{\lambda-\mu}{\sqrt{\upsilon^2+\rho^2}}\right)\right]\left[\Phi\left(\frac{\theta_2-\alpha}{\sqrt{\sigma_2^2+\tau^2}}\right)\right]$$
$$=\Phi\left(\overline{A_1}\right)\Phi(B_2)\Phi(S_w)\Phi(A_2)$$

(6.3.17)

$$R(3)=\left\{\int_{-\infty}^{\infty}F_1(y)g(y)dy\right\}\left\{\int_{-\infty}^{\infty}\overline{F}_2(z)w_2(z)dz\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\int_{-\infty}^{\infty}F_2(y)g(y)dy+\int_{-\infty}^{\infty}F_2(z)w_2(z)dz\right\}$$
$$\left\{\int_{-\infty}^{\infty}\overline{F}_3(z)w_3(z)dz\int_{-\infty}^{\infty}\overline{H}(v)k(v)dv\int_{-\infty}^{\infty}\overline{F_3}(y)g(y)dy\right\}$$
$$=P(Z_1<0)[P(T_2\ge 0)P(Z\ge 0)P(Z_2<0)+P(T_2<0)][P(T_3\ge 0)P(Z\ge 0)P(Z_3\ge 0)]$$
$$=\Phi\left(\overline{A}_1\right)\left[\Phi(B_2)\Phi(S_w)\Phi\left(\overline{A}_2\right)+\Phi\left(\overline{B}_2\right)\right]\left[\Phi(B_3)\Phi(S_w)\Phi(A_3)\right]$$

(6.3.18)

Then the system reliability R_3 may be obtained from the equation (2.2.1) in terms of the above marginal reliabilities $R(1)$, $R(2)$ and $R(3)$ as given in (6.3.16), (6.3.17) and (6.3.18).

6.4 Graphical Representations

To make the things clear, graphs of R_3 are drawn in **Fig.** 6.1(a)-6.1(b), **Fig.** 6.2(a)-6.2(b), **Fig.** 6.3(a)-6.3(b) for selected values of the parameters for cold and warm standby systems in case of exponential, gamma and normal distributions. In **Fig.** 6.1(a)-6.1(b), **Fig.** 6.2(a)-6.2(b), **Fig.** 6.3(a)-6.3(b) show the graphs of R_3, taking the stress parameter α along the horizontal axis and the corresponding reliabilities along the vertical axis graphs are plotted for different parametric values. From these graphs one can read directly the values of reliabilities R_3 for intermediate values of α . From **Fig.** 6.1(a) and **Fig.** 6.1(b), it is seen that reliabilities are increasing with increasing the stress parameter in case of exponential distribution. Again from **Fig.** 6.2(a)-6.2(b), **Fig.** 6.3(a)-6.3(b), reliabilities are decreasing with increasing α in case of gamma and normal distributions. These graphs show that values of the system reliability become smaller in case of warm standby system than that of cold standby system for exponential, gamma and normal distributions.

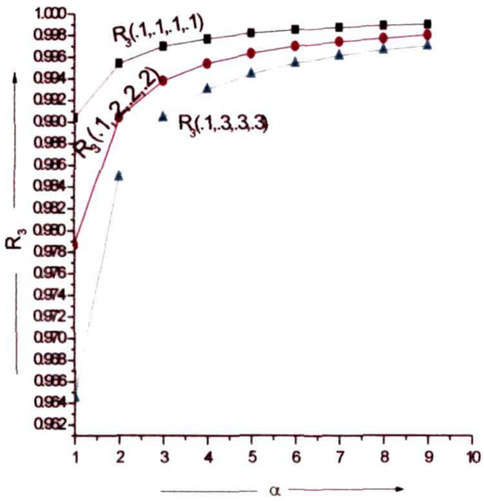

Fig. 6.1(a) R_3 for cold standby in case of Exponential Stress-Strength: Here $R_3(\rho,\theta_1,\theta_2,\theta_3)$

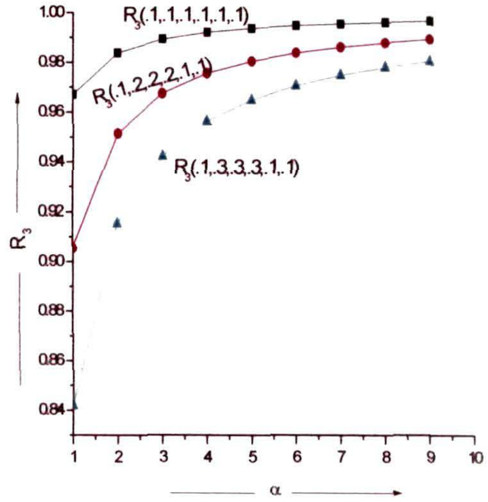

Fig. 6.1(b) R_3 for warm standby in case of Exponential Stress-Strength: Here $R_3(\rho,\theta_1,\theta_2,\theta_3,\beta_2,\beta_3)$

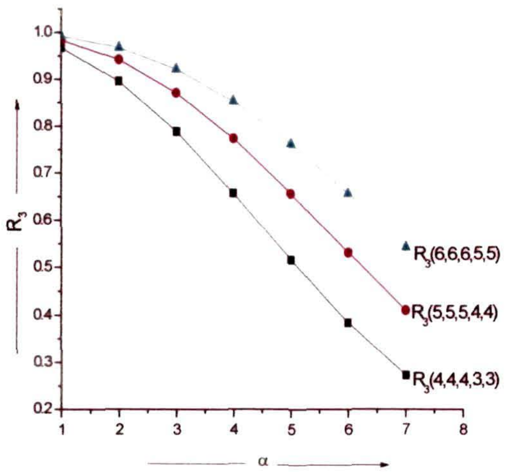

Fig. 6.2(a) R_3 for cold standby in case of Gamma Stress-Strength: Here $R_3(\theta_1,\theta_2,\theta_3,\lambda,\mu)$

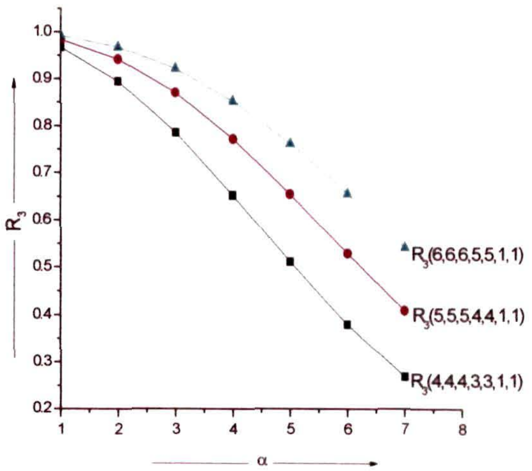

Fig. 6.2(b) R_3 for warm standby in case of Gamma Stress-Strength: Here $R_3(\theta_1,\theta_2,\theta_3,\lambda,\mu,\beta_2,\beta_3)$

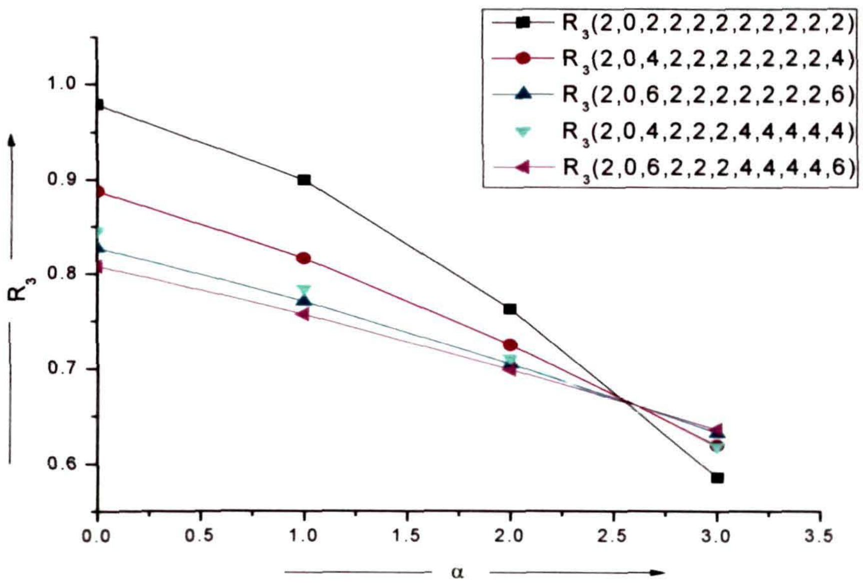

Fig. 6.3(a) R_3 for cold standby in case of Normal Stress-Strength:

Here $R_3(\lambda,\mu,\tau,\theta_1,\theta_2,\theta_3,\sigma_1,\sigma_2,\sigma_3,\upsilon,\rho)$

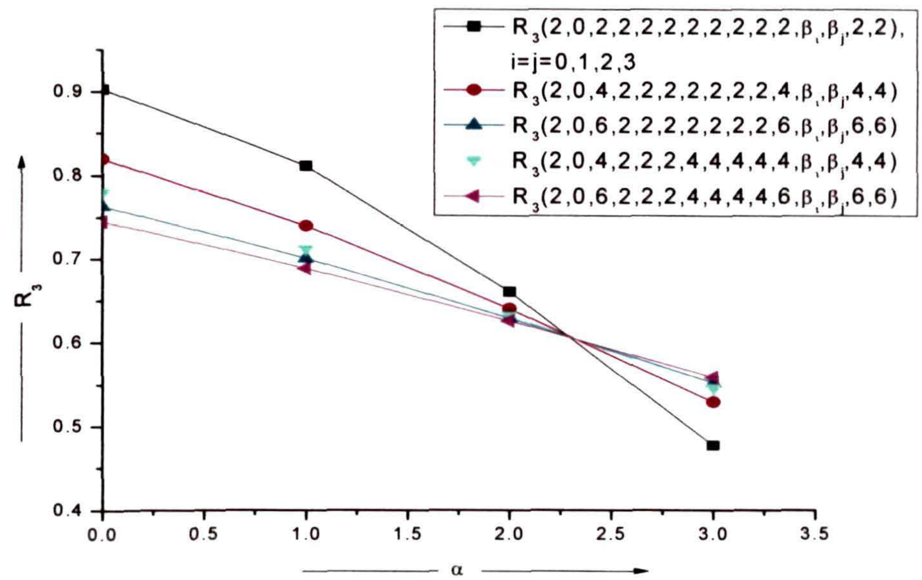

Fig. 6.3(b) R_3 for warm standby in case of Normal Stress-Strength:

Here $R_3(\lambda,\mu,\tau,\theta_1,\theta_2,\theta_3,\sigma_1,\sigma_2,\sigma_3,\upsilon,\rho,\beta_i,\beta_j,\gamma_2,\gamma_3)$ where $i=j=0,1,2,3$

6.5 Results and Discussions

For a few values of the parameters involved in the expressions of $R(r)$, $r = 1,2,3$ we evaluate $R(1)$, $R(2)$, $R(3)$ and R_3 for different distributions from their expressions are obtained.

From the **Table** 6.1 (cf. Appendix), here we observe that if mean stress α increases system reliability also increases. For example, if $\alpha = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9$ then the system reliability R_3 becomes 0.9904, 0.9954, 0.9970, 0.9977, 0.9982, 0.9985, 0.9987, 0.9989, 0.9990 respectively in case of cold standby system. Again in case of warm standby system if $\alpha = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9$ then we have the reliability (R_3) values are 0.9670, 0.9837, 0.9892, 0.9920, 0.9936, 0.9947, 0.9954, 0.9960 and 0.9964 respectively. If θ_1 increases the corresponding $R(1)$ decreases. The value of marginal reliability $R(1)$ becomes 0.9091, 0.8333, 0.7692 respectively for $\theta_1 = .1,.2,.3$. The marginal reliability $R(1)$ remains same for both cold and warm standby system. In case of cold standby system, if $\theta_1 = \theta_2 = .1$ then $R(1)$ and $R(2)$ become .0751 and 0.0062 respectively. But in case of warm standby system, $R(2)$ and $R(3)$ become 0.0376 and 0.0203 for $\theta_2 = \theta_3 = .1$. Further it is clear that the parameters β_2, β_3 in case of exponential distribution are seems to be very sensitive for warm standby system. Hence the values of the system reliability in case of warm standby system become smaller than that of cold standby system.

From the **Table** 6.2 (cf. Appendix), it is seen that if the stress parameter α increases then the system reliability R_3 decreases. In case of cold standby system the values of R_3 becomes 0.9677, 0.8958, 0.7884, 0.6563, 0.5158, 0.3840, 0.2725 respectively for $\alpha = 1,2,3,4,5,6,7$. Again in case of warm standby system the value of the system reliability becomes 0.9675, 0.8947, 0.7856, 0.6520, 0.5108, 0.3793, 0.2687 respectively for $\alpha = 1,2,3,4,5,6,7$. When the strength parameter θ_1 increases then the marginal reliability $R(1)$ increases for both cold and warm standby system. For example, when $\theta_1 = 4,\ 5,\ 6$ then $R(1)$ becomes 0.9375, 0.9688 and 0.9844 respectively. Here also noted that $R(1)$ remains same for both the systems. When the strength and stress parameter λ and μ of the switch increases then also the reliability R_3 increases. In case of cold standby system, for $\lambda = 3, 4, 5$ and $\mu = 3, 4, 5$ the reliability values are 0.8958, 0.9420, 0.9677 respectively. Again in case of warm standby system when the mean strength of the switch $\lambda = 3, 4, 5$ and mean stress of the switch $\mu = 3, 4, 5$ then R_3 becomes 0.8947, 0.9418, 0.9677 respectively. In case of gamma distribution, β_2, β_3 are seems to be very sensitive for warm standby system. Here also the values of the system reliability become smaller in case of warm standby system than that of cold standby system.

From the tabulated values of **Table** 6.3 (cf. Appendix), we observe that when the mean stress α increases then the values of the system reliability R_3 decreases. For instance, in case of cold standby system $\alpha = 0, 1, 2, 3$ the reliability values R_3 become 0.9789, 0.8989, 0.7623 and 0.5857 respectively. Again in case of warm standby system R_3 becomes 0.9028, 0.8104, 0.6606, 0.4770 respectively for $\alpha = 0, 1, 2, 3$. On the other hand, $R(1)$ decreases with the increase of standard deviation σ_1. It is seen that the marginal reliability $R(1)$ remains same for both cold and warm standby system. The increase in the strength (υ) and stress (ρ) parameter of the switch also decreases the system reliability. For example, in case of cold standby system, when $\upsilon = 2$, $R_3 = 0.9789$ and when $\upsilon = 4$, $R_3 = 0.8878$. For $\rho = 2, 4, 6$ the system reliability R_3 becomes 0.9789, 0.8878 and 0.8275 respectively. Again in case of warm standby system $R_3 = 0.9028$ for $\upsilon = 2$ and $R_3 = 0.8196$ for $\upsilon = 4$. For $\rho = 2, 4, 6$ the system reliability R_3 becomes 0.9028, 0.8196 and 0.7632 respectively. In case of normal distribution, we observe that the parameters γ_2, γ_3 are seems to be very sensitive for warm standby system. Hence the values of the system reliability of warm standby system become smaller than that of cold standby system.

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Chapter 7

Stress-Strength Model with Standby Redundancy and Cascade Redundancy

Chapter 7

Stress-Strength Model with Standby Redundancy and Cascade Redundancy

7.1 Introduction

Increase in the complexity of jobs performed increases the complexity of the devices (eg. Computer, Satellite, Plans, Missiles etc.) which increases the number of essential components in it. An increase in the number of essential components (i.e., components in series) decreases the reliability but the importance of jobs carried out by such complex devices requires that they should be highly reliable. So, the problem of increasing the reliability of a device is a real problem.

The strength of a component [Raghavachar, Kesava Rao and Pandit (1983), Rekha, and Shyam Sunder (1997), Shooman (1968)] can obviously be defined as the minimum stress required causing the component (or system) failure by considering the situation where a component works under the impact of stresses. If the stress equals or exceeds the strength of the component, it fails; otherwise it works. In practical situations, the magnitude of the stress is random, with considerable variations.

By cascade redundancy (Pandit and Sriwastav, 1975) we mean a standby redundancy where a standby component taking the place of a failed component is subjected to a modified value of the preceding stress. We assume that this modified value of stress is equal to 'k' times the stress on the preceding (failed) component. Here 'k' is called attenuation factor which is generally assumed to be a constant for all the components or a parameter having different fixed values for different components (Pandit and Sriwastav, 1975). But an attenuation factor may be a random variable also (Pandit and Sriwastav, 1978). Here we shall assume that k is a constant through it may be changing from component to component or even it may be a random variable (Gogoi and Borah, 2012).

Here we have assumed that stress-strength of all the components in the system are independent. Sriwastav and Kakati (1981) have assumed that the components stress-strengths are similarly distributed. But in general the stress distributions will be different from the strength distributions not only in parameter values but also in forms since stresses are independent of strengths and the two are governed by different physical conditions. They have considered a cascade system with dissimilar distributions of $X's$ and $Y's$ but not for stress-strength model. So in this chapter we have considered stress-strength model for dissimilar continuous distributions.

This chapter is organized as follows: Section 7.2 is devoted for mathematical models. In Sub-Section 7.2.1 and 7.2.2, an n-standby and an n-cascade system, respectively, are considered. In Section 7.3, we have assumed different particular forms of density functions for the stress-strength components and the system reliability is obtained. We have considered in Sub-Section 7.3.1, strength follows one-parameter exponential distribution and stress follows two-parameter exponential distribution, in Sub-Section 7.3.2, strength follows one-parameter exponential distribution and stress follows two-parameter gamma distribution, in Sub-Section 7.3.3, strength is Lindley and stress is one-parameter gamma distribution and in Sub-Section 7.3.4, strength is Lindley and stress is two-parameter gamma distribution. For all the cases we have obtained the general expressions of reliability for an n-standby system. In Sub-Section 7.3.5, the general expressions of reliability for n-cascade system are obtained when the strength follows one-parameter exponential distribution and stress follows two-parameter gamma distribution. In Sub-Section 7.3.5.A, a special case is considered when X_i's are one-parameter i.i.d exponential strength with parameter λ and stress follows two-parameter gamma distribution and the system reliability R_n for n-cascade system is obtained. In Sub-Section 7.3.6, reliability expressions of 3-cascade system is obtained when the strength follows one-parameter exponential distribution and stress of the components follows Lindley distributions. The reliability expressions are not simple enough to reflect the changes in reliability of different systems with change in parameters. To observe the change in the values of reliabilities with different parameters involved some numerical values of reliabilities are tabulated in **Table** 7.1 and **Table** 7.7 (cf. Appendix). In Section 7.4 some graphs are plotted for selected values of the parameters to facilitate the direct reading of reliability. Section 7.5 deals with the results and discussions.

7.2 Mathematical Formulation of the Model

7.2.1 An n-Standby System

Consider an n-standby system i.e., in an n-standby system, initially there are n-components, out of which only one is working under impact of stresses and the remaining $(n-1)$ are standbys. Whenever the working component fails one from standbys takes its place and is subjected to impact of stresses and the system works. The system fails when all the components fail. For a detailed description of such a system one may refer to Gogoi, Borah and Sriwastav (2010), Kakati (1983), Pandit and Sriwastav (1975) and Sriwastav and Kakati (1981).

Symbolically, let $X_1, X_2, ..., X_n$, be a set of n independent random variables, representing the strengths of the n-components arranged in order of activation in the system and let $Y_1, Y_2, ..., Y_n$, be another set of independent random variables, representing the stresses on the n-components respectively then the system reliability R_n of the system is given by the equation (2.2.1) where the marginal reliability $R(r)$ is the contribution to the reliability of the system by the r^{th} component may be defined as

$$R(r) = \Pr[X_1 < Y_1, X_2 < Y_2, ..., X_{r-1} < Y_{r-1}, X_r \geq Y_r]$$

Let $f_i(x)$ and $g_i(y)$ be the probability density functions (p.d.f.) of X_i and Y_i, $i = 1, 2, ..., n$ respectively then

$$R(r) = \left[\int_{-\infty}^{\infty} F_1(y) g_1(y) dy\right]\left[\int_{-\infty}^{\infty} F_2(y) g_2(y) dy\right] ... \left[\int_{-\infty}^{\infty} F_{r-1}(y) g_{r-1}(y) dy\right]\left[\int_{-\infty}^{\infty} \overline{F}_r(y) g_r(y) dy\right] \quad (7.2.1)$$

where $F_i(x)$ is the cumulative distribution function (c.d.f.) of X_i, i.e.,

$$F_i(x)=\int_{-\infty}^{x} f_i(x)dx \quad \text{and} \quad \overline{F}_i(x)=1-F_i(x)$$

The following four conditions for strength i.e., $f_i(x)$ and stress i.e., $g_i(y)$, have been considered for this investigation.

- Strength follows one-parameter exponential and stress follows two-parameter exponential distribution
- Strength follows one-parameter exponential and stress follows two-parameter gamma distribution
- Strength follows Lindley distribution and stress follows one-parameter gamma distribution
- Strength follows Lindley distribution and stress follows two-parameter gamma distribution

7.2.2 An n-Cascade System

An n-cascade system is a special type of n-standby system (Pandit and Sriwastav, 1975). Let $X_1, X_2, \ldots, X_n$ be the strengths of n-components in the order of activation and let $Y_1, Y_2, \ldots, Y_n$ are the stresses working on them. In cascade system after every failure the stress is modified by a factor k which is called attenuation factor such that

$$Y_2 = kY_1, Y_3 = kY_2 = k^2 Y_1, \ldots, Y_i = k^{i-1} Y_1 \ etc.$$

Then the reliability R_n of the system is defined by the equation (2.2.1)

where $R(r)=\Pr\left[X_1 < Y_1, X_2 < kY_1, \ldots, X_{r-1} < k^{r-2}Y_1, X_r \geq k^{r-1}Y_1\right]$

$R(r)$ is the marginal reliability due to the r^{th} component or we can write for cascade system,

$$R(r)=\int_{-\infty}^{\infty}\left[F_1(y_1)F_2(ky_1)F_3(k^2y_1)\ldots\overline{F}_r(k^{r-1}y_1)\right]g(y_1)dy_1 \tag{7.2.2}$$

We consider here the following two cases for this system

- One-parameter exponential strength and two-parameter gamma stress
- One-parameter exponential strength and Lindley stress

7.3 Reliability for Specific Distributions

When stress-strength follow particular distributions, we can find the $R(r)$, $r = 1,2,...,n$ and thereby obtain the system reliability R_n. In the following six subsections we assume different particular distributions for the stress-strength involved.

7.3.1 One -parameter Exponential strength and Two-parameter Exponential stress

Let $f_i(x)$ be the one-parameter exponential strength with mean $\frac{1}{\lambda_i}$ and $g_i(y)$ be the two-parameter exponential (Krishnamoorthy, Mukherjee and Guo, 2007) stress with parameter μ_i and θ_i respectively, $i = 1,2,...,n$ then we have the following probability density functions

$$f_i(x,\lambda) = \begin{cases} \lambda_i e^{-\lambda_i x_i}; \ x_i \geq 0, \ \lambda_i \geq 0 \\ 0, \qquad otherwise \end{cases}$$

and

$$g_i(y;\mu,\theta) = \begin{cases} \frac{1}{\theta_i} e^{-\frac{(y_i-\mu_i)}{\theta_i}} \ ; \ y_i > \mu_i, \ \mu_i \geq 0, \ \theta_i > 0 \\ 0, \qquad otherwise \end{cases}$$

Then from (7.2.1) we get,

$$R(1) = \frac{e^{-\lambda_1 \mu_1}}{1+\lambda_1\theta_1} \tag{7.3.1}$$

$$R(2)=\left[1-\frac{e^{-\lambda_1\mu_1}}{1+\lambda_1\theta_1}\right]\frac{e^{-\lambda_2\mu_2}}{1+\lambda_2\theta_2} \tag{7.3.2}$$

$$R(3)=\left[1-\frac{e^{-\lambda_1\mu_1}}{1+\lambda_1\theta_1}\right]\left[1-\frac{e^{-\lambda_2\mu_2}}{1+\lambda_2\theta_2}\right]\frac{e^{-\lambda_3\mu_3}}{1+\lambda_3\theta_3} \tag{7.3.3}$$

Therefore in general,

$$R(r)=\left[1-\frac{e^{-\lambda_1\mu_1}}{1+\lambda_1\theta_1}\right]\left[1-\frac{e^{-\lambda_2\mu_2}}{1+\lambda_2\theta_2}\right]\ldots\left[1-\frac{e^{-\lambda_{r-1}\mu_{r-1}}}{1+\lambda_{r-1}\theta_{r-1}}\right]\frac{e^{-\lambda_r\mu_r}}{1+\lambda_r\theta_r}$$

$$=\frac{e^{-\lambda_r\mu_r}}{1+\lambda_r\theta_r}\prod_{i=1}^{r}\left(1-\frac{e^{-\lambda_{i-1}\mu_{i-1}}}{1+\lambda_{i-1}\theta_{i-1}}\right),\qquad \text{Here considering, } \lambda_0=\theta_0=0 \tag{7.3.4}$$

For some particular values of the parameters we have tabulated some values of $R(1)$, $R(2)$, $R(3)$ and R_3 in **Table** 7.1 (cf. Appendix).

7.3.2 One -parameter Exponential strength and Two-parameter Gamma stress

Let $f_i(x)$ be one-parameter exponential strength with mean $\frac{1}{\lambda_i}$ and $g_i(y)$ be the two-parameter gamma stress with parameters μ_i and θ_i respectively, $i=1,2,\ldots,n$, then we have the following probability density functions

$$f_i(x,\lambda)=\begin{cases}\lambda_i e^{-\lambda_i x_i};\ x_i\ge 0,\ \lambda_i\ge 0\\ 0, \qquad otherwise\end{cases}$$

and

$$g_i(y;\mu,\theta)=\begin{cases}\dfrac{1}{\theta_i^{\mu_i}\Gamma\mu_i}y_i^{\mu_i-1}e^{-\frac{y_i}{\theta_i}},\ y_i,\mu_i,\theta_i>0\\ 0, \qquad otherwise\end{cases}$$

Then from (7.2.1) we get,

$$R(1)=\frac{1}{(1+\lambda_1\theta_1)^{\mu_1}} \tag{7.3.5}$$

$$R(2)=\left[1-\frac{1}{(1+\lambda_1\theta_1)^{\mu_1}}\right]\left[\frac{1}{(1+\lambda_2\theta_2)^{\mu_2}}\right] \tag{7.3.6}$$

$$R(3)=\left[1-\frac{1}{(1+\lambda_1\theta_1)^{\mu_1}}\right]\left[1-\frac{1}{(1+\lambda_2\theta_2)^{\mu_2}}\right]\left[\frac{1}{(1+\lambda_3\theta_3)^{\mu_3}}\right] \tag{7.3.7}$$

In general,

$$R(r)=A\prod_{i=1}^{r}\left[1-\frac{1}{(1+\lambda_{i-1}\theta_{i-1})^{\mu_{i-1}}}\right] \quad \text{where, } A=\frac{1}{(1+\lambda_r\theta_r)^{\mu_r}};\ \text{Here considering } \lambda_0=\theta_0=0 \tag{7.3.8}$$

Substituting the values of $R(r)$, $r=1,2,3,\ldots,n$ we can obtain R_n, the reliability of the system. A few numerical values of $R(1)$, $R(2)$, $R(3)$ and R_3 are tabulated in **Table** 7.2 (cf. Appendix) for different values of the parameters.

7.3.3 Strength is Lindley and stress is One-parameter Gamma distribution

Let $f_i(x)$ be the strength of Lindley distribution [Ghitany, Atieh and Nadarajah (2008), Mutairi, Ghitany and Kundu (to appear)] with parameter θ_i and $g_i(y)$ be one-parameter gamma stress with parameter m_i respectively, $i=1,2,\ldots,n$, then we have the following probability density functions

$$f_i(x;\theta)=\begin{cases}\dfrac{\theta_i^2}{1+\theta_i}(1+x_i)e^{-\theta_i x_i};\ x_i\rangle 0,\ \theta_i\rangle 0\\ 0, \qquad \textit{otherwise}\end{cases}$$

and

$$g_i(y)=\begin{cases}\dfrac{1}{\Gamma(m_i)}e^{-y_i}y_i^{m_i-1};\ y_i\ge 0,\ m_i\ge 1\\ 0, \qquad otherwise\end{cases}$$

Then from (7.2.1) we get,

$$R(1)=\frac{1}{(1+\theta_1)^{m_1}}+\frac{\theta_1 m_1}{(1+\theta_1)^{m_1+2}} \tag{7.3.9}$$

$$R(2)=\left[\frac{\theta_1^2}{(1+\theta_1)^{m_1+1}}+\frac{\theta_1^2 m_1}{(1+\theta_1)^{m_1+2}}\right]\left[\frac{1}{(1+\theta_2)^{m_2}}+\frac{\theta_2 m_2}{(1+\theta_2)^{m_2+2}}\right] \tag{7.3.10}$$

$$R(3)=\left[\frac{\theta_1^2}{(1+\theta_1)^{m_1+1}}+\frac{\theta_1^2 m_1}{(1+\theta_1)^{m_1+2}}\right]\left[\frac{\theta_2^2}{(1+\theta_2)^{m_2+1}}+\frac{\theta_2^2 m_2}{(1+\theta_2)^{m_2+2}}\right]\left[\frac{1}{(1+\theta_3)^{m_3}}+\frac{\theta_3 m_3}{(1+\theta_3)^{m_3+2}}\right] \tag{7.3.11}$$

In general,

$$R(r)=\left[\frac{\theta_1^2}{(1+\theta_1)^{m_1+1}}+\frac{\theta_1^2 m_1}{(1+\theta_1)^{m_1+2}}\right]\left[\frac{\theta_2^2}{(1+\theta_2)^{m_2+1}}+\frac{\theta_2^2 m_2}{(1+\theta_2)^{m_2+2}}\right]\cdots\left[\frac{1}{(1+\theta_r)^{m_r}}+\frac{\theta_r m_r}{(1+\theta_r)^{m_r+2}}\right] \tag{7.3.12}$$

Combining the terms $R(r)$, $r=1,2,3,\dots,n$ the reliability of the system R_n, can be obtained from (2.2.1). **Table** 7.3 (cf. Appendix) shows a few numerical values of $R(1)$, $R(2)$, $R(3)$ and R_3 for different values of the parameters.

7.3.4 Strength is Lindley and stress is Two-parameter Gamma distribution

Let $f_i(x)$ be the strength of Lindley distribution with parameter θ_i and $g_i(y)$ be two-parameter gamma stress with parameters μ_i and λ_i respectively, $i = 1,2,\ldots,n$, then we have the following probability density functions

$$f_i(x;\theta) = \begin{cases} \dfrac{\theta_i^2}{1+\theta_i}(1+x_i)e^{-\theta_i x_i}; \ x_i \rangle 0, \ \theta_i \rangle 0 \\ 0, \qquad otherwise \end{cases}$$

and

$$g_i(y;\mu,\lambda) = \begin{cases} \dfrac{1}{\lambda_i^{\mu_i}\Gamma(\mu_i)} y_i^{\mu_i - 1} e^{-\frac{y_i}{\lambda_i}}; \ y_i, \mu_i, \lambda_i > 0 \\ 0, \qquad otherwise \end{cases}$$

Then from (7.2.1) we get,

$$R(1) = \frac{1}{(1+\lambda_1\theta_1)^{\mu_1}} + \frac{\lambda_1\theta_1\mu_1}{(1+\theta_1)(1+\lambda_1\theta_1)^{\mu_1+1}} \tag{7.3.13}$$

$$R(2) = \left[\frac{\theta_1^2}{(1+\theta_1)(1+\lambda_1\theta_1)^{\mu_1}} + \frac{\lambda_1\theta_1^2\mu_1}{(1+\theta_1)(1+\lambda_1\theta_1)^{\mu_1+1}}\right]\left[\frac{1}{(1+\lambda_2\theta_2)^{\mu_2}} + \frac{\lambda_2\theta_2\mu_2}{(1+\theta_2)(1+\lambda_2\theta_2)^{\mu_2+1}}\right] \tag{7.3.14}$$

$$R(3) = \left[\frac{\theta_1^2}{(1+\theta_1)(1+\lambda_1\theta_1)^{\mu_1}} + \frac{\lambda_1\theta_1^2\mu_1}{(1+\theta_1)(1+\lambda_1\theta_1)^{\mu_1+1}}\right]\left[\frac{\theta_2^2}{(1+\theta_2)(1+\lambda_2\theta_2)^{\mu_2}} + \frac{\lambda_2\theta_2^2\mu_2}{(1+\theta_2)(1+\lambda_2\theta_2)^{\mu_2+1}}\right] \left[\frac{1}{(1+\lambda_3\theta_3)^{\mu_3}} + \frac{\lambda_3\theta_3\mu_3}{(1+\theta_3)(1+\lambda_3\theta_3)^{\mu_3+1}}\right] \tag{7.3.15}$$

In general,

$$R(r)=\left[\frac{\theta_1^2}{(1+\theta_1)(1+\lambda_1\theta_1)^{\mu_1}}+\frac{\lambda_1\theta_1^2\mu_1}{(1+\theta_1)(1+\lambda_1\theta_1)^{\mu_1+1}}\right]\left[\frac{\theta_2^2}{(1+\theta_2)(1+\lambda_2\theta_2)^{\mu_2}}+\frac{\lambda_2\theta_2^2\mu_2}{(1+\theta_2)(1+\lambda_2\theta_2)^{\mu_2+1}}\right]$$
$$\ldots\left[\frac{1}{(1+\lambda_r\theta_r)^{\mu_r}}+\frac{\lambda_r\theta_r\mu_r}{(1+\theta_r)(1+\lambda_r\theta_r)^{\mu_r+1}}\right]$$

(7.3.16)

Table 7.4 (cf. Appendix) shows a few numerical values of $R(1)$, $R(2)$, $R(3)$ and R_3 for different values of the parameters.

7.3.5 One-parameter Exponential strength and Two-parameter Gamma stress for Cascade system

Let $X_1, X_2, \ldots, X_n$ be one-parameter exponential strength i.e., $f_i(x)$ with mean $\frac{1}{\lambda_i}$ and Y_1 be a two-parameter gamma stress then we have the following probability density functions

$$f_i(x,\lambda)=\begin{cases}\lambda_i e^{-\lambda_i x_i}; & x_i \geq 0,\ \lambda_i \geq 0\\ 0, & otherwise\end{cases}$$

and

$$g(y_1;\mu,\theta)=\begin{cases}\frac{1}{\theta^{\mu}\Gamma\mu}y_1^{\mu-1}e^{-\frac{y_1}{\theta}}; & y_1,\mu,\theta\rangle 0\\ 0, & otherwise\end{cases}$$

Then from (7.2.2) we have,

$$R(1)=\frac{1}{(1+\lambda_1\theta)^{\mu}} \tag{7.3.17}$$

$$R(2)=\frac{1}{(1+\lambda_2 k\theta)^{\mu}}-\frac{1}{(1+\lambda_1\theta+\lambda_2 k\theta)^{\mu}} \tag{7.3.18}$$

$$R(3)=\frac{1}{(1+\lambda_3 k^2\theta)^{\mu}}-\frac{1}{(1+\lambda_2 k\theta+\lambda_3 k^2\theta)^{\mu}}-\frac{1}{(1+\lambda_1\theta+\lambda_3 k^2\theta)^{\mu}}+\frac{1}{(1+\lambda_1\theta+\lambda_2 k\theta+\lambda_3 k^2\theta)^{\mu}} \tag{7.3.19}$$

In general,

$$\begin{aligned}R(r)=&\frac{1}{(1+\lambda_r k^{r-1}\theta)^{\mu}}-\frac{1}{(1+\lambda_1\theta+\lambda_r k^{r-1}\theta)^{\mu}}-\dots\\&+(-1)^{r+1}\frac{1}{(1+\lambda_1\theta+\lambda_2 k\theta+\dots+\lambda_{r-1}k^{r-2}\theta+\lambda_r k^{r-1}\theta)^{\mu}}\end{aligned} \tag{7.3.20}$$

Substituting the values of $R(r)$, $r=1,2,3,\dots,n$ we can obtain R_n, the reliability of the system. A few numerical values of $R(1)$, $R(2)$, $R(3)$ and R_3 are tabulated in **Table** 7.5 (cf. Appendix) for different values of the parameters.

7.3.5.A Special Case

When $X_1, X_2, \dots, X_n$ are one-parameter i.i.d. exponential strength with parameter λ then we have,

$$R(1)=\frac{1}{(1+\lambda\theta)^{\mu}} \tag{7.3.21}$$

$$R(2)=\frac{1}{(1+\lambda k\theta)^{\mu}}-\frac{1}{(1+\lambda\theta+\lambda k\theta)^{\mu}} \tag{7.3.22}$$

$$R(3)=\frac{1}{(1+\lambda k^2\theta)^{\mu}}-\frac{1}{(1+\lambda k\theta+\lambda k^2\theta)^{\mu}}-\frac{1}{(1+\lambda\theta+\lambda k^2\theta)^{\mu}}+\frac{1}{(1+\lambda\theta+\lambda k\theta+\lambda k^2\theta)^{\mu}} \tag{7.3.23}$$

Then in general,

$$
\begin{aligned}
R(r) = & \frac{1}{\left(1+\lambda k^{r-1}\theta\right)^{\mu}} - \frac{1}{\left(1+\lambda\theta+\lambda k^{r-1}\theta\right)^{\mu}} - \ldots \\
& + (-1)^{r+1} \frac{1}{\left(1+\lambda\theta+\lambda k\theta+\ldots+\lambda k^{r-2}\theta+\lambda k^{r-1}\theta\right)^{\mu}}
\end{aligned}
\tag{7.3.24}
$$

The expression of $R(r+1)$, can be obtained from that of $R(r)$, in a similar way as obtained by Sriwastav (1976). We note that:

A typical term in $R(r)$ can be written as,

$$\frac{1}{\left[1+\lambda\theta g_{rl}(k')\right]^{\mu}},\ l = 0,1,2,\ldots,\left(2^{r-1}-1\right)$$

where $g_{rl}(k')$ is an appropriate sum of k''s, $i = 1,2,\ldots,(r-1)$. Each of these terms gives rise to two terms of $R(r+1)$, one positive and the other negative. We get the first term by multiplying each term in $g_{rl}(k')$ by k. The second term is got by adding $\lambda\theta$ to the denominator of the first term. Thus from the term,

$$\frac{1}{\left[1+\lambda\theta g_{rl}(k')\right]^{\mu}},\ \text{we get the two terms as}$$

$$\frac{1}{\left[1+\lambda\theta g_{rl}(k'^{+1})\right]^{\mu}} \quad \text{and} \quad -\frac{1}{\left[1+\lambda\theta+\lambda\theta g_{rl}(k'^{+1})\right]^{\mu}} \quad \text{for } R(r+1).$$

Combining the terms $R(r)$, $r = 1,2,3,\ldots,n$ the reliability of the system R_n, can be obtained from the equation (2.2.1). **Table** 7.6 (cf. Appendix) shows a few numerical values of $R(1)$, $R(2)$, $R(3)$ and R_3 for different values of the parameters.

7.3.6 One-parameter Exponential strength and Lindley stress for Cascade system

Let $f_i(x)$ be the one-parameter exponential strength with mean $\frac{1}{\lambda_i}$ and Y_1 be the stress of the Lindley distribution with parameter θ, then we have the following probability density functions

$$f_i(x,\lambda)=\begin{cases}\lambda_i e^{-\lambda_i x_i};\ x_i\geq 0,\ \lambda_i\geq 0\\ 0, \qquad otherwise\end{cases}$$

and

$$g(y_1,\theta)=\begin{cases}\frac{\theta^2}{1+\theta}(1+y_1)e^{-\theta y_1};\ y_1\rangle 0,\ \theta\rangle 0\\ 0, \qquad otherwise\end{cases}$$

Then from (7.2.2) we have,

$$R(1)=\frac{\theta^2}{(1+\theta)}\left[\frac{1}{\lambda_1+\theta}+\frac{1}{(\lambda_1+\theta)^2}\right] \tag{7.3.25}$$

$$R(2)=\frac{\theta^2}{(1+\theta)}\left[\frac{1}{(\lambda_2 k+\theta)}+\frac{1}{(\lambda_2 k+\theta)^2}-\frac{1}{(\lambda_1+\lambda_2 k+\theta)}-\frac{1}{(\lambda_1+\lambda_2 k+\theta)^2}\right] \qquad 7.3.26)$$

$$R(3)=\frac{\theta^2}{(1+\theta)}\left[\begin{array}{l}\frac{1}{(\lambda_3 k^2+\theta)}+\frac{1}{(\lambda_3 k^2+\theta)^2}+\frac{1}{(\lambda_1+\lambda_2 k+\lambda_3 k^2+\theta)}+\frac{1}{(\lambda_1+\lambda_2 k+\lambda_3 k^2+\theta)^2}\\ -\frac{1}{(\lambda_2 k+\lambda_3 k^2+\theta)}-\frac{1}{(\lambda_2 k+\lambda_3 k^2+\theta)^2}-\frac{1}{(\lambda_1+\lambda_3 k^2+\theta)}-\frac{1}{(\lambda_1+\lambda_3 k^2+\theta)^2}\end{array}\right]$$

(7.3.27)

Then the system reliability R_3 for a 3-cascade system from the equation (2.2.1) is given by $R_3=R(1)+R(2)+R(3)$. We have tabulated some numerical values of $R(1)$, $R(2)$, $R(3)$ and R_3 in **Table** 7.7 (cf. Appendix) for different values of the parameters.

7.4 Graphical Representations

Some graphs are plotted in **Fig.** 7.1(a)-7.1(b), **Fig.** 7.2(a)-7.2(b), **Fig.** 7.3(a)-7.3(b), **Fig.** 7.4(a)-7.4(b), **Fig.** 7.5(a)-7.5(b), **Fig.** 7.6, **Fig.** 7.7, by taking different parameters along the horizontal axis and the corresponding reliability along the vertical axis for different parametric values. In **Fig.** 7.1(a)-7.1(b) taking $\lambda_1 = \lambda_2 = \lambda_3$ along the horizontal axis and the corresponding $R(1)$ and R_3 along the vertical axis graphs are plotted for different fixed values of $\mu_1, \mu_2, \mu_3, \theta_1, \theta_2, \theta_3$. In **Fig.** 7.2(a)-7.2(b) taking $\theta_1 = \theta_2 = \theta_3$ along the horizontal axis and the corresponding $R(1)$ and R_3 along the vertical axis graphs are plotted for different fixed values of $\lambda_1, \lambda_2, \lambda_3, \mu_1, \mu_2, \mu_3$. Again in **Fig.** 7.3(a)-7.3(b) taking $m_1 = m_2 = m_3$ along the horizontal axis and the corresponding $R(1)$ and R_3 along the vertical axis graphs are plotted for different fixed values of $\theta_1, \theta_2, \theta_3$. Some graphs are drawn for different fixed values of $\lambda_1, \lambda_2, \lambda_3, \mu_1, \mu_2, \mu_3$ in **Fig.** 7.4(a)-7.4(b) by taking $\theta_1 = \theta_2 = \theta_3$ along the horizontal axis and the corresponding $R(1)$ and R_3 along the vertical axis. From the above mentioned **Fig.**, it is observed that the marginal reliability $R(1)$ and the system reliability R_3 decrease with increasing different stress-strength parameters. In **Fig.** 7.5(a) and **Fig.** 7.5(b) graphs are plotted for different fixed values of $\lambda_1, \lambda_2, \lambda_3, \theta, \mu$ by taking k along the horizontal axis and the corresponding R_3 along the vertical axis. Again in **Fig.** 7.6 taking $\lambda = \mu$ along the horizontal axis and the corresponding R_3 along the vertical axis graphs are plotted for different fixed values of k and θ. Also in **Fig.** 7.7 taking θ along the horizontal axis and the corresponding R_3 along the vertical axis graphs are plotted for different fixed values of $k, \lambda_1, \lambda_2, \lambda_3$. From these graphs of system reliability, as expected, it is also seen that R_3 decreases steadily with increasing the attenuation factor k. These graphs may also be used for reading the values of reliability, corresponding to intermediate values of the parameter.

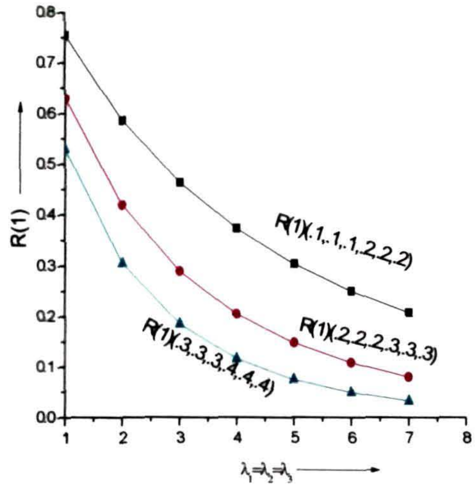

Fig. 7.1(a) Graph of $R(1)$ for fixed values of μ_1, μ_2, μ_3, θ_1, θ_2, θ_3 i.e., $R(1)(\mu_1, \mu_2, \mu_3, \theta_1, \theta_2, \theta_3)$ for Sub-Section 7.3.1

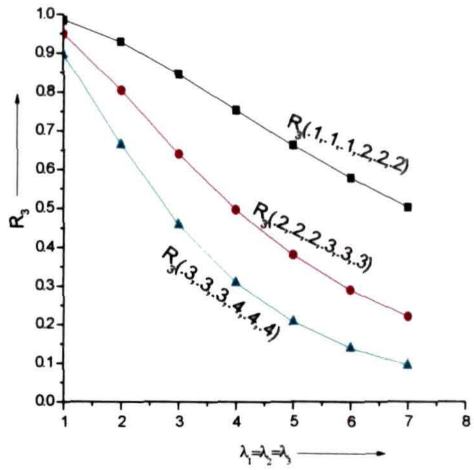

Fig. 7.1(b) Graph of R_3 for fixed values of μ_1, μ_2, μ_3, θ_1, θ_2, θ_3 i.e., $R_3(\mu_1, \mu_2, \mu_3, \theta_1, \theta_2, \theta_3)$ for Sub-Section 7.3.1

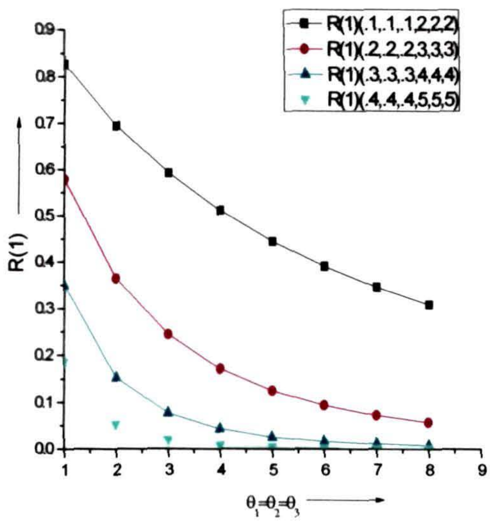

Fig. 7.2(a) Graph of $R(1)$ for fixed values of λ_1, λ_2, λ_3, μ_1, μ_2, μ_3 i.e., $R(1)(\lambda_1, \lambda_2, \lambda_3, \mu_1, \mu_2, \mu_3)$ for Sub-Section 7.3.2

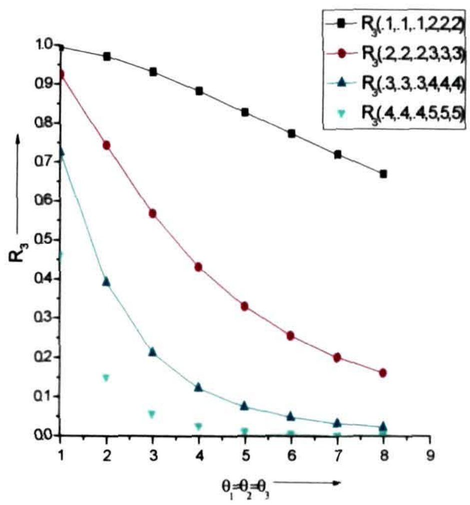

Fig. 7.2(b) Graph of R_3 for fixed values of λ_1, λ_2, λ_3, μ_1, μ_2, μ_3 i.e., $R_3(\lambda_1, \lambda_2, \lambda_3, \mu_1, \mu_2, \mu_3)$ for Sub-Section 7.3.2

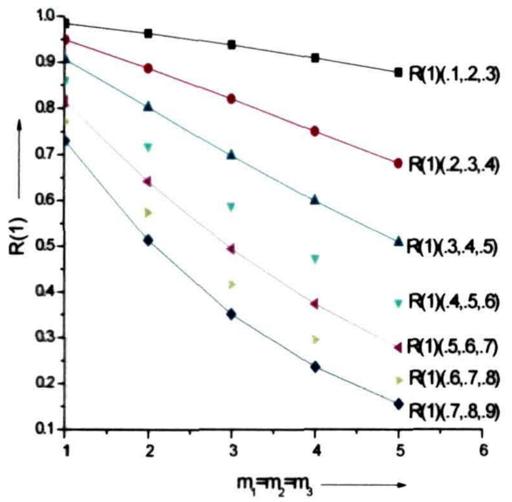

Fig. 7.3(a) Graph of $R(1)$ for fixed values of θ_1, θ_2, θ_3 i.e., $R(1)(\theta_1,\theta_2,\theta_3)$ for Sub-Section 7.3.3

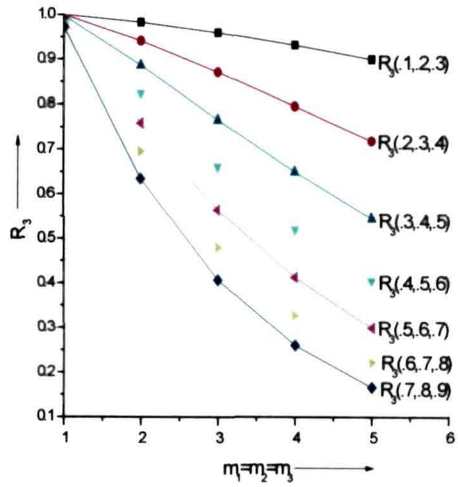

Fig. 7.3(b) Graph of R_3 for fixed values of θ_1, θ_2, θ_3 i.e., $R_3(\theta_1,\theta_2,\theta_3)$ for Sub-Section 7.3.3

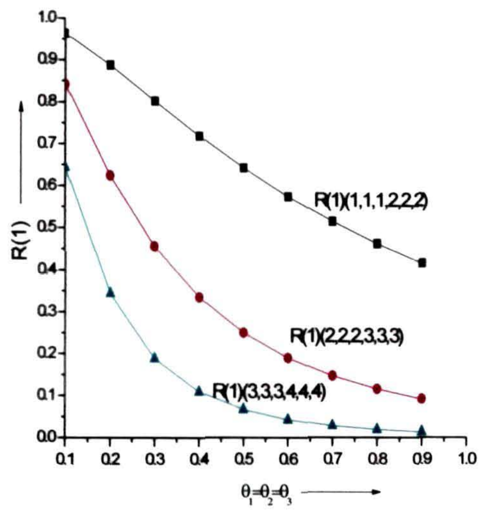

Fig. 7.4(a) Graph of $R(1)$ for fixed values of λ_1, λ_2, λ_3, μ_1, μ_2, μ_3 i.e., $R(1)(\lambda_1,\lambda_2,\lambda_3,\mu_1,\mu_2,\mu_3)$ for Sub-Section 7.3.4

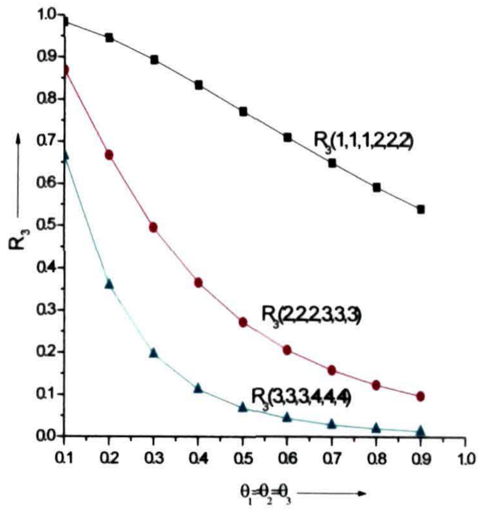

Fig. 7.4(b) Graph of R_3 for fixed values of λ_1, λ_2, λ_3, μ_1, μ_2, μ_3 i.e., $R_3(\lambda_1,\lambda_2,\lambda_3,\mu_1,\mu_2,\mu_3)$ for Sub-Section 7.3.4

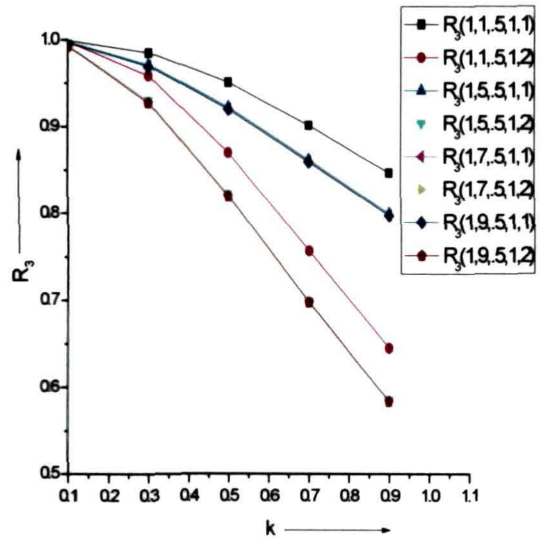

Fig. 7.5(a) Graph of R_3 for fixed values of λ_1, λ_2, λ_3, θ, μ i.e., $R_3(\lambda_1,\lambda_2,\lambda_3,\theta,\mu)$ for Sub-Section 7.3.5

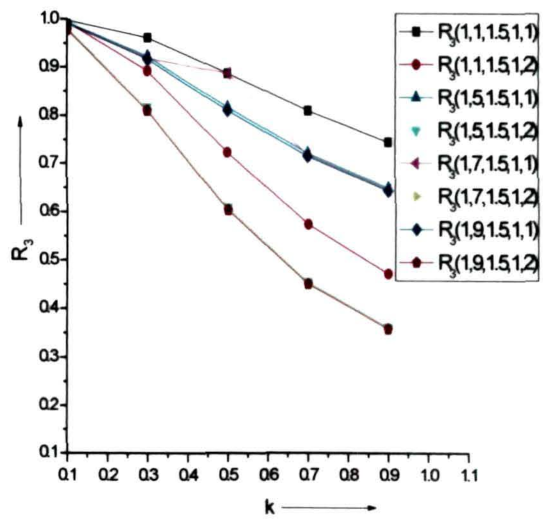

Fig. 7.5(b) Graph of R_3 for fixed values of λ_1, λ_2, λ_3, θ, μ i.e., $R_3(\lambda_1,\lambda_2,\lambda_3,\theta,\mu)$ for Sub-Section 7.3.5

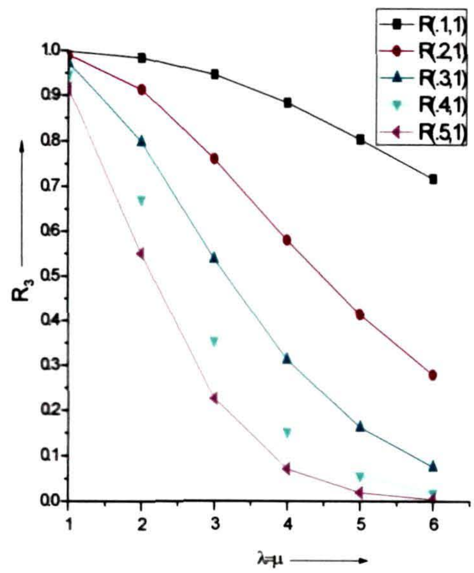

Fig. 7.6 Graph of R_3 for fixed values of k and θ i.e., $R_3(k,\theta)$ for Sub-Section 7.3.5.A

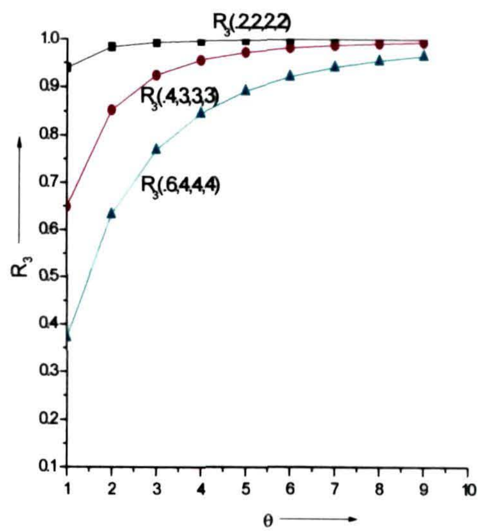

Fig. 7.7 Graph of R_3 for fixed values of k, λ_1, λ_2, λ_3 i.e., $R_3(k,\lambda_1,\lambda_2,\lambda_3)$ for Sub-Section 7.3.6

7.5 Results and Discussions

For some specific values of the parameters involved in the expressions of $R(r)$, $r = 1,2,3$ we evaluate the marginal reliabilities $R(1)$, $R(2)$, $R(3)$ and the system reliability R_3 for each cases of different distributions from their expressions obtained in the last section.

From the above **Table** 7.1 (cf. Appendix) we have seen that the system reliability R_3 decreases when the strength parameter is constant with increasing stress parameter. For instance, if the strength parameters $\lambda_1 = \lambda_2 = \lambda_3 = 1$ are fixed and the stress parameter $\mu_1 = \mu_2 = \mu_3 = \begin{cases} .1 \\ .2 \\ .3 \end{cases}$ and $\theta_1 = \theta_2 = \theta_3 = \begin{cases} .2 \\ .3 \\ .4 \end{cases}$ increases then the system reliabilities $R_3 = \begin{cases} 0.9851 \\ 0.9493 \\ 0.8956 \end{cases}$ increases. Increasing stress and strength parameter decreases the system reliability. When strength parameter increases with some fixed stress parameter also decreases the reliability. For example, if strength parameter $\lambda_1 = 1,2,3,4$ increases with fixed stress parameter $\mu_1 = \mu_2 = \mu_3 = \theta_1 = \theta_2 = \theta_3 = .1$ the system reliabilities $R_3 = 0.9944$, $R_3 = 0.9679$, $R_3 = 0.9204$, $R_3 = 0.8584$ decreases.

Table 7.2 (cf. Appendix) stated above we can conclude that when $\theta_1, \theta_2, \theta_3$ increases with increasing some values of λ_1, λ_2, λ_3, μ_1, μ_2, μ_3 the system reliability R_3 decreases. For example, if $\theta_1 = \theta_2 = \theta_3 = 1$, $\lambda_1 = \lambda_2 = \lambda_3 = .1$, $\mu_1 = \mu_2 = \mu_3 = 2$ then the system reliability $R_3 = 0.9948$ and if $\theta_1 = \theta_2 = \theta_3 = 2$, $\lambda_1 = \lambda_2 = \lambda_3 = .2$, $\mu_1 = \mu_2 = \mu_3 = 3$ then the system reliability $R_3 = 0.7433$.

From the above **Table** 7.3 (cf. Appendix) we have seen that when the strength parameter $\theta_1, \theta_2, \theta_3$ increases with increasing some stress parameter m_1, m_2, m_3 then the system reliability R_3 decreases. For example, if $\theta_1 = .1, \theta_2 = .2, \theta_3 = .3, m_1 = m_2 = m_3 = 1$ then the system reliability $R_3 = 0.9999$ and when $\theta_1 = .4, \theta_2 = .5, \theta_3 = .6, m_1 = m_2 = m_3 = 2$ then $R_3 = 0.8234$. By proper choice of the different parameters very high system reliability $R_3 = 0.9999$ can be achieved.

When stress is two-parameter gamma distribution and strength is Lindley distribution, a few values of $R(1), R(2), R(3)$ and R_3 have been computed and presented in **Table** 7.4 (cf. Appendix) for different values of stress-strength parameters. It is to be observed that, system reliability R_3 decreases with increasing stress-strength parameters. Also we have seen that some fixed values of the stress parameter λ_1, λ_2, λ_3, μ_1, μ_2, μ_3 with increasing the values of the strength parameter $\theta_1, \theta_2, \theta_3$ we get the system reliability R_3 decreases. For instance, $\lambda_1 = \lambda_2 = \lambda_3 = 1$, $\mu_1 = \mu_2 = \mu_3 = 2$, $\theta_1 = \theta_2 = \theta_3 = .1$ then $R_3 = 0.9839$ and if $\lambda_1 = \lambda_2 = \lambda_3 = 1$, $\mu_1 = \mu_2 = \mu_3 = 2$, $\theta_1 = \theta_2 = \theta_3 = .2$ then $R_3 = 0.9455$.

For different values of $\lambda_1, \lambda_2, \lambda_3, \theta, \mu$ and k we have tabulated the values of $R(1), R(2), R(3)$ and R_3 in **Table** 7.5 (cf. Appendix). From the tabulated values of R_3, it is observe that, the reliabilities are decreases with some fixed values of $\lambda_1 = 1, \theta = 1$ if the strength parameter λ_2, λ_3, stress parameter μ and the attenuation factor k (i.e., k=.1, .3, .5, .7, .9) increases. For example, when the strength parameter $\lambda_2 = 1$, $\lambda_3 = .5$, $\mu = 1$ and k=.1, .3, .5, .7, .9 then the system reliability becomes 0.9993, 0.9853, 0.9505, 0.9011, 0.8462 respectively. Again when $\lambda_2 = 1$, $\lambda_3 = .5$, $\mu = 2$ and k=.1, .3, .5, .7, .9 then the values of the system reliability become 0.9977, 0.9584, 0.8695, 0.7565, 0.6451 respectively. But when the strength parameter $\lambda_2 = 5$, $\lambda_3 = 1.5$, $\mu = 1$ and k=.1, .3, .5, .7, .9 then R_3 becomes 0.9932, 0.9226, 0.8168, 0.7205, 0.6491 respectively. Again the values of the reliability become 0.9812, 0.8169, 0.6094, 0.4541, 0.3611 respectively for $\lambda_2 = 5$, $\lambda_3 = 1.5$, $\mu = 2$ and k=.1, .3, .5, .7, .9. By proper choice of the parameters a high reliability $R_3 = 0.9993$ can be achieved.

From the tabulated values of **Table** 7.6 (cf. Appendix), we observed here that the change in the values of reliability is as expected. When the strength parameter (λ) and the stress parameter (μ) increases with increasing the attenuation factor k also then the values of the system reliability decreases. For instance, if $\lambda = 1$, $\mu = 1$, $k = .1, .2, .3, .4, .5$ with fixed values $\theta = 1$, then the system reliability R_3 becomes 0.9985, 0.9901, 0.9724, 0.9463, and 0.9144 respectively. Again when $\lambda = 3$, $\mu = 3$, $k = .1, .2, .3, .4, .5$ with fixed values $\theta = 1$, then the values of the reliability become 0.9453, 0.7600, 0.5376, 0.3533, and 0.2258 respectively.

From the **Table** 7.7 (cf. Appendix), we observed that, when the strength parameter λ_1 increases then there are significant decrease in the values of the marginal reliabilities $R(1)$. For example, if $\lambda_1 = 2, 3, 4$ then the values of the marginal reliability becomes 0.2222, 0.1563 and 0.1200 respectively. Again if the stress parameter θ increases with strength parameter $\lambda_1 = \lambda_2 = \lambda_3 = 2,3,4$ and attenuation factor $k = .2,.4,.6$ then the system reliability R_3 also increases. For example, if $\theta = 1$, $k = .2, .4, .6$ and $\lambda_1 (= \lambda_2 = \lambda_3) = 2, 3, 4$ then R_3 becomes 0.9396, 0.6491 and 0.3711 respectively. Again for $\theta = 2$, $k = .2, .4, .6$ and $\lambda_1 (= \lambda_2 = \lambda_3) = 2, 3, 4$ the values of the system reliability R_3 becomes 0.9837, 0.8526 and 0.6324 respectively.

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Chapter 8

Cascade Model for Warm Standby System with Imperfect Switching

Chapter 8

Cascade Model for Warm Standby System with Imperfect Switching

8.1 Introduction

As discussed in Chapter 2, in a stress-strength (S-S) model the components of a system work under the impact of stresses. A component fails if the stress on it exceeds its strength. Imperfect switching is also discussed in Chapter 1.

The components are under stress-whether active or standbys. Generally, the stress working on a component and the stress working on it when it was a standby may be quite different. For example, a standby component may fail due to humidity, corrosion etc. whereas the same component when activated may be facing mechanical stress, voltage fluctuations, vibrations etc. Similarly the switch may also under different kind of stress. That is, stress working on active component, standby components and the switch may be quite different from one another.

As mentioned in Chapter 7, Cascade reliability model is a stress-strength model with a particular type of redundancy. When an active component fails, the component taking its place faces an attenuated stress which is k times the stress on the preceding component, where k is called an attenuation factor which may be a constant or a random variable [Pandit and Sriwastav (1975, 1978)].

In this chapter we have considered an n-cascade system for warm standbys with imperfect switching. We have assumed to be cascade models for active as well as standby components and also for the switch. The system starts with one active component and a number of warm standbys. When the active component fails a component from standbys is put in its place by a device, called switch, if the switch is working. But if the switch has already failed, no component is put in place of the failed component then the system will fail. If the switch has not failed, it will bring a standby, which if not already failed as standby, will work and the system will work. But if this component has already failed as a standby and if switch is still working then another component is put in its place and so on.

Many authors have studied stress-strength model eg., Kakati (1983), Kapur and Lamberson (1977) etc. Studies of warm standby system for time-to-failure (TTF) models are considered by several authors including Srinivasan and Gopalan (1973), Gopalan (1975b), Gopalan and Venkatchalam (1977), Usha (1979) etc. Warm standby in S-S model is studied by Sriwastav and Dutta (1989), imperfect switching in S-S model is discussed by Sriwastav (2004), Bhowal (1999), Sriwastav and Dutta (1984) etc., cascade model for warm standby system is studied by Bhowal (1999), cascade model with imperfect switching is studied by Sriwastav (1992). But there have not been any studies for warm standby with imperfect switching in cascade model.

This chapter is organized as follows: In Section 8.2, general mathematical formulation of the model is developed and reliability of an n-cascade model for warm standby is obtained. In section 8.3, we have assumed different particular forms of density functions for the stress-strength components and the switch and the system reliability of a 4-cascade system R_4 is obtained. In Sub-section 8.3.1, stress-strength for the active component, standby component and the switch follow exponential distribution and the marginal reliability $R(1)$, $R(2)$, $R(3)$, $R(4)$ and system reliability R_4 for a 4-cascade system is obtained. In Sub-section 8.3.2, stress-strength for the active component and the switch follow exponential distribution and standby component follow gamma distribution and the marginal reliability expressions $R(1)$, $R(2)$, $R(3)$ and $R(4)$ are obtained. Also we evaluate the system reliability R_4 for a 4-cascade system in this case. The reliability, in each case, are estimated numerically and presented in tabular form in **Table** 8.1 and **Table** 8.2 (cf. Appendix). Results and discussions are devoted to Section 8.4.

8.2 Mathematical Formulation of the Model

Let us consider an n-cascade system (Bhowal, 1999) where, in the beginning one component is working and the remaining $(n-1)$ are warm standbys.

Let $X_1, X_2, \ldots, X_n$ be n random variables representing the strength of the n-components, arranged in the order of activation. Let Y_1 be a random variable representing the stress on the first component. The attenuation factor k is assumed to be constant. Let $Z_2, Z_3, \ldots, Z_n$ be the $(n-1)$ stresses on the 2nd, 3rd, ..., n^{th} components respectively, when they are as warm standbys. Let U and V be the strength and stress of the switch. We assume that X_i, Y_1, Z_j are all independent random variables (Sriwastav, 2004).

Let $R(i)$, $i = 1,2,\ldots,n$ be the marginal reliability due to the i^{th} component and R_n be the system reliability (Pandit and Sriwastav, 1975) which is given by the equation (2.2.1). Now the i^{th} active component fails if $X_i < k^{i-1}Y_1$ and j^{th} standby component fails if $X_j < Z_j$. The system fails when all the components have failed, either in operation or as standbys. The expressions for $R(i)$, $i = 1,2,\ldots,n$ for the system considered here are as follows

$$R(1) = P[X_1 \geq Y_1] \tag{8.2.1}$$

$$\begin{aligned} R(2) &= P[X_1 < Y_1, (U \geq V)\{X_2 \geq Z_2 \; and \; X_2 \geq kY_1\}] \\ &= P(U \geq V)P(X_2 \geq Z_2)P(X_1 < Y_1, X_2 \geq kY_1) \end{aligned} \tag{8.2.2}$$

$$\begin{aligned} R(3) &= P\begin{bmatrix} X_1 < Y_1, (U \geq V)\{X_2 \geq Z_2 \; and \; X_2 < kY_1 \; or \; X_2 < Z_2\} \\ (U \geq V)\{X_3 \geq Z_3 \; and \; X_3 \geq k^2 Y_1\} \end{bmatrix} \\ &= [P(U \geq V)]^2 \begin{bmatrix} P(X_2 \geq Z_2, X_3 \geq Z_3)P(X_1 < Y_1, X_2 < kY_1, X_3 \geq k^2 Y_1) \\ + P(X_2 < Z_2, X_3 \geq Z_3)P(X_1 < Y_1, X_3 \geq k^2 Y_1) \end{bmatrix} \end{aligned} \tag{8.2.3}$$

$$R(4) = P\begin{bmatrix} X_1 < Y_1, (U \geq V)\{X_2 \geq Z_2 \; and \; X_2 < kY_1 \; or \; X_2 < Z_2\} \\ (U \geq V)\{X_3 \geq Z_3 \; and \; X_3 < k^2 Y_1 or \; X_3 < Z_3\} \\ (U \geq V)\{X_4 \geq Z_4 \; and \; X_4 \geq k^3 Y_1\} \end{bmatrix}$$

$$= [P(U \geq V)]^3 \begin{bmatrix} P(X_2 \geq Z_2, X_3 \geq Z_3, X_4 \geq Z_4) P(X_1 < Y_1, X_2 < kY_1, X_3 < k^2 Y_1, X_4 \geq Z_4) \\ + P(X_2 < Z_2, X_3 \geq Z_3, X_4 \geq Z_4) P(X_1 < Y_1, X_3 < k^2 Y_1, X_4 \geq k^3 Y_1) \\ + P(X_2 \geq Z_2, X_3 < Z_3, X_4 \geq Z_4) P(X_1 < Y_1, X_2 < kY_1, X_4 \geq k^3 Y_1) \\ + P(X_2 < Z_2, X_3 < Z_3, X_4 \geq Z_4) P(X_1 < Y_1, X_4 \geq k^3 Y_1) \end{bmatrix}$$

(8.2.4)

Then in general we have,

$$R(i) = [P(U \geq V)]^{i-1} \begin{bmatrix} P(X_2 \geq Z_2, X_3 \geq Z_3, \ldots, X_i \geq Z_i) P(X_1 < Y_1, X_2 < kY_1, \ldots, X_i \geq k^{i-1} Y_1) \\ + P(X_2 < Z_2, X_3 \geq Z_3, \ldots, X_i \geq Z_i) P(X_1 < Y_1, X_3 < k^2 Y_1, \ldots, X_i \geq k^{i-1} Y_1) \\ + P(X_2 \geq Z_2, X_3 < Z_3, X_4 \geq Z_4, \ldots, X_i \geq Z_i) \\ P(X_1 < Y_1, X_2 < kY_1, X_4 < k^3 Y_1, \ldots, X_i \geq k^{i-1} Y_1) \\ + \ldots + P(X_2 < Z_2, X_3 < Z_3, \ldots, X_{i-1} < Z_{i-1}, X_i \geq Z_i) P(X_1 < Y_1, X_i \geq k^{i-1} Y_1) \end{bmatrix}$$

(8.2.5)

Let $f_i(x), g_i(y) \; and \; w_j(z)$ be the probability density functions of X, Y_i, Z_j, $i = 1,2,\ldots,n; \; j = 2,3,\ldots,n$. Similarly $k(u)$ and $s(v)$ be the p.d.f's of U and V respectively. The reliability $R(s)$ of the switch is given by

$$R(s) = P(U \geq V) = \int_{-\infty}^{\infty} \bar{k}(v) s(v) dv, \qquad (8.2.6)$$

where $k(u)$ is the distribution function of U and $\bar{k}(u) = 1 - k(u)$.

Since all the components work independently we can write the above expressions (8.2.1) to (8.2.4) as

$$R(1) = \int_{-\infty}^{\infty} \overline{F}_1(y) g(y) dy \tag{8.2.7}$$

$$R(2) = R(s) \int_{-\infty}^{\infty} \overline{F}_2(z) w_2(z) dz \int_{-\infty}^{\infty} F_1(y) \overline{F}_2(ky) g(y) dy \tag{8.2.8}$$

$$R(3) = [R(s)]^2 \left\{ \begin{aligned} &\left[\int_{-\infty}^{\infty} \overline{F}_2(z) w_2(z) dz \int_{-\infty}^{\infty} \overline{F}_3(z) w_3(z) dz \int_{-\infty}^{\infty} F_1(y) F_2(ky) \overline{F}_3(k^2 y) g(y) dy \right] \\ &+ \left[\int_{-\infty}^{\infty} F_2(z) w_2(z) dz \int_{-\infty}^{\infty} \overline{F}_3(z) w_3(z) dz \int_{-\infty}^{\infty} F_1(y) \overline{F}_3(k^2 y) g(y) dy \right] \end{aligned} \right\} \tag{8.2.9}$$

$$R(4) = [R(s)]^3 \left\{ \begin{aligned} &\left[\begin{aligned} &\int_{-\infty}^{\infty} \overline{F}_2(z) w_2(z) dz \int_{-\infty}^{\infty} \overline{F}_3(z) w_3(z) dz \int_{-\infty}^{\infty} \overline{F}_4(z) w_4(z) dz \\ &\int_{-\infty}^{\infty} F_1(y) F_2(ky) F_3(k^2 y) \overline{F}_4(k^3 y) g(y) dy \end{aligned} \right] \\ &+ \left[\begin{aligned} &\int_{-\infty}^{\infty} F_2(z) w_2(z) dz \int_{-\infty}^{\infty} \overline{F}_3(z) w_3(z) dz \int_{-\infty}^{\infty} \overline{F}_4(z) w_4(z) dz \\ &\int_{-\infty}^{\infty} F_1(y) F_3(k^2 y) \overline{F}_4(k^3 y) g(y) dy \end{aligned} \right] \\ &+ \left[\begin{aligned} &\int_{\infty}^{\infty} \overline{F}_2(z) w_2(z) dz \int_{-\infty}^{\infty} F_3(z) w_3(z) dz \int_{-\infty}^{\infty} \overline{F}_4(z) w_4(z) dz \\ &\int_{-\infty}^{\infty} F_1(y) F_2(ky) \overline{F}_4(k^3 y) g(y) dy \end{aligned} \right] \\ &+ \left[\begin{aligned} &\int_{-\infty}^{\infty} F_2(z) w_2(z) dz \int_{-\infty}^{\infty} F_3(z) w_3(z) dz \int_{-\infty}^{\infty} \overline{F}_4(z) w_4(z) dz \\ &\int_{-\infty}^{\infty} F_1(y) \overline{F}_4(k^3 y) g(y) dy \end{aligned} \right] \end{aligned} \right\} \tag{8.2.10}$$

Then in general we can write,

$$R(i) = [R(s)]^{i-1} \left\{ \begin{array}{l} \left[\begin{array}{l} \int_{-\infty}^{\infty} \overline{F}_2(z) w_2(z) dz \int_{-\infty}^{\infty} \overline{F}_3(z) w_3(z) dz \ldots \int_{-\infty}^{\infty} \overline{F}_r(z) w_r(z) dz \\ \int_{-\infty}^{\infty} F_1(y) F_2(ky) F_3(k^2 y) \ldots \overline{F}_{r-2}(k^{r-1} y) g(y) dy \end{array} \right] \\ + \left[\begin{array}{l} \int_{-\infty}^{\infty} F_2(z) w_2(z) dz \int_{-\infty}^{\infty} F_3(z) w_3(z) dz \ldots \int_{-\infty}^{\infty} \overline{F}_r(z) w_r(z) dz \\ \int_{-\infty}^{\infty} F_1(y) F_3(k^2 y) \ldots F_{r-3}(k^{r-2} y) \overline{F}_{r-2}(k^{r-1} y) g(y) dy \end{array} \right] \\ + \ldots + \left[\begin{array}{l} \int_{-\infty}^{\infty} F_2(z) w_2(z) dz \ldots \int_{-\infty}^{\infty} F_{r-1}(z) w_{r-1}(z) dz \\ \int_{-\infty}^{\infty} \overline{F}_r(z) w_r(z) dz \int_{-\infty}^{\infty} F_1(y) \overline{F}_{r-2}(k^{r-1} y) g(y) dy \end{array} \right] \end{array} \right\}$$

(8.2.11)

where $F_i(x)$ is the cumulative distribution function of X_i and $\overline{F_i}(x) = 1 - F_i(x)$, $i = 1,2,\ldots,n$. Substituting back the expression of $R(i)$, $i = 1,2,\ldots,n$ from (8.2.7) to (8.2.11) in (2.2.1) we get the system reliability R_n.

8.3 Stress-Strength follows Specific Distributions

When stress-strength follows particular distributions, we can find $R(i)$, $i = 1,2,\ldots,n$ and thereby obtain the system reliability R_n.

8.3.1 Exponential Stress-Strength

Let $f_i(x)$, $g(y)$, $w_j(z)$, $k(u)$ and $s(v)$ be all exponential densities with means $\frac{1}{\theta_i}, \frac{1}{\alpha}, \frac{1}{\beta_j}, \frac{1}{\lambda}$ and $\frac{1}{\gamma}$ respectively ($i = 1,2,\ldots,n$; $j = 2,3,\ldots,n$).

Then from equation (8.2.6),

$$R(s)=\frac{1}{1+\mu}, \text{ where } \mu=\frac{\lambda}{\gamma} \tag{8.3.1}$$

Similarly $R(1)$, $R(2)$, $R(3)$ and $R(4)$ may be obtained from (8.2.7), (8.2.8), (8.2.9) and (8.2.10) respectively as

$$\begin{aligned} R(1) &= \int_0^\infty \overline{F}_1(y)g(y)dy = \int_0^\infty e^{-\theta_1 y}\alpha e^{-\alpha y}dy \\ &= \frac{\alpha}{\alpha+\theta_1} \\ &= \frac{1}{1+\rho_1}, \quad where \quad \rho_1=\frac{\theta_1}{\alpha} \end{aligned} \tag{8.3.2}$$

$$R(2)=\left(\frac{1}{1+\mu}\right)\left(\frac{1}{1+\psi_2}\right)\left[\frac{1}{1+k\rho_2}-\frac{1}{1+\rho_1+k\rho_2}\right] \tag{8.3.3}$$

$$R(3)=\left(\frac{1}{1+\mu}\right)^2\left[\begin{array}{l} \frac{1}{1+\psi_2}\frac{1}{1+\psi_3}\left\{\begin{array}{l}\frac{1}{1+k^2\rho_3}-\frac{1}{1+\rho_1+k^2\rho_3}-\frac{1}{1+k\rho_2+k^2\rho_3} \\ +\frac{1}{1+\rho_1+k\rho_2+k^2\rho_3}\end{array}\right\} \\ +\frac{\psi_2}{1+\psi_2}\frac{1}{1+\psi_3}\left\{\frac{1}{1+k^2\rho_3}-\frac{1}{1+\rho_1+k^2\rho_3}\right\} \end{array}\right] \tag{8.3.4}$$

$$
\begin{aligned}
R(4)=\left(\frac{1}{1+\mu}\right)^{3}\Bigg[\; & \frac{1}{1+\psi_2}\frac{1}{1+\psi_3}\frac{1}{1+\psi_4}\left\{\begin{aligned}&\frac{1}{1+k^3\rho_4}-\frac{1}{1+\rho_1+k^3\rho_4}-\frac{1}{1+k\rho_2+k^3\rho_4}\\&+\frac{1}{1+\rho_1+k\rho_2+k^3\rho_4}-\frac{1}{1+k^2\rho_3+k^3\rho_4}\\&+\frac{1}{1+\rho_1+k^2\rho_3+k^3\rho_4}+\frac{1}{1+k\rho_2+k^2\rho_3+k^3\rho_4}\\&-\frac{1}{1+\rho_1+k\rho_2+k^2\rho_3+k^3\rho_4}\end{aligned}\right\}\\
&+\frac{\psi_2}{1+\psi_2}\frac{1}{1+\psi_3}\frac{1}{1+\psi_4}\left\{\begin{aligned}&\frac{1}{1+k^3\rho_4}-\frac{1}{1+\rho_1+k^3\rho_4}-\frac{1}{1+k^2\rho_3+k^3\rho_4}\\&+\frac{1}{1+\rho_1+k^2\rho_3+k^3\rho_4}\end{aligned}\right\}\\
&+\frac{1}{1+\psi_2}\frac{\psi_3}{1+\psi_3}\frac{1}{1+\psi_4}\left\{\begin{aligned}&\frac{1}{1+k^3\rho_4}-\frac{1}{1+\rho_1+k^3\rho_4}-\frac{1}{1+k\rho_2+k^3\rho_4}\\&+\frac{1}{1+\rho_1+k\rho_2+k^3\rho_4}\end{aligned}\right\}\\
&+\frac{\psi_2}{1+\psi_2}\frac{\psi_3}{1+\psi_3}\frac{1}{1+\psi_4}\left\{\frac{1}{1+k^3\rho_4}-\frac{1}{1+\rho_1+k^3\rho_4}\right\}\Bigg]
\end{aligned}
\tag{8.3.5}
$$

where, $\psi_i = \frac{\theta_i}{\beta_i}$, i=2,3,4 and $\rho_i = \frac{\theta_i}{\alpha}$, $i = 1,2,3,4$

Then ultimately the system reliability R_4 for a 4- cascade system may be obtained from the equation (2.2.1).

8.3.2 Exponential Stress-Strength for the Active Component and the Switch and Gamma Stress for the Standbys

Let $f_i(x)$, $g(y)$, $k(u)$ and $s(v)$ be all exponential densities with means $\frac{1}{\theta_i}, \frac{1}{\alpha}, \frac{1}{\lambda}$ and $\frac{1}{\gamma}$ respectively. Also let $w_j(z)$ be Gamma density with shape parameter β_j and scale parameter unity. Then $R(1)$, $R(2)$, $R(3)$ and $R(4)$ may be obtained from (8.2.7), (8.2.8), (8.2.9) and (8.2.10) respectively as

$$R(1)=\int_0^\infty \overline{F}_1(y)g(y)dy=\int_0^\alpha e^{-\theta_1 y}\alpha e^{-\alpha y}dy=\frac{\alpha}{\alpha+\theta_1}$$
$$=\frac{1}{1+\rho_1}\ \textit{where}\ \rho_1=\frac{\theta_1}{\alpha}$$

(8.3.6)

$$R(2)=\left(\frac{1}{1+\mu}\right)\frac{1}{(1+\theta_2)^{\beta_2}}\left[\frac{1}{1+k\rho_2}-\frac{1}{1+\rho_2+k\rho_2}\right] \tag{8.3.7}$$

$$R(3)=\left(\frac{1}{1+\mu}\right)^2\left[\begin{array}{l}\dfrac{1}{(1+\theta_2)^{\beta_2}}\dfrac{1}{(1+\theta_3)^{\beta_3}}\left\{\begin{array}{l}\dfrac{1}{1+k^2\rho_3}-\dfrac{1}{1+\rho_1+k^2\rho_3}-\dfrac{1}{1+k\rho_2+k^2\rho_3}\\+\dfrac{1}{1+\rho_1+k\rho_2+k^2\rho_3}\end{array}\right\}\\ \qquad +\left\{1-\dfrac{1}{(1+\theta_2)^{\beta_2}}\right\}\dfrac{1}{(1+\theta_3)^{\beta_3}}\left\{\dfrac{1}{1+k^2\rho_3}-\dfrac{1}{1+\rho_1+k^2\rho_3}\right\}\end{array}\right]$$

(8.3.8)

$$
R(4)=\left(\frac{1}{1+\mu}\right)^{3}\left[\begin{aligned}
&\frac{1}{(1+\theta_2)^{\beta_2}}\frac{1}{(1+\theta_3)^{\beta_3}}\frac{1}{(1+\theta_4)^{\beta_4}}\left\{\begin{aligned}
&\frac{1}{1+k^3\rho_4}-\frac{1}{1+\rho_1+k^3\rho_4}\\
&-\frac{1}{1+k\rho_2+k^3\rho_4}+\frac{1}{1+\rho_1+k\rho_2+k^3\rho_4}\\
&-\frac{1}{1+k^2\rho_3+k^3\rho_4}+\frac{1}{1+\rho_1+k^2\rho_3+k^3\rho_4}\\
&+\frac{1}{1+k\rho_2+k^2\rho_3+k^3\rho_4}\\
&-\frac{1}{1+\rho_1+k\rho_2+k^2\rho_3+k^3\rho_4}
\end{aligned}\right\}\\
&+\left\{1-\frac{1}{(1+\theta_2)^{\beta_2}}\right\}\frac{1}{(1+\theta_3)^{\beta_3}}\frac{1}{(1+\theta_4)^{\beta_4}}\left\{\begin{aligned}
&\frac{1}{1+k^3\rho_4}-\frac{1}{1+\rho_1+k^3\rho_4}\\
&-\frac{1}{1+k^2\rho_3+k^3\rho_4}+\frac{1}{1+\rho_1+k^2\rho_3+k^3\rho_4}
\end{aligned}\right\}\\
&+\frac{1}{(1+\theta_2)^{\beta_2}}\left\{1-\frac{1}{(1+\theta_3)^{\beta_3}}\right\}\frac{1}{(1+\theta_4)^{\beta_4}}\left\{\begin{aligned}
&\frac{1}{1+k^3\rho_4}-\frac{1}{1+\rho_1+k^3\rho_4}-\frac{1}{1+k\rho_2+k^3\rho_4}\\
&+\frac{1}{1+\rho_1+k\rho_2+k^3\rho_4}
\end{aligned}\right\}\\
&+\left\{1-\frac{1}{(1+\theta_2)^{\beta_2}}\right\}\left\{1-\frac{1}{(1+\theta_3)^{\beta_3}}\right\}\frac{1}{(1+\theta_4)^{\beta_4}}\left\{\frac{1}{1+k^3\rho_4}-\frac{1}{1+\rho_1+k^3\rho_4}\right\}
\end{aligned}\right]
\tag{8.3.9}
$$

where, $\rho_i=\dfrac{\theta_i}{\alpha}$, $i=1,2,3,4$

Then ultimately the system reliability R_4 for a 4- cascade system may be obtained from the equation (2.2.1).

8.4 Results and Discussions

It is observed from the **Table** 8.1 (cf. Appendix) that the reliability decreases with increase in k values if the other parameter remains constant. Also we observe that for $k<1$, reliability will be very high for all values of μ, ρ and ψ. For instance, when $k=0.1,\ 0.3,\ 0.5,\ 1.5$ then the system reliability becomes $R_4=0.9904,\ 0.9895,\ 0.9890$ and

0.9843 respectively. The table can be used for making a marginal reliability analysis. $R(1)$ decreases with increase in ρ_1 and k. From the table we observe that when $k = 0.1$, $\rho_1 = 0.1$ then $R(1) = 0.9091$ but when $k = 0.3$, $\rho_1 = 0.2$ then $R(1) = 0.8333$. When ρ_2 increases $R(2)$ also increases but $R(2)$ decreases with increasing ψ_2. Again when ρ_3 and ψ_3 together increase there is significant change in $R(3)$. For example, when $k = 0.1$, $\rho_1 = \rho_2 = \rho_3 = \rho_4 = 0.1$, $\psi_2 = \psi_3 = \psi_4 = 0.1$ then $R(3) = 0.0074$. Again when $k = 0.1$, $\rho_1 = \rho_2 = \rho_3 = \rho_4 = 0.2$, $\psi_2 = \psi_3 = \psi_4 = 0.3$ then $R(3) = 0.0276$. We may say that in this case reliability increases with increase in ψ values. Similar conclusion may be drawn for ψ_4. Increase in the values of $k, \rho_1, \rho_2, \rho_3, \rho_4, \psi_2, \psi_3, \psi_4$ the values of the reliability decreases.

From the **Table** 8.2 (cf. Appendix), it is observe that reliability R_4 decreases with increasing k and μ values, if the other parameter remains constant. When ρ_1 increases then the values of the marginal reliability $R(1)$ decreases. For example, $R(1) = 0.9091$ for $\rho_1 = 0.1$ and $R(1) = 0.8333$ for $\rho_1 = 0.2$. But when ρ_2 increases $R(2)$ also increases. Again when ρ_3 and ρ_4 increases $R(3)$ and $R(4)$ also increases. When β_2 and θ_2 increases $R(2)$ decreases. Again when β_3, β_4, θ_3 and θ_4 increase we see increase in $R(3)$ and $R(4)$ values. For example, when $k = 0.1$, $\mu = 0.1$, $\rho_1 = \rho_2 = \rho_3 = \rho_4 = 0.2$, $\theta_2 = \theta_3 = \theta_4 = 0.3$, $\beta_2 = \beta_3 = \beta_4 = 1.0$ then $R(3) = 0.0273$ and $R(4) = 0.0058$. Again when $k = 0.1$, $\mu = 0.1$, $\rho_1 = \rho_2 = \rho_3 = \rho_4 = 0.2$, $\theta_2 = \theta_3 = \theta_4 = 0.5$, $\beta_2 = \beta_3 = \beta_4 = 1.5$ then $R(3) = 0.0355$ and $R(4) = 0.0148$.

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Chapter 9

Summary and Future Works

Chapter 9

Summary and Future works

In this study, some stress-strength models in the interference theory of reliability have been investigated. A summary of our findings and certain propositions for future works related to these studies have been highlighted.

9.1 General Summary

In most of the studies of interference models, in evaluating the reliability of the system, only its stress-strength is considered. But in real life situation, time is an important factor which affects the reliability of the system. Therefore it is very much essential to bring time into the model directly. The motivation for studying this, an n-standby system has been considered and assumed that the number of stresses faced by the system in time $(0,t)$ follows a Poisson distribution in Chapter 2. Reliability can now be defined as the probability that the system working under impact of stresses will survive up to time t, when the stresses impinging on the system arrive as a Poisson process. Exponential, gamma, normal and Weibull stress-strength distributions have been studied to obtain the reliability. Tables for numerical values of reliability for some specific values of the parameters of the resulting distribution are also computed and some reliability graphs are drawn. Numerical values of reliability for some specific parametric values for exponential, gamma, normal and Weibull distribution shows that reliability decreases for increasing in the values of the parameter.

Repair facility has been studied in Chapter 3. To evaluate the reliability of the system at the N^{th} cycle of the stress, a 3-component standby system with a single repair facility with imperfect switch has been considered. In this case, most commonly used distributions in reliability theory viz. exponential, gamma and normal distributions have been studied. Also numerical values of reliability at the N^{th} cycle for the considering distribution shows that the values of the reliability are on expected line. That is, it is clear from the tables of numerical values of reliability $R_3(N)$, $N = 3,4,5$, that the reliability decreases when the number of cycle N increases. This should be the case, because if we use one component more than one or two times than its life goes to decrease.

Generally, it has been found that the parameters of stress-strength distributions are assumed to be constant. But in many situations this assumption may not be true and the parameters themselves may be random variables. In other words, the distributions with fixed parameters may not represent the stress and strength distributions adequately; distribution with random parameters may represent the situations better. Therefore, in Chapter 4, an attempt has been made to study identical stress-strength model with random parameters where stress-strength are exponential variates and one of the parameters (stress or strength) is assumed to be a random with known prior distribution, other parameters remaining constant. Reliability for uniform prior and two-point prior distribution is also computed for the parameters concerned. To study the reliability numerical values of R_1, R_2, R_3, R_4 have been obtained for uniform prior and two-point prior distributions. From tables of numerical values of R_1, R_2, R_3, R_4 it is clear that the reliabilities are steadily increasing with μ increases and reliabilities are decreases (increases) with increasing p for $\lambda_1 > \lambda_2(\lambda_1 < \lambda_2)$. Again reliabilities are decreases with increasing values of λ and reliabilities are increases (decreases) with increasing q, if $\mu_1 > \mu_2(\mu_1 < \mu_2)$.

In Chapter 5, a comparative study has been performed between warm and cold standby system with imperfect switching for identical strength. Various marginal reliability and system reliability for exponential, gamma and normal distributions have been derived in both warm and cold standby systems. Numerical marginal reliability values $R(1)$, $R(2)$, $R(3)$ and system reliability R_3 have been computed for selected values of the parameters. It is noted from the tables that the values of the system reliability become smaller in case of warm standby system than that of cold standby system. In comparison between warm and cold standby system it has been found that cold standby system is better to get the high system reliability. Same conclusion can be drawn in Chapter 6 for identical stress. In this chapter, to make the things clear, graphical set up has also been performed for cold and warm standby systems in case of exponential, gamma and normal distributions.

A number of techniques are to enhance the system reliability. But redundancy is the technique to achieve high reliability goals with any amount of maintenance. This is one of the means to achieve highly reliable systems with less dependable units. Therefore, in Chapter 7, redundancy has been made to study for stress-strength models. An n-standby and an n-cascade systems have been considered for our study when stress-strength are assumed to be dissimilar. We have considered four cases namely strength follows one-parameter exponential distribution and stress follows two-parameter exponential distribution, strength follows one-parameter exponential distribution and stress follows two-parameter gamma distribution, strength follows Lindley distribution and stress follows one-parameter gamma, strength follows Lindley distribution and stress follows two-parameter gamma distribution for an n-standby system. For all the four cases, it is clear from the numerical values of reliability that the values of the system reliability R_3 decreases with increasing different stress-strength parameters. Again for an n-cascade system we have considered only two cases, viz. strength follows one-parameter exponential distribution and stress follows two-parameter gamma distribution and strength follows one-parameter exponential distribution and stress follows Lindley distributions. From the numerical values of reliability for above two cases, it has been observed that the values of the system reliability R_3 decreases steadily with increasing the attenuation factor k. Similar conclusion can be drawn for the reliability from the graphical techniques.

Problem of warm standby with imperfect switching for stress-strength models has been studied in Chapter 5 and Chapter 6. But in Chapter 8, warm standby system for cascade model with imperfect switching has been performed. Reliability expressions have been derived when the stress-strength of the components and the switch follow particular distributions. Considering stress-strength for the active component, standby component and the switch follow exponential distributions and stress-strength for the active component and the switch follow exponential distribution and standby component follow gamma distribution then the marginal reliability $R(1)$, $R(2)$, $R(3)$, $R(4)$ and system reliability R_4 for a 4-cascade system have been obtained. Tables for numerical values of reliability of the resulting distribution are also computed. Numerical values of reliability for exponential and gamma distribution shows that, reliability decreases with increasing the attenuation factor k if the other parameter remains constant.

9.2 Future Works

Along this line of research on reliability theory some possible works to consider in the future.

From the survey of literature, very few studies are there for repairable systems in interference. With proper redundancy and repair, reliability can be achieved very close to one. For repairable system, more than reliability, availability of the system and cascade model with repair is not included in this thesis. Therefore, study on availability and cascade system with repair is also a new field in near future. Several authors have studied stress-strength as mixture of distributions. But one of the interesting modifications of reliability theory is the concept of progressive repairable system where stress-strength is mixture of distributions.

The reliability growth model is a structured process used to discover reliability deficiencies through testing, analyzing such deficiencies and implementation of corrective measures to lower the rate of occurrence. Some of the important advantages of the reliability growth model include assessments of achievement and projecting the product reliability trends. Therefore, in the development or even during testing of a large system some corrective actions (including repair) will be helpful for the future with a view of enhancing the reliability of the system.

In the absence of hard data for different systems discussed in this thesis, simulation technique will be helpful for future to estimate the system reliability or even availability and other characteristics of reliability, such as failure rate (FR), mean time to failure (MTTF), etc.

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Appendix

List of Tables

Table 2.1 Reliability $R_i(t)$, $i = 1,2,3,4$ at time 't' when stress-strength follows exponential distribution

αt	ρ	$R_1(t)$	$R_2(t)$	$R_3(t)$	$R_4(t)$
1 (t=1, $\alpha = 1$)	.5	0.7165	0.8758	0.9465	0.9773
	1	0.6065	0.7582	0.8529	0.9114
	1.5	0.5488	0.6805	0.7754	0.8430
	2	0.5134	0.6275	0.7162	0.7848
	2.5	0.4895	0.5894	0.6710	0.7372
2 (t=1, $\alpha = 2$ or t=2, $\alpha = 1$)	.5	0.5134	0.7416	0.8684	0.9350
	1	0.3679	0.5518	0.6898	0.7894
	1.5	0.3012	0.4458	0.5672	0.6664
	2	0.2636	0.3808	0.4849	0.5755
	2.5	0.2397	0.3375	0.4273	0.5084
3 (t=1, $\alpha = 3$ or t=3, $\alpha = 1$)	.5	0.3679	0.6131	0.7766	0.8766
	1	0.2231	0.3905	0.5369	0.6572
	1.5	0.1653	0.2843	0.3986	0.5031
	2	0.1353	0.2256	0.3158	0.4027
	2.5	0.1173	0.1891	0.2624	0.3350
4 (t=1, $\alpha = 4$ or t=2, $\alpha = 2$ or t=4, $\alpha = 1$)	.5	0.2636	0.4979	0.6801	0.8065
	1	0.1353	0.2707	0.4060	0.5301
	1.5	0.0907	0.1778	0.2719	0.3668
	2	0.0695	0.1312	0.1999	0.2721
	2.5	0.0574	0.1043	0.1569	0.2134
5 (t=1, $\alpha = 5$ or t=5, $\alpha = 1$)	.5	0.1889	0.3987	0.5853	0.7295
	1	0.0821	0.1847	0.3001	0.4166
	1.5	0.0498	0.1095	0.1812	0.2601
	2	0.0357	0.0753	0.1238	0.1789
	2.5	0.0281	0.0568	0.0919	0.1325

Table 2.2 Reliability $R_i(t)$, $i = 1,2,3,4$ at time 't' when stress-strength follows gamma distribution

αt	m	l	$R_1(t)$	$R_2(t)$	$R_3(t)$	$R_4(t)$
2 (t=1, $\alpha = 2$ or t=2, $\alpha = 1$)	1	1	0.3679	0.5518	0.6898	0.7894
	1	2	0.2231	0.3068	0.3852	0.4578
	2	1	0.6065	0.8340	0.9335	0.9744
	3	1	0.7788	0.9492	0.9891	0.9978
3 (t=1, $\alpha = 3$ or t=3, $\alpha = 1$)	1	1	0.2231	0.3905	0.5369	0.6572
	1	2	0.1054	0.1647	0.2258	0.2873
	2	1	0.4724	0.7381	0.8792	0.9472
	3	1	0.6873	0.9128	0.9780	0.9948
4 (t=1, $\alpha = 4$ or t=2, $\alpha = 2$ or t=4, $\alpha = 1$)	1	1	0.1353	0.2707	0.4060	0.5301
	1	2	0.0498	0.0871	0.1291	0.1746
	2	1	0.3679	0.6438	0.8162	0.9111
	3	1	0.6065	0.8719	0.9631	0.9902
5 (t=1, $\alpha = 5$ or t=5, $\alpha = 1$)	1	1	0.0821	0.1847	0.3001	0.4166
	1	2	0.0235	0.0456	0.0724	0.1036
	2	1	0.2865	0.5551	0.7482	0.8672
	3	1	0.5353	0.8280	0.9446	0.9838
6 (t=1, $\alpha = 6$ or t=2, $\alpha = 3$ or t=3, $\alpha = 2$ or t=6, $\alpha = 1$)	1	1	0.0498	0.1245	0.2178	0.3205
	1	2	0.0111	0.0236	0.0400	0.0602
	2	1	0.2231	0.4742	0.6781	0.8173
	3	1	0.4724	0.7824	0.9228	0.9753
7 (t=1, $\alpha = 7$ or t=7, $\alpha = 1$)	1	1	0.0302	0.0830	0.1557	0.2421
	1	2	0.0052	0.0121	0.0218	0.0345
	2	1	0.1738	0.4019	0.6085	0.7631
	3	1	0.4169	0.7360	0.8981	0.9648
8 (t=1, $\alpha = 8$ or t=2, $\alpha = 4$ or t=4, $\alpha = 2$ or t=8, $\alpha = 1$)	1	1	0.0183	0.0549	0.1099	0.1801
	1	2	0.0025	0.0062	0.0118	0.0194
	2	1	0.1353	0.3383	0.5413	0.7063
	3	1	0.3679	0.6898	0.8708	0.9522

Table 2.3 Reliability $R_i(t)$, $i = 1,2,3,4$ at time 't' when stress-strength follows normal distribution

αt	μ	σ	$R_1(t)$	$R_2(t)$	$R_3(t)$	$R_4(t)$
1 (t=1, $\alpha = 1$)	1	.5	0.8307	0.9562	0.9890	0.9968
	1	1	0.7868	0.9302	0.9777	0.9922
	2	.5	0.9639	0.9980	0.9999	1.0000
	2	1	0.9244	0.9913	0.9990	0.9998
2 (t=1, $\alpha = 2$ or t=2, $\alpha = 1$)	1	.5	0.6900	0.8985	0.9687	0.9877
	1	1	0.6191	0.8448	0.9400	0.9730
	2	.5	0.9290	0.9949	0.9997	0.9999
	2	1	0.8544	0.9783	0.9970	0.9992
3 (t=1, $\alpha = 3$ or t=3, $\alpha = 1$)	1	.5	0.5731	0.8330	0.9401	0.9718
	1	1	0.4871	0.7535	0.8902	0.9424
	2	.5	0.8954	0.9907	0.9993	0.9998
	2	1	0.7898	0.9615	0.9937	0.9977
4 (t=1, $\alpha = 4$ or t=2, $\alpha = 2$ or t=4, $\alpha = 1$)	1	.5	0.4761	0.7638	0.9042	0.9496
	1	1	0.3833	0.6627	0.8316	0.9031
	2	.5	0.8631	0.9855	0.9987	0.9995
	2	1	0.7301	0.9417	0.9890	0.9953
5 (t=1, $\alpha = 5$ or t=5, $\alpha = 1$)	1	.5	0.3954	0.6942	0.8626	0.9229
	1	1	0.3016	0.5764	0.7675	0.8592
	2	.5	0.8319	0.9794	0.9979	0.9990
	2	1	0.6749	0.9194	0.9829	0.9917
6 (t=1, $\alpha = 6$ or t=2, $\alpha = 3$ or t=3, $\alpha = 2$ or t=6, $\alpha = 1$)	1	.5	0.3285	0.6263	0.8166	0.8937
	1	1	0.2373	0.4968	0.7009	0.8148
	2	.5	0.8018	0.9724	0.9968	0.9984
	2	1	0.6238	0.8950	0.9753	0.9871
7 (t=1, $\alpha = 7$ or t=7, $\alpha = 1$)	1	.5	0.2729	0.5615	0.7677	0.8644
	1	1	0.1867	0.4249	0.6340	0.7731
	2	.5	0.7728	0.9646	0.9955	0.9976
	2	1	0.5766	0.8691	0.9663	0.9814
8 (t=1, $\alpha = 8$ or t=2, $\alpha = 4$ or t=4, $\alpha = 2$ or t=8, $\alpha = 1$)	1	.5	0.2266	0.5006	0.7171	0.8368
	1	1	0.1469	0.3611	0.5686	0.7365
	2	.5	0.7449	0.9562	0.9939	0.9966
	2	1	0.5330	0.8420	0.9559	0.9748

Table 2.4 Reliability $R_i(t)$, $i = 1,2,3,4$ at time 't' when stress-strength follows Weibull distribution

αt	b	θ	c	λ	$R_1(t)$	$R_2(t)$	$R_3(t)$	$R_4(t)$
.1 (t=1, α = .1 or t=.1, α = 1)	.9	.7	.8	.1	0.7670	0.7769	0.7854	0.7926
	.8	.6	.7	.2	0.6373	0.6506	0.6600	0.6668
	.7	.5	.6	.3	0.5365	0.5495	0.5573	0.5620
	.6	.4	.5	.4	0.4648	0.4764	0.4825	0.4857
.2 (t=1, α = .2 or t=.2, α = 1)	.9	.7	.8	.1	0.6940	0.7120	0.7274	0.7407
	.8	.6	.7	.2	0.5766	0.6007	0.6181	0.6307
	.7	.5	.6	.3	0.4855	0.5089	0.5234	0.5323
	.6	.4	.5	.4	0.4205	0.4416	0.4529	0.4590
.3 (t=1, α = .3 or t=.3, α = 1)	.9	.7	.8	.1	0.6280	0.6523	0.6734	0.6917
	.8	.6	.7	.2	0.5218	0.5544	0.5783	0.5960
	.7	.5	.6	.3	0.4393	0.4711	0.4911	0.5037
	.6	.4	.5	.4	0.3805	0.4090	0.4248	0.4334
.4 (t=1, α = .4 or t=.4, α = 1)	.9	.7	.8	.1	0.5682	0.5976	0.6232	0.6456
	.8	.6	.7	.2	0.4721	0.5114	0.5408	0.5626
	.7	.5	.6	.3	0.3975	0.4358	0.4604	0.4762
	.6	.4	.5	.4	0.3443	0.3787	0.3981	0.4090
.5 (t=1, α = .5 or t=.5, α = 1)	.9	.7	.8	.1	0.5142	0.5474	0.5766	0.6022
	.8	.6	.7	.2	0.4272	0.4717	0.5053	0.5307
	.7	.5	.6	.3	0.3596	0.4030	0.4314	0.4499
	.6	.4	.5	.4	0.3115	0.3505	0.3729	0.3857
.6 (t=1, α = .6 or t=.6, α = 1)	.9	.7	.8	.1	0.4652	0.5013	0.5332	0.5615
	.8	.6	.7	.2	0.3865	0.4348	0.4719	0.5002
	.7	.5	.6	.3	0.3254	0.3725	0.4039	0.4247
	.6	.4	.5	.4	0.2819	0.3242	0.3490	0.3636
.7 (t=1, α = .7 or t=.7, α = 1)	.9	.7	.8	.1	0.4210	0.4590	0.4930	0.5233
	.8	.6	.7	.2	0.3492	0.4007	0.4404	0.4711
	.7	.5	.6	.3	0.2944	0.3442	0.3779	0.4006
	.6	.4	.5	.4	0.2551	0.2997	0.3265	0.3425

Table 3.1 Reliability $R_3(N)$, $N = 3,4,5$ when stress-strength of the switch and the components follow exponential distribution

q	$a = 1/(1+\rho)$	$b = 1/(1+\upsilon)$	$R_3(3)$	$R_3(4)$	$R_3(5)$
.1	.9091	.8333	.9552	.9407	.9265
	.6667	.7692	.7864	.7257	.6697
	.5000	.6667	.5783	.4816	.4010
	.4000	.5882	.4264	.3207	.2412
	.3333	.5263	.3199	.2185	.1493
.2	.9091	.8333	.9552	.9407	.9264
	.6667	.7692	.7859	.7246	.6681
	.5000	.6667	.5772	.4795	.3983
	.4000	.5882	.4251	.3185	.2386
	.3333	.5263	.3186	.2166	.1472
.3	.9091	.8333	.9552	.9407	.9263
	.6667	.7692	.7850	.7227	.6651
	.5000	.6667	.5754	.4758	.3933
	.4000	.5882	.4229	.3146	.2339
	.3333	.5263	.3164	.2132	.1435
.4	.9091	.8333	.9552	.9406	.9262
	.6667	.7692	.7838	.7197	.6603
	.5000	.6667	.5728	.4703	.3856
	.4000	.5882	.4198	.3090	.2270
	.3333	.5263	.3134	.2082	.1381
.5	.9091	.8333	.9551	.9404	.9259
	.6667	.7692	.7823	.7157	.6536
	.5000	.6667	.5694	.4630	.3751
	.4000	.5882	.4159	.3014	.2176
	.3333	.5263	.3059	.2017	.1309
.6	.9091	.8333	.9551	.9403	.9256
	.6667	.7692	.7805	.7105	.6445
	.5000	.6667	.5654	.4536	.3614
	.4000	.5882	.4110	.2920	.2056
	.3333	.5263	.3048	.1936	.1219
.7	.9091	.8333	.9550	.9401	.9251
	.6667	.7692	.7783	.7041	.6328
	.5000	.6667	.5606	.4423	.3445
	.4000	.5882	.4053	.2806	.1912
	.3333	.5263	.2991	.1840	.1112
.8	.9091	.8333	.9550	.9398	.9245
	.6667	.7692	.7757	.6964	.6182
	.5000	.6667	.5550	.4288	.3242
	.4000	.5882	.3987	.2674	.1745
	.3333	.5263	.2927	.1728	.0990

Table 3.2 Reliability $R_3(N)$, $N = 3,4,5$ when stress-strength of the switch and the components follow gamma distribution

l	m	c	d	q	a	b	$R_3(3)$	$R_3(4)$	$R_3(5)$
2	2	2	3	.1	.5000	.6875	.6003	.5061	.4267
4	2	3	2		.1875	.3125	.0858	.0378	.0167
2	4	2	5		.8125	.8906	.9397	.9204	.9014
4	6	3	5		.7461	.7734	.8371	.7888	.7434
6	4	5	3		.2539	.2266	.0756	.0320	.0135
2	2	2	3	.2	.5000	.6875	.5991	.5038	.4236
4	2	3	2		.1875	.3125	.0853	.0373	.0163
2	4	2	5		.8125	.8906	.9395	.9200	.9009
4	6	3	5		.7461	.7734	.8369	.7883	.7426
6	4	5	3		.2539	.2266	.0755	.0318	.0134
2	2	2	3	.3	1	.6875	1	1	1
4	2	3	2		.6875	.3125	.4839	.3799	.2982
2	4	2	5		1	.8906	1	1	1
4	6	3	5		.7461	.7734	.8365	.7874	.7412
6	4	5	3		.2539	.2266	.0752	.0316	.0132
2	2	2	3	.4	.5625	.6875	.6390	.5461	.4662
4	2	3	2		.2500	.3125	.1116	.0527	.0249
2	4	2	5		.8125	.8906	.9390	.9185	.8983
4	6	3	5		.7461	.7734	.8359	.7860	.7388
6	4	5	3		.4414	.2266	.1829	.1036	.0586
2	2	2	3	.5	.5000	.6875	.5905	.4855	.3977
4	2	3	2		.1875	.3125	.0819	.0338	.0139
2	4	2	5		.8125	.8906	.9386	.9173	.8960
4	6	3	5		.7461	.7734	.8353	.7841	.7354
6	4	5	3		.4414	.2266	.1827	.1033	.0583
2	2	2	3	.6	.5000	.6875	.5861	.4752	.3825
4	2	3	2		.1875	.3125	.0801	.0320	.0126
2	4	2	5		.8125	.8906	.9381	.9156	.8927
4	6	3	5		.7461	.7734	.8344	.7816	.7307
6	4	5	3		.2539	.2266	.0739	.0301	.0122
2	2	2	3	.7	.5000	.6875	.5808	.4627	.3637
4	2	3	2		.1875	.3125	.0780	.0298	.0112
2	4	2	5		.8125	.8906	.9374	.9136	.8884
4	6	3	5		.7461	.7734	.8334	.7785	.7246
6	4	5	3		.2539	.2266	.0733	.0294	.0116
2	2	2	3	.8	.5000	.6875	.5747	.4479	.3413
4	2	3	2		.1875	.3125	.0755	.0273	.0095
2	4	2	5		.8125	.8906	.9367	.9111	.8829
4	6	3	5		.7461	.7734	.8323	.7747	.7168
6	4	5	3		.2539	.2266	.0726	.0285	.0109

Table 3.3 Reliability $R_3(N)$, $N=3,4,5$ when stress-strength of the switch and the components follow normal distribution

μ	σ	α	τ	q	a	b	$R_3(3)$	$R_3(4)$	$R_3(5)$
3	1	2	1.5	.1	.9831	.8664	.9932	.9910	.9887
1	.5	1.5	4		.8145	.6420	.8136	.7596	.7091
2	3	5	1		.7365	.9998	.9997	.9994	.9991
3	6	.5	4		.6891	.5483	.6350	.5457	.4690
1	6	3.5	5		.5653	.7538	.7117	.6352	.5668
3	1	2	1.5	.2	.9831	.8664	.9932	.9910	.9887
1	.5	1.5	4		.8145	.6420	.8136	.7595	.7089
2	3	5	1		.7365	.9998	.9991	.9981	.9971
3	6	.5	4		.6891	.5483	.6348	.5454	.4686
1	6	3.5	5		.5653	.7538	.7106	.6330	.5638
3	1	2	1.5	.3	.9831	.8664	.9932	.9910	.9887
1	.5	1.5	4		.8145	.6420	.8135	.7593	.7086
2	3	5	1		.7365	.9998	.9982	.9958	.9932
3	6	.5	4		.6891	.5483	.6346	.5449	.4678
1	6	3.5	5		.5653	.7538	.7089	.6292	.5582
3	1	2	1.5	.4	.9831	.8664	.9932	.9910	.9887
1	.5	1.5	4		.8145	.6420	.8134	.7589	.7081
2	3	5	1		.7365	.9998	.9969	.9923	.9871
3	6	.5	4		.6891	.5483	.6342	.5441	.4665
1	6	3.5	5		.5653	.7538	.7064	.6235	.5497
3	1	2	1.5	.5	.9831	.8664	.9932	.9910	.9887
1	.5	1.5	4		.8145	.6420	.8132	.7585	.7072
2	3	5	1		.7365	.9998	.9953	.9876	.9784
3	6	.5	4		.6891	.5483	.6338	.5429	.4647
1	6	3.5	5		.5653	.7538	.7032	.6159	.5377
3	1	2	1.5	.6	.9831	.8664	.9932	.9910	.9887
1	.5	1.5	4		.8145	.6420	.8131	.7579	.7061
2	3	5	1		.7365	.9998	.9933	.9814	.9666
3	6	.5	4		.6891	.5483	.6332	.5414	.4621
1	6	3.5	5		.5653	.7538	.6994	.6061	.5220
3	1	2	1.5	.7	.9831	.8664	.9932	.9910	.9887
1	.5	1.5	4		.8145	.6420	.8128	.7572	.7046
2	3	5	1		.7365	.9998	.9909	.9738	.9513
3	6	.5	4		.6891	.5483	.6326	.5395	.4587
1	6	3.5	5		.5653	.7538	.6948	.5942	.5024
3	1	2	1.5	.8	.9831	.8664	.9932	.9910	.9887
1	.5	1.5	4		.8145	.6420	.8126	.7563	.7026
2	3	5	1		.7365	.9998	.9881	.9645	.9322
3	6	.5	4		.6891	.5483	.6319	.5372	.4544
1	6	3.5	5		.5653	.7538	.6895	.5800	.4787

Table 4.1 Values of R_1, R_2, R_3, R_4 for exponential stress-strength when strength parameter λ is random and uniformly distributed in the range (a, b)

a	b	μ	R_1	R_2	R_3	R_4
1	2	1	0.4055	0.6466	0.7898	0.8751
		3	0.6694	0.8907	0.9639	0.9881
		5	0.7708	0.9475	0.9880	0.9972
		7	0.8245	0.9692	0.9946	0.9991
		9	0.8578	0.9798	0.9971	0.9996
3	4	1	0.2231	0.3965	0.5312	0.6358
		3	0.4625	0.7111	0.8447	0.9165
		5	0.5889	0.831	0.9305	0.9714
		7	0.6672	0.8893	0.9632	0.9877
		9	0.7204	0.9218	0.9781	0.9939
5	6	1	0.1542	0.2846	0.3949	0.4881
		3	0.3533	0.5818	0.7296	0.8251
		5	0.4766	0.7261	0.8567	0.9249
		7	0.5603	0.8067	0.9150	0.9626
		9	0.6209	0.8563	0.9455	0.9794
7	8	1	0.1178	0.2217	0.3134	0.3942
		3	0.2859	0.4901	0.6359	0.7400
		5	0.4002	0.6402	0.7842	0.8706
		7	0.4830	0.7327	0.8618	0.9285
		9	0.5456	0.7935	0.9061	0.9574
9	10	1	0.0953	0.1815	0.2925	0.3301
		3	0.2401	0.4226	0.5612	0.6666
		5	0.3450	0.5709	0.7189	0.8159
		7	0.4244	0.6687	0.8093	0.8902
		9	0.4866	0.7364	0.8647	0.9305

Table 4.2 Values of R_1, R_2, R_3, R_4 for exponential stress-strength when strength parameter λ is random having two-point distribution

p	λ_1	λ_2	μ	R_1	R_2	R_3	R_4
0.1	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.5167	0.7664	0.8871	0.9454
	2	1	3	0.7350	0.9298	0.9814	0.9951
0.2	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.5333	0.7822	0.8984	0.9526
	2	1	3	0.7200	0.9216	0.9780	0.9939
0.3	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.5500	0.7975	0.9089	0.9590
	2	1	3	0.7050	0.9130	0.9743	0.9924
0.4	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.5667	0.8122	0.9186	0.9647
	2	1	3	0.6900	0.9039	0.9702	0.9908
0.5	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.5833	0.8264	0.9277	0.9699
	2	1	3	0.6750	0.8944	0.9657	0.9888
0.6	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.6000	0.8400	0.9360	0.9744
	2	1	3	0.6600	0.8844	0.9607	0.9866
0.7	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.6167	0.8531	0.9437	0.9784
	2	1	3	0.6450	0.8740	0.9553	0.9841
0.8	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.6333	0.8656	0.9507	0.9819
	2	1	3	0.6300	0.8631	0.9493	0.9813
0.9	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.6500	0.8775	0.9571	0.9850
	2	1	3	0.6150	0.8518	0.9429	0.9780

Table 4.3 Values of R_1, R_2, R_3, R_4 for exponential stress-strength when stress parameter μ is random and uniformly distributed in the range (c,d)

c	d	λ	R_1	R_2	R_3	R_4
1	2	1	0.5945	0.8356	0.9333	0.9730
		3	0.3306	0.5519	0.7000	0.7992
		5	0.2292	0.4059	0.5421	0.6471
		7	0.1755	0.3202	0.4395	0.5379
		9	0.1422	0.2642	0.3608	0.4586
2	3	1	0.7123	0.9172	0.9762	0.9932
		3	0.4530	0.7008	0.8364	0.9105
		5	0.3323	0.5542	0.7024	0.8013
		7	0.2625	0.4561	0.5988	0.7041
		9	0.2169	0.3868	0.5198	0.6239
3	4	1	0.7769	0.9503	0.9890	0.9975
		3	0.5375	0.7861	0.9011	0.9543
		5	0.4111	0.6532	0.7958	0.8797
		7	0.3328	0.5549	0.7030	0.8019
		9	0.2796	0.4810	0.6261	0.7307
4	5	1	0.8177	0.9668	0.9939	0.9989
		3	0.5994	0.8395	0.9357	0.9742
		5	0.4732	0.7225	0.8538	0.9230
		7	0.3909	0.6290	0.7740	0.8624
		9	0.3330	0.5551	0.7033	0.8021
5	6	1	0.8458	0.9762	0.9963	0.9994
		3	0.6467	0.8752	0.9559	0.9844
		5	0.5234	0.7729	0.8918	0.9484
		7	0.4397	0.6861	0.8241	0.9014
		9	0.3791	0.6145	0.7607	0.8513
6	7	1	0.8665	0.9822	0.9976	0.9997
		3	0.6839	0.9001	0.9684	0.9900
		5	0.5649	0.8107	0.9177	0.9642
		7	0.4812	0.7309	0.8604	0.9276
		9	0.4192	0.6626	0.8040	0.8862
7	8	1	0.8822	0.9861	0.9983	0.9998
		3	0.7141	0.9183	0.9767	0.9933
		5	0.5998	0.8398	0.9359	0.9743
		7	0.5170	0.7667	0.8873	0.9456
		9	0.4544	0.7023	0.8376	0.9114

Table 4.4 Values of R_1, R_2, R_3, R_4 for exponential stress-strength when stress parameter μ is random having two-point distribution

q	μ_1	μ_2	λ	R_1	R_2	R_3	R_4
0.1	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.4833	0.7331	0.8621	0.9287
	2	1	3	0.2650	0.4598	0.6029	0.7082
0.2	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.4667	0.7156	0.8483	0.9191
	2	1	3	0.2800	0.4816	0.6268	0.7313
0.3	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.4500	0.6975	0.8336	0.9085
	2	1	3	0.2950	0.5030	0.6496	0.7530
0.4	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.4333	0.6789	0.8180	0.8969
	2	1	3	0.3100	0.5239	0.6715	0.7733
0.5	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.4167	0.6597	0.8015	0.8842
	2	1	3	0.3250	0.5444	0.6925	0.7924
0.6	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.4000	0.6400	0.7840	0.8704
	2	1	3	0.3400	0.5644	0.7125	0.8103
0.7	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.3833	0.6197	0.7655	0.8554
	2	1	3	0.3550	0.5840	0.7317	0.8269
0.8	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.3667	0.5989	0.7460	0.8391
	2	1	3	0.3700	0.6031	0.7500	0.8425
0.9	1	1	1	0.5000	0.7500	0.8750	0.9375
	1	2	2	0.3500	0.5775	0.7254	0.8215
	2	1	3	0.3850	0.6218	0.7674	0.8569

Table 5.1 Marginal and system reliability of cold and warm standby system for identical strength in case of exponential distribution

θ	ρ	α_1	α_2	α_3	$R(1)$	**Cold standby**			**Warm standby**				
						$R(2)$	$R(3)$	R_3	β_2	β_3	$R(2)$	$R(3)$	R_3
0.1	0.1	0.3	0.3	0.3	.7500	.1705	.0387	.9592	0.1	0.1	.0852	.0523	.8875
0.1	0.1	0.5	0.5	0.5	.8333	.1263	.0191	.9787	0.1	0.1	.0631	.0363	.9328
0.1	0.1	1.1	1.1	1.1	.9167	.0694	.0053	.9914	0.1	0.1	.0347	.0187	.9701
0.1	0.1	1.3	1.3	1.3	.9286	.0603	.0039	.9928	0.1	0.1	.0301	.0161	.9748
0.2	0.1	0.3	0.3	0.3	.6000	.2182	.0793	.8975	0.2	0.2	.1091	.0744	.7835
0.2	0.2	0.3	0.3	0.3	.6000	.2000	.0667	.8667	0.2	0.2	.1000	.0667	.7667
0.2	0.2	0.5	0.5	0.5	.7143	.1701	.0405	.9248	0.2	0.2	.0850	.0526	.8502
0.2	0.2	1.1	1.1	1.1	.8462	.1085	.0139	.9685	0.2	0.2	.0542	.0306	.9310
0.2	0.2	1.3	1.3	1.3	.8667	.0963	.0107	.9737	0.2	0.2	.0481	.0267	.9416
0.3	0.1	0.3	0.3	0.3	.5000	.2273	.1033	.8306	0.3	0.3	.1136	.0826	.6963
0.3	0.2	0.3	0.3	0.3	.5000	.2083	.0868	.7951	0.3	0.3	.1042	.0738	.6780
0.3	0.3	0.3	0.3	0.3	.5000	.1923	.0740	.7663	0.3	0.3	.0962	.0666	.6627
0.3	0.3	0.5	0.5	0.5	.6250	.1803	.0520	.8573	0.3	0.3	.0901	.0581	.7732
0.3	0.3	1.1	1.1	1.1	.7857	.1295	.0213	.9366	0.3	0.3	.0648	.0377	.8882
0.3	0.3	1.3	1.3	1.3	.8125	.1172	.0169	.9466	0.3	0.3	.0586	.0335	.9046
0.4	0.1	0.3	0.3	0.3	.4286	.2226	.1157	.7669	0.4	0.4	.1113	.0846	.6245
0.4	0.2	0.3	0.3	0.3	.4286	.2041	.0972	.7298	0.4	0.4	.1020	.0753	.6059
0.4	0.3	0.3	0.3	0.3	.4286	.1884	.0828	.6998	0.4	0.4	.0942	.0678	.5906
0.4	0.4	0.3	0.3	0.3	.4286	.1749	.0714	.6749	0.4	0.4	.0875	.0616	.5776
0.4	0.4	0.5	0.5	0.5	.5556	.1764	.0560	.7879	0.4	0.4	.0882	.0581	.7018
0.4	0.4	1.1	1.1	1.1	.7333	.1397	.0266	.8996	0.4	0.4	.0698	.0416	.8447
0.4	0.4	1.3	1.3	1.3	.7647	.1285	.0216	.9148	0.4	0.4	.0643	.0375	.8665
0.5	0.1	0.3	0.3	0.3	.3750	.2131	.1211	.7091	0.5	0.5	.1065	.0835	.5651
0.5	0.2	0.3	0.3	0.3	.3750	.1953	.1017	.6720	0.5	0.5	.0977	.0743	.5469
0.5	0.3	0.3	0.3	0.3	.3750	.1803	.0867	.6420	0.5	0.5	.0901	.0667	.5319
0.5	0.4	0.3	0.3	0.3	.3750	.1674	.0747	.6171	0.5	0.5	.0837	.0605	.5192
0.5	0.5	0.3	0.3	0.3	.3750	.1562	.0651	.5964	0.5	0.5	.0781	.0553	.5085
0.5	0.5	0.5	0.5	0.5	.5000	.1667	.0556	.7222	0.5	0.5	.0833	.0556	.6389
0.5	0.5	1.1	1.1	1.1	.6875	.1432	.0298	.8606	0.5	0.5	.0716	.0433	.8024
0.5	0.5	1.3	1.3	1.3	.7222	.1337	.0248	.8807	0.5	0.5	.0669	.0396	.8287

Table 5.2 Marginal and system reliability of cold and warm standby system for identical strength in case of gamma distribution

θ	α_1	α_2	α_3	λ	μ	$R(1)$	Cold standby			Warm standby				
							$R(2)$	$R(3)$	R_3	β_2	β_3	$R(2)$	$R(3)$	R_3
1	1	1	1	1	1	.5000	.1250	.0313	.6563	1	1	.0625	.0391	.6016
1	2	2	2	2	1	.2500	.1406	.0791	.4697	2	2	.0352	.0313	.3165
1	3	3	3	3	1	.1250	.0957	.0733	.2940	3	3	.0120	.0116	.1486
2	1	1	1	1	1	.7500	.0938	.0117	.8555	1	1	.0703	.0242	.8445
2	2	2	2	2	1	.5000	.1875	.0703	.7578	2	2	.0938	.0645	.6582
2	3	3	3	3	1	.3125	.1880	.1131	.6136	3	3	.0587	.0514	.4227
3	1	1	1	1	1	.8750	.0547	.0034	.9331	1	1	.0479	.0086	.9314
3	2	2	2	2	1	.6875	.1611	.0378	.8864	2	2	.1108	.0525	.8507
3	3	3	3	3	1	.5000	.2188	.0957	.8145	3	3	.1094	.0786	.6880
4	1	1	1	1	1	.9375	.0293	.0009	.9677	1	1	.0275	.0025	.9675
4	2	2	2	2	1	.8125	.1143	.0161	.9428	2	2	.0928	.0280	.9333
4	3	3	3	3	1	.6563	.1974	.0594	.9130	3	3	.1295	.0701	.8559
5	1	1	1	1	1	.9688	.0151	.0002	.9841	1	1	.0147	.0006	.9841
5	2	2	2	2	1	.8906	.0731	.0060	.9697	2	2	.0651	.0119	.9676
5	3	3	3	3	1	.7734	.1533	.0304	.9572	3	3	.1186	.0451	.9371
6	1	1	1	1	1	.9844	.0077	.00006	.9921	1	1	.0076	.0001	.9921
6	2	2	2	2	1	.9375	.0439	.0021	.9835	2	2	.0412	.0044	.9831
6	3	3	3	3	1	.8555	.1082	.0137	.9773	3	3	.0926	.0234	.9714

Table 5.3 Marginal and system reliability of cold and warm standby system for identical strength in case of normal distribution: Here $\lambda = 2,\ \alpha_1 = \alpha_2 = \alpha_3 = \beta_2 = \beta_3 = \mu = 0$

θ	σ	τ_1	τ_2	τ_3	υ	ρ	$R(1)$	**Cold standby**			**Warm standby**				
								$R(2)$	$R(3)$	R_3	γ_2	γ_3	$R(2)$	$R(3)$	R_3
1	1	2	2	2	1	2	.6726	.1793	.0478	.8998	2	2	.0587	.0611	.7925
1	2	4	4	4	2	4	.5885	.1629	.0451	.7965	4	4	.0670	.0551	.7106
1	3	6	6	6	3	6	.5593	.1521	.0414	.7528	6	6	.0671	.0504	.6767
2	1	2	2	2	1	2	.8145	.1231	.0186	.9561	2	2	.0228	.0309	.8682
2	2	4	4	4	2	4	.6726	.1481	.0326	.8534	4	4	.0485	.0474	.7685
2	3	6	6	6	3	6	.6172	.1458	.0345	.7975	6	6	.0558	.0476	.7206
3	1	2	2	2	1	2	.9101	.0666	.0049	.9816	2	2	.0060	.0095	.9256
3	2	4	4	4	2	4	.7488	.1265	.0214	.8967	4	4	.0318	.0358	.8164
3	3	6	6	6	3	6	.6726	.1359	.0275	.8360	6	6	.0445	.0423	.7595
4	1	2	2	2	1	2	.9632	.0289	.0008	.9929	2	2	.0011	.0018	.9661
4	2	4	4	4	2	4	.8145	.1016	.0127	.9288	4	4	.0189	.0238	.8571
4	3	6	6	6	3	6	.7245	.1232	.0209	.8686	6	6	.0339	.0356	.7940
5	1	2	2	2	1	2	.9873	.0102	.0001	.9976	2	2	.0001	.0002	.9877
5	2	4	4	4	2	4	.8682	.0770	.0068	.9520	4	4	.0101	.0139	.8923
5	3	6	6	6	3	6	.7720	.1086	.0153	.8959	6	6	.0248	.0282	.8250
6	1	2	2	2	1	2	.9964	.0030	.000008	.9993	2	2	.00001	.00001	.9964
6	2	4	4	4	2	4	.9101	.0550	.0033	.9685	4	4	.0049	.0073	.9223
6	3	6	6	6	3	6	.8145	.0933	.0107	.9184	6	6	.0173	.0212	.8529

Table 6.1 Marginal and system reliability of cold and warm standby system for identical stress in case of exponential distribution

α	ρ	θ_1	θ_2	θ_3	$R(1)$	Cold standby			Warm standby				
						$R(2)$	$R(3)$	R_3	β_2	β_3	$R(2)$	$R(3)$	R_3
1	0.1	0.1	0.1	0.1	.9091	.0751	.0062	.9904	0.1	0.1	.0376	.0203	.9670
1	0.1	0.2	0.2	0.2	.8333	.1263	.0191	.9787	0.1	0.1	.0421	.0302	.9056
1	0.1	0.3	0.3	0.3	.7692	.1614	.0339	.9645	0.1	0.1	.0403	.0324	.8419
2	0.1	0.1	0.1	0.1	.9524	.0412	.0018	.9954	0.1	0.1	.0206	.0108	.9837
2	0.1	0.2	0.2	0.2	.9091	.0751	.0062	.9904	0.1	0.1	.0250	.0174	.9515
2	0.1	0.3	0.3	0.3	.8696	.1031	.0122	.9849	0.1	0.1	.0258	.0201	.9154
3	0.1	0.1	0.1	0.1	.9677	.0284	.0008	.9970	0.1	0.1	.0142	.0073	.9892
3	0.1	0.2	0.2	0.2	.9375	.0533	.0030	.9938	0.1	0.1	.0178	.0122	.9674
3	0.1	0.3	0.3	0.3	.9091	.0751	.0062	.9904	0.1	0.1	.0188	.0145	.9423
4	0.1	0.1	0.1	0.1	.9756	.0216	.0004	.9977	0.1	0.1	.0108	.0055	.9920
4	0.1	0.2	0.2	0.2	.9524	.0412	.0018	.9954	0.1	0.1	.0137	.0094	.9755
4	0.1	0.3	0.3	0.3	.9302	.0590	.0037	.9930	0.1	0.1	.0147	.0113	.9563
5	0.1	0.1	0.1	0.1	.9804	.0175	.0003	.9982	0.1	0.1	.0087	.0044	.9936
5	0.1	0.2	0.2	0.2	.9615	.0336	.0012	.9963	0.1	0.1	.0112	.0076	.9803
5	0.1	0.3	0.3	0.3	.9434	.0485	.0025	.9944	0.1	0.1	.0121	.0093	.9648
6	0.1	0.1	0.1	0.1	.9836	.0147	.0002	.9985	0.1	0.1	.0073	.0037	.9947
6	0.1	0.2	0.2	0.2	.9677	.0284	.0008	.9970	0.1	0.1	.0095	.0064	.9836
6	0.1	0.3	0.3	0.3	.9524	.0412	.0018	.9954	0.1	0.1	.0103	.0078	.9705
7	0.1	0.1	0.1	0.1	.9859	.0126	.0001	.9987	0.1	0.1	.0063	.0032	.9954
7	0.1	0.2	0.2	0.2	.9722	.0246	.0006	.9974	0.1	0.1	.0082	.0055	.9859
7	0.1	0.3	0.3	0.3	.9589	.0358	.0013	.9961	0.1	0.1	.0090	.0068	.9747
8	0.1	0.1	0.1	0.1	.9877	.0111	.0001	.9989	0.1	0.1	.0055	.0028	.9960
8	0.1	0.2	0.2	0.2	.9756	.0216	.0004	.9977	0.1	0.1	.0072	.0049	.9877
8	0.1	0.3	0.3	0.3	.9639	.0317	.0010	.9966	0.1	0.1	.0079	.0060	.9778
9	0.1	0.1	0.1	0.1	.9890	.0099	.00009	.9990	0.1	0.1	.0049	.0025	.9964
9	0.1	0.2	0.2	0.2	.9783	.0193	.0003	.9980	0.1	0.1	.0064	.0043	.9890
9	0.1	0.3	0.3	0.3	.9677	.0284	.0008	.9970	0.1	0.1	.0071	.0054	.9802

Table 6.2 Marginal and system reliability of cold and warm standby system for identical stress in case of gamma distribution

α	θ_1	θ_2	θ_3	λ	μ	$R(1)$	**Cold standby**			**Warm standby**				
							$R(2)$	$R(3)$	R_3	β_2	β_3	$R(2)$	$R(3)$	R_3
1	4	4	4	3	3	.9375	.0293	.0009	.9677	1	1	.0275	.0025	.9675
1	5	5	5	4	4	.9688	.0151	.0002	.9841	1	1	.0147	.0006	.9841
1	6	6	6	5	5	.9844	.0077	.00006	.9921	1	1	.0076	.0001	.9921
2	4	4	4	3	3	.8125	.0762	.0071	.8958	1	1	.0714	.0107	.8947
2	5	5	5	4	4	.8906	.0487	.0027	.9420	1	1	.0472	.0040	.9418
2	6	6	6	5	5	.9375	.0293	.0009	.9677	1	1	.0288	.0013	.9677
3	4	4	4	3	3	.6563	.1128	.0194	.7884	1	1	.1057	.0236	.7856
3	5	5	5	4	4	.7734	.0876	.0099	.8710	1	1	.0849	.0120	.8703
3	6	6	6	5	5	.8555	.0618	.0045	.9218	1	1	.0609	.0053	.9216
4	4	4	4	3	3	.5000	.1250	.0313	.6563	1	1	.1172	.0348	.6520
4	5	5	5	4	4	.6367	.1157	.0210	.7734	1	1	.1120	.0232	.7720
4	6	6	6	5	5	.7461	.0947	.0120	.8528	1	1	.0932	.0131	.8524
5	4	4	4	3	3	.3633	.1157	.0368	.5158	1	1	.1084	.0391	.5108
5	5	5	5	4	4	.5000	.1250	.0313	.6563	1	1	.1211	.0331	.6542
5	6	6	6	5	5	.6230	.1174	.0221	.7626	1	1	.1156	.0233	.7619
6	4	4	4	3	3	.2539	.0947	.0353	.3840	1	1	.0888	.0366	.3793
6	5	5	5	4	4	.3770	.1174	.0366	.5310	1	1	.1138	.0379	.5286
6	6	6	6	5	5	.5000	.1250	.0313	.6563	1	1	.1230	.0322	.6553
7	4	4	4	3	3	.1719	.0712	.0295	.2725	1	1	.0667	.0301	.2687
7	5	5	5	4	4	.2744	.0996	.0361	.4101	1	1	.0964	.0369	.4078
7	6	6	6	5	5	.3872	.1186	.0364	.5422	1	1	.1168	.0370	.5410

Table 6.3 Marginal and system reliability of cold and warm standby system for identical stress in case of normal distribution: Here $\mu = 0, \theta_1 = \theta_2 = \theta_3 = \lambda = 2$

α	τ	σ_1	σ_2	σ_3	υ	ρ	$R(1)$	Cold standby			Warm standby						
								$R(2)$	$R(3)$	R_3	β_2	β_3	γ_2	γ_3	$R(2)$	$R(3)$	R_3
0	2	2	2	2	2	2	.7602	.1386	.0801	.9789	0	0	2	2	.1053	.0372	.9028
0	4	2	2	2	2	4	.6726	.1481	.0670	.8878	0	0	4	4	.0996	.0474	.8196
0	6	2	2	2	2	6	.6241	.1464	.0570	.8275	0	0	6	6	.0914	.0477	.7632
0	2	4	4	4	4	2	.6726	.1481	.0670	.8878	0	0	2	2	.0996	.0474	.8196
0	4	4	4	4	4	4	.6382	.1474	.0600	.8455	0	0	4	4	.0940	.0479	.7801
0	6	4	4	4	4	6	.6092	.1450	.0538	.8081	0	0	6	6	.0884	.0473	.7450
1	2	2	2	2	2	2	.6382	.1756	.0852	.8989	1	1	2	2	.1120	.0602	.8104
1	4	2	2	2	2	4	.5885	.1629	.0645	.8158	1	1	4	4	.0959	.0551	.7394
1	6	2	2	2	2	6	.5628	.1536	.0539	.7703	1	1	6	6	.0864	.0511	.7003
1	2	4	4	4	4	2	.5885	.1629	.0645	.8158	1	1	2	2	.0959	.0551	.7394
1	4	4	4	4	4	4	.5702	.1564	.0569	.7835	1	1	4	4	.0892	.0523	.7116
1	6	4	4	4	4	6	.5551	.1505	.0509	.7565	1	1	6	6	.0835	.0497	.6884
2	2	2	2	2	2	2	.5000	.1901	.0722	.7623	2	2	2	2	.0950	.0656	.6606
2	4	2	2	2	2	4	.5000	.1682	.0566	.7247	2	2	4	4	.0841	.0562	.6403
2	6	2	2	2	2	6	.5000	.1560	.0487	.7047	2	2	6	6	.0780	.0512	.6292
2	2	4	4	4	4	2	.5000	.1682	.0566	.7247	2	2	2	2	.0841	.0562	.6403
2	4	4	4	4	4	4	.5000	.1595	.0509	.7104	2	2	4	4	.0798	.0526	.6324
2	6	4	4	4	4	6	.5000	.1523	.0464	.6987	2	2	6	6	.0762	.0497	.6258
3	2	2	2	2	2	2	.3618	.1756	.0483	.5857	3	3	2	2	.0635	.0517	.4770
3	4	2	2	2	2	4	.4115	.1629	.0451	.6195	3	3	4	4	.0670	.0504	.5289
3	6	2	2	2	2	6	.4372	.1536	.0419	.6326	3	3	6	6	.0671	.0481	.5524
3	2	4	4	4	4	2	.4115	.1629	.0451	.6195	3	3	2	2	.0670	.0504	.5289
3	4	4	4	4	4	4	.4298	.1493	.0391	.6183	3	3	4	4	.0672	.0488	.5459
3	6	4	4	4	4	6	.4449	.1505	.0408	.6361	3	3	6	6	.0669	.0472	.5590

Table 7.1 Marginal reliability $R(1)$, $R(2)$, $R(3)$ and system reliability R_3 when strength is one-parameter exponential and stress is two-parameter exponential distribution

λ_1	λ_2	λ_3	μ_1	μ_2	μ_3	θ_1	θ_2	θ_3	$R(1)$	$R(2)$	$R(3)$	R_3
1	1	1	.1	.1	.1	.2	.2	.2	.7540	.1855	.0456	.9851
1	1	1	.2	.2	.2	.3	.3	.3	.6298	.2332	.0863	.9493
1	1	1	.3	.3	.3	.4	.4	.4	.5292	.2491	.1173	.8956
2	2	2	.1	.1	.1	.2	.2	.2	.5848	.2428	.1008	.9284
2	2	2	.2	.2	.2	.3	.3	.3	.4190	.2434	.1414	.8038
2	2	2	.3	.3	.3	.4	.4	.4	.3049	.2119	.1473	.6641
3	3	3	.1	.1	.1	.2	.2	.2	.4630	.2486	.1335	.8452
3	3	3	.2	.2	.2	.3	.3	.3	.2888	.2054	.1461	.6403
3	3	3	.3	.3	.3	.4	.4	.4	.1848	.1507	.1228	.4583
4	4	4	.1	.1	.1	.2	.2	.2	.3724	.2337	.1467	.7528
4	4	4	.2	.2	.2	.3	.3	.3	.2042	.1625	.1293	.4961
4	4	4	.3	.3	.3	.4	.4	.4	.1158	.1024	.0906	.3088
5	5	5	.1	.1	.1	.2	.2	.2	.3033	.2113	.1472	.6618
5	5	5	.2	.2	.2	.3	.3	.3	.1472	.1255	.1070	.3797
5	5	5	.3	.3	.3	.4	.4	.4	.0744	.0688	.0637	.2069
6	6	6	.1	.1	.1	.2	.2	.2	.2495	.1872	.1405	.5772
6	6	6	.2	.2	.2	.3	.3	.3	.1076	.0960	.0857	.2892
6	6	6	.3	.3	.3	.4	.4	.4	.0486	.0463	.0440	.1389
7	7	7	.1	.1	.1	.2	.2	.2	.2069	.1641	.1301	.5012
7	7	7	.2	.2	.2	.3	.3	.3	.0795	.0732	.0674	.2202
7	7	7	.3	.3	.3	.4	.4	.4	.0322	.0312	.0302	.0936
1	1	1	.1	.1	.1	.1	.1	.1	.8226	.1459	.0259	.9944
2	2	2	.1	.1	.1	.1	.1	.1	.6823	.2168	.0689	.9679
3	3	3	.1	.1	.1	.1	.1	.1	.5699	.2451	.1054	.9204
4	4	4	.1	.1	.1	.1	.1	.1	.4788	.2496	.1301	.8584
.1	.1	.1	.2	.2	.2	1	1	1	.8911	.0970	.0106	.9987
.1	.1	.1	.2	.2	.2	2	2	2	.8168	.1496	.0274	.9939
.1	.1	.1	.2	.2	.2	3	3	3	.7540	.1855	.0456	.9851
.1	.1	.1	.2	.2	.2	4	4	4	.7001	.2099	.0630	.9730
.1	.1	.1	1	1	1	.1	.1	.1	.8959	.0933	.0097	.9989
.1	.1	.1	2	2	2	.1	.1	.1	.8106	.1535	.0291	.9932
.1	.1	.1	3	3	3	.1	.1	.1	.7335	.1995	.0521	.9811
.1	.1	.1	4	4	4	.1	.1	.1	.6637	.2232	.0751	.9620

Table 7.2 Marginal reliability $R(1)$, $R(2)$, $R(3)$ and system reliability R_3 when strength is one-parameter exponential and stress is two-parameter gamma distribution

θ_1	θ_2	θ_3	λ_1	λ_2	λ_3	μ_1	μ_2	μ_3	$R(1)$	$R(2)$	$R(3)$	R_3
1	1	1	.1	.1	.1	2	2	2	.8264	.1434	.0249	.9948
1	1	1	.2	.2	.2	3	3	3	.5787	.2438	.1027	.9252
1	1	1	.3	.3	.3	4	4	4	.3501	.2275	.1479	.7255
1	1	1	.4	.4	.4	5	5	5	.1859	.1514	.1232	.4605
2	2	2	.1	.1	.1	2	2	2	.6944	.2122	.0648	.9715
2	2	2	.2	.2	.2	3	3	3	.3644	.2316	.1472	.7433
2	2	2	.3	.3	.3	4	4	4	.1526	.1293	.1096	.3915
2	2	2	.4	.4	.4	5	5	5	.0529	.0501	.0475	.1505
3	3	3	.1	.1	.1	2	2	2	.5917	.2416	.0986	.9319
3	3	3	.2	.2	.2	3	3	3	.2441	.1845	.1395	.5682
3	3	3	.3	.3	.3	4	4	4	.0767	.0708	.0654	.2130
3	3	3	.4	.4	.4	5	5	5	.0194	.0190	.0187	.0571
4	4	4	.1	.1	.1	2	2	2	.5102	.2499	.1224	.8825
4	4	4	.2	.2	.2	3	3	3	.1715	.1421	.1177	.4312
4	4	4	.3	.3	.3	4	4	4	.0427	.0409	.0391	.1227
4	4	4	.4	.4	.4	5	5	5	.0084	.0083	.0083	.0250
5	5	5	.1	.1	.1	2	2	2	.4444	.2469	.1372	.8285
5	5	5	.2	.2	.2	3	3	3	.1250	.1094	.0957	.3301
5	5	5	.3	.3	.3	4	4	4	.0256	.0249	.0243	.0749
5	5	5	.4	.4	.4	5	5	5	.0041	.0041	.0041	.0123
6	6	6	.1	.1	.1	2	2	2	.3906	.2380	.1451	.7737
6	6	6	.2	.2	.2	3	3	3	.0939	.0851	.0771	.2561
6	6	6	.3	.3	.3	4	4	4	.0163	.0160	.0157	.0480
6	6	6	.4	.4	.4	5	5	5	.0022	.0022	.0022	.0066
7	7	7	.1	.1	.1	2	2	2	.3460	.2263	.1480	.7203
7	7	7	.2	.2	.2	3	3	3	.0723	.0671	.0623	.2017
7	7	7	.3	.3	.3	4	4	4	.0108	.0107	.0106	.0321
7	7	7	.4	.4	.4	5	5	5	.0013	.0013	.0013	.0013
8	8	8	.1	.1	.1	2	2	2	.3086	.2134	.1475	.6695
8	8	8	.2	.2	.2	3	3	3	.0569	.0537	.0506	.1612
8	8	8	.3	.3	.3	4	4	4	.0075	.0074	.0074	.0223
8	8	8	.4	.4	.4	5	5	5	.0008	.0008	.0007	.0023

Table 7.3 Marginal reliability $R(1)$, $R(2)$, $R(3)$ and system reliability R_3 when strength is Lindley and stress is one-parameter gamma distribution

θ_1	θ_2	θ_3	m_1	m_2	m_3	$R(1)$	$R(2)$	$R(3)$	R_3
.1	.2	.3	1	1	1	.9842	.0150	.0007	.9999
.1	.2	.3	2	2	2	.9630	.0188	.0010	.9829
.1	.2	.3	3	3	3	.9376	.0209	.0012	.9597
.1	.2	.3	4	4	4	.9088	.0216	.0012	.9316
.1	.2	.3	5	5	5	.8775	.0213	.0011	.8999
.2	.3	.4	1	1	1	.9491	.0461	.0041	.9993
.2	.3	.4	2	2	2	.8873	.0495	.0046	.9415
.2	.3	.4	3	3	3	.8198	.0471	.0041	.8711
.2	.3	.4	4	4	4	.7502	.0417	.0033	.7951
.2	.3	.4	5	5	5	.6810	.0352	.0023	.7185
.3	.4	.5	1	1	1	.9058	.0810	.0107	.9976
.3	.4	.5	2	2	2	.8018	.0747	.0095	.8860
.3	.4	.5	3	3	3	.6976	.0612	.0067	.7655
.3	.4	.5	4	4	4	.5987	.0467	.0042	.6497
.3	.4	.5	5	5	5	.5084	.0339	.0024	.5448
.4	.5	.6	1	1	1	.8601	.1140	.0200	.9941
.4	.5	.6	2	2	2	.7185	.0909	.0140	.8234
.4	.5	.6	3	3	3	.5876	.0646	.0081	.6603
.4	.5	.6	4	4	4	.4728	.0428	.0041	.5197
.4	.5	.6	5	5	5	.3757	.0270	.0019	.4046
.5	.6	.7	1	1	1	.8148	.1429	.0309	.9886
.5	.6	.7	2	2	2	.6420	.0992	.0176	.7587
.5	.6	.7	3	3	3	.4938	.0616	.0082	.5637
.5	.6	.7	4	4	4	.3731	.0357	.0034	.4122
.5	.6	.7	5	5	5	.2780	.0197	.0013	.2990
.6	.7	.8	1	1	1	.7715	.1670	.0426	.9811
.6	.7	.8	2	2	2	.5737	.1016	.0198	.6951
.6	.7	.8	3	3	3	.4158	.0555	.0076	.4790
.6	.7	.8	4	4	4	.2956	.0283	.0026	.3266
.6	.7	.8	5	5	5	.2071	.0138	.0008	.2217
.7	.8	.9	1	1	1	.7307	.1865	.0544	.9717
.7	.8	.9	2	2	2	.5136	.1001	.0209	.6346
.7	.8	.9	3	3	3	.3514	.0484	.0067	.4066
.7	.8	.9	4	4	4	.2357	.0219	.0019	.2596
.7	.8	.9	5	5	5	.1555	.0095	.0005	.1657

Table 7.4 Marginal reliability $R(1)$, $R(2)$, $R(3)$ and system reliability R_3 when strength is Lindley and stress is two-parameter gamma distribution

λ_1	λ_2	λ_3	μ_1	μ_2	μ_3	θ_1	θ_2	θ_3	$R(1)$	$R(2)$	$R(3)$	R_3
1	1	1	2	2	2	.1	.1	.1	.9630	.0204	.0004	.9839
1	1	1	2	2	2	.2	.2	.2	.8873	.0548	.0034	.9455
1	1	1	2	2	2	.3	.3	.3	.8018	.0834	.0087	.8938
1	1	1	2	2	2	.4	.4	.4	.7185	.1017	.0144	.8346
1	1	1	2	2	2	.5	.5	.5	.6420	.1110	.0192	.7721
1	1	1	2	2	2	.6	.6	.6	.5737	.1135	.0224	.7096
1	1	1	2	2	2	.7	.7	.7	.5136	.1115	.0242	.6493
1	1	1	2	2	2	.8	.8	.8	.4611	.1068	.0247	.5926
1	1	1	2	2	2	.9	.9	.9	.4151	.1006	.0244	.5401
2	2	2	3	3	3	.1	.1	.1	.8418	.0266	.0008	.8692
2	2	2	3	3	3	.2	.2	.2	.6247	.0401	.0026	.6674
2	2	2	3	3	3	.3	.3	.3	.4554	.0366	.0029	.4949
2	2	2	3	3	3	.4	.4	.4	.3348	.0284	.0024	.3656
2	2	2	3	3	3	.5	.5	.5	.2500	.0208	.0017	.2726
2	2	2	3	3	3	.6	.6	.6	.1900	.0150	.0012	.2061
2	2	2	3	3	3	.7	.7	.7	.1468	.0107	.0007	.1583
2	2	2	3	3	3	.8	.8	.8	.1153	.0077	.0005	.1235
2	2	2	3	3	3	.9	.9	.9	.0918	.0056	.0003	.0977
3	3	3	4	4	4	.1	.1	.1	.6439	.0210	.0006	.6656
3	3	3	4	4	4	.2	.2	.2	.3433	.0148	.0006	.3588
3	3	3	4	4	4	.3	.3	.3	.1886	.0073	.0002	.1962
3	3	3	4	4	4	.4	.4	.4	.1092	.0034	.0001	.1128
3	3	3	4	4	4	.5	.5	.5	.0666	.0016	.0004	.0682
3	3	3	4	4	4	.6	.6	.6	.0424	.0008	.00001	.0433
3	3	3	4	4	4	.7	.7	.7	.0281	.0004	.000006	.0285
3	3	3	4	4	4	.8	.8	.8	.0192	.0002	.000002	.0195
3	3	3	4	4	4	.9	.9	.9	.0135	.0001	.000001	.0137

Table 7.5 Marginal reliability $R(1)$, $R(2)$, $R(3)$ and system reliability R_3 for one-parameter exponential(λ) strength and two-parameter gamma(μ, θ) stress, where $\lambda_1 = 1$, $\theta = 1$

λ_2	λ_3	k	μ	$R(1)$	$R(2)$	$R(3)$	R_3
1	.5	.1	1	.5000	.4329	.0664	.9993
		.3		.5000	.3344	.1509	.9853
		.5		.5000	.2667	.1839	.9505
		.7		.5000	.2179	.1832	.9011
		.9		.5000	.1815	.1647	.8462
		.1	2	.2500	.5997	.1480	.9977
		.3		.2500	.4027	.3057	.9584
		.5		.2500	.2844	.3351	.8695
		.7		.2500	.2088	.2977	.7565
		.9		.2500	.1581	.2370	.6451
5	.5	.1	1	.5000	.2667	.2310	.9977
		.3		.5000	.1143	.3571	.9714
		.5		.5000	.0635	.3587	.9221
		.7		.5000	.0404	.3211	.8615
		.9		.5000	.0280	.2714	.7994
		.1	2	.2500	.2844	.4592	.9936
		.3		.2500	.0784	.6018	.9302
		.5		.2500	.0322	.5393	.8216
		.7		.2500	.0163	.4326	.6989
		.9		.2500	.0094	.3260	.5854
7	.5	.1	1	.5000	.2179	.2794	.9973
		.3		.5000	.0787	.3912	.9699
		.5		.5000	.0404	.3799	.9203
		.7		.5000	.0246	.3350	.8596
		.9		.5000	.0165	.2810	.7975
		.1	2	.2500	.2088	.5340	.9928
		.3		.2500	.0446	.6337	.9283
		.5		.2500	.0163	.5535	.8199
		.7		.2500	.0077	.4398	.6976
		.9		.2500	.0042	.3300	.5843
9	.5	.1	1	.5000	.1815	.3156	.9971
		.3		.5000	.0575	.4117	.9692
		.5		.5000	.0280	.3915	.9194
		.7		.5000	.0165	.3423	.8588
		.9		.5000	.0109	.2859	.7968
		.1	2	.2500	.1581	.5843	.9924
		.3		.2500	.0278	.6497	.9275
		.5		.2500	.0094	.5598	.8192
		.7		.2500	.0042	.4429	.6971

		.9		.2500	.0023	.3317	.5840
1	1.5	.1	1	.5000	.4329	.0649	.9978
		.3		.5000	.3344	.1265	.9609
		.5		.5000	.2667	.1207	.8874
		.7		.5000	.2179	.0912	.8090
		.9		.5000	.1815	.0624	.7439
		.1	2	.2500	.5997	.1436	.9933
		.3		.2500	.4027	.2399	.8926
		.5		.2500	.2844	.1882	.7226
		.7		.2500	.2088	.1146	.5735
		.9		.2500	.1518	.0631	.4712
5	1.5	.1	1	.5000	.2667	.2265	.9932
		.3		.5000	.1143	.3083	.9226
		.5		.5000	.0635	.2533	.8168
		.7		.5000	.0404	.1801	.7205
		.9		.5000	.0280	.1211	.6491
		.1	2	.2500	.2844	.4468	.9812
		.3		.2500	.0784	.4885	.8169
		.5		.2500	.0322	.3271	.6094
		.7		.2500	.0163	.1877	.4541
		.9		.2500	.0094	.1017	.3611
7	1.5	.1	1	.5000	.2179	.2742	.9920
		.3		.5000	.0787	.3397	.9184
		.5		.5000	.2667	.1207	.8874
		.7		.5000	.0246	.1910	.7156
		.9		.5000	.0165	.1281	.6446
		.1	2	.2500	.2088	.5200	.9789
		.3		.2500	.0446	.5171	.8116
		.5		.2500	.0163	.3385	.6049
		.7		.2500	.0077	.1930	.4507
		.9		.2500	.0042	.1043	.3586
9	1.5	.1	1	.5000	.1815	.3098	.9913
		.3		.5000	.0575	.3587	.9162
		.5		.5000	.0280	.2815	.8094
		.7		.5000	.0165	.1970	.7135
		.9		.5000	.0109	.1319	.6427
		.1	2	.2500	.1581	.5694	.9775
		.3		.2500	.0278	.5317	.8094
		.5		.2500	.0094	.3438	.6032
		.7		.2500	.0042	.1953	.4495
		.9		.2500	.0023	.1055	.3578

Table 7.6 Marginal reliability $R(1)$, $R(2)$, $R(3)$ and system reliability R_3 for Special Case: where $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$ (identical), $\theta = 1$

λ	k	μ	$R(1)$	$R(2)$	$R(3)$	R_3
1	.1	1	.5000	.4329	.0656	.9985
	.2		.5000	.3788	.1113	.9901
	.3		.5000	.3344	.1379	.9724
	.4		.5000	.2976	.1487	.9463
	.5		.5000	.2667	.1478	.9144
2	.1	2	.1111	.5968	.2761	.9840
	.2		.1111	.4237	.3780	.9128
	.3		.1111	.3135	.3737	.7982
	.4		.1111	.2394	.3196	.6701
	.5		.1111	.1875	.2522	.5508
3	.1	3	.0156	.4426	.4871	.9453
	.2		.0156	.2339	.5105	.7600
	.3		.0156	.1373	.3847	.5376
	.4		.0156	.0868	.2509	.3533
	.5		.0156	.0580	.1522	.2258
4	.1	4	.0016	.2591	.6218	.8826
	.2		.0016	.0944	.4839	.5799
	.3		.0016	.0420	.2684	.3120
	.4		.0016	.0214	.1285	.1515
	.5		.0016	.0119	.0581	.0716
5	.1	5	.0001	.1316	.6717	.8034
	.2		.0001	.0312	.3824	.4137
	.3		.0001	.0102	.1515	.1618
	.4		.0001	.0041	.0516	.0558
	.5		.0001	.0019	.0169	.0189
6	.1	6	.000008	.0596	.6572	.7168
	.2		.000008	.0088	.2703	.2792
	.3		.000008	.0021	.0742	.0763
	.4		.000008	.0006	.0175	.0181
	.5		.000008	.0002	.0041	.0043

Table 7.7 Marginal reliability $R(1)$, $R(2)$, $R(3)$ and system reliability R_3 when strength follows one-parameter exponential and stress follows Lindley distribution

θ	k	λ_1	λ_2	λ_3	$R(1)$	$R(2)$	$R(3)$	R_3
1	.2	2	2	2	.2222	.4219	.2954	.9396
	.4	3	3	3	.1563	.2159	.2769	.6491
	.6	4	4	4	.1200	.1136	.1375	.3711
2	.2	2	2	2	.4167	.4151	.1519	.9837
	.4	3	3	3	.3200	.2971	.2354	.8526
	.6	4	4	4	.2593	.1943	.1788	.6324
3	.2	2	2	2	.5400	.3626	.0908	.9933
	.4	3	3	3	.4375	.3074	.1792	.9240
	.6	4	4	4	.3673	.2290	.1724	.7687
4	.2	2	2	2	.6222	.3144	.0600	.9966
	.4	3	3	3	.5224	.2959	.1373	.9556
	.6	4	4	4	.4500	.2408	.1543	.8452
5	.2	2	2	2	.6803	.2753	.0425	.9981
	.4	3	3	3	.5859	.2783	.1076	.9718
	.6	4	4	4	.5144	.2416	.1352	.8912
6	.2	2	2	2	.7232	.2440	.0316	.9988
	.4	3	3	3	.6349	.2599	.0862	.9810
	.6	4	4	4	.5657	.2369	.1180	.9206
7	.2	2	2	2	.7562	.2186	.0244	.9992
	.4	3	3	3	.6737	.2423	.0705	.9866
	.6	4	4	4	.6074	.2297	.1031	.9403
8	.2	2	2	2	.7822	.1978	.0194	.9994
	.4	3	3	3	.7052	.2263	.0586	.9901
	.6	4	4	4	.6420	.2214	.0906	.9539
9	.2	2	2	2	.8033	.1805	.0158	.9996
	.4	3	3	3	.7312	.2118	.0495	.9926
	.6	4	4	4	.6710	.2127	.0800	.9637

Table 8.1 Marginal reliability $R(1)$, $R(2)$, $R(3)$, $R(4)$ and system reliability R_4 for a 4-cascade system in case of exponential distribution

k	μ	ρ_1	ρ_2	ρ_3	ρ_4	ψ_2	ψ_3	ψ_4	$R(1)$	$R(2)$	$R(3)$	$R(4)$	R_4
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	.9091	.0737	.0074	.0002	.9904
0.3	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	.9091	.0710	.0094	.0004	.9895
0.5	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	.9091	.0684	.0111	.0003	.9890
1.5	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	.9091	.0575	.0126	.0051	.9843
0.1	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	.9091	.0676	.0062	.0001	.9830
0.3	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	.9091	.0651	.0079	.0003	.9821
0.5	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	.9091	.0627	.0093	.0003	.9814
1.5	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	.9091	.0527	.0106	.0039	.9763
0.1	0.1	0.2	0.2	0.2	0.2	0.1	0.1	0.1	.8333	.1328	.0154	.0006	.9822
0.3	0.1	0.2	0.2	0.2	0.2	0.1	0.1	0.1	.8333	.1238	.0220	.0011	.9802
0.5	0.1	0.2	0.2	0.2	0.2	0.1	0.1	0.1	.8333	.1156	.0265	.0031	.9785
1.5	0.1	0.2	0.2	0.2	0.2	0.1	0.1	0.1	.8333	.0848	.0228	.0089	.9497
0.1	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.2	.9091	.0619	.0096	.0008	.9815
0.3	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.2	.9091	.0597	.0110	.0005	.9803
0.5	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.2	.9091	.0575	.0120	.0008	.9795
0.1	0.1	0.2	0.2	0.2	0.2	0.3	0.3	0.3	.8333	.1124	.0273	.0046	.9775
1.5	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.2	.9091	.0483	.0119	.0053	.9746
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.3	0.1	.9091	.0737	.0062	.0012	.9902
0.1	0.1	0.1	0.1	0.1	0.1	0.3	0.1	0.1	.9091	.0624	.0167	.0003	.9885
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.3	.9091	.0737	.0074	.0001	.9903
0.1	0.1	0.2	0.2	0.2	0.5	0.2	0.5	0.1	.8333	.1218	.0180	.0066	.9796
0.1	0.2	0.1	0.1	0.1	0.3	0.1	0.2	0.5	.9091	.0676	.0057	.0004	.9828
0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	.7692	.1172	.0338	.0052	.9255
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	.6667	.1016	.0427	.0129	.8239
0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	.5882	.0742	.0339	.0137	.7101
0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	.5263	.0508	.0210	.0086	.6067
1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	.4545	.0279	.0082	.0025	.4932
1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	.4000	.0155	.0030	.0006	.4192

Table 8.2 Marginal reliability $R(1)$, $R(2)$, $R(3)$, $R(4)$ and system reliability R_4 for a 4-cascade system when the active component and the switch follow exponential distribution and the standby component follow gamma distribution

k	μ	ρ_1	ρ_2	ρ_3	ρ_4	θ_2	θ_3	θ_4	β_2	β_3	β_4	$R(1)$	$R(2)$	$R(3)$	$R(4)$	R_4
0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.4	1.0	1.0	1.0	.9091	.0676	.0105	.0021	.9872
0.5	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.4	1.0	1.0	1.0	.9091	.0627	.0132	.0031	.9850
1.0	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.4	1.0	1.0	1.0	.9091	.0574	.0142	.0044	.9807
1.5	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.4	1.0	1.0	1.0	.9091	.0527	.0130	.0042	.9748
0.1	0.2	0.1	0.1	0.1	0.1	0.2	0.3	0.4	1.0	1.0	1.0	.9091	.0619	.0088	.0016	.9799
0.5	0.2	0.1	0.1	0.1	0.1	0.2	0.3	0.4	1.0	1.0	1.0	.9091	.0575	.0111	.0024	.9777
1.0	0.2	0.1	0.1	0.1	0.1	0.2	0.3	0.4	1.0	1.0	1.0	.9091	.0526	.0119	.0034	.9736
1.5	0.2	0.1	0.1	0.1	0.1	0.2	0.3	0.4	1.0	1.0	1.0	.9091	.0483	.0110	.0032	.9684
0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.3	0.4	0.5	0.5	0.5	.8333	.1334	.0144	.0016	.9811
0.5	0.1	0.2	0.2	0.2	0.2	0.2	0.3	0.4	0.5	0.5	0.5	.8333	.1161	.0252	.0050	.9746
1.0	0.1	0.2	0.2	0.2	0.2	0.2	0.3	0.4	0.5	0.5	0.5	.8333	.0988	.0272	.0094	.9593
1.5	0.1	0.2	0.2	0.2	0.2	0.2	0.3	0.4	0.5	0.5	0.5	.8333	.0851	.0218	.0069	.9403
0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.4	0.5	0.5	0.5	.8333	.1223	.0121	.0012	.9977
0.5	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.4	0.5	0.5	0.5	.8333	.1064	.0212	.0039	.9609
1.0	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.4	0.5	0.5	0.5	.8333	.0906	.0229	.0073	.9468
1.5	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.4	0.5	0.5	0.5	.8333	.0780	.0183	.0053	.9297
0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.1	0.5	0.5	.8333	.1435	.0066	.0005	.9834
0.1	0.1	0.1	0.1	0.1	0.1	0.5	0.5	0.4	1.0	1.0	1.0	.9091	.0541	.0173	.0056	.9804
0.1	0.1	0.5	0.1	0.1	0.1	0.2	0.3	0.4	1.0	1.0	1.0	.6667	.0676	.0382	.0075	.7724
0.1	0.2	0.2	0.2	0.2	0.2	1.0	1.0	1.0	0.5	0.5	0.5	.8333	.0947	.0259	.0064	.9540
0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.3	0.4	1.0	1.0	1.0	.8333	.1218	.0207	.0041	.9758
0.5	0.1	0.2	0.2	0.2	0.2	0.2	0.3	0.4	1.0	1.0	1.0	.8333	.1060	.0286	.0078	.9679
0.1	0.1	0.2	0.2	0.2	0.2	0.3	0.3	0.3	1.0	1.0	1.0	.8333	.1124	.0273	.0058	.9730
0.1	0.1	0.2	0.2	0.2	0.2	0.5	0.5	0.5	1.5	1.5	1.5	.8333	.0795	.0355	.0148	.9484
0.5	0.1	0.2	0.2	0.2	0.2	0.5	0.5	0.5	1.5	1.5	1.5	.8333	.0692	.0370	.0178	.9395

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