INVESTIGATION OF THE BASIC PARAMETERS OF THE ELECTRON AND THE POSITRONIUM MASS SPECTRA INVOKING VARIOUS MODELS

A thesis submitted in partial fulfilment of the requirements for award of the degree of Doctor of Philosophy

SOVAN GHOSH

Registration No: 063 of 2010

Department of Physics
School of Sciences
Tezpur University
Napaam, Tezpur – 784028
Assam, India

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Let me not pray to be sheltered from dangers but to be fearless when facing them

- Rabindranath Tagore

Dedicated to my parents

Swapna Ghosh (maa)

And

Manimohan Ghosh (baba)

Whose unending sacrifice
made me able to carry on
the research work
and
to write this thesis

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Abstract

The electron is the first sub-atomic particle discovered. It was discovered by J. J. Thomson in 1897. It is a charged lepton, which does not decay into more fundamental particles. In the Standard model of particle physics it is called as a point particle. Different experimental facts have described the properties of the electron. Mass, charge, spin, magnetic moment, electric dipole moment, gyromagnetic ratio and the size or radius of the electron are the important parameters, which are dealt to connect experimental facts with the theoretical aspects of the electron.

Depending on those properties several models of the electron are proposed theoretically. To depict a picture of the electron, the process of proposing electron models started by Lorentz immediately after the discovery of this particle. Lorentz-Abraham-Poincare model, Zitterbewegung model, Relativistic Spinning Sphere model and Dynamical Spinning Sphere model are some recent amongst all the models. These models are based on the different electromagnetic phenomenon and they talk about different sizes or radii of the electron. Models discuss basically either sub-structures of the electron or some sort of structures depending on properties and mathematical formulations about the electron.

The radii of the electron are revealed from different models to give the proper picture of the electron. They lie on a long-range scale. Though all these radii are originated from the different electromagnetic phenomenon, they have some common features, which co-relate them as well as those basic phenomena.

Recent work about the size of the electron, anomalous magnetic moment and the gyromagnetic ratio provide us a good platform to test the theoretical results and also to refine the approaches for the electron models. An effort is given here to co-relate all these models and the radii. The parameters and the models are studied with

an aim to propose a possible new model covering as much as of the previous problems regarding the models.

After the work for the models and the parameters of the electron, we have done some investigations of the mass spectra of the positronium has been done. It is a quasi-stable bound state of the electron and the positron. This study is actually a shifting from semi-classical works of the electron to the quantum mechanical domain. Fine structure constant connects them in a unique way. Also the positronium is the very next step after the study of electron-positron and in that regard also this study is important. In this thesis we have studied the mass spectra for the S-wave of the positronium. In question of structural matter, the positron resembles the electron. Hence the immediate next one is their bound state, which is studied here. Thus our approach makes a bridge amongst the electron, the positron and the positronium. Moreover it has been shown a step forward from classical to quantum mechanical era.

The **Chapter 1** is the introductory chapter about the electron. Here we have started with the discussion about the basic building blocks of the Nature. A little touch is given to the Standard Model of particle physics too. Then the discovery of the electron is discussed with the historical accounts about the early experiments and the theories. In this connection the experiment of J. J. Thomson has been described elaborately. Next we have studied about the properties or the parameters of the electron with their experimental values. Charge, mass, spin, magnetic moment, gyromagnetic ratio, size are discussed there.

In the **Chapter 2**, different models of the electron have been discussed according to the properties of the electron. They are mainly classified into the structural and the sub-structural models. The structural models are further classified into point-like, extended, and extended body with point-like charge models. Lorentz-Abraham-Poincare model, Compton model, Bunge model, Zitterbewegung model, Relativistic Spinning Sphere model, Dynamical Spinning Sphere model are noteworthy amongst all these models. Unified Composite model of the particles in the discussion of sub-quark particles. Some limitations about old-fashioned Classical model, Blinder model, Semi-classical Ring model, and Semi-classical Tachyonic model are discussed here.

The **Chapter 3** describes about the eight different radii of the electron. We have discussed their origin and the significance in the behaviour of the electron. Several relations amongst those radii are established. Following the trend of the relations of radii involving fine-structure constant, we offered the mathematical formalism of the charge radius with the order in agreement with the LEP result from CERN. In addition, the relations between Rydberg constant and the electron structure are also attempted. These relations not only unite the different radii of the electron, but plays significant role to co-relate different electromagnetic phenomena and the aspects to give better explanation for the various models of the electron.

In the **Chapter 4**, the properties of the electron are discussed in terms of the fine structure constant. This dimensionless factor is found to be accountable for the relations among various radii of the electron. A current-loop has been developed in connection with the rotation of the charge around the axis of rotation and that work is extended to calculate the corresponding self-magnetic field also. The current-loop and the magnetic fields are expressed in a form with the intrinsic properties of the electron. It is noteworthy that all the current-loop and the magnetic field expressions have been come out in α-quantized manner. The behaviour of the charge particle in the external magnetic field is also shown with the help of α-quantization. Incorporating the α-mass leap proposal of MacGregor for fermion, we calculated the radius of the muon and the tau for the particles with electromagnetic nature. In the calculation, the velocity of the charge in the α-quantized manner is also followed. But the consequent velocity for the classical electron radius exceeds the speed of light and to control that fact we propose the classical electron radius as a length-contracted form of the Compton radius of the electron.

In the **Chapter 5**, the electromagnetic mass of the electron is discussed along with its magnetic moment and the spinning sphere model of the electron. In this chapter, a co-relation between the charge and the mass is also established in the light of the electromagnetic mass of the electron. In addition to that, we expressed the energy of the electron here in terms of the charge and the mass together.

In the **Chapter 6**, we have tried to give a model of the electron incorporating the basic features of this charged lepton. We considered basic features of the Relativistic Spinning Sphere model of the electron. Then the motion of the charge is also regarded here and we have tried to formulate the path of the rotation of the charge. Starting with the magnetic field radius calculation, we have arrived to a new radius of the electron, which is composed of the classical and the Compton radius of the electron. Composite radius gives the hint of a helical path that can be considered for the rotation of the charge. The rotation of the charge produces the current and the magnetic field in result. We calculated the number of turns of the helical path also in the frame of a Compton-sized spherical model. There we incorporated the recent anomalous magnetic moment and the gyromagnetic ratio values to compare the model with the recent measurements. The radius of the path decreases towards the pole of the sphere and the charge returns in a similar way followed towards the equator. It is also shown that though the charge is moving, it is mostly found at the equator.

Using the approach of the exponential series of $\frac{\alpha}{2\pi}$, one can get the exact radius in different levels in the sphere to represent the rotation of charge, which can actually be verified in order with gyromagnetic ratio expression. Relativistic Spinning Sphere model is connected with the Dynamical Spinning Sphere model and the Zitterbwewegung model of the electron to provide the conditions of the new model. More explicitly this model can co-relate the basic models with the logic of changing the size. In fact, the conflict between the point and the extended models can be solved with this model.

In the **Chapter 7**, we have discussed about the positronium, the quasi-stable bound state of the electron and the positron. The discovery of the positronium is described here. Then we used the assumption of harmonic oscillator wave function and calculated the Hamiltonian for the same. The kinetic energy, one photon exchange potential and the confinement potential are calculated with the corresponding wave function. These provide us the way to reach the mass spectra of the S-wave of positronium in the framework of the non-relativistic models. Hence for each wave function the kinetic energy and the potentials are calculated with the help of computer programming. The diagonalised mass matrix has re-produced the spectra for the singlet and the triplet of the S-wave positronium.

In the **Chapter 8**, we have discussed about the conclusion came out from the works done in the previous chapters. Here we also have discussed about some of the future possible work regarding this subject.

DECLARATION

I hereby declare that the thesis entitled **"Investigation of the Basic Parameters of the Electron and the Positronium Mass Spectra Invoking Various Models"** being submitted to Tezpur University, Tezpur, Assam in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy, has previously not formed the basis for the award of any degree, diploma, associateship, fellowship or any other similar title or recognition.

Date: 29.06.2022

Place: Napaam, Tezpur

(Sovan Ghosh)
Department of Physics
Tezpur University
Tezpur – 784028
Assam, India

TEZPUR UNIVERSITY

CERTIFICATE

This is to certify that the thesis entitled **Investigation of the Basic Parameters of the Electron and the Positronium Mass Spectra Invoking Various Models,** submitted to the School of Sciences, Tezpur University in partial fulfilment for the award of the degree of Doctor of Philosophy in Physics is a record of research work carried out by Mr. Sovan Ghosh under my supervision and guidance.

All help received by him from various sources have been duly acknowledged.

No part of this thesis has been submitted elsewhere for award of any other degree.

Signature of Supervisor: (J. K. Sarma)

Designation: Professor

School: Sciences

Department: Physics

Place: Napaam, Tezpur. **Date:** 22.6.2012

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During my journey in the educational highway I realised the fact that though at the end of the day the degree or diploma goes to someone's name, it is not possible for one to complete the process without anyone's help. At least I can say that I am unable to face all those problems whether in research, or moral support or the financial help. So many faces and names are there and it would be very much injustice if I forget single one. Hence let me thank to all my friends, well-wishers, colleagues and relatives for your continuous support and encouragement through out not only this research period, but all the way long beginning with the day by my first teacher, my Maa (mother). Here in this small space of thesis I will be able to mention only few names and if I miss one please forgive me for my unintentional mistake.

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Date: 29.06.2012 *Sovan Ghosh*

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You know it would be sufficient to really understand the electron

- Albert Einstein

Chapter 1

Introduction

What are the matters made of? This is the most fundamental question arose in the mind of the people at the dawn of the rational thinking of the human civilization. The preliminary thinking was developed considering air, water, earth as the basis. But, the inquisitive nature of human race did not allow them to be satisfied with the scenario. Scientists continued to get the fundamental building blocks of Nature. In the process the electron was discovered and it was followed by other particles. In the twentieth century physics these particles claim the most important role. But the properties of the electron are yet to be well-explained with a definite picture. Here in this chapter we have discussed about the discovery of the electron and the properties. This chapter bears almost an introduction of the electron and its properties which will create a platform for the next study.

1.1 The journey begins

Anaximenes's model of the fundamental structure of matter [1] is the primary footstep to the search for basic building blocks of the universe. Later Mendeleev's periodic table introduced more than hundred chemical elements. But, the first clearly identified sub-atomic particle in the history of physics is the electron. Following that, the proton and the neutron were discovered. In process a huge number of particles were discovered by 1960 so that the beautiful garden of the particles soon became a jungle [2]. Physicists felt the necessity to classify all the particles according to their properties. In 1961 M. Gellman and Y. Ne'eman independently proposed the Eightfold Way of particles to put the baryons and mesons into weird geometrical patterns according to their charge and strangeness [2].

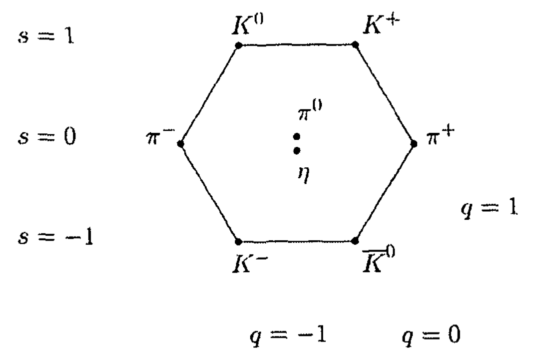

Figure 1.1: Meson octet in the eightfold way

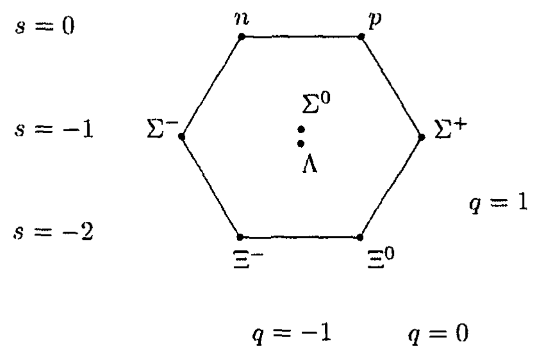

Figure 1.2: Baryon octet in the eightfold way

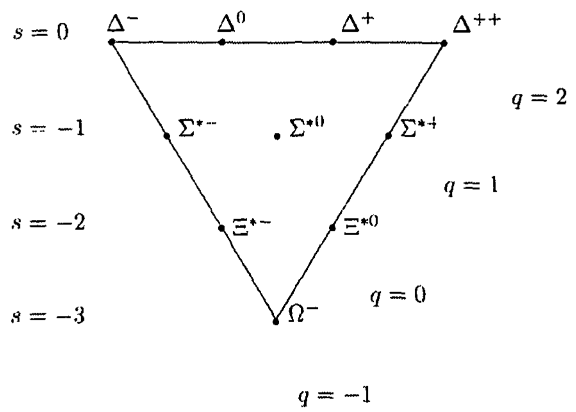

Figure 1.3: Baryon decuplet in the eightfold way

All the particles are classified due to their interacting nature and the interactions take place according to four fundamental forces. Hadrons are controlled by strong force and gluon plays the role of the mediator for them. They are discussed in Chromodynamics theory. Two kinds of hadrons are there. They are baryons and mesons. Mesons are bosons in nature and they are composed of a quark and an anti-quark. Baryons are classified into nucleons and hyperons. Baryons are composed of three quarks or three anti-quarks. The neutron and the proton are the two nucleons. A neutron is carrying an udd quark composition whereas a proton carries an uud quark composition. $\Lambda^0, \overline{\Lambda^0}, \Sigma^+, \Sigma^-, \Sigma^0, \Xi^0, \Omega^-$ are the hyperons. More intense investigation is going on to study the structure of hadrons using different QCD evolution equations [3-10].

Table 1.1: Four fundamental forces and their mediators

(This table is adapted from ref. [2])

Forces	**Strength**	**Theory**	**Mediator**
Strong	1	Chromodynamics	Gluon
Electromagnetic	10^{-2}	Electrodynamics	Photon
Weak	10^{-13}	Flavoudynamics	W and Z
Gravitational	10^{-42}	Geometrodynamics	Graviton

Electromagnetic force involves leptons. The corresponding mediator is the photon and the concerned theory is electrodynamics. Leptons indeed undergo the weak interaction too. The electron, the muon, the tau and their corresponding neutrinos are the members of the lepton family. They have the smaller masses in comparison to the hadron masses. Neutrinos were first described as the massless and chargeless particles. Recent observations and theories give the signature of the mass of the neutrino and about its mass mixing [11-13].

The dynamics of the known sub-atomic particles is given by the Standard Model (SM) of particle physics. In 1960, S. Glashow introduced the electroweak theory, which is a combination of electromagnetic and weak interactions. S. Weinberg and A. Salam incorporated the Higgs mechanism to Glashow's theory and proposed the standard model in the present version. According to the standard model, all matter is made out of three kinds of elementary particles; e.g. leptons, quarks and mediators. The mediators, leptons and quarks are described in the standard model. Quarks and leptons are observed not to decay into more fundamental particles. The meadiators are field particles. Structure of the standard model has a gauge group $SU(3)\times SU(2)\times U(1)$ that incorporates the strong force and unifies the electromagnetic and the weak interactions [14].

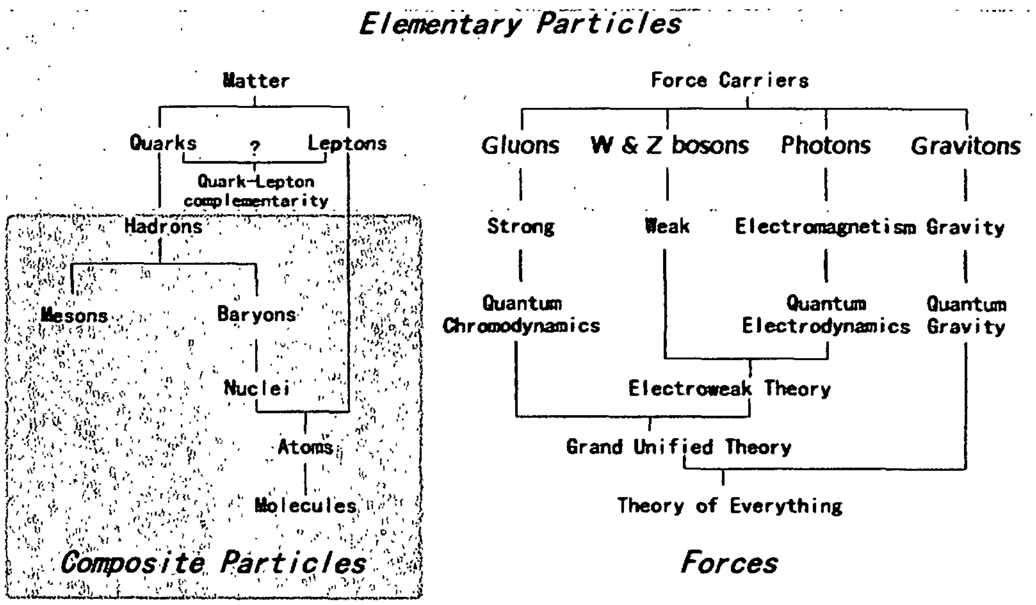

Figure 1.4: Matter and forces

The preon and other substructure models of the particles recommend for more fundamental states, which are the components of the existing lepton and quark families [15-18]. Indeed the modified version of the substructure model is described recently incorporating a new theory of sub-chromodynamics.

Photons are the most commonly known mediator and are emitted or absorbed during an electromagnetic interaction. Gluons are considered to be exchanged between colours of quarks, which incorporate the chromodynamics in the standard model. In weak interactions mediators are W and Z bosons. But the graviton is hypothetical and yet to be detected experimentally.

At this point, all the particles are well set in the standard model of particle physics with a huge success of the proposal. To explain the spontaneous symmetry breaking and the basis of mass of the universe, the Higgs boson was proposed by P. Higgs. This is yet to be discovered in the laboratories. In CERN the Large Hadron Collider (LHC) [19], the biggest machine of the human civilization is now in the search of this God's particle. Identification of Higgs boson would be the greatest triumph of the mankind to have an overview of the birth of the universe.

Table 1.2: Lepton classification

Generations	Lepton	Charge	Electron-lepton number	Muon-lepton number	Tau-lepton number
First	e	-1	1	0	0
	ν_e	0	1	0	0
Second	μ	-1	0	1	0
	ν_μ	0	0	1	0
Third	τ	-1	0	0	1
	ν_τ	0	0	0	1

1.2 Discovery of the electron

It is known in general that J. J. Thomson is the person responsible for the birth of the microphysics with his discovery of the electron. But it is notable that the discoveries in science are not the individual performances. Physicists attempted in between 1850-1900 to explain the nature of the charged bodies. That long list includes great and pioneering minds in the world of physics. In fact, at Cavendish laboratory people were trying and the others were also on their way with different approaches, according to the historical account by O. Lodge and W. Kauffman, which were discussed in "Histories of the Electron" book [20].

In 1856, W. E. Weber along with R. Kohlrausch recommended that the ratio of the electrostatic and the electromagnetic units produce a number that can be identified as the speed of light known at that time.

G. J. Stoney was first to use the term 'electron' to represent the fundamental unit of electric charge [21-22]. In 1874 and 1881, Stoney suggested the minimum quantity of electricity as one of the key physical units. He also mentioned that it may be the basis of a complete body of systematic units and called it as "electron" or "atom of electricity" [23]. The quantity of electricity traversing an electrolyte during electrolysis is described as "electron" by Stoney.

Table 1.3: Steps of the discovery of the electron

Evidence	Discoverer
Electric atom theory of electromagnetism	Weber
Optical dispersion by mechanical oscillators	Helmholtz
Optical dispersion by electric oscillators	Lorentz
Theory of motion of charged particles	Heaviside, Poynting, Larmor
Electromagnetic mass	Thomson
Atom of electricity, the electron	Stoney
Faraday's laws imply a unit of electricity, electron	Helmholtz
Maxwell's continuum electromagnetic theory	Maxwell, Hertz
Cathode rays, attempts to explain	Crookes, Goldstein, Lenard, Perrin
Estimates the size of the electron	Richarz, Ebert, Stoney
Mobility of carriers in gaseous conduction	Townsend, Schuster
m/e for cathode rays, suggests rays as corpuscles	Thomson
Reconciliation of Maxwell's and atomic theories of electromagnetism	Lorentz
Magnetic splitting of spectral lines	Zeeman, Lorentz

Contemporary to Stoney, H. Helmholtz also observed that in the case of electrolytes each valency must be charged with a minimum quantity of electricity, which is non-divisible and known as "valency-charge". H. A. Lorentz predicted in his theory that the atom might consist of charged particles. His colleague and former student P. Zeeman was busy with the study of the spectrum of the element sodium in a magnetic field. Magnetic splitting of spectral lines observed by him in 1896 clearly advocated the indication of the likely sizes of the unitary charges [24]. He noticed that the widening of the D-lines of the spectrum of sodium is proportional to the magnetic field. Lorentz picked up the numerical factor from this relationship and used it to figure out the value of the ratio of the mass to charge of the carriers of electric charge in atoms [24]. F. Richarz, H. Ebert and G. J. Stoney also attempted to calculate the size of the electron from the emitting luminous vapour using the kinetic theory of gas [20].

Using the potential difference V between anode and cathode, E. Wichert reached upper limit of the kinetic energy of the particles in terms of eV and defined the magnetic deflection of cathode rays [25]. He used a collimated beam of cathode rays, which got deflected transversely by high-frequency coils, but separated from one another in the direction of the beam. Thus he reached the charge to mass ratio using the kinetic energy and the magnetic deflection. In 1897, Wichert was more specific about the value of e than Thomson [25].

Kaufmann also went on almost in the same line that of Wichert to produce the *e/m* only in 1897. He used the β-rays from radioactive sources. But the result was not good enough.

1.3 Thomson's experiment

Though large number of physicists was involved in the process, the discovery of the particle "electron" is recognized appropriately in the name of the British Physicist Sir Joseph John Thomson [21]. He described the cathode rays and derived the famous formulation of "m/e" of the electron [22]. The speed of the cathode rays was the first concern of Thomson and in 1894 he measured it as 200km/s, but he has thrown that out due to some faults. In 1897, he detected the deflection of the cathode rays by electric forces between the rays and electrified metal plates. The nature of the deflection was away from the negatively charged plate and towards the positively charged plate [21].

Figure 1.5: Sir Joseph John Thomson

In the cathode ray tube, the ray particles pass through a deflection region where they are subjected to some electric and magnetic forces acting at right angles to their original direction. There after they travel through a longer force-free region and strike at the end of the tube. Using Newton's second law of force, Thomson arrived at the interpretation of his own measurements and formulated the work as

$$\text{Displacement of ray at the end of tube} = \frac{\text{Force on ray particle} \bullet \text{Length of deflection region} \bullet \text{Length of drift region}}{\text{Mass of ray particle} \bullet (\text{Velocity of ray particle})^2}.$$

Newton's second law states regarding the force on a body and its consequence with the acceleration as

$$F = ma. \qquad \textbf{1.1}$$

In Thomson's experiment the force was $F = 10^{-11}$ dyne, and the mass of the electron is about 9×10^{-28} gm. These ensure the acceleration of about $a = \frac{F}{m} = 1.1 \times 10^{16}$ cm/s^2.

Consequently the very high speed was calculated as 1.1×10^{10} cm/s only after a micro second time.

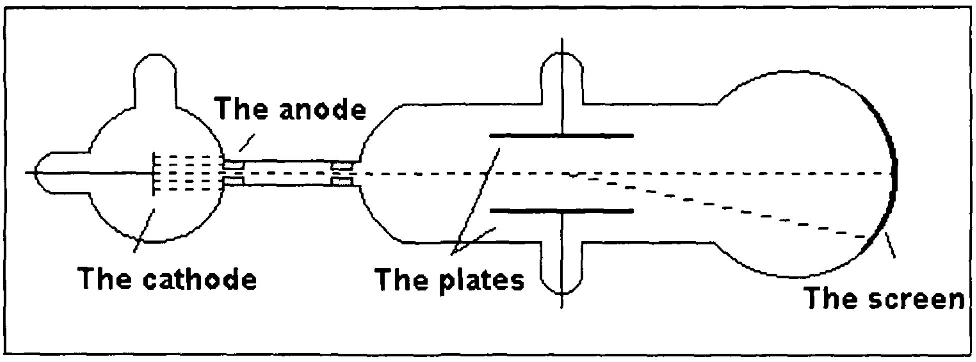

Figure 1.6: Schematic diagram of Thomson's experiment

If the force F is exerted on the cathode-ray particles, acting in a transverse direction to the motion of the ray, then the particles will experience acceleration in the direction of magnitude $a = \frac{F}{m}$. The force will be operated for a time t and the velocity perpendicular to their original motion comes out to be

$$v_{perp} = ta = t\frac{F}{m}. \quad \textbf{1.2}$$

The deflection region is of length l. If the particles travel the deflection region with a component of velocity v in the original direction of ray, then the time during which the particles are accelerated is

$$t = \frac{l}{v}. \quad \textbf{1.3}$$

Using equation 1.3 into 1.2 for t, *perpendicular velocity comes out to be*

$$v_{prep} = \frac{Fl}{mv}. \quad \textbf{1.4}$$

After the deflection region, the ray particles travel through the drift region of length L with a velocity v in a deflected direction from the original direction. The time spent in the drift region is

$$T = \frac{L}{v}. \quad \textbf{1.5}$$

But simultaneously the velocity of the ray particles is v_{prep} in perpendicular direction to their original direction. Hence the displacement was counted as

$$d = Tv_{prep}. \tag{1.6}$$

Using equations 1.4 and 1.6 jointly the displacement comes out as

$$d = \left(\frac{L}{v}\right) \times \left(\frac{Fl}{mv}\right) = \frac{FlL}{mv^2}. \tag{1.7}$$

Equation 1.7 provides the extent of the displacement of the ray particles from the straight direction.

Electric forces were introduced by the parallel, charged metal plates. In this experiment, the length and the width of the metal plates were considered much greater than their separation so that any effect of the plate edges can be very easily ignored. So the electric force here is at right angle to the axis of cathode ray. To be précised about the force, one can say that if the cathode-ray particles have electric charge e, the exerted electric force by electric field E on the particles is

$$F_{elec} = eE. \tag{1.8}$$

Consequently the displacement of the ray at the end of the tube will be

$$d_{elec} = \frac{eElL}{mv^2}. \tag{1.9}$$

Or in words

$$\text{Displacement of ray by electric field} = \frac{\text{Charge of ray particle} \bullet \text{Electric field} \bullet \text{Length of deflection region} \bullet \text{Length of drift region}}{\text{Mass of ray particle} \bullet (\text{Velocity of ray particle})^2}.$$

Thomson treated the cathode rays as streams of individual particles. He was successful to get exactly the magnetic force on the moving particles and hence calculated the displacement of cathode ray due to a magnetic field at a right angle to its direction.

The magnetic force by a magnetic field B on a particle with charge e and velocity v is given as

$$F_{mag} = evB. \tag{1.10}$$

By means of the expression of magnetic force from equation 1.10 in equation 1.7, displacement of the ray due to magnetic force at the end of tube comes out as

$$d_{mag} = \frac{eBlL}{mv}. \qquad \mathbf{1.11}$$

Or in words

$$\text{Displacement of ray by magnetic field} = \frac{\begin{matrix}\text{Charge of} & \text{Magnetic} & \text{Length of defle -} & \text{Length of drift} \\ \text{ray particle} \bullet & \text{field} \bullet & \text{- ction region} & \bullet \text{ region}\end{matrix}}{\text{Mass of ray particle} \bullet \text{Velocity of ray particle}}.$$

As a result the ratio of the magnetic deflection and the electric deflection can be figured out as

$$\frac{\text{Magnetic deflection}}{\text{Electric deflection}} = \frac{\text{Magnetic field}}{\text{Electric Field}} \bullet \text{Velocity}.$$

This gives magnitude of the velocity as

$$v = \left(\frac{E}{B}\right)\left(\frac{d_{mag}}{d_{elec}}\right). \qquad \mathbf{1.12}$$

Here comes out the aim of Thomson's experiment as the ratio of the mass to charge using equation 1.12 in equation 1.11 as

$$\frac{m}{e} = \frac{B^2 lLd_{elec}}{d_{mag}}. \qquad \mathbf{1.13}$$

On the 30th April 1897, J. J. Thomson announced the results of his experiments on cathode rays and according to him the rays were negatively charged subatomic particles, which were a universal constituent of matter [23]. Thomson also argued that the mass-to-charge ratio of cathode rays depended neither on the chemical composition of the gas within the cathode ray tube nor on the material of the tube's electrodes [24]. He named them as "corpuscles". In 1899, he again spoke about his corpuscle theory at the British Association Meeting and then it was accepted only after two years of his so-called announcement [23]. In this regard, we must say that though Stoney used the word "electron" [25-26], Thomson disagreed to approve it and R. A. Millikan too disapproved [27]. But other physicists affirmed the name "electron" ignoring Thomson and Millikan's opposition.

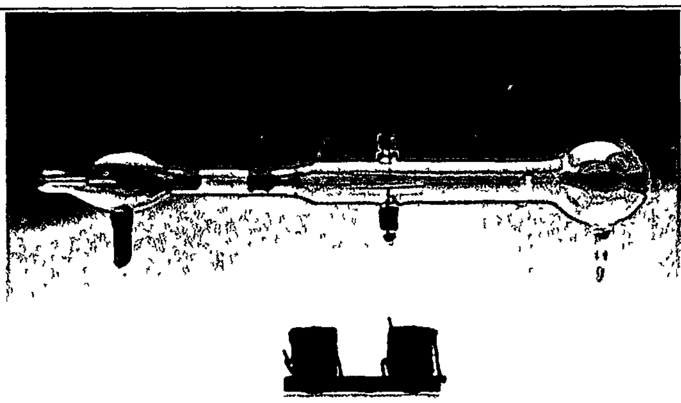

Figure 1.7: One of the cathode ray tubes used by Thomson

1.4 Properties of the electron

More than a century ago electron was discovered. By this time different properties of the electron have been discovered. Here we are going to have a quick look on some important properties of the electron. When we are going to describe those properties of the electron, sometimes the other unit systems are also described. But when we have concentrated in our own work there only Gaussian system is used.

The electron is a charged lepton. This refers to the significant spectroscopic properties of the electron. Till the time of this inscription, the electron is known as an elementary particle with very low mass. Thomson found the charge of the electron to be negative. From three different cathode ray tubes he measured the ratio $m/e \approx 0.4$, 0.5 and 0.9×10^{-11} kg/C. From his measurement of the unit of charge on an ion, the magnitude of charge comes out to be $e = 2.2 \times 10^{-19}$ C and this leads one to infer the mass-value as $m \approx 1.4 \pm 0.5 \times 10^{-30}$ kg [27]. These estimations were within a factor of two with recent accepted values. K. Woltz gave a comprehensive list including his own work on $e/m = 1.764(3) \times 10^{11}$ C/kg. But more accurate results were given later in 1916 by Millikan during the precision measurement of charge e and $\hbar$. He found the charge of the electron as $e = 1.592 \times 10^{-19}$ C and Planck's constant is $\hbar = 1.054 \times 10^{-34}$ Js. Along with the above calculations; Woltz's measurements provide the mass as $m = 9.025 \times 10^{-31}$ kg. Again with the development of the quantum electrodynamics using the general considerations by F. J. Dyson, it can be shown that the radiative corrections top the motion of the electron can be made finite in all orders with the suitable use of the charge and mass renormalization [28].

Recent data of the electron properties from Particle Data Group [29] are given as:

Mass $m = 0.510998910 \pm 0.000000013$ MeV,

Magnetic moment anomaly $\frac{(g-2)}{2} = (1159.65218073 \pm 0.00000028) \times 10^{-6}$,

Electric dipole moment $d = (0.07 \pm 0.07) \times 10^{-26}$ e-cm and

Mean life $\tau > 4.6 \times 10^{26}$ yr.

Mass

Mass of the electron is found extremely low from the various experimental measurements. But it is a question of ambiguity what actually the electron-mass is. Earlier it was thought that the entire mass of the electron was electromagnetic. But the reformed concept is apart from that. According to A. Pais, the mass of the electron is not purely electromagnetic in nature [30-31]. Again, he also confessed that the cause of the mass of the electron is still beyond our knowledge [30]. M. H. MacGregor expressed that no completely electromagnetic structure is available and the electromagnetic framework is needed for the electron just to hold it together [31]. In the current scenario, the origin of the mass of the electron is a big puzzle. Hence the concepts of the electromagnetic mass and the mechanical mass both are regarded in recent works.

In a recent measurement of the g-factor the electron's mass has been determined with more accuracy. Currently in atomic mass unit the mass of the electron is presented as 0.0005485799092(4)u [29, 32]. Theoretical and experimental approaches with electron properties for more than a century state that four different kinds of mass or equivalent energy are attributed to the electron. They are electrostatic self-energy (W_E), magnetic self-energy (W_H), mechanical mass (W_M) and gravitational mass (W_G) [33]. In different units the mass of the electron is expressed below according to the experimental evidences [34].

Table 1.4: Electron-mass in different units

Symbol	Numerical Value	Unit
m_e	$9.10938215(45)\times10^{-31}$	kg
$m_e = A_r(e)$	$5.4857990943(23)\times10^{-4}$	u
$m_e c^2$	$8.18710438(41)\times10^{-14}$	J
$m_e c^2$	$0.510998910(13)$	MeV

Charge

Charge is one of the intrinsic properties of the electron and this is a fundamental quantity of Nature. For the electron the charge is negative. The numerical value of the elementary charge of the electron is $1.602176487(40)\times10^{-19}$ C [34]. This is the key factor behind the behaviour of the electron. Due to its charge, the electron is involved in the electromagnetic interactions. The size of the charge of the electron or the charge radius of the electron is yet to be précised though it is confirmed by the LEP results of CERN that the size is even less than 10^{-19} m or 10^{-17} cm [35]. Charge of the electron has a crucial role in describing the electron models. Some models show a classical distribution of charge [36]. They are surface and volume distributions of charge. The other models advocates for a point-charge [37]. It controls the current, magnetic field and magnetic moment of the electron. As a fundamental quantity of Nature the electric charge is also involved in the description of the fine structure constant.

Spin

The spin of the electron is a mysterious angular momentum for which no actual physical picture is available yet [38]. Experimentally, in 1921 spin was first exposed when O. Stern and W. Gerlach experimented with the silver atoms passing through a magnetic field and observed a non-classical distribution of silver atoms on photo-plate. Hypothesis of the spin of the electron was proposed by G. E. Uhlenbeck and S. A. Goudsmit in the framework of a small rigid rotating body. But W. E. Pauli did the most influential study over the matter theoretically. Pauli described the spin as "... a classically not describable two-valuedness". This "two-valued quantum degree of freedom" allowed him to formulate the famous 'Pauli exclusion principle'.

Uhlenbeck and Goudsmit's results met a favourable response by the work of L. H. Thomas. With the advancement of quantum physics, the spin is regarded as a quantum property of the electron instead of being a classical one [38]. The magnetic moment and the spin are related in the current representation of the picture according to the standard model of particles.

Classically the spin is the rotation of the particle around its axis. It is called there as angular momentum. Classical and semi-classical models incorporate the spin with the rotation only. N. Bohr proposed a fundamental quantum unit of orbital angular momentum [39] in terms of $\hbar$, which is the Planck's constant divided by 2π. Being a quantum mechanical property, the spin can take only discrete values. The spin of the electron is $\frac{\hbar}{2}$ or $-\frac{\hbar}{2}$. In particle physics, depending on the spin the particles are classified in two classes: bosons with integral spin and fermions with half-integral spin. Our concerned electron is a half-integral spin particle.

Magnetic moment

From the hypothesis of the spinning electron the magnetic moment of the electron is defined as $\frac{eh}{4\pi mc}$ [40], where symbols have their usual meanings. Without radiative corrections, the intrinsic magnetic moment of the electron is given by the Bohr magneton only as $\mu_0 = \frac{e\hbar}{2mc}$ [41]. The magnetic moment of any of the three charged leptons $(\ell = e, \mu, \tau)$ is known as

$$\mu_\ell = g_\ell \frac{e}{2m_\ell} s, \qquad 1.14$$

where g_ℓ is the g-factor of the particle, m is its mass and s is its spin [34]. Otherwise one can mark the magnetic moment in terms of Bohr magneton as

$$\mu = -\frac{g}{2} \mu_B \frac{s}{\hbar/2}, \qquad 1.15$$

with $\frac{g}{2} = 1$ for a point electron in a renormalizable Dirac explanation. According to QED predictions it is considered that the vacuum fluctuations and polarization slightly increase this value. For the lepton substructures this value could deviate from the QED or Dirac predictions [42-43].

Anomalous magnetic moment of the electron a_e is one of the simplest quantities, which can be calculated very precisely [44]. This is measured experimentally as $a_e = 1159652188.4(4.3)\times 10^{-12}$. It plays a crucial role to test the validity of QED.

g-factor

The g-value is a dimensionless measure of the moment. This is the magnetic moment in units of the Bohr magneton for the electron [43]. The g-factor for a Dirac point particle with $g = 2$, can be expressed as

$$\frac{g}{2} = 1 + a_{QED}(\alpha) + a_{hadronic} + a_{weak} + a_{new},$$

where $a_{QED}(\alpha) \approx 10^{-3}$ is the anomalous magnetic moment and a function of the fine-structure constant. Hadronic and weak are calculated accordingly and within the Standard model, whereas the last term can cause deviation from Dirac or QED prediction with a substructure idea of the electron [42] which is a subject of this thesis and beyond Standard model exposure.

For free electron, the g-factor can be expressed as

$$g_e = \frac{2\mu_e}{\mu_B} = 2(1 + a_e), \qquad \textbf{1.16}$$

where μ_e is the magnetic moment, μ_B is the Bohr magneton and a_e is the electron magnetic moment anomaly [45]. The numerical value of the factor is given in "The 1986 adjustment of the fundamental physical constants" as 2.002319304386(20) [45].

Mean life

The mean life of the electron is tested in different experiments for the years. Measured value of the mean life is as $\tau_e > 4.2(2.4)\times 10^{24}$ yr. according to the experimental outcome on the electron stability and non-paulian transitions in Iodine atoms from Gran Sasso National Laboratory of INFN [46]. But very recent observation recorded as a lower limit > 1.22×10^{26} yr for the mean life time of the electron decay via the branch $e^- \rightarrow \gamma + \nu_e$. With single Ge detector, the best limit till is counted as 1.93×10^{26} yr [47].

Size and shape

Size of the electron is a real enigma yet. Some of the modern approaches regard the electron as a point particle [48]. The standard model of particle physics also supports them. But classical theories argue against. In fact, from different phenomenon the radius of the electron is measured are different. They vary in the range of 10^{-11} m to 10^{-15} m [33, 49]. Compton calculated the radius of the electron in his way using classical electrodynamics as well as the scattering nature. He found that the magnitude of the diameter of the electron is comparable with the wavelength of the shortest γ rays and thus the radius of the electron came out in his calculation as 2×10^{-10}cm or 2×10^{-12}m [50]. It is also confirmed in the same article that the radius of the electron is the same in all atoms.

Lorentz calculated the size of the corpuscle of Thomson's corpuscle as $\sim10^{-13}$cm or $\sim10^{-15}$m. Though the size of the electron in term of radius is same in all the atoms, the sizes predicted by different electromagnetic phenomenon are different. The expressions of those sizes are called as different radii [33, 49]. They are listed below as:

R_0 = Classical radius,

R_C = Compton radius,

R_{QMC} = Quantum mechanical Compton radius,

R_{QMC}^{α} = QED-corrected quantum mechanical Compton radius,

R_{em} = Electromagnetic radius,

R_H = Magnetic field radius,

R_{QED} = QED charge distribution for a bound electron,

R_E = Charge radius.

As we are going to have detailed study about the radii and the sizes, here we are not going in depth. Therefore the size of the electron is really an enigmatic thing that is yet to be explained properly. The puzzling size of the electron also results in the shape of the electron. Shape of the electron is a question of ambiguity. If it is regarded as point particle, the shape will be meaningless. But extended models offer different shapes for the electron. Amongst them, spinning spheres are well-known. The idea of spherical electron was first done by Lorentz and others and later it was continued up to MacGregor, Rivas and other contemporaries. Compton advocated about the ring model. According to the string theory the particle's shape is given by the corresponding vibration.

Table 1.5: Some of the fundamental physical constants related to the electron (This table is adapted from ref. [45])

Quantity	Symbol	Value	Unit
Speed of light in vacuum	c	299 792 458	ms^{-1}
Planck constant	h	6.626 075 5(40)	10^{-34} Js
Elementary charge	e	1.602 177 33(49)	10^{-19} C
Bohr magneton	μ	25812.805 6(12)	Ω
Fine-structure constant	α	7.297 353 08(33)	10^{-3}
Rydberg constant	R_∞	10 973731.534(13)	m^{-1}
Bohr radius	R_{em}	0.529 177 249(24)	10^{-10} m
Electron mass	m_e	9.109 389 7(54) 5.485 799 03(13) 0.510 999 06(15)	10^{-31} kg 10^{-4} u MeV
Compton wavelength	λ_C	2.426 310 58(22)	10^{-12} m
Classical electron radius	R_0	2.817 940 92(38)	10^{-15} m
Thomson cross section	σ_e	0.665 246 16(18)	10^{-28} m^2
Electron magnetic moment anomaly	a_e	1.159 652 193(10)	10^{-3}
Electron g-factor	g	2.002 319 304 386(20)	-

Electric dipole moment

Electric Dipole Moment (EDM) of the electron is one very important observable for the testing of the CP violations. The presence of an electric dipole moment can be traced by placing the particle of interest in an electric field E and measuring the corresponding incremental energy according to $W = -d.E$. Several theoretical models predicted the electron's EDM. The standard model predicts the magnitude of EDM as $|d_e| < 10^{-38}$ e cm, when supersymmetric models predict as $|d_e| < 10^{-27}$ e cm. Some of the models predict a range of EDM. Amongst them lepton-flavour changing model gives the range over 10^{-29} e cm to 10^{-26} e cm. In a more précised form, the left-right symmetry models claims the range of 10^{-28} e cm to 10^{-26} e-cm, whereas Higgs models predict $|d_e|$ in the range 10^{-28} e cm to 3×10^{-27} e cm [51].

Size, shape, magnetic moment, mass etc. are the properties of the electron and they are the evidences, which are not matching with the point particle theory. They not only state of the behaviour of this charged lepton, but also advocate an extended structure, which is classically acceptable. Consequently physicists tried to figure out a clear picture of the structure or the substructure of the electron which falls under the "beyond standard model physics" now. Experimentally to get the exact size and shape of the electron is a tough job and that is a limitation which prompts people to propose more theories of models of the electron. We therefore have studied the proposed models and tried to give an account in our own way without violating the behavioural nature of the electron. In this regard the key points to be discussed are given below:

- Different models of the electron
- Size of the electron from different electromagnetic phenomenon
- Electron properties in the light of fine structure constant
- Electromagnetic mass of the electron
- Helical motion and spinning sphere model of the electron

At the end we have extended our study up to the positronium (bound state of the electron and the positron) mass spectra, so that we can even have a picture of the immediate next status after the free electron.

1.5 Concluding remarks

Studying all the above facts, it is seen that standard model cannot explain the electron completely. Therefore a new model is necessary to explain all the properties of the electron. Point particle structure is neither explainable from geometrical point of view, nor from the particle aspects. No proper explanation of EDM is also possible from the point particle theory. Hence this is a humble attempt where we are going to discuss and study the electron properties in the light of classical approach.

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Truth can be stated in a thousand different ways, yet each one can be true.

-Swami Vivekananda

Chapter 2

Models of the′electron

The electron was identified in 1897 as the first subatomic particle [1]. But the structure of the electron is still a subject of debate. The properties of the electron discussed in the previous chapter illustrate the different aspects of this particle. Depending on them, several models of the electron [2-4] have been proposed theoretically. Real experimental features say that the electron does not decay into other particle. But again it is questionable in this regard that whether it is our experimental limitation to probe the 0.511 MeV/c^2 particles or not. If it is our limitation to break that small mass, then what can be the exact measures? To get the answer, throughout the last century, good numbers of theories were proposed. Models have been developed both in classical and quantum mechanical ways. Some of the models are at the boundary of the two and are known as semi-classical approaches. Roughly the models are either point-particle models or extended models.

Depending on the present day experimental facts some of the existing models claim the electron to be a point particle. Basically the very low mass of the electron is responsible for this one to be counted as a point particle. Again as it does not decay into some more elementary particles this suggests it to be a point like particle. On the other hand the treatment of the electron is better studied in quantum physics and that obviously put forward the claim of a very tiny entity. Hence in the standard model it is considered as a point-like one.

2.1 Classification of the models of the electron

Classical results and sometimes its features have shown the behaviour of the electron as an extended particle. Idea of the extended structure of the electron is also advocated by number of physicists [3-4]. When an extended electron is supposed, there arises the question about the nature of its charge. Two different types of extended models of the electron are considered in this regard. Some of the models represent the charge as glued over the entire outer surface of the structure [2, 4, 5]. On the other hand, several structures are there with an extended electron and a point charge [6-8]. Hence P. Lancini proposes the classification of the models of the electron [9], which we have listed below:

A. Point-like models

A point-like electron actually does not imply any structure and some works are devoted in the support of it [10]. They refer to the explanation of the properties of the electron without structure [11-13]. The standard model and other quantum mechanical models support a point-like electron.

B. Extended models

Several models propose an actual extended structure of the electron [2, 4, 5, 14-19]. They argue of complete electromagnetic structure. Indeed these models say about the surface and volume charge distribution of the electron.

C. Extended models with point-like charge

The third kind refers to the extended body with a point charge [6-8]. In these models spherical structure is emphasized with a point-like charge on the sphere and charge is regarded not to be glued over the entire body.

Some other models are also there which does not fall in these three kinds and they are chiefly sub-structure models. Lepton and quark sub-structure models are proposed in good number of articles [20-25]. Preon model, Rishon model and unified composite models fall in this category.

Also there are some other theories, which deal with the extra dimensions or above the known four dimensions [9, 26-27] and they provide the picture of the electron in their own ways.

We are going to discuss some important models of the electron in each category.

2.2 Point-like models

2.2.1 Visser model

In a classical model [11] of the electron proposed by M. Visser, ordinary electromagnetism coupled to the neo-Newtonian classical gravity is described. Here a charge point is considered in an electromagnetic field. Non-gravitation matter density is regarded to be

$$\rho = m_0 \delta(r) + \frac{1}{8\pi}\frac{1}{4\pi\varepsilon_0 c^2}\frac{Q^2}{r^4}. \qquad \textbf{2.1}$$

Here m_0 is the bare mass and $\frac{1}{8\pi}\frac{1}{4\pi\varepsilon_0 c^2}\frac{Q^2}{r^4}$ is the electromagnetic mass density, with Q as the total charge and ε_0 as the free-space permittivity. In Geometrodynamic units [11] $G \equiv c \equiv \frac{1}{4\pi\varepsilon_0} = 1$ is regarded to represent the gravitational and the electromagnetic mass density together. Hence for a dimensionless variable ψ the differential equation comes out as

$$\Delta\psi = \left(2\pi m_0 \delta^3(r) + \frac{1}{4}\frac{Q^2}{r^4}\right)\psi. \qquad \textbf{2.2}$$

The general solution of this equation is integrated out as

$$\psi(r) = \frac{\cosh(\kappa - Q/2r)}{\cosh(\kappa)}, \qquad \textbf{2.3}$$

where κ is the integration constant and related to the mass of the system. The bare mass is expressed as

$$m_0 = -Q. \qquad \textbf{2.4}$$

The total energy comes out to be equal to the gravitational mass of the system and is identified here as

$$m = Q\tanh(\kappa). \qquad \textbf{2.5}$$

Visser's calculation for bare mass in rationalized MKSA units show that

$$m_0 = -\frac{Q}{\sqrt{4\pi\varepsilon_0 G}} = -(1.16\times10^{10}\,kg/C)Q. \qquad \textbf{2.6}$$

With $Q = e = 1.602\times10^{-19}\,C$, the bare mass is figured out as

$$m_0 = -1.85\times10^{-9}kg = -1.04\times10^{18}GeV/c^2. \qquad \textbf{2.7}$$

Simultaneously, this bare mass is expressed in terms of Planck mass when connected through the fine structure constant as [11]

$$m_0 = -\sqrt{\alpha}M_{Planck}. \qquad \textbf{2.8}$$

But experimentally measured mass is not in agreement with this value of the bare mass concept given by the author. Firstly, the negative sign of this mass is not explained. Secondly, the magnitude of the electron mass measured is too small compared to the mass expressed in this model. This very high value of the mass makes this model of no use. In the recent scenario, the gravitational mass W_G associated with the electron is regarded as negligibly small and hence it is expressed as $W_G = 0$. But the concept of mechanical mass is grown up with the certain logical steps and it takes around 99.9% of the total mass as expressed by MacGregor [7].

Another important property spin of the electron is completely ignored in this model. What role it can play in this model is not tested at all. The fact is that the known $\frac{\hbar}{2}$ spin of the electron is not fitting with a huge massive electron. If one tries to put this calculated bare mass and the known spin together the result indicates to a rest body. Similarly the magnetic moment of the electron, which plays the crucial role in the behaviours of the electron, is also omitted completely. These limitations weaken the model proposed by Visser.

2.2.2 Blinder model

In a recent work S. M. Blinder proposed a classical electron model [12] even without a structure. In this model, the self-energy of a point charge and that of a dipole are focussed keenly. The electron is considered here as a point particle. This model regards the total energy as electromagnetic and in consequence that offers an electromagnetic origin of the angular momentum. This allows the parameterisation of permittivity within the range of two-third of classical electron radius [12]. The angular momentum is set as

$$S = \frac{1}{c^2}\int r\times(E\times H)d^3r, \qquad \textbf{2.9}$$

where E is the electric field, H is the magnetic field and c is the speed of light in free space. The electric field energy is calculated here as $W_{elec} = \frac{3}{4}mc^2$ and the magnetic self-energy as $W_{mag} = \frac{1}{4}mc^2$ with m as the mass of the electron.

The size or the radius of the electron in the model is not précised. If the radius or the size is used as classical electron radius, this is not clear that how it can be put again as a point particle with structure-less picture. Huge electric energy or mass is also questionable in today's conditions when the charge is confined within a length $<10^{-19}$ m or $<10^{-17}$cm [8]. Again as the recent experimental results tell us of the length of the charge radius to be $<10^{-17}$cm [8], Blinder's model loses its strength.

2.2.3 Massless point charge model

This model is developed on the Abraham-Lorentz equation for a point electron [13], which is expressed as

$$m\ddot{r} = (m_0 + \delta m)\ddot{r} = \frac{2e^2}{3c^3}\frac{d\ddot{r}}{dt} + F, \qquad \textbf{2.10}$$

where

$$\delta m = \frac{4e^2}{3\pi c^3}\int_0^{k_c^{*}} dk = \frac{4\alpha m_*}{3\pi^{1/2}}, \qquad \textbf{2.11}$$

with $e = e_*\sqrt{\alpha}$, $k_c^{*} = \sqrt{\pi}/r_*$, m_* is the mass and r_* is the radius of the electron. Here F is some external force driving the electron and e_* is the true or bare electronic charge. The bare mass is given as

$$m_0 = m - \delta m \approx -\alpha m_* . \qquad \textbf{2.12}$$

Using Puthoff model [28] W. C. Daywitt arrived at the conclusion that the zero-point agitation of the Planck particles within the degenerate negative energy Planck vacuum creates zero-point electromagnetic field that exists in the free space. According to this model the driving force e_*E_{zp} is responsible for the mass of the electron, and consequently the point charge e_* and the radius too. Here E_{zp} is the zero point electric field.

This model is mounted on a massless idea, whereas the mass of the electron is playing the significant role in the leading models of the electron. Secondly, though Daywitt claims it as a massless one, the model in fact is not so. Neither the size nor the spin of the electron is précised with this model. None of the electromagnetic properties of the electron is also tested within this model. As a crucial point, it can be pointed out that Daywitt marked $\frac{e^2}{mc^2}$ as Compton radius, but it is known commonly as classical electron radius and the Compton radius is mathematically expressed as $\frac{\hbar}{mc}$. Again the classical radius and the Compton radius are at a gap of the order of 10^2 or exactly by the factor of the fine structure constant, which can incorporate a huge change in the corresponding calculation.

2.3 Extended models

2.3.1 Lorentz-Abraham model

H. A. Lorentz constructed a particle electrodynamics [2], and tried to put the macroscopic phenomena of electromagnetism and optics in terms of microscopic behaviour of the electrons. Before Lorentz stepped into the business, his predecessors had tried with the interaction between the charges. But he did it the other way via the electromagnetic field.

If the current density j and the charge density ρ are related as

$$j = \rho v, \qquad \textbf{2.13}$$

and ρ is defined by

$$e = \int \rho d^3\tau, \qquad \textbf{2.14}$$

then the electric and magnetic fields can be produced by the Maxwell's equations. Here $d^3\tau$ represents the volume distribution of the electron within which the charge is distributed and v is the linear velocity of the charge. The force exerted by the fields E and B on a charged particle is then expressed as

$$f = \rho\left(E + \frac{v}{c} \times B\right). \qquad \textbf{2.15}$$

As Maxwell's equations are also applied in this case, the above expression is called as Maxwell-Lorentz equation [2].

The system was considered with the atoms or the ions to which the electrons are bound elastically. In other words, this is a physical system of a charged harmonic oscillator. The radiation is emitted by the oscillator with units of ergs/sec at the rate of

$$R = \frac{2}{3} \cdot \frac{e^2}{c^3} \cdot a^2 , \qquad \textbf{2.16}$$

where a is the acceleration of the charge. This loss of energy results to a damping force as

$$F_{rad} = \frac{2}{3} \cdot \frac{e^2}{c^3} \cdot \frac{da}{dt} . \qquad \textbf{2.17}$$

With a rigid spherical structure, M. Abraham got a purely electromagnetic electron of the classical electron radius [2]. The momentum of the electron is then given by the Poynting vector and is written as

$$p = \frac{1}{c^2} \int S d^3 r , \qquad \textbf{2.18}$$

where

$$S = \frac{c}{4\pi} E \times B . \qquad \textbf{2.19}$$

Here r is the radius of the spherical structure. This gave the momentum of the electron due to the Coulomb field of an electric field moving with velocity v as

$$p_{elm} = \frac{4}{3} m_{elm} v , \qquad \textbf{2.20}$$

where m_{elm} was the electromagnetic mass considered [2]. Then the total momentum of the electron is calculated out as

$$p = \left(m_0 + \frac{4}{3} m_{elm} \right) v = mv , \qquad \textbf{2.21}$$

where $m_0 v = p_{neutral}$ is the momentum of uncharged part of the electron.

The force exerted on the electron is also aimed to be calculated and for that we start with the Newton's law of motion

$$m\dot{v} = F_{ext} , \qquad \textbf{2.22}$$

if there is no radiation. But as the charge particle's acceleration is associated with radiation, the equation 2.22 can be given in a modified form as

$$m\dot{v} = F_{ext} + F_{rad} \,. \qquad \textbf{2.23}$$

One may arrive now the exerted force by using equation 2.17 in equation 2.23 as

$$m\left(\dot{v} - \frac{2}{3}\frac{e^2}{c^3}\ddot{v}\right) = F_{ext} \,. \qquad \textbf{2.24}$$

This is called as Abraham-Lorentz equation of motion [29].

This model gives the charge distribution as rigid and spherically symmetric. This consequently refers to the question of self-force, which can be incorporated; since due to Coulomb's law each part of the charged sphere repels all other parts. The self-force can be written as [2]

$$F_{self} = \int \rho\left(E + \frac{v}{c} \times B\right) d^3 r \,. \qquad \textbf{2.25}$$

If the electron is to be purely electromagnetic in structure, then for an external force employed on it, can be balanced as

$$F_{self} + F_{exterenal} = 0 \,. \qquad \textbf{2.26}$$

Dirac pointed out the great problems regarding this model. The electromagnetic origin of the Lorentz model is discarded [30]. He cited the example of the neutron mass, which claims the independence of electromagnetism for mass. Also in the theory of the positron, the idea of the electromagnetic mass no longer stands. This way, the self-force treatment is with diverging self-energy. Secondly the damping could make it to zero energy state that we cannot get for the electron. These problems made this model weak and opened the area for new speculations.

2.3.2 Allen model

In the meeting of royal society, H. S. Allen proposed the case for a ring electron [14]. He discussed the properties and the work done by others and gave his own conclusion to the properties in a ring type electron. He was in favour of an electron in the form of a current circuit capable of producing magnetic effects. According to him, exerting electrostatic forces, the electron behaves like a small magnet. The important outcomes of this model are discussed by him. According to him: there is no loss of energy by radiation, the ring electron gives good explanation of the facts of paramagnetism, a small amount of ionisation of gases produced by X-rays would be able to have an explanation and Bohr's theory of origin of series lines in spectra may be restated to apply to the ring electron and so on.

2.3.3 Old classical model

In the paper of D. Lynden-Bell [5] the old-fashioned electron model is discussed partially and he also pointed out the weakness of the model. We here did the job a bit extensively. The problem of the large velocity is discussed in short in that paper. It is a model of the electron given with uniform surface charge density.

The angular momentum of the rapidly rotating uniformly charged sphere is expressed as [5]

$$L = \frac{2}{9} \cdot \frac{e^2}{c} \cdot \frac{v}{c}. \qquad \textbf{2.27}$$

The spin angular momentum of the electron is $\frac{\hbar}{2}$. If this is employed in equation 2.27, the velocity of the sphere comes out as

$$v = \frac{9}{4} \alpha^{-1} c, \qquad \textbf{2.28}$$

where α is the fine structure constant. Hence the velocity of the sphere exceeds the velocity of light. This indicates the inconsistency of the proposed model due to the violation of the postulate of the special theory of relativity. If we consider with the same structure that describes the equation 2.27, the spin angular momentum comes out to be as

$$L = \frac{2}{9} \alpha \frac{v\hbar}{c}. \qquad \textbf{2.29}$$

Hence the radius of the electron is resulted as

$$R = \frac{2}{9} R_0, \qquad \textbf{2.30}$$

where $R_0 = \frac{e^2}{mc^2}$, the classical electron radius.

The energy of the sphere in the electric field is given as [5]

$$\varepsilon_e = \frac{e^2}{2R}, \qquad \textbf{2.31}$$

where, r is the concerned radius. The energy in the magnetic field is known as

$$\varepsilon_m = \frac{e^2 v^2}{9Rc^2}. \qquad \textbf{2.32}$$

Consequently the total energy due to the electric and the magnetic field is given as the sum of the equation 2.31 and 2.32

$$\varepsilon = \frac{e^2}{2R}\left(1+\frac{2}{9}\frac{v^2}{c^2}\right). \qquad \textbf{2.33}$$

At the speed limit of $v=c$, we have the radius $R_C = \frac{\hbar}{mc}$, which is known as Compton radius [7]. Using Compton radius at maximum velocity, the total energy can be calculated from equation 2.33 as

$$\varepsilon_C = \frac{11}{18}\cdot\frac{e^2}{R_C}. \qquad \textbf{2.34}$$

With the use of Compton radius in equation 2.32, we have the energy

$$\varepsilon_C = \frac{11}{18}\alpha\cdot mc^2. \qquad \textbf{2.35}$$

If the velocity is very less than the velocity of the light; i.e. $v << c$, we have

$$\varepsilon \approx \frac{e^2}{2R}. \qquad \textbf{2.36}$$

Using the classical electron radius, $R_0 = \frac{e^2}{mc^2}$, the energy can be calculated from equation 2.36 as

$$\varepsilon_0 = \frac{mc^2}{2}. \qquad \textbf{2.37}$$

For electromagnetic radius of the electron $R_{em} = \frac{\hbar^2}{me^2}$, we can write from 2.33

$$\varepsilon_{em} = \frac{\alpha^2 mc^2}{2}\left(1+\frac{2v^2}{9c^2}\right). \qquad \textbf{2.38}$$

If the velocity $v<<c$, then for electromagnetic radius following equation 2.36 we can have a modified version of equation 2.38 as

$$\varepsilon_0 = \frac{\alpha^2 mc^2}{2}. \qquad \textbf{2.38-a}$$

It is well known now that there are eight well-defined radii of the electron [7, 31] and they all are derived from different electromagnetic aspects. Above-mentioned energy equations 2.35, 2.37 and 2.38 are for Compton radius, classical electron radius and the electromagnetic radius respectively and these three radii belong to the family of those eight radii. Hence equation 2.33 can be considered as a generalized form of total energy. Therefore, $\varepsilon_{\max}$ and $\varepsilon_{\min}$ can be set with employing the conditions $v = c$ and $v << c$ in equation 2.33. From equations 2.35, 2.37 and 2.38-a we know that the radius corresponds to $\varepsilon_{\max}$ is R_C and radii concerned to $\varepsilon_{\min}$ are R_0 and R_{em} respectively. This concludes the range of electron size within the frame of R_{em} to R_0, which essentially rejects all other radii. Also the energy comes out from equation 2.28 with the classical radius

$$\varepsilon = 10558.0625mc^2 . \qquad \textbf{2.39}$$

This is completely an absurd value for the energy as the maximum energy is regarded as mc^2.

As equation 2.33 refers to a relation between the radius and the velocity, the radius can be defined in terms of energy as

$$R = \frac{e^2}{2\varepsilon}\left(1 + \frac{2v^2}{9c^2}\right). \qquad \textbf{2.40}$$

The maximum possible energy we can account for the particle is mc^2. Using this in equation 2.40 one will get the velocity $v = \frac{3}{\sqrt{2}}c$ for the classical electron radius R_0 which is quite an impossible value according to special theory of relativity. Similarly, spinning sphere models follow the form of the angular momentum as

$$L = I\omega , \qquad \textbf{2.41}$$

where I is the moment of inertia and ω is the angular velocity. This gives the expression of angular momentum of a sphere as

$$L = \frac{2}{5}mvR . \qquad \textbf{2.42}$$

Equation 2.42 leads to the velocity of the electron as

$$v \approx 114c , \qquad \textbf{2.43}$$

which is quite absurd.

The smearing out charge over the entire sphere is a huge drawback that we have proved here from classical aspects only. Firstly, this offers a tachyonic electron, which is in contradiction with the special theory of relativity. Secondly, this rejects the other electromagnetic phenomenon, which advocate for the electron size. Thirdly, the recent experimental measurement of the radius of charge of the electron is also against to this idea. All the above problems occur with this model due to its charge distribution with a surface charge $\sigma = 4\pi R^2$ [5]. This refers to the fact that the models of the spinning sphere with the charge distribution over the sphere no longer stands good. At this juncture, the charge can be put as a very tiny object residing in a small place and can be treated as a point-charge. This confirms the experimental measurements from LEP [8] too.

2.3.4 Compton model

A. H. Compton calculated the scattering coefficient of high frequency radiation assuming rigid charged electron. He started the work [15] with the scattering coefficient per unit mass of the substance given by Thomson's work as

$$\frac{\sigma}{\rho} = \frac{8\pi}{3} \cdot \frac{Ne^4}{m^2C^4}, \qquad 2.44$$

where σ is the ratio of the scattered to the incident energy per unit volume of the material, ρ is its density and N is the number of electrons in unit mass of the substance. According to him, the scattering would depend upon the structural "form" of the electron. For the simplest structure of the rigid, uniform, spherical shell of the electricity, the scattering coefficient is calculated as [15]

$$\frac{\sigma}{\rho} = \frac{8\pi}{3} \cdot \frac{e^4 N}{m^2C^4} \frac{\sin^4\left(\frac{2\pi a}{\lambda}\right)}{\left(\frac{2\pi a}{\lambda}\right)^4}, \qquad 2.45$$

where a is the radius of the spherical shell and λ is the wave-length of the incident beam.

For the flexible spherical electron, this result comes out as

$$\frac{\sigma}{\rho} = 2\pi NL^2 \int_0^\pi \frac{I_0}{I} \sin\theta d\theta, \qquad \textbf{2.46}$$

with intensity I of the incident beam. When the beam is scattered by an electron with an angle θ by an unpolarized beam of γ-rays, the distance at which the intensity of the scattered beam is measured [15] is given by L. On the other hand, the ring electron assumption gives the mass scattering coefficient as

$$\frac{\sigma}{\rho} = \frac{8\pi}{3} \frac{Ne^4}{m^2 C^4} \left\{ 1 - a\left(\frac{a}{\lambda}\right)^2 + b\left(\frac{a}{\lambda}\right)^4 - c\left(\frac{a}{\lambda}\right)^6 + \ldots \right\}, \qquad \textbf{2.47}$$

where a, b, c are the constants.

Comparing the experimental and the theoretical facts and getting the dissimilar results, Compton came to the conclusion that, only potential justification of the dissymmetry can be accounted by presuming that the scattering particles have dimensions comparable with the wave-length of the rays which they scatter. This concludes that the diameter of the electron is comparable in magnitude with the wave-length of the shortest γ-rays and the radius of the electron comes out as about 2×10^{-10} cm [15]. Continuation of his work came in good agreement with the experimental values for the absorption of high frequency radiation in aluminium if the electron is taken to be a ring of radius $1.85 \pm 0.05 \times 10^{-10}$ cm [16].

His work gave the very fundamental point about the size of the electron. Indeed Compton's work about the shape is acceptable one, as it does not go for any mathematical trouble. With the limited experimental set up it was a great triumph of him to calculate the scattering coefficient per unit mass and the size and shape of the electron. The striking fact is that, both in theoretical and experimental view point his results were quite good. Size and shape of the electron was given by his work, but the mass and the spin are the other important intrinsic properties remaining and if those would be calculated by him today we need not to write down the necessity of a complete picture of the electron.

2.3.5 Lortz model

In this proposal the electron is portrayed by the combination of a model of relativistic continuum mechanics and vacuum electrodynamics [19]. The equations of the vacuum electrodynamics in SI units are given as

$$\nabla . B = 0 , \tag{2.48}$$

$$\nabla . E + \partial_t B = 0 , \tag{2.49}$$

$$\nabla . E = \frac{q}{\varepsilon_0} , \tag{2.50}$$

$$\mu_0 j = \nabla \times B - \frac{1}{c^2} \partial_t E , \tag{2.51}$$

and

$$j = qv . \tag{2.52}$$

Here E and B are the electric and magnetic fields, j is the current density, q is the charge density, v is the corresponding velocity of charge, c is the speed of light in free space and μ_0 is the free space permeability.

In this model extended charge is considered in order to avoid divergence of its self-energy. The model is axis symmetric. The electromagnetic field is poloidal [19], the current density and the flow are toroidal, while the mass and the charge densities are axis symmetric scalars.

The total energy of the system is

$$U = Mc^2 = U_M + U_E + U_B , \tag{2.53}$$

where mechanical energy is

$$U_M = c^2 \int \rho_0 \gamma^2 d^3\tau , \tag{2.54}$$

and the electromagnetic energies

$$U_E = \frac{\varepsilon_0}{2} \int E^2 d^3\tau \tag{2.55}$$

and

$$U_B = \frac{1}{2\mu_0} \int B^2 d^3\tau . \tag{2.56}$$

Here ρ_0 is the mass density and $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ and $d^3\tau$ gives the distribution of the energy within the structure. This work has been extended with internal and external solutions also. It was concluded with the fact that for the electron model, the quantization is not always necessary, and sometimes the description by stationary states of continuum mechanics may be simpler.

2.3.6 Semi-classical Sirota model

The semi-classical radius of the electron is calculated by N. N. Sirota [32] in ring like charged body model. Here the ring is considered to be rotating as a rotating body with angular velocity ω around the axis of rotation. The radius is calculated here equating the compressive force due to the magnetic field originated from the rotation of the charge and the centrifugal force due to the rotating volume.

The centrifugal force is denoted as $F_C = \frac{mv^2}{r}$ and the compressive force is $F_H = \frac{1}{2}\mu_0 \Phi_l^2$, where the function $\Phi_l = \frac{I}{2}$ and the current is given as $I = \frac{ev}{2\pi r}$. Consequently equating these two forces we have

$$\frac{mv^2}{r} = \frac{\mu_0 e^2 v^2}{32\pi^2 r^2}. \quad \textbf{2.57}$$

This evolves the semi-classical radius as

$$r = \frac{\mu_0 e^2 v^2}{32\pi^2 m}. \quad \textbf{2.58}$$

For the rotating particle, the angular momentum is known as

$$L = mvr. \quad \textbf{2.59}$$

Again the angular momentum in the case of the electron is $\frac{\hbar}{2}$. Hence the velocity comes out to be as

$$v \approx 10^4 \text{ m/s}. \quad \textbf{2.60}$$

Using a velocity as $v = c$, we get the angular momentum as

$$L \approx 27.51 \times 10^{-22} \text{ J-s}. \quad \textbf{2.61}$$

But the expected order is 10^{-34} which is far from the above result. This prompts us to question about this particular model as a partly failure.

2.3.7 Semi-classical model with tachyonic matter

An extended rotating object [33] was considered by G. N. Ramachandran and his colleagues to describe the model of the electron spin. The object is composed of two different parts. The core is with the linear velocity less than the speed of light in free space, i.e. c and this part is enclosed within a matter of the tachyonic matter [33] with velocity greater than c. This idea was introduced to get a better stability of the particle. This is an interesting presentation with the scheme of two different sorts of velocity according to the distance from the centre of the object. The mass of the electron is presented here as

$$E = mc^2 = \frac{4}{3}\pi k c^2 R^3, \qquad 2.62$$

and the spin is given as

$$S = \hbar\sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)} = \pi^2 kcR^5 / 5R_C. \qquad 2.63$$

Here R is the radius of the spherical object. In both the expressions of the mass and the spin, the factor k is present. From equation 2.62, k seems to be the mass density of the object, though it is not well-explained in the article [32].

The key limitation of this model comes in terms of the use of the tachyonic matter. If instead of tachyonic matter, the velocity could be brought down up to c or less, this model may be a better choice then. Secondly, this model incorporates the lowest radius as Compton radius R_C whereas upper limit goes up to $\sqrt{2}R_C$ only. Thus this model rejects the other electromagnetic phenomena related to the electron and also the other radii.

2.3.8 Semi-classical picture of the electron spin

The relation between the spin angular frequency of the electron and its rest energy is developed with the help of its semi-classical model. Here the charge density and the mass density both are considered to be directly varying with the volume of the sphere, which prompts the distribution of the charge over the sphere

[34]. A close value of the spin angular momentum is deduced. But how the electrostatic energy will account for the rest mass of the electron is not discussed in the model. Again the distribution pattern of the mass and the charge is expressed there as $\rho_m \propto \rho_e \propto r^3$. ρ_m is the relativistic mass density and the ρ_e is the charge density distribution. But it is not explained whether the distributions follow any kind of relation between the mass and the charge or not.

2.4 Extended models with point-like charge

2.4.1 Bunge model

Without executing Foldy-Wouthuysen transformation [35-37], a mean position operator with a smooth motion is derived by M. Bunge to illustrate the picture of the electron [6]. This is actually a model of the electron, which represents the extended electron. This is a Compton-sized model with a point charge and distributed mass. L. L. Foldy and S. A. Wouthuysen gave the mean-position operator that Bunge says the center-of-mass operator. The operator given by them is not oscillatory and its time derivative is proportional to the momentum, and this is read as [6]

$$x' = e^{-\iota s} x e^{\iota s}, \qquad \textbf{2.64}$$

where x is the point where charge is concentrated. The displacement operator proposed by Bunge as

$$T = e^{\iota s}, \qquad \textbf{2.65}$$

where $s = \frac{\Lambda}{2}\sum_{0}^{3} {}_{\mu}\gamma^{\mu} \frac{\partial}{\partial x^{\mu}}$, $\Lambda = \frac{\hbar}{m_0 c}$, $\mu = 0,1,2,3$ and $\gamma = \begin{vmatrix} 0 & \sigma \\ -\sigma & 0 \end{vmatrix}$.

This gives a model of the electron within the frame of the standard representation of the one particle theory. This is a mixed model of the concepts of extended and the point particle. Here Lorentz force has been employed to the terminus of the trembling vector and that portrays the charge of Dirac's electron [6] at that point and its oscillation around the mean position co-ordinate with amplitude of the Compton radius. This model again tells about the spreading of the mass over a region of dimensions of a Compton wavelength.

But this model does not predict a sharp boundary or the exact path of the motion of the charge. Again how long it can go or where the charge cannot go is not mentioned. M. H. MacGregor also mentioned about the similarity of this model with his relativistic spinning sphere model of the electron with the limitations of the detailed relativistic structure and the equatorial location of the charge [7].

2.4.2 Relativistic spinning sphere model

Relativistic spinning sphere model [7, 38-39] of the electron depicts the picture of merely a classical structure of the particle, which involves the spectroscopic properties of the electron at the first order of the fine structure constant, α.

Framing the characteristic features, the sphere is considered to be made of the non-interacting but rigid mechanical mass along with the point charge e, residing at the equatorial zone of the sphere. This spherical model successfully transforms under the Lorentz transformation too. The observed electron spin, the total quantum-mechanical spin and the accurate gyromagnetic ratio are reproduced exactly by this model. MacGregor has shown the context of the different energies corresponding to the electron structure. The spinning mass is taken here as $m_s = \frac{3}{2} m_0$, where m_0 is the rest mass of the electron. Starting with the relationship between the mass and the spin angular momentum of the electron, the RSS model is developed.

MacGregor considered the well-known facts about the electron to build up the relativistic spinning sphere model of the electron. They are: a) the electron is spinning, b) the spin is quantized, c) the spin is $J = \frac{1}{2}\hbar$ and it is very large with respect to the mass of the electron, d) the spherical shape of the electron is not changed due to relativistic rotation.

As the relativistic rotation incorporates the increase in mass, the rest mass m_0 of an element of ring is changed according to

$$m(r) = \frac{m_0(r)}{\sqrt{1-\omega^2 r^2 / c^2}}, \qquad \textbf{2.66}$$

where ω is the angular velocity of the spinning ring and r is the distance of the element from the axis of rotation. For a sphere with radius R, the mass-density of the sphere is figured out as

$$D = \frac{m(r)}{V_{sphere}} = \frac{3m_0(r)}{4\pi R^3\sqrt{1-\omega^2 r^2/c^2}}, \qquad 2.67$$

where V_{sphere} is the volume of the entire sphere. To get the relativistic mass of a spinning sphere, it is easier to explore the axially centred cylindrical mass elements. The volume of the cylindrical element becomes

$$V(r) = 4\pi\sqrt{R^2 - r^2}\,rdr, \qquad 2.68$$

where R is the radius of the sphere and r is the distance of the element from the axis of rotation. So the spinning mass of the element of the ring can be calculated as

$$m_s = \frac{3m_0}{R^3}\sqrt{\frac{R^2 - r^2}{1-\omega^2 r^2/c^2}}\,rdr. \qquad 2.69$$

Therefore for the entire sphere, the spinning mass is

$$M_s = \frac{3M_0}{R^3}\int_0^R \sqrt{\frac{R^2 - r^2}{1-\omega^2 r^2/c^2}}\,rdr, \qquad 2.70$$

where M_0 is the non-spinning mass. With the equatorial velocity c of the spinning sphere, the angular velocity attains the highest value without violating the special theory of relativity as

$$\omega = \frac{c}{R}. \qquad 2.71$$

This value of ω reduces the above equation 2.69 into

$$M_s = \frac{3M_0}{R^3}\int_0^R Rrdr \qquad 2.72$$

which implies

$$M_s = \frac{3}{2}M_0. \qquad 2.73$$

MacGregor has established the model keeping the theory of relativity in his mind. That helped him to write down separately the conditions of non-rotating and the rotating frames. The relativistic moment of inertia of the spinning sphere about the axis of rotation comes out as

$$I = \frac{3}{4}m_0 R^2 = \frac{1}{2}mR^2. \qquad 2.74$$

In the conclusion, it is shown that the mechanical mass constitutes 99.9% of the observed mass.

2.4.3 Dynamical spinning sphere model

A classical model, called dynamical spinning sphere model, for a spinning electron is proposed by M. Rivas in the framework of kinematical formalism [8, 40-41]. The dynamics of the system is expressed in terms of some arbitrary evolution parameter τ though the Lagrangian is independent of τ. In the generalized Lagrangian, some kinematical variables will be the time derivatives of some other kinematical variables depending on the nature of the higher order derivatives. Thus the dynamical variables will no longer be of second order, rather fourth order or of higher, which advocate the condition for the charge position of a spinning particle.

In this formalism, author has made good use of Galilei group of space-time transformation [40] to represent the dynamics of the elementary particles. The action of a group element $g \equiv (b, \vec{a}, \vec{v}, \vec{\alpha})$ on a space-time point $x \equiv (t, \vec{r})$, is represented by $x' = gx$. $\vec{a}$ is the space parameter and b represents the time parameter whereas the velocity parameter is $\vec{v}$ and $\vec{\alpha}$ is dimensionless. The corresponding expression for the x' is given as

$$x' = \exp(bH)\exp(\vec{a}.\vec{P})\exp(\vec{v}.\vec{K})\exp(\vec{\alpha}.\vec{J})x, \qquad \textbf{2.75}$$

where H, $\vec{P}$, $\vec{K}$ and $\vec{J}$ are the generators of the Poincare group. This is a rotation of the point, followed by a pure Galilei transformation and a space and time translation.

The Lagrangian for the non-relativistic spinning elementary particle is given as

$$L = T\dot{t} + \vec{R}.\dot{r} + \vec{U}.\dot{u} + \vec{V}.\dot{\rho} \qquad \textbf{2.76}$$

and the functions are $T = \frac{\partial L}{\partial \dot{t}}$, $R_i = \frac{\partial L}{\partial \dot{r}^i}$, $U_i = \frac{\partial L}{\partial \dot{u}^i}$ and $V_i = \frac{\partial L}{\partial \dot{\rho}^i}$. In general, they will be the functions of the ten kinematical variables $(t, \vec{r}, \vec{u}, \vec{\rho})$ and the homogeneous functions of zero degree of the derivatives $(\dot{t}, \dot{r}, \dot{u}, \dot{\rho})$. Similarly the relativistic condition is described as

$$L = T\dot{t} + \vec{R}.\dot{r} + \vec{U}.\dot{u} + \vec{W}.\dot{w} \;, \qquad \textbf{2.77}$$

where $T=\frac{\partial L}{\partial \dot{t}}$, $R_i=\frac{\partial L}{\partial \dot{r}^i}$, $U_i=\frac{\partial L}{\partial \dot{u}^i}$ and $W_i=\frac{\partial L}{\partial \omega^i}$. In general, they will be the functions of the ten kinematical variables $\left(t,\vec{r},\vec{u},\vec{\alpha}\right)$ and the homogeneous functions of zero degree of the derivatives $\left(\dot{t},\dot{r},\dot{u},\dot{\alpha}\right)$. Here t represents the time parameter, $\vec{r}$ represents the position, $\vec{u}$ represents the velocity of the particle as $u=c$ and the orientation is given by $\vec{\alpha}$. This model describes the magnetic moment and the spin of the electron in a better way. This is concerned with the orientation part and ultimately describes the spinning sphere model by a fourth order differential equation using Frenet-Serret differential equations [42].

The mass of the particle got less attention in this work and that is a weaker point left by this model, as the mechanical mass plays a crucial role in the electron's total energy. This model has been built more or less considering the Zitterbewewgung model of the electron. But if structural phenomena to be considered, then the limits of the motion should also be defined as well, which is not in this case.

2.5 Other models

2.5.1 The electron in a (3+3)-dimensional space-time

The electron is considered for this model by P. Lancini as a mass-less particle moving in a (3 + 3) dimensional space-time [9, 26]. This was first introduced by P. Demers, R. Mignani and E. Recami and later by E. A. B. Cole [26]. Concept of vacuum polarization is used here. When the electron moves in the extra-dimensional space-time, it is subjected to an attractive force toward the standard space-time. These extra dimensions assist the electron to be associated with a two-dimensional motion, which can produce the existence of the spin as well as the magnetic moment of the electron.

From the Klein-Gordon equation [9], the mass of the electron is attained as an integral constant. The electron is considered here in the extra-dimensional time plane. There it is attracted toward standard space-time. When a negatively charged particle leaves toward the "time" space of the 6-D space-time, immediately a positively charged hole arises at the 4-D space-time left by the particle. Due to the vacuum polarization, an attractive force is presumed along the radial time direction in the extra-dimensional plane and this is considered to be responsible for the spin and the magnetic moment of the electron.

2.5.2 Classical electromagnetic and scalar fields

The electron is regarded here as a composite model of classical electromagnetic and scalar fields [43]. This is a coupled system of two bosons, photon and mass-less boson with spin 0. The electron is described in this approach as a classical electromagnetic-scalar wave related to the equation of motion. The slightly generalized classical Maxwell equations are considered in a specific medium that models the relativistic atom along with the half-spin representation [43]. Charge is here considered as a secondary quality, generated by the interacting electromagnetic-scalar fields. So, at the starting, there are two bosons. One is having spin 1 and the other one is having spin 0. Therefore, the pair of Dirac equations are described here as a double bosonic system.

The electric $\left(\varepsilon(\vec{x})\right)$ and the magnetic $\left(\mu(\vec{x})\right)$ permeabilities are expressed in terms of the mass, m_0 and Φ, the interaction potential as

$$\varepsilon(\vec{x}) = 1 - \frac{\Phi(\vec{x}) + m_0}{\omega} \quad \text{and} \quad \mu(\vec{x}) = 1 - \frac{\Phi(\vec{x}) + m_0}{\omega}. \qquad \textbf{2.78}$$

Here ω is the angular frequency. Thus the mass is related thus with the electromagnetic phenomenon here to couple the two bosons here. Author also gave the statement that the electron's states are the linear combinations of the electromagnetic-scalar field in the quark model of the hadrons [43].

2.5.3 Zitterbewegung model

As the basis of the electron spin and the magnetic moment, Zitterbewegung motion is supposed to be responsible. Several works have been done in the field of Zitterbewegung [44-54]. This is a local circulatory motion of the electron. Though this has been proposed independently by many physicists, E. Schrodinger's contribution is the pioneering [44]. Schrodinger identified the highly oscillatory motion by investigating the behavior of charge [44].

He started with the Hamiltonian for the free electron-positron system as

$$H = c\vec{\alpha}.\vec{p} + mc^2\beta, \qquad \textbf{2.79}$$

where $\vec{\alpha}$ and β satisfy the properties

$$\{\alpha_i, \alpha_j\} = 2\delta_{ij} I \ \ (\text{i, j=1,2,3}) \qquad \textbf{2.80}$$

and $\{\alpha_i, \beta\} = 0$, $\beta^2 = I$. The momentum vector $\vec{p}$ and the position vector (which is a conjugate co-ordinate) $\vec{x}$ are considered with

$$[x_i, x_j] = 0 = [p_i, p_j], \qquad \textbf{2.81}$$

$$[x_i, p_j] = i\hbar\delta_{ij} I, \qquad \textbf{2.82}$$

and to commute with $\vec{\alpha}$ and β.

To represent the Heisenberg picture all these relations can be considered at any one time and the time derivative of any of these operators (say A), which do not depend on the time explicitly, can be expressed as

$$\frac{dA}{dt} = i[H, A]/\hbar. \qquad \textbf{2.83}$$

Hence

$$\frac{d\vec{p}}{dt} = 0 \text{ and } \frac{dH}{dt} = 0, \qquad \textbf{2.84}$$

while

$$\frac{d\vec{x}}{dt} = c\vec{\alpha} \qquad \textbf{2.85}$$

and

$$-i\hbar\frac{d\vec{\alpha}}{dt} = [H, \vec{\alpha}] = -\{H, \vec{\alpha}\} + 2H\vec{\alpha} = -2c\vec{p} + 2H\vec{\alpha}. \qquad \textbf{2.86}$$

Hence the last equation can be re-written as

$$-i\hbar\frac{d\vec{\alpha}}{dt} = 2H\vec{\eta}, \qquad \textbf{2.87}$$

expressing

$$\eta = \vec{\alpha} - cH^{-1}\vec{p}. \qquad \textbf{2.88}$$

At this point Schrodinger noted that

$$-i\hbar\frac{d\eta}{dt} = -i\hbar\frac{d\vec{\alpha}}{dt} = 2H\vec{\eta}, \qquad \textbf{2.89}$$

so that

$$\vec{\eta}(t) = e^{2iHt/\hbar}\vec{\eta}_0 . \qquad 2.90$$

The $\vec{\eta_0}$ in the above equation is a constant operator and is represented as

$$\vec{\eta}_0 = \vec{\eta}(0) = \vec{\alpha}(0) - cH^{-1}\vec{p} . \qquad 2.91$$

One can verify it simply that

$$\{H, \vec{\eta}\} = 0 = \{H, \vec{\eta}_0\} , \qquad 2.92$$

so that it can be written using equation 2.90,

$$\vec{\eta}(t) = \vec{\eta}_0 e^{-2iHt/\hbar} . \qquad 2.93$$

Taking together the equations 2.86, 2.89 and 2.93, Schrodinger obtained the expression

$$\frac{d\vec{x}}{dt} = c\vec{\alpha} = c^2 H^{-1}\vec{p} + c\vec{\eta}_0 e^{-2iHt/\hbar} . \qquad 2.94$$

Integrating we get

$$\vec{x}(t) = \vec{a} + c^2 H^{-1}\vec{p}t + \frac{1}{2}i\hbar c\vec{\eta}_0 H^{-1} e^{-2iHt/\hbar} , \qquad 2.95$$

where $\vec{a}$ is a constant operator of the integration and is expressed as

$$\vec{a} = \vec{x}(0) - \frac{1}{2}\hbar c\vec{\alpha}(0)H^{-1} + \frac{1}{2}\hbar c^2 H^{-2}\vec{p} . \qquad 2.96$$

Therefore equation 2.94 can be written now as

$$\vec{x}(t) = \vec{x}_A(t) + \vec{\xi}(t) \qquad 2.97$$

with

$$\vec{x}_A(t) = \vec{a} + c^2 H^{-1}\vec{p}t . \qquad 2.98$$

This can be considered as the position-operator of a relativistic point-mass. The other part of the equation 2.96 is

$$\vec{\xi}(t) = \frac{1}{2}i\hbar c\vec{\eta}_0 H^{-1} e^{2iHt/\hbar} = \frac{1}{2}i\hbar c\vec{\eta} H^{-1} . \qquad 2.99$$

This depicts a high-frequency Zitterbewewgung motion superimposed on the macroscopioc motion associated with $\vec{x}_A$. The Zitterbewegung motion has the characteristic amplitude $\frac{\hbar}{2mc}$, which is half of the Compton wavelength of the electron.

According to D. Hestenes [47-49], Schrodinger's work raised the question about the possible physical significance which can be attributed to the Zitterbewegung. He pointed out the hints in three different ways stated below as:

A. the Zitterbewegung is a mathematical artifact of the one-particle Dirac theory that does not appear in a correctly formulated quantum field theory.

B. the Zitterbewegung is an erratic motion of the electron due to random electron-positron pair creation and annihilation.

C. the Zitterbewegung is a localized helical motion of the electron with an orbital angular momentum that can be identified with the electron spin.

Barut and his colleagues [44-45] continued their work incorporating the relative motion in the centre-of-mass rest frame. With $\vec{p} = 0$, the Hamiltonian and its inverse become

$$H_r = mc^2\beta \text{ and } H^{-1} = \frac{\beta}{mc^2}, \quad \textbf{2.100}$$

where $\beta = \beta(0)$ is a constant of the motion. Then the relative co-ordinate ξ takes the form

$$\vec{\xi}(t)\,|_{\vec{p}=\vec{0}} = \frac{1}{2} i\lambda\vec{\alpha}(0)\beta e^{-2i\beta ct/\lambda}, \quad \textbf{2.101}$$

with λ as the Compton wavelength. Starting with the facts

$$\frac{d\vec{x}_A}{dt} = \vec{0} \text{ and } \vec{p} = \vec{0} \Rightarrow \vec{P}_{\text{rel}} = \vec{P}_{\text{charge}}, \quad \textbf{2.102}$$

the relative motion or Zitterbewegung is described in terms of the variables $\vec{Q}(t)$ and $\vec{P}(t)$ with Hamiltonian H_r. From the commutation relations they arrived at the harmonic oscillator with

$$\frac{d^2\vec{Q}}{dt^2} + \omega^2\vec{Q} = \vec{0} \quad \textbf{2.103}$$

and

$$\frac{d^2\vec{P}}{dt^2} + \omega^2\vec{P} = \vec{0}, \quad \textbf{2.104}$$

where

$$\omega = \frac{2c}{\lambda} = \frac{2mc^2}{\hbar}. \quad \textbf{2.105}$$

The general solutions come out as

$$\vec{Q}(t)=\vec{Q}(0)\cos\omega t+\frac{\lambda^2}{2\hbar}\vec{P}(0)\sin\omega t \qquad \textbf{2.106}$$

and

$$\vec{P}(t)=\vec{P}(0)\cos\omega t-\frac{2\hbar}{\lambda^2}\vec{Q}(0)\sin\omega t\,. \qquad \textbf{2.107}$$

Proceeding in this way, the states and the operators associated with the Zitterbewegung are obtained. They considered a compact phase space, with three degrees of freedom for a point carrying a charge e and thus derived the relative co-ordinate of the charge as

$$\vec{\xi}\frac{1}{2}i\frac{\hbar}{mc}\vec{\alpha}\beta=\frac{1}{2}i\frac{\lambda}{u}\vec{\alpha}\beta=\frac{1}{u}\vec{Q}\,, \qquad \textbf{2.108}$$

where λ represents the Compton wavelength and it is related to u with the angular frequency by the relation

$$\omega=u\frac{2c}{\lambda}\,. \qquad \textbf{2.109}$$

To develop the model of the electron with the mass, spin and the electricity of the particle, Hestenes [47] begins with the clue that the electron spin is an orbital angular momentum in some instantaneous internal system of the electron, which is called by him as the rest system of the electron. The properties of the electron are considered in this modeling as given below:

(1) the electron is a massless point particle,

(2) the electron undergoes Zitterbewegung with an intrinsic orbital angular momentum or spin of fixed magnitude $s=\frac{\hbar}{2}$,

(3) the Zitterbewegung frequency can be identified with the electron de Broglie frequency

$$\omega_0=\frac{mc^2}{\hbar}\,, \qquad \textbf{2.110}$$

(4) the electron has an electric charge e and

(5) the total free electron self-energy is given by

$$mc^2=E_0+U_0\,, \qquad \textbf{2.111}$$

where E_0 and the U_0 are the kinetic energy and the self energy. This mass-less point particle model is possible only with a velocity of c, which compels a Compton-sized electron only. Again the kinetic energy is also calculable from the above consideration as

$$E_0 = \frac{mc^2}{2}, \tag{2.112}$$

and the self-energy as

$$U_0 = \frac{mc^2}{2}. \tag{2.113}$$

2.5.4 Unified composite model

Scheme of substructure of the fundamental particles was grown up in the last century [20-25]. Sometimes, they are called as Preon model [23], and with some different approach, sometimes they are called as Rishon model [25]. These sorts of endeavors display the particles in more elementary way. Though the experimental confirmation for them is a big deal and as they are not verified till, they may remain proposals only. Unified composite model of all fundamental particles and forces is one of these approaches. The concept of substructures introduces the possibility of smaller size and greater energy than in the present scenario. As well they claim for some more elementary particle and hence it is a beyond standard model concept.

A unified model of the Nambu-Jona-Lasinio type is described by H. Terazawa [20]. This is based upon the relation between the fine structure constant and the Newtonian gravitational constant. This relation is given as

$$\alpha = \frac{3\pi}{\left(\sum Q^2\right)\ln\left(\dfrac{4\pi}{\kappa_0 N_0 G m^2}\right)}. \tag{2.114}$$

This is known as G-α relation. Re-arranging G-α relation from equation 2.114, one can express the mass of the particle in terms of the fine structure constant and G as

$$m = \left(\frac{4\pi}{\kappa_0 N_0 G}\right)^{\frac{1}{2}} \exp\left[-\frac{3\pi}{2\alpha.\sum Q^2}\right]. \tag{2.115}$$

The platform gets ready with above equations. Testing with them for leptons and quarks one can now put a sub-quark model [21] as below:

$$\nu_e = (w_1 h_1 C_0),\quad \nu_\mu = (w_1 h_2 C_0),\quad \nu_\tau = (w_1 h_3 C_0),$$
$$e = (w_2 h_1 C_0),\quad \mu = (w_2 h_2 C_0),\quad \tau = (w_2 h_3 C_0),$$
$$u_i = (w_1 h_1 C_i),\quad c_i = (w_1 h_2 C_i),\quad t_i = (w_1 h_3 C_i),$$
$$n_i = (w_2 h_1 C_i),\quad s_i = (w_2 h_2 C_i),\quad b_i = (w_2 h_3 C_i),$$

for i =1, 2, 3.

This model consists of an iso-doublet of spinor subquarks with charges $\pm\frac{1}{2}$, w_1, w_2 and a Pati-Salam color-quartet of scalar subquarks with charges $+\frac{1}{2}$ and $-\frac{1}{6}$, C_0 and $C_i (i = 1, 2, 3)$. w_1 and w_2 are called the "wakems" which are responsible for weak and electromagnetic interactions. C_0 and C_i are called as "chroms", which stand for colors. They are the singlet and triplet under SU(3) color symmetry. This is consisting of the spinor and scalar subquarks with the charge $+\frac{1}{2}$, w_1 and C_0. The left-handed w_L and the right-handed w_{1R} and w_{2R} are a doublet and singlets of the Weinberg-Salam SU (2) respectively. *N*-plet of the unknown H-symmetry is formed by the h_i's. Therefore, with chroms one can set SU(4) symmetry for sub-colors, which can be described by the quantum subchromodynamics (QSCD), the Yang-Mills gauge theory of SU(4). Subquark charges satisfy Nishijima-GellMann rule of $Q = I_w + (B - L)/2$ and the anomaly-free condition of $\sum Q_w = \sum Q_C = 0$ [22].

Unified composite model presents the electron as a composite S-wave ground state of the spinor sub-quark w_2 of charge $-\frac{1}{2}$ and spin 0. The quantum numbers of the electron are then constituted by those of sub-quarks. In this regard the properties of the electron are described there. Therefore, the spinor sub-quark and the sub-color singlet should come out from the electron. But till the date no experiment is devised to make this possible at least. Until and unless any decay channel of the electron is identified, it is almost impossible to comment positively on this sub-quark phenomenon.

2.6 Concluding remarks

Here we have tried to discuss different kinds of models of the electron, so that the idea can be grown in which way one can proceed. This incorporates both theoretical and experimental facts. But it was not possible for us to describe all the models given till the date. One can have a look for further study in references [55-62]. Some of the above-mentioned models are in fact carrying some information, which do not stand in the modern day physics, and we have tried to figure out their problems. The above study shows us very distinctly that two sorts of approaches are possible with the present experimental conditions. First kind is to go along with the standard model predictions, though it is difficult to explain the finite mass, the spin and the charge of the electron with the model. Second one is to chose the extended model with point-like charge and indeed this sort of ideas have better configurations with the existing properties.

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No problem is too small or too trivial if we can really do something about it.
– Richard Phillips Feynman

Chapter 3

The radii of the electron and their relations

Following J. J. Thomson's discovery of the electron in 1897, varieties of experimental and theoretical conditions have been made to clarify its dynamic and static properties throughout the last century that we have illustrated partially in the chapter 1. Theoretically, different shapes and models were presumed for the electron. A brief prologue about the models of the electron is given in the previous chapter. Those models are based upon the properties of the electron, which reveal the different sizes of the electron.

Depending upon the various models, we get different types of radii of the electron. Here we have discussed their origin and the significance in the behaviour of the electron. It is noteworthy that though their origins may be very different, they can be connected well together. We attempt here to put different types of the radii of the electron in such a manner that the originating phenomena can also be united. Following the trend of the relations of the radii involving fine-structure constant, we can offer the mathematical formalism of the charge radius with the order in agreement with the LEP result from CERN. In addition, we attempted to get a relation between Rydberg constant and the electron structure.

3.1 Classical radius

According to the theory of Thomson [1], for a charged particle in uniform motion with velocity v, the corresponding electromagnetic field will have a kinetic energy

$$T_{elm} = f \cdot \frac{e^2}{R_0 c^2} \cdot \frac{v^2}{2}, \quad \textbf{3.1}$$

where f is a numerical factor that depends on the charge distribution within the sphere of radius R_0 and total charge e. Comparing the known form of the kinetic energy as $T_{elm} = \frac{1}{2} mv^2$ with equation 3.1, we have [1]

$$R_0 = f \cdot \frac{e^2}{mc^2}. \quad \textbf{3.2}$$

Abraham-Lorentz-Poincare model [1] also describes classical electron radius. This is a model with spherically symmetric charge distribution. Classical electron radius is worked out from this model, when the self-energy of the charged sphere is equated to its total energy. For surface distribution of charge, radius becomes $R_0 = \frac{1}{2} \cdot \frac{e^2}{mc^2}$ and for volume distribution the radius comes out as $R_0 = \frac{3}{5} \cdot \frac{e^2}{mc^2}$. The factors $\frac{3}{5}$ or $\frac{1}{2}$ depend on the nature of the distribution of the charge and it is denoted by f in equation 3.2. The generalized version of the classical electron radius is given [2] as

$$R_0 = \frac{e^2}{mc^2}. \quad \textbf{3.3}$$

The value of R_0 is 2.82×10^{-13} cm.

The classical electron radius is also involved in the scattering of radiation by a free charge, shown by Thomson. This scattering cross-section $\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 = \frac{8\pi}{3} R_0^2$ is also called as Thomson cross-section [2-3]. Hence a classical distribution of charge should have a radius of this order if its electrostatic self-energy is equal to the electron mass [3]. As it is mentioned above, the classical electron radius is roughly the size of the electron would have its mass to be completely due to its electrostatic potential energy. But the idea of its mass being completely due to its electrostatic potential energy is not supported nowadays. In fact, a small contribution of electromagnetic mass is also witnessed [1-2]. The electromagnetic mass is represented by J. Schwinger in terms of electromagnetic self-energy [4]. In modern classical-limit theories, e.g. in non-relativistic Thomson scattering and the relativistic Klein-Nishina formula [5], classical electron radius is used. Also this is the length scale at which renormalization becomes important in quantum electrodynamics.

3.2 Compton radius

Compton radius lies at the borderline of the classical and the quantum physics. In some of the articles, it is told as the least possible classical radius and classical physics does not go beyond this [6-7]. As reduced Planck's constant, $\hbar$ is involved, this is also considered as quantum mechanical measurement. Compton radius of an elementary particle is the length scale at which relativistic quantum field theory works. In other words, Compton radius of the electron is the characteristic length scale of QED. Experimentally Compton's work provided 2×10^{-10} cm [8] or in a more modified form $(1.85\pm0.05)\times10^{-10}$ cm [9] as the radius of the electron.

Energy of an elementary particle can be written with the help of particle nature as well as the wave nature of the particle. The Einstein equation of the energy is known as

$$E = mc^2 \tag{3.4}$$

and the Planck-Einstein relation

$$E = \hbar\omega . \tag{3.5}$$

Here $E, m, \hbar, \omega$ and c are the energy of the electron, mass of the electron, Planck constant, angular velocity of the particle and speed of light in free space respectively. From equations 3.4 and 3.5, we have $mc^2 = \hbar\omega$ and angular velocity becomes

$$\omega = \frac{mc^2}{\hbar} . \tag{3.6}$$

If the rotational motion of the particle is associated with a linear velocity $v = c$, equation 3.6 gives the corresponding Compton radius as

$$R_C = \frac{\hbar}{mc} . \tag{3.7}$$

From Compton-effect, the difference between the wavelength λ and λ_0 gives $\Delta\lambda = \lambda - \lambda_0 = \frac{h}{mc}(1-\cos\theta)$ and $\frac{h}{mc}$ divided by 2π is Compton radius of electron. Here λ, λ_0, h, m, c and θ are the wavelength after scattering, initial wavelength, Planck constant, rest mass of the electron, speed of light in free space and the scattering angle respectively. Compton radius of electron is denoted as R_C and it is measured as $R_C = 3.86\times10^{-11}$ cm. In different classical electron models, R_C is directly used to get the spin $\frac{\hbar}{2}$. The difference between the wavelengths and the introduction of $\hbar$ put the Compton radius in the region of wave-nature. Schwinger's idea of total mass of electron, comprising of the mechanical mass and the electromagnetic mass, supports this.

3.3 Electromagnetic radius

Electrostatic energy of the electron [2] is expressed as

$$W_E = \frac{e^2}{R}, \qquad \textbf{3.8}$$

where e is the charge of the electron and R is the corresponding radius of the spherical electron. W_E was considered as the total energy during the derivation of the classical electron radius. But this provides only the electrostatic part. Hence the magnetic part was required and was added later. With the introduction of the magnetic moment of electron, the expression comes out as [10]

$$W_{em} = \frac{e^2}{R} + \frac{\mu^2}{c^2R^3}, \qquad \textbf{3.9}$$

where μ is the magnetic moment, R is the radius and c is the velocity of light in free space. The electromagnetic condition is regarded here.

If we consider the entire energy as electromagnetic energy, then the equation 3.9 can be re-written as

$$mc^2 = \frac{e^2}{R} + \frac{\mu^2}{c^2R^3}. \qquad \textbf{3.10}$$

The spin component is calculated as [10]

$$S = \frac{e\mu}{c^2R}. \qquad \textbf{3.11}$$

Using equation 3.11 in equation 3.10 with the replacement of μ, we have

$$mc^2 = \frac{e^2}{R}\left[1+\left(\frac{Sc}{e^2}\right)^2\right], \quad 3.12$$

and radius becomes

$$R_{em} = \frac{e^2}{mc^2}\left[1+\left(\frac{Sc}{e^2}\right)^2\right]. \quad 3.13$$

Charge of the electron is of the very small value whereas c is very high. Therefore $\frac{e^2}{c}$ becomes a negligibly small quantity with respect to the spin of the electron. So we have $S >> \frac{e^2}{c}$. Using this approximation and putting mass = m, charge = e, and spin = $\hbar$, the electromagnetic radius [10] becomes

$$R_{em} = \frac{\hbar^2}{me^2}. \quad 3.14$$

This radius is $\sim 10^4$ times larger than classical electron radius and $\sim 10^2$ times larger than Compton radius. This is also known as quantum Bohr radius of hydrogen atom. Due to the involvement of the magnetic moment, this radius becomes larger than the Compton radius and the classical radius.

3.4 QED charge distribution for a bound electron

According to the scattering experiments, the electron is regarded as a point-particle, but its appearance in atomic bound states is not point-like and the Lamb shift [11] experiment demands the presence of the electric charge over a region of space that is comparable to R_C [2]. Hence the electron bound-state charge distribution radius, deduced from the Lamb shift experiments, is quite large. In the hydrogen atom, the charge on the electron appears to be spread out over a large region of space compared to the intrinsic size of the charge itself. QED calculations give accurate magnitude of the effect, but not a very clear explanation. Zitterbewegung motion [12-13], revealed by the Lamb shift is a phenomenon that shows a large electron charge distribution radius R_{QED} [2]. Vacuum polarization [11, 14] is another standard QED effect, which leads to a Coulomb polarization of the vacuum state by the charge e, where this polarization broadens over a distance that is comparable to the Compton radius R_C. This broadening of the electric field of the charge and the spatial location of the charge specify the effective bound-state QED charge radius $R_{QED} \cong R_C$ [2].

3.5 Quantum mechanical Compton radius

Relativistic spinning sphere model of the electron is a semi-classical approach that deals with a classical electron without violating QED. Relativistic moment of inertia [2] of the spinning sphere is $I = \frac{3}{4} m_0 R_C^2 = \frac{3}{2} m R_C^2$ where $m = \frac{3}{2} m_0$ and R_C is Compton radius of the sphere. Here m_0 is the rest mass of the electron and m is the spinning mass. The observed angular momentum of the electron is $J = I\omega = \frac{\hbar}{2}$, with $R_C = \frac{\hbar}{mc}$ and $\omega = \frac{mc^2}{\hbar}$.

The relation between the total angular momentum vector and the total angular momentum quantum number is given as $J = \sqrt{j(j+1)}\hbar$. As $j = \frac{1}{2}$ for the electron, the value comes out as

$$J = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\hbar = \frac{\sqrt{3}}{2}\hbar . \qquad \textbf{3.15}$$

Therefore, one can write the expression of the total angular momentum together with the moment of inertia and the angular velocity as

$$\frac{\sqrt{3}}{2}\hbar = I\omega . \qquad \textbf{3.16}$$

Relativistically spinning sphere model gives the result as

$$J = \frac{1}{2} mRc . \qquad \textbf{3.17}$$

Hence the Compton radius is modified with this quantum mechanical behaviour as

$$R_{QMC} = \sqrt{3} R_C . \qquad \textbf{3.18}$$

Quantum mechanical Compton radius confirms the quantum mechanical behaviour of the electron and its value is $R_{QMC} = 6.69 \times 10^{-11}$ cm.

3.6 QED-corrected quantum mechanical Compton radius

To make theoretical prediction of the magnetic moment of the electron consistent with the experimental results, Schwinger introduced a correction to the magnetic moment [4] known as Schwinger correction [2]. This gives a more or less an accurate value with the form as [2, 4]

$$\mu = \frac{e\hbar}{2mc}\left(1+\frac{\alpha}{2\pi}\right). \qquad \textbf{3.19}$$

This form of the magnetic moment portrays the composite nature of the mass of the electron, which is considered to be a sum of the mechanical mass and the electromagnetic mass.

With Schwinger correction, the spinning mass can be written as,

$$m_s = m\left(1-\frac{\alpha}{2\pi}\right), \qquad \textbf{3.20}$$

where $\frac{\alpha}{2\pi}$ is the Schwinger correction term and the electromagnetic mass is termed as

$$m\cdot\frac{\alpha}{2\pi} = \Delta m . \qquad \textbf{3.21}$$

Quantum mechanical Compton radius is modified again by MacGregor [2] with the introduction of Schwinger correction and QED-corrected quantum mechanical Compton radius is expressed as

$$R_{QMC}^{\alpha} = \sqrt{3}R_C\left(1+\frac{\alpha}{2\pi}\right). \qquad \textbf{3.22}$$

3.7 Magnetic field radius

Four different kinds of mass and equivalently energy are attributed to the electron. They are electrostatic self-energy (W_E), magnetic self-energy (W_H), mechanical mass (W_M) and gravitational mass (W_G). Electrostatic self-energy is expressed in equation 3.8. This involves only the electric part. Magnetic self-energy is the energy due to the self-magnetic field of the charge of the electron. The 99.9% [2] of the electron-energy is the mechanical mass which is the observed electron mass. Gravitational mass is negligible at the sub-atomic length-scale and magnetic self-energy is about only 0.1% of the total energy of the electron [2].

Rotation of charge gives birth to current and magnetic field. Magnetic field introduces the magnetic self-energy W_H. The corresponding radius is known as magnetic field radius and it is represented by the symbol R_H.

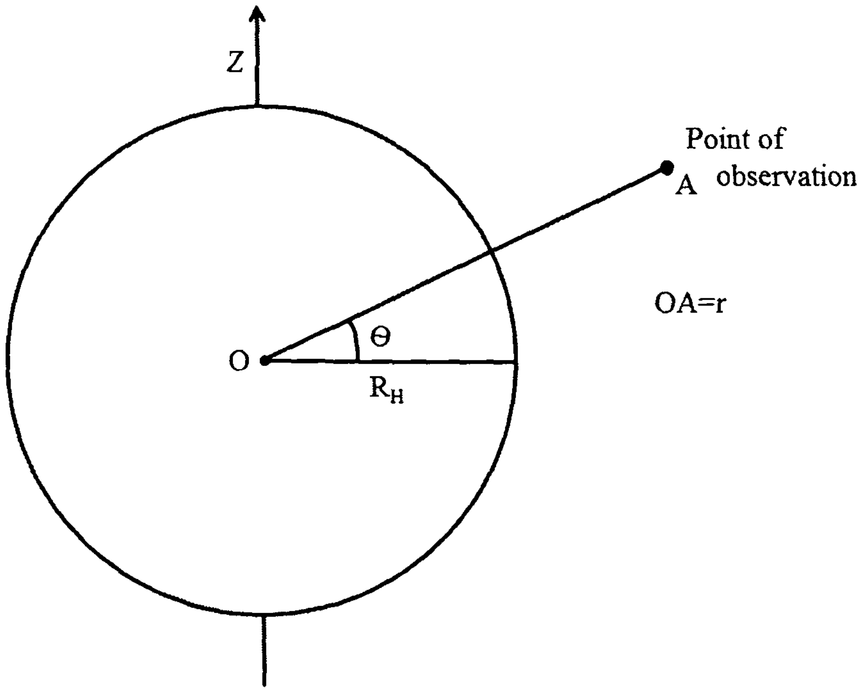

Figure 3.1: Magnetic field radius in the framework of spherical electron

The representation of the magnetic moment of the electron can be set with the help of a current loop. Using polar co-ordinates and orienting the axis of the current loop along the z-axis, the asymptotic magnetic field components [2-3] are obtained as

$$H_r = \frac{2\mu Cos\theta}{r^3} \text{ and } H_\theta = \frac{\mu Sin\theta}{r^3}. \qquad \textbf{3.23}$$

Here μ is the magnetic moment, θ is the angle between the z-axis and the point of observation, r is the distance of the point of observation from the origin of the co-ordinate. Magnetic self-energy W_H is represented as $W_H = \frac{1}{2}\int_V B.Hd^3x$. Here B is the magnetic field and H is the auxiliary magnetic field.

It can be divided into parts as W_H^{Ext} and W_H^{Int} depending on [2] relativistic spinning sphere radius $r > R_H$ or $r < R_H$. Hence $W_H = W_H^{Ext} + W_H^{Int}$. When $r > R_H$, external self-energy W_H^{Ext} will be the energy and when $r < R_H$, the corresponding energy will be W_H^{Int}. So [2]

$$W_H^{Ext} = \frac{\mu^2}{8\pi}\int_{R_H}^{\infty}\int_0^{\pi}\left(\frac{1}{r^6}\right)\left(3\mathrm{Cos}^2\theta + 1\right)2\pi r^2 \mathrm{Sin}\theta d\theta\, dr = \frac{\mu^2}{3R_H^3} \qquad \textbf{3.24a}$$

$$W_H^{Int} \geq \frac{\mu^2}{8\pi}\int_0^{R_H}\int_0^{\pi}\left(\frac{1}{r^6}\right)\left(3\mathrm{Cos}^2\theta + 1\right)2\pi r^2 \mathrm{Sin}\theta d\theta\, dr = \frac{\mu^2}{3R_H^3} \qquad \textbf{3.24b}$$

Addition of equations 3.24a and 3.24b produces

$$W_H^{Tot} \geq \frac{2\mu^2}{3R_H^3}. \qquad \textbf{3.25}$$

We consider here the 'equal' sign only to calculate with the minimum energy.

As we have mentioned already, the electromagnetic self-energy of a free electron can be described in terms of electromagnetic mass is a small correction to the mechanical mass [2]. Hence we have Schwinger correction term as $\Delta m \cong m.\frac{\alpha}{2\pi}$ [2]. So magnetic-self energy is written as

$$W_H = c^2 \Delta m. \qquad \textbf{3.26}$$

Equating the expressions of 3.25 and 3.26 for W_H, we have

$$mc^2 \cdot \frac{\alpha}{2\pi} = \frac{2\mu^2}{3R_H^3}. \qquad \textbf{3.27}$$

As fine-structure constant is $\alpha = \frac{e^2}{\hbar c}$, from equation 3.27, we have

$$mc^2 \cdot \frac{e^2}{\hbar c} \cdot \frac{1}{2\pi} = \frac{2\mu^2}{3R_H^3}. \qquad \textbf{3.28}$$

Therefore, the magnetic field radius can be expressed as

$$R_H^3 = \frac{4\pi\mu^2}{\alpha mc^2}. \qquad \textbf{3.29}$$

The formulation of the magnetic moment of the electron is known as

$$\mu = \frac{e\hbar}{2mc}. \qquad \textbf{3.30}$$

Using the expression of the magnetic moment from equation 3.30 and writing the fine structure constant in terms of e, $\hbar$ and c in the equation 3.29, one can get the simple form of the magnetic field radius as

$$R_H = \pi^{1/3}\left(\frac{\hbar}{mc}\right). \qquad \textbf{3.31}$$

The bracketed term in the right hand side of the equation 3.31 is known to us according to equation 3.7 as the Compton radius of the electron. This prompts us to write the equation 3.31 as

$$R_H = \pi^{1/3} R_C. \qquad \textbf{3.31a}$$

Schwinger-corrected form of the magnetic moment of the electron can be written as

$$\mu = \frac{e\hbar}{2mc}\left(1 + \frac{\alpha}{2\pi}\right). \qquad \textbf{3.32}$$

Using the equation 3.32 in equation 3.29, the expression of R_H is obtained as

$$R_H^3 = \frac{4\pi}{3mc^2\alpha}\left\{\left(1 + \frac{\alpha}{2\pi}\right)^2 \frac{e^2\hbar^2}{4m^2c^2}\right\}. \qquad \textbf{3.33}$$

With the help of equation 3.7, we re-write equation 3.33 as

$$R_H^3 = \frac{\pi}{3} R_C^3 \left(1 + \frac{\alpha}{2\pi}\right)^2. \qquad \textbf{3.34}$$

In equation 3.34, magnetic field radius R_H is expressed in terms of Compton radius.

An approximation of $\frac{\pi}{3} \approx 1$ can produce a simpler form of the magnetic field radius as

$$R_H^3 \cong R_C^3 \left(1 + \frac{\alpha}{2\pi}\right)^2. \qquad \textbf{3.35}$$

Therefore, the magnetic field radius and the Compton radius can be re-written from the equation 3.35 as

$$R_H = R_C \left(1 + \frac{\alpha}{2\pi}\right)^{2/3}. \qquad \textbf{3.36}$$

Equation 3.36 represents the magnetic field radius in terms of Compton radius and fine structure constant. Indeed, it can be said that equation 3.36 is a Schwinger corrected version of magnetic field radius, which has been expressed in equation 3.31a.

3.8 Charge radius

The electron is a charged lepton and its charge plays the most significant role in the electromagnetic behaviour. Dynamics of a charged particle is in generally employed to express the nature of the electron. The electric dipole moment, the magnetic moment and a small fraction of the mass are directly dependent on the charge of the electron. The scattering properties of the electron also insist a vastly smaller radius for its electric charge [15]. In different models of the electron, the charge gets the importance due to the facts of the electrodynamics. Formerly it was assumed that the charge is smeared over the entire electron. In some models, the charge is considered to reside in the equatorial zone [16] also. Several classical and semi-classical models follow the idea of localized charge. But the exact measurement of the size of the charge of the electron is yet to be done. Quantum electrodynamics defines it as a point charge. Recent LEP experiment predicts that the charge of the electron is confined within a region of $R_E < 10^{-17}$ cm or $R_E < 10^{-19}$ m [17]. So the charge radius, R_E is very small compared to R_C or R_0.

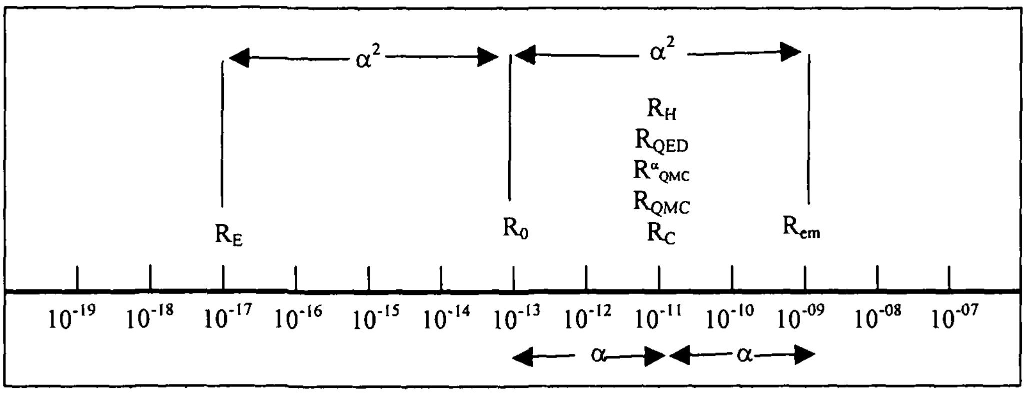

Figure 3.2: Range of the electron radii

Table 3.1: Eight different electron radii

Symbol	Name	Expression
R_0	Classical radius	$\frac{e^2}{mc^2}$
R_C	Compton radius	$\frac{\hbar}{mc}$
R_{QMC}	Quantum mechanical Compton radius	$\sqrt{3}\frac{\hbar}{mc}$
R_{QMC}^{α}	QED-corrected quantum mechanical Compton radius	$\sqrt{3}\left(1+\frac{\alpha}{2\pi}\right)\frac{\hbar}{mc}$
R_{em}	Electromagnetic radius	$\frac{\hbar^2}{me^2}$
R_H	Magnetic field radius	$\geq 0.106 R_C$
R_{QED}	QED charge distribution for a bound electron	$\cong R_C$
R_E	Charge radius	Yet to be calculated

3.9 Gravitational radius

Gravitational electron radius has been proposed by M. Zagoni [18]. This radius is considered as

$$R_G = \frac{Gm}{c^2} \,. \qquad \textbf{3.37}$$

The important condition linked up with this is

$$R_G = R_0 \alpha^r . \qquad \textbf{3.38}$$

where r is denoting the order of α.

The derivation leads to a value of

$$\frac{r}{2} = 10.0019 \pm 0.0001 . \qquad \textbf{3.39}$$

This refers to the fact that the order of α will go up to

$$r \approx 20. \qquad \textbf{3.40}$$

Equation 3.40 describes, together with equation 3.38, the value of the gravitational electron radius as

$$R_G \approx 10^{-53}\,\text{cm}. \qquad \textbf{3.41}$$

This is almost an improbable question with present facilities. Even it is difficult to imagine such a small length and this makes this radius insignificant. It can be noted that, it is even smaller than the Planck's length. Hence this radius is of no-use in the rest of the thesis, we have just mentioned this here as an information.

3.10 Analysis of classical, Compton and electromagnetic radii

Classical radius is expressed mathematically with three universal constants, e, m and c. Indeed Compton radius and the Electromagnetic radius are also formed with the help of these universal constants and Planck's constant are also involved there.

Table 3.2: Basic factors related to different radii

Radii	Power of e	Power of $\hbar$	Power of c	Power of m
R_0	2	0	-2	-1
R_C	0	1	-1	-1
R_{em}	-2	2	0	-1

From table 3.2, a general feature of the radius is seen. The radius or the length is inversely proportional to the mass from which we can predict that

$$\text{Length} \propto \frac{1}{\text{Mass}}, \text{ or}$$

$$\text{Length} = \frac{\text{Constant}}{\text{Mass}}.$$

Considering the total mass as a combination of mechanical mass and electromagnetic mass, we get

$$\text{Length} = \frac{\text{Constant}}{\left(1 + \dfrac{\alpha}{2\pi}\right)\text{Mass}}.$$

Hence the radius or the size of the electron is influenced by the electromagnetic effect also. Fine structure constant is also an electromagnetic phenomenon, which confirms that these radii are related to each other due to some electromagnetic phenomenon. Table 3.2 also shows, as the size decreases, the velocity is decreases. But the power of $\hbar$ is increases with the decreasing radii.

3.11 Mathematical formalism of charge radius

Classical electron radius and Compton radius are connected with the help of the fine structure constant as

$$R_0 = \alpha R_C . \qquad \textbf{3.42}$$

Similarly, the relation between Compton radius and electromagnetic radius is also governed by the fine structure constant as

$$R_C = \alpha R_{em} . \qquad \textbf{3.43}$$

If we want to continue our observation of this behaviour of the electron radii, then we have two options: either it would be greater than electromagnetic radius or it should be lower than classical radius. The first condition can be omitted, as it would be a large figure for the tiny electron. Hence we follow the second condition, which prompts a smaller radius than classical electron radius. We already mentioned that result from LEP predicts the charge radius as $R_E < 10^{-17}$ cm [17]. Figure 3.1 shows us that the region of $R_E < 10^{-17}$ cm coincides with the second order of the fine structure constant starting from classical electron radius. Hence the charge radius can be expressed as

$$R_E = \alpha^2 R_0 = \frac{e^6}{m\hbar^2 c^4} . \qquad \textbf{3.44}$$

Generally, the radii are derived from the energy equations. The α^2 term plays for the magnetic part of the radius. It is to be noted that $\alpha = \frac{1}{137}$ which reduces any term when it is multiplied by α . Now, re-arranging equation 3.44, we can have the total energy of the electron as

$$E = \frac{e^2}{\alpha^{-2} R_E} . \qquad \textbf{3.45}$$

Therefore, equation 3.44 can take the form of the charge radius of the electron and that is a mathematical proposal in agreement with the experimental facts from LEP.

3.12 Relations among different radii

Relations of R_C with R_{QMC}, R_{QMC}^{α} and R_H are stated respectively in equations 3.18, 3.22 and 3.31. Relation between R_C and R_{QED} [2] is given as

$$R_C \cong R_{QED}\,. \qquad \textbf{3.46}$$

Using the relations 3.3, 3.42 and 3.44 together, we have

$$R_E = \alpha^2 R_0 = \alpha^3 R_C\,. \qquad \textbf{3.47}$$

Using the relations 3.42 and 3.35, the relation between R_H and R_0 can be written as

$$R_H^3 = \alpha^{-3}\left(1+\frac{\alpha}{2\pi}\right)^2 R_0^3\,. \qquad \textbf{3.48}$$

Similarly the relation between R_H and R_{em} can be brought out by using equation 3.43 into equation 3.35 as

$$R_H^3 = \alpha^3\left(1+\frac{\alpha}{2\pi}\right)^2 R_{em}^3\,. \qquad \textbf{3.49}$$

Using equation 3.18 into equation 3.35, we have

$$R_H^3 = \frac{R_{QMC}^3}{3\sqrt{3}}\left(1+\frac{\alpha}{2\pi}\right)^2. \qquad \textbf{3.50}$$

A similar replacement of R_C by R_{QMC}^{α} in equation 3.35 with the help of equation 3.22 leads to

$$R_H^3 = \frac{{R_{QMC}^{\alpha}}^3}{3\sqrt{3}}\left(1-\frac{\alpha}{2\pi}\right). \qquad \textbf{3.51}$$

From equations 3.47 and 3.48, we relate magnetic field radius with charge radius as

$$R_H^3 = \alpha^{-9} R_E^3\left(1+\frac{\alpha}{2\pi}\right)^2. \qquad \textbf{3.52}$$

Equation 3.36 states about the relation between Compton radius and magnetic field radius. Expanding $\left(1+\frac{\alpha}{2\pi}\right)^{2/3}$ binomially and neglecting the higher order terms, we get the simpler form of the relation as

$$R_H \cong R_C\left(1+\frac{\alpha}{3\pi}\right). \qquad 3.53$$

Combining equations 3.42 and 3.53, we can bind R_H, R_C and R_0 in a single equation as

$$R_H - R_C = \frac{R_0}{3\pi}. \qquad 3.54$$

Using equation 3.43 in the equation 3.53 we have the form

$$R_H = \alpha\frac{R_C}{3\pi} + \alpha R_{em}. \qquad 3.55$$

Taking α out from left hand side of the equation 3.55 and using equation 3.43, we have

$$\frac{R_H R_{em}}{R_C} = \frac{R_C}{3\pi} + R_{em}. \qquad 3.56$$

Re-arranging equation 3.56 we get the quadratic equation for Compton radius as

$$R_C^2 + 3\pi R_{em} R_C - 3\pi R_H R_{em} = 0. \qquad 3.57$$

Combining equations 3.42, 3.43, 3.18, 3.22 and 3.53, we can write

$$R_C = \alpha^{-1} R_0 = \alpha R_{em} = \frac{R_{QMC}}{\sqrt{3}} = \frac{R_{QMC}^{\alpha}}{\sqrt{3}\left(1+\frac{\alpha}{2\pi}\right)} = \frac{R_H}{\left(1+\frac{\alpha}{3\pi}\right)}. \qquad 3.58$$

Equation 3.58 relates all these six radii in a single one. This relation also states how any two of them are related. We express those relations in the form of ratio in table-3.3. We have aimed to make connection amongst all radii via Compton radius, as it is the significant point between the classical and the quantum domain.

To relate the above radii, the equations involve c, velocity of light in free space, α fine structure constant. α itself involves e, electric charge of electron, $\hbar$, Planck's constant divided by 2π and c. Of these three, e carries the intrinsic property of the electron. Hence, it can be concluded that, charge of the electron plays a significant role for its structure and in the relations amongst electron radii. The linear velocities of the charge of the electron corresponding to the above-discussed radii can also be calculated at ease using different relations amongst the radii. Indeed the properties, which involve the radius of the electron, will follow these relations too.

Table 3.3: Relations in terms of α among different radii

Ratio	α involved relation
$\frac{R_0}{R_C}$	α
$\frac{R_C}{R_{em}}$	α
$\frac{R_0}{R_{em}}$	α^2
$\frac{R_{QMC}}{R_C}$	$\sqrt{3}$
$\frac{R_{QMC}^{\alpha}}{R_C}$	$\sqrt{3}\left(1+\frac{\alpha}{2\pi}\right)$
$\frac{R_{QMC}}{R_{em}}$	$\sqrt{3}\alpha$
$\frac{R_{QMC}^{\alpha}}{R_{em}}$	$\sqrt{3}\alpha\left(1+\frac{\alpha}{2\pi}\right)$
$\frac{R_H}{R_C}$	$1+\frac{\alpha}{3\pi}$
$\frac{R_0}{R_H}$	$\frac{\alpha}{1+\frac{\alpha}{3\pi}}$

3.13 Rydberg constant and the electron radii

Rydberg constant [19] represents the limiting value of the highest wave number i.e. the inverse wavelength, of any photon that can be emitted from the hydrogen atom. For $n = 1$, the wave-number [20] comes out to be $\frac{1}{\lambda} = \text{Constant}\sqrt{R}$ and Rydberg constant is read as the energy only. Rydberg constant not only connects fine-structures of the electronic energy levels of the corresponding spectroscopic radiations [21]. But it also provides a link between the wave nature and the particle nature of the electron by putting a limit of the highest wave-number corresponding to a photon involved in spectroscopic radiation. Fine structure constant and the Planck's constant fix Rydberg constant as [19]

$$R_\infty = \alpha^2 \frac{mc}{2h}. \qquad \textbf{3.59}$$

α^2, m, c and $\hbar$ are the parameters which relate Rydberg constant with the electron. α-quantized results of the electron radii prompt the involvement of Rydberg constant with the electron radii.

The equations 3.7 along with 3.59 provide the relation between the Rydberg constant and the Compton radius as

$$R_\infty = \frac{\alpha^2}{4\pi R_C}. \qquad \textbf{3.60}$$

Using the relation 3.42 in the equation 3.60, we have

$$R_\infty = \frac{\alpha^3}{4\pi R_0}. \qquad \textbf{3.61}$$

Similarly, from equations 3.43 and 3.60 for electromagnetic radius, we arrive at

$$R_\infty = \frac{\alpha}{4\pi R_{em}}. \qquad \textbf{3.62}$$

In the same way, using the equation 3.47 in equation 3.61, we get

$$R_\infty = \frac{\alpha^5}{4\pi R_E}. \qquad \textbf{3.63}$$

The equations 3.60 to 3.63 show the behaviour and the relation of the different radii of the electron with Rydberg constant. They reveal the α-quantized nature of the behaviour. Involvement of this constant with the size of the electron shows that the spectral lines and the fine structure constant have great electromagnetic impact on the size and the structure of the electron.

3.14 Concluding remarks

The long-range scale of the radii of the electron is mentioned here, which shows the enigmatic nature of the electron and its behaviour in some definite conditions. We have developed the relations amongst those radii of the electron and have reported the results in reference [22-23]. The relations brought not only the radii under the single scheme, but also their electromagnetic origins.

To introduce QED-corrected quantum mechanical Compton radius R^{α}_{QMC}, the concept of electromagnetic mass is used. This prompts the connection between mechanical and the electromagnetic mass from the above-discussed relations. This also provides the behaviour in mechanical as well as electromagnetic way.

From the Schwinger-corrected definite form of magnetic field radius R_H, the relations of R_H with other radii are developed. The calculation of magnetic field radius gives the signature of a slightly distorted spherical model of the electron. This model is following the relativistic spinning sphere model and does not violate QED.

Magnetic field radius of the electron is also related with classical radius R_0, Compton radius R_C and electromagnetic radius R_{em} by α-associated terms. Hence the importance of α is realized as relating all sorts of phenomena of the electron. The concept of α-quantized mass leap is developed by MacGregor [24]. Here our approach proposes the α-quantization of the radii of the electron. All these α-quantized factors will be discussed in details in next chapter.

They show how much impact the fine-structure constant leaves in the lepton-structure. In fact, in the calculation of current-loop for the different electron radii also, the α-quantized nature is being followed [22-23, 25].

A significant finding of this part is the mathematical formulation of charge radius of the electron [23], which is based upon the α-quantized nature of the radii of the electron and the LEP results.

All the α-quantized radii are connected together with Rydberg constant, which gives the signature of accurate measurement of classical electron radius. We developed here the relations of Rydberg constant with the other radii of the electron too and they all show the α-quantized nature.

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The only real valuable thing is intuition - Albert Einstein

Chapter 4

α-quantization of the properties of the electron

In the previous chapter, we have discussed about the radii of the electron and observed that they follow the α-quantized nature. Here in the present chapter we shall discuss how the properties of the electron are influenced by the fine structure constant α. In the calculation of the current-loop, we have used different radii and have observed that the nature of the α-quantization is followed. Following the current-loop, the magnetic fields are also seen with α-quantized behaviour.

There we arrive at a new form of the expression of the current and the magnetic field in terms of the charge, the mass and the spin of the electron. We also tested the behaviour of the charge in external magnetic field for different radii. Using the α-quantized mass of the particles, we have attempted to calculate the radius of the muon and the tau. The linear velocities of the charge when it rotates in the surface of the electron with different radii for a spherical electron are also calculated here and they bear the signature of a striking result that classical radius is a length-contracted form of Compton radius of the electron.

4.1 Fine structure constant

Fine structure constant is a magic number with which physicists are obsessed to spend countless hours [1]. This is a dimensionless number and mathematically formulated as $\alpha = \frac{e^2}{\hbar c}$. MacGregor accounted for the double mystery of the fine structure constant to clarify its range of domain along with the mystery of its origin

[1]. To define the range of fine structure constant A. Czarnecki very nicely articulates it as connection between colour of a rose and the hardness of oak via electromagnetism [2]. First, it was described in the article by A. Somerfield [3]. This plays a crucial role in the characteristic features of the electron and this is the coupling constant in the electromagnetic interaction. The deviations of the spectral lines of the atomic spectral lines from the Bohr model introduced the fine structure constant. Here, in this chapter, we are going to observe how fine structure constant affects the different phenomena of the electron.

4.2 α-quantization of the radii of the electron

In the previous chapter, we discussed about the size of the electron, which in deed is a very important property of the electron. The puzzling electromagnetic behaviour of the electron offers eight different measurements of its size in terms of the radii of the electron [4-5]. All these radii are lying on a long-range scale.

Classical radius was expressed by both Thomson and Lorentz [4, 6] aiming the electron as a pure electromagnetic one. We have the expression of classical radius of the electron in the previous chapter and let us recapitulate it as

$$R_0 = \frac{e^2}{mc^2}. \qquad \textbf{4.1}$$

Similarly, we can look back to the other radii also. Compton radius [4] is formulated as

$$R_C = \frac{\hbar}{mc}. \qquad \textbf{4.2}$$

Mathematically equations 4.1 and 4.2 are connected as

$$R_0 = \alpha R_C. \qquad \textbf{4.3}$$

Fine structure constant is relating here classical radius and Compton radius. The relation is considered to be between two phenomena than between two radii of the electron. Electromagnetic radius of the electron [5] is known as

$$R_{em} = \frac{\hbar^2}{me^2}. \qquad \textbf{4.4}$$

In the similar way, equations 4.2 and 4.3 relate two more different electromagnetic phenomena in a form with fine structure constant as

$$R_C = \alpha R_{em}. \qquad \textbf{4.5}$$

Equations 4.3 and 4.5 describe two α-quantized steps amongst three different radii of the electron. Using equation 4.5 in equation 4.3, we can get

$$R_0 = \alpha^2 R_{em}. \qquad \textbf{4.6}$$

In equation 4.6 we arrive at the second order of difference in α between two radii.

α-quantized behaviour of R_0, R_C and R_{em} are shown in equations 4.3, 4.5 and 4.6. Out of all eight radii of electrons, R_{em} is the largest one. Numerically, R_C, R_{QMC}, R_{QMC}^{α}, R_{QED} and R_H are close in results. Mathematical formalism of charge radius is yet to be précised and this is expected to be equal to LEP results or even less than that [7]. One can check easily that 10^{-17} cm is another second order α-quantized state of classical radius and we have $\alpha^2 R_0 \sim 10^{-17}$. Relating these points in the previous chapter, we have given a proposal of mathematical formulation of charge radius of the electron as

$$R_E = \alpha^2 R_0, \qquad \textbf{4.7}$$

which has been expressed in reference [8]. Though we have discussed about the radii of the electron and their relations in the previous chapter, we have recapitulated them for the sake of the discussion about the other properties of the electron.

4.3 Rotation of the charge and α-quantization of the current

The charge passing per unit time per unit area is known as current, $I = \frac{Q}{T}$, where Q is the charge and T is the time by which Q amount of charge passes unit area. To deal with the electron, we say the charge as e. When a small charge e rotates in a circular path of radius R with linear velocity v around the axis of rotation, the current comes out as

$$I = \frac{ev}{2\pi R}. \qquad \textbf{4.8}$$

As we are studying relativistically spinning sphere model [4] and the charge is assumed to be rotating in the equator of the sphere with a velocity of c, we get current-loop corresponding to each radius for this electron model.

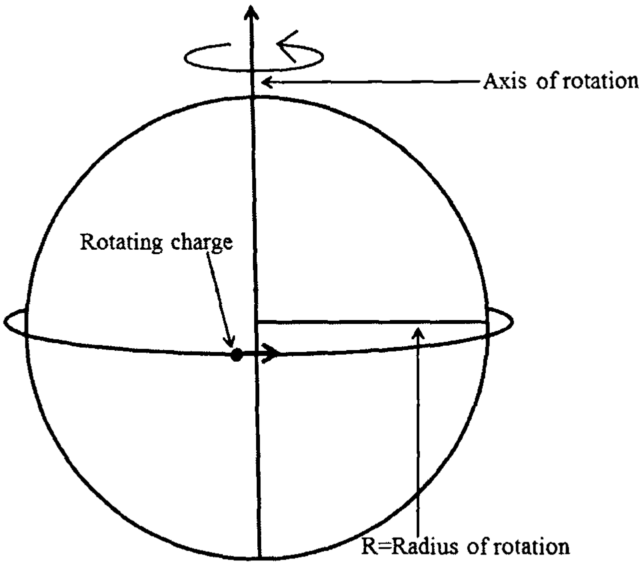

Figure 4.1: Rotation of the charge of the electron around the axis of rotation

Using the expression of Compton radius of the electron from equation 4.2 in the equation 4.8, we have the current-loop

$$I_C = \frac{ec}{2\pi R_C} = \frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right), \quad \textbf{4.9}$$

where e, m and $\frac{\hbar}{2}$ are the charge, the mass and the spin of the electron respectively. Therefore, in other words, this current-loop can be written in terms of the charge, the mass and the spin of the electron as

$$I_C = \frac{c^2}{4\pi}\left(\frac{\text{Charge.Mass}}{\text{Spin}}\right).$$

Using the mathematical expression of classical radius from equation 4.1 in the equation 4.8, the corresponding current-loop can be calculated as

$$I_0 = \frac{ec}{2\pi R_0} = \alpha^{-1}\left[\frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right)\right]. \quad \textbf{4.10}$$

Similarly, using equation 4.4 for the expression of the electromagnetic radius of the electron in equation 4.8, one can get the as

$$I_{em} = \frac{ec}{2\pi R_{em}} = \alpha\left[\frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right)\right]. \qquad \textbf{4.11}$$

Quantum mechanical Compton radius and QED-corrected quantum mechanical Compton radius are defined by other authors [4]. The mathematical expression of quantum mechanical Compton radius [4] is

$$R_{QMC} = \sqrt{3}\frac{\hbar}{mc}, \qquad \textbf{4.12}$$

when putting equation 4.12 into equation 4.8 the concerned current-loop can be given by

$$I_{QMC} = \frac{1}{\sqrt{3}}\left[\frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right)\right]. \qquad \textbf{4.13}$$

More accurate calculation [4] provides QED-corrected quantum mechanical Compton radius of the electron as

$$R^{\alpha}_{QMC} = \sqrt{3}\left(1+\frac{\alpha}{2\pi}\right)\frac{\hbar}{mc}. \qquad \textbf{4.14}$$

Current-loop for R^{α}_{QMC} can be calculated by using equation 4.14 in the equation 4.8 as

$$I^{\alpha}_{QMC} \approx \frac{1}{\sqrt{3}}\left(1-\frac{\alpha}{2\pi}\right)\left[\frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right)\right]. \qquad \textbf{4.15}$$

Magnetic moment of the electron was calculated by G. E. Uhlenbeck and S. A. Goudsmit as $\mu = \frac{e\hbar}{2mc}$ [4]. But the experimental results differed from the theoretical by 0.01%. Solution to this problem was given by J. Schwinger in 1949 [4, 9]. From the virtual emission and absorption of light quanta, the logarithmically divergent self-energy of a free electron arises. The electromagnetic self-energy of a free electron can be described as electromagnetic mass of the electron and this must be added to the mechanical mass of the electron to give the experimental mass. This electromagnetic mass is the above-mentioned correction to the mechanical mass of the electron. Hence the corrected magnetic moment written as [4]

$$\mu = \frac{e\hbar}{2mc}\left(1 + \frac{\alpha}{2\pi}\right), \qquad \textbf{4.16}$$

with $\alpha \approx \frac{1}{137}$ is the fine-structure constant and $\frac{\alpha}{2\pi}$ is known as Schwinger correction term [4, 9]. Magnetic field and current are related as

$$\mu = \frac{IA}{c}, \qquad \textbf{4.17}$$

where $A = \pi R^2$ is the area of the current-loop with the radius R.

Putting equation 4.16 into 4.17, we can get the modified version of the current-loop for Compton radius from equation 4.9, as

$$I_C = \left[\frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right)\right]\left(1 + \frac{\alpha}{2\pi}\right), \qquad \textbf{4.18}$$

In a similar way, for classical radius, the Schwinger-corrected form of current-loop is calculated from equations 4.10, 4.16 and 4.17

$$I_0 = \alpha^{-1}\left[\frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right)\right]\left(1 + \frac{\alpha}{2\pi}\right), \qquad \textbf{4.19}$$

The Schwinger-corrected version of the current-loop for the electromagnetic radius, quantum mechanical Compton radius and the QED-corrected quantum mechanical Compton radius respectively are

$$I_{em} = \alpha\left[\frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right)\right]\left(1 + \frac{\alpha}{2\pi}\right), \qquad \textbf{4.20}$$

$$I_{QMC} = \frac{1}{\sqrt{3}}\left[\frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right)\right]\left(1 + \frac{\alpha}{2\pi}\right), \qquad \textbf{4.21}$$

and

$$I_{QMC}^{\alpha}=\frac{1}{\sqrt{3}}\left[\frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right)\right]. \qquad 4.22$$

4.4 Magnetic field for the different electron radii

In electrodynamics, current I can also be written with the help of current density J as

$$I=\int J.da, \qquad 4.23$$

where da is the area of the element. For magnetic field B, we have

$$\nabla\times B=\frac{4\pi}{c}J. \qquad 4.24$$

According to Stoke's theorem, for a surface S, closed by the curve C

$$\int_S(\nabla\times B).da=\oint_C B.dl, \qquad 4.25$$

where dl is the small line element on the curve C. Using equations 4.23 and 4.24 in equation 4.25, we get Ampere's law

$$\oint_C B.dl=\frac{4\pi}{c}I. \qquad 4.26$$

Magnetic field is expressed in terms of current by equation 4.26. Hence using equation 4.26 we can extend our results of current for the magnetic fields also. For R_C, Ampere's law for Compton radius can be calculated by putting equation 4.9 into equation 4.26 as

$$\oint_C B_C.dl=c\left(\frac{em}{\frac{\hbar}{2}}\right). \qquad 4.27$$

Similar calculations using equations 4.10, 4.11, 4.13 and 4.15 for R_0, R_{em}, R_{QMC} and R_{QMC}^{α} respectively in equation 4.26, we get the magnetic fields as

$$\oint_C B_0.dl=\alpha^{-1}c\left(\frac{em}{\frac{\hbar}{2}}\right), \qquad 4.28$$

$$\oint_C B_{em}.dl = \alpha c\left(\frac{em}{\frac{\hbar}{2}}\right), \qquad 4.29$$

$$\oint_C B_{QMC}.dl = \frac{c}{\sqrt{3}}\left(\frac{em}{\frac{\hbar}{2}}\right), \qquad 4.30$$

and

$$\oint_C B_{QMC}^{\alpha}.dl = \frac{c\left(1-\frac{\alpha}{2\pi}\right)}{\sqrt{3}}\left(\frac{em}{\frac{\hbar}{2}}\right). \qquad 4.31$$

In the previous section we found the α-quantized nature from the expressions of the current for the different electron radii. It is noteworthy that Schwinger corrected magnetic moment can be used for the expression of the magnetic fields also. Hence using equations 4.18 - 4.22 one can get the corrections of the magnetic field equations 4.27 - 4.31 for Ampere's law as

$$\oint_C B_C.dl = c\left(\frac{em}{\frac{\hbar}{2}}\right)\left(1+\frac{\alpha}{2\pi}\right), \qquad 4.32$$

$$\oint_C B_0.dl = \alpha^{-1}c\left(\frac{em}{\frac{\hbar}{2}}\right)\left(1+\frac{\alpha}{2\pi}\right), \qquad 4.33$$

$$\oint_C B_{em}.dl = \alpha c\left(\frac{em}{\frac{\hbar}{2}}\right)\left(1+\frac{\alpha}{2\pi}\right), \qquad 4.34$$

$$\oint_C B_{QMC}.dl = \frac{c}{\sqrt{3}}\left(\frac{em}{\frac{\hbar}{2}}\right)\left(1+\frac{\alpha}{2\pi}\right) \qquad 4.35$$

and

$$\oint_C B^{\alpha}_{QMC}.dl = \frac{c}{\sqrt{3}}\left(\frac{em}{\frac{\hbar}{2}}\right). \tag{4.36}$$

Equations 4.32 – 4.36 produce Ampere's law in terms of the charge, the mass and the spin of the electron. But mathematically, B is inside the integral and a product with line element dl. To get the value of B separately, long straight current carrying wire's approximation [10-11] is used which gives

$$B = \frac{2I}{cR}. \tag{4.37}$$

As equation 4.37 is a modified version of equation 4.26, the equations 4.32 to 4.36 can be modified respectively as

$$B_C = \frac{c}{2\pi R_C}\left(\frac{em}{\frac{\hbar}{2}}\right)\left(1+\frac{\alpha}{2\pi}\right), \tag{4.38}$$

$$B_0 = \frac{\alpha^{-1}c}{2\pi R_0}\left(\frac{em}{\frac{\hbar}{2}}\right)\left(1+\frac{\alpha}{2\pi}\right), \tag{4.39}$$

$$B_{em} = \frac{\alpha c}{2\pi R_{em}}\left(\frac{em}{\frac{\hbar}{2}}\right)\left(1+\frac{\alpha}{2\pi}\right), \tag{4.40}$$

$$B_{QMC} = \frac{c}{\sqrt{3}}\frac{1}{2\pi R_{QMC}}\left(\frac{em}{\frac{\hbar}{2}}\right)\left(1+\frac{\alpha}{2\pi}\right) \tag{4.41}$$

and

$$B^{\alpha}_{QMC} = \frac{c}{\sqrt{3}}\frac{1}{2\pi R^{\alpha}_{QMC}}\left(\frac{em}{\frac{\hbar}{2}}\right). \tag{4.42}$$

Here we get the expressions for magnetic field corresponding to five different radii of the electron, when the charge is in a rotational motion with linear velocity c.

4.5 Generalized current-loop and magnetic field

For a rotational motion of charge on relativistic spinning sphere model, we have the equations 4.22 to 4.26 of the current-loop. The remarkable thing is that, all of these five expressions carry a common factor $\frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right)$. We say this common factor as generalized current

$$I_G = \frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right). \qquad \textbf{4.43}$$

In fact, all of the above current-loops, i.e. equations 4.18 to 4.22, can be re-written respectively in terms of the generalized current-loop as

$$I_C = I_G\left(1+\frac{\alpha}{2\pi}\right), \qquad \textbf{4.44}$$

$$I_0 = \alpha^{-1} I_G\left(1+\frac{\alpha}{2\pi}\right), \qquad \textbf{4.45}$$

$$I_{em} = \alpha I_G\left(1+\frac{\alpha}{2\pi}\right), \qquad \textbf{4.46}$$

$$I_{QMC} = \frac{1}{\sqrt{3}} I_G\left(1+\frac{\alpha}{2\pi}\right) \qquad \textbf{4.47}$$

and

$$I_{QMC}^{\alpha} = \frac{I_G}{\sqrt{3}}. \qquad \textbf{4.48}$$

In the equations 4.38 to 4.42 of magnetic field also, the term-generalized current-loop is present. Hence equations 4.38 to 4.42 can be re-written as

$$B_C = \frac{2I_G}{c^2 R_C}\left(1+\frac{\alpha}{2\pi}\right), \qquad \textbf{4.49}$$

$$B_0 = \frac{2\alpha^{-1} I_G}{c^2 R_0}\left(1+\frac{\alpha}{2\pi}\right), \qquad \textbf{4.50}$$

$$B_{em} = \frac{2\alpha I_G}{c^2 R_{em}}\left(1+\frac{\alpha}{2\pi}\right), \qquad \textbf{4.51}$$

$$B_{QMC} = \frac{2I_G}{\sqrt{3}c^2 R_{QMC}}\left(1+\frac{\alpha}{2\pi}\right) \qquad 4.52$$

and

$$B_{QMC}^{\alpha} = \frac{2I_G}{\sqrt{3}c^2 R_{QMC}^{\alpha}}. \qquad 4.53$$

The current-loop equations 4.44 to 4.48 for different radii can now be related with each other as

$$I_C = \alpha I_0 = \alpha^{-1} I_{em} = \sqrt{3} I_{QMC} = \sqrt{3}\left(1+\frac{\alpha}{2\pi}\right) I_{QMC}^{\alpha}. \qquad 4.54$$

Using equations 4.49 to 4.53, we have similar relations for the self-magnetic field produced for those above-mentioned radii as

$$B_C = \alpha^2 B_0 = \alpha^{-2} B_{em} = 3B_{QMC} = 3\left(1+\frac{\alpha}{2\pi}\right) B_{QMC}^{\alpha}. \qquad 4.55$$

As the current for different radii are found to be α-quantized, magnetic moment of the electron can be expressed with as [8]

$$\mu = \alpha^{-1}\frac{eR_0}{2} = \frac{eR_C}{2} = \alpha\frac{eR_{em}}{2}, \qquad 4.56\text{-a}$$

which can be re-written with Schwinger-corrected magnetic moment as

$$\mu = \alpha^{-1}\frac{eR_0}{2}\left(1+\frac{\alpha}{2\pi}\right) = \frac{eR_C}{2}\left(1+\frac{\alpha}{2\pi}\right) = \alpha\frac{eR_{em}}{2}\left(1+\frac{\alpha}{2\pi}\right). \qquad 4.56\text{-b}$$

4.6 Rotation of the charge in external magnetic field

In the section 4.3, the rotation of the charge is considered and the self-magnetic field originated due to its own rotation is discussed. In that case, no external magnetic field was regarded. In this section, we are going to observe the behaviour of the rotating charge in an external magnetic field using the electron radii. Indeed the behaviour of the charge in uniform [12] magnetic field and non-uniform magnetic field [13] are analysed by H. Goldstein and R. J. Deissler.

If a rotating charged non-conducting ring is considered, the lagrangian for the system can be written as [13]

$$\mathrm{L} = \frac{1}{2}mv^2 + \frac{1}{2}mR^2\omega^2 + \frac{\omega e R^2}{2c}B(z), \qquad 4.57$$

where e is the charge of the electron, m is the mass of the electron, R is the radius of rotation of the charge and ω is the angular velocity. Hence the generalized angular momentum [13] is

$$\mathrm{L} = mR^2\omega + \frac{eR^2}{2c}B(z)\ . \tag{4.58}$$

Using the angular momentum of electron as $L = \frac{\hbar}{2}$, the z-component of magnetic field from equation 4.58 can be written as

$$B(z) = \frac{2c}{eR^2}\left[\frac{\hbar}{2} - mR^2\omega\right]. \tag{4.59}$$

Using the expression of fine structure constant $\alpha = \frac{e^2}{\hbar c}$, equation 4.59 can be re-written as

$$B(z) = \alpha^{-1}\left[\frac{e}{R^2} - \frac{2m\omega e}{\hbar}\right]. \tag{4.60}$$

Using Compton radius from equation 4.2, the corresponding magnetic field can be calculated in the form of

$$B_C(z) = \alpha^{-1}\left(\frac{em}{\frac{\hbar}{2}}\right)\left[\frac{mc^2}{2\hbar} - \omega\right] = \frac{\hbar}{\mu}\left[\frac{mc^2}{2\hbar} - \omega\right]. \tag{4.61}$$

Equation 4.61 holds one form $\frac{\text{Charge. Mass}}{\text{Spin}}$, where the charge, the mass and the spin of the electron together. In the above sections, we have already noticed this factor for our work with the self-magnetic field. In fact, the fine structure constant can be analysed as

$$\alpha = \left(\frac{em}{\frac{\hbar}{2}}\right)\frac{\mu}{\hbar}. \tag{4.62}$$

Fine structure constant and g-factor are related as [4]

$$\frac{g}{2} = 1 + \frac{\alpha}{2\pi}. \tag{4.63}$$

Using equation 4.63 between fine structure constant and g-factor in equation 4.62 we have

$$\frac{em}{\frac{\hbar}{2}} = \frac{(g-2)\pi\hbar}{\mu}. \qquad \textbf{4.64}$$

Equation 4.64 refers to the connection of the electron's intrinsic properties with the g-factor.

Using classical radius form equation 4.1 in equation 4.60, we have

$$B_0(z) = \alpha^{-1}\left(\frac{em}{\frac{\hbar}{2}}\right)\left[\alpha^{-2}\frac{mc^2}{2\hbar} - \omega\right] = \frac{\hbar}{\mu}\left[\alpha^{-2}\frac{mc^2}{2\hbar} - \omega\right]. \qquad \textbf{4.65}$$

In a similar way, using equation 4.4 for electromagnetic radius, we have

$$B_{em}(z) = \alpha^{-1}\left(\frac{em}{\frac{\hbar}{2}}\right)\left[\alpha^{2}\frac{mc^2}{2\hbar} - \omega\right] = \frac{\hbar}{\mu}\left[\alpha^{2}\frac{mc^2}{2\hbar} - \omega\right]. \qquad \textbf{4.66}$$

Equations 4.61, 4.65 and 4.66 are not only carrying the term containing the charge, the mass and the spin, but also they are α-quantized.

Using equation 4.61 and expressing current as $I = \frac{cBR}{2}$ from equation 4.37, with an approximation of long straight current carrying wire, one can get the current contribution for Compton radius as

$$I_C = \alpha^{-1}\frac{c^2}{4}\left(\frac{em}{\frac{\hbar}{2}}\right)\left[1 - \frac{2\omega R_C}{c}\right]. \qquad \textbf{4.67}$$

Similarly, for classical radius we have the current contribution from equation 4.65 as

$$I_0 = \alpha^{-1}\frac{c^2}{4}\left(\frac{em}{\frac{\hbar}{2}}\right)\left[\alpha^{-1} - \frac{2\omega R_0}{c}\right]. \qquad \textbf{4.68}$$

Current contribution can be calculated from equation 4.66 for electromagnetic radius as

$$I_{em} = \alpha^{-1}\frac{c^2}{4}\left(\frac{em}{\frac{\hbar}{2}}\right)\left[\alpha - \frac{2\omega R_{em}}{c}\right]. \qquad 4.69$$

When the ring oscillates around the z-co-ordinate, $\omega = 0$. Then the corresponding magnetic fields can be given respectively as

$$B_C = \frac{e}{R_0 R_C}, \qquad 4.70$$

$$B_0 = \alpha^{-2}\frac{e}{R_0 R_C} \qquad 4.71$$

and

$$B_{em} = \alpha^2 \frac{e}{R_0 R_C} = \frac{e}{R_{em} R_C}. \qquad 4.72$$

For $\omega = 0$, equation 4.61 can also be written as

$$B_C(z) = \alpha^{-1}\frac{mc^2}{2\hbar}\left(\frac{em}{\frac{\hbar}{2}}\right). \qquad 4.73$$

In a similar way equations 4.65 and 4.66 can be re-written respectively as

$$B_0(z) = \alpha^{-3}\frac{mc^2}{2\hbar}\left(\frac{em}{\frac{\hbar}{2}}\right) \qquad 4.74$$

and

$$B_{em}(z) = \alpha\frac{mc^2}{2\hbar}\left(\frac{em}{\frac{\hbar}{2}}\right). \qquad 4.75$$

Using the relation 4.64 of g-factor with the intrinsic properties of the electron one can get the magnetic field from equation 4.73 as

$$B_C = \alpha^{-1}\frac{\pi mc^2(g-2)}{2\mu}. \qquad 4.76$$

Similarly, the magnetic field for classical electron radius can be written from equation 4.74 as

$$B_0 = \alpha^{-3} \frac{\pi mc^2 (g-2)}{2\mu} \quad \textbf{4.77}$$

and from equation 4.75 for electromagnetic radius we have

$$B_{em} = \alpha^2 \frac{\pi mc^2 (g-2)}{2\mu}. \quad \textbf{4.78}$$

Equations 4.73 to 4.75 gave magnetic field for $\omega = 0$ with α-quantization, whereas equations 4.76 to 4.78 express magnetic field for $\omega = 0$ with α-quantization in terms of g-factor.

The current comes out from equation 4.73 as

$$I_C = \frac{\alpha^{-1} c^2}{4} \left(\frac{em}{\frac{\hbar}{2}} \right). \quad \textbf{4.79}$$

In a similar pattern, equations 4.74 and 4.75 produce current respectively as

$$I_0 = \frac{\alpha^{-2} c^2}{4} \left(\frac{em}{\frac{\hbar}{2}} \right) \quad \textbf{4.80}$$

and

$$I_{em} = \frac{c^2}{4} \left(\frac{em}{\frac{\hbar}{2}} \right). \quad \textbf{4.81}$$

The α-quantization nature is remaining invariant for current even when the angular velocity is set to be zero also, is seen in equations 4.79 to 4.81.

4.7 α-quantized mass-leap and radii of the muon and the tau

The muon and the tau are in the lepton family along with the electron. In the mass tree, they are in higher positions compared to the electron. But other properties are of similar nature. MacGregor has put all of them in a well-calculated connection [1]. In QED, the fine structure constant α is the coupling constant. Comparison of the electron with the other particle mass data set produces two different α- quantized masses, and they appear in two different forms know as fermionic with half-integral spin and bosonic with integral spin. α-quantized steps of the masses for fermions and the bosons are shown by MacGregor [1, 14] respectively as

$$m_f = \frac{3}{2}\frac{m_e}{\alpha} \qquad \textbf{4.82}$$

and

$$m_b = \frac{m_e}{\alpha}. \qquad \textbf{4.83}$$

Equation 4.82 expresses the mass quanta, which is created in the "α-leap" from the electron to the muon and equation 4.83 expresses the mass quantum that is created as a part of a hadronically bound particle-antiparticle pair in the "α-leap" from an electron-positron pair to the pion, where m_e is the electron mass [1].

The factor $\frac{m_e}{\alpha}$ is found in the expression of current-loop for classical radius, i.e. equation 4.19 and by re-writing equation 4.19, we have

$$I_0 = \left[\frac{c^2}{4\pi}\left(\frac{e\frac{m_e}{\alpha}}{\frac{\hbar}{2}}\right)\right]\left(1+\frac{\alpha}{2\pi}\right), \qquad \textbf{4.84}$$

where $m_e = m =$ mass of the electron. Using equation 4.82 in equation 4.84 we get

$$I_0 = \left[\frac{c^2}{6\pi}\left(\frac{em_\mu}{\frac{\hbar}{2}}\right)\right]\left(1+\frac{\alpha}{2\pi}\right), \qquad \textbf{4.85}$$

where m_μ is the mass of the muon. Comparing the current-loop expression for the muon in a similar way with that of the electron, we have the radius of the muon as

$$R_\mu = \frac{3}{2}\frac{\hbar}{m_\mu c}. \qquad \textbf{4.86}$$

Compton radius of the electron is known as $R_C = \frac{\hbar}{m_e c}$ with m_e as the mass of the electron. Equation 4.86 looks like Compton radius of the electron. Also right hand side carries a dimension of length that is essential for radius. Hence R_μ can be called as the radius of the muon. Mass of the tau is almost 17 times of the mass of the muon. Therefore in the same way with the help of equation 4.19 and the α-leap of the fermionic mass from 4.82, we have

$$I_0 = \frac{2}{51}\left[\frac{c^2}{4\pi}\left(\frac{em_\tau}{\frac{\hbar}{2}}\right)\right]\left(1+\frac{\alpha}{2\pi}\right). \qquad \textbf{4.87}$$

Again, comparing the current-loop expression for the tau in a similar way with that of the electron, the radius of the tau comes out as

$$R_\tau = \frac{51}{2}\frac{\hbar}{m_\tau c}. \qquad \textbf{4.88}$$

This equation 4.88 gives a form of the radius just like equation 4.61 and this also looks like Compton radius.

The first α-quantized mass-leap for fermion is given by equation 4.82. With the fermionic mass, the moment of inertia for a ring can be written as

$$I_f = \frac{1}{2}m_f R_f^2, \qquad \textbf{4.89}$$

where R_f is the radius of gyration. Using equation 4.82 in equation 4.89 for m_f, one can get the moment of inertia for the muon as

$$I_f = \frac{3}{4}\frac{m_e}{\alpha}R_f^2. \qquad \textbf{4.90}$$

Thus the angular momentum can be written as

$$L_f = \frac{3}{4}\frac{m_e}{\alpha}v_f R_f. \qquad \textbf{4.91}$$

Using the angular momentum value as $\frac{\hbar}{2}$ in equation 4.91 we will arrive at the form $\frac{\text{Charge. Mass}}{\text{Spin}}$, which we have derived already to deal with the current-loop as

$$\frac{em_e}{\frac{\hbar}{2}} = \frac{4}{3}\cdot\frac{e\alpha}{v_f R_f}. \qquad \textbf{4.92}$$

The left-hand side of the equation 4.92 describes the properties of the electron, whereas the right hand side is controlled by the properties of the immediate next fermion in the α-leap of masses of the elementary particles or in other words the properties of the muon. Replacing compact form of the electron properties using equation 4.67 in the equation 4.43 we get the generalized current for the muon

$$I_G = \frac{c^2}{3\pi}\left(\frac{e\alpha}{v_f R_f}\right). \qquad \textbf{4.93}$$

4.8 α-quantization of the velocity

The relativistic moment of inertia of the spinning sphere is [4]

$$I = \frac{1}{2} m_S R^2, \qquad \textbf{4.94}$$

where m_s is the total mass of the spinning sphere. Spinning mass becomes $m_S = \frac{3}{2}m$ for higher velocity, with m being the non-spinning rest mass [4]. For smaller values of angular velocity, the spinning mass and non-spinning mass are equal, i.e. $m_S = m$. With the increasing angular velocity ω, the spinning mass m_S increases. Hence the relativistic moment of inertia is [4]

$$I = \frac{3}{4} mR^2 = \frac{1}{2} m_S R^2. \qquad \textbf{4.95}$$

Compton radius, R_C gives $\frac{\hbar}{2}$ spin with the linear velocity c. So the angular momentum

$$L = I\omega = \frac{1}{2} m_S R_C^2 \omega = \frac{\hbar}{2}. \qquad \textbf{4.96}$$

The relation between the linear velocity and the angular velocity is $v = r\omega$. We are using here the suffixes according to the notations of the concerned radii. For classical radius, we have the expression of velocity as

$$v_0 = R_0 \omega_0 \qquad \textbf{4.97}$$

where ω_0 is the corresponding angular momentum.

The spin angular momentum will follow the pattern of the equation 4.96 and hence for classical radius we can write it as

$$\frac{1}{2} m_S R_0^2 \omega_0 = \frac{\hbar}{2}. \qquad \textbf{4.98}$$

Then the angular velocity is

$$\omega_0 = \frac{\hbar}{m_S R_0^2} = \frac{v_0}{R_0}\,. \qquad \textbf{4.99}$$

So the velocity will be

$$v_0 = \frac{\hbar}{m_S R_0}\,. \qquad \textbf{4.100}$$

Therefore velocity v_0 is calculated with the help of the equation 4.1 as

$$v_0 = \alpha^{-1} c\,. \qquad \textbf{4.101}$$

But this is not possible, as it is well known from special theory of relativity that velocity of light is the highest velocity. As at the very beginning of the deduction, classical radius, one relativistic approach was equated to one non-relativistic scheme this problem arises. So it is proved here that for a relativistic spinning sphere, classical radius does not stand at all with relativistic moment of inertia. If we have to use classical radius for relativistic spinning sphere model, we must have to introduce it in some other way. On the other hand the above result gives the indication for smaller radius and higher velocity also.

As the radii are decreased, the velocities are increased. Now with the behaviour of the other parameters, it is clear that the velocity of the particle is in fraction of c. The velocity of the Compton-sized electron is well- known as c. Hence one can predict the other way round that the Classical electron radius is the contracted length of Compton radius of the electron with the help of α-quantized relation 4.3, $R_0 = \alpha R_C$. Hence the velocity corresponding to classical electron radius becomes

$$v_0 = c\sqrt{1-\alpha^2}\,. \qquad \textbf{4.102}$$

Now, we have the velocity of classical radius in terms of c and is closer to c, but it is not greater than c. This is in agreement with the special theory of relativity.

In the similar way, for magnetic field radius, the velocity and the spin can be written respectively as

$$v_H = R_H \omega_H \qquad \textbf{4.103}$$

and

$$\frac{1}{2} m_S R_H^2 \omega_H = \frac{\hbar}{2}. \qquad \textbf{4.104}$$

Using the simplified form of the relation between R_C and R_H, we have

$$R_H = R_C\left(1 + \frac{\alpha}{3\pi}\right). \qquad \textbf{4.105}$$

Then along with $R_C = \frac{\hbar}{m_S c}$ we have

$$v_H = \frac{\hbar}{m_S R_H} = \frac{\hbar}{m_S R_C\left(1 + \frac{\alpha}{3\pi}\right)} \cong c\left(1 - \frac{\alpha}{3\pi}\right). \qquad \textbf{4.106}$$

For quantum mechanical Compton radius following a similar pattern of the relation between linear velocity and the angular velocity, we start with the relation

$$v_{QMC} = R_{QMC}\,\omega_{QMC} \qquad \textbf{4.107}$$

and the corresponding angular momentum is

$$\frac{1}{2} m_S R_{QMC}^2 \omega_{QMC} = \frac{\hbar}{2}. \qquad \textbf{4.108}$$

Proceeding in the similar way, we have

$$v_{QMC} = \frac{\hbar}{m_S R_{QMC}}. \qquad \textbf{4.109}$$

Using equation 4.12 and $R_C = \frac{\hbar}{m_S c}$ we have

$$v_{QMC} = \frac{c}{\sqrt{3}}. \qquad \textbf{4.110}$$

To continue the same sort of calculations, the linear velocity and the angular velocity for QED-corrected quantum mechanical Compton radius are related as

$$v_{QMC}^{\alpha} = R_{QMC}^{\alpha}\,\omega_{QMC}^{\alpha} \qquad \textbf{4.111}$$

and the corresponding angular momentum is given

$$\frac{1}{2} m_S R_{QMC}^2 \omega_{QMC} = \frac{\hbar}{2}. \qquad \textbf{4.112}$$

In the similar way, we have

$$v_{QMC}^{\alpha} = \frac{\hbar}{m_S R_{QMC}^{\alpha}}. \qquad \textbf{4.113}$$

Using equation 4.14 and relativistic moment of inertia corrected Compton radius $R_C = \frac{\hbar}{m_S c}$ we have

$$v_{QMC}^{\alpha} = \frac{c}{\sqrt{3}}\left(1 + \frac{\alpha}{2\pi}\right)^{-1} \cong \frac{c}{\sqrt{3}}\left(1 - \frac{\alpha}{2\pi}\right). \qquad \textbf{4.114}$$

Electromagnetic radius is given in equation 4.4. The relation between the corresponding linear velocity and the angular velocity is written

$$v_{em} = R_{em}\omega_{em} \qquad \textbf{4.115}$$

and the concerned angular momentum is

$$\frac{1}{2} m_S R_{em}^2 \omega_{em} = \frac{\hbar}{2}. \qquad \textbf{4.116}$$

Following previous way, we have the velocity for the charge associated with the electron, when electromagnetic radius is concerned, as

$$v_{em} = \alpha c. \qquad \textbf{4.117}$$

4.9 Concluding remarks

Fine structure constant is found here as the connecting parameter amongst the different radii of the electron. Regarding this matter, it is noteworthy that all these radii or the sizes of the electron are derived or calculated from different electromagnetic phenomenon. Hence the correlation through α is actually a link amongst those basic phenomena, which differ from each other in the way of happening. The α-quantization of the current loop or magnetic field simply follows the nature of the relation amongst the radii. But the mass-leap says about the contribution of the fine structure constant for the mass, life-time [1, 14-15] of the concerned particle. Hence when we connect α-leap of the mass and the α-quantization of the radii, current and magnetic field, the basic features of the electron are visualised in terms of the fine structure constant. The other prediction of the classical electron radius as a "length-contracted form of the Compton radius" also gets strong platform in the association of the α-related facts and the velocity of the charge can be well defined then for a classical sized-electron.

α-quantization of these properties of the electron for different radii is quite significant to connect between different electromagnetic phenomena. It is also clear from the above results that whenever the properties of the electron are changed or measured due to its various radii, fine structure constant controls the entire matter.

Again a striking behaviour of the electron properties we have got here by representing current and magnetic field in terms of charge, mass and spin together. It is commonly known that current and magnetic field are dependent on the charge. But our results have shown their dependence on the mass and the spin also. Therefore the three intrinsic properties of the electron contribute on the current and the magnetic field. This reflects a hidden nature of the mass and the spin and we are going to discuss them in the next chapter.

The results from the external magnetic field have shown two major points. Firstly, the magnetic field and the corresponding current are shown there in a manner where α-quantization is maintained. Three radii are used there and the α-quantization nature is very clear from the results. Secondly, the magnetic field and the current are expressed with the factor consisting of three intrinsic properties of the electron, which we got for the self-magnetic field of the electron [16] also.

Another significant observation is that, the α-quantization property remains invariant from the definite value of angular velocity to zero angular velocity also. Hence this can be concluded that α-quantization is connected to the intrinsic nature of the particle, which gets affirmed with equation 4.62. Hence equation 4.62 is important to study the nature of the electron properties with the help of α. Lande *g*-factor is also related to α, which ensures a better measurement for intrinsic properties of the electron and corresponding structure.

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Memory is deceptive because it is coloured by today's events
- Albert Einstein

Chapter 5

Electromagnetic mass of the electron

As in this thesis, we are working with the different properties of the electron, it is important for us to explain them also invoking the spinning sphere model of it. The electron is known as a tiny charged particle having a small amount of finite mass, and its charge makes it involve into electromagnetic interactions. This raises the question of the origin of its mass, whether it has some electromagnetic nature or not. Concept of the electromagnetic mass was primarily developed in the last half of the nineteenth century, and has been carried till the date from various approaches. Here, we have discussed some of those approaches and later have focussed on the mass corresponding to the charge of the electron in the framework of the spinning sphere of the electron. There, it is shown how the charge and the mass contribute together in total energy of the electron. We observed also here, how the charge and the mass behave in the relativistic speed.

5.1 Mass and electromagnetic mass

The mass of the electron is a fundamental constant [1]. But mass is not the only fundamental property associated with it. The charge and the spin are also attached with it and they play some crucial role in the electromagnetic behaviour of the electron. According to Newton "The quantity of any matter is the measure of it by its density and volume conjointly.... This quantity is what I shall understand by the term mass of a body...." [2]. Around the year 1900, physicists became interested about the possibility of the electromagnetic origin of the part or of all the mass of the electron [3].

Thomson described the electromagnetic mass [4] as $m_{elm} = f\frac{e^2}{Rc^2}$, where f is a numerical factor of order 1 and this factor depends on the charge distribution within the spherical structure with radius R. He argued this for one electromagnetic particle, which is in uniform motion with velocity v with the electromagnetic field having a kinetic energy $T_{elm} = f\frac{e^2v^2}{2Rc^2}$. After Thomson's work, the electron model and properties were treated by M. Abraham and H. A. Lorentz. They developed the radiation reaction force on an extended electron. J. H. Poincaré gave the idea that it is impossible for the charged particles to be held together without the presence of any other attractive and non-electromagnetic forces. Hence Poincaré stresses provided the condition for a mass to be added to the electromagnetic mass of the particle to get the observed mass [5].

Electromagnetic mass is also discussed by M. Born and L. Infeld [6]. They described the mass m as $mc^2 = 1.2361\frac{e^2}{r_0}$, where $r_0 = \sqrt{ae}$. Here a^{-1} is the absolute field and e is the charge. Their result also has showed the mass as self-energy of the electron that can justify the suppression of the associated quantum mechanical terms [6].

Dealing with the same problem, R. P. Feynman says that the electromagnetic mass can be expressed with the help of the energy of the electric field as $U_{elec} = \frac{3}{4}m_{elec}c^2$. Here he has considered the high velocity particle of course and m_{elec} is defined as the electromagnetic mass [7].

Classical electrodynamics supports the idea of electromagnetic origin of the part of the mass of the electron as the rest mass of a charged particle is greater than that of its uncharged twin and hence one can express the total mass as a sum of its mechanical mass and electromagnetic mass [8] in the way $m = m_{mech} + m_{em}$. Indeed D. J. Griffiths and R. E. Owen described that the electromagnetic mass of a charged particle of specified size, shape and charge can be obtained in three ways: a) from the electrostatic energy of the particle, b) using the momentum of the particle and c) from the self-force of the particle [8].

A. M. Luiz illustrated the idea of electromagnetic mass of spheroidal bodies [9]. The uniform charge distribution is considered there. The body is assumed to be moving with a constant velocity v in an empty space. The total linear momentum is shown as $p = \frac{v}{6\pi c^2}\iiint E^2 dV = mv$, where c is velocity of light in free space, E is the quasi-static electric field and m is the electromagnetic mass and this is expressed by Luiz as $m = \frac{2e^2}{3Rc^2}$ with R as the radius of the body [9].

Thus electromagnetic mass is a puzzle to the electrodynamics as well as in electron-physics. If the problem is attacked to solve, we must have in our mind that it must be accommodated in the present theory of the electron and its models along with the current experimental background, and in the next section we are going to discuss them.

5.2 Present scenario of charge radius

LEP experiments indicate that the charge of the electron is distributed over a small radius of the order of 10^{-19} m or 10^{-17} cm [10], so that it can be considered as point-like. The explanation of related scattering by QED [11-12] too demand that the charge of the electron is concentrated with a smaller mass as compared to the total mass of the electron. This prompts us to link between the mass and the charge of an elementary particle.

The experimental result of magnetic moment of the electron is not found to match with the magnetic moment when only the mechanical mass of electron is considered [13-14] in theoretical calculation. Schwinger proposed a correction term $m \cdot \frac{\alpha}{2\pi}$ [13-14] to compensate the difference between the theoretical and experimental results. This compensating mass is termed as electromagnetic mass of the electron [13-14]. It may be noted that $\alpha(= \frac{e^2}{\hbar c})$ is the so-called fine structure constant coupling the strength of interaction between the electron and the photon [15].

In the standard relativistically spinning sphere model [14] of the electron, the charge is regarded to be confined in a very small region. We consider here the charge part of the electron to be in a tiny space of radius R_E, which is smaller than all other known radii [10, 14, 16]. In fact, out of the eight different known electron radii, $R_E \sim$

10^{-19} m is the smallest one and the next is classical electron radius $R_0 \sim 10^{-15}$ m, which is 10^4 times larger than R_E.

With all these facts and figures, now we are ready to incorporate the electromagnetic mass of the electron in the spinning sphere model and it can give us the probable answer about the electromagnetic origin of the mass of the electron.

5.3 Charge of the electron and the magnetic self-energy

Rotation of a charged particle around its axis of rotation gives rise to a current-loop

$$I = \frac{e}{T}. \quad \textbf{5.1}$$

Here e is the charge and T is the time period of rotation. If the linear velocity of the charge is v and radius of rotation is R, the time period T can be written as

$$T = \frac{2\pi R}{v}. \quad \textbf{5.2}$$

Putting equation 5.2 in equation 5.1, we have the expression of current in terms of velocity and the radius as

$$I = \frac{ev}{2\pi R}. \quad \textbf{5.3}$$

This current-loop introduces a magnetic field B and according to Ampere's law, B can be written as [17]

$$B = \frac{2I}{cR}. \quad \textbf{5.4}$$

Using equation 5.3 in equation 5.4

$$B = \frac{ev}{\pi c R^2}. \quad \textbf{5.5}$$

Here we have the magnetic field related to the radius of rotation and the velocity of the charge in equation 5.5. The rotation is through free space permeability and the permeability for free space in Gaussian units has been considered $\mu = 1$. This helps us to step for auxiliary magnetic field as

$$H = B. \quad \textbf{5.6}$$

With the help of equation 5.5 and 5.6 we have the expression for auxiliary magnetic field due to the rotation of the charge

$$H = \frac{ev}{\pi c R^2}. \quad 5.7$$

Magnetic self-energy is the energy, which is contained in the magnetic field associated with the magnetic moment of the electron. Magnetic field and moment are results of current, which is originated due to the motion of the charge. Thus considering the rotation of the charge of the electron we can continue for magnetic self-energy. Hence the magnetic self-energy of the above system will be read with the help of equations 5.5, 5.6 and 5.7 as

$$W_H = \frac{1}{2}\int H \cdot B d^3x = \frac{1}{2} \cdot \frac{e^2 v^2}{\pi^2 c^2 R^4} \int d^3x. \quad 5.8$$

To be more specific for relativistic spinning sphere model, if we choose $v = c$, the magnetic self-energy comes out as

$$W_H = \frac{e^2}{2\pi^2} \int \frac{1}{R^4} d^3x. \quad 5.9$$

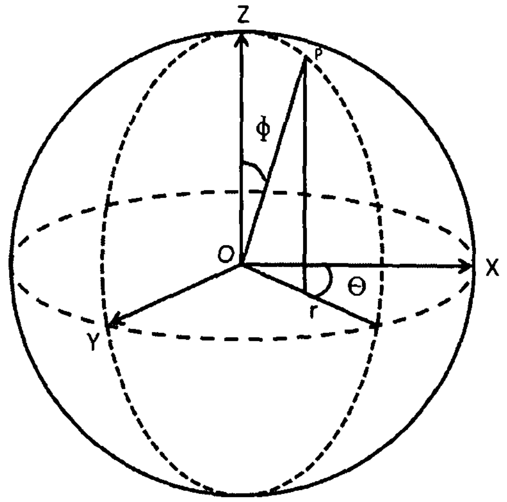

Figure 5.1: Spherical polar co-ordinates

If $\int d^3x$ is expressed in terms of spherical polar co-ordinate system and considering the orientation of the current-loop along the z-axis we have

$$\int_0^R \int_0^\pi \int_0^{2\pi} r^2 \sin\theta \, dr \, d\theta \, d\phi = \frac{4}{3}\pi R^3. \quad 5.10$$

Using equation 5.10 in equation 5.9 we have

$$W_H = \frac{2e^2}{3\pi R}. \qquad \textbf{5.11}$$

In the calculation of relativistic spinning sphere model [14], the correction of magnetic moment [13] is used. Hence the magnetic self-energy [14] is written with the help of electromagnetic mass of the electron as

$$W_H = m \cdot \frac{\alpha}{2\pi} c^2, \qquad \textbf{5.12}$$

where $m \cdot \frac{\alpha}{2\pi}$ is known as electromagnetic mass and $\frac{\alpha}{2\pi}$ is the Schwinger correction.

Equating equation 5.11 with equation 5.12 for the magnetic self-energy, we arrive at

$$m.\frac{\alpha}{2\pi} c^2 = \frac{2e^2}{3\pi R}. \qquad \textbf{5.13}$$

Hence the charge can be written as

$$e = \frac{c}{2}\sqrt{3m\alpha R}. \qquad \textbf{5.14}$$

Here, we have not considered any of the known form of radii of the electron to represent R. So now we can get the expression of the radius R as

$$R = \frac{4}{3}\alpha^{-1}\frac{e^2}{mc^2}. \qquad \textbf{5.15}$$

In the chapter 3 and the chapter 4, we have seen the form of classical radius as

$$R_0 = \frac{e^2}{mc^2}. \qquad \textbf{5.16}$$

Using equation 5.16 in equation 5.15, we have

$$R = \frac{4}{3}\alpha^{-1} R_0. \qquad \textbf{5.17}$$

Again, we have seen earlier in the chapter 3 and the chapter 4, that the fine structure constant related classical radius with Compton radius as

$$R_0 = \alpha R_C. \qquad \textbf{5.18}$$

Using equation 5.18 in equation 5.17, we get the radius as

$$R = \frac{4}{3} R_C . \qquad 5.19$$

This shows that the charge and the mass can be related for a spherical electron theory with the help of fine structure constant. Basic constant c is also related with this formulation and that prompts us about the fact that the charge and the mass of the electron can be co-related only in a relativistic frame. Thus the above conditions demand a radius of the sphere as slightly higher than Compton radius.

The expression of total energy is obtained using equation 5.13

$$E = (m + \frac{2e^2}{3\pi R^2})c^2 . \qquad 5.20$$

It is to be mentioned here that the mass can only be the rest mass of the sphere and it is less than the observed mass of the electron.

In equation 5.9, we have derived the magnetic self-energy considering the velocity $v = c$. But it is completely a special condition. Hence one can calculate the charge-mass relation and the corresponding radius using an arbitrary velocity v for a general condition. Now with the arbitrary velocity v, equation 5.11 can be re-written as

$$W_H = \frac{2e^2 v^2}{3\pi c^2 R} . \qquad 5.21$$

Equating equation 5.21 with equation 5.12 we have the expression of the charge as

$$e = \frac{c^2}{2v}\sqrt{3m\alpha R} . \qquad 5.22$$

Now two conditions are attached with this equation 5.22. The change of the velocity will obviously affect the body. We know that the mass is increased with the increasing velocity in relativistic condition. This keeps the radius and the charge un-affected in equation 5.22. Now the change in charge can also be traced if we fix the radius and the mass. Or in other words the charge will be decreasing with the increasing velocity, which can be observed in the table 5.1.

This is very clear from the mathematical observations that the amount of charge decreases with increasing velocity for a constant mass whereas vice-versa is true for the mass with constant charge. The increase of the mass is a known condition with the relativistic velocity. Thus below the velocity c, special theory of relativity will not be violated if one advocates for decreasing charge vide above-mentioned conditions. This also ensures the fact provided in equation 5.20. The total energy is a constant. We have observed it to be composed of two components, the charge and the mass. Then obviously the increment in one component results in the decrement of the other. But when the velocity reaches c, it behaves in opposite manner.

Table 5.1: Behaviour of the charge and the mass with relativistic velocity

Velocity of the charge	Charge (Mass fixed)	Mass (Charge fixed)
$\frac{c}{100}$	$50c\sqrt{3m\alpha R}$	$\frac{1}{7500}\frac{e^2}{\alpha c^2 R}$
$\frac{c}{90}$	$45c\sqrt{3m\alpha R}$	$\frac{1}{6075}\frac{e^2}{\alpha c^2 R}$
$\frac{c}{80}$	$40c\sqrt{3m\alpha R}$	$\frac{1}{4800}\frac{e^2}{\alpha c^2 R}$
$\frac{c}{70}$	$35c\sqrt{3m\alpha R}$	$\frac{1}{3675}\frac{e^2}{\alpha c^2 R}$
$\frac{c}{60}$	$30c\sqrt{3m\alpha R}$	$\frac{1}{2700}\frac{e^2}{\alpha c^2 R}$
$\frac{c}{50}$	$25c\sqrt{3m\alpha R}$	$\frac{1}{1875}\frac{e^2}{\alpha c^2 R}$
$\frac{c}{40}$	$20c\sqrt{3m\alpha R}$	$\frac{1}{1200}\frac{e^2}{\alpha c^2 R}$
$\frac{c}{30}$	$15c\sqrt{3m\alpha R}$	$\frac{1}{675}\frac{e^2}{\alpha c^2 R}$
$\frac{c}{20}$	$10c\sqrt{3m\alpha R}$	$\frac{1}{300}\frac{e^2}{\alpha c^2 R}$
$\frac{c}{10}$	$5c\sqrt{3m\alpha R}$	$\frac{1}{75}\frac{e^2}{\alpha c^2 R}$
$\frac{c}{5}$	$\frac{5}{2}c\sqrt{3m\alpha R}$	$\frac{4}{75}\frac{e^2}{\alpha c^2 R}$
$\frac{c}{2}$	$c\sqrt{3m\alpha R}$	$\frac{1}{3}\frac{e^2}{\alpha c^2 R}$

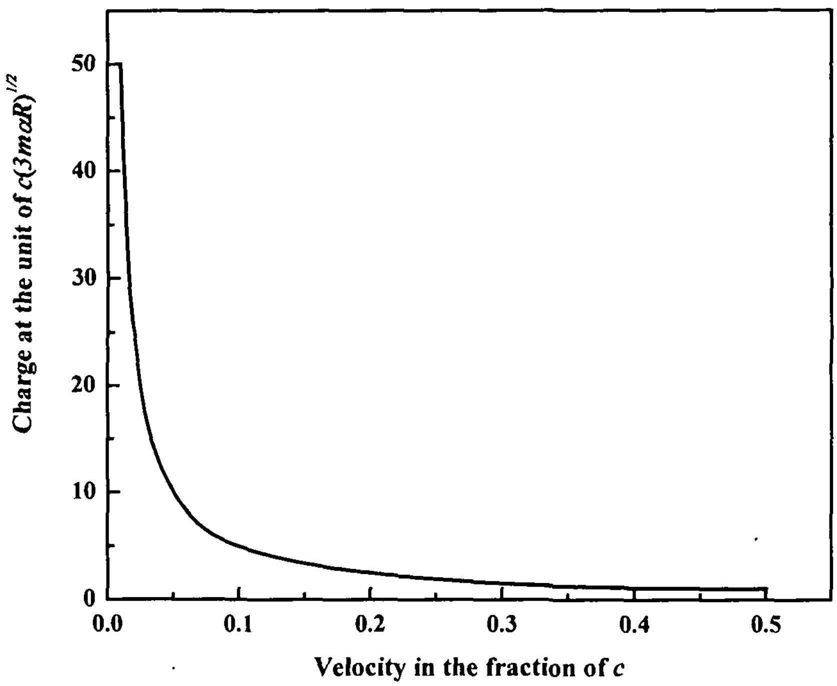

Figure 5.2: Behaviour of the charge in relativistic velocity

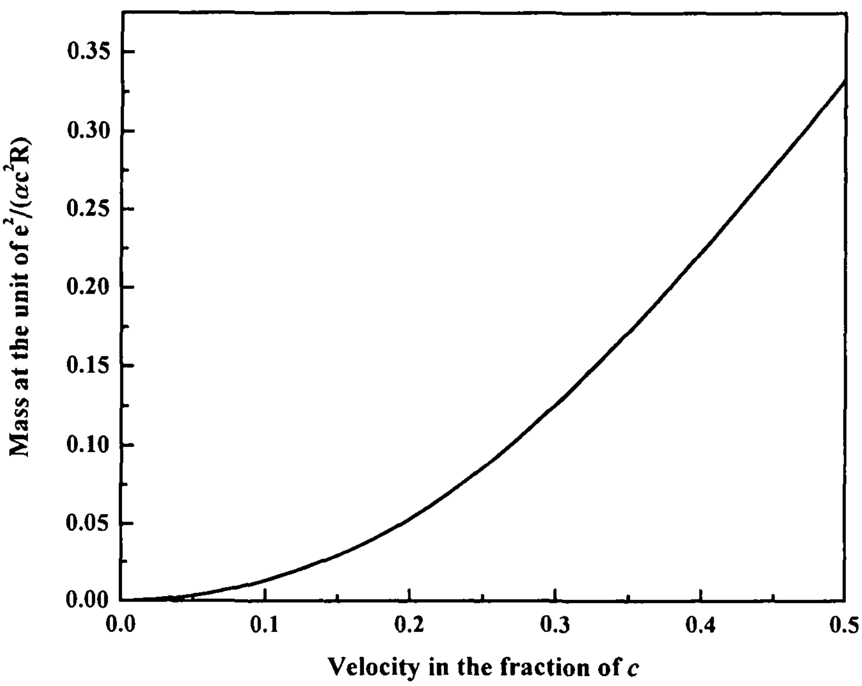

Figure 5.3: Behaviour of the charge in relativistic velocity

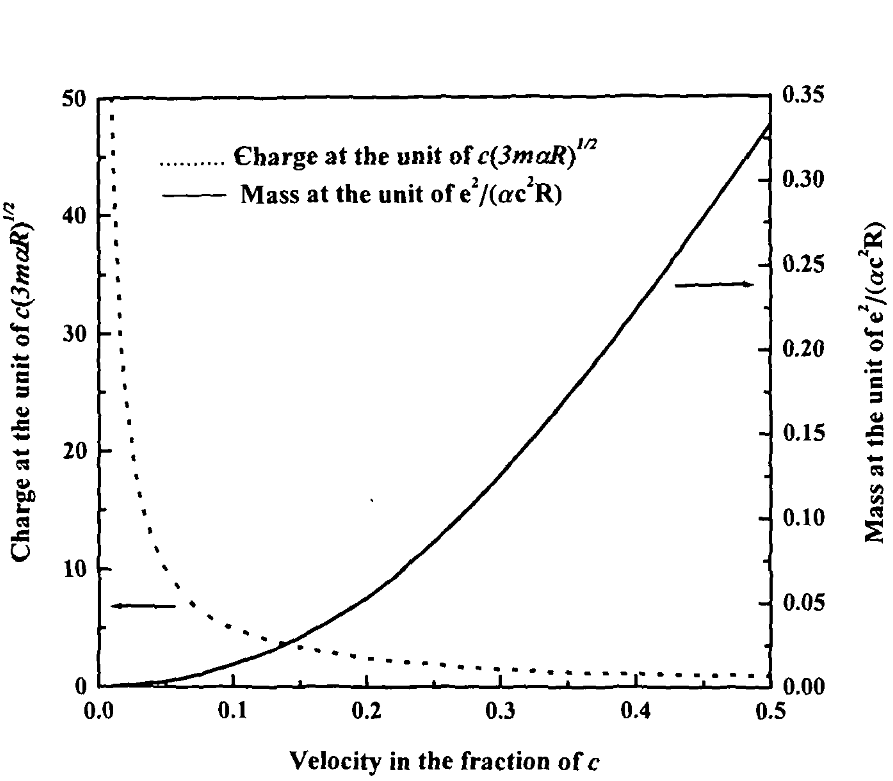

Figure 5.4: Behaviour of the charge and the mass in relativistic velocity

Here in figure 5.2 we get the behaviour of the charge of the electron when it is in relativistic linear velocity. It shows the amount of the charge is decreasing, when the mass is considered as constant. But when the charge is taken as constant, we get the increasing nature in the amount of the mass with the relativistic linear velocity of the charge in the figure 5.3. In the figure 5.4 we have given the overlapping of the figure 5.2 and the figure 5.3 only to show the nature of the two curves in a single picture. The dotted curve in the figure 5.4 shows the decreasing in the amount of the charge whereas the solid line shows the increasing in the amount of the mass.

5.4 Concluding remarks

The electromagnetic mass is expressed by us as the mass responsible for the existence of the charge. The relation between the charge and the mass of the electron is shown here in the framework of spinning sphere model of the electron. The magnetic self-energy gives 0.07% of the total mass of the electron and the charge radius is also smaller than all other radii of the electron. These two facts are completely supporting the calculation given here. This relation between the charge and the mass confirms the fact that without mass no charge can exist. Indeed, no charged particle in the particle physics is known without mass. Our work strengthens the above fact.

We observed that at relativistic speed, for a fixed size of spinning sphere electron, the amount of the charge decreases with increasing velocity, when the mass is constant. With similar conditions if the charge is considered to be constant, the amount of mass increases with increasing velocity. This gives a signature of the fact that the charge may transform into the mass or the equivalent energy when the velocity approaches *c*. Therefore, one can predict some critical point just below the *c* velocity, where the charge and the mass are no longer different entity. Fine structure constant is shown as the cause of the conversion between the charge and the mass.

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If the facts don't fit the theory, change the facts -Albert Einstein

Chapter 6

Helical motion of the charge and spinning sphere model

In the previous chapters, we have studied the different properties of the electron, specially the radii of the electron and the other radii-involved properties in the framework of spinning sphere model of the electron. Depending on them, we are going to depict a picture of spinning sphere model of the electron, which can co-relate the different electromagnetic phenomena, as well as can be linked up with current experimental consequences. Study of the energy of the electron leads here to the new kind of radius of the electron and that gives the diagram of a helical motion of the charge. With this motion, we found the structure of the electron in agreement with the experimental results of the magnetic moment and the gyromagnetic ratio. The proposed model is expected to connect different models.

6.1 Magnetic self-energy and composite radius

Four different kinds of mass, or equivalent energy are attached to the electron. They are electrostatic self-energy (W_E), magnetic self-energy (W_H), *mechanical mass* (W_M) *and the gravitational mass* (W_G).

It is about only 0.1% of the total energy of the electron [1]. It is the energy contained within the magnetic field, associated with the magnetic moment [1]. This concept can be used to develop the electromagnetic part of the desired model of the electron. We have seen in equation 3.25 of chapter 3, that the total magnetic self-energy is calculated in relativistic spinning sphere model as

$$W_H = \frac{2\mu^2}{3R_H^3}, \qquad \textbf{6.1}$$

where $\mu\left(=\frac{e\hbar}{2mc}\right)$ is the magnetic moment and R_H is magnetic field radius. This is also in close approximation with the calculation of F. Rasetti and E. Fermi [1]. Magnetic field radius is closer to Compton radius in size.

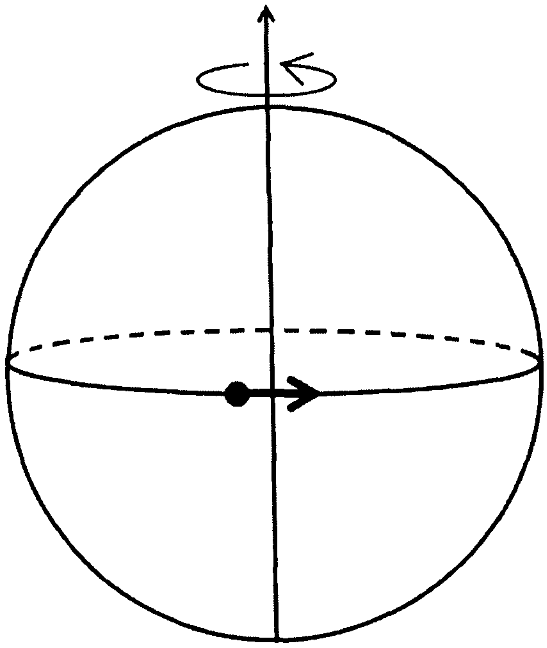

Figure 6.1: Relativistic spinning sphere with a tiny charge at the equator

To match the theoretical and the experimental values of the magnetic moment of the electron, J. Schwinger introduced a correction term, which is known as Schwinger-correction [2]. In terms of the energy, the Schwinger-correction can be expressed as [1]

$$W_H \approx m \cdot \frac{\alpha}{2\pi} c^2 . \quad \textbf{6.2}$$

Equating the expressions 6.1 and 6.2 for magnetic self-energy, we have

$$R_H^3 \cong R_C^3 \left(1+\frac{\alpha}{2\pi}\right)^2 . \quad \textbf{6.3}$$

Re-arranging and re-combining the terms of equation 6.3 we get a composition of classical electron radius and Compton radius as

$$R_H^3 = R_C R_{C0}^2 , \quad \textbf{6.4}$$

where we have introduced R_{C0} as addition in length only and mathematically it can be written as

$$R_{C0} = \left(R_C + \frac{R_0}{2\pi} \right). \qquad \textbf{6.5}$$

As R_{C0} is defined basically consisting classical electron radius and Compton radius [3], let us call this as Composite radius. From the relations amongst the radii of the electron, we know that fine structure constant relates classical radius and Compton radius from equation 3.42

$$R_0 = \alpha R_C. \qquad \textbf{6.6}$$

Using equation 6.6 in equation 6.5 we can express the composite radius of the electron in terms of the Compton radius as

$$R_{C0} = R_C \left(1 + \frac{\alpha}{2\pi} \right). \qquad \textbf{6.7}$$

Indeed, we can say now in that composite radius is the Schwinger-corrected Compton radius. Re-arrangement of the form of composite radius from equation 6.7 tells us about the peripheral length with composite radius as

$$2\pi R_{C0} = 2\pi R_C + R_0. \qquad \textbf{6.8}$$

The left-hand side of equation 6.8 consists of two terms, out of which the first one describes the circumference of a circle with Compton radius. The second term is the length equals to classical electron radius only. Hence one can conclude that these two terms together represent a helical path with first part covering horizontal distance and the second part vertical distance.

6.2 Helical motion of the charge

With this new composite radius, now we can try to calculate the current-loop for the rotation of the charge. Considering the rotation of the charge-centre around the mass-centre in this helical path, we get the dynamics of the charge of the electron model. But to be very specific, this is not the structure we are dealing exclusively with. Rather this gives us the signature of the electromagnetic nature of the electron on a Compton-sized spinning sphere. To establish the current-loop, we must be as accurate as possible about the distance travelled by the charge of the electron. The charge is considered to be confined in a very tiny place on the equatorial zone of the surface of the sphere obeying scattering incidents. In fact the electric charge is known to be confined within the charge radius of the electron, R_E [1, 4]. Hence we consider the length for one complete turn due to the rotation, is equal to a distance $R_E + 2\pi R_{C0}$. When the number of turns will be increasing the length can be calculated with the term $R_E + 2n\pi R_{C0}$. If the charge is having uniform velocity v, the total time required to complete n number of turn of rotation is

$$T = \frac{R_E + 2n\pi R_{C0}}{v}. \qquad \textbf{6.9}$$

Using equation 6.9 in the definition of current $I = \frac{e}{T}$, the current-loop contribution for the rotation of the charge in helical path can be written

$$I = \frac{ev}{R_E + 2n\pi R_{C0}}. \qquad \textbf{6.10}$$

So, ultimately the denominator of the equation 6.10 employs classical radius, Compton radius and charge radius of the electron.

The relation between the magnetic moment and the current is known as [5]

$$\mu = \frac{IA}{c}, \qquad \textbf{6.11}$$

where A is the area covered by the charge during the rotation. This area is concerned with the helical path and the corresponding area can be considered as

$$A = 2\pi(n-1)R_C R_0. \qquad \textbf{6.12}$$

Therefore the magnetic moment can be written with equations 6.10, 6.11 and 6.12 together as

$$\mu = \frac{2(n-1)\pi ev R_C R_0}{c(R_E + 2n\pi R_{C0})}. \qquad \textbf{6.13}$$

Using the approximation of infinitely long current carrying wire for the helical motion, the magnetic field comes out as $B = \frac{2I}{cR}$ [6]. Therefore, in the present situation we have the magnetic field as

$$B = \frac{2}{cR_{C0}}\left[\frac{ev}{R_E + 2n\pi R_{C0}}\right]. \qquad \textbf{6.14}$$

Number of turn, n is chosen here arbitrarily.

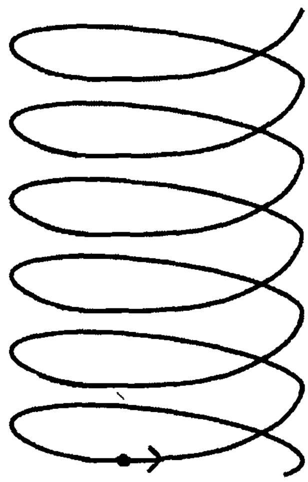

Figure 6.2: Helical motion of the charge

Employing the condition of *n*, we obtain the lower limit of *n* as 1 for the existence of the helical motion. Here we must remember that the magnetic field *B* and the velocity *v* depend on the concerned turn of rotation. For a Compton-sized model, the maximum height of the helical path within the sphere can be written as

$$h_{max} = 2R_C , \qquad \textbf{6.15}$$

as the distance between the two pole is $2R_C$. Again as the two successive turns are at a distance of R_0, the maximum length or the vertical height can be calculated with the help of the classical electron radius with *n* number of turns as

$$h = (n-1)R_0 . \qquad \textbf{6.16}$$

Equating equations 6.15 with 6.16 and using equation 6.6, we can have the upper limit of the number of turns as

$$n = 1 + \frac{2}{\alpha} . \qquad \textbf{6.17}$$

Hence the range of *n* goes from 1 to $1 + \frac{2}{\alpha}$.

Magnetic field is originated due to the rotation of the charge in the helical path. This field at the end of the first turn will be

$$B_1 = \frac{2}{R_{C0}} \left[\frac{ev_1}{R_E + 2\pi R_{C0}} \right], \qquad \textbf{6.18}$$

where v_1 is the primary linear velocity of the charge. As the length of the path and the time-taken are very short, the magnetic field will affect in successive turns. Hence B_1 will act on the charge as an external magnetic field. Behaviour of the charged particle in uniform and non-uniform magnetic field is well studied by H. Goldstein [7] and R. J. Deissler [5]. We have used the condition [5] for the charge in a non-uniform magnetic field considering the charge in the magnetic field originated from the last turn.

The hypothesis of spinning electron indeed is related with angular momentum and magnetic moment [8-9]. The generalized angular momentum of the system will be [5]

$$L = mR_C v + \frac{eR_{C0}^2 B}{2c}, \qquad \textbf{6.19}$$

m is the mass of the particle. Hence after the first turn, the generalized angular momentum will be

$$L_1 = mR_C v_1 + \frac{eR_{C0}^2 B_1}{2c}. \qquad \textbf{6.20}$$

Magnetic field, B_1 initiates the force on the charge particle in the second turn. So the B_1-initiated Lorentz force F_{L1} will act on the second turn and will introduce velocity v_2. The force F_{L1} is

$$F_{L1} = ev_2 B_1. \qquad \textbf{6.21}$$

Though the Lorentz forces act, the charge continues with same circular path. This gives the hint of another force which balances the Lorentz force. It is very clear from the nature of the second force that it would be a centripetal force. At the same time the Lorentz force due to the previous turn will be subtracted and it can directly be derived from equation 6.20 as

$$F_{C1} = \frac{L_1 v_1}{R_C^2} - \frac{eB_1 v_1}{2c}. \qquad \textbf{6.22}$$

Equating equations 6.21 and 6.22, we have the relations between v_2 and v_1 as

$$v_2 = v_1\left[\frac{L_1}{eB_1 R_C^2} - \frac{1}{2c}\right]. \qquad \textbf{6.23}$$

v_2 will be the velocity at the second turn and this will govern the picture for the next turn. Therefore according to equation 6.14, the magnetic field originated at the end of the second turn, $n = 2$ is

$$B_2 = \frac{2}{cR_{C0}}\left(\frac{ev_2}{R_E + 4\pi R_{C0}}\right). \qquad \textbf{6.24}$$

Using equations 6.23 and 6.18 together in equation 6.24, one can have a modified version of equation 6.24 as

$$B_2 = B_1\left(\frac{R_E + 2\pi R_{C0}}{R_E + 4\pi R_{C0}}\right)\left[\frac{L_1}{eB_1R_C^2} - \frac{1}{2c}\right]. \qquad \textbf{6.24-a}$$

Following equation 6.19, generalized angular momentum after the turn $n = 2$ will be

$$L_2 = mR_Cv_2 + \frac{eR_{C0}^2B_2}{2c}. \qquad \textbf{6.25}$$

Using the expression for v_2 and B_2 from equations 6.23 and 6.24 respectively in equation 6.25, we have

$$L_2 = \left[\frac{L_1}{eB_1R_C^2} - \frac{1}{2c}\right]\left[mv_1R_C + \frac{eR_{C0}^2B_1}{2c}\left(\frac{R_E + 2\pi R_{C0}}{R_E + 4\pi R_{C0}}\right)\right]. \qquad \textbf{6.26}$$

For $n = 3$, a similar set of equations can be derived as

$$v_3 = v_2\left[\frac{L_2}{eR_C^2B_2} - \frac{1}{2c}\right], \qquad \textbf{6.27}$$

$$B_3 = B_1\left(\frac{R_E + 2\pi R_{C0}}{R_E + 6\pi R_{C0}}\right)\left[\frac{L_1}{eR_C^2B_1} - \frac{1}{2c}\right]\left[\frac{L_2}{eR_C^2B_2} - \frac{1}{2c}\right] \qquad \textbf{6.28}$$

and

$$L_3 = \left[\frac{L_1}{eR_C^2B_1} - \frac{1}{2c}\right]\left[\frac{L_2}{eR_C^2B_2} - \frac{1}{2c}\right]\left[mv_1R_C + \frac{eR_{C0}^2B_1}{2c}\left(\frac{R_E + 2\pi R_{C0}}{R_E + 6\pi R_{C0}}\right)\right]. \qquad \textbf{6.29}$$

From equation 6.29 it is seen that L_3 is carrying the contributions from L_1 and L_2. Similarly the B_1, B_2, L_1 and L_2 are contributing in B_3.

Carrying on for the next turn, we have similar set of equations given below:

$$v_4 = v_3\left[\frac{L_3}{eR_C^2B_3} - \frac{1}{2c}\right], \qquad \textbf{6.30}$$

$$B_4 = B_1\left(\frac{R_E + 2\pi R_{C0}}{R_E + 8\pi R_{C0}}\right)\left[\frac{L_1}{eR_C^2 B_1} - \frac{1}{2c}\right]\left[\frac{L_2}{eR_C^2 B_2} - \frac{1}{2c}\right]\left[\frac{L_3}{eR_C^2 B_3} - \frac{1}{2c}\right] \quad 6.31$$

and

$$L_4 = \left[\frac{L_1}{eR_C^2 B_1} - \frac{1}{2c}\right]\left[\frac{L_2}{eR_C^2 B_2} - \frac{1}{2c}\right]\left[\frac{L_3}{eR_C^2 B_3} - \frac{1}{2c}\right]$$

$$\cdot\left[mv_1 R_C + \frac{eR_{C0}^2 B_1}{2c}\left(\frac{R_E + 2\pi R_{C0}}{R_E + 6\pi R_{C0}}\right)\right]. \quad 6.32$$

The nature of equations for the fourth turn is similar to those of first, second and third turns of the charge. They give the equations for the n-th turn as

$$v_n = v_{n-1}\left[\frac{L_{n-1}}{eR_C^2 B_{n-1}} - \frac{1}{2c}\right], \quad 6.33$$

$$B_n = B_1\left(\frac{R_E + 2\pi R_{C0}}{R_E + 2n\pi R_{C0}}\right)\left[\frac{L_1}{eR_C^2 B_1} - \frac{1}{2c}\right]\left[\frac{L_2}{eR_C^2 B_2} - \frac{1}{2c}\right]\cdots\left[\frac{L_{n-1}}{eR_C^2 B_{n-1}} - \frac{1}{2c}\right] \quad 6.34$$

and

$$L_n = \left[\frac{L_1}{eR_C^2 B_1} - \frac{1}{2c}\right]\left[\frac{L_2}{eR_C^2 B_2} - \frac{1}{2c}\right]\cdots\cdots$$

$$\cdots\cdots\left[\frac{L_{n-1}}{eR_C^2 B_{n-1}} - \frac{1}{2c}\right]\left[mv_1 R_C + \frac{eR_{C0}^2 B_1}{2c}\left(\frac{R_E + 2\pi R_{C0}}{R_E + 2n\pi R_{C0}}\right)\right]. \quad 6.35$$

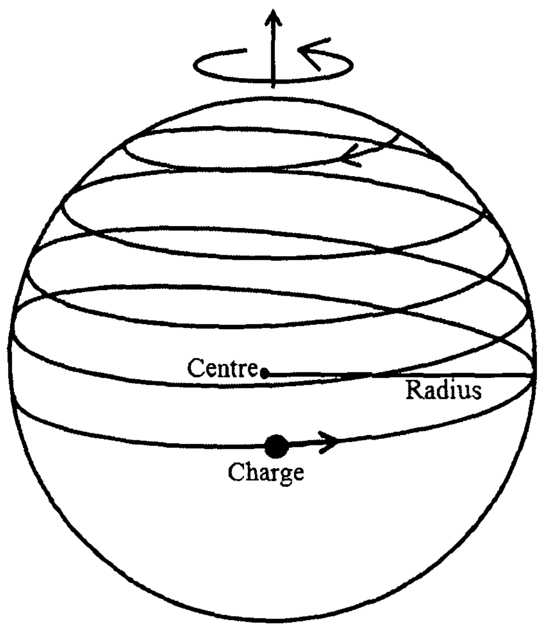

Figure 6.3: Relativistic spinning sphere with helical motion of the charge

Equations 6.33 to 6.35 provide us the n-th order of the velocity, the magnetic field and the angular momentum of the charge. In our recent contribution [10] we have worked out the above part.

6.3 Modified composite radius with higher orders of α

Equation 6.7 predicts a helical path of the charge that we have discussed in the previous section. But at this juncture, it is questionable, whether the helical motion of the charge is restricted within the sphere and follows the spherical structure or not. We have observed from equations 6.5 and 6.7 that the first two terms of the newly given composite radius or modified Compton radius are representing Compton and classical radii of the electron which are related with each other via first order α-quantized relation in equation 6.6. The structure of composite radius and the α-quantization between the two radii in equation 6.6 together leave the signature of the α-quantized turns on the helical path within the spherical structure. It is known that α provides a fraction $\frac{1}{137}$ and hence the increasing order in α gives smaller values. Consequently, if a charge-particle takes a helical path starting from equatorial zone, it would obviously follow lower radii towards the polar region of the sphere. Therefore, to get a complete helical path in terms of composite radius, one can include the higher order terms of α to equation 6.7 to provide a more accurate result [11]

$$R_{C0} = R_C\left[1 + \frac{\alpha}{2\pi} + K_2\left(\frac{\alpha}{2\pi}\right)^2 + K_3\left(\frac{\alpha}{2\pi}\right)^3 + K_4\left(\frac{\alpha}{2\pi}\right)^4 + \ldots\right], \qquad \textbf{6.36}$$

where K_2, K_3 and K_4 are the numerical constants associated with those terms.

The first term in the right hand side of equation 6.36 is Compton radius and the second one is classical radius. Then the other terms would contribute for the next lower radii after classical radius. Indeed, every higher order term decreases the length of the radii. This tells us of a smaller periphery away from the centre. One can imagine the particular circular disks with these radii to constitute the sphere. Thus, it is a logical way to describe the helical motion of the charge on a spherical structure. In other words they provide some particular levels in the structure.

Table 6.1: Number of levels and distances of the levels from the pole

Number of levels	Distances from the pole
1st	R_C
2nd	αR_C
3rd	$\alpha^2 R_C$
4th	$\alpha^3 R_C$
5th	$\alpha^4 R_C$
……	……
n-th	$\alpha^{n-1} R_C$

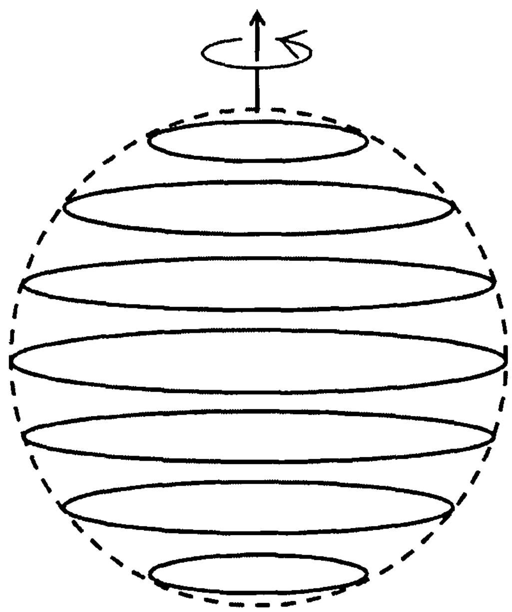

Figure 6.4: Circular disks in a sphere

From equation 6.36, we see that the 1st level is at equator with radius R_C and the R_C distance away from the pole. The 2nd level is $R_0 = \alpha R_C$ distance away from the pole. The 3rd level is $\alpha^2 R_C$ away from the pole, whereas the 4th level is $\alpha^3 R_C$ away from the pole and so on. This shows that the charge touches only some particular heights away from the equator and they lie on a specific mathematical series.

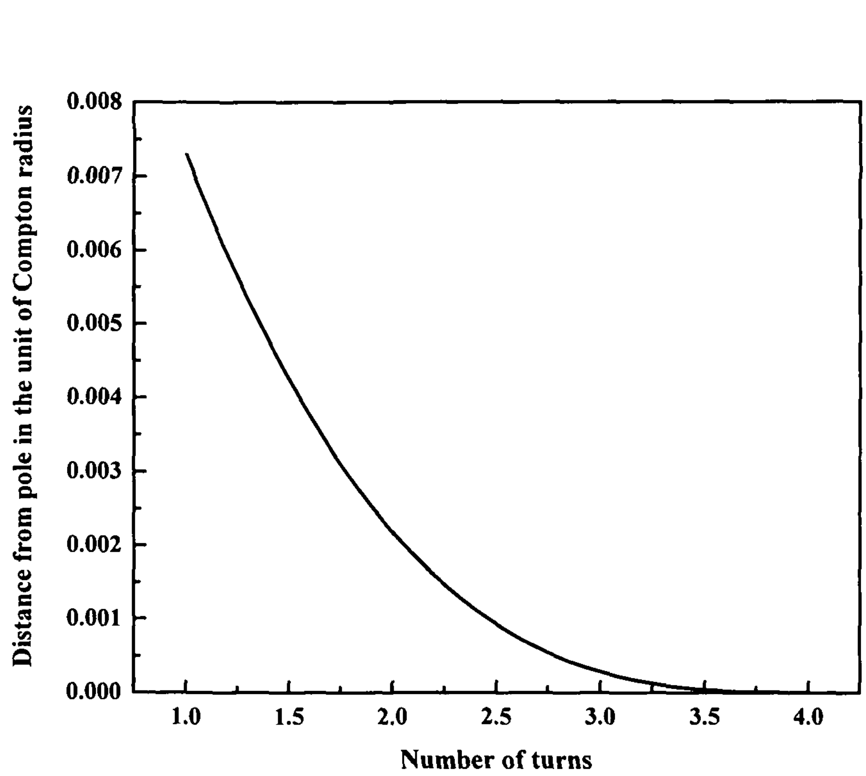

Figure 6.5: Number of turns vs its distances from the pole

6.4 Modified helical motion and the magnetic moment

The magnitude of the fundamental intrinsic magnetic moment of the electron without the radiative correction is defined as $\mu = \frac{e\hbar}{2mc}$ [12]. This is the zeroth-order magnetic moment of the electron. It was given by Uhlenbeck and Goudsmit [1]. Later it was realized that the accurate magnetic moment of the electron is approximately 0.01% greater than this value. In the Schwinger-corrected form of the magnetic moment of the electron, this 0.01% correction was included and provided with the help of the fine structure constant as

$$\mu = \frac{e\hbar}{2mc}\left(1 + \frac{\alpha}{2\pi}\right), \qquad 6.37$$

with $\frac{\alpha}{2\pi}$ multiplied with Bohr magneton gives the Schwinger correction [1-2].

In QED, the measurement of the magnetic moment of the electron states about the interaction of the electron with the fluctuating vacuum [13-15]. Combining equations 6.7 and 6.37, we have the magnetic moment of the electron as

$$\mu = \frac{eR_{C0}}{2}. \qquad \textbf{6.38}$$

As R_{C0} is composed of R_C and R_0, one can re-write the expression of the magnetic moment in equation 6.38 as the sum of magnetic moments due to R_C and R_0 to

$$\mu = \frac{eR_C}{2} + \frac{eR_0}{4\pi}. \qquad \textbf{6.39}$$

The factor $1+\frac{\alpha}{2\pi}$ made it possible to express the magnetic moment with R_C and R_0.

This factor also connects the g-factor and the fine structure constant as [1, 16]

$$\frac{g}{2} = 1 + \frac{\alpha}{2\pi}. \qquad \textbf{6.40}$$

Equation 6.40 states also about the dependence of the g-factor on α [17]. In fact, with the recent results, g-factor can be expressed more accurately as [18]

$$\frac{g}{2} = 1 + \left(\frac{\alpha}{2\pi}\right) - 0.3284790\left(\frac{\alpha}{\pi}\right)^2 + 1.1765\left(\frac{\alpha}{\pi}\right)^3 - 0.8\left(\frac{\alpha}{\pi}\right)^4. \qquad \textbf{6.41}$$

It is noteworthy in this regard that more accuracy in the value of g-factor refers to the change in the value of the magnetic moment also. Hence the structure of this composite radius also changes accordingly. Indeed equation 6.7 can be re-written with the help of equation 6.40 as

$$R_{C0} = \frac{g}{2}R_C. \qquad \textbf{6.42}$$

Using equation 6.6, which describes the relation between R_0 and R_C, along with equation 6.39, we can have the relation of classical radius to Compton radius in terms of the g-factor as

$$\frac{R_0}{R_C} = 2\pi\left(\frac{g}{2} - 1\right). \qquad \textbf{6.43}$$

This actually gives us hint about the fact that the relations amongst the different radii can go in the higher order of α if one obeys equation 6.40 of the g-factor.

Again the factor $\frac{g-2}{2}$ is related with the anomalous magnetic moment of the electron a and the Bohr magneton μ_B as [19]

$$a = \frac{\mu}{\mu_B} - 1 = \frac{g-2}{2}. \qquad \textbf{6.44}$$

Equations 6.42 and 6.44 together provide the condition

$$R_{C0} = (1+a)R_C. \qquad \textbf{6.45}$$

If we compare now equation 6.45 with equation 6.36, we have the form of anomalous magnetic moment as

$$a = \frac{\alpha}{2\pi} + K_2\left(\frac{\alpha}{2\pi}\right)^2 + K_3\left(\frac{\alpha}{2\pi}\right)^3 + K_4\left(\frac{\alpha}{2\pi}\right)^4 + \ldots\ldots . \qquad \textbf{6.46}$$

Here K_2, K_3 and K_4 are the numerical constants associated with those terms. We have not used the K_1, as we have followed the suffixes from the order of α. Contemporary expression of the anomalous magnetic moment of the electron from the experimental facts for higher order of the fine structure constant is given as

$$a_e(\text{QED}) = C_e^{(2)}\left(\frac{\alpha}{\pi}\right) + C_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_e^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + \ldots, \qquad \textbf{6.47}$$

where $C_e^{(i)}$s are the co-efficients and the first one was calculated by Schwinger [2, 20]. Equations 6.46 and 6.47 are of same pattern and the orders of α is symmetric.

The mass component associated with the anomalous magnetic moment is the Schwinger-corrected mass [1]

$$\Delta m = m \cdot \frac{\alpha}{2\pi}. \qquad \textbf{6.48}$$

Combining equations 6.40, 6.44 and 6.48, we have the electromagnetic mass, as we described in the chapter 5, in terms of g-factor and the anomalous magnetic moment of the electron as

$$\Delta m = m\left(\frac{g}{2} - 1\right) = ma. \qquad \textbf{6.49}$$

Recent measurement of the g-factor leaves impact on the Δm. Thus it ensures more accurate measurement of both electromagnetic and mechanical mass of the electron. As we are describing Δm to be the electromagnetic mass, it can now be précised from equations 6.49 and 6.47 as

$$\Delta m = m\left[C_e^{(2)}\left(\frac{\alpha}{\pi}\right)+C_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2+C_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3+C_e^{(8)}\left(\frac{\alpha}{\pi}\right)^4+.....\right]. \qquad 6.50$$

Equation 6.50 is identical with equation 6.48 in addition with the next three orders of correction. The corresponding energy is then expressed with the help of equation 6.50 as

$$W_H \cong mc^2\left[C_e^{(2)}\left(\frac{\alpha}{\pi}\right)+C_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2+C_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3+C_e^{(8)}\left(\frac{\alpha}{\pi}\right)^4+.....\right]. \qquad 6.51$$

In the same way, one can re-write the magnetic moment as

$$\mu = \frac{e\hbar}{2mc}\left[1+C_e^{(2)}\left(\frac{\alpha}{\pi}\right)+C_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2+C_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3+C_e^{(8)}\left(\frac{\alpha}{\pi}\right)^4+.....\right]. \qquad 6.52$$

Using equation 6.52 in equation 6.1 and equating with equation 6.51 we have

$$R_H^3 = \frac{1}{6c}\alpha R_C^3\frac{\left[1+C_e^{(2)}\left(\frac{\alpha}{\pi}\right)+C_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2+C_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3+C_e^{(8)}\left(\frac{\alpha}{\pi}\right)^4+.....\right]^2}{\left[C_e^{(2)}\left(\frac{\alpha}{\pi}\right)+C_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2+C_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3+C_e^{(8)}\left(\frac{\alpha}{\pi}\right)^4+.....\right]}. \qquad 6.53$$

Introducing the relation between classical radius and Compton radius we get the combination of two radii as

$$R_H^3 = \frac{1}{6c}R_0R_C^2\frac{\left[1+C_e^{(2)}\left(\frac{\alpha}{\pi}\right)+C_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2+C_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3+C_e^{(8)}\left(\frac{\alpha}{\pi}\right)^4+.....\right]^2}{\left[C_e^{(2)}\left(\frac{\alpha}{\pi}\right)+C_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2+C_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3+C_e^{(8)}\left(\frac{\alpha}{\pi}\right)^4+.....\right]}. \qquad 6.54$$

For the convenience of our calculation, we write equation 6.54 as

$$R_H^3 = \frac{1}{6c}R_0R_C^2\frac{[1+a_e]^2}{a_e}, \qquad 6.55$$

where

$$a_e = \left[C_e^{(2)}\left(\frac{\alpha}{\pi}\right)+C_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2+C_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3+C_e^{(8)}\left(\frac{\alpha}{\pi}\right)^4+.....\right]$$

is the recent calculation of the anomalous magnetic moment from QED which we have seen equation 6.47. Indeed, this is of the same pattern with equation 6.46. But as we have different numerical constants in equation 6.46 and 6.47, let us take any one form amongst equation 6.46 and 6.47 and define it in a more generalised symbol χ instead of a or a_e.

$$\chi = \left[C_e^{(2)}\left(\frac{\alpha}{\pi}\right) + C_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_e^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + \ldots\ldots \right]. \qquad 6.56$$

Therefore in a more précised form equation 6.54 can be written as

$$R_H^3 = SR_0R_{C0\chi}^2, \qquad 6.57$$

where

$$S = \frac{1}{6c\chi} \qquad 6.58$$

and

$$R_{C0\chi} = R_C(1+\chi). \qquad 6.59$$

This equation 6.59 [3] reveals here the new expression of the composite radius of the electron. One can get this sort of expression also by putting the higher order terms in the right hand side of equation 6.7. Independent of the way of arriving at the point of the present condition, the helical path will be there. Here, with this new pattern of composite radius, we can continue for the helical motion of the charge in the similar way we did earlier.

So, the total time required for the motion of the charge of the electron will now be modified from equation 6.9 with the help of equation 6.59 as

$$T = \frac{R_E + 2n\pi R_{C0\chi}}{v}. \qquad 6.60$$

Therefore the corresponding current will be modified from equation 6.10 according to equation 6.60 as

$$I = \frac{ev}{R_E + 2n\pi R_{C0\chi}}. \qquad 6.61$$

The magnetic moment will now be read in a modified form of equation 6.13 with the help of equation 6.59 as

$$\mu = \frac{2(n-1)\pi ev\chi R_C^2}{c\left(R_E + 2n\pi R_{C0_\chi}\right)}. \qquad 6.62$$

The number of turns can now be calculated as

$$n = 1 + \frac{2}{\chi}. \qquad 6.63$$

Thus, with the new form of composite radius, we can have the new sets of equations of velocity, angular momentum and the magnetic field for different turns of rotation as given below.

Magnetic field after the first turn will be calculated as

$$B_1 = \frac{2}{R_{C0_\chi}}\left[\frac{ev_1}{R_E + 2n\pi R_{C0_\chi}}\right]. \qquad 6.64$$

Equation 6.64 is the modified form of equation 6.18. Similarly, equation 6.20 can be modified as below in equation 6.65

$$L_1 = mR_C v_1 + \frac{eR_{C0_\chi}^2 B_1}{2c}. \qquad 6.65$$

The velocity v_2 will retain the form of equation 6.23

$$v_2 = v_1\left[\frac{L_1}{eR_C^2 B_1} - \frac{1}{2c}\right].$$

Equation 6.23 is modified as

$$B_2 = B_1\left(\frac{R_E + 2\pi R_{C0_\chi}}{R_E + 4\pi R_{C0_\chi}}\right)\left[\frac{L_1}{eR_C^2 B_1} - \frac{1}{2c}\right]. \qquad 6.66$$

Equation 6.26 takes the modified form as

$$L_2 = \left[\frac{L_1}{eR_C^2 B_1} - \frac{1}{2c}\right]\left[mv_1 R_C + \frac{eR_{C0_\chi}^2 B_1}{2c}\left(\frac{R_E + 2\pi R_{C0_\chi}}{R_E + 4\pi R_{C0_\chi}}\right)\right]. \qquad 6.67$$

For n-th turn, equation 6.23 remains same, but equations 6.34 and 6.35 are expressed in corrected version below in equations 6.68 and 6.69.

$$v_n = v_{n-1}\left[\frac{L_{n-1}}{eR_C^2 B_{n-1}} - \frac{1}{2c}\right],$$

$$B_n = B_1\left(\frac{R_E + 2\pi R_{C0_\chi}}{R_E + 2n\pi R_{C0_\chi}}\right)\left[\frac{L_1}{eR_C^2 B_1} - \frac{1}{2c}\right]\left[\frac{L_2}{eR_C^2 B_2} - \frac{1}{2c}\right]\cdots\left[\frac{L_{n-1}}{eR_C^2 B_{n-1}} - \frac{1}{2c}\right]. \qquad 6.68$$

and

$$L_n = \left[\frac{L_1}{eR_C^2 B_1} - \frac{1}{2c}\right]\left[\frac{L_2}{eR_C^2 B_2} - \frac{1}{2c}\right]\cdots$$

$$\cdots\left[\frac{L_{n-1}}{eR_C^2 B_{n-1}} - \frac{1}{2c}\right]\left[mv_1 c + \frac{eR_{C0_\chi}^2 B_1}{2c}\left(\frac{R_E + 2\pi R_{C0_\chi}}{R_E + 2n\pi R_{C0_\chi}}\right)\right]. \qquad 6.69$$

All the turns of the rotation of the charge are expressed in above equations 6.64 to 6.69. This helical path of the motion of the charge is of a similar category, which is described by dynamical spinning sphere model of the electron [4]. There, it is defined as the motion of the charge-centre around the centre of mass. The Zitterbewegung motion [21-25], which was introduced by Schrodinger [23] and developed by others portrays an analogous picture. We have seen that the experimental observations are providing the higher order corrections of the magnetic moment and *g*-factor.

As the charge is following the Compton-sized path at the equator, it promotes the extended electron structure. Again at pole, it touches almost a point-like behaviour. Therefore extended and point-like structure can be connected with this feature. Thus we have come to know that the charge will take the smaller radius during the rotation, away from the equator and a bigger radius towards the equator. So one can imagine here the sphere as a system composed of several circular disks according to the spherical structure.

As the charge is moving in a relativistic velocity and the distance required to be covered in the levels away from the equator is less, the charge gets more time to be at the equator. Because it can pass very fast through such small lengths. Hence the maximum probability of finding the charge is in the equatorial zone of the spherical structure.

6.5 Generalized spinning mass

As we have worked here with a relativistic but arbitrary velocity instead of considering *c*-velocity, we must look for a generalized spinning mass for the relativistic spinning sphere. MacGregor considered cylindrical mass elements in a spinning sphere [1] with the volume

$$V(r) = 4\pi\sqrt{R^2 - r^2}\,rdr, \qquad 6.70$$

where r is the distance of the cylindrical element from the axis of rotation and R is the radius of the sphere. This gives the total mass of the spinning sphere is

$$M_S = \frac{3M_0}{2}\int_0^R \sqrt{\frac{R^2 - r^2}{1 - \omega^2 r^2 / c^2}}\,rdr. \qquad 6.71$$

For $\omega = \frac{c}{R}$, the total mass comes out as

$$M_S = \frac{3}{2}M_0, \qquad 6.72$$

with M_S as spinning mass and the M_0 as the rest mass of the electron.

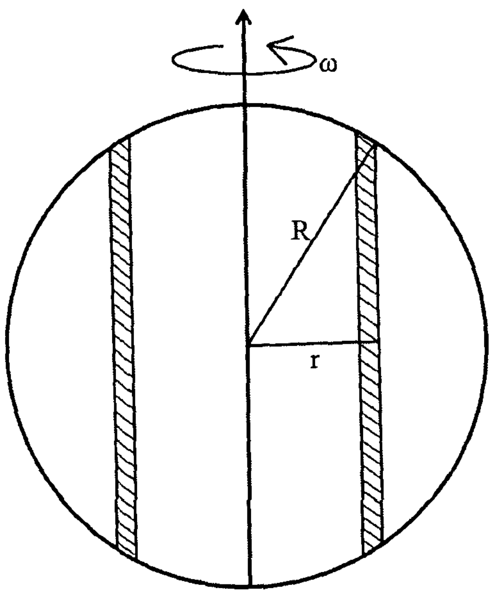

Figure 6.6: Relativistic spinning sphere with cylindrical mass-strip

If we consider the arbitrary velocity v, the angular velocity would be

$$\omega = \frac{v}{R}. \qquad 6.73$$

Hence the spinning mass can be expressed as

$$M_S = \frac{3M_0}{2}\int_0^R \sqrt{\frac{R^2 - r^2}{1 - \frac{v^2}{c^2}\frac{r^2}{R^2}}}\,rdr. \qquad 6.74$$

Integrating within the limit we have the generalized form of the spinning mass as

$$M_S = \frac{3M_0}{R^3}\left[\frac{R^3 d^2}{2} - \frac{R^3 d}{4}(1-d)^2 \log\left\{-v^2 R^2 (d-1)^2\right\}\right], \qquad 6.75$$

where $d = \frac{c}{v}$ is a ratio. For $d = 1$ or $v = c$, equation 6.75 takes exactly the same form of 6.72, derived by MacGregor.

6.6 Concluding remarks

Here we have developed a model, which is in fact a modified version of the relativistic spinning sphere model of the electron. The stress is given in the charge part of the electron. We have designed the helical motion of the charge from the nature of the Composite radius of the electron. This motion is again re-constructed with the correction due to anomalous magnetic moment and the *g*-factor. The theoretical approach to explain the higher order of the anomalous magnetic moment is given. The generalized spinning mass is also calculated.

The fact is that, though the work was started with the classical approach, we have finally arrived the quantized-states chosen by the charge during the rotation. As the residue magnetic field will affect on the charge at the pole and the energy retained at pole will be less than that given by the magnetic field, the charge would return towards the equator. Thus, we have the spherical structure with the helical motion of the charge. Zitterbewegung is a kind of helical motion and this model connects the Zitterbewegung with classical and semi-classical approaches. Again, this also predicts the connection amongst the point-like and the extended particle model. It is quite interesting that circular vortex streamlines gave a similar pattern of the electron structure from different sorts of calculations done by A. Martin in his article [26]. His work revealed the circular streamlines centered on the vertical axis. Hence a helical nature is seen in that work which supports our calculations. Our result for the shape of the electron is also in agreement with the recent measurement of the shape of the electron [27]. Indeed the aspheric nature of the shape of the electron is also reflected by the composite radius of the electron and the spinning sphere structure studied in this thesis.

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When everything is easy one quickly gets stupid -Maxim Gorky

Chapter 7

Positronium mass spectra

In the earlier chapters, we have concentrated over the semi-classical physics to depict a picture of the electron. The immediate next turn is the positron, the anti particle of the electron, and the behaviour of the positron will be similar to that of the electron as the properties concerned are matching with each other. The Positronium is a bound state of the electron and the positron. Here we attempt to obtain the mass spectrum of the S-wave positronium in the framework of non-relativistic models. One photon exchange potential is considered here. We tried to calculate the corresponding wave function considering the positronium as a harmonic oscillator. In this chapter, an attempt has been made to obtain the mass spectrum of the S-wave positronium in the frame work of non-relativistic models. A good agreement is obtained with the masses provided from the combination of the electron and the positron masses.

7.1 Bound state

Bound states are composed of generally two particles. A good number of bound states are present in recent particle physics. Positronium, Charmonium, Quarkonium and Bottomonium are the well-known bound states in the current scenario [1]. Positronium is a quasi-stable bound system [2]. This is an electron-positron bound state. It is its own anti-particle [3]. This system offers unique opportunities to test the understanding of bound states in the framework of QED [4]. S. Mohorovicic predicted the chance of existence of the positronium [5]. It was discovered by M. Deutsch and it is denoted as Ps [6]. Gross spectroscopic structure of the positronium is equivalent to that of hydrogen. The reduced mass of the positronium is half that of the electron [5]. Two major states of the positronium are

$1\,^1S_0$ and $1\,^3S_1$, which are discussed later as singlet and triplet respectively. $1\,^1S_0$ is known as para-positronium and $1\,^3S_1$ is known as ortho positronium [7].

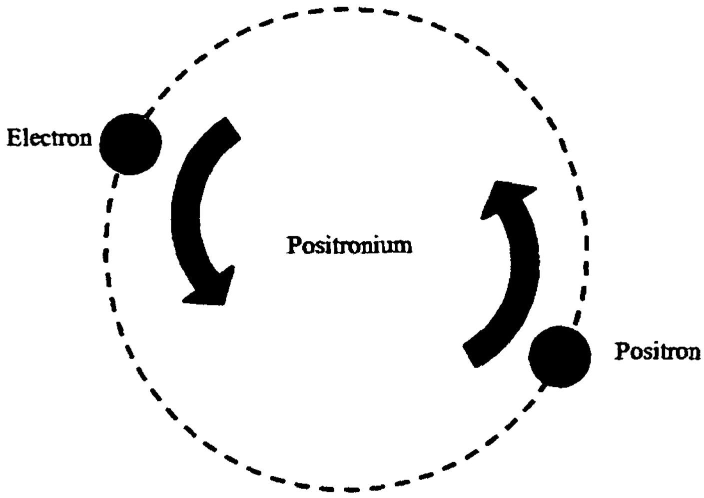

Figure 7.1: Positronium, the electron-positron bound state

Positronium states are given as $n\,^{(2s+1)}l_j$. Here n is the principal quantum number, s is the spin quantum number and l is the orbital quantum number. For $l=0$, we have positronium S-wave, for $l=1$, we have positronium P-wave and for $l=2$, we have positronium D-wave. Therefore for principal quantum number, $n=1$, the state will be read as $1\,^{(2s+1)}l_j$. The total angular momentum quantum number is j. If we aim to have positronium S-wave states, we must have to work with $l=0$ condition and it will be written as $^{(2s+1)}S_j$. To determine the value of j, now we require the spin quantum number s. For singlet state s is 0, whereas for triplet state s is 1. Thus for $s=0$ and $l=0$, we get $j=0$. This prompts us to write the concerned S-wave state as $^1S_0$. If we consider $s=1$ with $l=0$, we have $j=1$ and this gives $^3S_1$ triplet state.

We have attempted here to produce the mass of the positronium states in terms of its wave-functions in the framework of potential model problem [8]. Mesons are also the bound states of quarks and anti-quarks [1] and they are represented in terms of quarks and anti-quarks invoking potential model problem [9-11]. We have tried to continue our work in the same way including the properties of the electron and the positron in the place of quark and anti-quark.

7.2 Non-relativistic model of the positronium

We consider here the positronium as a harmonic oscillator potential. The Hamiltonian of the non relativistic positronium model is

$$H = K + V_{conf}(\vec{r}) + V_{opep}(\vec{r}), \qquad 7.1$$

where K is the kinetic energy term, V_{conf} is the confinement potential and V_{opep} is the one-photon exchange potential.

In case of meson bound states, one gluon exchange potential is considered and for the electron-positron bound state, one photon exchange potential is considered. The expression of the kinetic energy is given as [8, 12]

$$K = \sum_{e^-e^+}\left(M_{e^-} + \frac{P_{e^-}^2}{2M_{e^-}}\right) - K_{CM}, \qquad 7.2$$

where $M_{e^-} = M_{e^+} = \text{mass of the electron}$. K is sum of the kinetic energies of the electron and the positron, including the rest mass minus the kinetic energy of the centre of mass of the total system. The confinement term represents the effect of QED and it is given as [8]

$$V_{conf}(\vec{r}) = -a_c r, \qquad 7.3$$

where a_c is the confinement strength. The term $\vec{r}$ is the relative distance between the electron and the positron. The nature of the confinement potential is central in nature. The central part of two-body potential due to one photon exchange potential is expressed as

$$V_{opep}^{cent}(\vec{r}) = \alpha\left[\frac{1}{r} - \frac{\pi}{M_{e^-}M_{e^+}}\left(1 + \frac{2}{3}\overrightarrow{\sigma_{e^-}}.\overrightarrow{\sigma_{e^+}}\right)\delta(\vec{r})\right], \qquad 7.4$$

where α is the fine structure constant, M_{e^-} is the mass of the electron, M_{e^+} is the mass of the positron, σ_{e^-} and σ_{e^+} are the Pauli spin matrices.

7.3 Harmonic oscillator wave function

The general wave function of the harmonic oscillator is [8]

$$\psi_{nlm}(r,\theta,\phi)=N(\alpha r)^{l}L_{n}^{l+1/2}(x)e^{-\frac{\alpha^{2}r^{2}}{2}}Y_{lm}(\theta,\phi), \qquad 7.5$$

where

$$|N|^{2}=\frac{2\alpha^{3}n!}{\sqrt{\pi}}\frac{2^{[2(n+l)+1]}}{(2n+2l+1)!}(n+l)! \qquad 7.6$$

and

$$L_{n}^{l+1/2}(x)=\frac{e^{x}x^{-(l+1/2)}}{n!}\frac{d^{n}}{dx^{n}}\left[e^{-x}x^{l+1/2+n}\right]. \qquad 7.7$$

Here $L_{n}^{l+1/2}(x)$ are associated Laguerre polynomials, $\alpha=\frac{1}{b}$ and N is the normalization constant. Symbols have their usual meanings. It is noteworthy that $\alpha=\frac{1}{b}$ is not the fine structure constant, b is the oscillator parameter. The harmonic oscillator wave function for the 0S state is [8]

$$\psi_{0S}=\frac{2}{\pi^{1/4}b^{3/2}}e^{\left[-\frac{r^{2}}{2b^{2}}\right]}Y_{00}(\theta,\phi). \qquad 7.8$$

Here 0S, 1S, 2S, 3S and 4S represent the harmonic oscillator wave functions for S wave bound states for radial quantum numbers n = 0, 1, 2, 3 and 4. As we are dealing here with the positronium, we shall concentrate on the S-wave positronium.

7.4 Kinetic energy matrix elements

As the positronium is a bound state of the electron and the positron, the kinetic energy of it can be calculated only when both the electron and the positron will be contributing to it. In the matrix element $\langle e^{-}e^{+}\,|\,K\,|\,e^{-}e^{+}\rangle$, we have the electron-positron wave-function $|e^{-}e^{+}\rangle$. The non-relativistic expression for the kinetic energy of the electron-positron system is expressed in equation 7.2. The matrix element $\left\langle e^{-}e^{+}\,|-\frac{\nabla^{2}}{2\mu}|\,e^{-}e^{+}\right\rangle$ can be evaluated to calculate the kinetic energy. μ is the reduced mass of the system and this can be expressed as

$$\mu = \frac{M_{e^-} M_{e^+}}{M_{e^-} + M_{e^+}} \quad . \qquad 7.9$$

The Laplace operator ∇^2 can be expressed in spherical coordinates as

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) - \frac{L^2}{\hbar^2 r^2} \, . \qquad 7.10$$

For 0S state, the matrix element can be written as [11]

$$\left\langle \psi_{0S} \left| -\frac{\nabla^2}{2\mu} \right| \psi_{0S} \right\rangle = -\frac{1}{2\mu}\int_0^\infty \left(\frac{2}{\pi^{1/4} b^{3/2}}\right)^2 4\pi r^2 \exp\left[-r^2/2b^2 \left\{\frac{2}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2}\right\}\right] \exp\left[-r^2/2b^2\right] dr$$

$$= \frac{3}{4b^2 \mu} \, , \qquad 7.11$$

where b is the oscillator parameter and μ is the reduced mass. Thus we can have the kinetic energy matrix elements for various states as $\left(\left\langle \psi_{nS} \left| -\frac{\nabla^2}{2\mu} \right| \psi_{nS} \right\rangle_{n=0,1,2,3,4}\right)$ and they are listed below:

Table 7.1: Calculated kinetic energy matrix element

Matrix element	Results
$\left\langle \psi_{0S} \left\| -\frac{\nabla^2}{2\mu} \right\| \psi_{0S} \right\rangle$	$\frac{3}{4b^2\mu}$
$\left\langle \psi_{1S} \left\| -\frac{\nabla^2}{2\mu} \right\| \psi_{1S} \right\rangle$	$\frac{7}{4b^2\mu}$
$\left\langle \psi_{2S} \left\| -\frac{\nabla^2}{2\mu} \right\| \psi_{2S} \right\rangle$	$\frac{11}{4b^2\mu}$
$\left\langle \psi_{3S} \left\| -\frac{\nabla^2}{2\mu} \right\| \psi_{3S} \right\rangle$	$\frac{15}{4b^2\mu}$
$\left\langle \psi_{4S} \left\| -\frac{\nabla^2}{2\mu} \right\| \psi_{4S} \right\rangle$	$\frac{19}{4b^2\mu}$

7.5 Central one-photon exchange potential matrix elements

The central one-photon exchange potential is written in equation 7.4. Now the matrix elements can be expressed as

$$\left\langle e^-e^+\middle|V_{opep}^{cent}\middle|e^-e^+\right\rangle = \alpha\left[\left\langle e^-e^+\middle|\frac{1}{r}\middle|e^-e^+\right\rangle - \frac{\pi}{M_{e^-}M_{e^+}}\left(1+\frac{2}{3}\overrightarrow{\sigma_{e^-}}.\overrightarrow{\sigma_{e^+}}\right)\left\langle e^-e^+\middle|\delta^3(\vec{r})\middle|e^-e^+\right\rangle\right],$$

where $\overrightarrow{\sigma_{e^-}}.\overrightarrow{\sigma_{e^+}} = -3$ for ${}^1S_0$ and $\overrightarrow{\sigma_{e^-}}.\overrightarrow{\sigma_{e^+}} = 1$ for ${}^3S_1$. In the central one-photon exchange potential matrix elements, we see two associated parts. They are $\left\langle e^-e^+\middle|\frac{1}{r}\middle|e^-e^+\right\rangle$ and $\left\langle e^-e^+\middle|\delta^3(\vec{r})\middle|e^-e^+\right\rangle$. The first one is the Coulombic potential matrix element and the second one is the radial part of the $\delta^3(\vec{r})$ matrix elements.

The matrix element $\left\langle e^-e^+\middle|\frac{1}{r}\middle|e^-e^+\right\rangle$ for 0S state can be written as

$$\left\langle \psi_{0S}\psi_{0S}\middle|\frac{1}{r}\middle|\psi_{0S}\psi_{0S}\right\rangle = \left(\frac{1}{\pi b^2}\right)^3 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-\left(r_1^2+r_2^2\right)/2b^2\right]$$

$$.\frac{1}{|r_1-r_2|}\exp\left[-\left(r_1^2+r_2^2\right)/2b^2\right]d^3r_1d^3r_2.$$

We use here the transformation $\vec{r} = \vec{r_1} - \vec{r_2}$ and $\vec{R} = \frac{(\vec{r_1}+\vec{r_2})}{2}$, where $\vec{r_1}$ and $\vec{r_2}$ are the distance of the electron and the positron from the centre of the system. Performing the integration over the centre of mass coordinates $\vec{R}$ and evaluating the Gaussian integral, we have the matrix elements as

$$\text{Matrix elements} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-\left(R^2+r^2\right)/b^2\right]\frac{1}{r}d^3r d^3R$$

$$= \left(\frac{1}{\pi b^2}\right)^3 \pi^{3/2}b^3 4\pi\int_0^{\infty}\exp\left[-r^2/b^2\right]r dr.$$

The potential due to $\delta^3(\vec{r})$ provides two sorts of matrix elements, one is of radial part and other is of angular matrix elements. The $\delta^3(\vec{r})$ potential is given by [8]

$$\delta^3(\vec{r}) = \frac{\delta(r_{e^-} - r_{e^+})}{r_{e^-}^2}\sum_{k=0}^{\infty}\frac{2k+1}{4\pi}Y^k(e^-)Y^k(e^+),$$

where Y^k's are the renormalized spherical harmonics. The matrix element

$\langle \psi_{0S}\psi_{0S}|\delta^3(r)|\psi_{0S}\psi_{0S}\rangle =$

$$\left(\frac{1}{\pi b^2}\right)^3 \frac{1}{4\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp[-(r_i + r_j)/2b^2]\frac{\delta(r_i - r_j)}{r_i^2}\exp[-(r_i^2 + r_j^2)/2b^2]d^3r_i d^3r_j = \frac{1}{2\sqrt{2}b^3\pi^{3/2}}.$$

The matrix elements of the one-photon exchange potential for Coulombic potential and the radial part are given below:

Table 7.2: Diagonal matrix elements for Coulombic potential and radial part

n	$\langle \psi_{nS}\|\frac{1}{r}\|\psi_{nS}\rangle$	$\langle \psi_{nS}\|\delta^3(r)\|\psi_{nS}\rangle$
0	$\frac{2}{b\sqrt{\pi}}$	$\frac{1}{2\sqrt{2}b^3\pi^{3/2}}$
1	$\frac{5}{3b\sqrt{\pi}}$	$\frac{41}{128\sqrt{2}b^3\pi^{3/2}}$
2	$\frac{89}{60b\sqrt{\pi}}$	$\frac{8257}{32768\sqrt{2}b^3\pi^{3/2}}$
3	$\frac{381}{280b\sqrt{\pi}}$	$\frac{448697}{2097152\sqrt{2}b^3\pi^{3/2}}$
4	$\frac{25609}{20160b\sqrt{\pi}}$	$\frac{405918745}{2147483648\sqrt{2}b^3\pi^{3/2}}$

7.6 Confinement potential matrix elements

Again confinement potential for the positronium system we already have described in equation 7.3 as

$$V_{conf}(\vec{r}) = -a_c r,$$

where a_c is the confinement strength of the system.

The diagonal matrix elements $\left(\langle \psi_{nS}|r|\psi_{nS}\rangle_{n=0,1,2,3,4}\right)$ are expressed in the table below:

Table7.3: Calculated confinement potential matrix elements

Matrix element	Results
$\langle 0S\vert r\vert 0S\rangle$	$\frac{2b}{\sqrt{\pi}}$
$\langle 1S\vert r\vert 1S\rangle$	$\frac{3b}{\sqrt{\pi}}$
$\langle 2S\vert r\vert 2S\rangle$	$\frac{15b}{4\sqrt{\pi}}$
$\langle 3S\vert r\vert 3S\rangle$	$\frac{35b}{8\sqrt{\pi}}$
$\langle 4S\vert r\vert 4S\rangle$	$\frac{315b}{64\sqrt{\pi}}$

7.7 S-wave spectroscopy

We have described the full Hamiltonian of the S-wave positronium system in equation 7.1. Individual terms are also given above. The relative wave function for 0S state becomes [8]

$$\psi_{0S} = \left(\frac{1}{\pi b^2}\right)^{3/4} \exp\left(\frac{-r^2}{2b^2}\right),$$

where b is the oscillator parameter. In computing the mass of the positronium, we have diagonalized the Hamiltonian matrix $\left(\langle \psi_{iS}|r|\psi_{jS}\rangle_{i,j=0,1,2,\ldots,10}\right)$ in the relative space. The total energy or the mass of the positronium is obtained by calculating the energy eigen values of the Hamiltonian in the harmonic oscillator basis. Therefore, we see here that the wave function depends on the oscillator parameter b.

As the electron mass and the positron mass are same and they are given as $M_{e^-} = M_{e^+} = 0.511\,\text{MeV}$, the positronium mass can have a maximum value of 1.022 MeV. Varying the b-parameters we got the values closer to expected positronium mass of 1.022 MeV at around b = 4.0 picometer. Using the above parameters we got the value of the positronium S-wave as 1.029145 MeV for $^3S_1$ state, which is very close to the above-mentioned 1.022 MeV.

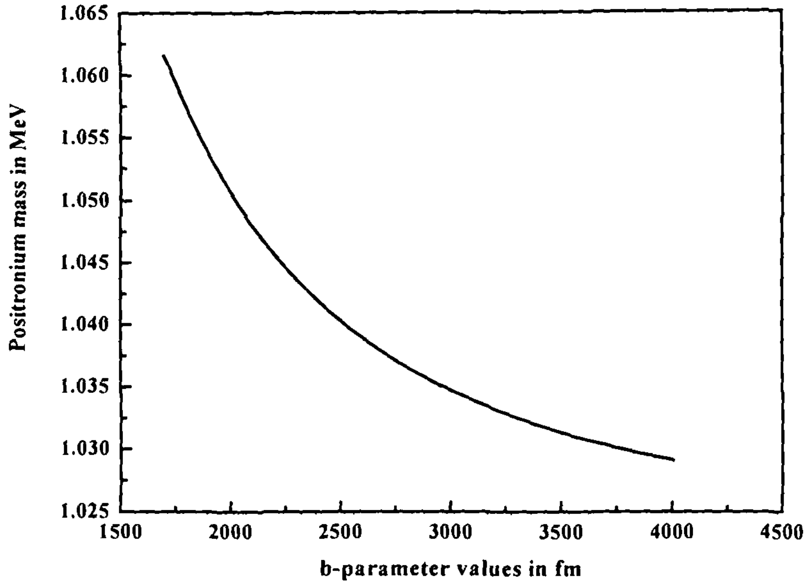

Figure 7.2: Positronium mass vs. b-parameter

Table 7.4: Parameters and values used

Parameters	values
b	4.0 pm
M_{e^-,e^+}	0.511MeV
a_c	260MeVfm^{-1}
α	0.00729927

7.8 Concluding remarks

We started this part of the positronium work with the inspiration from quark-antiquark bound state problems. The meson system involves huge mass compared to the electron or the positron masses. Again the coupling constant is used for the positronium system is the α instead of α_s for the meson. The α_s is very large compared to the fine structure constant as the strong force is very strong compared to the electromagnetic force. Hence the only possible condition was *b*, the oscillator parameter, which could control the situation to produce positronium mass spectra with logical theory. There we get a large *b* for positronium compared to the *b* values for mesons. It is 4 picometer and physically acceptable as it falls in the order of the

Compton radius of the electron, or the Compton wavelength. The value of the b parameter gives the signature of the positronium of picometer order. We have worked on the Compton-sized electron. Hence it is also a favourable result to advocate for the positronium structure.

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Never express yourself more clearly than you are able to think

- Niels Bohr

Chapter 8

Conclusions

In this thesis, we have concentrated on the 'enigmatic' particle electron and its classical and semi-classical structural models in the light of the properties of the electron. The size and the shape of this particle have been focussed and we have studied the radii of the electron from different electromagnetic phenomena. Consequently, we established some relations amongst different radii of the electron [1]. Our study has *confirmed the involvement of the fine structure constant in the* electron radii and we found the relations between any two radii are α-quantized [2]. The radii of the electron are related in terms of the fine structure constant. Thus all the known radii are related and this reflects the relations amongst the originating electromagnetic phenomena indeed. Depending on the α-quantized relations of the radii, we propose the mathematical form of the charge radius of the electron, which is in agreement with the measurement and the prediction of the experimental facts [1-2].

As the fine structure constant connects the different radii of the electron, it controls all the properties involving the size of the electron. Thus for different radii, we get α-quantized results for current-loops and magnetic fields too. We have introduced here the current-loop of the electron in terms of its intrinsic properties - the charge, the mass and the spin in a compact form and this form is found to exist in all the current-loops and the magnetic fields for different radii [3]. Thus we get a generalised current-loop form for the electron which is independent of phenomena and it functions as a general feature of the electron. The similar nature of the current-loop expression is observed in the external magnetic field also [4]. The work for current-loop is also extended for the muon and the tau [4] with the help of α-quantized mass feature. The linear velocity of the charge with spinning sphere model is calculated for different radii and we found there classical radius as contracted form of Compton radius [1]. We have established the relations of the radii of the electron with the Rydberg constant also.

The anomalous magnetic moment and the Schwinger correction got importance in our work here. Regarding the electromagnetic mass of the electron, we have focussed on the Schwinger-corrected mass. This led us to the expression of the energy in terms of the charge and the mass [5]. Our observations show that below the linear velocity c, charge decreases and the mass increases with the increasing velocity [6].

Using the magnetic self-energy, we have extracted a new kind of radius of the electron, which is composed of classical and Compton radius [7]. This is done in the frame work of the relativistic spinning sphere. In turn, we expressed it as modified Compton radius also [8]. Incorporating the higher order terms for this radius we have composite radius which produces magnetic moment in good agreement with current experimental observation [9]. This radius introduces a helical path of the motion of the charge [5, 9]. The charge is shown to follow the helical path with a rotational motion. The charge continues the rotational motion in the helical path with decreasing order of radii in each turn. Those radii in the turns are specific and they follow the orders of α, multiplied with the Compton radius. We have described the entire set of radii in the helical path as composite radius. This is also related to the higher orders of the anomalous magnetic moment of the electron. We expressed the spinning mass of the relativistic spinning sphere with arbitrary velocity.

This model also offers explanation of the different sizes of the electron and connects various models of it. This shows how the charge can be related to a point or extended electron model. Indeed, when the charge takes turn at the equator, it shows the behaviour of an extended particle, but as it arrives at the pole, the point-like nature is revealed. Hence this model obeys experimental stands as well as touches the theoretical points with the help of fine structure constant. The charge lies mostly at equatorial zone.

We have considered at the final phase of the thesis, positronium as a harmonic oscillator and then calculated its mass for the bound system with the help of one photon exchange potential. This leaves the signature of the structural behaviour of the positronium along with its mass spectra and it is also shown that the positronium is also in the order of the Compton-size.

For future, the work of the model of the electron can be associated with the wave functions corresponding to the specific energies within the structure of the electron. The model of the electron can also be tested for other leptons. For positronium P and D waves also, one can observe the various parameters to get a complete picture of the positronium structure. Then it can confirm the co-relation of the structure of the electron model with that of the positronium.

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Appendix

1 Harmonic Oscillator Wave Function

The harmonic oscillator wave function is given as

$$\psi_{nlm} = N(\alpha r)^l L_n^{l+\frac{1}{2}}(x) e^{-\frac{\alpha^2 r^2}{2}} Y_{lm}(\theta, \phi)$$

Here

$$|N|^2 = \frac{2\alpha^3 n!}{\sqrt{\pi}} \frac{2^(2(n+l)+1)}{(2n+2l+1)!}(n+l)!$$

and

$$L_n^{l+\frac{1}{2}}(x) = \frac{exp(x)x^{-(l+\frac{1}{2})}}{n!} \frac{d^n}{dx^n}[exp(-x)x^{l+\frac{1}{2}+n}]$$

$$\frac{d^n}{dx^n} = \sum_{s=0}^{\infty} \frac{n!}{(n-s)!} \frac{1}{s!} \frac{d^{n-s}}{dx^{n-s}}[e^{-x}] \frac{d^s}{dx^s}[x^{l+\frac{1}{2}+n}]$$

We know that $N = 2n + l$.
Leibnitz theorem says

$$\frac{d^n}{dx^n}[A(x)B(x)] = \sum_{s=0}^{\infty} \frac{n!}{(n-s)!} \frac{1}{s!} \frac{d^{n-s}}{dx^{n-s}}[A(x)] \frac{d^s}{dx^s}[B(x)]$$

Case (i)

$n = 0$, $l = 0$ and $N = 0$

$N = (\frac{4\alpha^3}{\sqrt{\pi}})^{\frac{1}{2}}$.

$L_0^{\frac{1}{2}}(x) = 1$

Using N, $L_0^{\frac{1}{2}}(x)$ we have

$$\psi_{000} = (\frac{4\alpha^3}{\sqrt{\pi}})^{\frac{1}{2}}.e^{[\frac{-\alpha^2 r^2}{2}]} \frac{1}{\sqrt{4\pi}}$$

Using $\alpha = \frac{1}{b}$, we get

$$\psi_{000} = (\frac{1}{\pi b^2})^{\frac{3}{4}} e^{-\frac{r^2}{2b^2}}.$$

Here b is the oscillator parameter.
S-wave functions upto $n = 5$ are given below:

$$\psi_{0S} = (\frac{1}{\pi b^2})^{\frac{3}{4}} e^{-\frac{r^2}{2b^2}}$$

$$\psi_{1S} = \sqrt{\frac{2}{3}}(\frac{1}{\pi b^2})^{\frac{3}{4}}(\frac{3}{2} - \frac{r^2}{b^2})e^{-\frac{r^2}{2b^2}}$$

$$\psi_{2S} = \sqrt{\frac{1}{120}}(\frac{1}{\pi b^2})^{\frac{3}{4}}(4\frac{r^4}{b^4} - 20\frac{r^2}{b^2} + 15)e^{-\frac{r^2}{2b^2}}$$

$$\psi_{3S} = \sqrt{\frac{8}{105}}(\frac{1}{\pi b^2})^{\frac{3}{4}}\frac{1}{6}(-\frac{r^6}{b^6} + \frac{21}{2}\frac{r^4}{b^4} - \frac{105}{4}\frac{r^2}{b^2} + \frac{105}{8})e^{-\frac{r^2}{2b^2}}$$

$$\psi_{4S} = \sqrt{\frac{16}{945}}(\frac{1}{\pi b^2})^{\frac{3}{4}}\frac{1}{24}(\frac{r^8}{b^8} - 18\frac{r^6}{b^6} + \frac{189}{2}\frac{r^4}{b^4} - \frac{315}{2}\frac{r^2}{b^2} + \frac{945}{16})e^{-\frac{r^2}{2b^2}}$$

$$\psi_{5S} = \sqrt{\frac{32}{10395}}(\frac{1}{\pi b^2})^{\frac{3}{4}}\frac{1}{12}(-\frac{r^{10}}{b^{10}} + \frac{55}{2}\frac{r^8}{b^8} - \frac{495}{2}\frac{r^6}{b^6} + \frac{3465}{4}\frac{r^4}{b^4} - \frac{17325}{16}\frac{r^2}{b^2} + \frac{10395}{32})e^{-\frac{r^2}{2b^2}}$$

2 Calculation to find out confinement potential

$$V_{conf}(r_{ij}) = -a_c r_{ij}$$

$$\begin{aligned}
< 0S|r_{ij}|0S > &= \int_{-\infty}^{\infty} (\frac{1}{\pi b^2})^{\frac{3}{2}} e^{-\frac{r^2}{b^2}} r d^3 r \\
&= (\frac{1}{\pi b^2})^{\frac{3}{2}} 4\pi \int_0^{\infty} e^{-\frac{r^2}{b^2}} r^3 dr \\
&= (\frac{1}{\pi b^2})^{\frac{3}{2}} 4\pi \int_0^{\infty} e^{-t^2} b^3 t^3 b dt \\
&= (\frac{1}{\pi b^2})^{\frac{3}{2}} \frac{4\pi b^4}{2} \int_0^{\infty} e^{-t^2} t^3 dt \\
&= \frac{2b}{\sqrt{\pi}} \qquad (1)
\end{aligned}$$

3 Evaluattion of $\delta^3(r)$ matrix elements

The radial part of thc $\delta(r_{ij})$ potential is given by

$$\delta^3(r_{ij}) = \frac{\delta(r_i - r_j)}{r_i^2} \sum_{k=0}^{\infty} \frac{2k+1}{4\pi} y^k(i).Y^k(j),$$

where $Y^{(k)}$'s are the renormalized spherical harmonic.

For S-wave $k = 0$. Therefore, $\sum_{k=0}^{\infty} \frac{2k+1}{4\pi} y^k(i).Y^k(j) = \frac{1}{4\pi}$. The matrix element $< \psi_{0S}\psi_{0S}|\delta^3(r_{ij})|\psi_{0S}\psi_{0S} >= I$ is evaluated below:

$$\begin{aligned}
I &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\frac{1}{\pi b^2})^{\frac{3}{4}} e^{-\frac{r_1^2}{2b^2}} (\frac{1}{\pi b^2})^{\frac{3}{4}} e^{-\frac{r_2^2}{2b^2}} \frac{1}{4\pi} \cdot \frac{\delta(r_1 - r_2)}{r_i^2} \cdot (\frac{1}{\pi b^2})^{\frac{3}{4}} e^{-\frac{r_1^2}{2b^2}} (\frac{1}{\pi b^2})^{\frac{3}{4}} e^{-\frac{r_2^2}{2b^2}} d^3 r_1 d^3 r_2 \\
&= (\frac{1}{\pi b^2})^3 \frac{1}{4\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{(r_1^2+r_2^2)}{2b^2}} . \frac{\delta(r_1 - r_2)}{r_i^2} \cdot e^{-\frac{(r_1^2+r_2^2)}{2b^2}} d^3 r_1 d^3 r_2 \\
&= (\frac{1}{\pi b^2})^3 \frac{1}{4\pi} \int_0^{\infty} e^{-\frac{r_1^2}{b^2}} \cdot \frac{1}{r_1^2} \cdot e^{-\frac{r_2^2}{b^2}} 4\pi r_1^2 4\pi r_1^2 d^3 r_1 d^3 r_2 \quad [as\ r_1 = r_2] \\
&= (\frac{1}{\pi b^2})^3 4\pi \int_0^{\infty} e^{-\frac{2r_1^2}{b^2}} r_1^2 dr_1 \qquad (2)
\end{aligned}$$

The substitution $\frac{2r_1^2}{b^2} = t^2$ givcs us $dr_1 = \frac{b}{\sqrt{2}} dt$. Using this substitution in equation (2), we have

$$
\begin{aligned}
I &= (\frac{1}{\pi b^2})^3 4\pi \int_0^\infty e^{-t^2} \frac{b^2 t^2}{2} \frac{b}{\sqrt{2}} dt \\
&= \frac{1}{2\sqrt{2}} \frac{1}{b^3 \pi^{3/2}}.
\end{aligned}
\tag{3}
$$

4 Approximation for kinetic energy kernels

The non-relativistic expression for the kinetic energy of the electron-positron bound system is given by

$$
K = \sum_{i=1}^{2} (m_i + \frac{P_i^2}{2m_i}) - K_{CM} \tag{4}
$$

Here $i = 1$ denotes the electron and $i = 2$ expresses the positron in the equation (4). The total mass of the system is $M = m_1 + m_2$, where m_1 is the mass of the electron and m_2 is the mass of the positron. The position vectors corresponding to m_1 and m_2 are r_1 and r_2 respectively. The relative distance between r_1 and r_2 is

$$
r = r_1 - r_2. \tag{5}
$$

Therefore, the position vector of the centre of mass of the system is

$$
R = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}. \tag{6}
$$

Therefore r_1 and r_2 are expressed in terms of R, r, m_1 and m_2 as

$$
r_1 = R + \frac{m_2 r}{m_1 + m_2}. \tag{7}
$$

and

$$
r_2 = R - \frac{m_1 r}{m_1 + m_2}. \tag{8}
$$

respectively.

The reduced mass of the system is defined as

$$
\mu = \frac{m_1 m_2}{m_1 + m_2}. \tag{9}
$$

Let the momentum of the electron be P_1. Then $P_1 = m_1\dot{r}_1$ and that can be expressed by using equations (7) and (9) as

$$P_1 = m_1\dot{R} + \mu\dot{r}. \tag{10}$$

Similarly, the momentum of the positron can be written from equations (8) and (9)as

$$P_2 = m_2\dot{R} - \mu\dot{r}. \tag{11}$$

Therefore we have the sum of the kinetic energy for the electron and the positron as

$$\begin{aligned} \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} &= \frac{1}{2m_1}[m_1\dot{R} + \mu\dot{r}]^2 + \frac{1}{2m_2}[m_2\dot{R} - \mu\dot{r}] \\ &= \frac{P_R^2}{2M} + \frac{P_r^2}{2\mu} \end{aligned} \tag{12}$$

Here $P_R = M\dot{R}$ is the kinetic energy of the centre of mass of the system and $P_r = \mu\dot{r}$ is the kinetic energy for the reduced mass of the system.

Hence we have,

$$\frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} - \frac{P_R^2}{2M} = \frac{P^2}{2\mu} \tag{13}$$

Now for a non-relativistic S-wave we have the kinetic energy kernel according to equation (4) and $P_i \rightarrow i\hbar\nabla_i$. Hence the matrix element can be formulated as $< e^-e^+| - \frac{\nabla^2}{2\mu}|e^-e^+ >$.

List of Publications:

1. Ghosh, S., Devi, M. R., Choudhury, A. and Sarma, J. K. Self Magnetic Field and Current-loop of Electron with Five Different Radii and Intrinsic Properties, *Int. J. App. Phys.* **1**(2) 91--100, 2011.

2. Ghosh, S., Choudhury, A. and Sarma, J. K. Relations of the electron radii and electron model, *Int. J. Phys.* **4**(2) 125--140, 2011.

3. Ghosh, S., Choudhury, A. and Sarma, J. K. Electromagnetic mass and charge in the framework of spinning sphere model of electron, *Int. J. App. Phys.* **1**(2) 119--123, 2011.

4. Ghosh, S., Choudhury, A. and Sarma, J. K. External magnetic field with different radii of electron and intrinsic properties of electron invoking the spinning sphere model of electron, *Pac-Asi. J. Maths.* **5**(2) 109--115, 2011.

5. Ghosh, S., Choudhury, A. and Sarma, J. K. Radii of electrons and their α-quantized relations, *Indian. J. Phys.* DOI 10.1007/s12648-012-0083-5.

6. Ghosh, S., Choudhury, A. and Sarma, J. K. Radius of electron, magnetic moment and helical motion of the charge of electron, *Apeiron* (accepted).

7. Ghosh, S., Choudhury, A. and Sarma, J. K. Electromagnetic mass and charge of the electron at the relativistic speed (communicated).

8. Ghosh, S., Choudhury, A. and Sarma, J. K. Magnetic self-energy and helical motion of charge invoking spinning sphere model of electron (communicated).

9. Ghosh, S., Choudhury, A. and Sarma, J. K. Magnetic moment of the electron in higher order from the modified Compton radius (communicated).

Presented research papers:

1. Electromagnetic Energy, Spin and Velocity of the Electron in the Framework of the Uniformly Charged Spinning Sphere at 57th Annual Technical Session of Assam Science Society in Gauhati University (oral)

2. Magnetic Self-Energy and Helical Motion of Charge Invoking Spinning Sphere Model of Elecron at 99th Indian Science Congress at KIIT University (Poster). This poster was awarded as the best poster in Physical Sciences at 99th Indian Science Congress at KIIT University.

3. Behaviour of External Magnetic Field with Different Radii of Electron and Intrinsic Properties of Electron 99th Indian Science Congress at KIIT University (Poster)

4. Helical Motion of Charge and Electron Structure at National Workshop on "Nuclear And Atomic Techniques Based Pure And Applied Sciences" at Department of Tezpur University in collaboration with UGC-DAE during 01-03 February 2011. (Poster)

5. The radii of the electron and their α-quantized relations at 7th PANE National Conference during Oct 2010 at Department of Physics, Manipur University at Imphal, Manipur (Oral)

6. Self Magnetic Field (internal) of Electron and the intrinsic properties 97th Indian Science Congress at Kerala University from 3rd to 7th January, 2010 (Oral)

Great things are not done by impulse, but by a series of small things brought together - *Vincent Van Gogh*

ADDENDA

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Self Magnetic Field and Current-loop of Electron with Five Different Radii and Intrinsic Properties

Sovan Ghosh, Maibam Ricky Devi, Amarjyoti Choudhury and Jayanta Kumar Sarma

Dept. of Physics, Tezpur University, Tezpur-784 028, Assam, India
E-mail: gsovan@gmail.com, deviricky@gmail.com, ajc@tezu.ernet.in, jks@tezu.ernet.in

Abstract

Here we study the behavior of the self-magnetic field and current due to the rotations of charge in the semi-classical modified relativistic spinning sphere model of electron. Though the original model is Compton-sized, we have tested below and above Compton-radius and α-quantized results come out as consequence.

Keywords: Electron-radii, Electron-model, α-quantization.

Introduction

Studies of the properties of the charged particles can be explored by probing about its behavior in uniform [1] and non-uniform [2] magnetic fields. Charged particles are studied along with its dynamics for more than a century. The dynamics of charged particles was focused by Maxwell who gave birth to electrodynamics [3]. Maxwell's macroscopic theory [3] was replaced by Lorentz's microscopic theory [3] after the discovery of electron.

Electron is the lightest charged particle and this is called as a point particle in the Standard Model of Particle Physics. But different electromagnetic phenomenon revealed eight different electron radii (Table-I) [4] [5] which give the signature of some extended electron model. Lorentz and Abraham made the first attempt to arrive at the structure of electron with the help of the dynamics of its charge [3]. Afterwards quite a large number of models of electron were proposed. [3] [6]. Relativistic Spinning Sphere model by M.H. MacGregor [4] [6] is proposed in recent-days. This is a semi-classical model which correlates the spectroscopic properties of the electron accurately to first order of α. This behaves as a relativistically spinning mechanical sphere of matter with an equatorial point charge e [4]. In this article we have considered the rotation of the equatorial charge with the speed c and examined the nature of current and magnetic field produced due to that rotation. Charge e is in fact residing in a very small space compared to the volume of the electron as charge radius $R_E < 10^{-19}\,m$ which is predicted from recent LEP experiment [6] [7].

Table I: Eight different electron radii.

Symbols	Name	Values
R_0	Classical electron radius	$\frac{e^2}{mc^2}$
R_C	Compton radius	$\frac{\hbar}{mc}$
R_{QMC}	Quantum mechanical Compton radius	$\sqrt{3}\frac{\hbar}{mc}$
R^{α}_{QMC}	QED-corrected quantum mechanical Compton radius	$\sqrt{3}(1+\frac{\alpha}{2\pi})\frac{\hbar}{mc}$
R_{em}	Classical electromagnetic radius	$\frac{\hbar^2}{mc^2}$
R_H	Magnetic field radius	$\geq 0.106 R_C$
R_{QED}	Observed QED charge distribution for a bound electron	$\approx R_C$
R_E	Charge-radius of electron	Yet to be calculated

In this process we don't consider any external field. Formulating the current-loop calculation of the charge within the radius of electron we proceed here. Out of the eight different electron radii, five are formulated with $\alpha, \hbar, e, c, m$, where α is fine structure constant, $\hbar$ is reduced Planck's constant, e is charge of electron, m is mass of the electron and c is the velocity of light in free space. Hence we use classical electron radius, Compton radius of electron, Quantum mechanical Compton radius, QED-corrected Quantum mechanical Compton radius and electromagnetic radius of electron to study the magnetic field originated from the rotation of the charge on the equator of the relativistic spinning sphere.

Rotation of charge and Ampere's law

Charge passing per unit time per unit area is known as current, $I = \frac{Q}{T}$, where Q is the charge and T is the time by which Q amount of charge passes unit area. To deal with electron we say the charge as e. When a small charge e rotates in a circular path of radius R with linear velocity v around the axis of rotation, the current comes out as

$$I = \frac{ev}{2\pi R} \tag{1}$$

In electrodynamics, current I can also be written with the help of current density J as

$$I = \int J.da, \tag{2}$$

where da is the area of the element. For magnetic field B, we have

$$\nabla \times B = \mu_0 J. \tag{3}$$

where μ_0 is the free space permittivity. According to Stoke's theorem, for a surface S, closed by the curve C

$$\int_S (\nabla \times B).da = \oint_C B.dl \tag{4}$$

where dl is the small line element on the curve C. Using equation (2) and (3) together in equation (4) we get Ampere's law

$$\oint_C B.dl = \mu_0 I \tag{5}$$

As we are studying RSS model and the charge is assumed to be rotating in the equator of the sphere with a velocity of c, we get current-loop corresponding to each radius for relativistically spinning spherical electron model.

Compton radius of electron is known as $R_C = \frac{\hbar}{mc}$, where $\hbar$ is reduced Planck's constant, m is the mass of electron, c is the velocity of light in free space.

Using Compton radius in equation (1), we have

$$I_C = \frac{ec}{2\pi R_C} = \frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right) \tag{6}$$

where e is the charge of electron, m is the mass of electron and $\frac{\hbar}{2}$ is the spin of electron. Therefore in other words this current-loop can be written in terms of three intrinsic properties (charge, mass and spin) of electron as

$$I_C = \frac{c^2}{4\pi}\left(\frac{Charge.Mass}{Spin}\right).$$

Classical electron radius is mathematically expressed as $R_0 = \frac{e^2}{mc^2}$. This is also known as Thomson scattering cross-section or Lorentz-radius. The current-loop for classical electron radius is

$$I_0 = \frac{ec}{2\pi R_0} = \alpha^{-1}\left[\frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right)\right]. \tag{7}$$

Classical electromagnetic radius $\frac{\hbar^2}{me^2}$ is also known as Bohr radius of the hydrogen atom. This is a larger one than the classical electron radius and the Compton radius of electron. The current-loop expression for this radius comes out as

$$I_{em} = \frac{ec}{2\pi R_{em}} = \alpha\left[\frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right)\right]. \tag{8}$$

Quantum mechanical Compton radius (R_{QMC}) and QED-corrected quantum mechanical Compton radius (R^{α}_{QMC}) are defined by M. H. MacGregor [4]. The formalism of quantum mechanical spin and magnetic moment projection factors lead to an electron radius $R_{QMC} = \sqrt{3}\frac{\hbar}{mc}$, quantum mechanical Compton radius of electron. Hence the current-loop of quantum mechanical Compton radius is calculated as

$$I_{QMC} = \frac{1}{\sqrt{3}}\left[\frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right)\right] \tag{9}$$

Applying magnetic self-energy corrections R_{QMC} becomes $R^{\alpha}_{QMC} = \sqrt{3}(1+\frac{\alpha}{2\pi})\frac{\hbar}{mc}$, QED-corrected quantum mechanical Compton radius. The corresponding current-loop comes out as

$$I^{\alpha}_{QMC} \approx \frac{1}{\sqrt{3}}(1-\frac{\alpha}{2\pi})\left[\frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right)\right]. \tag{10}$$

For R_C, Ampere's law can be calculated by putting equation (6) into equation (5) as

$$\oint_C B_C.dl = \mu_0[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})]. \tag{11}$$

Similar calculations using equations (7), (8), (9), and (10) for R_0, R_{em}, R_{QMC} and R^{α}_{QMC} respectively produce

$$\oint_C B_0.dl = \mu_0\alpha^{-1}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})], \tag{12}$$

$$\oint_C B_{em}.dl = \mu_0\alpha[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})], \tag{13}$$

$$\oint_C B_{QMC}.dl = \frac{\mu_0}{\sqrt{3}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})], \tag{14}$$

and

$$\oint_C B^{\alpha}_{QMC}.dl = \frac{\mu_0(1-\frac{\alpha}{2\pi})}{\sqrt{3}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})]. \tag{15}$$

Magnetic moment of electron was calculated by Uhlenbeck and Goudsmit as $\mu = \frac{e\hbar}{2mc}$. But the experimental results differed from the theoretical by 0.01%. Solution to this problem was given by Schwinger in 1949 [4] [8]. From the virtual emission and absorption of light quanta the logarithmically divergent self-energy of a free electron arises. The electromagnetic self-energy of a free electron can be described as electromagnetic mass of the electron and this must be added to the mechanical mass of the electron to give the experimental mass. This electromagnetic mass is the above-mentioned correction to the mechanical mass of the electron. Hence the corrected magnetic moment written as $\mu = \frac{e\hbar}{2mc}(1+\frac{\alpha}{2\pi})$, where $\alpha \approx \frac{1}{137}$ is the fine-structure constant and $\frac{\alpha}{2\pi}$ is known as Schwinger correction term [4][8].

Using the total mass of electron (= mechanical mass + electromagnetic mass), the expressions (6) – (10) can be re-written with the introduction of $m(1+\frac{\alpha}{2\pi})$

$$I_C = [\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{16}$$

$$I_0 = \alpha^{-1}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{17}$$

$$I_{em} = \alpha[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{18}$$

$$I_{QMC} = \frac{1}{\sqrt{3}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{19}$$

$$I_0 = \frac{1}{\sqrt{3}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})]. \tag{20}$$

In the same way the corrections can be made in expressions (11) - (15) for Ampere's law as

$$\oint_C B_C.dl = \mu_0[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{21}$$

$$\oint_C B_0.dl = \mu_0\alpha^{-1}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{22}$$

$$\oint_C B_{em}.dl = \mu_0\alpha[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{23}$$

$$\oint_C B_{QMC}.dl = \frac{\mu_0}{\sqrt{3}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}) \tag{24}$$

and

$$\oint_C B^{\alpha}_{QMC}.dl = \frac{\mu_0}{\sqrt{3}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})]. \tag{25}$$

Approximation for long straight wire and B in terms of charge, mass and spin

The expressions (21) – (25) produce Ampere's law in terms of charge, mass and spin of the electron. But mathematically B is inside the integral and having product with line element dl. To get the value of B separately long straight current carrying wire's approximation [9] is used here which gives

$$B = \frac{\mu_0 I}{2\pi R}. \tag{26}$$

As equation (26) is a modified version of equation (5), the expressions (21) – (25) can be modified respectively as

$$B_C = \frac{\mu_0}{2\pi R_C}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{27}$$

$$B_0 = \frac{\mu_0 \alpha^{-1}}{2\pi R_0}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{28}$$

$$B_{em} = \frac{\mu_0 \alpha}{2\pi R_{em}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{29}$$

$$B_{QMC} = \frac{1}{\sqrt{3}}\frac{\mu_0}{2\pi R_{QMC}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{30}$$

$$B_{QMC}^{\alpha} = \frac{1}{\sqrt{3}}\frac{\mu_0}{2\pi R_{QMC}^{\alpha}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})]. \tag{31}$$

Generalized current-loop and magnetic field

For a rotational motion of charge on RSS model, we have the current-loop expressions (16) – (20). The remarkable thing is that all of these five expressions carry a common factor $[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})]$. We say this common factor as generalized current $I_G = [\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})]$. In fact all of the above current-loops (equations (16) – (20)) can be re-written respectively in terms of the generalized current-loop as

$$I_C = I_G(1+\frac{\alpha}{2\pi}), \tag{32}$$

$$I_0 = \alpha^{-1} I_G(1+\frac{\alpha}{2\pi}), \tag{33}$$

$$I_{em} = \alpha I_G(1+\frac{\alpha}{2\pi}), \tag{34}$$

$$I_{QMC} = \frac{1}{\sqrt{3}} I_G (1 + \frac{\alpha}{2\pi}), \tag{35}$$

$$I_{QMC}^{\alpha} = \frac{I_G}{\sqrt{3}}. \tag{36}$$

In the expressions (27) - (31) of magnetic field also the term generalized current-loop is present. Hence the equations (27) – (31) can be re-written as

$$B_C = \frac{\mu_0 I_G}{2\pi R_C}(1 + \frac{\alpha}{2\pi}), \tag{37}$$

$$B_0 = \frac{\alpha^{-1} \mu_0 I_G}{2\pi R_0}(1 + \frac{\alpha}{2\pi}), \tag{38}$$

$$B_{em} = \frac{\alpha \mu_0 I_G}{2\pi R_{em}}(1 + \frac{\alpha}{2\pi}), \tag{39}$$

$$B_{QMC} = \frac{1}{\sqrt{3}} \frac{\mu_0 I_G}{2\pi R_{QMC}}(1 + \frac{\alpha}{2\pi}), \tag{40}$$

$$B_{QMC}^{\alpha} = \frac{1}{\sqrt{3}} \frac{\mu_0 I_G}{2\pi R_{QMC}}. \tag{41}$$

The current-loop expressions ((32) – (36)) for different radii can now be related with each other as

$$I_C = \alpha I_0 = \alpha^{-1} I_{em} = \sqrt{3} I_{QMC} = \sqrt{3}(1 + \frac{\alpha}{2\pi}) I_{QMC}^{\alpha}. \tag{42}$$

Using the equations (37) – (41) we have similar relation for the self-magnetic field produced for those above-mentioned radii as

$$B_C = \alpha^2 B_0 = \alpha^{-2} B_{em} = 3 B_{QMC} = 3(1 + \frac{\alpha}{2\pi}) B_{QMC}^{\alpha}. \tag{43}$$

α - quantized mass-leap and approximation for radius of muon and tau

In QED, the fine structure constant α is a coupling constant too. Comparison of the electron to the other particle mass data set has been yielded two different α-quantized masses, and they appear in two different forms know as fermionic (with half integral spin) and bosonic (with integral spin). These two α-masses are (1) $m_f = \frac{3}{2}\frac{m_e}{\alpha} = 105 MeV$ mass quanta that is created in the "α-leap" from the electron to the muon; (2) $m_b = \frac{m_e}{\alpha} = 70 MeV$ mass quantum that is created as part of a hadronically bound particle-antiparticle pair in the "α-leap" from an electron-positron pair to the pion (where m_e is the electron mass) [10].

The factor $(\frac{m_e}{\alpha})$ is found in the expression of current-loop for classical electron radius (equation (17)) and by re-writing equation (17), we have

$$I_0 = [\frac{c^2}{4\pi}(\frac{e\frac{m_e}{\alpha}}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \quad (44)$$

where $m_e = m =$ mass of electron. Using $m_f = \frac{3}{2}\frac{m_e}{\alpha}$ in equation (44) we get

$$I_0 = [\frac{c^2}{6\pi}(\frac{em_\mu}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \quad (45)$$

where m_μ is the mass of muon. Comparing the current-loop expression for muon in a similar way to electron we have the radius of muon as

$$R_\mu = \frac{3}{2}\frac{\hbar}{m_\mu c} \quad (46)$$

Compton radius of electron is known as $R_C = \frac{\hbar}{m_e c}$ with m_e as the mass of the electron. Equation (46) looks like the Compton radius of electron. Also right hand side carries a dimension of length which is essential for radius. Hence R_μ can be called as radius of muon.

Mass of tau is almost 17 times of the mass of the muon. Therefore in the same way with the help of equation (17) and the α-leap of the fermionic mass, we have

$$I_0 = \frac{2}{51}[\frac{c^2}{4\pi}(\frac{em_\tau}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}). \quad (47)$$

Therefore radius of tau comes out as

$$R_\tau = \frac{51}{2}\frac{\hbar}{m_\tau c}. \quad (48)$$

This equation (48) gives a form of radius just like equation (46) and this also looks like Compton radius.

Conclusion

Current (equations (16) – (20)) and magnetic field (equations (27) – (31)) are expressed in terms of three intrinsic properties of electron; i.e. charge, mass and spin. This is an interesting feature that mass and spin are also involved in describing current and magnetic field.

With the help of equations (32) – (36) the relation amongst the current-loops is developed in equation (42). Similar relation amongst the equations (37) – (41) is derived in equation (43).

It is also remarkable that though Compton radius, classical electron radius and electromagnetic radius are originated from different electromagnetic phenomenon, their current and the magnetic field are expressed in a generalized way with α-quantization. α-leap of the mass of elementary particles is discussed by MacGregor [10] and that helped to formulate equations (44) – (48) about muon and tau. Equations (46) and (48) prompt to suggest lower radius and higher mass-density for rising in mass of leptons.

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RELATIONS OF THE ELECTRON RADII AND ELECTRON MODEL

S. Ghosh[1], A. Choudhury[1] & J. K. Sarma[1]
[1]*HEP Laboratory, Department of Physics, Tezpur University, Napaam, Tezpur–784 028, Assam, India*

Abstract: *Varieties of experimental and theoretical considerations indicate eight different types of radii of electron. Here we attempt to find relations between different types of radii of electron Also we explore the physical significances of α-quantization of electron radii The velocities for different radii are also calculated here. In addition, relations between Rydberg constant and electron structure are also attempted.*

Keywords: *Electron-radii, α-quantization.*

1. INTRODUCTION

Since the discovery of electron in 1897, various approaches have been made to explain its different dynamic and static properties. Theoretically different shapes were assumed for electron and with those assumptions various sizes of the particle were calculated. But at the beginning of the 20th century little attention was paid to electron radius due to the diverging electromagnetic self-energy [1]. Thomson introduced electron radius as $R = f\frac{e^2}{mc^2}$ [1] with e as the charge of electron, m as the mass of the electron, c–velocity of the light in free space and f being a numerical factor. Later, approaches to give model of electron were led by Lorentz and Abraham. Lorentz replaced Maxwell's macroscopic theory by his microscopic ideas and correspondingly determined the upper limit of the size of electron (at that time) [1]. With the introduction of quantum theory of particles and its application to Electrodynamics, the inclusion of QED correction [2][3] was made in the radius of electron. Theory of Compton scattering also suggests a different radius of the electron. LEP experiments in CERN gives the signature of the size of the charge-radius of the electron as $R_E < 10^{-17}$cm [4]. The modern-day particle-physics regards electron as a point particle. But different electromagnetic phenomenon revealed different radii of electron. Eight of them [3][5], as shown in Table 1, are considered in this paper to seek further information and concepts related to the size of electron. The fact is that some of the above-mentioned radii follow classical electromagnetism and the rest follow quantum mechanical approaches.

*Corresponding author gsovan@gmail com

But it is significant that the classical and quantum mechanical results are related to each other.

Table 1
Eight Different Electron Radii

Symbols	*Name*	*Values*
R_0	Classical electron radius	$\frac{e^2}{mc^2}$
R_C	Compton radius	$\frac{h}{mc}$
R_{QMC}	Quantum mechanical Compton radius	$\sqrt{3}\frac{\hbar}{mc}$
R^{α}_{QMC}	QED-corrected quantum mechanical Compton radius	$\sqrt{3}\left(1+\frac{\alpha}{2\pi}\right)\frac{\hbar}{mc}$
R_{em}	Classical electromagnetic radius	$\frac{\hbar^2}{me^2}$
R_H	Magnetic field radius	$\geq 0.106\ R_C$
R_{QED}	Observed QED charge distribution for a bound electron	$\cong R_C$
R_E	Radius of the electric charge on the electron	Yet to be calculated

1.1. Classical Electron Radius

According to the theory of Thomson, for a charged particle in uniform motion with velocity v, the corresponding electromagnetic field will have a kinetic energy

$$T_{elm} = f\frac{e^2}{R_0 c^2}\frac{v^2}{2} \tag{1}$$

where f is a numerical factor that depends on the charge distribution within the sphere of radius R_0 and total charge e. Comparing the known form of kinetic energy as $T_{elm} = \frac{1}{2}mv^2$ with equation (1), we have [1]

$$R_0 = f\frac{e^2}{mc^2} \tag{2}$$

Abraham-Lorentz-Poincare model is also described with classical electron radius. This is a model of spherically symmetric charge distribution. Classical electron radius is calculated from this model, when the self-energy of the charged sphere is equated with its total energy. For surface distribution of charge, radius becomes $R_0 = \frac{1}{2}mc^2$ and for volume distribution the radius comes out as $R_0 = \frac{3}{5}\frac{e^2}{mc^2}$. The $\frac{3}{5}$ or $\frac{1}{2}$ depends on the nature of the distribution.

of the charge and this factor is denoted be f in equation (2). The generalized version of classical electron radius is given [3] as

$$R_0 = \frac{e^2}{mc^2}. \tag{3}$$

The value of R_0 is 2.82×10^{-13} cm. The classical electron radius is also involved in the scattering of radiation by a free charge, shown by Thomson. This scattering cross-section $\left(\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 = \frac{8\pi}{3}R_0^2\right)$ is also called as Thomson cross-section [3][6]. Hence a classical distribution of charge totaling the electronic charge should have a radius of this order if its electrostatic self-energy is equal to the electron mass [6]. As it is mentioned above, the classical electron radius is roughly the size; the electron would need to have for its mass to be completely due to its electrostatic potential energy. But the idea of its mass being completely due to its electrostatic potential energy is not supported nowadays. In fact a small contribution of electromagnetic mass is also witnessed [1][3]. For point particle an infinite self-energy is a doubtful, which admits physically meaningless solutions and violations of causality [2]. In modern classical-limit theories; e.g. in non-relativistic Thomson scattering and the relativistic Klein-Nishina formula, classical electron radius is used. Also this is the length scale at which renormalization becomes important in quantum electrodynamics.

1.2. Compton Radius

Compton radius comes out at the boundary of classical and quantum physics. It is being said as the lowest possible classical radius and classical physics don't go beyond this [7][8]. As this involves Planck's constant, $\hbar$, this is considered as quantum mechanical measurement also. Compton radius of an elementary particle is the length scale at which relativistic quantum field theory works. In other words, Compton radius of electron is the characteristic length scale of QED. Energy of an elementary particle can be written with the help of particle nature as well as the wave nature of the particle; i.e. with the Einstein equation

$$E = mc^2 \tag{4}$$

and the Planck-Einstein relation

$$E = \hbar\omega \tag{5}$$

From equations (4) and (5), we have $mc^2 = \hbar\omega$ and angular velocity becomes

$$\omega = \frac{mc^2}{\hbar}. \tag{6}$$

If the rotational motion of the particle is considered to be characterized with a velocity of $v = c$, equation (6) gives the corresponding radius (Compton radius) as

$$R_c = \frac{\hbar}{mc}. \tag{7}$$

From Compton-effect the difference between the wavelength λ and λ_0 gives $\Delta\lambda = \lambda - \lambda_0 = \frac{h}{mc}(1 - \cos\theta)$ and $\frac{h}{mc}$ divided by 2π is Compton radius of electron. Compton radius of electron is denoted as R_C and it measures as $R_C = 3.86\times10^{-11}$ *cm*. In different classical electron models, R_C is directly used to get the spin $\frac{\hbar}{2}$. The difference between the wavelengths and the introduction of $\hbar$ put the Compton radius in the region of wave-nature. As well as we can predict from the introduction of the Compton radius as a difference of the two wavelengths that the Compton-sized electron is a composite model of two different wavelengths matters. Schwinger's idea of total mass of electron, comprising of the mechanical mass and the electromagnetic mass, supports this.

1.3. Quantum Mechanical Compton Radius and QED-corrected Quantum Mechanical Compton Radius

MacGregor introduced two modified forms of Compton radius of electron. These are quantum mechanical Compton radius, R_{QMC} and QED-corrected quantum mechanical Compton radius R^{α}_{QMC}[3]. Relativistic spinning sphere model of electron is a semi-classical approach that deals with a classical electron without violating QED. Relativistic moment of inertia [3] of the spinning sphere is $I = \frac{3}{4}m_0c^2 = \frac{3}{2}mc^2$ where $m = \frac{3}{2}m_0$ and R_C is Compton radius of the sphere. The angular momentum is $J = I\omega = \frac{\hbar}{2}$ with $R_C = \frac{\hbar}{mc}$ and $\omega = \frac{mc^2}{\hbar}$. But the quantum mechanical formalism of angular momentum vectors shows that the total spin angular momentum of the electron is

$$J = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\hbar = \frac{3}{2}\hbar \tag{8}$$

Hence the R_C is modified with this quantum mechanical behavior as

$$R_{QMC} = \sqrt{3}R_C\,. \tag{9}$$

Quantum mechanical Compton radius confirms the quantum mechanical behavior of electron and it is written as $R_{QMC} = 6.69\times10^{-11}$ cm.

As the mass of the electron is considered to be composed of mechanical mass and electromagnetic mass, the spinning mass can be written as, $m_s = m\left(1 - \frac{\alpha}{2\pi}\right)$, where $\frac{\alpha}{2\pi}$ is the Schwinger correction and the electromagnetic mass is termed as

$$m.\frac{\alpha}{2\pi} = \Delta m\cdot \tag{10}$$

Quantum mechanical Compton radius is modified again with the introduction of Schwinger correction and QED-corrected quantum mechanical Compton radius is expressed as

$$R^{\alpha}_{QMC} = \sqrt{3}R_C\left(1+\frac{\alpha}{2\pi}\right). \tag{11}$$

From different scattering experiments electron is regarded as a point-particle, but its manifestation in atomic bound states is not point-like and the Lamb shift experiment claims that the electric charge is smeared out over a region of space that is comparable to R_C. In fact electron bound-state charge distribution radius, deduced from the Lamb shift experiments is very large. In the hydrogen atom, the charge on the electron appears to be spread out over a large region of space compared to the intrinsic size of the charge itself. QED calculations give accurate magnitude of the effect but not a very clear explanation. Zitterbewegung motion revealed by the Lamb shift [9][10] is a phenomenon that shows a large electron charge distribution radius R_{QED}. Vacuum polarization is another standard QED effect, which leads to a Coulomb polarization of the vacuum state by the charge e, where this polarization extends over a distance that is comparable to the Compton radius R_C. This broadening of the electric field of the charge and the spatial location of the charge indicates for an effective bound-state QED charge radius $R_{QED} \cong R_C$.

1.4. Magnetic field radius

Rotation of charge gives birth to current and magnetic field. Magnetic field introduces the magnetic self-energy. The radius, around which this magnetic field is considered, is known as magnetic field radius. This magnetic field, according to Ampere's hypothesis arises from the motion of the electric charge e, and is asymptotic magnetic self-energy W_H, apply all the way in to a magnetic field radius R_H. Hence the magnetic self-energy is expressed as $W_H \geq \frac{2\mu^2}{3R_H^3}$ with μ as the anomalous magnetic moment of the electron [3]. On the other hand the expression of magnetic self-energy consists of electromagnetic mass of electron and the velocity of light in free space. Modern day idea of electromagnetic mass was first introduced by J. Schwinger to match the theoretical value magnetic moment of electron with the experimental data [2]. Therefore magnetic field radius is responsible for the magnetic field around the electron and this is parameter, which has greater impact on the magnetic moment of electron.

1.5. Charge radius of electron

Electron is a charged particle and this charge is related to the other properties also. Being the charged particle electron follows the dynamics of charged particles. It is a matter of fact that the dipole moment, magnetic moment and a small fraction of mass also depend upon the charge of the electron. The scattering properties of the electron also insist a vastly smaller radius for its electric charge [11]. In different models of electron the charge gets the importance due to its dynamics. Earlier idea was of a distribution of charge over the entire electron. In some models charge is considered to reside in the equatorial zone [12] also. Several classical and semi-classical models follow the idea of localized charge. But the exact measurement of the size of the charge of the electron is yet to be done. Quantum electrodynamics defines it as a point charge. Recent LEP experiment predicts that the charge of the electron is confined within a region of $R_E < 10^{-17}$ cm or $R_E < 10^{-19}$ m [4]. So charge radius, R_E is very small compared to R_C or R_0.

1.6. Electromagnetic Radius

The electrostatic contribution of the electron to its energy in terms of electromagnetic radius [5] is written as

$$W_E = \frac{e^2}{R} \tag{12}$$

With the introduction of the magnetic moment of electron, the expression for energy becomes

$$mc^2 = \frac{e^2}{R} + \frac{\mu^2}{c^2 R^3}, \tag{13}$$

where μ is the magnetic moment, R is the radius and c is the velocity of light in free space. Therefore the Spin component comes out as

$$S = \frac{e\mu}{c^2 R}. \tag{14}$$

Using equation (14) in equation (13) with the replacement of μ, we have

$$mc^2 = \frac{e^2}{R}\left[1 + \left(\frac{Sc}{e}\right)^2\right] \tag{15}$$

and radius becomes

$$R_{em} = \frac{e^2}{mc^2}\left[1 + \left(\frac{Sc}{e}\right)^2\right]. \tag{16}$$

With the mass = m, charge = e, and spin = $\hbar$, the electromagnetic radius is read as

$$R_{em} = \frac{\hbar^2}{me^2}. \tag{17}$$

This is also known as quantum Bohr radius of hydrogen atom. Due to the involvement of the magnetic moment this radius becomes larger than the Compton radius and the classical radius.

2. α QUANTIZATION OF CLASSICAL, COMPTON AND ELECTROMAGNETIC RADIUS OF ELECTRON

Classical electron radius or Lorentz radius is expressed in equation (1). Compton radius of electron is given by equation (7), with the inclusion of $\hbar$, the reduced Planck's constant. Equation (17) gives R_{em}, the electromagnetic radius of electron [5]. Using equation (3) and (7), we have

$$R_0 = \frac{e^2}{mc^2} = \alpha R_C \tag{18}$$

Similarly from (7) and (17), we get

$$R_C = \frac{h}{mc} = \alpha R_{em} \tag{19}$$

First order α-leap for electron radii is introduced by equation (18) and (19). Combination of equation (18) and (19) gives us the second order α-leap as

$$R_0 = \alpha^2 R_{em} \tag{20}$$

In Compton effect, one needs to apply quantum field theoretical approach in order to capture QED related phenomena of Compton effect. Hence large R_{em} needs to be squeezed to a point where relativistic field theoretical approach starts to matter. In the process, if R_{em} (i.e. hydrogen atom radius) is squeezed by $\alpha\left(\frac{1}{137}\right)$, we arrive the Compton-radius, R_C. Again in R_0, no quantum mechanical effects are considerable, where as in R_C, elements of uncertainty principle pushes up its size. With the analysis from Table 2 also it can be defined that these three radii are α-quantized. Secondly charge radius of electron that is $R_E < 10^{-19}$m (10^{-17}cm) according to LEP experiments in CERN [4]. Hence with the help of our analysis charge-radius of electron can be approximated as

$$R_E = \alpha^2 R_0 = \frac{e^6}{mh^2c^4} \tag{21}$$

First and second order of -quantization of electron radii are shown in Figure 1, which contains all the eight radii mentioned here in centimeter-scale. From this figure also our prediction of the form of charge-radius, R_E is supported.

Table 2
Basic Factors Related to Different Radii

Radii	*Power of e*	*Power of ħ*	*Power of c*	*Power of m*
R_0	2	0	-2	-1
R_C	0	1	-1	-1
R_{em}	-2	2	0	-1

From Table 2 a general feature of radius is seen. Radius or length is inversely proportional to mass from which we can predict that

$$\text{Length} \propto \frac{1}{\text{Mass}}.$$

Therefore.

$$\text{Length} = \frac{\text{Constant}}{\text{Mass}}.$$

Considering the total mass as a combination of mechanical mass and electromagnetic mass

$$\text{Length} = \frac{\text{Constant}}{\left(1+\frac{\alpha}{2\pi}\right)\text{Mass}}.$$

Hence radius or size of electron is influenced by electromagnetic effect also. Fine structure constant is also an electromagnetic phenomenon, which confirms that these radii are related to each other due to some electromagnetic phenomenon.

3. MAGNETIC SELF-ENERGY AND MAGNETIC FIELD RADIUS

The representation of the magnetic moment of the electron can be given by a current loop. *Using polar (r, θ, z) co-ordinate and orienting the axis of the current loop along the z-axis,* the asymptotic magnetic field components [3][6] are obtained as

$$H_r = \frac{2\mu Cos\theta}{r^3},\ H_0 = \frac{\mu Sin\theta}{r^3} \tag{22}$$

Magnetic self-energy W_H is represented as $W_H = \frac{1}{2}\int_V B.Hd^3x$.

Magnetic self-energy can be divided into parts as Magnetic self-energy can be divided into parts as W_H^{Ext} and W_H^{Int} depending on [3] relativistic spinning sphere radius $r > R_H$ or $r < R_H$. Hence $W_H = W_H^{Ext} + W_H^{Int}$.

When $r > R_H$, external self-energy W_H^{Ext} will be the energy and when $r < R_H$, the corresponding energy will be W_H^{Int}.

So $$W_H^{Ext} = \frac{\mu^2}{8\pi}\int_{R_H}^{\infty}\int_0^{\pi}\left(\frac{1}{r^6}\right)\left(3Cos^2\theta+1\right)2\pi r^2 Sin\theta d\theta dr = \frac{\mu^2}{3R_H^3} \tag{23}$$

$$W_H^{Ext} \geq \frac{\mu^2}{8\pi}\int_0^{R_H}\int_0^{\pi}\left(\frac{1}{r^6}\right)\left(3Cos^2\theta+1\right)2\pi r^2 Sin\theta d\theta dr = \frac{\mu^2}{3R_H^3} \tag{24}$$

We consider here the 'equal' sign only to calculate with the minimum energy.

Addition of equations (23) and (24) produces

$$W_H^{Tot} = \frac{2\mu^2}{3R_H^3}. \tag{25}$$

As we have mentioned already, the electromagnetic self-energy of a free electron can be described in terms of electromagnetic mass and this electromagnetic mass is a small correction to the mechanical mass [2][3]. Hence we have Schwinger correction term as , $\Delta m \cong m.\frac{\alpha}{2\pi}$ [1]. So magnetic-self energy is written as

$$W_H = c^2\,\Delta m. \tag{26}$$

Equating the expressions of (25) and (26) for W_H, we have

$$mc^2\frac{\alpha}{2\pi} = \frac{2\mu^2}{3R_H^3}. \tag{27}$$

As fine-structure constant is $\alpha = \dfrac{e^2}{\hbar c}$, from equation (27), we have

$$mc^2\frac{e^2}{\hbar c}\frac{1}{2\pi} = \frac{2\mu^2}{3R_H^3}. \tag{28}$$

Schwinger-corrected form of magnetic moment of electron is

$$\mu = \frac{e\hbar}{2mc}\left(1+\frac{\alpha}{2\pi}\right). \tag{29}$$

Using the equation (29) in equation (28), the expression of R_H is obtained as

$$R_H^3 = \frac{4\pi}{3mc^2\alpha}\left\{\left(1+\frac{\alpha}{2\pi}\right)^2\frac{e^2h^2}{4m^2c^2}\right\}. \tag{30}$$

With the help of equation (7), we re-write equation (30) as

$$R_H^3 = \frac{\pi}{3}R_C^3\left(1+\frac{\alpha}{2\pi}\right)^2. \tag{31}$$

In equation (31), magnetic field radius R_H is expressed in terms of Compton radius. Approximating $\frac{\pi}{3} \approx 1$ and re-writing $\left(1+\frac{\alpha}{2\pi}\right)^2 = \left(1+\frac{\alpha}{\pi}+\frac{\alpha^2}{4\pi^2}\right)$ in equation (31), we obtain

$$R_H^3 \cong R_C^3\left(1+\frac{\alpha}{\pi}+\frac{\alpha^2}{4\pi^2}\right). \tag{32}$$

Equation (32) reflects the fact that magnetic field radius is Compton radius modified by zeroth, first and second order terms of electromagnetic fine structure constant.

Using the equation between classical and Compton electron radius in equation (32), we arrive at the form

$$R_H^3 = R_C \cdot R_{C0} R_{C0} \tag{33}$$

where, $$R_{C0} = \left(R_C + R_0'\right) = \left(R_C + \frac{R_0}{2\pi}\right) = R_C\left(1+\frac{\alpha}{2\pi}\right). \tag{34}$$

The above relation (33) and (34) states that the magnetic field due to the charge of electron is effective within a volume made of R_C and R_{C0}.

4. RELATIONS AMONG DIFFERENT RADII

Relations of R_C with R_{QMC}, $R^{\alpha}{}_{QMC}$, R_0, R_{em} and R_H are stated in equations (9), (11), (18), (19) and (32). Relation between R_C and R_{QED} [3] is known as

$$R_C = \cong R_{QED}. \tag{35}$$

Exact formulation of the charge-radius of electron is yet to be done. According to our proposal of α-quantized configuration of R_E, the relation between R_C and R_E can be written as

$$R_E = \alpha^2 R_0 = \alpha^3 R_C. \tag{36}$$

Replacement of R_C by R_0 in equation (32) with the help of equation (18) gives the relation between R_H and R_0 as

$$R_H^3 = \alpha^{-3}\left(1 + \frac{\alpha}{\pi} + \frac{\alpha^2}{4\pi^2}\right) R_0^3. \tag{37}$$

Similarly using the relation (19) in equation (32), we get the relation between R_H and R_{em}

$$R_H^3 = \alpha^3\left(1 + \frac{\alpha}{2\pi}\right)^2 R_{em}^3. \tag{38}$$

Using equation (9) into equation (32), we have

$$R_H^3 = \frac{R_{QMC}^3}{3\sqrt{3}}\left(1 + \frac{\alpha}{2\pi}\right)^2. \tag{39}$$

A similar replacement of R_C by R_{QMC} in equation (32) with the help of equation (11) leads to

$$R_H^3 = \frac{R_{QMC}^{\alpha\,3}}{3\sqrt{3}}\left(1 - \frac{\alpha}{2\pi}\right). \tag{40}$$

From equations (21) and (32) we relate the magnetic field radius with the charge-radius of electron as

$$R_H^3 = \alpha^{-9} R_E^3 \left(1 + \frac{\alpha}{2\pi}\right)^2. \tag{41}$$

Looking for a simpler form of equation (32) we take cube root of R_C and it results as

$$\frac{R_H}{R_C} = \left(1 + \frac{\alpha}{2\pi}\right)^{\frac{2}{3}} \cong \left(1 + \frac{\alpha}{3\pi}\right). \tag{42}$$

Using equation (18) into equation (42) we can bind R_H, R_C and R_0 in a single equation as

$$R_H - R_C = \frac{R_0}{3\pi}. \tag{43}$$

Equation (24) and the α-quantized relation $\alpha = \frac{R_C}{R_{em}}$ form a quadratic equation of R_C as

$$R_C^2 + 3\pi R_{em} R_C - 3\pi R_H R_{em} = 0. \tag{44}$$

From equations (9), (11), (18), (19) and (42) we can write

$$R_C = \alpha^{-1} R_0 = \alpha R_{em} = \frac{R_{QMC}}{\sqrt{3}} = \frac{R_{QMC}^{\alpha}}{\sqrt{3}\left(1 + \frac{\alpha}{2\pi}\right)} = \frac{R_H}{\left(1 + \frac{\alpha}{2\pi}\right)}. \tag{45}$$

Equation (45) relates all these six radii in a single one. This relation also states how any two of them are related. We express those relations in the form of ratio in Table 3.

Table 3
Relations in Terms of Among Different Radii

Ratio	α *involved relations*
$\frac{R_0}{R_C}$	α
$\frac{R_C}{R_{em}}$	α
$\frac{R_0}{R_{em}}$	α^2
$\frac{R_{QMC}}{R_C}$	$\sqrt{3}$
$\frac{R_{QMC}^{\alpha}}{R_C}$	$\sqrt{3}\left(1 + \frac{\alpha}{2\pi}\right)$
$\frac{R_{QMC}}{R_{em}}$	$\sqrt{3}\alpha$
$\frac{R_{QMC}^{\alpha}}{R_{em}}$	$\sqrt{3}\alpha\left(1 + \frac{\alpha}{2\pi}\right)$
$\frac{R_H}{R_C}$	$\left(1 + \frac{\alpha}{3\pi}\right)$
$\frac{R_0}{R_H}$	$\frac{\alpha}{\left(1 + \frac{\alpha}{3\pi}\right)}$

To relate the above radii, the equations involve c (velocity of light in free space) and α (fine structure constant). α itself involves e (electric charge of electron), $\hbar$ (Planck's constant divided by 2π) and c. Of these three, charge, e carries the intrinsic property of electron. Hence it can be concluded that charge of electron plays a significant role for electron's structure and in the relations amongst electron radii.

Another importance of these relations is that, the velocity of the electron for those different radii can be brought out by keeping the spin, $\frac{\hbar}{2}$ and we are going to do this in the next section.

5. VELOCITIES FOR DIFFERENT RADII

The relativistic moment of inertia of the spinning sphere is $I = \frac{1}{2} m_s R^2$, where m_s is the total mass of the spinning sphere. Spinning mass becomes $m_s = \frac{3}{2} m$ for higher velocity, with m being the non-spinning rest mass [3]. For smaller values of angular velocity, !, the spinning mass and non-spinning mass are equal; i.e: $m_s = m$. With the increasing angular velocity, ω, the spinning mass, m_s increases. Hence the relativistic moment of inertia $I = \frac{3}{4} mR^2 = \frac{1}{2} m_s R^2$.

Compton radius, R_C gives $\frac{\hbar}{2}$ spin with the linear velocity c. So

$$L = I\omega = \frac{1}{2} m_s R_C^2 \omega = \frac{\hbar}{2} \tag{46}$$

Following $v = r\omega$ and different suffixes of the radii the notations are used here. For classical electron radius (R_0), $v_0 = R_0 \omega_0$ with ω_0 as the corresponding angular momentum.

The spin angular momentum will be $\frac{1}{2} m_s R_0^2 \omega_0 = \frac{\hbar}{2}$. Then the angular velocity is $\omega_0 = \frac{\hbar}{m_s R_0^2} = \frac{v_0}{R_0}$. So the velocity will be $v_0 = \frac{\hbar}{m_s R_0}$. With $R_0 = \frac{e^2}{m_s c^2}$, the velocity v_0 becomes

$$v_0 = \alpha^{-1} c. \tag{47}$$

But this is not possible; as it is well known from special theory of relativity that velocity of light is the highest velocity. As at the very beginning of the deduction the classical electron radius, one relativistic approach was equated to one non-relativistic scheme this problem arises. So it is proved here that for a relativistic spinning sphere the classical electron radius does not stand at all with relativistic moment of inertia. If we have to use classical electron radius for relativistic spinning sphere model, we must have to introduce it in some other way. On the other hand the above result gives the indication for smaller radius and higher velocity also.

This problem can be solved in a different way. As the size of the radii is decreased, the velocity is increased. Now with the behavior of the other parameters it is clear that the velocity of the particle is in fraction of c. Hence one can write as $v_0 = \beta c$. The velocity of the Compton-sized electron is well- known as c. Hence one can predict the other way round that the Classical electron radius is the contracted length form of Compton radius of electron with the help of α-quantized relation (6), $R_0 = \alpha R_C$. Hence the velocity corresponding to classical electron radius becomes

$$v_0 = c\sqrt{1-\alpha^2}. \tag{48}$$

Now we have the velocity of the classical electron radius in terms of c and closed to c, but it is not greater than c. This is in agreement with the special theory of relativity.

In the similar way for magnetic field radius the velocity and spin can be written as $V_H = R_H\omega_H$ and $\frac{1}{2}m_s R_H^2\omega_H = \frac{\hbar}{2}$. Hence using equation (24) and $R_C = \frac{\hbar}{m_s c}$ we have

$$v_H = \frac{\hbar}{m_s R_H} = \frac{\hbar}{m_s\left[R_C + \frac{\alpha}{3\pi}\right]} \cong c\left(1 - \frac{\alpha}{3\pi}\right). \tag{49}$$

For quantum mechanical Compton radius we start with $v_{QMC} = R_{QMC}\cdot\omega_{QMC}$ and $\frac{1}{2}m_s R_{QMC}^2\cdot\omega_{QMC} = \frac{\hbar}{2}$. Proceeding in the similar way we have $v_{QMC} = \frac{\hbar}{m_s R_{QMC}}$. Using expression (9) and $v_{QMC} = \frac{\hbar}{m_s R_{QMC}}$ we have

$$v_{QMC} = \frac{c}{\sqrt{3}}. \tag{50}$$

For QED-corrected quantum mechanical Compton radius we have $v_{QMC}^{\alpha} = R_{QMC}^{\alpha}\cdot\omega_{QMC}^{\alpha}$ and $\frac{1}{2}m_s R_{QMC}^2\,\omega_{QMC} = \frac{\hbar}{2}$ In the similar way we have $v_{QMC}^{\alpha} = \frac{\hbar}{m_s R_{QMC}^{\alpha}}$. Using expression (11) and relativistic moment of inertia corrected Compton radius $R_C = \frac{\hbar}{m_s c}$ we have

$$v_{QMC}^{\alpha} = \frac{c}{\sqrt{3}}\left(1+\frac{\alpha}{2\pi}\right)^{-1} \cong \frac{c}{\sqrt{3}}\left(1-\frac{\alpha}{2\pi}\right). \tag{51}$$

Electromagnetic radius is $R_{em} = \frac{\hbar^2}{me^2}$ and $R_{em} = \alpha^{-1}R_C$. Then with the help of $v_{em} = R_{em}\omega_{em}$ and $\frac{1}{2}m_s R_{em}^2\omega_{em} = \frac{\hbar}{2}$. we have

$$v_{em} = \alpha c \tag{52}$$

6. RYDBERG CONSTANT AND ELECTRON RADII

Rydberg constant represents the limiting value of the highest wave number (the inverse wavelength) of any photon that can be emitted from the hydrogen atom. For $n = 1$, the wave number [13] comes out to be $\frac{1}{\lambda}$ = Constant$\sqrt{R}$ and the Rydberg constant is read as the energy only. Rydberg constant not merely connects fine-structures of the electronic energy levels of the corresponding spectroscopic radiations [14]. It also provides a link between the wave and particle nature of the electron by putting a limit of highest wavenumber corresponding to a photon involved in spectroscopic radiation. Fine-structure constant and the Planck's constant fix the Rydberg constant as [15]

$$R_\infty = \alpha^2 \frac{mc}{2h}. \tag{53}$$

α^2, m, c, h are the parameters which relate Rydberg constant with electron. α-quantized results of electron radii prompt the involvement of Rydberg constant with electron radii.

Therefore using equations (18), (34) and (53) we have

$$R_\infty = -\frac{\alpha}{2R_C}\left(1 - \frac{R_{C0}}{R_C}\right). \tag{54}$$

Hence Rydberg constant gives here the energy of the model we predicted in this paper with the help of equation (54). Involvement of this constant with the size of the electron shows that the spectral lines and the fine-structure constant have great electromagnetic impact on the size and the structure of the electron.

Table 4
Electron Radii and Corresponding Velocities

Radius	*Velocity (m/s)*
R_C	c
R_H	$c\left(1 - \frac{\alpha}{3\pi}\right)$
R_{QMC}	$\frac{c}{\sqrt{3}}$
R^{α}_{QMC}	$\frac{c}{\sqrt{3}}\left(1 - \frac{\alpha}{2\pi}\right)$
R_{em}	αc
R_0	$c\sqrt{1-\alpha^2}$

7. CONCLUSION

The above relations co-relate the different phenomena, which produce those particular radii of electron. With reference to RSS model developed by MacGregor, we aimed to bind the different physical aspects of electron in a single model. As throughout the paper, we have used the Schwinger correction for mass, it appears distinct which part of the radii is responsible for which sort of mass (mechanical or electromagnetic).

Lorentz, Compton and Bohr radii of electron are α-quantized. The magnetic field radius of electron is also related with R_0, R_C and R_{em} by α-associated terms. Hence the importnace of is realized as relating all sorts of phenomena of electron. The concept of α-quantized mass leap is developed by MacGregor [16]. Here our approach proposes the α-quantization of the radii of the electron.

This shows how much impact the fine-structure constant leaves in the lepton-structure. In fact in the calculation of current-loop for different electron radii also the α-quantized nature is being followed [17].

Here the tentative α-quantized form of charge radius of electron R_E is proposed, the measurement of which is yet to be precised experimentally.

The velocities for different radii are calculated here. This conceptually can put the fact that change in velocities of electron can produce different forms of electron according to sizes and different electromagnetic phenomenon take place and the vice-versa. The value of the velocity calculated for R_0 in the formal way helps us to predict that the classical electron radius is a Lorentz contracted Compton radius of electron.

From the Schwinger-corrected definite form of the magnetic field radius, the relations of R_H with other radii are developed. The calculation of magnetic field radius gives the signature of a slightly distorted spherical model of electron.

This model is following the RSS model and does not violate QED. The addition (equation (34)) of R_0 and R_C produces a new radius, we say as Composite radius of electron, R_{C0}.

All the α-quantized radii are connected together with Rydberg constant, which gives the signature of accurate measurement of the classical electron radius. The best-measured constant [14][15] is described here as the energy of the model we predicted for electron with radii R_C and R_{C0}.

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Electromagnetic Mass and Charge in the Framework of Spinning Sphere Model of Electron

***Sovan Ghosh, Amarjyoti Choudhury and Jayanta Kumar Sarma**

Dept. of Physics, Tezpur University, Tezpur–784 028, Assam, India
E-mail: ajc@tezu.ernet.in, jks@tezu.ernet.in
**Corresponding Author E-mail: gsovan@gmail.com*

Abstract

The indication of LEP results about the distribution of charge of electron over a very small region of radius $\sim 10^{-19}$m is used in the Relativistic spinning sphere (RSS) model of electron. Electromagnetic mass of electron was introduced in QED to compensate the difference between the theoretical and the experimental results of magnetic moment. This prompts us to interpret the charge of electron together with magnetic self-energy, LEP results and RSS model of electron. It is also predicted that the particles with zero mass will not contain any charge.

Keywords: Electron, Charge-mass equivalence, Charge-energy equivalence

Introduction

LEP experiments indicate that the charge of the electron is distributed over a small radius $\sim 10^{-19}$ m or 10^{-17} cm [1] so that the charge distribution could be considered as point-like. The explanation of related scattering by QED [2-3] too demand that the charge of electron is concentrated with a smaller mass as compared to the total mass of electron. This prompts us to link between mass and charge of an elementary particle.

The experimental result of magnetic moment of electron is not found to match with the magnetic moment when only the mechanical mass of electron is considered [4-5] in theoretical calculation. Schwinger proposed a correction term $m.\frac{\alpha}{2\pi}$ [4-5] to compensate the difference between the theoretical and experimental results. This compensating mass is termed as electromagnetic mass of electron [4-5]. It may be noted that $\alpha(=\frac{e^2}{\hbar c})$ is the so-called fine structure constant coupling the strength of interaction between electron and photon [6].

In the standard RSS (Relativistically Spinning Sphere) model [5] of the electron, the charge is considered to be confined in a very small region and this is consistent with the demands of LEP experiments and QED theory. Thus we allow the charge part of the electron to be a small sphere of radius R_E, which is smaller than all other known radii [1] [5] [7]. In fact out of the eight different known electron radii, R_E (~ 10^{-19} m) is the smallest one and the next is classical electron radius R_0(~ 10^{-15} m), which is 10^4 times larger than R_E.

Magnetic self-energy and charge of electron

Magnetic self-energy is the energy, which is contained in the magnetic field associated with the magnetic moment of electron. Magnetic field and moment are results of current, which is originated due to the motion of the charge. Hence considering the rotation of the charge of electron we can proceed for magnetic self-energy.

Rotation of a charge particle around its axis of rotation gives rise to a current-loop

$$I = \frac{e}{T}. \tag{1}$$

Here e is the charge and T is the time period of rotation. If the velocity is v and radius of rotation is R, the time period T can be written as

$$T = \frac{2\pi R}{v}. \tag{2}$$

Putting equation (2) in (1) we have the expression of current in terms of velocity and the radius as

$$I = \frac{ev}{2\pi R}. \tag{3}$$

This current-loop introduces a magnetic field B and according to Ampere's law, B can be written as

$$B = \frac{\mu_0 I}{2\pi R} \tag{4}$$

Using equation (3) in (4)

$$B = \mu_0 \frac{ev}{(2\pi R)^2}. \tag{5}$$

Here we have the magnetic field related with the radius of rotation and the velocity through free space permeability. This helps us to step for auxiliary magnetic field and it is related with B as

$$H = \frac{B}{\mu_0}. \tag{6}$$

With the help of equation (5) and (6) we have the expression for auxiliary magnetic field due to the rotation of the charge

$$H = \frac{ev}{(2\pi R)^2}. \tag{7}$$

Hence the magnetic self-energy of the system will be read as

$$W_d = \frac{1}{2}\int H.Bd^3x = \frac{\mu_0}{2}\frac{e^2v^2d^3x}{(2\pi R)^4}. \tag{8}$$

To be more specific for relativistic spinning sphere model, if we choose $v = c$, the magnetic self-energy comes out as

$$W_H = \frac{\mu_0}{2}\int\frac{e^2c^2}{(2\pi R)^4}d^3x. \tag{9}$$

If $\int d^3x$ is expressed in terms of spherical polar co-ordinate system and considering the orientation of the current-loop along the z-axis we have

$$\int_0^R\int_0^\pi\int_0^{2\pi} r^2 \sin\theta dr d\theta d\phi = \frac{4}{3}\pi R^3. \tag{10}$$

Using equation (10) in (9) we have

$$W_H = \frac{\mu_0 e^2 c^2}{24\pi^3 R}. \tag{11}$$

In the calculation of RSS (Relativistic spinning sphere) model [5] the correction of magnetic moment [4] is used. Hence the magnetic self-energy [5] is written with the help of electromagnetic mass of electron as

$$W_H = m.\frac{\alpha}{2\pi}c^2, \tag{12}$$

where $m.\frac{\alpha}{2\pi}$ is known as electromagnetic mass and $\frac{\alpha}{2\pi}$ is the Schwinger correction.

Equating (11) with (12) for the magnetic self-energy, we arrive at

$$m.\frac{\alpha}{2\pi} = \frac{\mu_0 e^2}{24\pi^3 R}. \tag{13}$$

Hence charge can be written as

$$e = \sqrt{\frac{m.\alpha.12\pi^2 R}{\mu_0}}. \tag{14}$$

This shows that charge is the expression of a special phenomenon permeated in the matrix of mass which is switched on by permeability μ_0 of the matter. Thus μ_0 is the enabling or stirring parameter in the exchange of mass into charge and vice-versa.

The expression of total energy is obtained using the equation (13)

$$E = (m + \frac{\mu_0 e^2}{24\pi^3 R})c^2. \tag{15}$$

In terms of Coulomb force the energy can be written as

$$E = (m + \frac{F.R}{6\pi^2})c^2. \tag{16}$$

Conclusions

Equations (15) and (16) give the total energy in which the first term corresponds to the mass and the second term to the charge or the mass equivalent to charge. The so-called electromagnetic mass [5] is expressed in this article as the mass responsible for the existence of charge. Thus we establish charge-mass equivalence in equation (15) and the direct relation between the charge and mass is established in the equation (14). Hence equation (15) can be told as charge-mass-energy relation or extended form of mass-energy relation with the help of electromagnetic mass. The magnetic self-energy gives 0.07% of the total mass of the electron and the charge radius is also smaller than all other radii of electron. These two facts are completely supporting the calculation lead in this article. In other words the charge is a small fraction in electron with some sort of relation with its mass as expressed in equations (14), (15) and (16). Equations (13) and (14) also predict that no charge can be contained in zero-mass particles or charge can't reside without mass.

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EXTERNAL MAGNETIC FIELD WITH DIFFERENT RADII OF ELECTRON AND INTRINSIC PROPERTIES OF ELECTRON INVOKING THE SPINNING SPHERE MODEL OF ELECTRON

S. GHOSH, A. CHOUDHURY & J. K. SARMA

ABSTRACT: Several models of electron are proposed for last one century depending on the properties of electron. Different electromagnetic phenomenon also revealed the different radii. For different radii the behavior of charged spinning sphere (electron) is one interesting topic and being discussed here in under non-uniform magnetic field. It also reveals the nature of how the external magnetic field can get affected for the different electron radii and lead to the results, which include the intrinsic properties of electron in a special manner. These results are α-quantized, too.

1. INTRODUCTION

Properties of electron are well studied in the particle physics. Since its discovery in 1897 by J. J. Thomson many attempts have been made by the scientists to give a proper picture of electron depending on its different properties, which are shown with the help of different experimental facts [1]. Different radii of electron [2][3] are described by several electromagnetic phenomena, out of which some follow classical behaviour and rest take the quantum path. Some works on electron properties and models [2][4][5] have been done recently. Relativistic spinning sphere model [2] by MacGregor and Dynamical spinning sphere model [5] by Martin Rivas are two of them.

According to RSS model [2] the charge of the electron is residing in a very tiny place on the spherical electron. Moreover the charge is not glued over the entire surface of the sphere and this is in agreement with QED. This model states about a Compton-sized electron that carries the tiny charge. But the size of this charge is a real enigma. The charge radius is recently predicted by LEP experiment in CERN as $R_E < 10^{-19}$ or $R_E < 10^{-17}$ cm [5]. A very recent approach is taken by the current authors to predict theoretically the charge radius of electron [6].

Again a rotating charge constitutes corresponding magnetic field, which are calculated for different radii of electron and reported very recently by our group [5].

Keywords: Electron model. Electron radii, α-quantization

This magnetic field is calculated due to its own rotation in the absence of any external field.

On the other hand if the charge is rotating under the influence of some external magnetic field, the body will experience the force. In fact the behaviour of charge in uniform [7] magnetic field and non-uniform magnetic field [8] are analysed by Goldstein and Deissler. Hence here we attempt other way round to observe how the external magnetic field being treated by the self-magnetic field originated due to rotation of charge of electron for three different radii of electron. As the RSS model is we are dealing with, we consider Compton radius along with classical electron radius and electromagnetic radius keeping the fact in mind that classical electron radius and electromagnetic radius are related to the Compton-radius in a -quantized manner

2. Rotation of Charge in External Magnetic Field

If a rotating charged non-conducting ring is considered, the lagrangian for the system can be written as [8]

$$\mathcal{L} = \frac{1}{2} mv^2 + \frac{1}{2} mR^2\omega^2 + \frac{\omega e R^2}{2c} B(z), \qquad (1)$$

where e is the charge of electron, m is the mass of electron, R is the radius of rotation of the charge and ω is the angular velocity. Hence the generalized angular momentum [8] is

$$L = mR^2\omega + \frac{eR^2}{2c} B(z), \qquad (2)$$

Using the angular momentum of electron as $L = \frac{\hbar}{2}$, the z-component of magnetic field from equation (2) can be written as

$$B(z) = \frac{2c}{eR^2}\left[\frac{\hbar}{2} - mR^2\omega\right]. \qquad (3)$$

Using the expression of fine structure constant $\alpha = \frac{e^2}{\hbar c}$, equation (3) can be re-written as

$$B(z) = \alpha^{-1}\left[\frac{e}{R^2} - \frac{2m\omega e}{\hbar}\right]. \qquad (4)$$

If we use Compton radius, $R = R_C = \frac{\hbar}{mc}$, corresponding magnetic field comes out as

$$B_C(z) = \alpha^{-1}\left(\frac{em}{\frac{h}{2}}\right)\left[\frac{mc^2}{2\hbar} - \omega\right] = \frac{\hbar}{\mu}\left[\frac{mc^2}{2\hbar} - \omega\right]. \tag{5}$$

The second term in equation (5) holds the charge, mass and spin of electron together. In our recent work [9] also this particular factor is noticed during the calculation of current and self-magnetic field. Also the α-quantization nature for different properties of electron and lepton is well-known fact [6][10][11,[12]. In fact the fine structure constant can be analysed as

$$\alpha = \left(\frac{em}{\frac{h}{2}}\right)\frac{\mu}{\hbar}. \tag{6}$$

Fine structure constant and g-factor are related as [2]

$$\frac{g}{2} = 1 + \frac{\alpha}{2\pi}. \tag{7}$$

Using the relation between fine structure constant and g-factor in equation (6) we have

$$\left(\frac{em}{\frac{h}{2}}\right) = \frac{(g-2)\pi\hbar}{\mu}. \tag{8}$$

Equation (8) refers to the connection of electron's intrinsic properties with the g-factor.

Using classical electron radius $R = R_0 = \frac{e^2}{mc^2}$ in equation (4) we have

$$B_0(z) = \alpha^{-1}\left(\frac{em}{\frac{\hbar}{2}}\right)\left[\alpha^{-2}\frac{mc^2}{2\hbar} - \omega\right] = \frac{h}{\mu}\left[\alpha^{-2}\frac{mc^2}{2\hbar} - \omega\right]. \tag{9}$$

In similar way, for electromagnetic radius $R = R_{em} = \frac{\hbar^2}{me^2}$ we have

$$B_{em}(z) = \alpha^{-1}\left(\frac{em}{\frac{\hbar}{2}}\right)\left[\alpha^2\frac{mc^2}{2\hbar} - \omega\right] = \frac{\hbar}{\mu}\left[\alpha^2\frac{mc^2}{2\hbar} - \omega\right]. \tag{10}$$

Equations (5), (9) and (10) are not only carrying the term containing charge, mass and spin, as well they are α-quantized also.

Using equation (5) and expressing current as $I = \frac{cBR}{2}$, with an approximation of long straight current carrying wire, one can get the current contribution for Compton radius as

$$I_C = \alpha^{-1}\frac{c^2}{4}\left(\frac{em}{\frac{\hbar}{2}}\right)\left[1 - \frac{2\omega R_C}{c}\right]. \tag{11}$$

Similarly for classical electron radius we have the current contribution from equation (9) as

$$I_0 = \alpha^{-1}\frac{c^2}{4}\left(\frac{em}{\frac{\hbar}{2}}\right)\left[\alpha^{-1} - \frac{2\omega R_0}{c}\right]. \tag{12}$$

Current contribution can be calculated from equation (10) for electromagnetic radius as

$$I_{em} = \alpha^{-1}\frac{c^2}{4}\left(\frac{em}{\frac{\hbar}{2}}\right)\left[\alpha - \frac{2\omega R_{em}}{c}\right]. \tag{13}$$

When the ring oscillates around the z-co-ordinate, $\omega \doteq 0$. Then the corresponding magnetic fields can be given respectively as

$$B_C = \frac{e}{R_0 R_C}, \tag{14}$$

$$B_0 = \alpha^{-2}\frac{e}{R_0 R_C}, \tag{15}$$

$$B_{em} = \alpha^2\frac{e}{R_0 R_C} = \frac{e}{R_{em} R_C}. \tag{16}$$

For $\omega = 0$, equation (5) can also be written as

$$B_C(z) = \alpha^{-1} \frac{mc^2}{2\hbar}\left(\frac{em}{\frac{\hbar}{2}}\right). \tag{17}$$

In similar way equation (9) and (10) can be re-written respectively as

$$B_0(z) = \alpha^{-3} \frac{mc^2}{2\hbar}\left(\frac{em}{\frac{\hbar}{2}}\right), \tag{18}$$

$$B_{em}(z) = \alpha \frac{mc^2}{2\hbar}\left(\frac{em}{\frac{\hbar}{2}}\right). \tag{19}$$

Using the relation (8) of g-factor with the intrinsic properties of electron one can get the magnetic field from equation (17) as

$$B_C = \alpha^{-1} \frac{\pi mc^2 (g-2)}{2\mu}. \tag{20}$$

Similarly the magnetic field for classical electron radius can be written from equation (18), as

$$B_0 = \alpha^{-3} \frac{\pi mc^2 (g-2)}{2\mu}, \tag{21}$$

and from equation (19) for electromagnetic radius we have

$$B_{em} = \alpha^{2} \frac{\pi mc^2 (g-2)}{2\mu}. \tag{22}$$

Equations (17)-(19) gave magnetic field for $\omega = 0$ with α-quantization, where as equations (20)-(22) express magnetic field for $\omega = 0$ with α-quantization in terms of g-factor.

The current comes out from equation (17) as

$$I_C = \frac{\alpha^{-1} c^2}{4} \left(\frac{em}{\frac{h}{2}} \right). \tag{23}$$

In similar way equations (18) and (19) produces current respectively as

$$I_0 = \frac{\alpha^{-2} c^2}{4} \left(\frac{em}{\frac{h}{2}} \right) \tag{24}$$

and

$$I_{em} = \frac{c^2}{4} \left(\frac{em}{\frac{h}{2}} \right). \tag{25}$$

The α-quantization nature is remaining invariant for current even when the angular velocity is set to be zero also, is seen in equations (23)-(25).

3. Conclusion

The results shown here predict two major points. Firstly the magnetic field and the corresponding current are shown here in a manner where α-quantization is maintained. Three radii are used there and the α-quantization nature is very clear from the results. Secondly the magnetic field and the current are expressed with the factor consisting of three intrinsic properties of electron. Earlier this sort of nature we got for the self-magnetic field of electron [9]. Here we are getting the same for the external field also.

Another significant thing is noticed here that the α-quantization property remains invariant from the definite value of angular velocity to a zero angular velocity also. Hence this can be concluded that α-quantization is connected to the intrinsic nature of the particle which gets affirmed with the equation (6). Fine structure constant or α is in generally expressed with charge of electron, Planck's constant and velocity of light in free space. It is noteworthy that these three also play a significant role in the electron-structure, size and other properties. Hence the equation (6) is important to study the nature of electron properties with the help of α. Lande g-factor is also related to α, which ensures a better measurement for intrinsic properties of electron and corresponding structure.

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S. Ghosh
Dept. of Physics, Tezpur University,
Napaam, Tezpur-784 028, Assam, India.
E-mail: gsovan@gmail.com

Amarjyoti Choudhury
Dept. of Physics, Tezpur University,
Napaam, Tezpur-784 028, Assam, India.
E-mail: ajc@tezu.ernet.in

Jayanta Kumar Sarma
Dept. of Physics, Tezpur University,
Napaam, Tezpur-784 028, Assam, India.
E-mail: jks@tezu.ernet.in

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ORIGINAL PAPER

Radii of electrons and their α-quantized relations

S Ghosh*, A Choudhury and J K Sarma

Department of Physics, Tezpur University, Tezpur 784 028, Assam, India

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Abstract: Varieties of experimental and theoretical considerations indicate eight different types of radii of an electron like classical electron radius, Compton radius (electron), electromagnetic radius (electron) etc. Here we attempt to discuss the α-quantized relations among different types of radii of electrons, where α is the fine structure constant. In addition, α-quantized results for current and magnetic fields are also focused here.

Keywords: Electron radii; α-Quantization; Electron structure

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1. Introduction

Since the discovery of electrons in 1897, various approaches have been made to explain its nature [1]. But the structure of electrons is yet to be discovered [2]. In fact to find the hadron and lepton structures works are going on. For hadrons, the structure functions have been calculated in theoretical ways [3]. An electron is stated as a point particle in the Standard Model of Physics. But it seems that anything point is an improbable entity. The different dynamic and static properties of an electron have revealed some facts about its extended size and shape [1, 2]. Various approaches have been made to give the exact structure of an electron. Thomson gave the idea of classical electron radius, R_0 considering classical electrodynamics and later this was re-constructed by Lorentz, Abraham and Poincare [1]. Hence R_0 is also known as Thomson-scattering length or Lorentz radius.

With the help of Compton scattering experiment the Compton radius of an electron, R_C has been introduced by equating Einstein energy equation and Planck–Einstein relation [1, 2, 4] and this is approximately 10^2 times larger than the classical electron radius. From the calculation of magnetic self-energy, the magnetic field radius R_H is determined [4]. Application of electron self-energy is also used in a wider range [5]. QED equivalent electron radius R_{QED}, which comes from Lamb shift [4], is approximately equal to R_C [4]. By considering the quantum mechanical formalism of angular momentum of electrons MacGregor has introduced a corrected version of Compton radius of electron as quantum mechanical Compton radius R_{QMC}. Again considering Schwinger correction [6] of magnetic moment of electrons to R_{QMC}, we have QED-corrected quantum mechanical Compton radius R^{α}_{QMC} [4].

Equating the total energy of the electron to the electrostatic contribution of the electron and its magnetic moment, we have arrived at the electromagnetic radius of electron R_{em} [7]. This is also known as Bohr radius of hydrogen atom [7]. The charge radius of the electron R_E is yet to be calculated precisely. The Relativistic Spinning Sphere (RSS) model describes a point charge in an extended Compton-sized electron [4]. The Dynamical Spinning Sphere model also supports the same idea [8]. Recent LEP experiments in CERN give the signature of the size of the charge-radius of the electron as $R_E < 10^{-19}$ m (10^{-17} cm) [8].

2. α-Quantized relations

Classical electron radius is known as [1]

$$R_0 = \frac{e^2}{mc^2} \tag{1}$$

where, e is the charge of electron, m is the mass of the electron and c is the speed of light in free-space. The Compton radius of electron can be written as [1, 2, 4]

*Corresponding author, E-mail gsovan@gmail com

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$$R_C = \frac{\hbar}{mc}. \tag{2}$$

Hence from Eqs. (1) and (2) we have

$$R_0 = \alpha R_C \tag{3}$$

where fine structure constant $\alpha = \frac{e^2}{\hbar c}$. The electromagnetic radius is given by [7]

$$R_{em} = \frac{\hbar^2}{me^2}. \tag{4}$$

Relating Eqs. (2) and (4) we have

$$R_C = \alpha R_{em}. \tag{5}$$

Equations (3) and (5) describe two α-quantized steps amongst three different radii of electron which will be clear in Fig. 1. Using Eq. (5) in Eq. (3) we get

$$R_0 = \alpha^2 R_{em}. \tag{6}$$

In Eq. (6) we arrive at the next order of difference in α.

α-Quantized behavior of R_0, R_C and $R_{em}$are shown in Eqs. (3), (5) and (6). Out of all eight radii of electrons R_{em} is the largest one. Being concerned with numerical values of the electron radii, one can find that R_C, R_{QMC}, R^{α}_{QMC}, R_{QED} and R_H are close in results. Remaining is charge radius of the electron and this radius is expected according to LEP results of 10^{-17} cm (10^{-19} m) order or even less than that [6]. It is to be mentioned that 10^{-17} cm (10^{-19}m) is another α-quantized state and we have $\alpha^2 R_0 \sim 10^{-17}$. Hence it can be written that

$$R_E \leq \alpha^2 R_0. \tag{7}$$

From Eqs. (3) and (6), we get

$$R_0 R_{em} = R_C^2. \tag{8}$$

Using Eqs. (6) and (7) (taking $R_E = \alpha^2 R_0$only), we obtain

$$R_E R_{em} = R_0^2. \tag{9}$$

We observe that Eqs. (3), (5), (6) and (7) give the relations between two radii, but Eqs. (8) and (9) give those of three radii.

3. α-Quantization of current and magnetic field

α-Quantized results of electron radii inspire us to examine the other properties of the electron. Magnetic moment of the electron is invariant for any radii and it is given by [4]

$$\mu = \frac{eh}{2mc}. \tag{10}$$

From Eqs. (1) and (10) we obtain

$$\mu = \alpha^{-1}\frac{eR_0}{2}. \tag{11}$$

Using the α-quantization property of radii of electron from Eqs. (3), (6) and (7) in Eq. (11) we can write

$$\mu = \alpha^{-1}\frac{eR_0}{2} = \frac{eR_C}{2} = \alpha\frac{eR_{em}}{2} = \alpha^{-3}\frac{eR_E}{2}. \tag{12}$$

Again the rotating charge produces current and for the different electron radii the current contribution come out as [9]

$$I_C = \frac{ec}{2\pi R_C}, \tag{13a}$$

$$I_0 = \alpha^{-1}\frac{ec}{2\pi R_C}, \tag{13b}$$

$$I_{em} = \alpha\frac{ec}{2\pi R_C} \tag{13c}$$

and

$$I_E = \alpha^{-3}\frac{ec}{2\pi R_C}. \tag{13d}$$

Current contributions resulting from Eqs. (13a–d) are distinctly shown as α-quantized where factor of α is multiplied with I_C to produce the currents for other radii. Hence the α-quantization in current is very distinct here.

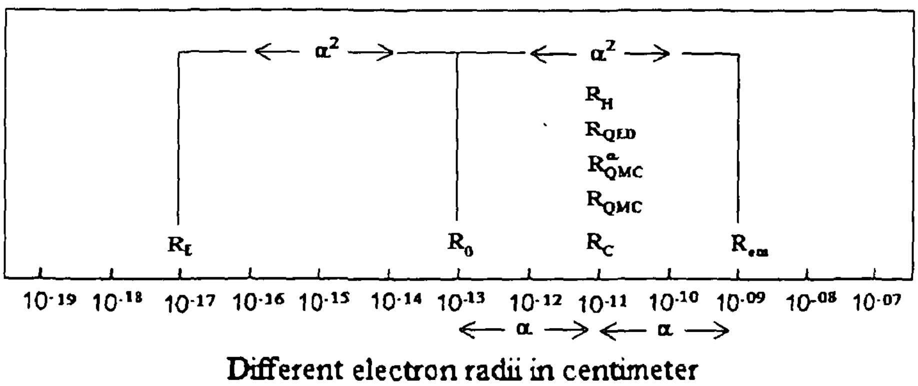

Fig. 1 α-Quantized behavior of the different electron radii

Table 1 α-Quantized radii of electron and corresponding current and magnetic field

Radius	Current	Magnetic field
R_0	$I_0 = \alpha^{-1} I_C$	$B_0 = \alpha^{-2} B_C$
R_C	I_C	B_C
R_{em}	$I_{em} = \alpha I_C$	$B_{em} = \alpha^2 B_C$
R_E	$I_E = \alpha^{-3} I_C$	$B_E = \alpha^{-6} B_C$

Table 2 α-Quantized factors of radii of electron and corresponding current and magnetic field

Radius	α-Quantized factor for radii	α-Quantized factor for I	α-Quantized factor for B
R_0	α	α^{-1}	B^{-2}
R_C	1	1	1
R_{em}	α^{-1}	α	α^2
R_E	α^3	α^{-3}	α^{-6}

Similarly, magnetic field and current are related [10] as

$$B = \frac{2I}{cR} \tag{14}$$

Putting the results of Eqs (13a–d) in Eq (14), (also following the relations among the current-loops from [9]) the magnetic fields are obtained as

$$B_C = \frac{2I_C}{cR_C}, \tag{15a}$$

$$B_0 = \alpha^{-2} \frac{2I_C}{cR_C}, \tag{15b}$$

$$B_{em} = \alpha^2 \frac{2I_C}{cR_C} \tag{15c}$$

and

$$B_E = \alpha^{-6} \frac{2I_C}{cR_C} \tag{15d}$$

4. Conclusions

Our calculations show the α-quantization among several radii of electrons, their current contributions and the magnetic fields It is shown that all the eight radii of electrons are α-quantized α-Quantized relations also enable us to connect the different electromagnetic phenomena, which are responsible for the origin of those electron radii Corresponding current and magnetic fields are also related accordingly The results for α-quantized factors for different radii of the electron strengthen our proposal of the form of charge-radius R_E Interestingly the value for that radius is consistent with LEP results [8] The fine-structure constant deals a role of connector between any two radii and this reflects the fact that those different phenomena are also connected through α only

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Radius of electron, magnetic moment and helical motion of the charge of electron

S. Ghosh[1], **A. Choudhury** and **J. K. Sarma**
HEP Laboratory, Department of Physics, Tezpur University, Tezpur, Assam, 784 028, India
[1] *gsovan@gmail.com*

Abstract

Depending on different electromagnetic phenomenon, several models of electron are described by the scientists for more than a century. Electromagnetic phenomenon revealed eight different electron radii, which are related with each other in -quantized way. Leading from one - quantized relation amongst classical electron radius and Compton radius of electron, composite radius is defined. Higher order corrections to magnetic moment and g-factor are used to describe more accurate and a generalised form of composite radius. Depending on the generalised composite radius the helical model of electron is developed which is a modified relativistic spinning sphere model but with slightly aspheric nature.

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Keywords: Electron-model, Magnetic moment, Electron radius

1 Introduction

Electron was discovered in 1897 by J. J. Thomson. After the discovery of electron several models of the electron have been proposed [1], [2]. The proposals are based on the properties of the electron, which are indeed enigmatic. They are roughly divided into three classes, in which the electron is regarded as: a) A strictly point-like particle; b) An actual extended particle; c) An extended-like particle in which the position of the point-like charge is distinct from the particle center-of-mass [2].

As electron is a charged lepton, its properties involve electromagnetic phenomena. Different electromagnetic phenomena revealed eight different radii of electron [3],[4]. The models of electron are also related to the size of the particle or the radii and hence with electromagnetic phenomenon as those different electromagnetic phenomena are the origin of the radii. Relativistic spinning sphere model of electron introduces the spectroscopic way to treat electron model in a semi-classical manner which involves a spherical structure of the particle with tiny charge and mass without violating QED theory [3].

It is noteworthy that the strictly point-like models face the problem with classical formalism and the velocity goes beyond c. Again the extended model in which the charge is glued over the entire body violates QED. Hence the extended body with a point like charge is more approachable. Relativistic spiining sphere model [3] is of that type. Here we are introducing the way to co-relate relativistic spinning sphere model with the semi-classical helical motion of charge, which in other words can be said as type of zitterbewegung motion. Zitterbewegung model of electron was [5], [6], [7] and [8], originally proposed by Schrodinger [5] is also carrying the feature of an extended-like particle with a point-like charge that is distinct from the behaviour of its center-of-charge and center-of-mass.

Indeed the hypothesis of spinning electron or a fast rotating particle incorporates an angular momentum and a magnetic moment to the electron [9], [10]. This magnetic moment was originally introduced due to Dirac equation and calculated of one Bohr magneton [11]. Again the g-factor coming out from magnetic moment is related to the fine structure constant, which is claimed to be one of the most accurately measured constants. These leave impact on the facts and figures of the spectroscopic properties of electron. Hence starting with a semi-classical model of electron we can proceed to the QED-corrected region of particles to describe the electromagnetic phenomenon with the help of different electron radii and also in connection with the fine-structure constant.

In the framework of relativistic spinning sphere model we have incorporated the helical motion of point-like charge of the electron with the help of the fine structure constant and the recent measurements of anomalous magnetic moment of the electron.

2 Magnetic self-energy and composite radius of electron

Four different kinds of mass (or equivalently energy) are attributed to electron. They are electrostatic self-energy (W_E), magnetic self-energy (W_H), mechanical mass (W_M) and gravitational mass (W_G). Magnetic self-energy is about only 0.1% of the total energy of electron [3].

Magnetic self-energy of an electron is the energy contained in the magnetic field, associated with the magnetic moment [3]. Therefore using this concept we develop the electromagnetic part of desired model. According to RSS model of electron [3], which is in close approximation with the calculation of Rasetti and Fermi [3], the total magnetic self-energy of the electron comes out as

$$W_H = \frac{2\mu^2}{3{R_H}^3}, \tag{1}$$

where $\mu (= \frac{e\hbar}{2mc})$ is the magnetic moment and R_H is the magnetic field radius of electron [3]. Magnetic field radius is closer to Compton radius in size. To match the theoretical and experimental values of magnetic moment of electron, in 1948 J. Schwinger introduced a correction term, which is known as Schwinger-corrected mass term [12]. Schwinger-correction can be expressed in terms of energy as

$$W_H \simeq m.\frac{\alpha}{2\pi}c^2. \tag{2}$$

Equating the expressions (1) and (2) for magnetic self-energy, we have

$${R_H}^3 \simeq {R_C}^3(1+\frac{\alpha}{2\pi})^2 \tag{3}$$

Re-arranging and re-combining the terms of equation (3) we resolve a composition (only addition in length) of classical electron radius and Compton electron radius as

$${R_H}^3 = R_C {R_{C0}}^2, \tag{4}$$

where

$$R_{C0} = (R_C + \frac{R_0}{2\pi}) \tag{5}$$

As R_{C0} is defined basically with the classical electron radius and Compton radius, we say this as Composite radius of electron. Fine structure constant, $\alpha = \frac{e^2}{\hbar c}$ relates the classical electron radius and Compton radius [3][13] in the way

$$R_0 = \alpha R_C. \tag{6}$$

Equation (6) indicates α-quantizedrelation among the two radii of electron. In fact RSS (relativistic spinning sphere) model, given by M. H. MacGregor [3] correlates the spectroscopic properties of the electron accurately to first order in α. Results of some other properties of the electron are also observed with α-quantization in some recent works [14][15][16].

Using the relation (6) in equation (5) we can define composite radius in another way as

$$R_{C0} = R_C(1 + \frac{\alpha}{2\pi}). \tag{7}$$

3 Magnetic moment of electron, g-factor and composite radius

Magnitude of the fundamental intrinsic magnetic moment of electron without the radiative corrections is defined as $\frac{e\hbar}{2mc}$ [17]. Hence this is also being known as zeroth-order value for electron magnetic moment [3]. In QED the measurement of magnetic moment of electron states the interaction of electron with the fluctuating vacuum. This also ensures of substructure of electron [18][19][20]. This zeroth-order of electron magnetic moment was given by Uhlenbeck and Goudsmit [3]. Later it was realised that the actual magnetic moment for electron is approximately 0.01% larger than this value. This concludes in a corrected form of magnetic moment as [3]

$$\mu = \frac{e\hbar}{2mc}(1 + \frac{\alpha}{2\pi}) \tag{8}$$

where, α is the fine structure constant and $\frac{\alpha}{2\pi}$ is the famous Schwinger correction [12]. Combining equations (7) and (8), we have the magnetic moment of electron as

$$\mu = \frac{eR_{C0}}{2}. \tag{9}$$

Hence one can say that

$$\mu = \frac{eR_C}{2} + \frac{eR_0}{4\pi}. \tag{10}$$

The factor $(1+\frac{\alpha}{2\pi})$ made it possible to express the magnetic moment with R_C and R_0 . The factor $(1+\frac{\alpha}{2\pi})$ is also connecting the g-factor and the fine structure constant as [3]

$$\frac{g}{2} = 1 + \frac{\alpha}{2\pi} \tag{11}$$

and equation (11) states that about the dependence of g-factor on α [21].

In fact with recent result, a more accurate g is expressed as [22]

$$\frac{g}{2} = 1 + (\frac{\alpha}{2\pi}) - 0.3284790(\frac{\alpha}{\pi})^2 + 1.1765(\frac{\alpha}{\pi})^3 - 0.8(\frac{\alpha}{\pi})^4. \tag{12}$$

It is to be mentioned that more accurate value of g means the change in the value of magnetic moment also. Hence the structure of this composite radius also changes and exactly this is expressed from both equations (11) and (12) in equation (13)

$$R_{C0} = \frac{g}{2}R_C. \tag{13}$$

With the help of g factor and equation (13) the ratio of classical electron radius and Compton radius can be concluded as

$$\frac{R_0}{R_C} = 2\pi(\frac{g}{2} - 1). \tag{14}$$

Again the factor $(\frac{g}{2}-1)$ is related with the anomalous magnetic moment of electron and the Bohr magneton as [21]

$$a = \frac{\mu}{\mu_B} - 1 = \frac{g-2}{2}. \tag{15}$$

where,a is the anomalous magnetic moment of electron.

Using equations (13) and (15) together, one can conclude that,

$$R_{C0} = (1+a)R_C. \tag{16}$$

Electromagnetic mass of electron is defined as

$$\Delta m = m.\frac{\alpha}{2\pi}. \tag{17}$$

Combination of equations (11), (15) and (17) produce the expression of electromagnetic mass in terms of anomalous magnetic moment as

$$\Delta m = m(\frac{g}{2} - 1) = ma. \quad (18)$$

At the present situation anomalous magnetic moment is expressed from experimental facts up to higher order of fine structure constant as [24] [25]

$$a_e(QED) = C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ..., \quad (19)$$

where, $C_e^{(i)}$ s are the co-efficients and the first one was calculated by Schwinger in 1948 [12][24].

Hence the recent measurement of g-factor is also got affected with these values, which in turn leaves impact on the electromagnetic mass, Δm too. This in fact offers us not only to measure the electromagnetic mass of electron more accurately, also ensures the more accurate measurement of mechanical mass of electron and the ultimately the more prcised values of spin too.

Therefore the electromagnetic mass can be calculated with the corrected higher order terms as

$$\Delta m = m[C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...], \quad (20)$$

which is exactly identical with equation (17), only with more accurate measurement. The corresponding energy is then expressed in with the help of equation (20)

$$W_H \simeq mc^2[C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...]. \quad (21)$$

In the same way one can re-write the magnetic moment as

$$\mu = \frac{e\hbar}{2mc}[C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...]. \quad (22)$$

Using equations (22) in equation (1) and equating with equation (21) we have

$$R_H{}^3 = \frac{2}{3}\alpha R_C{}^3 \frac{[1 + C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...]^2}{[C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...]}. \quad (23)$$

Using equation (6) in equation (23) again we have the combination of two radii

$$R_H{}^3 = \frac{2}{3}R_0R_C{}^2\frac{[1 + C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...]^2}{[C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...]}. \quad (24)$$

For the convenience of our calculation equation (24) can be written as

$$R_H{}^3 = \frac{2}{3}R_0R_C{}^2\frac{[1+\chi]^2}{\chi}, \quad (25)$$

where,

$$\chi = [C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...]. \quad (26)$$

In a more prcised form we express equation (26) as

$$R_H{}^3 = SR_0R_C{}^2(1+\chi)^2 = SR_0[R_C(1+\chi)]^2 = SR_0R_{C0\chi}{}^2, \quad (27)$$

where,

$$S = \frac{2}{3}\chi^{-1} \quad (28)$$

and

$$R_{C0\chi} = R_C(1+\chi). \quad (29)$$

Equation (29) reveals here the new expression of composite radius of electron. The first term in the left hand side of equation (29) is only Compton radius part, but the second term χR_C involves α -quantized terms of electron radii. Hence the nature of helical motion can be invariant like the preliminary version of composite radius and this χR_C part will take care of the distance between two successive turns.

We developed the nature of helical motion of charge and electron model with the composite radius in a companion paper. Continuing the similar effect for the corrected pattern of composite radius we can have the total time required for the motion of the charge as

$$T = \frac{R_E + 2n\pi R_{C0\chi}}{v}. \quad (30)$$

The corresponding current therefore can be written as

$$I = \frac{ev}{R_E + 2n\pi R_{C0\chi}}. \quad (31)$$

Current and magnetic moment are related as [25]

$$\mu = \frac{IA}{c}, \tag{32}$$

where, A is the corresponding area. The corresponding magnetic moment comes out to be

$$\mu = \frac{2(n-1)\pi ev\chi {R_C}^2}{c(R_E + 2n\pi R_{C0\chi})}. \tag{33}$$

Using the approximation of infinite long current carrying wire (in Gaussian) the magnetic field can be calculated for n-th arbitrary term as

$$B = \frac{2}{R_{C0\chi}}[\frac{ev}{R_E + 2n\pi R_{C0\chi}}], \tag{34}$$

where, v is the corresponding velocity.

Number of turns n and the velocity of the charged particle are chosen here arbitrarily in the way of developing this dynamics. For the existence of the helical motion the lower limit can be chosen for the number of turn as $n = 1$ and the Compton-sized model can have a maximum length of the helical path as

$$h_{max} = 2R_C. \tag{35}$$

On the other hand the maximum length in terms of χR_C can be written as

$$h = 2(n-1)\pi\chi R_C. \tag{36}$$

Therefore we get the range of n as the upper limit of n comes out by equating the equations (35) and (36) as

$$n = 1 + \frac{1}{\chi}. \tag{37}$$

So n ranges from 1 to $1 + \frac{1}{\chi}$.

At the end of the first turn the magnetic field will be generated as

$$B_1 = \frac{2}{R_{C0\chi}}[\frac{ev_1}{R_E + 2n\pi R_{C0\chi}}], \tag{38}$$

where, v_1 is the primary velocity. Magnetic field will now act on the charge as external magnetic field so that we can consider the sphere as sum of rotating rings.

The generalized angular momentum of the system [26] will be read as

$$L = mR_C v + \frac{eR_{C0}^2 B}{2c}. \tag{39}$$

Hence after the first turn the generalized angular momentum will be

$$L_1 = mR_C v_1 + \frac{eR_{C0_X}^2 B_1}{2c}. \tag{40}$$

L_1 will initiate the force F_{L1} which will act on the charge in the second turn. The force F_{L1} is

$$F_{L1} = ev_2 B_1. \tag{41}$$

The force for which the charge continues the circulatory motion with the same radius is

$$F_{C1} = \frac{L_1 v_1}{R_C^2} - \frac{eB_1 v_1}{2c}. \tag{42}$$

Equating these two forces from equations (41) and (42) we have

$$v_2 = v_1 [\frac{L_1}{eB_1 R_C^2} - \frac{1}{2c}]. \tag{43}$$

Therefore the magnetic field originated at the end of second term, $n = 2$ is

$$B_2 = \frac{2}{R_{C0_X}} B_1 (\frac{R_E + 2\pi R_{C0_X}}{R_E + 4\pi R_{C0_X}}) [\frac{L_1}{eB_1 R_C^2} - \frac{1}{2c}]. \tag{44}$$

Hence the generalized angular momentum after $n = 2$ turn will be read as

$$L_2 = mR_C v_2 + \frac{eR_{C0_X}{}^2 B_2}{2c}. \tag{45}$$

Using v_2 and B_2 from equations (43) and (44) in equation (45) we have

$$L_2 = [\frac{L_1}{eB_1 R_C^2} - \frac{1}{2c}][mv_1 R_C + \frac{eR_{C0_X} B_1}{c}(\frac{R_E + 2\pi R_{C0_X}}{R_E + 4\pi R_{C0_X}})]. \tag{46}$$

In a similar manner we can go up to n-th order and have equations for v_n, B_n and L_n respectively as

$$v_n = v_{n-1} [\frac{L_{n-1}}{eB_{n-1} R_C^2} - \frac{1}{2c}], \tag{47}$$

$$B_n = B_1(\frac{R_E + 2\pi R_{C0\chi}}{R_E + 2n\pi R_{C0\chi}})[\frac{L_1}{eR_C^2 B_1} - \frac{1}{2c}][\frac{L_2}{eR_C^2 B_2} - \frac{1}{2c}]$$

$$[\frac{L_3}{eB_3 R_C^2} - \frac{1}{2c}]........[\frac{L_{n-1}}{eB_{n-1}R_C^2} - \frac{1}{2c}]. \tag{48}$$

and

$$L_n = [\frac{L_1}{eB_1 R_C^2} - \frac{1}{2c}][\frac{L_2}{eB_2 R_C^2} - \frac{1}{2c}]..$$

$$..[\frac{L_{n-1}}{eB_{n-1}R_C^2} - \frac{1}{2c}][mv_1 R_C + \frac{eR_{C0\chi}{}^2 B_1}{2c}(\frac{R_E + 2\pi R_{C0\chi}}{R_E + 2n\pi R_{C0\chi}})]. \tag{49}$$

Here with equations (47)-(49) the velocity, magnetic field and the angular momentum for the n-th order are derived. Relativistic spinning sphere model with corrections from anomalous magnetic moment is modified here.

4 Conclusion

With the introduction of composite radius, the results vary from those of the RSS model. RSS model is developed on Compton radius only. But here classical electron radius is also taking part when magnetic moment is taken with only Schwinger correction. Therefore the results change, but in a regular pattern as fine structure constant, α is controlling the difference between Compton and classical electron radius. Relation of α with the g-factor leads us to connect anomalous magnetic moment with this semi-classical idea of model of electron through the composite radius and Compton radius. The combination of g and α also makes the connection between the QED calculations to the semi-classical approaches. When the anomalous magnetic moment and its recent corrected forms are used, composite radius is also changed and we got a generalised form of the helical motion for relativistic spinning sphere. The structure with radii R_C and $R_{C0\chi}$ is not one exact sphere, rather one can say as aspheric in nature, which is also supported by one recent observation [27]. It is quite interesting that a similar pattern of the electron structure from different sorts of calculations was shown by A. Martin in his article [28].

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ADDENDUM-A

Points raised by the examiners and the responses of the author

Examiner's comment	Author's response
Examiner 1: Comment 1: It is always of interest to compare new theoretical work with pertinent experimental data, although this not always possible. In this connection, the calculation of a helical charge configuration for the electron is carried out in Chapter 6 of the thesis, which leads to an asymmetrical charge distribution. Then the experimental work in Ref. 27 (see page 133) is cited here (see page 131) as evidence of this asymmetrical shape. However, the conclusion of the authors of Ref. 27 is that their highly accurate measurement of the charge itself (via the electric dipole moment) is, within experimental errors, consistent with a symmetrical shape. Thus the discussion of Ref. 27 in the thesis (page 131) should probably be modified. The fact that the electron itself has a dipole magnetic moment serves as evidence that the overall electron shape is asymmetrical. But the intrinsic shape of the tiny point charge on the electron seems to be symmetric.	Though the aspheric nature is mentioned in Ref. 27 of the Chapter 6, in the page 131, the authors of Ref. 27 also mentioned that their measurement of the electric dipole moment is within the experimental errors, consistent with a symmetrical shape. The electron dipole moment is the responsible factor for the overall asymmetric shape of the electron. Though in our work the shape of the tiny charge appears to be symmetric, the shape of the electron, however, is inferred a little aspheric due to the helical motion of the charge. This is in agreement with Ref. 27.

Examiner 3: Comment 1:	
On page 13 the author states: *the spin of the electron is a mysterious angular momentum for which no actual physical picture is available yet, ... the spin is regarded as a quantum property of the electron instead of being a classical one* and makes a reference to a publication of 1986. This is not true. Since Corben's book, authors like Lavy-Leblond, Barut et coll., Nikitin and myself have devoted some effort in this direction to state that there are many plausible classical interpretations of the spin, once the independent degrees of freedom have been properly chosen. What is a quantum property is that the angular momentum of elementary matter, when measured, is quantized. It is equivalent to say that the energy of hydrogen atom is not a classical property because its spectrum is discrete. Material systems have energy and angular momentum, in the classical and in the quantum mechanical description. Angular momentum conservation is a fundamental conservation law such as the energy conservation, because space is three-dimensional and isotropic. But its quantum description produces a discrete spectrum. This wrong statement is contained in many excellent books on	The property spin is not discussed in the thesis on page 13, it is in page 14. He/ She mentioned some efforts by some authors to give the plausible explanations of the classical interpretations of the spin. However, we believe that no consistent classical picture of spin has emerged till date. On the other hand quantum explanation of the electron spin is well known. Reviewer himself /herself quoted Pauli to state that the two-valuedness of spin has no classical explanation. To address this raised point we can tell that nowhere in this thesis it is not claimed that explanation of spin is impossible classically, rather it is mentioned that no actual classical physical picture is present with us. Indeed the quantization and hence the two-valuedness of spin angular momentum of the electron has not been explained classically.

quantum mechanics and it is usually attributed to Pauli. But what Pauli wrote was that the two valuedness of spin has no classical explanation, i.e., the spin is quantized and not that the spin cannot be explained classically.	
Examiner 3: Comment 2: The author quotes in chapter 2, where the different models are analyzed, the Lecture Notes of a course I delivered at New Delhi in 2007. This lecture course, which is a short transcription of my book, was devoted to a general formalism for describing elementary spinning particles, formalism which describes many models either relativistic and non relativistic. Among all the models there is one and only one relativistic model which satisfies Dirac's equation when quantized. This model is presented here as a spinning sphere model on pages 43-44, and that … has been built more or less considering the zitterbewegung model of the electron. In fact the model is a point-like model and it has no spherical shape. The charge of the electron is located at a point, which is therefore interpreted as the center of charge CC. But the center of mass, CM, becomes different point so that the CC moves around the CM suggesting this	Examiner 3 has written briefly about his/her own model describing the facts of Chapter 2. According to him/her it is a point-like model. He says that the author has not understood the model and his/her model is not used in the subsequent work. A model of the electron, which describes two different centers CC and CM, cannot be considered as a point model though he/she claimed to be so. That is why we have not put it in point model section. It is the fact that his/her model is not used extensively to propose the new model, but the nature of the proposed model is touching his/her model too. Obeying his/her advise now "dynamical spinning sphere" will be read as "dynamical spinning electron" in Chapter 2 , 2.4.3 subsection at page 43, in Chapter 6, page 129, paragraph 1 of the thesis and also in page I of Contents of the thesis. This should also be noticed that the Chapter 2 contains an extensive survey of different approaches of the

motion the so called zitterbewegung motion. It is a consequence of the separation of both points that the model offers a zitterbewegung-like structure. The separation between these points is half Compton's wavelength, $R = \hbar / 2mc$, and the frequency of this motion is $\omega = 2mc^2 / \hbar$, as in Dirac's theory, when quantized, because it satisfies Dirac's equation. I think the author has not understood the description of the model. Nevertheless, this model is not used in the subsequent work.	problem and there a good number of models are discussed which is addressed by Examiner 1 as an excellent summary of the various approaches to the problem. In the same point Examiner 2 also mentioned that the author painstakingly summarizes total 16 (sixteen) such models. Examiner 4 also mentioned about Chapter 2 that some limitations of some other models have also been discussed in the thesis.
Examiner 3: Comment 3: Chapter 3 is devoted to the different definitions of the radius of the electron according to a postulated structure, electromagnetic, gravitational, magneto-static, etc. and some corrections to them due to the anomalous gyromagnetic ratio. In some places it is not clear whether the radius is related to the mass or charge distribution. When the charge is located at a single point it thus corresponds to the mass distribution, but when the charge is distributed it is unclear.	Examiner 3 has mentioned about the Chapter 3 that when the charge is located at a single point it thus corresponds to the mass distribution, but when the charge is distributed is unclear. Here Examiner 3 has expressed his/her doubt over the matter of distribution of charge. Indeed distribution of charge and related radius, which is discussed in section 3.4 at page 61 in Chapter 3 of the thesis, is concerned only with a bound state. The concerned experimental facts are obtain-ed by Lamb Shift. But it is to be mentioned here that bound states are in general stable for very short period of time. On the other hand our work is

	about a stable and free electron model. Such kind of systems cannot follow a distribution of charge for a Compton-sized particle as it violates scattering property.
Examiner 3: Comment 4: Chapters 4, 5 and 6 deal basically with the model proposed by Mac Gregor in the book The enigmatic electron, with some peculiar modifications. The problem with Mac Gregor's model and its modification, in my opinion, is that it does not satisfy Dirac equation when quantized, so that its classical analysis has a limited scope. It is not only a matter of adjusting certain parameters to match the different observables with the experimental results. Most of the analysis done by Mac Gregor is concerned with experimental results of low energy physics, around or below 10 MeV, in which the scattering cross section predicts greater values for the electron radius.	During the description of Chapters 4, 5 and 6, Examiner 3 has expressed his/her doubt about the model of the electron proposed by M. H. MacGregor. The work over the relativistic spinning sphere model of the electron is a classical and to some extent semi-classical approach. Hence here the quantization is not addressed. The modified model proposed in this thesis is in good agreement with some of the contemporary experimental observations from LEP, CERN and Harvard University. Again the fact is that the point charge obeys the QED and hence one can expect to touch the Dirac equation quantizing this model.
Examiner 3: Comment 5: The notation of the different radii, R_0, R_C, R_H, etc., follow the definition given in that book, and are obtained by different physical assumptions concerning the	Examiner 3 commented about the study of the author regarding the different radii of the electron and calculations lead with them as, "The physical interest of this

energy of the electron, whether this energy is pure electrostatic, magnetostatic, Compton-like, etc. From this, the author makes some expansions of the different radii in terms of powers of the finite structure constant, considered as a ratio between the classical electrostatic electron radius and Compton radius. The original aspect is that it produces different results than in the spherical spinning model of Mac Gregor. The physical interest of this kind of gymnastics of formulae is rather doubtful, because it is not related to any suggested physical experiment.	kind of gymnastics of formulae is rather doubtful, because it is not related to any suggested physical experiment." We believe that our work is based on the logical interpretation of experimental data. The experimental observations are studied in theory to understand the original mechanisms. Afterwards those theories are tested against new experiments done with better accuracy. That is why perhaps the Examiner 1 noted: "It is always of interest to compare new theoretical work with pertinent experimental data, although this not always possible ".
Examiner 3: Comment 6: A paper of mine is cited extensively in various chapters and in the published papers by the author and collaborators, as the source that the experimental radius of the electron in scattering experiments is $R_E < 10^{-19}$ m. (The dynamical equation of the spinning electron J. Phys. A: Math. And General, 36, 4703 (2003), arxiv: physics/0112005). I am not an experimentalist. I got that number from various CERN reports and papers. For instance, the PRD paper by Bourilkov, arxiv:hep-ph/0002172, analyzes various LEP experiments in the	Examiner 3 advised in his/her report to follow arxiv: hep-ph/0002172 by Dimitri Bourilkov instead of his/her paper mentioned several times in the thesis to point out the electron's charge radius < 2.8×10^{-19} m. Throughout the thesis instead of the Ref. "Rivas, M. The dynamical equation of the spinning electron, *J. Phys. A: Math. and General* **36**(16), 4703--4715, 2003 " mentioned in the list below, "Bourilkov, D. Search for TeV Strings and new phenomena in Bhabha scattering at CERN LEP 2, *Phys. Rev. D* **62**(7) 076005, 2000 will be used

energy range from 4.2 to 16.2 TeV to derive un upper limit of the electron radius < 2.8×10^{-19} m. But this size depends on the energy of the beam. This limit decreases when the colliding energy increases, suggesting that the charge distribution of the electron has a point-like structure, thus contradicting all models with a finite charge distribution extension. My quoted paper suggest a different dynamical behaviour of the electron than the models analyzed in the manuscript. I think its quotation is misleading and should be replaced by any one from experiments at LEP. Instead of this quotation the author should have analyzed the historical evolution of the experimental value of the electron radius with the increasing energy of the colliding beams in scattering experiments, to justify how the point-like hypothesis for the charge is sustainable.	now to discuss about the CERN-result of the size of the charge radius of the electron in the following pages: 1. Chapter 1, Page 14, Paragraph 1, Ref. 35; 2. Chapter 2, Page 28, Paragraph 2, Ref. 8; 3. Chapter 2, Page 35, Paragraph 1, Ref. 8; 4. Chapter 3, Page 67, Paragraph 2, Ref. 17; 5. Chapter 3, Page 70, Paragraph 2, Ref. 17; 6. Chapter 4, Page 80, Paragraph 2, Ref. 7; 7. Chapter 5, Page 103, Paragraph 3,5 Ref. 10; 8. Chapter 6, Page 116, Paragraph 1, Ref. 4.
Examiner 3: Comment 7: In my opinion the author should have restrict to mention explicitly that in addition to an exposition of different classical electron models, the main subject of the thesis is the analysis of the different electron and positronium parameters, of a particular model. I	In the proposed model, Zitterbewegung and spinning sphere are combined through the helical motion of charge on a spherical structure. At equatorial zone of spherical structure the radius is R_C. This advocates for extended electron. Again when the interacting charge reaches polar

cannot understand how *the proposed model is expected to connect different models,* as is stated on page 113.	region, it behaves like a point particle. Depending on all these points the proposed model is expected to connect different models, as stated in page 113 in the first paragraph. The connection between the point-like and the extended pictures are expressed in page 129 in the 2nd and the 3rd paragraphs and the connection amongst various models via proposed model is discussed in page 131 in the subsection "6.6 Concluding remarks" also.